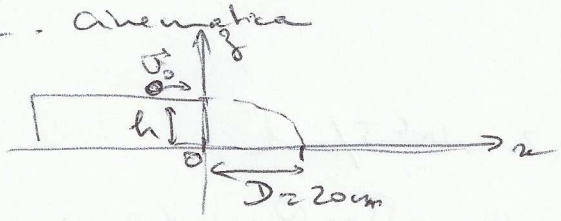
Esame del 03/02/2015 – Soluzioni.

I. cinematica



$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases} \quad \begin{cases} v_x = v_0 \\ v_y = -gt \end{cases} \quad \begin{aligned} x &= v_0 t \\ y &= -\tfrac{1}{2} g t^2 + h \end{aligned}$$

t_c per $x = 20 = D \rightarrow D = v_0 t_c \quad t_c = \dfrac{D}{v_0}$

b) $t_c = \dfrac{20}{50} = 0.4\,s$ a) $0 = -\tfrac{1}{2} g t_c^2 + h$

$$h = \tfrac{1}{2}\, 9.81 \times \left(\frac{20}{50}\right)^2 = 0.78\,m$$

a) $v_f = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + (g t_c)^2} = 3.95\,m/s$

II

Conservazione del momento angolare:

$$I_0 \omega = l m v$$

Conservazione dell'energia:

$$\tfrac{1}{2} I_0 \omega^2 = \tfrac{1}{2} m v^2$$

$$\rightarrow \frac{\omega}{v} = \frac{ml}{I_0} \quad , \quad \left(\frac{\omega}{v}\right)^2 = \frac{m}{I_0} \rightarrow \frac{m^2 l^2}{I_0^2} = \frac{m}{I_0}$$

$$m = \frac{I_0}{l^2} = \frac{M}{3} = 90\,g$$

III

Onde:

$$y_1 = A_0 \sin(kx - \omega t)$$
$$y_2 = A_0 \sin(kx - \omega t + \phi_2)$$

$$y = y_1 + y_2 = 2A_0 \cos\left(\frac{\phi_2}{2}\right) \cdot \sin\left(kx - \omega t - \frac{\phi_2}{2}\right)$$

$$A = 2A_0 \cos\left(\frac{\phi_2}{2}\right) = 9.24\,m \; ; \quad f = \frac{\omega}{2\pi} = 600\,Hz.$$

IM Montgolfiera.

A1. $Q = m_0 c \Delta T = \rho_0 V c \Delta T$

A2. $Q = 24\Pi\ J$

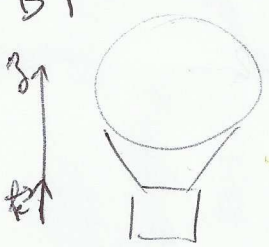
A3. combustione del propano : $Q_m = 2.2\,10^6\ J/mol$

$$Q = Q_m \times \frac{m_p}{M_p} \qquad m_p = M_p \cdot \frac{Q}{Q_M} = 480\ g$$

A4. potenza del bruciatore $P = 60\,kW = 60\,kJ/s$

$$t = \frac{Q}{P} = \frac{2.4\,10^7}{60\,10^3} = 400\,s.$$

B1



B1. $\vec{P}_\Pi = -\Pi g \vec{k}$; $\vec{P}_m = -mg\vec{k} = -\rho V g \vec{k}$

archimede $\vec{P}_A = \rho_0 V g \vec{k}$

B2. $\vec{P}_\Pi + \vec{P}_m + \vec{P}_A$ si proietta lungo l'asse z :

$$\rho_0 V g - \Pi g - \rho V g = g\left((\rho_0 - \rho)V - \Pi\right)$$

La forza risultante è diretta verso l'alto se la mongolfiera sale se : $(\rho_0 - \rho)V > \Pi$

$$: (\rho_0 - \rho) > \frac{\Pi}{V}$$

B3. attrito : $\vec{F}_f = -\eta \phi \vec{v}$

$\sum \vec{F} = m\vec{a} \longrightarrow \vec{P}_A + \vec{P}_\Pi + \vec{P}_m + \vec{F} = m\vec{a}$

senza attrito, se: $P_A - \Pi g - \rho V g > 0 \rightarrow a > 0$

moto uniformemente accelerato

con attrito $P_A - \Pi g - \rho V g - \eta \phi v = ma$

se $a > 0$, v aumenta finché

$P_A - \Pi g - \rho V g - \eta \phi v = 0 \rightarrow$ accelerazione nulla

velocità limite ←

B4. $\rho_0 V g - \rho_g V g - \eta \phi v_{lim} = 0$

$$\rightarrow v_{lim} = \frac{(\rho_0 - \rho_g) V g}{\eta \phi} \qquad v_{lim} = 7.5\ km/s\ !!$$