Elastic proton electron scattering

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Motivations

Elastic $ep$ scattering is a privileged tool for learning on the internal structure of the proton.

Unpolarized cross section, M. N. Rosenbluth (1950)
Polarization method by A.I. Akhiezer and M.P. Rekalo (1967).

Possible applications of $pe$ scattering (inverse kinematics):

- Proton charge radius measurement
- Polarized (anti)proton beams ($\bar{p}e^+$)
- Beam polarimeters for high energy polarized proton beams, Novisibirsk (1997).
Plan

• Formalism

• Kinematics and proton charge radius

• Polarization observables:
  ➢ Transfer coefficients
  ➢ Correlation coefficient

• Summary
Inverse Kinematics (Lab.)

\[ p(p_1) + e(k_1) \rightarrow p(p_2) + e(k_2) \]

- Inverse kinematics: projectile heavier than the target \( \rightarrow \) take into account the electron mass

- Specific kinematics:
  - very small scattering angles
  - very small transferred momenta

- Equivalent total energy, \( s = (p_1 + k_1)^2 \):

  \[ E_p = \frac{M_p}{m_e} \varepsilon_e \sim 2000 \varepsilon_e \]
Formalism

- Scattering amplitude: \( \mathcal{M} = \frac{e^2}{k^2} j_\mu J_\mu \)

- Hadronic current:

\[
J_\mu = \bar{u}(p_2) \left[ F_1(k^2) \gamma_\mu - \frac{1}{2M} F_2(k^2) \sigma_{\mu\nu} k_\nu \right] u(p_1)
\]

Beam \hspace{5cm} Target

Sachs form factors:

\[
G_M(k^2) = F_1(k^2) + F_2(k^2)
\]
\[
G_E(k^2) = F_1(k^2) - \tau F_2(k^2)
\]
\[
\tau = -k^2/(4M^2)
\]

- Leptonic current: \( j_\mu = \bar{u}(k_2) \gamma_\mu u(k_1) \)
Unpolarized cross section (lab.)

\[ \frac{d\sigma}{dk^2} = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2}{m^2 \bar{p}^2} = \frac{\pi \alpha^2}{2m^2 \bar{p}^2} \frac{D}{k^4}, \]

\[ D = k^2(k^2 + 2m^2)G_M^2(k^2) + 2 \left[ k^2M^2 + \frac{1}{1 + \tau} \left( 2mE + \frac{k^2}{2} \right)^2 \right] \left[ G_E^2(k^2) + \tau G_M^2(k^2) \right] \]

\( M \) (m): Proton (electron) mass,
\( E \): energy of incident proton beam.

- Diverges as \( k^{-4} \)
- Dominance of \( G_E \) at low \( Q^2 = -k^2 \).

Precise measurement of proton charge radius
Proton charge radius

- For small values of $Q^2 = -k^2$:

$$G_E(Q^2) = 1 - \frac{1}{6}Q^2 < r_c^2 > + O(Q^2)$$

$$< r_c^2 > = -6 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0}$$

- (Muonic) Hydrogen spectroscopy (Lamb shift):

$$\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3}m_r^3r_c^{-2}\delta_{l0}$$

Proton structure correction to the energy levels of atomic electron
Proton radius puzzle:


\[ r_c = 0.879(8) \]
\[ r_c = 0.84184(67) \]
\[ r_c = 0.8768(69) \]
\[ r_c = 0.895(18) \]

In ep scattering (●●●), precision on the measurement is strongly related to the fit function at \( Q^2 = 0 \).

Minimum value of \( Q^2 \) achieved is \( 0.004 \, \text{GeV}^2 \)

Proton radius measurement with pe elastic scattering

- \( E_p = 100 \text{ MeV} \rightarrow \) Below the pion threshold for pp reactions.
- The maximum of the momentum transfer squared:

\[
(Q^2)_{\text{max}} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}
\]

\( E_p = 100 \text{ MeV}, \quad (Q^2)_{\text{max}} = 0.2 \times 10^{-6} \text{ GeV}^2 \)

Extension to low \( Q^2 \) to gives severe constraints to the fitting Procedure of the slope of \( G_E \)
**pe elastic scattering at** $E_p = 100$ MeV

Differential cross section

\[ \mathcal{L} = 10^{32} \text{ cm}^{-2}\text{s}^{-1} \]

\# events = $25 \times 10^9$/s

\[ \Delta E_2 = E_{\text{scat.}} - E_{\text{beam}} \]

Momentum resolution of the order $10^{-4}$ for the scattered protons is needed

\[ \sin \theta_{p,\text{max}} = \frac{m}{M} \]
Conclusion 1

Possibility to accessing low $Q^2$ values with high statistics in $p\,e$ elastic scattering

→ precise measurement of $r_c$.

Second application:

Polarized (anti)proton beams

(high energy application)
Polarized antiprotons: why?

- Knowledge of the short range $\bar{p}p$ interaction (elastic scattering)
- Spin dependence of partonic processes
- Spin structure of the proton (annihilation into hadrons: pions, hyperons..)
- Transversity (Drell-Yan)
- Relative phase of proton electromagnetic form factors (annihilation into leptons)
- ......

(Reviews from J. Ellis, M. Anselmino, S. Brodsky, ...
Polarized (Anti)Proton Beam:

By repeated traversal of a beam through a polarized hydrogen target in a storage rings (Rathmann PRL 71 (1993))

- Spin Filter: selective removal through $pp$ scattering beyond the acceptance.
- Spin Flip: selective reversal the spin of the particle in one spin state.

Spin Transfer: from polarized electrons.

Provided 2.4% of polarization at $T=23$ MeV in ~ 90 min.
Polarization transfer coefficients:

- **Dirac density matrix:**
  \[
  \frac{\bar{u}(p) u(p)}{m_i} = \left( \frac{\vec{p} + m}{2} \right) (1 - \gamma_5 \hat{s})
  \]
  \[
  s_i^0 = \frac{\vec{p} \cdot \vec{\chi}_i}{m_i}, \quad \vec{s}_i = \vec{\chi}_i + \frac{\vec{p} \cdot \vec{\chi}_i \vec{p}}{m_i(E_i + m_i)}
  \]

- **Hadronic and leptonic tensors:**
  \[
  W_{\mu\nu} = J_\mu J^*_\nu = W^0_{\mu\nu} + W^1_{\mu\nu}(s_{p1}) + W^1_{\mu\nu}(s_{p2}) + W^2_{\mu\nu}(s_{p1}, s_{p2})
  \]
  \[
  L_{\mu\nu} = j_\mu j^*_\nu = L^0_{\mu\nu} + L^1_{\mu\nu}(s_{e1}) + L^1_{\mu\nu}(s_{e2}) + L^2_{\mu\nu}(s_{e1}, s_{e2})
  \]

- **Polarised cross section:**
  \[
  \frac{d\sigma}{dk^2} = \frac{d\sigma_{\text{unp}}}{dk^2} \left[ 1 + T_{\ell\ell} \chi_{\ell}^e \chi_{\ell}^p + T_{nn} \chi_n^e \chi_n^p + T_{tt} \chi_t^e \chi_t^p + T_{lt} \chi_{\ell}^e \chi_t^p + T_{tl} \chi_t^e \chi_{\ell}^p \right]
  \]
Polarization transfer coefficients

$l(t) = \text{longitudinal (transverse)}$

along the incident proton beam,

$n = \text{normal wrt scatt. plane.}$
Polarization transfer coefficients

\[ E = 23 \text{ MeV} \rightarrow T_{nn} = -3.8 \times 10^{-12} \]

\[ E = 10 \text{ GeV} \rightarrow T_{nn} = -1.2 \times 10^{-6} \]
Conclusion 2

Large polarization effects appear in $pe$ elastic scattering at energies between 10 GeV and 50 GeV.


\[
\text{Angular asymmetry} = C_{ij} P_i^{\text{targ.}} P_j^{\text{beam}}
\]

Analyzing power reaction requirements:

1- Smallest theoretical uncertainties as possible at the level of process amplitude.

2- Large analyzing power $C_{ij}$.

$\bar{p} e$ elastic scattering fulfills these requirements
Spin correlation coefficients (analyzing powers):

\[ \vec{p} + \vec{e} \rightarrow p + e \]

Hadronic and leptonic tensors:

\[ W_{\mu\nu} = J_\mu J^*_\nu = W^0_{\mu\nu} + W^1_{\mu\nu}(s_{p1}) + W^1_{\mu\nu}(s_{p2}) + W^2_{\mu\nu}(s_{p1}, s_{p2}) \]

\[ L_{\mu\nu} = j_\mu j^*_\nu = L^0_{\mu\nu} + L^1_{\mu\nu}(s_{e1}) + L^1_{\mu\nu}(s_{e2}) + L^2_{\mu\nu}(s_{e1}, s_{e2}) \]

Polarized cross section:

\[ \frac{d\sigma}{dk^2} = \frac{d\sigma^{unp}}{dk^2} \left[ 1 + C_{\ell\ell} \chi_{\ell}^e \chi_{\ell}^p + C_{nn} \chi_n^e \chi_n^p + C_{tt} \chi_t^e \chi_t^p + C_{\ell t} \chi_{\ell}^e \chi_t^p + C_{t\ell} \chi_t^e \chi_{\ell}^p \right] \]
Spin correlation coefficients

\( \theta_e = 0 \text{ mrad} \)
\( \theta_e = 10 \text{ mrad} \)
\( \theta_e = 30 \text{ mrad} \)
\( \theta_e = 50 \text{ mrad} \)

Long. electron

Trans. electron

Long. proton

Transv. proton

Cnn

Ctt

Clt

Ctl

Cll

E [GeV]

0 50 100 150

0 50 100 150

0 50 100 150

0 10 50 100 150
The figure of merit

\[ \left( \frac{\Delta P}{P} \right)^2 = \frac{2}{L t_m d\sigma / d\Omega d\Omega C_{ij}^2 P^2} \]

\[ F_{ij}^2 = \int \frac{d\sigma}{d\Omega} C_{ij}^2 d\Omega \]

Transverse (e)-longitudinal (p)

At \( E \sim 10 \text{ GeV}, \quad L = 10^{32} \text{ cm}^{-2}\text{s}^{-1} \)
\( \Delta p = 1\% \text{ in } t_m = 3\text{ min} \)
Conclusions

Relativistic description of proton-electron scattering: kinematics, differential cross section and polarization phenomena.

- Possibility to accessing low $Q^2$ values with high statistics $\rightarrow$ precise measurement of $r_c$.

- Polarization effects are large at energies in the GeV range: Possible applications to polarized physics for high energy (anti)proton beams.
Thank you for your attention
The figure of merit

\[ \mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A_{ij}^2(\theta_p), \quad \epsilon(\theta_p) = \frac{N_f(\theta_p)}{N_i} \]

\[ \left( \frac{\Delta P(\theta_p)}{P} \right)^2 = \frac{2}{N_i(\theta_p) \mathcal{F}^2(\theta_p) P^2} = \frac{2}{L t_m (d\sigma/d\Omega) d\Omega A_{ii}^2(\theta_p) P^2} \]

At \( E \sim 10 \text{ GeV}, \quad L = 10^{32} \text{ cm}^{-2} \text{ s}^{-1}, \quad \Delta P = 1\% \text{ in } t = 3 \text{ min} \]
Polarized antiprotons: how?

- Parity-violating (in flight) decay of anti-$\Lambda^0$ hyperons $P=45\%, I(p)\sim 10^4 \text{s}^{-1}$
  (FermiLab, A. Bravar, PRL 77 (1996), D.P. Grosnick, PRC 55,1159 (1997), NIMA290(1990))

- Stern-Gerlach separation in an inhomogeneous magnetic field (too expensive)

- Elastic scattering on C, LH2…

and also….
Methods for measuring proton charge radius

- **Hydrogen spectroscopy (CODATA, Lamb shift)**

- **Dirac** ➞ Energy levels of hydrogen electron depend only on the principal quantum number \( n \).

- Proton structure corrections (at leading order):

\[
\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_p^2 \delta_{l0}
\]

- Other QED effects to the Lamb shift: self energy, vacuum polarization, nuclear motion ...
Methods for measuring proton charge radius

- Muonic hydrogen spectroscopy (CODATA, Lamb shift)

PSI Experiment

*Nature* 466, 213-216 (8 July 2010)

X-ray timing and $2S_{1/2} - 2P_{3/2}$ transition spectra
Methods for measuring proton charge radius

• Hydrogen spectroscopy (CODATA, Lamb shift)

• Muonic Hydrogen spectroscopy (Lamb shift)

• Elastic e–p scattering to determine electric form factor:

\[< r_c^2 > = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \]