Anomalous magnetic moment of the muon and related low-energy projects

Standard Model (beyond) study: two frontiers in search

Energy frontier: highest energies,

 $E \gtrsim M_z$

Collider experiments (LHC, Tevatron): direct discovery of new particles, Higgs production, Z- and W-boson and Higgs decay modes, coupling contstants ... few signature events

High statistical experiments (e^+e^- and τ -decay data....): high precise data for low-energy observables

Precision/intensity frontier: low energies

 $(E \leq 5-10 \text{ GeV})$

allows indirect measurements SM parameters/ searches beyond SM:

(g-2), EDM, running electromagnetic fine structure constant $\alpha_{em}(s)$...

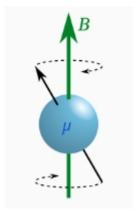




THE MAGNETIC MOMENT OF A LEPTON





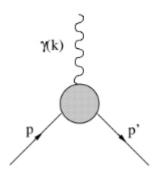


For a charged particle magnetic moment μ and spin s:

$$\overrightarrow{\mu} = \mathbf{g} \frac{q}{2m} \overrightarrow{s}$$

g (dimensionless) well-known factor
"gyromagnatic ratio"
("gyromagnatic factor")

In Quantum Field Theory the response of a charged lepton (say, a muon) to a static e.m. field is encoded in (k = p - p'):



$$\left\langle \mu(p')|J_{em}^{
u}(0)|\mu(p)
ight
angle =-ie\overline{u}(p')\Gamma^{
u}(p,p')u(p)$$

Lorentz invariance and e.m. current conservation constrain Γ^{v} -structure:

$$\Gamma^{
u}(p,p') = \gamma^{
u} F_1(k^2) + rac{i\sigma^{
u
ho} k_
ho}{2m} F_2(k^2) + ext{P-violating terms}$$

$$\Gamma^{
u}(p,p') = \gamma^{
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ho} k_
ho}{2m} F_2(k^2) + ext{P-violating terms}$$

Gyromagnetic factor g_{μ} related to form-factors $F_{1}(k^{2})$ and $F_{2}(k^{2})$:

$$g_{\mu}=2[F_1(0)+F_2(0)]$$

- * Electric charge conservation \Rightarrow Dirac FF $F_1(0) = 1$

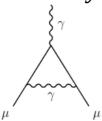


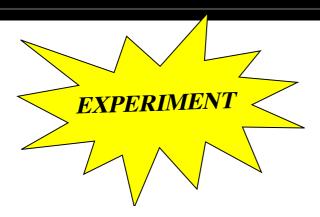
* At the tree level in SM Pauli FF
$$F_2(0) = 0 \Rightarrow g_{\mu} = g_{\mu}^{Dirac} \equiv 2$$

The muon anomalous magnetic moment (AMM)

$$a_{\mu} = rac{g_{\mu} - 2}{2} = F_2(0)$$

non-zero only at loop level, the first theoretically calculated value (g_e) by Julian Schwinger in 1948:





S. Corrodi, on behalf of the Muon g-2 collaboration B.L. Roberts, The history of the muon (g-2) experiments

THE MAGNETIC MOMENT OF THE MUON: HISTORY

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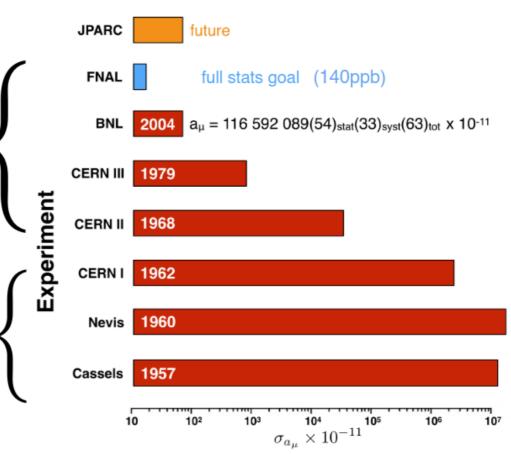
FNAL goal: 4 x improvement

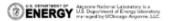
Storage Ring

Dilated lifetime measurement of a_{μ} , more precise

Stopped Muons

Stop muons in a magnetic field measurement of g_u directly







S. Corrodi, on behalf of the Muon g-2 collaboration B.L. Roberts, The history of the muon (g-2) experiments

Study of angular distribution of electrons in consecutive decays

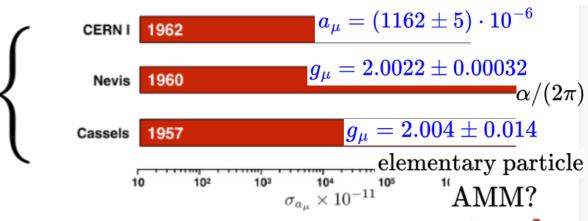
$$\pi^\pm o \mu^\pm o e^\pm \ igg| \pi^\pm o \mu^\pm +
u_\mu(\overline{
u}_\mu)$$

muons stopped in the presence of magnetic field and then decay

$$\mu^\pm o e^\pm
u_e (\overline{
u}_e) \overline{
u}_\mu (
u_\mu)$$

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S. Corrodi, on behalf of the Muon *q*-2 collaboration B.L. Roberts, The history of the muon (q-2) experiments

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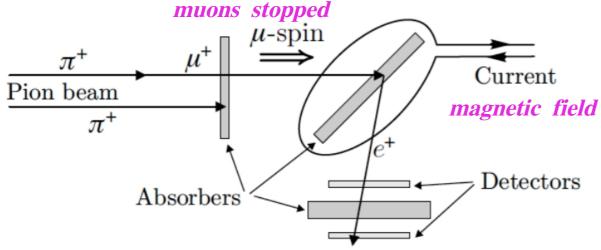
$$\pi^\pm o \mu^\pm o e^\pm$$

• The muon from pion decay at rest $\pi^{\pm}
ightarrow \mu^{\pm} + \overline{
u}_{\mu}(
u_{\mu})$

is born 100% polarized to conserve angular momentum, since the neutrino (antineutrino) is left (right)

handed

handed
• Spin in the presence of magnetic field precesses $\overrightarrow{\omega}_s = -\frac{qg\overrightarrow{B}}{2m_\mu} - \frac{qg\overrightarrow{B}}{\gamma m_\mu} (1-\gamma)$ $\gamma = \sqrt{1-v^2/c^2}$



Muons decay

$$\mu^\pm o e^\pm
u_e(\overline{
u}_e) \overline{
u}_\mu(
u_\mu)$$

• *Detects high energy positrons*

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Study of angular distribution of electrons in decays $\pi^\pm o \mu^\pm o e^\pm$

$$\pi^\pm o \mu^\pm o e^\pm$$

• The muon from pion decay at rest $\pi^{\pm} \to \mu^{\pm} + \overline{\nu}_{\mu}(\nu_{\mu})$

is born 100% polarized to conserve angular momentum, since the neutrino (antineutrino) is left (right)

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• Spin in the presence of magnetic field precesses $\overrightarrow{\omega}_s = -\frac{qg\overrightarrow{B}}{2m_\mu} - \frac{qg\overrightarrow{B}}{\gamma m_\mu} (1-\gamma)$ $\gamma = \sqrt{1-v^2/c^2}$

$$e^{+} \underbrace{\text{p}}^{\text{(at rest)}}_{\overline{\nu}_{u}} v_{e}$$

Muons decay

$$\mu^\pm o e^\pm
u_e (\overline{
u}_e) \overline{
u}_\mu (
u_\mu)$$

• *Detects high energy positrons*

high energy positrons have momentum along the muon spin \rightarrow detect highest energy positrons and time dependence give information about muon spin precession \rightarrow about muon magnetic moment

s(muon)-p(positron) correlation fundamental to all muon anomaly experiments

S. Corrodi, on behalf of the Muon q-2 collaboration B.L. Roberts, The history of the muon (q-2) experiments

Study of angular distribution of electrons in consecutive decays

$$\pi^\pm o \mu^\pm o e^\pm$$

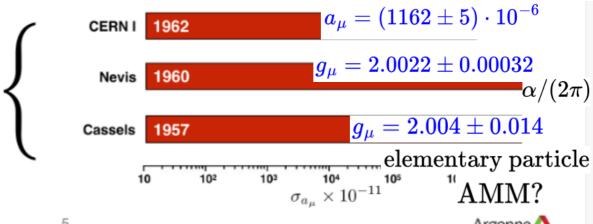
muons stopped in the presence of magnetic field and then decay

AMM
$$a_{\mu}=rac{g_{\mu}-2}{2}$$
 1957: No conclusion on AMM $g_{\mu}=2.004\pm0.014$ 1960: AMM not zero! $g_{\mu}=2.0022\pm0.00032$ $a_{\mu}^{LO;\;QED}=0.00016$

1962: F.J.M Farley proposed to measure the anomalous moment (directly) using a muon storage ring

Stopped Muons

Stop muons in a magnetic field measurement of g_u directly



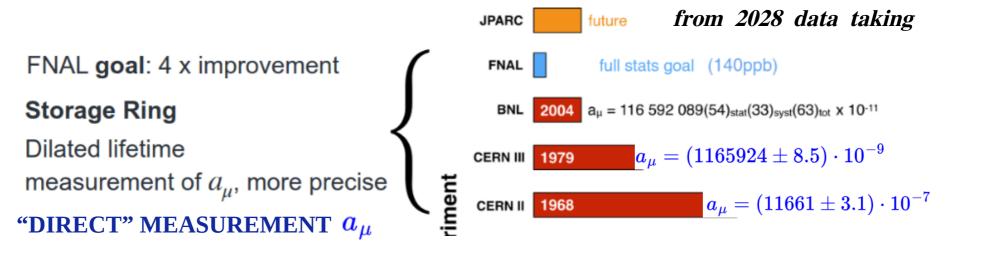




S. Corrodi, on behalf of the Muon g-2 collaboration

from 2028 data taking **JPARC** future FNAL goal: 4 x improvement **FNAL** full stats goal (140ppb) **Storage Ring** $a_{\mu} = 116 592 089(54)_{stat}(33)_{syst}(63)_{tot} \times 10^{-11}$ Dilated lifetime $a_{\mu} = (1165924 \pm 8.5) \cdot 10^{-9}$ CERN III 1979 riment measurement of a_{μ} , more precise $a_{\mu} = (11661 \pm 3.1) \cdot 10^{-7}$ **CERN II** 1968 "DIRECT" MEASUREMENT a_{μ}

S. Corrodi, on behalf of the Muon g-2 collaboration



 Inject polarized muons into a magnetic storage ring with a uniform vertical magnetic field

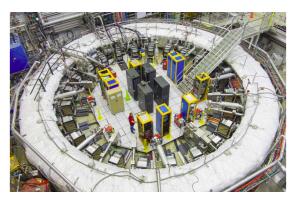
Proton bunch Pions
Polarized Muons
Target

- Both the muon spin and momentum precess:
 - muons circulate around the ring at the cyclotron frequency $\vec{\omega}_C = \frac{q}{vm} \vec{B}$
 - muon spin precession frequency $\vec{\omega}_S = \frac{q}{\gamma m_{\mu}} \vec{B} (1 + \gamma a_{\mu})$
- Muon anomaly is related to anomalous precession frequency: $\vec{\omega}_a \cong \vec{\omega}_S \vec{\omega}_C \cong a_\mu \frac{q}{m_\mu} \vec{B}$
- Measure B and ω_a to extract the anomaly
- Spin precession frequency is calculated based on decay time spectra of high energy positron

Fermilab Muon *g*-2 Experiment (E989)

https://arxiv.org/pdf/2402.15410 https://muon-g-2.fnal.gov/ MUonE2024, A. Driutti

- Uses the same magic-momentum measurement principle developed initially for the CERN III experiment \rightarrow strorage ring
- The same storage ring and muon injection principle of E821 at BNL
- + improved instrumentation for the magnetic field and muon spin precession frequency measurements
- March 26 July 7 2018 : Run1, 1.2 · BNL after data quality selection



Summer 2013



https://arxiv.org/pdf/2402.15410 https://muon-g-2.fnal.gov/ MUonE2024, A. Driutti

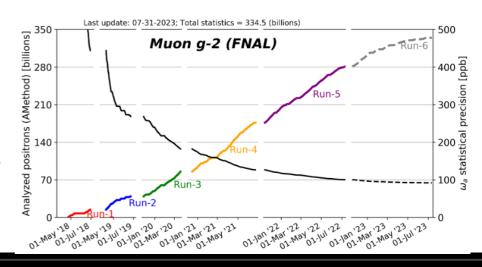
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- The result from the analysis of the Run-1/2/3 data confirmed result from BNL experiment

```
FNAL Run-1 + C + FNAL Run-2/3 + C + FNAL Run-1 + Run-
```

```
\mathbf{a}_{\mu}(\mathbf{FNAL}) = 116592040(54) \cdot 10^{-11} (460 \,\mathrm{ppb}) RUN 1 \mathbf{a}_{\mu}(\mathbf{FNAL}) = 116592055(24) \cdot 10^{-11} (203 \,\mathrm{ppb}) RUN 2/3 \mathbf{a}_{\mu}(\mathbf{BNL}) = 116592089(63) \cdot 10^{-11} (540 \,\mathrm{ppb}) \mathbf{a}_{\mu}(\mathbf{Exp}) = 116592061(41) \cdot 10^{-11} (350 \,\mathrm{ppb})
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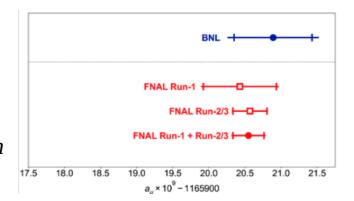
- With Run-2 and Run-3 data measurement achieved a factor 2 uncertainty reduction
- Completed all runs (collected > **21 BNL**): there is more data still to analyze!



https://muon-g-2.fnal.gov/

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- With Run-2 and Run-3 data measurement achieved a factor 2 uncertainty reduction
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• New (2023) FNAL g-2 result

$$a_{\mu} = 116592057(25) \times 10^{-11} (215 \text{ ppb})$$

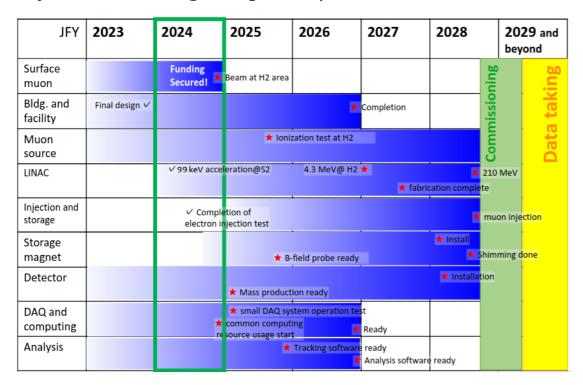
• Good agreement with FNAL Run-1, BNL

Next: analysis of Run-4/5/6 expects to achieve the <u>experimental goal of 140 ppb</u> (4*BNL precision)

New *g*−2 Experiment

Yuta Okazaki on behalf of the J-PARC muon g-2/EDM collaboration

Physics data taking is expected from 2029



JPARC experiment

(Japan Proton Accelerator Research Complex, Tokai)

measuring the muon spin precession in a uniform B-field

After 2 years of data taking, muon g-2 measurement at 450 ppb statistical uncertainty (comparable to BNL)

Final goal: 100 ppb

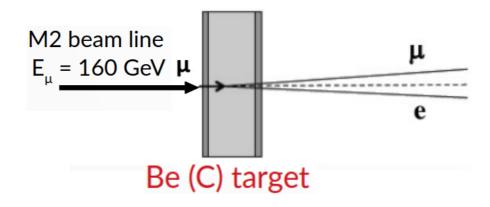
New MUonE Experiment

Letter of intent MuonE project

MUonE is a new experiment to measure the hadronic part of the running of the electromagnetic coupling constant in the spacelike region by the scattering of high-energy muons on atomic electrons on a low-Z target through the elastic process $\mu e \rightarrow \mu e$ [17].

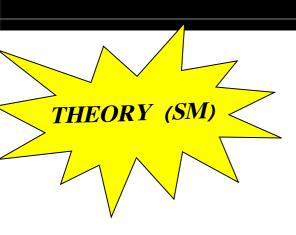


μ-e elastic scattering

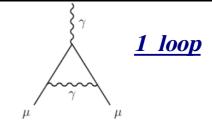


Measures directly the most "problematic" SM part of AMM

... will be discussed later after the theoretical part



<u>QED</u>

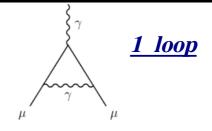


Current calculated value

Standard Model = QED + EW + QCD



<u>QED</u>



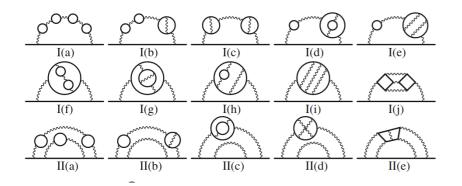
Current calculated value

$$Standard Model = QED + EW + QCD$$

QED: **5** loops

$$a_{\mu}^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765\,857\,420(13) \left(\frac{\alpha}{\pi}\right)^{2} + 24.050\,509\,85(23) \left(\frac{\alpha}{\pi}\right)^{3} + 130.8782(60) \left(\frac{\alpha}{\pi}\right)^{4} + 751.0(9) \left(\frac{\alpha}{\pi}\right)^{5} + \cdots$$

$$\sim 10000 > 100000$$

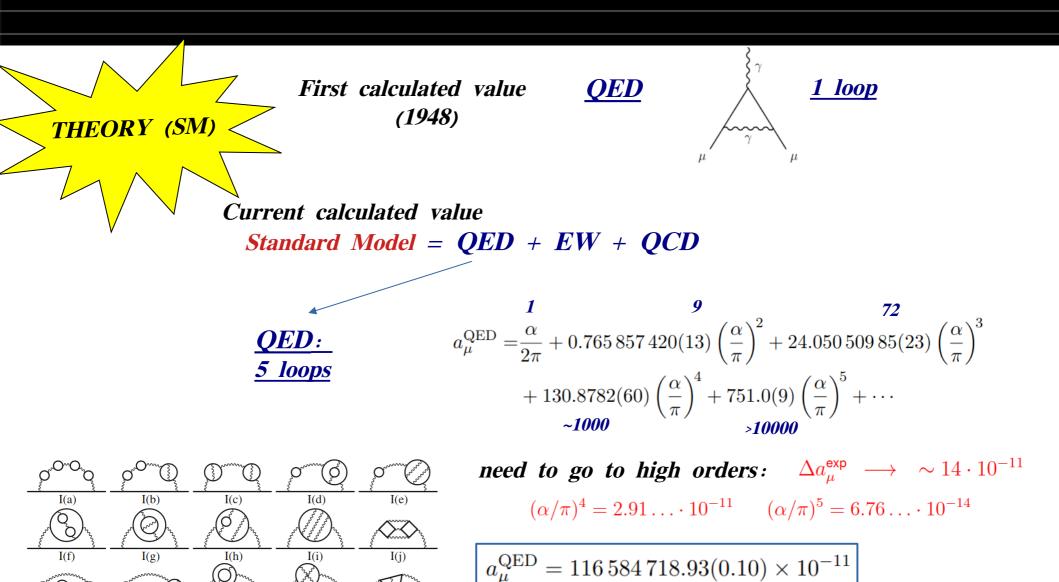


Complete numerical results available

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008)... Phys. Rev. Lett. 109,111808 (2012)

New independent numerical evaluation

S. Volkov, Phys. Rev. D 100, 096004 (2019); arXiv:2404.00649



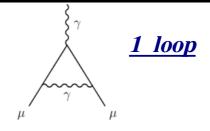
no uncertainty from theory at the level of precision reached by experiment

 $\left| a_{\mu}^{exp;WA} - a_{\mu}^{QED} = extstyle{7341(22)} \cdot 10^{-11}
ight| \; -$

> 99.9% AMM



<u>QED</u>

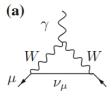


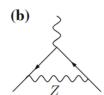
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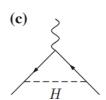
$$Standard Model = QED + EW + QCD$$

<u>Loop contributions involving</u> W, Z, Higgs particles + fermions

1-loop







W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

2-loop







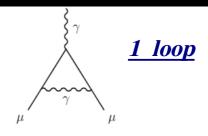
A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

T. Gribouk, A. Czarnecki, Phys. Rev. D 72, 053016 (2005)

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)



<u>QED</u>

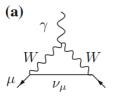


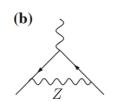
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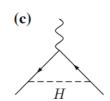
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<u>1-loop</u>







$$a_{\mu}^{\rm weak(1)} = 194.79(1) \cdot 10^{-11}$$

2-loop







$$(\mathbf{d}) \bigvee_{\mu} \int_{\nu_{\mu}}^{f} W$$

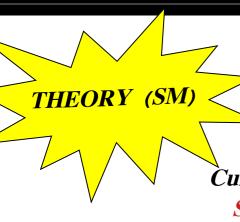
$$(f)_{\gamma}$$
 H
 μ
 γ , Z

Updated after higgs discovery

$$a_{\mu}^{\text{EW}} = 153.6(1.0) \times 10^{-11}$$

$$a_{\mu}^{exp;WA} - a_{\mu}^{QED} - a_{\mu}^{weak} = 7187(22) \cdot 10^{-11}$$

no uncertainty from theory at the level of precision reached by experiment



<u>QED</u>

 $\underbrace{\begin{array}{c}
\uparrow^{\gamma} \\
\mu\end{array}}$ 1 loop

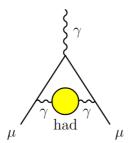
Current calculated value

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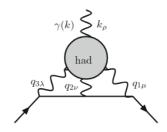
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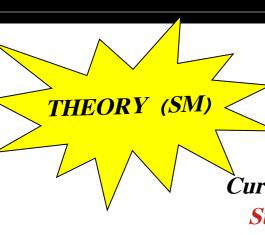
Hadronic (quark and gluon) loop contributions

Hadronic Vacuum
Polarization (HVP)



<u>Hadronic Light-by-Light</u> <u>scattering</u>





<u>QED</u>

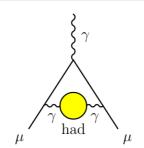
 $\downarrow_{\mu}^{\gamma} \qquad \underline{1 \ loop}$

Current calculated value

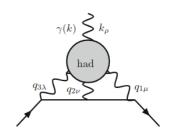
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Hadronic Vacuum
Polarization (HVP) LO



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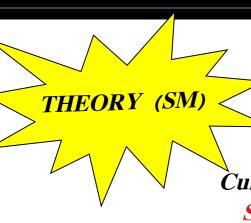


MAIN PROBLEM: integration over momentum from threshold up to ∞

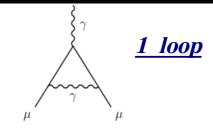
 $q^2 > 5$ (2) GeV² \rightarrow perturbative QCD, quark loops

below → models based on dispersion relation + experimental data for hadronic form factors (data driven method)

... a new approach, from 2019: Lattice QCD







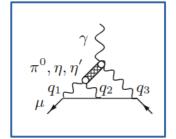
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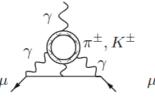
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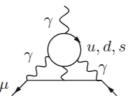


Hadronic (quark and gluon) loop contributions

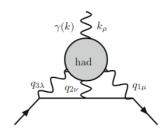
<u>Hadronic Light-by-Light</u> <u>scattering</u>



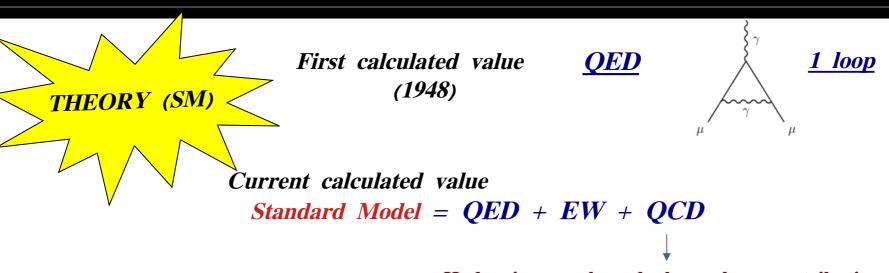






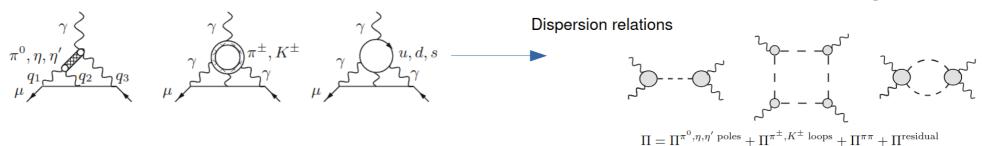


LO hLbL

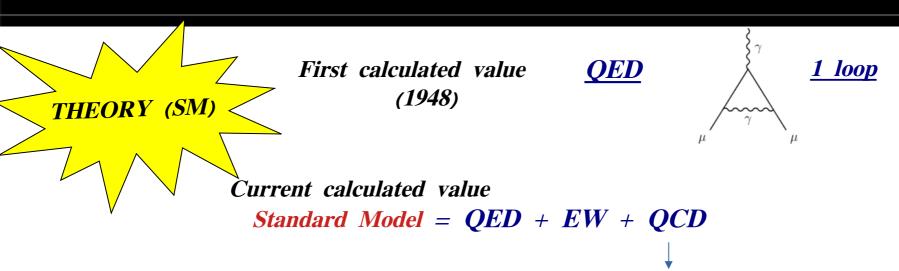


Hadronic (quark and gluon) loop contributions

<u>Hadronic Light-by-Light</u> <u>scattering</u>

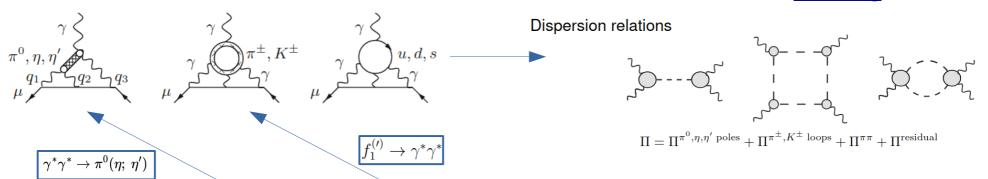


Needs input (transition form factors)



Hadronic (quark and gluon) loop contributions

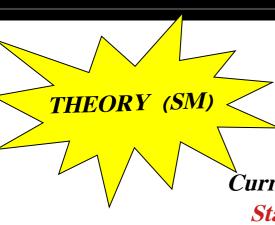
<u>Hadronic Light-by-Light</u> <u>scattering</u>



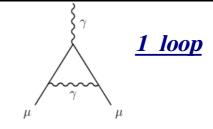
Needs input (transition form factors)

M. Hoferichter, B. Kubis and M. Zanke, JHEP07, 106 (2021); JHEP08, 209 (2023)

S. Holz, C. Hanhart, M. Hoferichter and B. Kubis, Eur. Phys. J. C 82, 434 (2022); Eur. Phys. J. C 82, 1159 (2022)



QED



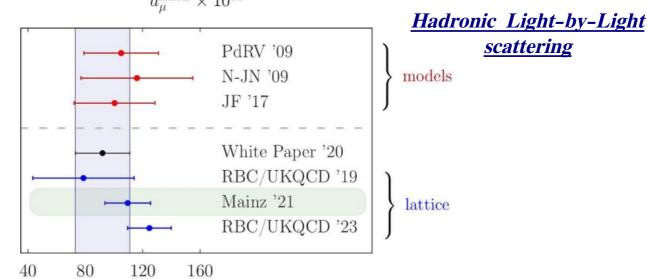
Current calculated value

$$Standard Model = QED + EW + QCD (strong interaction)$$

Hadronic (quark and gluon) loop contributions $a_{\mu}^{\mathrm{HLbL}} \times 10^{11}$

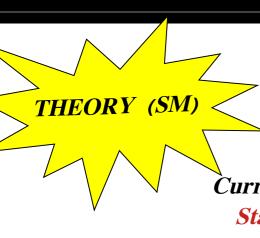
good agreement between lattice QCD and data driven method at $\simeq 20 \times 10^{-11}$

$$a_{\mu}^{hLbL} = 92(19) \cdot 10^{-11}$$



$$a_{\mu}^{exp;WA} - a_{\mu}^{QED} - a_{\mu}^{weak} - a_{\mu}^{hLbL} = 7095(29) \cdot 10^{-11}$$
 \blacksquare Uncertainty from theory starts to become significant

Need another factor of 2 for final Fermilab precision



<u>QED</u>

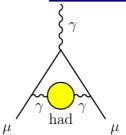
 $\underbrace{\begin{array}{c}
\gamma\\ \\
\mu
\end{array}}$ $\underline{1 \ loop}$

Current calculated value

$$Standard Model = QED + EW + QCD$$

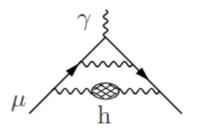
Hadronic (quark and gluon) loop contributions

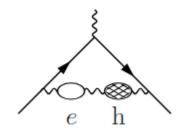
<u>Hadronic Vacuum</u> Polarization (HVP) LO

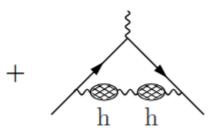


Main problem → low energy part of integrand

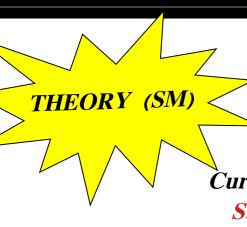
Hadronic Vacuum
Polarization (HVP) NLO



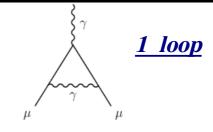




they can include the same loop of HVP-LO, so not considered here



<u>QED</u>

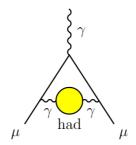


Current calculated value

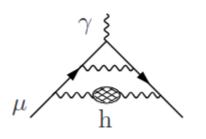
$$Standard Model = QED + EW + QCD$$

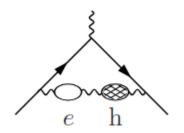
Hadronic (quark and gluon) loop contributions

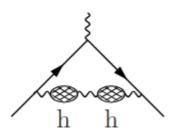
<u>Hadronic Vacuum</u> <u>Polarization (HVP) LO</u>



data driven method Lattice QCD

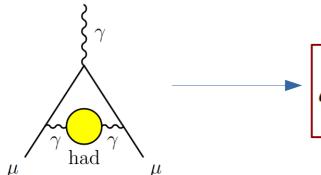






Hadronic Vacuum Polarization (HVP) LO

Data Driven Method

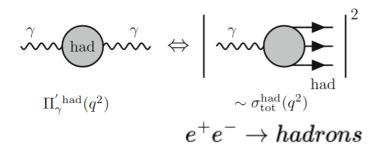


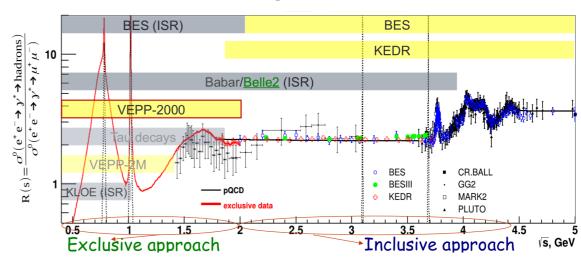
$$a_{\mu}^{\text{HVP}} = \frac{\alpha^2}{3\pi^2} \int_{M_{\pi}^2}^{\infty} \frac{K(s)}{s} R(s) \, ds \qquad R(s) = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}}$$

$$R(s) = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}}$$

$$R(s) = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}} 4\pi\alpha^2/(3s)$$

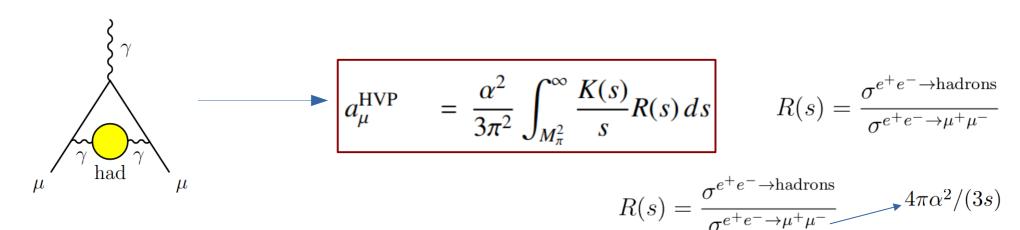
Dispersive Approach





Hadronic Vacuum Polarization (HVP) LO

Data Driven Method

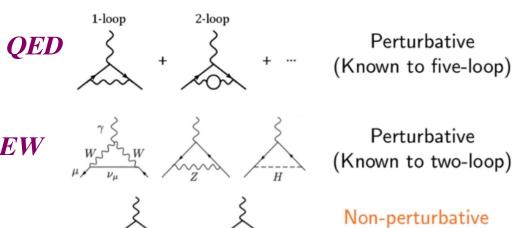


Dispersive Approach

$$a^{HVP;\ LO} = 6931(40) \cdot 10^{-11}$$
 $a^{HVP;\ NLO} = -98.3(7) \cdot 10^{-11}$
 $a^{HVP;\ NNLO} = 12.4(1) \cdot 10^{-11}$

Lattice QCD
$$a_{\mu}^{HVP;\,LO} = 7043(150) \cdot 10^{-11}$$

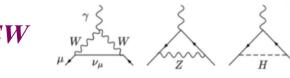
The uncertainty in the SM prediction





 $\sim 99.99\% \sim 0.001\%$

 $a_{\mu}^{\rm QED} = 116\,584\,718.93(0.10)\times 10^{-11}$



$$\sim 1~\mathrm{ppm}$$

$$\sim$$
0.2%

$$a_{\mu}^{\text{EW}} = 153.6(1.0) \times 10^{-11}$$

$$\sim 59$$
 ppm $\sim 84\%$

$$a_{\mu}^{HVP;\,LO}=6931(40)\cdot 10^{-11}$$

$$a_{\mu}^{HVP;\,LO} = 7043(150)\cdot 10^{-11}$$

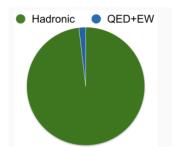


Non-perturbative

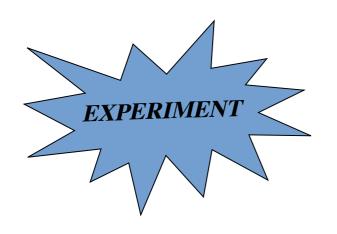
(Data-driven & lattice)



$$a_{\mu}^{\text{HLbL}} = 92(19) \times 10^{-11}$$



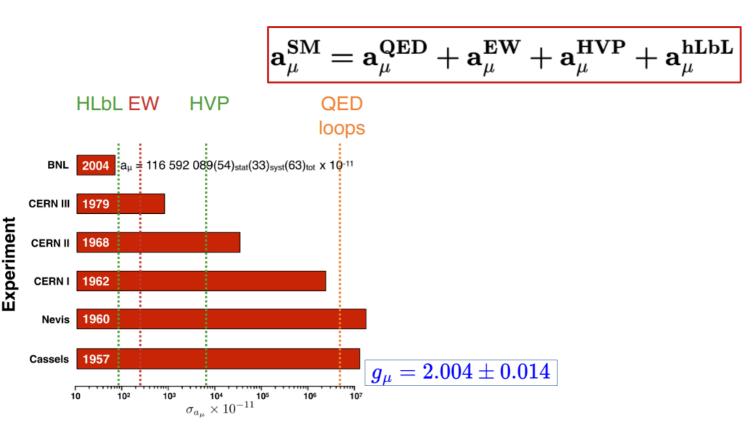
- Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to < 1%
- ▶ Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to \sim 20%, second largest uncertainty (now subdominant)





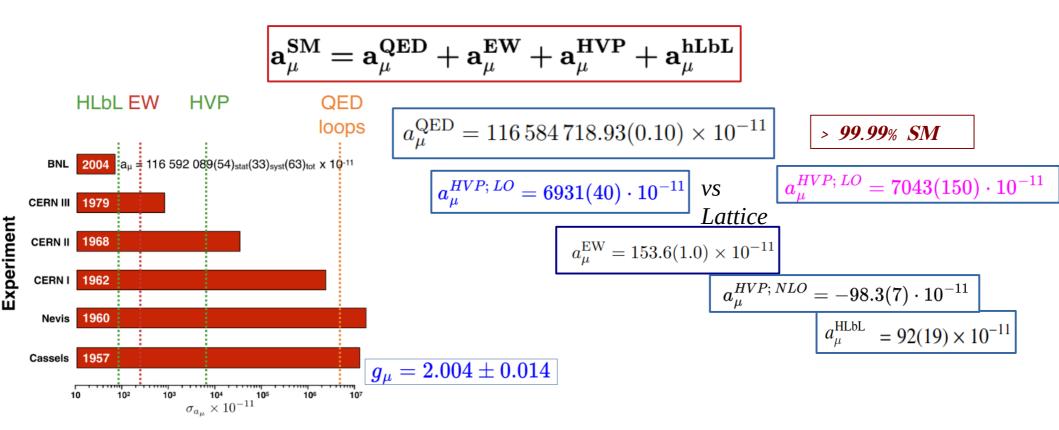


EXPERIMENT vs THEORY: precision



Precision has improved in more than in 10^5 times \rightarrow sensible to all SM contributions

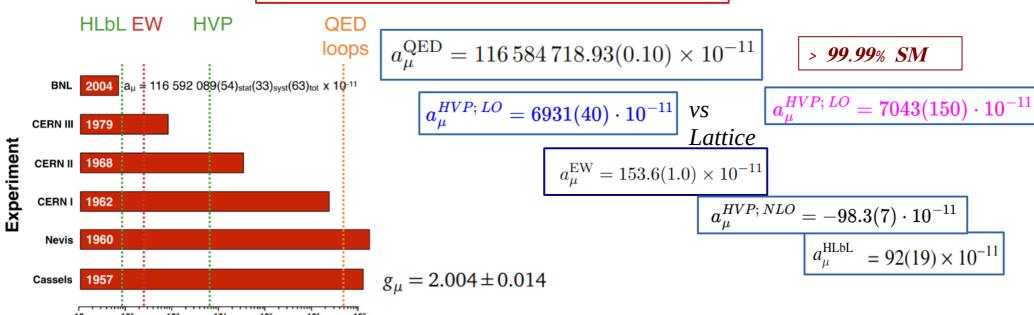
EXPERIMENT vs THEORY, history: precision



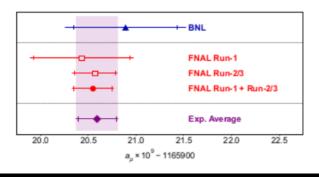
Precision has improved in more than in 10^5 times \rightarrow sensible to all SM contributions

EXPERIMENT vs THEORY, history: precision

$$\mathbf{a}_{\mu}^{ ext{SM}} = \mathbf{a}_{\mu}^{ ext{QED}} + \mathbf{a}_{\mu}^{ ext{EW}} + \mathbf{a}_{\mu}^{ ext{HVP}} + \mathbf{a}_{\mu}^{ ext{hLbL}}$$



2023, FNAL

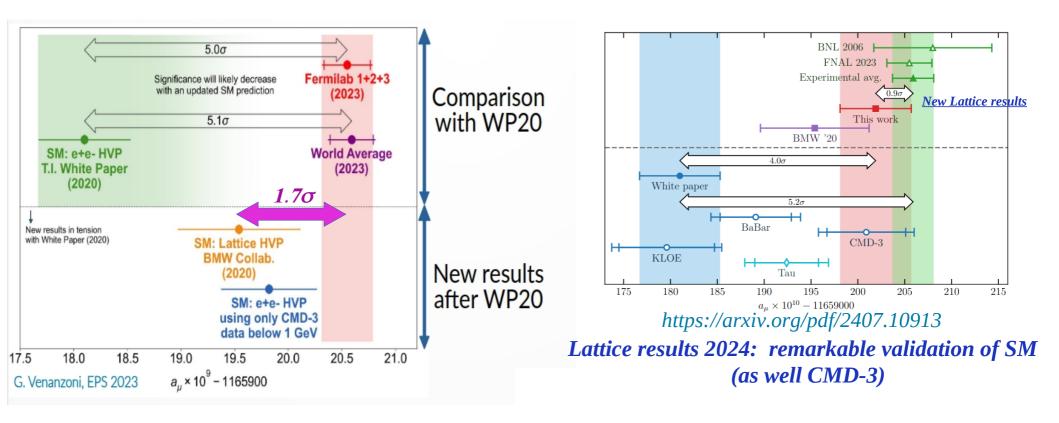


 $\sigma_{a_{\mu}} \times 10^{-11}$

$$\mathbf{a}_{\mu}^{\mathrm{exp;\,WA}} = \mathbf{116\;592\;059\;(22)\cdot 10^{-11}}$$

EXPERIMENT vs THEORY: 2024

$$\mathbf{a}_{\mu}^{ ext{exp; WA}} = \mathbf{116\ 592\ 059\ (22)} \cdot \mathbf{10^{-11}}$$



Until differences are understood no conclusions can be drawn about the SM validity

<u>REANALYZE EXPERIMENTAL DATA FOR HVP =></u> for low energy e+e- data !!!

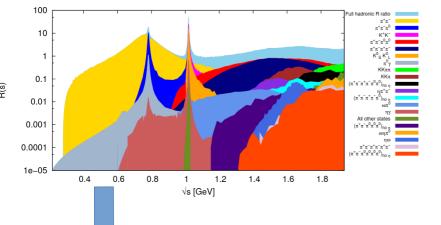
Data Driven method for HVP calculation

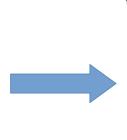
$$a_{\mu}^{\mathrm{HVP}} = \frac{\alpha^2}{3\pi^2} \int_{M_{\pi}^2}^{\infty} \frac{K(s)}{s} R(s) \, ds$$

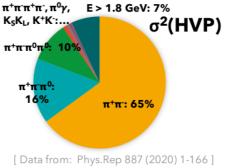


QED kernel function $K(s) \sim 1/s$

$$R(s) = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}}$$







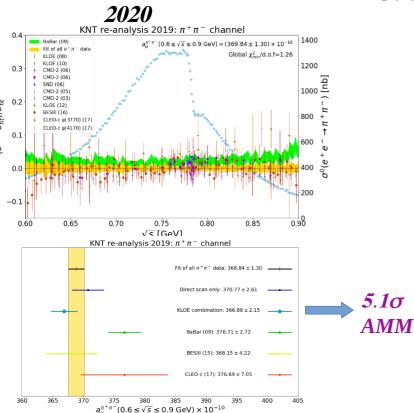
Add necessary precision Kashevarzi talk

 $e^+e^- \rightarrow \pi^+\pi^{--}$ gives main contribution to R(s)"golden channel"

Dedicated measurements of $e^+e^- \rightarrow$ hadrons.

- \leq 2 GeV = exclusive final states $(\pi^0 \gamma, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, K\bar{K}, K\bar{K}\pi, K\bar{K}2\pi, 2K\bar{K}, p\bar{p}, n\bar{n}...)$.
- ≥ 2 GeV = inclusive hadronic R-ratio (all hadrons).

"Golden channel" $\pi^+\pi^-$

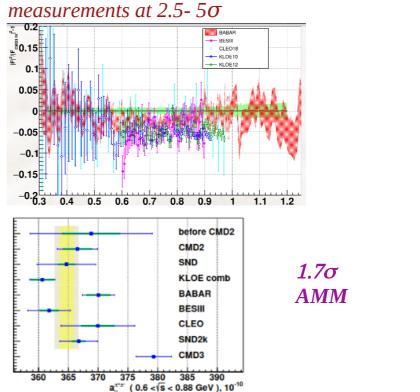


Tension between KLOE vs. BaBar, not at the level of the g-2 discrepancy!

CMD-3, 2023

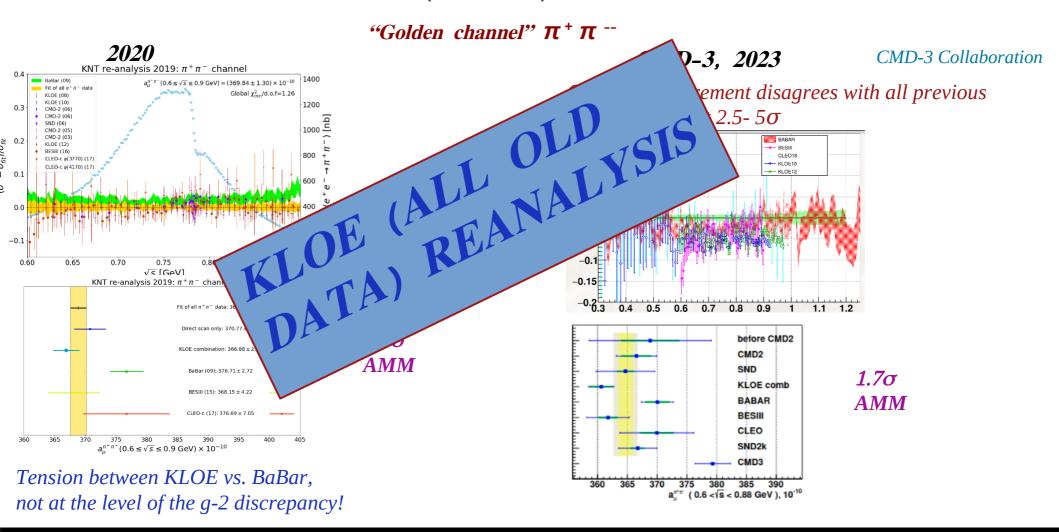
CMD-3 Collaboration

CMD-3 measurement disagrees with all previous



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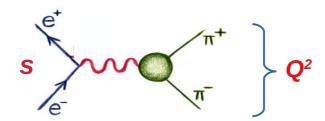
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Two methods from cross section measurement:

- · Direct energy scan fixed CM energy measurement of production cross section.
- Radiative return measure differential cross section with tagged ISR photon to reconstruct production cross section.

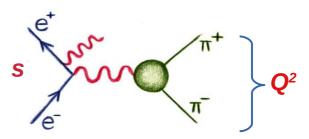
Energy scan experiments (Direct scanning)



- setting the beam energies of a collider to a given c.m. energy
- to collect data at other Q^2 requires changes and adjustments of accelerator settings:

$$Q^2 = s$$

Radiative return method (Initial State Radiation, ISR)



- fixed beam energy run
- events with a photon
- to collect data at other Q^2 does not require any change of accelerator settings:

$$Q^2 = s - 2E_\gamma \sqrt{s}$$

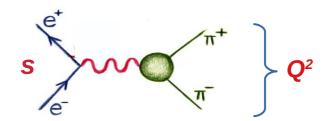
Dedicated measurements of $e^+e^- \rightarrow$ hadrons.

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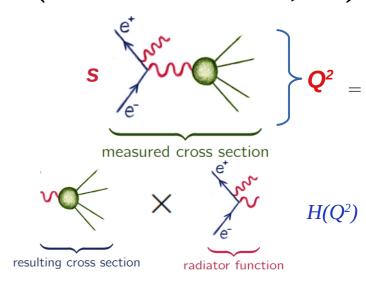
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$$Q^2 rac{d\sigma(e^+e^- o \pi^+\pi^-\gamma; s, Q^2)}{dQ^2} = \sigma(e^+e^- o \pi^+\pi^-; Q^2) imes H(s, Q^2)$$

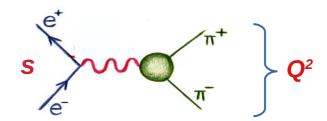
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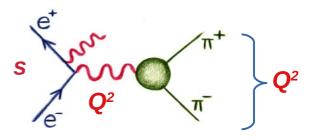
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$$Q^2 = s - 2E_\gamma \sqrt{s}$$

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 $H(Q^2)$ -radiator function, QED RC => <0.1%

Dedicated measurements of $e^+e^- \rightarrow$ hadrons.

- \leq 2 GeV = exclusive final states $(\pi^0 \gamma, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, K\bar{K}, K\bar{K}\pi, K\bar{K}2\pi, 2K\bar{K}, p\bar{p}, n\bar{n} \dots)$.
- ≥ 2 GeV = inclusive hadronic R-ratio (all hadrons).

Two methods from cross section measurement:

- Direct energy scan fixed CM energy measurement of production cross section.
- Radiative return measure differential cross section with tagged ISR photon to reconstruct production cross section.



Radiative Return ISR

ISR



Babar $(E_{CM} = \Upsilon(4s))$

- Comprehensive (almost all) exclusive final states measured below 2 GeV.
- High statistics, from-threshold measurements of π⁺π⁻.

BES-III ($E_{CM} = 2-5 \text{ GeV}$)

- High-precision measurement of $\pi^+\pi^-$ on ρ -resonance.
- Measurements of other modes, e.g. $\pi^+\pi^-\pi^0$, inclusive.

KLOE $(E_{CM} = \phi)$

- 3 high-precision measurements of $\pi^+\pi^-$ on ρ -resonance, using different methods.
- Combination results in most precise measurement of $\pi^+\pi^-$.

Others

- CLEO-c $(\pi^+\pi^-)$.
- Belle-II (hopefully in the near future).

Direct scan

SND and CMD-3 (Novosibirsk)

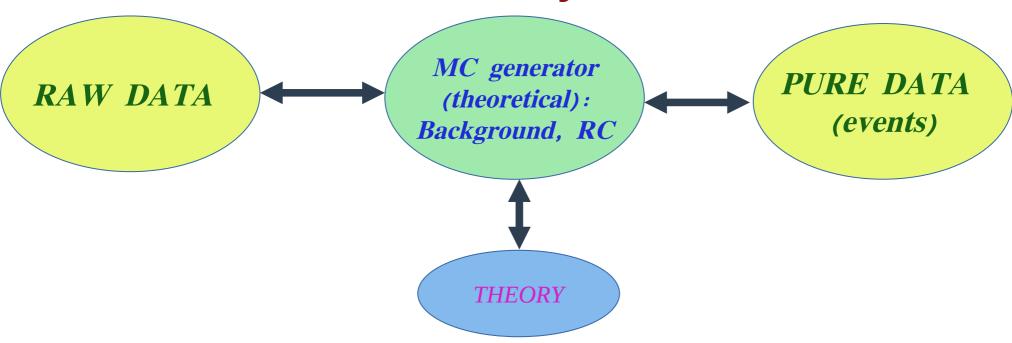
- Both located at VEPP-2000 machine.
- Comprehensive (almost all) exclusive final states measured below 2 GeV.

KEDR (Novosibirsk)

Inclusive measurement.

Plus, many older measurements from now inactive experiments...

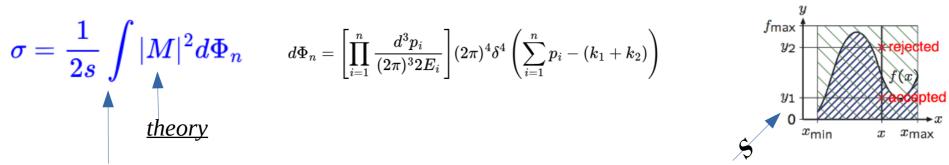
DATA analysis



Monte Carlo (MC) Event Generators

Monte Carlo Event Generator:

- realistic experimental cuts on the angles and energy of (incoming and outgoing) particles cannot be taken into account in an analytical calculation
- one has to use a MC event generator as a bridge between the theoretical results and experimental data
- a Monte Carlo generator allows to calculate various energy and angular distributions of final particles taking into account conservation laws.

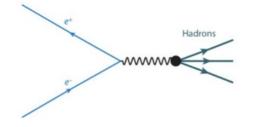


Monte Carlo method: estimated as a ratio under integrand and under max value

- For both direct and radiative return the main process $(e^+e^- \to X)$ has to be implemented with radiative corrections into a MC generator
- Main "irreducible background" are radiative corrections due to photon emission, either soft photon or collinear

Radiative Corrections: MC

Direct scanning



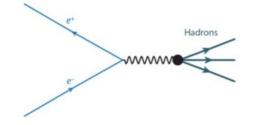
measured cross-section $\sigma(e^+e^-(\gamma_{ISR}^{invisable})
ightarrow hadrons(\gamma_{FSR}^{invisable}))$

- 2pions, radiative corrections account for ISR and FSR effects
- other channels: radiative correction accounts for ISR effects only



MC generator	Channel	Precision	Comment
MCGPJ (VEPP-2M, VEPP- 2000)	e+e- → e+e-,μ+μ-, π+π-,	0.2%	photon jets along all particles (collinear Structure function) with exact NLO matrix elements
BabaYaga@NLO (KLOE, BaBar, BESIII)	e+e- → e+e-,μ+μ-, γγ	0.1%	QED Parton Shower approach with exact NLO matrix elements

Direct scanning

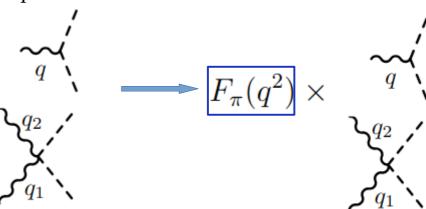


measured cross-section $\sigma(e^+e^-(\gamma_{ISR}^{invisable})
ightarrow hadrons(\gamma_{FSR}^{invisable}))$

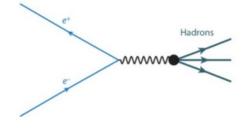
- 2pions, radiative corrections account for ISR and FSR effects
- other channels: radiative correction accounts for ISR effects only



- ISR radiative corrections: final-state independent, QED
- FSR radiative corrections: depend on final state, model dependent
- * two-pion mode: standard approach sQED*VMD: calculations without form-factor, then final amplitude is scaled by FF



Direct scanning



measured cross-section $\sigma(e^+e^-(\gamma_{ISR}^{invisable})
ightarrow hadrons(\gamma_{FSR}^{invisable}))$

- 2pions, radiative corrections account for ISR and FSR effects
- other channels: radiative correction accounts for ISR effects only

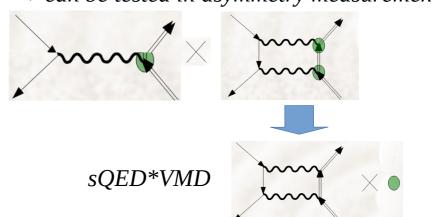
adjust for radiative corrections $\sigma(e^+e^- \to hadrons)$

- ISR radiative corrections: final-state independent, QED
- FSR radiative corrections: depend on final state, model dependent

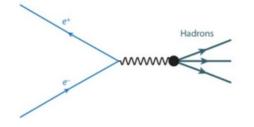
* two-pion mode: standard approach sQED*VMD \rightarrow can be tested in asymmetry measurements

either ISR*FSR or one-photon*two-photon process

only for hard photon RC



Direct scanning

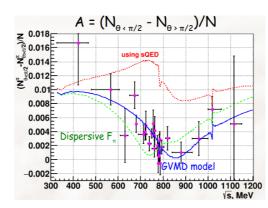


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ightarrow hadrons(\gamma_{FSR}^{invisable}))$

- 2pions, radiative corrections account for ISR and FSR effects
- other channels: radiative correction accounts for ISR effects only



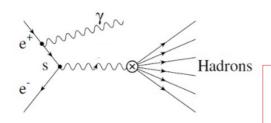
- ISR radiative corrections: final-state independent, QED
- FSR radiative corrections: depend on final state, model dependent
 - * two-pion mode: standard approach $sQED*VMD \rightarrow CMD-3$ tested in asymmetry measurements caused by one-photon*two-photon process
- box diagram was added to MC
- two models for pion form-factor conventional sQED*VMD approach gives ~1% inconsistency, the predicted shape is in tension with data



Radiative Corrections: MC

Radiative return method





$$\textit{measured cross-section} \quad \sigma(e^+e^-\gamma_{ISR}(\gamma_{ISR}^{invisable}) \rightarrow hadrons(\gamma_{FSR}^{invisable}))$$

adjust for FSR radiative corrections, box diagrams

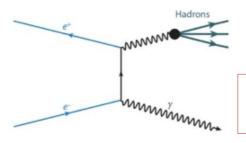
$$Q^{2} \frac{d\sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\gamma)}{dQ^{2}} = \sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-})H(Q^{2})$$

$$\sigma(e^+e^- o hadrons)$$

MC generator	Channel	Precision	Comment	
EVA (KLOE)	e⁺e⁻ →π⁺π⁻γ	O(%)	Tagged photon ISR at LO + Structure Function FSR: point-like pions	
AFKQED (BaBar)	e⁺e⁻ →π⁺π⁻γ, 	depends on the event selection (can be as good as Phokhara)	ISR at LO +Structure Function	RC and FSR are handled differently
PHOKHARA (KLOE, BaBar BESIII)	$e^+e^- \rightarrow \pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$, $4\pi\gamma$,	0.5%	ISR and FSR(sQED+Form Factor) at NLO	



more complicated for analysis



measured cross-section $\sigma(e^+e^-\gamma_{ISR}(\gamma_{ISR}^{invisable}) \rightarrow hadrons(\gamma_{FSR}^{invisable}))$

adjust for FSR radiative corrections, box diagrams

$$Q^{2} \frac{d\sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\gamma)}{dQ^{2}} = \sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-})H(Q^{2})$$

$$\sigma(e^+e^-
ightarrow hadrons)$$

Radiator function $H(Q^2) = ISR RC$: crucial ingredient for ISR analyses, quite complex analytic form

NNLO precise + HO correction to be estimated

Radiative return method

more complicated for analysis

e⁺ Hadrons

$$\textit{measured cross-section} \quad \sigma(e^+e^-\gamma_{ISR}(\gamma_{ISR}^{invisable}) \rightarrow hadrons(\gamma_{FSR}^{invisable}))$$

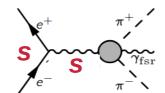
adjust for FSR radiative corrections, box diagrams

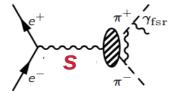
$$Q^{2} \frac{d\sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\gamma)}{dQ^{2}} = \sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-})H(Q^{2})$$

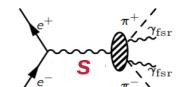
$$\sigma(e^+e^- o hadrons)$$

FSR problem: FSR irreducible background

FSR-only







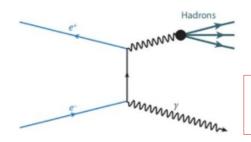
destroys ISR method

- to be excluded from analysis on base of MC
- to be included in MC, currently only in Phokhara/EVA
- model-dependent
- Phokhara: $sQED*VMD \rightarrow add$ other models

Radiative return method



more complicated for analysis



 $\textit{measured cross-section} \quad \sigma(e^+e^-\gamma_{ISR}(\gamma_{ISR}^{invisable}) \rightarrow hadrons(\gamma_{FSR}^{invisable}))$

adjust for FSR radiative corrections, box diagrams

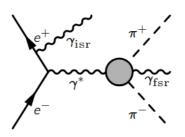
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$$\sigma(e^+e^- o hadrons)$$

FSR problem:

FSR irreducible background

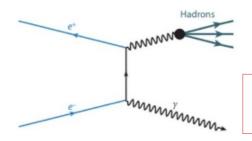
FSR-mixed



- OK for analysis, to be subtracted in calculation of $\sigma(e^+e^- \to \pi^+\pi^-)$.
- to be included in MC, currently only in Phokhara
- model-dependent
- Phokhara: $sQED*VMD \rightarrow add$ other models

Radiative return method

more complicated for analysis



 $\textit{measured cross-section} \quad \sigma(e^+e^-\gamma_{ISR}(\gamma_{ISR}^{invisable}) \rightarrow hadrons(\gamma_{FSR}^{invisable}))$

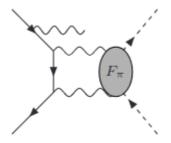
adjust for FSR radiative corrections, box diagrams

$$Q^{2} \frac{d\sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-}\gamma)}{dQ^{2}} = \sigma(e^{+}e^{-} \rightarrow \pi^{+}\pi^{-})H(Q^{2})$$

$$\sigma(e^+e^- o hadrons)$$

RC due to BOX diagrams





destroys ISR method

- to be excluded from analysis on base of MC
- to be included in MC, currently only in Phokhara
- model-dependent
- Phokhara: sQED*VMD, complex analytic form
- other models????

MC Generators: summary

MC generators for exclusive channels (exact NLO + Higher Order terms in some approx)

		,	
MC generator	Channel		
MCGPJ (VEPP-2M, VEPP- 2000)	$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \pi^+\pi^-,$	0.2%	photon jets along all particles (collinear Structure function) with exact NLO matrix elements
BabaYaga@NLO (KLOE, BaBar, BESIII)	e⁺e⁻ → e⁺e⁻,μ⁺μ⁻, γγ	0.1%	QED Parton Shower approach with exact NLO matrix elements

MC generators for ISR (from approximate to exact NLO)



For scan experiment: only MCGPJ available with declared 0.2% precision

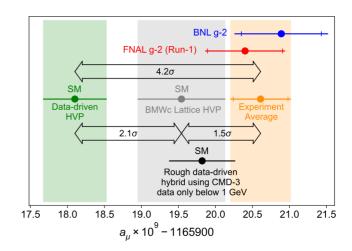
For ISR: only Phokhara with 0.5% precision

We need high-precision MC generators for radiative corrections at the experiment level:

- different experiments use different MC
- high-order radiative corrections are handled differently
- the first comparison done \rightarrow to be continued
- new MC generator MCMule, effort to understand KKMC' physics for two pion mode
- aim at 0.1% uncertainty \rightarrow theory (NNLO + higher order RC estimation + model FSR + box diagrms) + MC implementation

CONCLUSION

- ✓ Fermilab Muon g−2 Experiment (FNAL-E989) completed all runs:
 - → the Run-1/2/3 data analyzed and confirmed result from BNL experiment
 - → with Run-2 and Run-3 data measurement achieved a factor 2 uncertainty reduction
- ✓ Unfortunately, the theory situation is not quite in such a good shape:
 - → QED, EW contributions under control at the required level of precision
 - → the dominant sources of theory error are the hadronic contributions: HVP dominates the total uncertainty
- \checkmark New Lattice as well as CMD-3 results challenging old e+e- data
- ✓ Important tensions at the level of HVP:
 - → between KLOE and BABAR
 - → between CMD-3 and earlier experiments (incl. CMD-2!)
 - → between Lattice and data-based evaluations (except CMD-3)



Until differences are understood, and intense scrutiny of new/old results is complete, no conclusions can be drawn about the validity of SM estimates. A lot of work still be done...

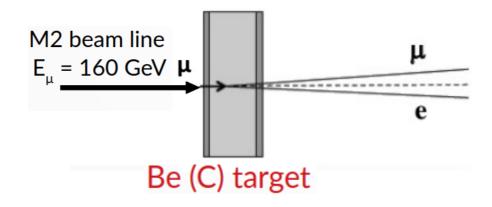
New MUonE Experiment

Letter of intent MuonE project

MUonE is a new experiment to measure the hadronic part of the running of the electromagnetic coupling constant in the spacelike region by the scattering of high-energy muons on atomic electrons on a low-Z target through the elastic process $\mu e \rightarrow \mu e$ [17].



μ-e elastic scattering



Measures directly the most "problematic" SM part of AMM

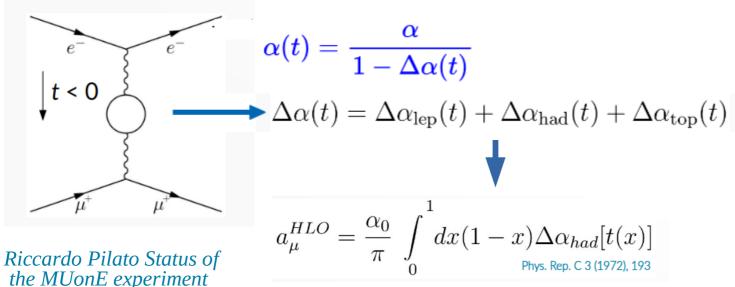
New MUonE Experiment

MÕNE

Letter of intent MuonE project

MUonE is a new experiment to measure the hadronic part of the running of the electromagnetic coupling constant in the spacelike region by the scattering of high-energy muons on atomic electrons on a low-Z target through the elastic process $\mu e \rightarrow \mu e$ [17].

μ-e elastic scattering



$$t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0$$

Measured differential cross section and extracted $\Delta \alpha_{had}(t)$ from its shape

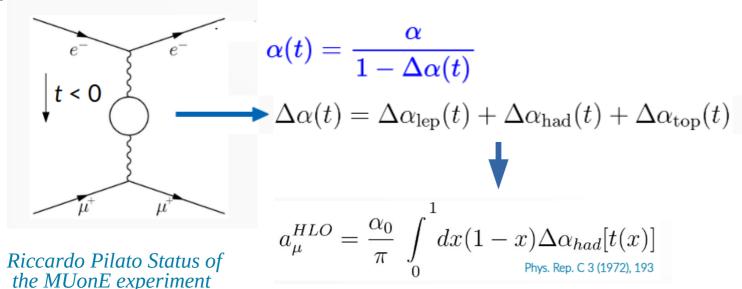
New MUonE Experiment

Uğne

Letter of intent MuonE project

MUonE is a new experiment to measure the hadronic part of the running of the electromagnetic coupling constant in the spacelike region by the scattering of high-energy muons on atomic electrons on a low-Z target through the elastic process $\mu e \rightarrow \mu e$ [17].

μ-e elastic scattering



$$t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0$$

MUonE Test Run at CERN North Area (M2 beam) Aug/Sep 2023 Requested 4 weeks of data taking in 2025

OUTLOOK

- ✓ The Fermilab experiment aims to reduce the BNL uncertainty by a factor four \Rightarrow final result in 2025
- ✓ New JPARC experiment, data taking from 2029, after 2 year run expected statistics comparable to BNL
- ✓ New MuonE experiment to provide an alternative way to measure HVP, expected 4 week run in 2025
- ✓ Improvements on SM theory/data side:
 - reanalyses of "old" data from BaBar, Belle II, KLOE, SND → mainly, the two pion mode
 - more sophisticated models than standard $sQED \rightarrow the two pion mode$
 - high order QED radiative corrections → the two pion mode
 - revision of Monte Carlo generators and their cross-check → mainly, the two pion mode
 - reexamination of τ -data-based evaluation of HVP

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The RadioMonteCarlow 2 Effort

```
Riccardo Aliberti 1 Paolo Beltrame 2 Ettore Budassi 3, 4

Carlo M. Carloni Calame 4 Gilberto Colangelo 5 Lorenzo Cotrozzi 2

Achim Denig 1 Anna Driutti 6, 7 Tim Engel 8 Lois Flower 2, 9

Andrea Gurgone 3, 4, 6 Martin Hoferichter 5 Fedor Ignatov 2

Sophie Kollatzsch 11, 10 Alberto Lusiani 15, 7 Stefan E. Müller 16

Jérémy Paltrinieri 2 Pau Petit Rosàs 2 Fulvio Piccinini 4 Alan Price 17

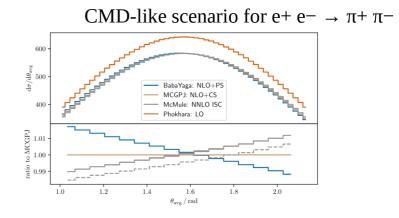
Lorenzo Punzi 7, 15 Marco Rocco 10, 18, 19 Olga Shekhovtsova 20, 21

Andrzej Siódmok 17 Adrian Signer 10, 11 Giovanni Stagnitto 22

Peter Stoffer 10, 11 Thomas Teubner 2 William J. Torres Bobadilla 2

Francesco P. Ucci 3, 4 Yannick Ulrich 2, 5 Graziano Venanzoni 2, 7
```

- Project page (theory, exp data, MC codes...): https://radiomontecarlow2.gitlab.io/
- The state of the art document https://arxiv.org/pdf/2410.22882



OUTLOOK

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- The state of the art document https://arxiv.org/pdf/2410.22882
- Twice per year Working Group Satellite Meeting: 12-14.11 2024, Liverpool, 7-9.05.2025 Pisa??
- PostDoc/PhD positions: https://www.jobs.ac.uk/job/DLO363/postdoctoral-research-associate-in-particle-physics-grade-7-8

If you are interested in joining the RadioMonteCarLow2 effort please contact Adrian Signer, Yannick Ulrich, and Graziano Venanzoni