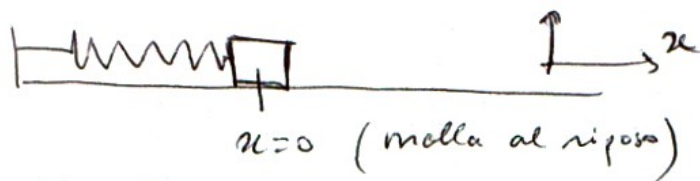


I. Meccanica.

Moto sul piano orizzontale.



1. $\vec{F} = -kx \vec{e}_x$

$$E_p = \frac{1}{2} kx^2$$

2. Conservazione dell'energia fra lo stato iniziale (molla compressa, cubo fermo) e un istante qualunque:

$$\frac{1}{2} ka^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

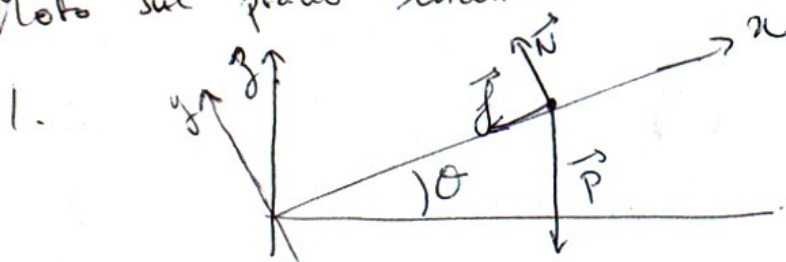
$$\rightarrow v^2 = \frac{k}{m} (a^2 - x^2), \quad v = \sqrt{\frac{k}{m} (a^2 - x^2)}$$

3. v massima per $x_0 = 0$

$$\rightarrow v_0 = \sqrt{\frac{k}{m}} a$$

4. $v_0 = \sqrt{\frac{10}{0.1}} \times 0.1 = 1 \text{ m/s}$

Moto sul piano inclinato



2. $\vec{P} + \vec{N} + \vec{f} = m\vec{a}$

su (y): $-P \cos \theta + N = 0 \quad \rightarrow N = mg \cos \theta$

$$f = \mu_d N \Rightarrow f = \mu_d mg \cos \theta$$

3.
$$W_f = \int_{x=0}^{x_f} \vec{f} \cdot d\vec{x} = \int_0^{x_f} -\mu_d mg \cos \theta dx = -\mu_d mg \cos \theta [x]_0^{x_f}$$

con $z = x \sin \theta$

$$W_f = -\mu_d mg \frac{\cos \theta}{\sin \theta} z = -\mu_d mg \cot \theta \cdot z$$

$$4. \quad \Delta E_m = W(f)$$

$$e \Delta E_m = \Delta E_c + \Delta E_p$$

altezza massima $\rightarrow v_y = 0$, $y = h$

$$\left(0 - \frac{1}{2} m v_0^2\right) + (mgh - 0) = -\mu_d mg \cotan \theta \cdot h$$

$$h = \frac{v_0^2}{2g(1 + \mu_d \cotan \theta)}$$

2. Idrodinamica

1. eq. di continuità : $v_B D_B^2 = v_A D_A^2$

th. di Bernoulli : $\frac{1}{2} \rho v_A^2 + p_A + \rho g z_A = \frac{1}{2} \rho v_B^2 + p_B + \rho g z_B$

$$v_B^2 - v_A^2 = 2 \frac{(p_A - p_B)}{\rho} + 2g(z_A - z_B)$$

$$v_B^2 \left(1 - \frac{D_B^4}{D_A^4}\right) = 2 \frac{(p_A - p_B)}{\rho} - 2g h'$$

portata $Q = v_B D_B^2 \frac{\pi}{4}$

$$Q = \frac{\pi D_B^2}{4} \sqrt{\frac{2(p_A - p_B)/\rho - 2g h'}{\left(1 - \frac{D_B^4}{D_A^4}\right)}}$$

2. $p_A - p_B = p_1 - \rho g(z_A - z_2) - (p_2 - \rho g(z_B - z_2))$

$$p_1 - p_2 = \rho_{Hg} g h$$

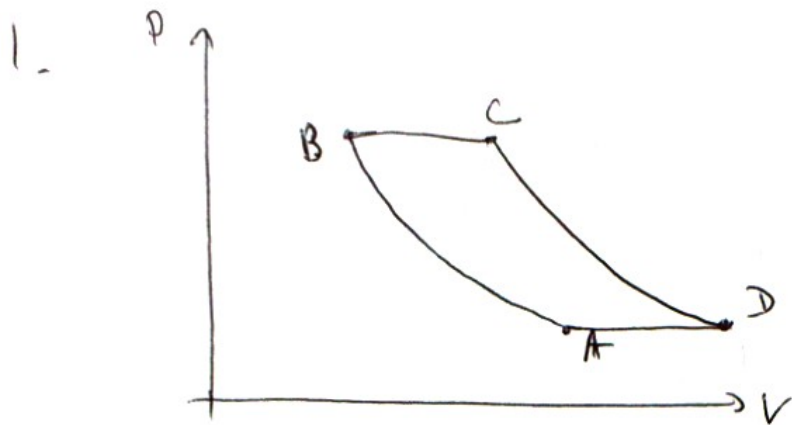
$$p_A - p_B = (\rho_{Hg} - \rho) g h + \rho g h'$$

$$Q = \frac{\pi D_B^2}{4} \sqrt{\frac{2(\rho_{Hg} - \rho) g h}{\left(1 - \frac{D_B^4}{D_A^4}\right)}}$$

$\rightarrow$ indipendente di h' !

$$v_B = 4 v_A = 9.7 \text{ m/s} , \quad Q = 172 \text{ L/s}$$

3. Termodinamica



2 a)	A	B	C	D
	$V_1 = 2 \text{ L}$	$P_2 = 7 \cdot 10^5 \text{ Pa}$	$P_3 = P_2$	$P_4 = P_1$
	$P_1 = 10^5 \text{ Pa}$	$P_1 V_1^\gamma = P_2 V_2^\gamma$	$T_3 = 1180 \text{ K}$	$V_4 = \left(\frac{P_3}{P_1}\right)^{1/\gamma} V_3$
	$T_1 = 300 \text{ K}$	$V_2 = \left(\frac{P_1}{P_2}\right)^{1/\gamma} V_1$	$V_3 = 1.12 \text{ L}$	$V_4 = 4.5 \text{ L}$
	$n_1 = \frac{P_1 V_1}{RT_1}$	$V_2 = 0.5 \text{ L}$		$T_4 = 676 \text{ K}$
	$n_1 = 0.08 \text{ mol}$	$T_2 = 526 \text{ K}$		

b) $C_p = \frac{7}{2} R$ $C_v = \frac{5}{2} R$

c) gas perfetto :

$$\frac{3}{2} k_B T = \frac{1}{2} m \overline{v^2}$$

$$\frac{3}{2} RT = \frac{1}{2} n_{\text{mol}} \overline{v^2}$$

$$v = \sqrt{\frac{3RT}{n_{\text{mol}}}} = 507 \text{ m/s}$$

3. y

Q

W

 $A \rightarrow B$ $Q_{AB} = 0$
adiabatica

$$\Delta U = W + Q = n C_V \Delta T$$

$$W_{AB} = n C_V (T_2 - T_1)$$

$$W_{AB} = 375,8 \text{ J}$$

 $B \rightarrow C$
isobara

$$\delta Q = n C_p dT$$

$$Q = n C_p (T_3 - T_2)$$

$$Q_{BC} = 1522,5 \text{ J}$$

$$\delta W = -P dV$$

$$W = -P_2 (V_3 - V_2)$$

$$= -nR(T_3 - T_2)$$

$$W = -435 \text{ J}$$

 $C \rightarrow D$ $Q = 0$

$$W_{CD} = n C_V (T_4 - T_3)$$

$$W_{CD} = -838 \text{ J}$$

 $D \rightarrow A$

$$Q_{DA} = n C_p (T_1 - T_4)$$

$$Q_{DA} = -875,3 \text{ J}$$

$$W_{DA} = -nR(T_1 - T_4)$$

$$W_{DA} = 250 \text{ J}$$

b) $W_{\text{tot}} = -64 \text{ J}, 2 \text{ J} \rightarrow$ il ciclo produce lavoroc) - rendimento $\varphi = -\frac{W_{\text{tot}}}{Q_{\text{ricevuta}}} = \frac{Q_{\text{tot}}}{Q_{\text{ricevuta}}}$

$$Q_{\text{ricevuta}} = Q_{BC} = n C_p (T_3 - T_2)$$

$$Q_{\text{rilasciata}} = Q_{DA} = n C_p (T_1 - T_4)$$

$$\varphi = 1 + \left(\frac{T_1 - T_4}{T_3 - T_2} \right)$$

$$\varphi = 42,5\%$$