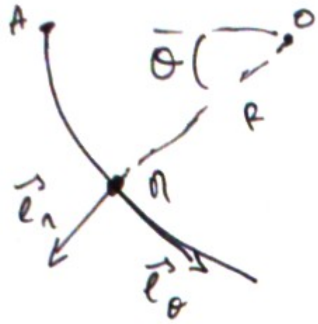
# Correzione dell'esame di Fisica I
18/09/2014

## I. Problema di meccanica

1/




2/ $\vec{OM} = R\,\vec{e}_r$

$\vec{v} = \frac{d\vec{OM}}{dt} = R\frac{d\vec{e}_r}{dt} = R\frac{d\vec{e}_r}{d\theta}\cdot\frac{d\theta}{dt}$

$\vec{v} = R\dot{\theta}\,\vec{e}_\theta$

$\vec{a} = \frac{d\vec{v}}{dt} = R\ddot{\theta}\,\vec{e}_\theta + R\dot{\theta}\frac{d\vec{e}_\theta}{dt}$

$\vec{a} = -R\dot{\theta}^2\,\vec{e}_r + R\ddot{\theta}\,\vec{e}_\theta$

3/ $\|\vec{OM}\| = R$

$\|\vec{v}\| = R\dot{\theta}$

$\|\vec{a}\| = \sqrt{(R\dot{\theta}^2)^2 + (R\ddot{\theta})^2}$

$= R\sqrt{\dot{\theta}^4 + \ddot{\theta}^2}$

4/


$\vec{P}$ peso

$\vec{R}_N$ reazione vincolare

5/ $\sum \vec{F} = m\vec{a}$ $\qquad \vec{P} + \vec{R}_N = m\vec{a}$

lungo $\vec{e}_r$ : $\begin{cases} mg\sin\theta - R_N = -mR\dot{\theta}^2 \\ \cancel{m}g\cos\theta = \cancel{m}R\ddot{\theta} \end{cases}$

— $\vec{e}_\theta$ :

6/ $\vec{P}$ conservativa :


$W_{\vec{P}_{A\to M}} = -\Delta E_p$ $\qquad E_p = -mgz + c$

con $E_p(z=0) = 0 \rightarrow E_p = -mgz$

$E_p = -mgR\sin\theta$

$W_{\vec{P}} = -\left(E_p(M) - E_p(A)\right)$

$W_{\vec{P}} = mgR\sin\theta$

$\vec{R}_N$ non conservativa ma sempre perpendicolare al moto:

$W_{\vec{R}_N} = 0$ .

7/ Teorema dell'energia cinetica: $\Delta E_c = W_{\vec{P}}$

o Conservazione dell'energia meccanica $\Delta E_m = \Delta E_c + \Delta E_p = W^F_{\text{non conserv.}} = 0$

$$\rightarrow \left(\frac{1}{2} m v_1^2 - 0\right) + \left(-mgR\sin\theta - 0\right) = 0$$

$$\rightarrow v^2 = 2gR\sin\theta$$

8/ $\frac{d(v^2)}{dt} = 2v\frac{dv}{dt}$

$\frac{d}{dt}(2gR\sin\theta) = 2gR\cos\theta\frac{d\theta}{dt}$ abbiamo anche (v. 3/) $v = R\dot{\theta}$

$\frac{d\theta}{dt} = \dot{\theta}$

$$\rightarrow 2R\dot{\theta}\cdot R\ddot{\theta} = 2gR\cos\theta\cdot\dot{\theta} \quad \rightarrow \quad R\ddot{\theta} = g\cos\theta.$$

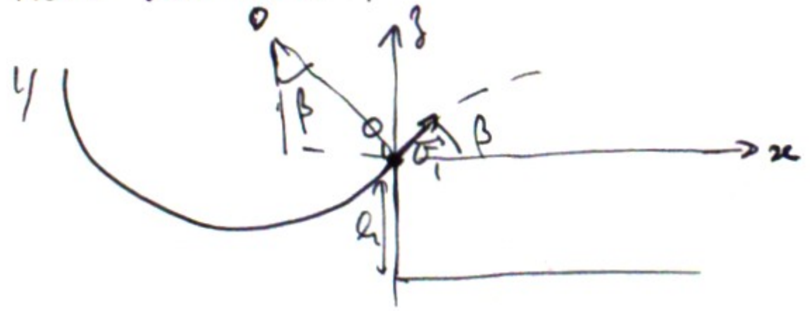
9/ $v = \sqrt{2gR\sin\theta}$

$v_{max} \rightarrow \sin\theta$ max. $\rightarrow \theta = \frac{\pi}{2}$ $\qquad v_{max} = \sqrt{2gR}$

$v_1^2 = 2gR\sin\left(\frac{\pi}{2}+\beta\right) = 2gR\cos\beta \qquad v_1 = v_{max}\sqrt{\cos\beta}$

Moto nell'aria

1/




$\vec{a}\begin{cases} a_x = 0 \\ a_z = -g \end{cases}$

$\vec{v}\begin{cases} v_x = v_{1x} = v_1\cos\beta \\ v_z = -gt + v_{1z} = -gt + v_1\sin\beta \end{cases} \qquad \vec{OM}\begin{cases} x = v_1\cos\beta\, t \\ z = -\frac{1}{2}gt^2 + v_1\sin\beta\, t \end{cases}$

2/ eq. traiettoria

$t = \frac{x}{v_1\cos\beta} \quad \rightarrow \quad z = -\frac{1}{2}\frac{g}{v_1^2\cos^2\beta}x^2 + \tan\beta\, x$

3/ caduta : $z_c = -h = -\frac{1}{2}\frac{g}{v_1^2\cos^2\beta}x_c^2 + \tan\beta\, x_c$

$$\rightarrow \frac{1}{2}\frac{g}{v_1^2\cos^2\beta}x_c^2 - \tan\beta\, x - h = 0$$

con $a = \frac{g}{2v_1^2\cos^2\beta}$, $x_c = \frac{\tan\beta \pm \sqrt{\tan^2\beta + 4ah}}{2a}$, $x_c > 0$.

## II Condotta forzata

1/ $P_A = P_{atm}$ , $P_B = P_A + \rho g (z_A - z_B)$ , $P_C = P_A + \rho g z_A$

2/ $D_v = v \pi R^2$ $(m^3/s)$ ; $D_m = \rho D_v = \rho v \pi R^2$ $(kg/s)$

3/

$$\frac{1}{2}\rho v_A^2 + \rho g z_A + P_A = \frac{1}{2}\rho v_E^2 + \rho g z_E + P_E$$

$P_A = P_E = P_{atm}$ ; $z_E = 0$

eq. di continuità : $v_A S_c = v S$ con $S \ll S_c$ $\rightarrow v_A \ll v$

$$\rightarrow \rho g z_A = \frac{1}{2}\rho v_E^2 = \frac{1}{2}\rho v^2$$ ($v$ costante lungo la condotta)

4/ $v = \sqrt{2 g z_A}$

5/ $v = \sqrt{2 \times 10 \times 200} = 63{,}2\ m/s$ (se $g = 10\ m/s^2$)

6/ $E_c = \frac{1}{2} m v^2 = \frac{1}{2}\rho V v^2$

$E_c = \frac{1}{2} 1000 \times 1 \times (63.2)^2 = 2 \cdot 10^6\ J$

7/ $P = \eta \frac{E_c}{t} = \frac{1}{2}\eta \cdot \rho \frac{V}{t} v^2 = \frac{1}{2}\eta \rho D_v v^2 = \frac{1}{2}\eta \rho \pi R^2 v^3$

8/ $P \approx 18.6\ MW$. $\left(P = \frac{1}{2} \times 0.75 \times 1000 \times \pi \times (0.25)^2 \times (63.2)^3\right)$

## III termodinamica.

1/ Calore specifico del succo + bicchiere : $C_s$

termicamente isolato : $Q_{tot} = 0$

$$C_s \Delta T + m c \Delta T + m l_f = 0$$

$$m = \frac{C_s (30-10)}{c(10-0) + l_f} = 29.6\ g$$

2/ sistema non isolato $\rightarrow m' > m$

massa "in eccesso" $m'' = m' - m = 0.3 \times 29.6 = 8.88\ g$

Calore ricevuto dal sistema :

$$Q = m'' L_f + m'' c (10-0) = 3.3\ kJ.$$