

Particle Detectors

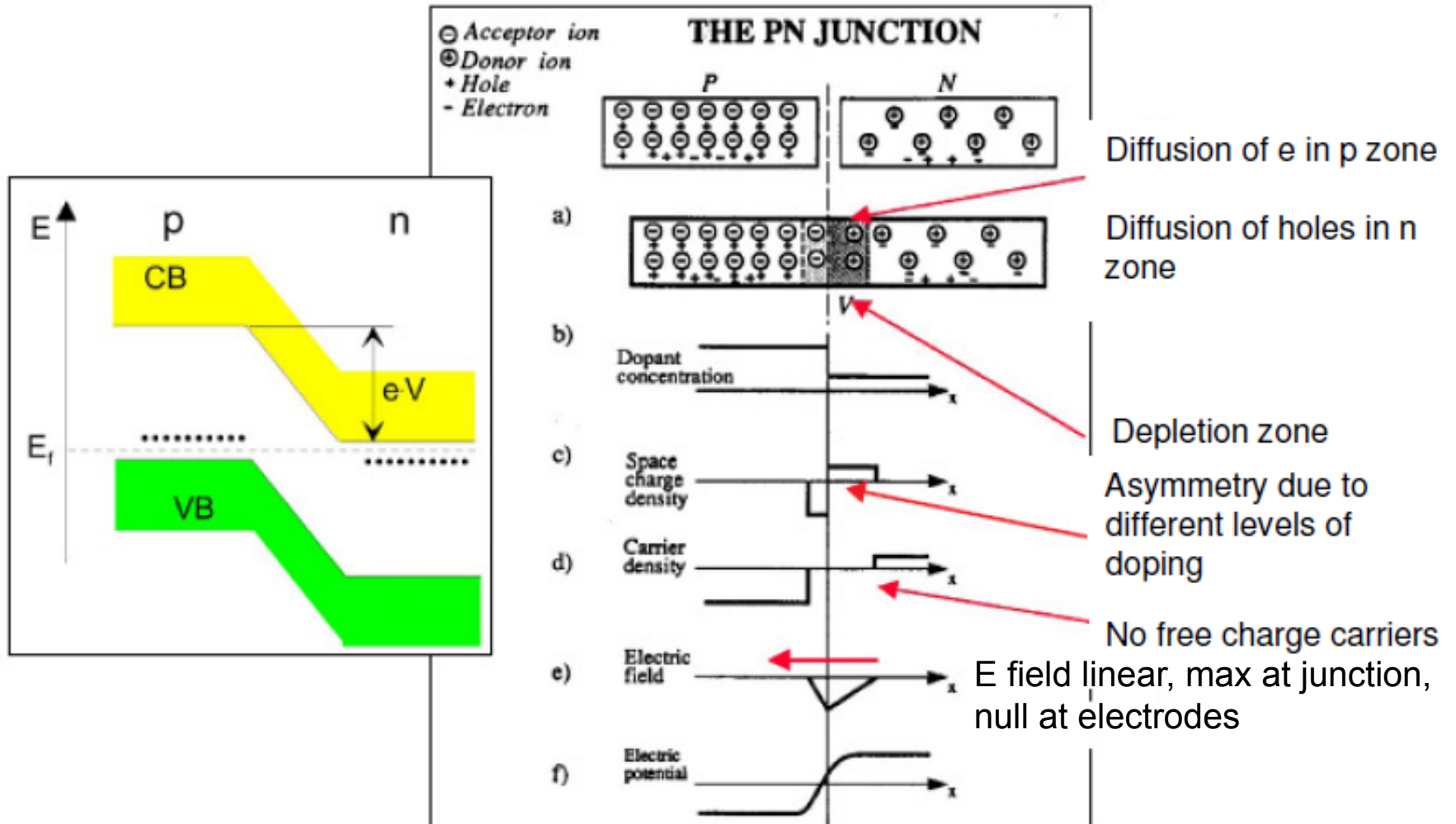
Lecture 19

23/05/18

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P-N junction: overview



Potential quadratically increasing in the space charge region between 0 at p and +V at n side

Rivelatori a stato solido

Capacità

Siccome abbiamo una carica associata alla giunzione dipendente dal voltaggio possiamo parlare di una **capacità di carica spaziale**:

$$C_j = \frac{dQ}{dV_B} = \frac{dQ}{dw} \frac{dw}{dV_B}$$

Abbiamo chiamato **w** la profondità della zona di svuotamento.

L' incremento di carica **dQ** si ha ai lati della giunzione a causa dell'allargamento di **w** **dw** causato dalla crescita **dV_B** del voltaggio di bias **V_B**. ➔

$$\begin{aligned} dQ &= qN_d dw \\ \frac{dw}{dV_B} &= \frac{dx_n}{dV_B} = \sqrt{\frac{q \epsilon N_a \cdot N_d}{2(N_a + N_d)V_B}} \\ \Rightarrow C_j &= \sqrt{\frac{\epsilon}{2\mu\rho V_B}} \end{aligned}$$

Capacità per unità di area.

L'ordine di grandezza di **C_j** è ~150÷200 pF/cm² per **V_B**~100V

Se **A** è l'area avremo **C_j**=(ε/w)A

Rivelatori a stato solido

- In condizioni di reverse bias, la corrente NON e' zero.
- Due contributi: correnti di generazione e correnti di diffusione

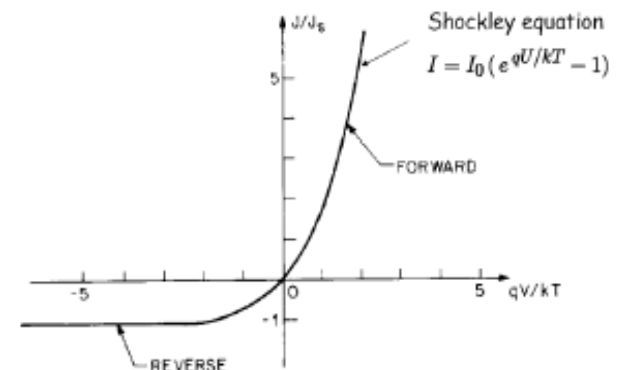
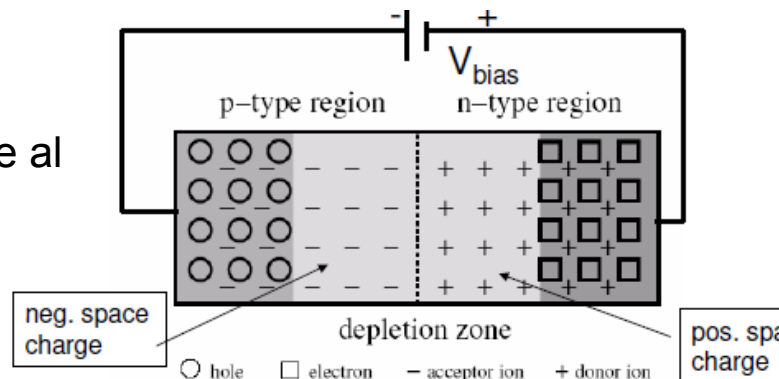
$j_{diff} = q(n_p/\tau_n)L_n + q(p_n/\tau_p)L_p$ (L = lunghezza di diffusione). Queste correnti di diffusione sono dovute alle cariche generate vicino alla zona di svuotamento e che diffondono nella zona di svuotamento stessa. Se τ_p e τ_n sono le vite medie dei p nella regione n e degli n nella regione p le lunghezze di diffusione saranno $L_n = (D_n\tau_n)^{1/2}$ e $L_p = (D_p\tau_p)^{1/2}$, essendo D la costante di diffusione.

$j_{gen} = (1/2)(n_i/\tau_o)d$ corrente dovuta alla carica generata nella zona di svuotamento. τ_o = vita media dei portatori minoritari nella zona di svuotamento: dipende da difetti e impurita' del reticolo che ricombinano/intrappolano la carica. Questa corrente è proporzionale a d , lo spessore svuotato, che a sua volta è proporzionale a $(V_B)^{1/2}$. n_i dipende fortemente dalla temperatura (raddoppia ogni 8°) → mantenere il silicio a temperatura costante.

Il contributo delle due correnti dipende dal materiale: nel Ge domina la corrente di diffusione, mentre nel Si domina la corrente di generazione.

Valori tipici nel Si, 35 nA/cm²

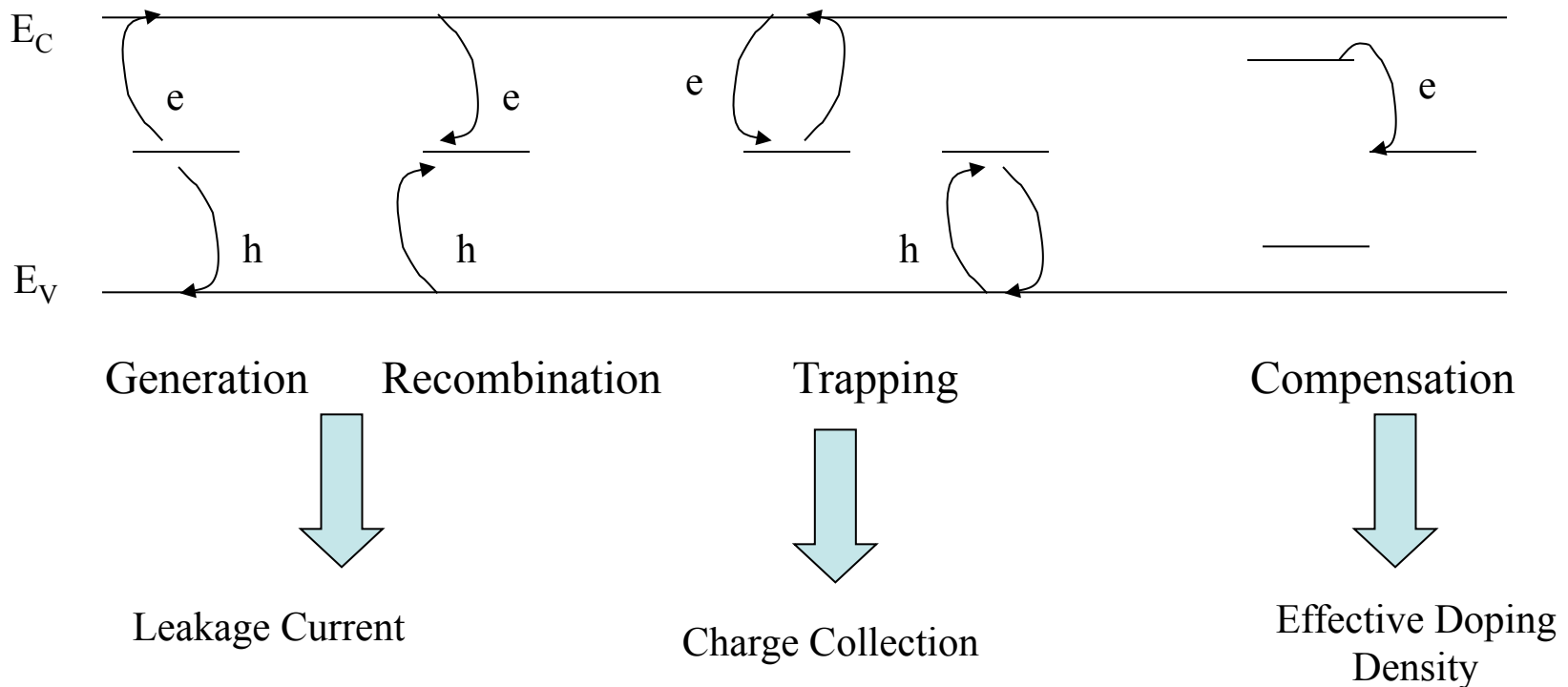
La corrente del rivelatore contribuisce al rumore dell'apparato



Effects of Defects

I difetti creano livelli di energia nella gap, che oltre a generare corrente di leakage, possono intrappolare elettroni e lacune per un tempo lungo rispetto a quello di raccolta oppure possono ricombinare coppie catturate dalla stessa trappola → **inefficienze di raccolta di carica**.

Fisicamente sono creati da impurita' per sostituzione, interstiziale, dislocamento di piani reticolari, cioe' da tutti quei fattori che "distorcono" il potenziale periodico "ideale" del reticolo del semiconduttore...p. es irraggiamento



La statistica di questi processi e' descritta da quella di Shockley Read Hall (che non facciamo)

Rivelatori a stato solido

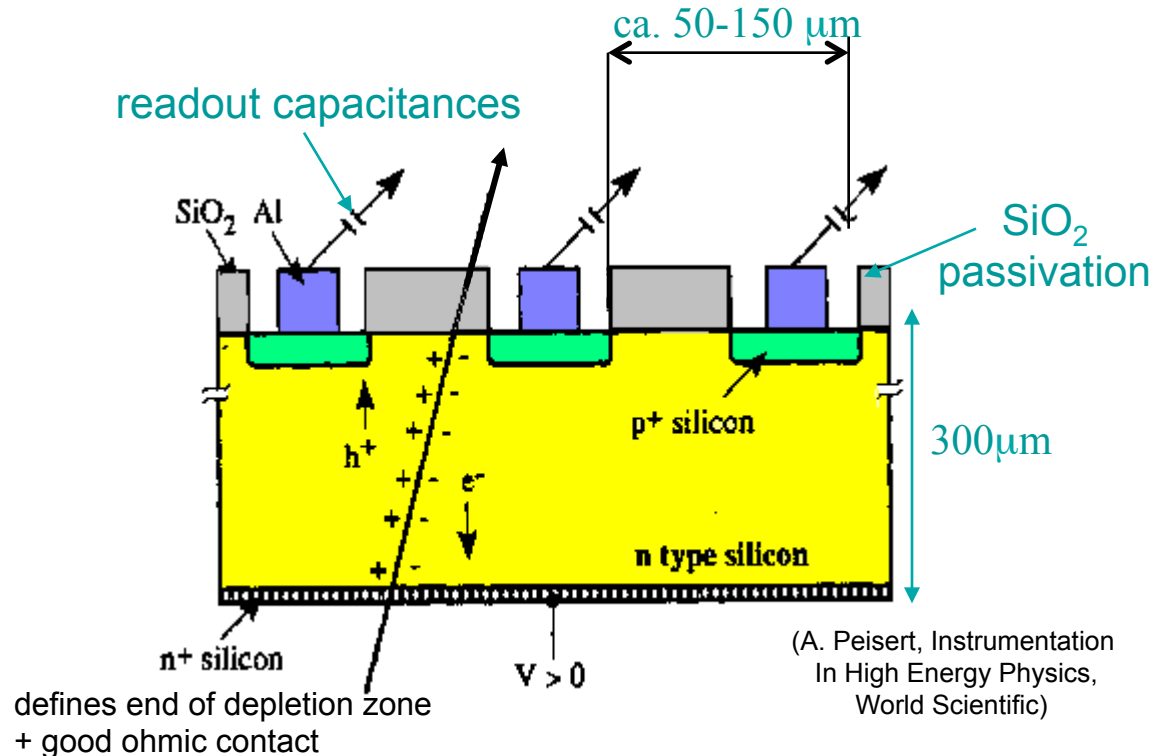
- ❑ Polarizzando inversamente il diodo (cioè applicando una $V_B \sim 100V$ dello stesso segno di V_d) → la sottile zona di carica spaziale si estende su tutto il diodo → diodo completamente svuotato.
 $d \sim x_n \sim (2\varepsilon V_B / qN_d)^{1/2}$ ed in termini della resistività ρ $d = (2\varepsilon V_B \rho \mu)^{1/2}$
- ❑ Il deposito di energia nella zona completamente svuotata, dovuto al passaggio della particella carica, crea delle coppie libere e-lacuna.
- ❑ Sotto l'influenza del campo elettrico, gli elettroni derivano verso il lato n, le lacune verso il lato p → si ha una corrente rivelabile
- ❑ la parte p serve :
 - ❖ per poter svuotare la parte n e quindi può essere molto sottile
 - ❖ per raccogliere le lacune che si sono formate

Rivelatori a stato solido

Possiamo usare la zona di carica spaziale svuotato come camera di ionizzazione. **Segmentando una o entrambe le faccie possiamo misurare la coordinata del punto di passaggio della particella ionizzante.**

Larghezza tipica microstrip 15 μm

Passo tipico 50-100 μm (da confrontare con quello delle camere a deriva e TPC, 1-2 mm)

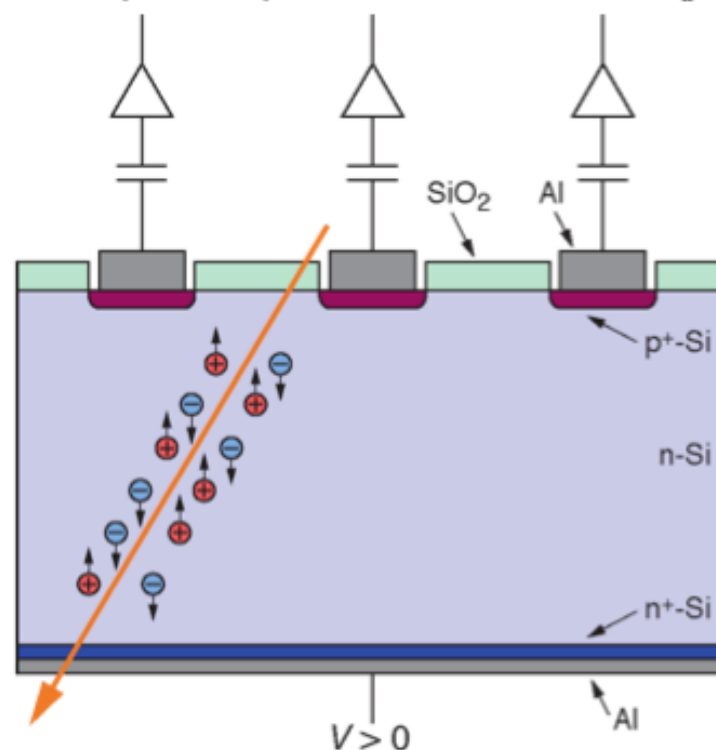


Rivelatori a stato solido

Through going charged particles create e^-h^+ pairs in the depletion zone (about 30.000 pairs in standard detector thickness). These charges drift to the electrodes. The drift (current) creates the signal which is amplified by an amplifier connected to each strip. From the signals on the individual strips the position of the through going particle is deduced.

A typical n-type Si strip detector:

- ★ p⁺n junction:
 $N_a \approx 10^{15} \text{ cm}^{-3}$, $N_d \approx 1-5 \cdot 10^{12} \text{ cm}^{-3}$
- ★ n-type bulk: $\rho > 2 \text{ k}\Omega\text{cm}$
→ thickness 300 μm
- ★ Operating voltage < 200 V.
- ★ n⁺ layer on backplane to improve ohmic contact
- ★ Aluminum metallization



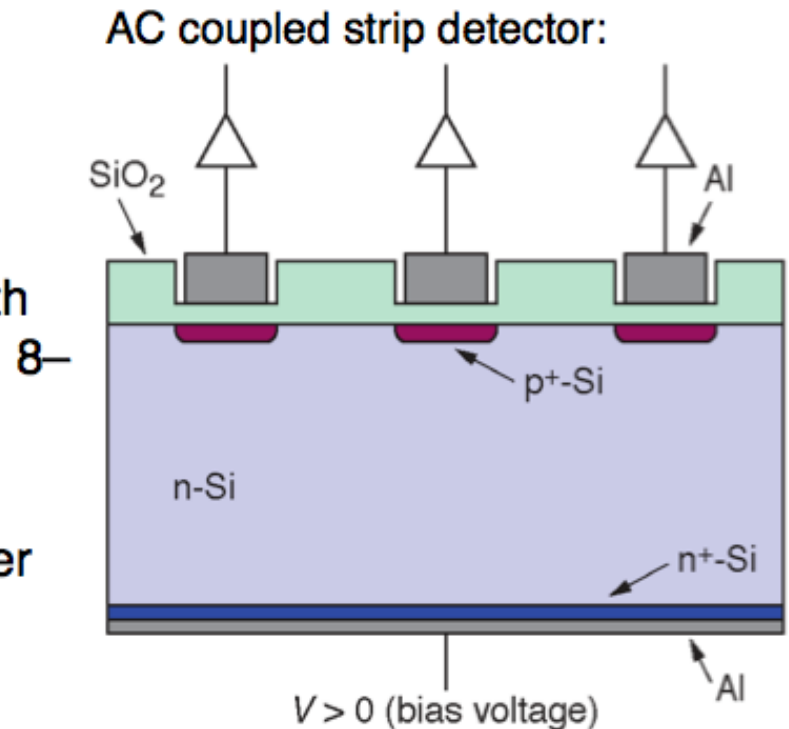
AC Coupling

La corrente delle strip va disaccoppiata dall'ingresso del circuito di lettura poiche' saturerebbe l'amplificatore.

Si inserisce un condensatore fra la strip e il circuito esterno, Puo' essere discreto o integrato

AC coupling blocks leakage current from the amplifier.

- ★ Integration of coupling capacitances in standard planar process.
- ★ Deposition of SiO_2 with a thickness of 100–200 nm between p+ and aluminum strip
- ★ Depending on oxide thickness and strip width the capacitances are in the range of 32 pF/cm.
- ★ Problems are shorts through the dielectric (pinholes). Usually avoided by a second layer of Si_3N_4 .



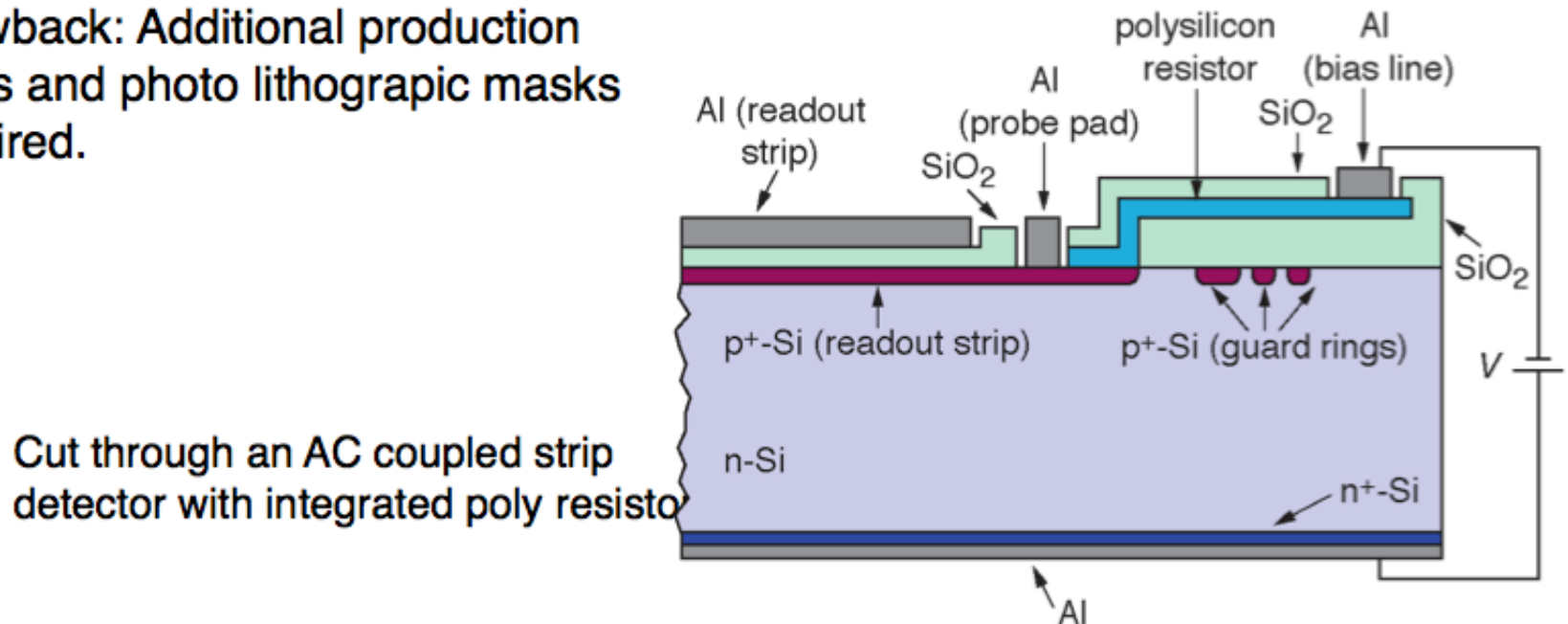
Several methods to connect the bias voltage: polysilicon resistor, punch through bias, FOXFET bias.

Alimentazione del detector (bias)

- Ciascuna strip va tenuta a tensione V (che puo' essere massa, per esempio sulla faccia di giunzione)
- La densita' di strip e' molto alta e le strip sono molto piccole (10 μm x qualche cm)
- Occorre una "linea" di alimentazione per ciascuna strip
- Non e' pensabile usare un "filo" per ogni strip
- Soluzione: linea di alimentazione (o bias) integrata nel rivelatore

Polysilicon Bias

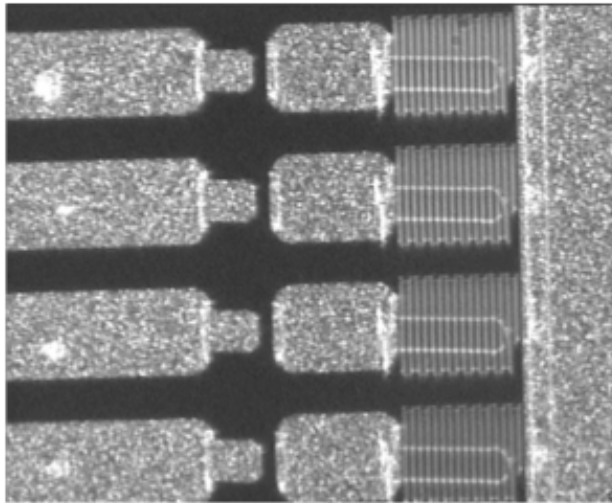
- ★ Deposition of polycrystalline silicon between p⁺ implants and a common bias line.
- ★ Sheet resistance of up to $R_s \approx 250 \text{ k}\Omega/\square$. Depending on width and length a resistor of up to $R \approx 20 \text{ M}\Omega$ is achieved ($R = R_s \cdot \text{length}/\text{width}$).
- ★ To achieve high resistor values winding poly structures are deposited.
- ★ Drawback: Additional production steps and photo lithographic masks required.



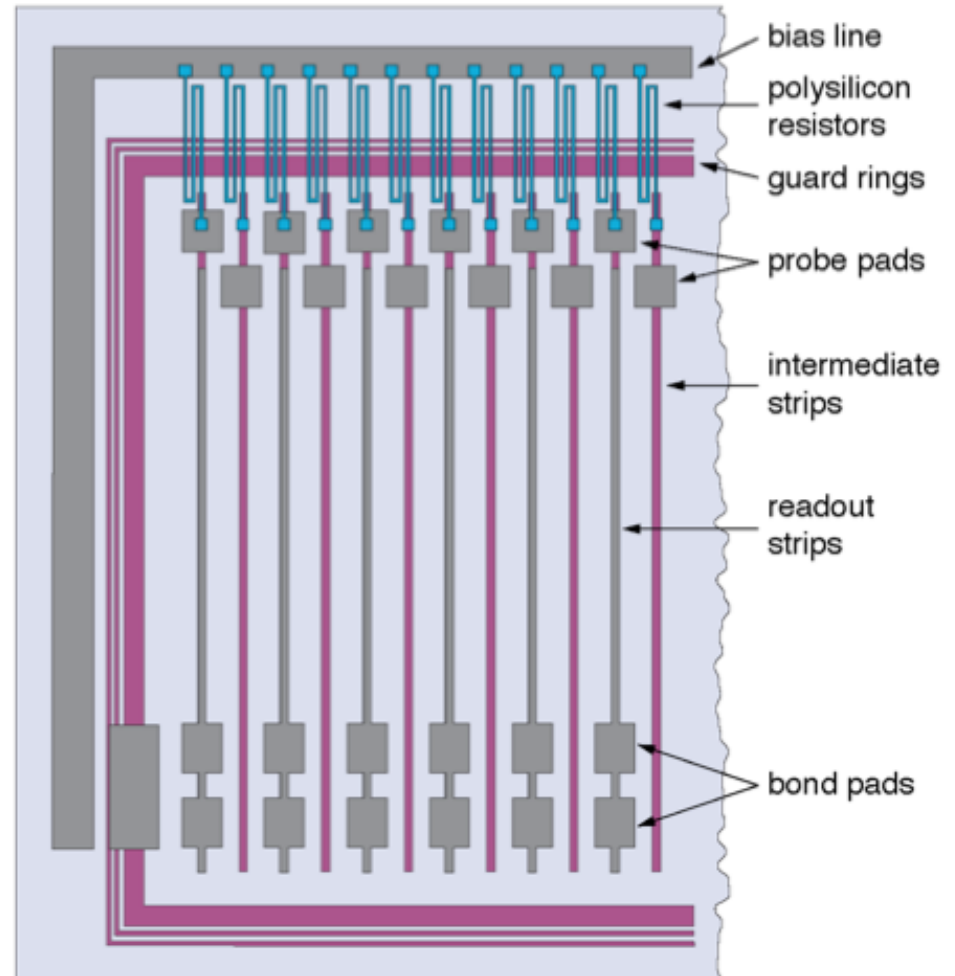
Polysilicon bias

Top view of a strip detector with polysilicon resistors:

CMS-Microstrip-Detektor: Close view of area with polysilicon resistors, probe pads, strip ends.

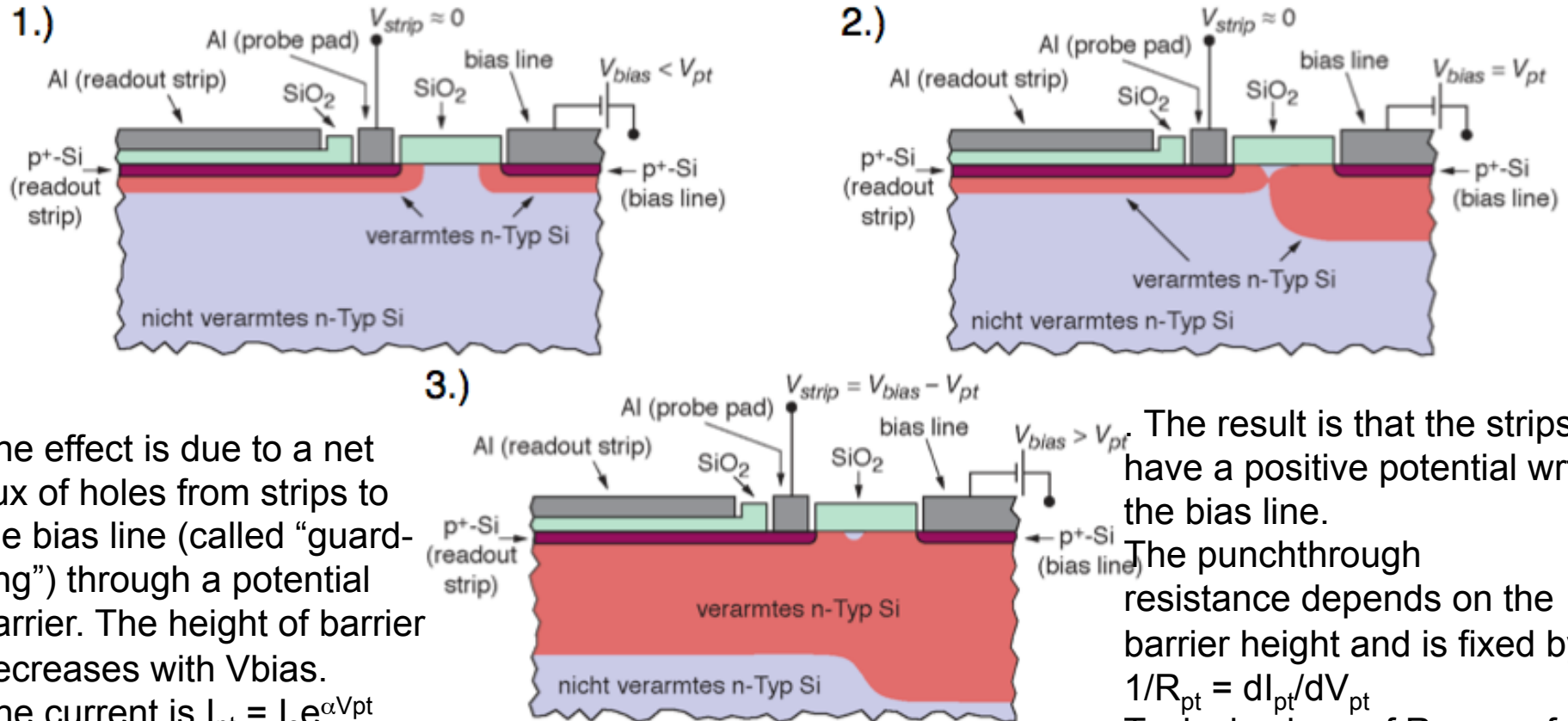


CMS Collaboration, HEPHY Vienna



Punch through bias

Punch through effect: Figures show the increase of the depletion zone with increasing bias voltage (V_{pt} = punch through voltage).



The effect is due to a net flux of holes from strips to the bias line (called “guard-ring”) through a potential barrier. The height of barrier decreases with V_{bias} .

The current is $I_{pt} = I_s e^{\alpha V_{pt}}$

The result is that the strips have a positive potential wrt the bias line.

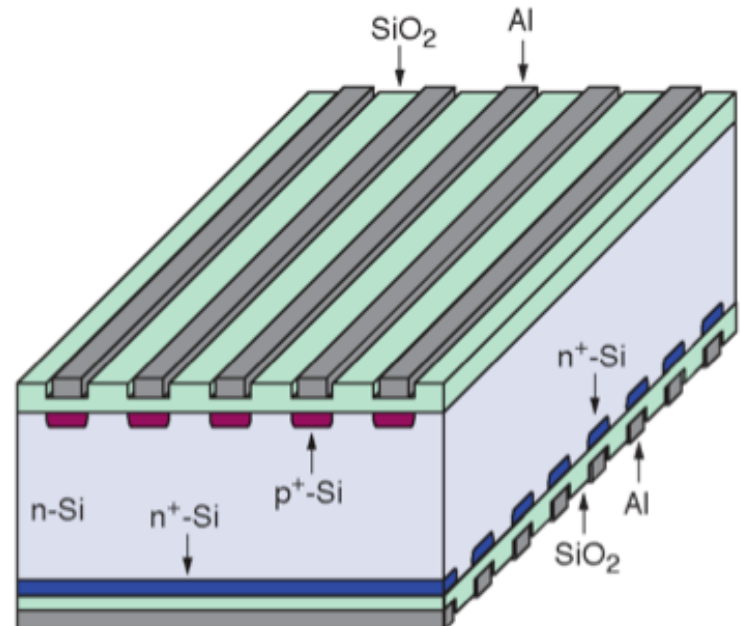
The punchthrough resistance depends on the barrier height and is fixed by $1/R_{pt} = dI_{pt}/dV_{pt}$. Typical values of R_{pt} are of O(1-10 GOhm)

Advantage: No additional production steps required.

silici doppia faccia

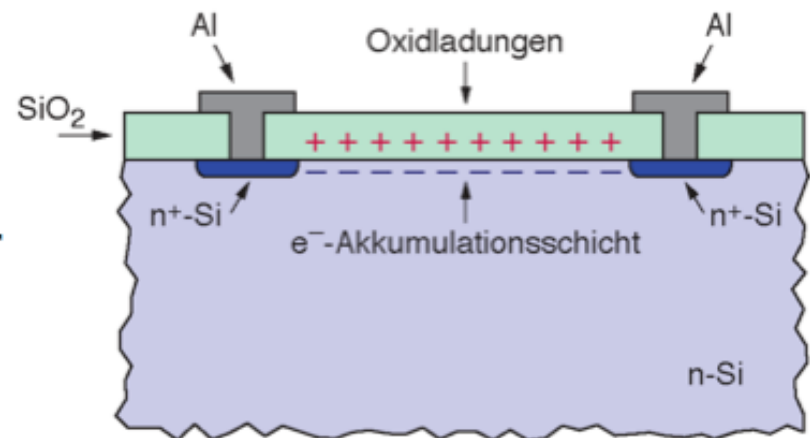
- ★ Single sided strip detector measures only one coordinate. To measure second coordinate requires second detector layer.
- ★ Double sided strip detector measures two coordinates in one detector layer (minimizes material).
- ★ In n-type detector the n^+ backside becomes segmented, e.g. strips orthogonal to p^+ strips.
- ★ Drawback: Production, handling, tests are more complicated and hence double sided detector are expensive.

Scheme of a double sided strip detector (biasing structures not shown):



Silici doppia faccia

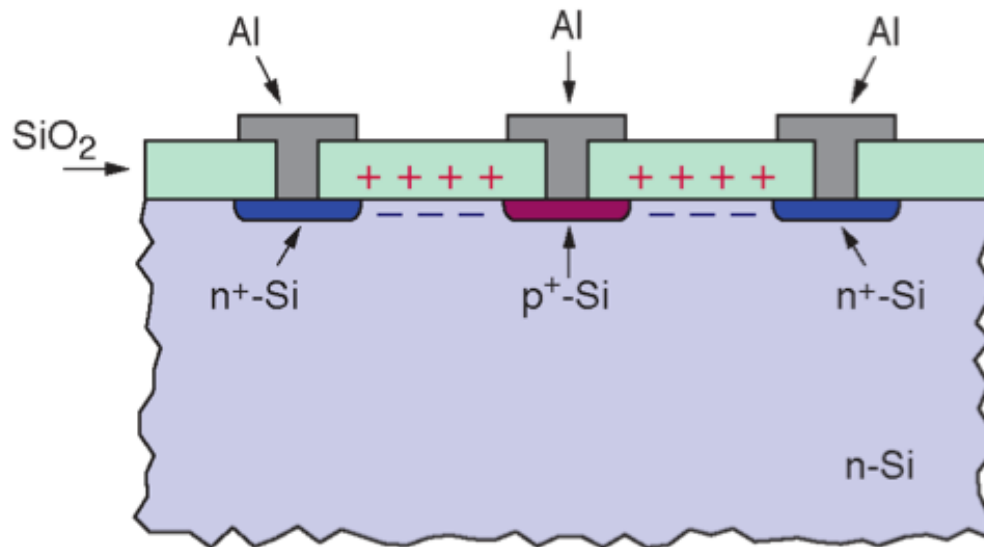
- ★ Problem with n^+ segmentation: Static, positive oxide charges in the Si-SiO₂ interface.
- These positive charges attract electrons. The electrons form an accumulation layer underneath the oxide.
- n^+ strips are no longer isolated from each other (resistance $\approx k\Omega$).
- Charges generated by through going particle spread over many strips.
- **No position measurement possible.**
- ★ Solution: Interrupt accumulation layer using p^+ -stops, p^+ -spray or field plates.



Positive oxide charges cause electron accumulation layer.

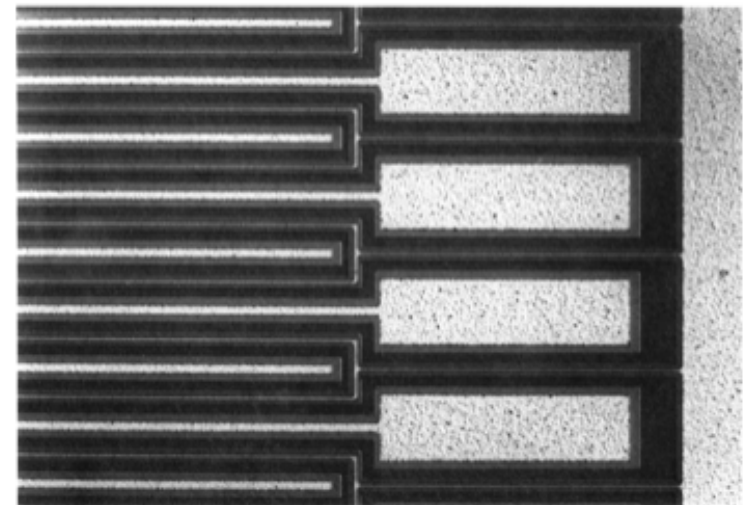
Silici doppia faccia

- ★ p⁺-implants (**p⁺-stops, blocking electrodes**) between n⁺-strips interrupt the electron accumulation layer.
- Interstrip resistance reach again GΩ.



A. Peisert, *Silicon Microstrip Detectors*,
DELPHI 92-143 MVX 2, CERN, 1992

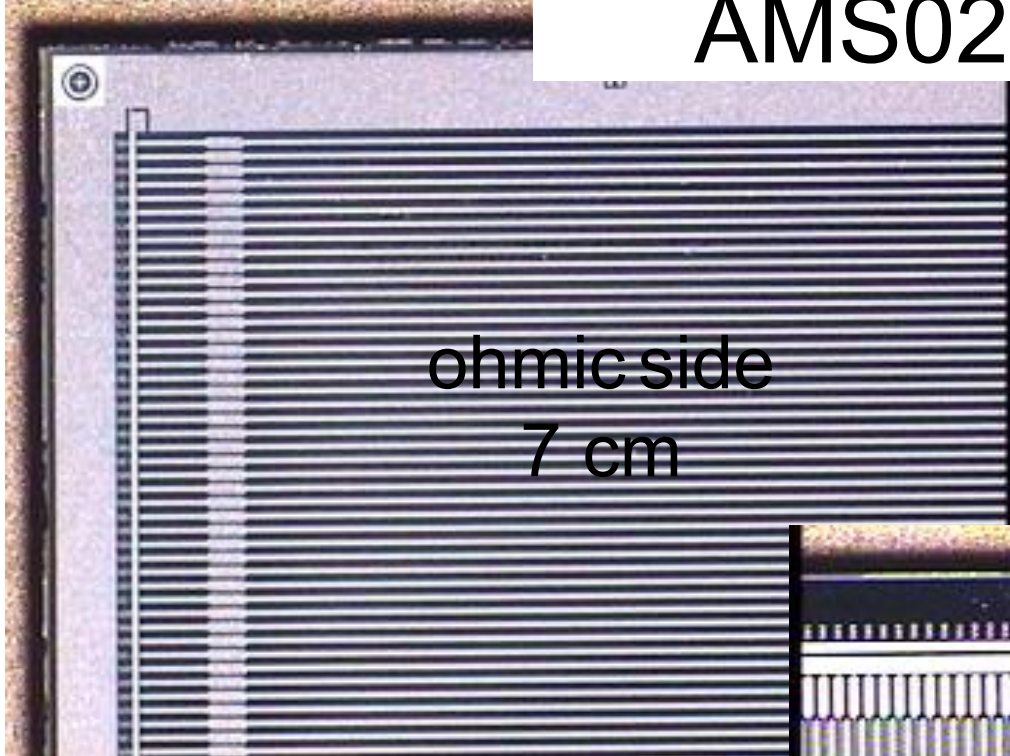
Picture showing the n⁺-strips and the p⁺-stop structure:



J. Kemmer and G. Lutz, *New Structures for Position Sensitive Semiconductor Detectors*,
Nucl. Instr. Meth. A **273**, 588 (1988)

AMS02 silicon detector

- 1264 p⁺ strip junctions on one side (junction side) with geometric pitch 55 μm
- 384 n⁺ (ohmic side) perpendicular to the p⁺ strips with geometric pitch 104 μm



ohmic side
7 cm

The basic units of the tracker are 300 μm -thick, n-doped, high resistivity $7 \times 4 \text{ cm}^2$ silicon sensors. 2500 pieces have been produced for the tracker assembly

I silici del tracciatore di AMS02 sono alimentati con punch through sulla faccia p e con surface through su quella n



Bending plane

junction side
4 cm

The strips are metallized with Al for with pads for ultrasonic bonding.

The strip width is 12 μm on P side and 40 μm on N side



110 μ

The sensors junctions operate over-depleted in reverse bias voltage regime to suppress the leakage current (noise source).

The operation bias voltage is 75 V



104 μ

Rivelatori a stato solido

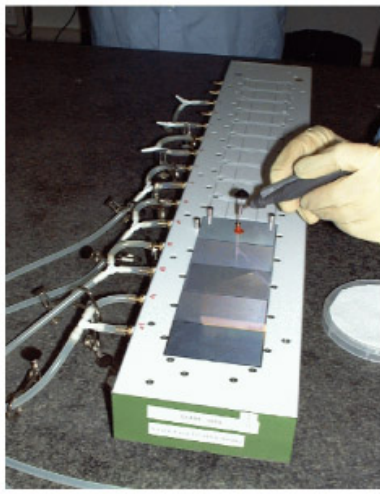
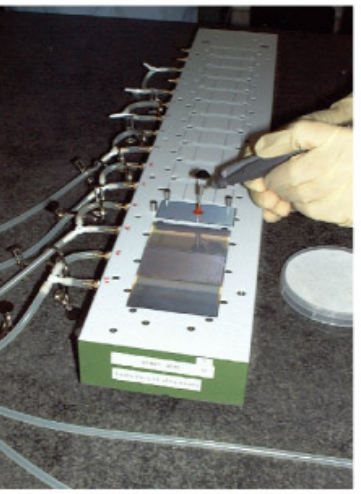
Risoluzione spaziale intrinseca (di singolo punto)

Dipende da:

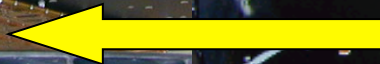
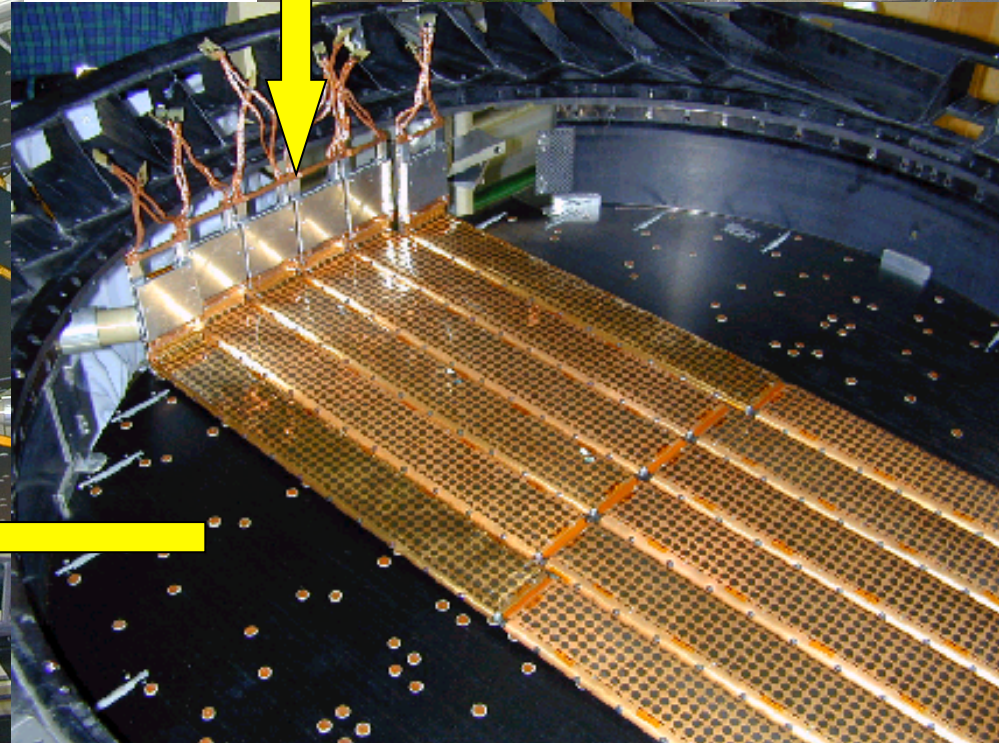
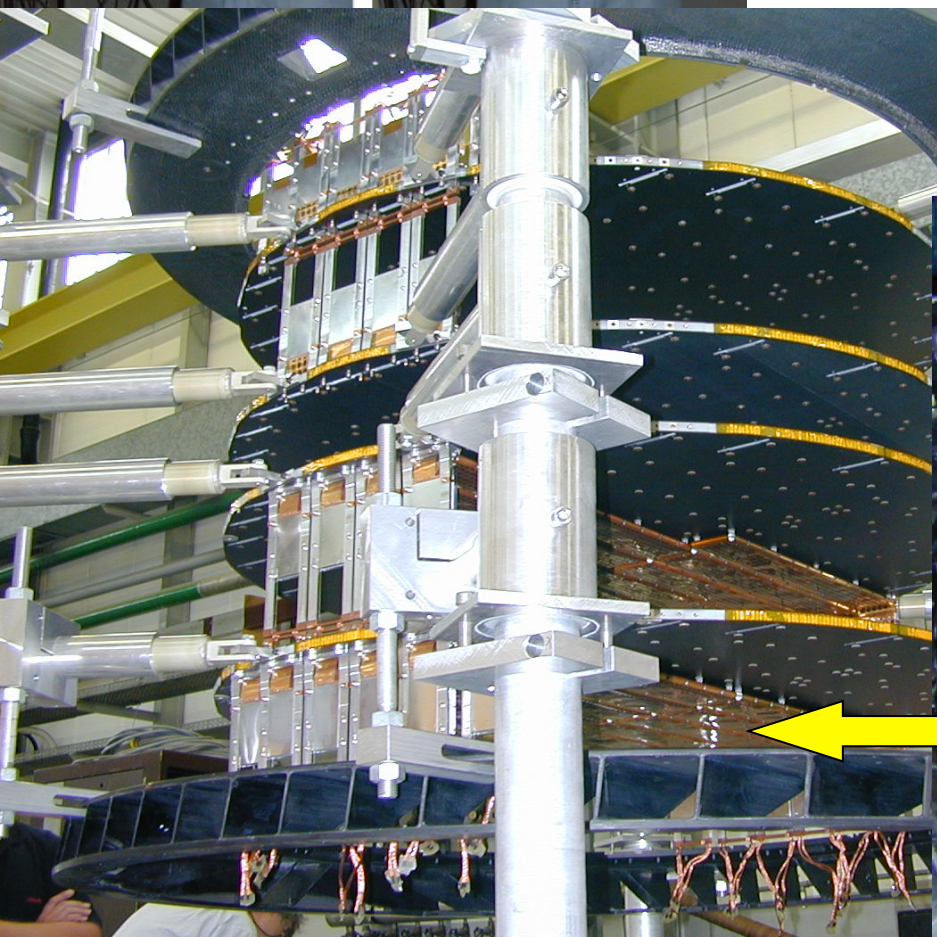
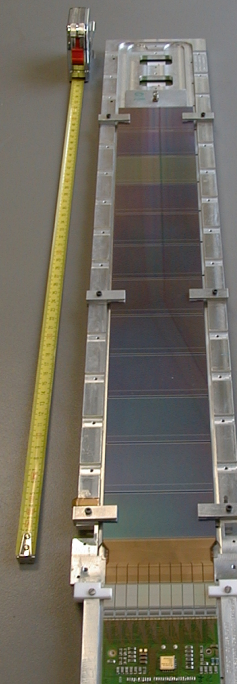
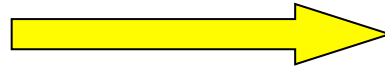
- ❖ **Processi fisici** quali le fluttuazioni della perdita di energia, la diffusione dei portatori di carica.
- ❖ **Fattori esterni** quali il numero di strip, il modo di lettura, il rumore dell'apparato (o meglio S/N)

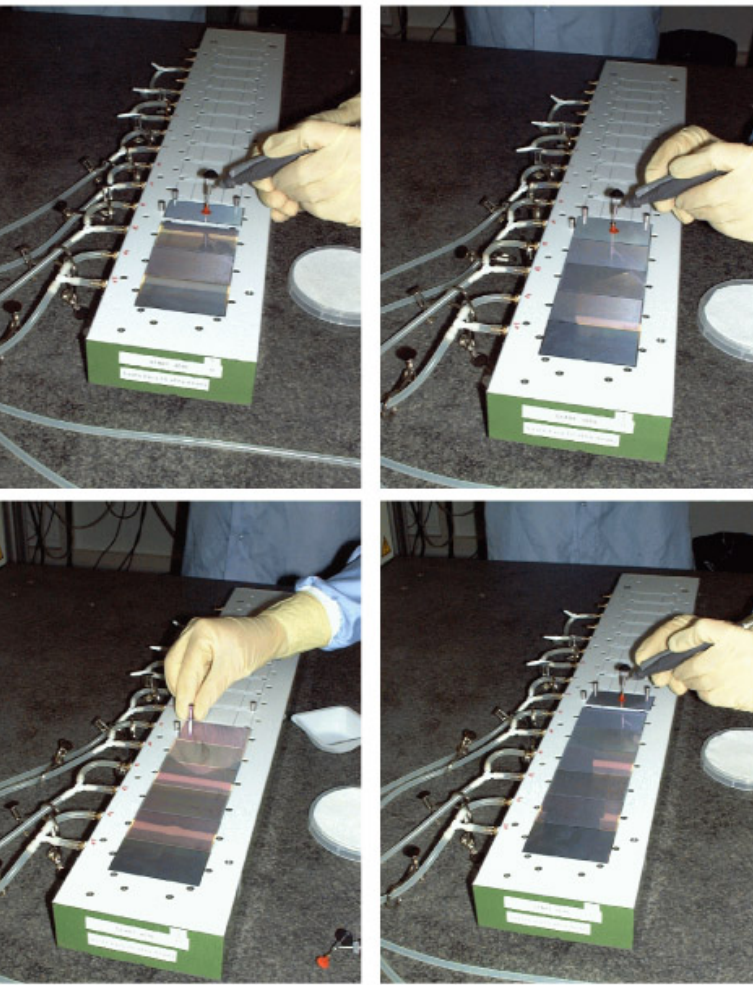
Si possono raggiungere precisioni fino a $\sim 3 \mu\text{m}$.

La risoluzione spaziale "totale" dipende in maniera cruciale dall'allineamento relativo dei differenti moduli che costituiscono l'apparato tracciante → necessarie misure continue di posizione relativa dei rivelatori



The tracker assembly





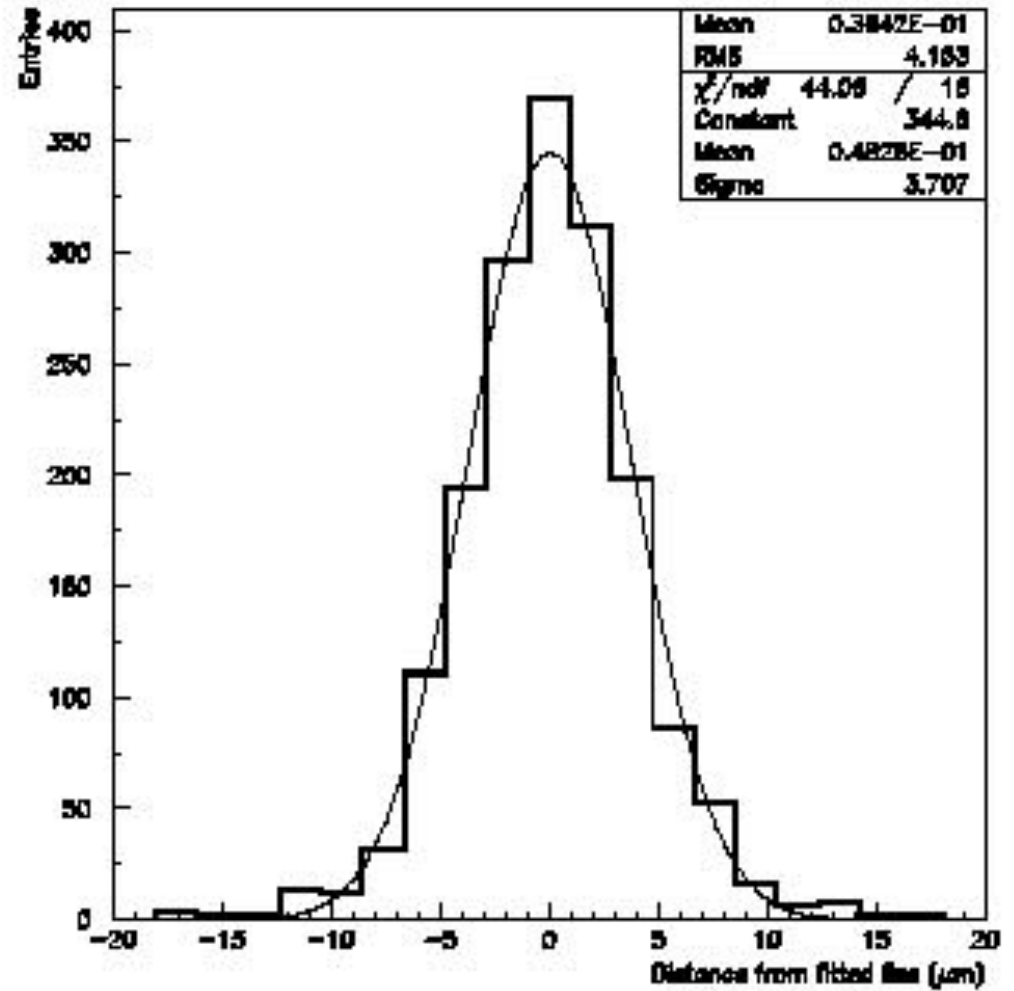
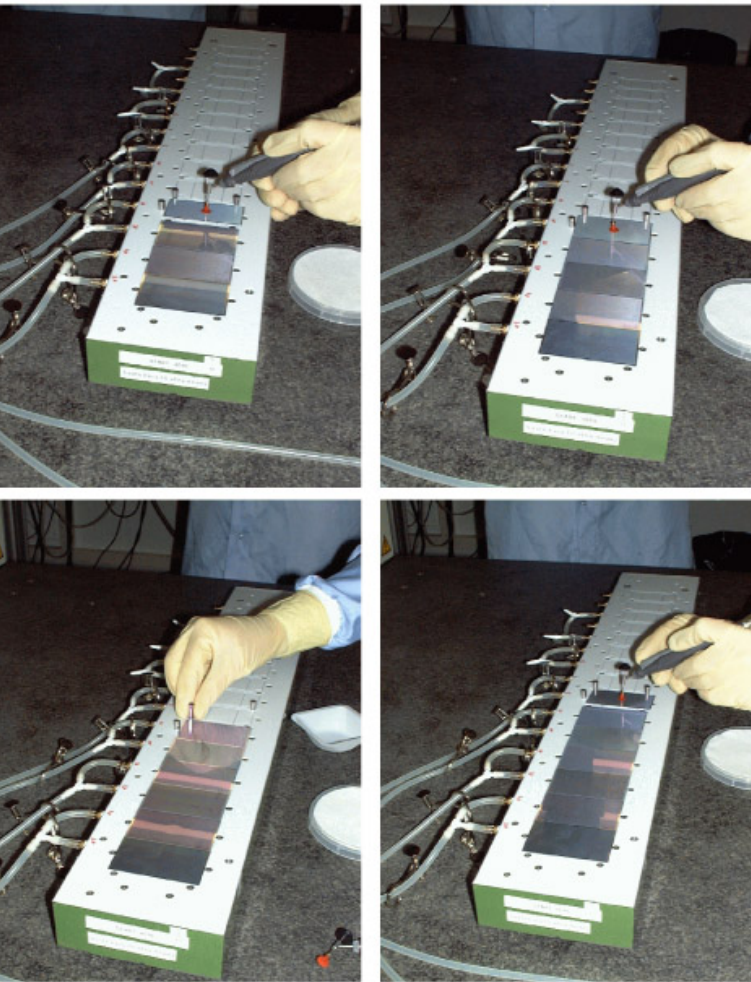
The fundamental unit of the tracker is a ‘**ladder**’: a set of mechanically and electrically connected silicon sensors.

To reach the highest position resolution, it is necessary to align the single sensors to a precision of few microns over a length up to 60 cm → special high precision jigs have been built to assemble the sensors.

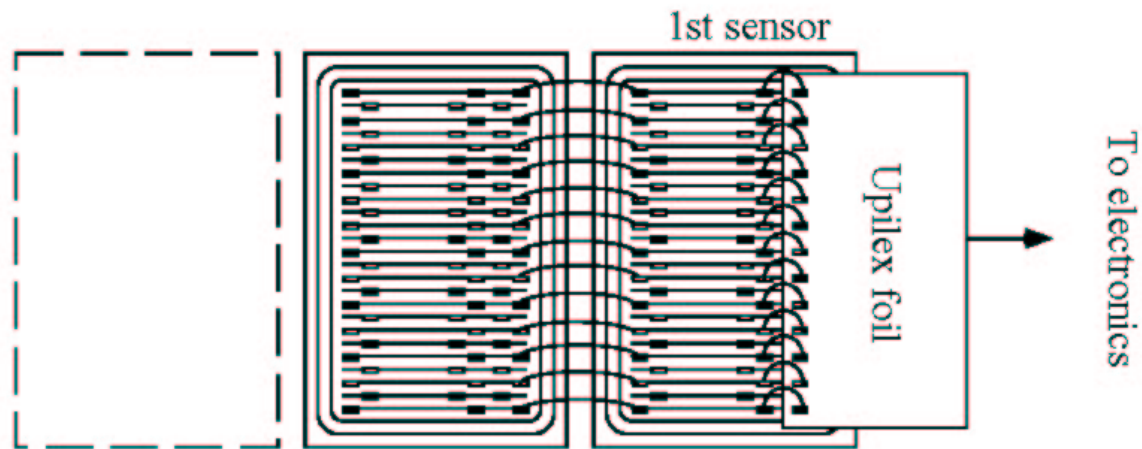
The tolerance on the sensors dimensions is less than 5 μm and the distance between 2 adjacent sensors is about 20 μm .

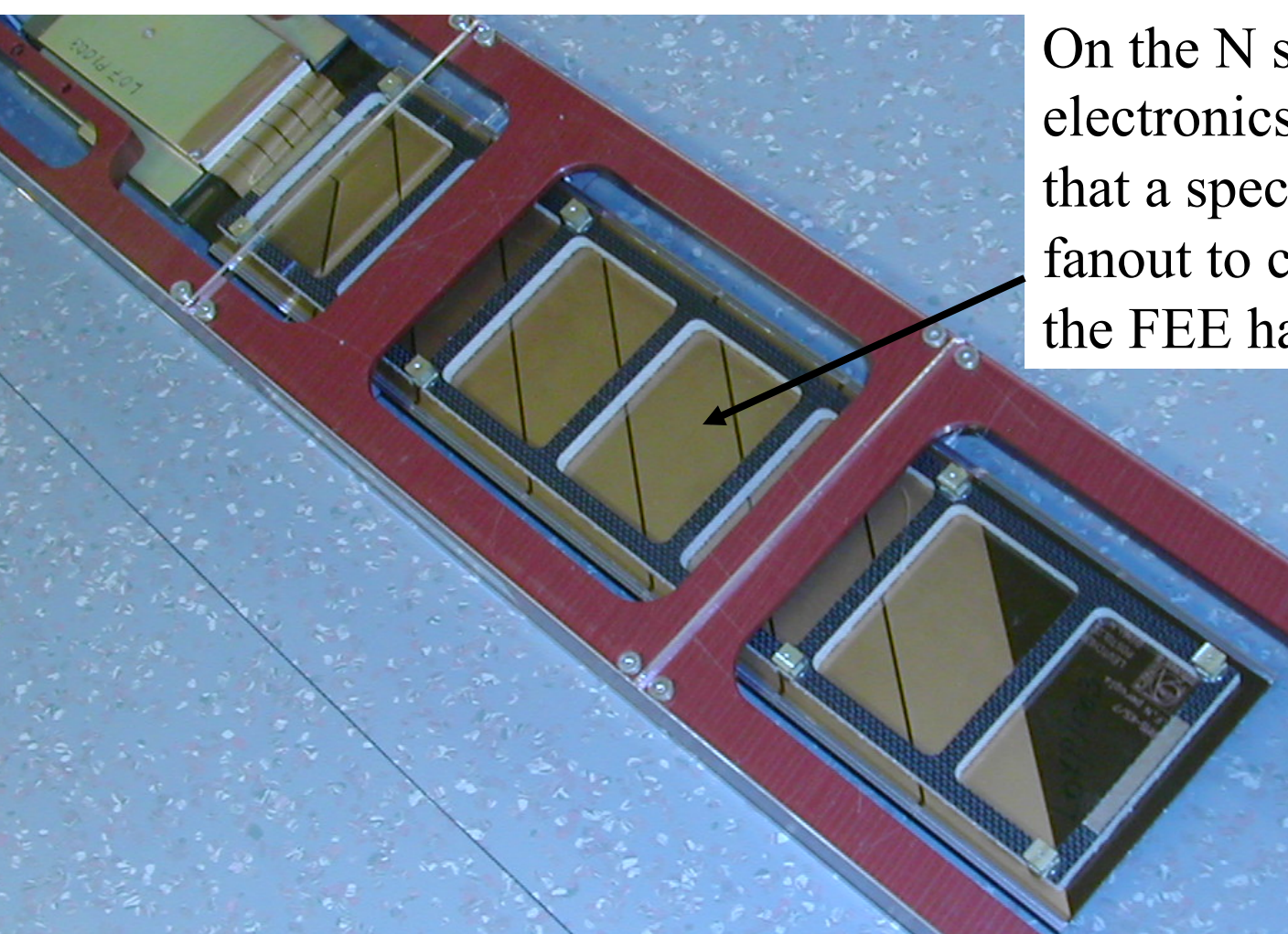
The final precision on alignment is better than 4 μm

The ladder assembly



To connect electrically all the sensors in a ladder, all the strips on the P side are daisy-chained with a 25 μm Al wire through ultrasonic bondings

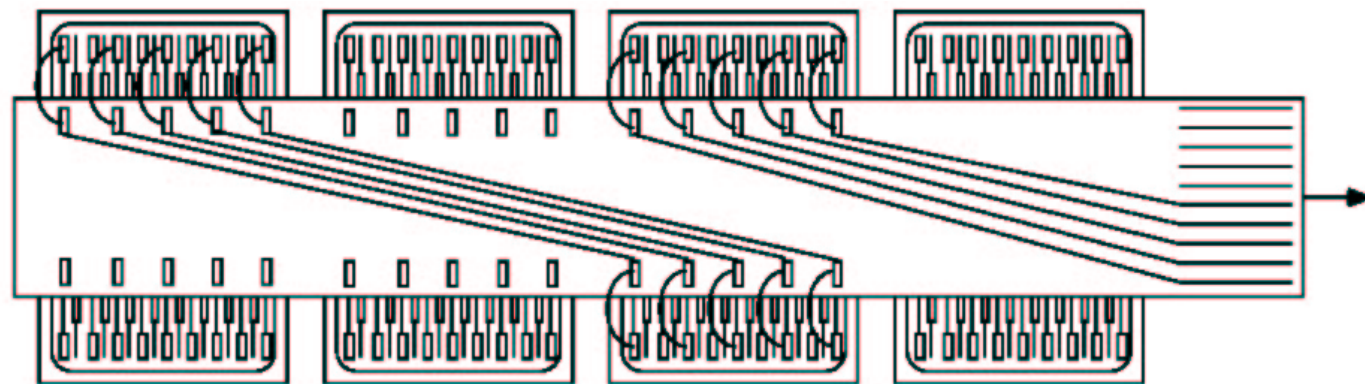


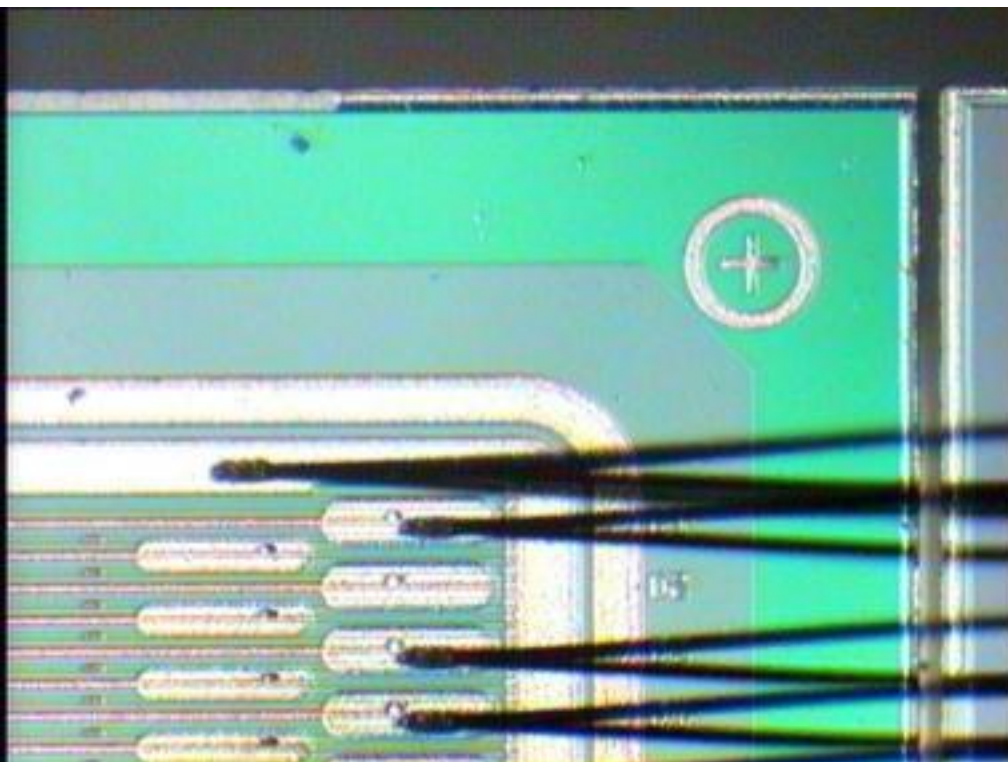


On the N side, the front-end electronics is at one end, so that a special upilex (kapton) fanout to connect the strips to the FEE has been developed

To reduce the number of readout channels, the odd and even strips are connected to same RO channel.

The ambiguity is solved with additional information from TOF

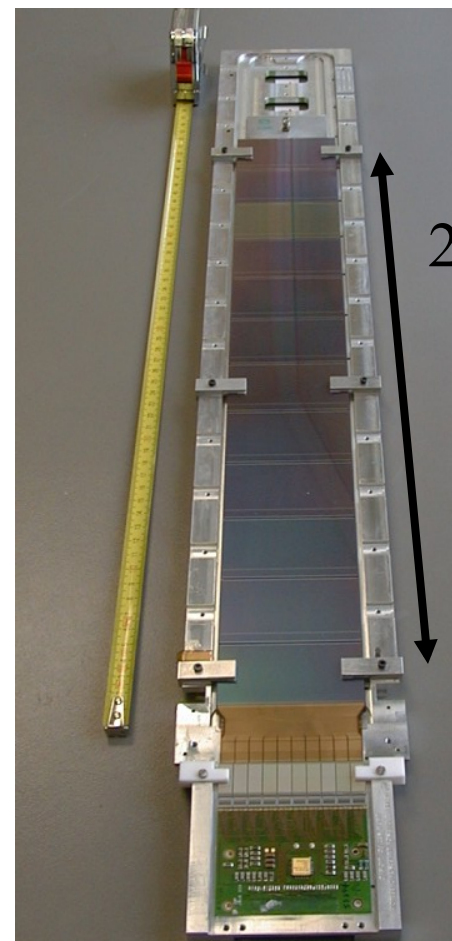
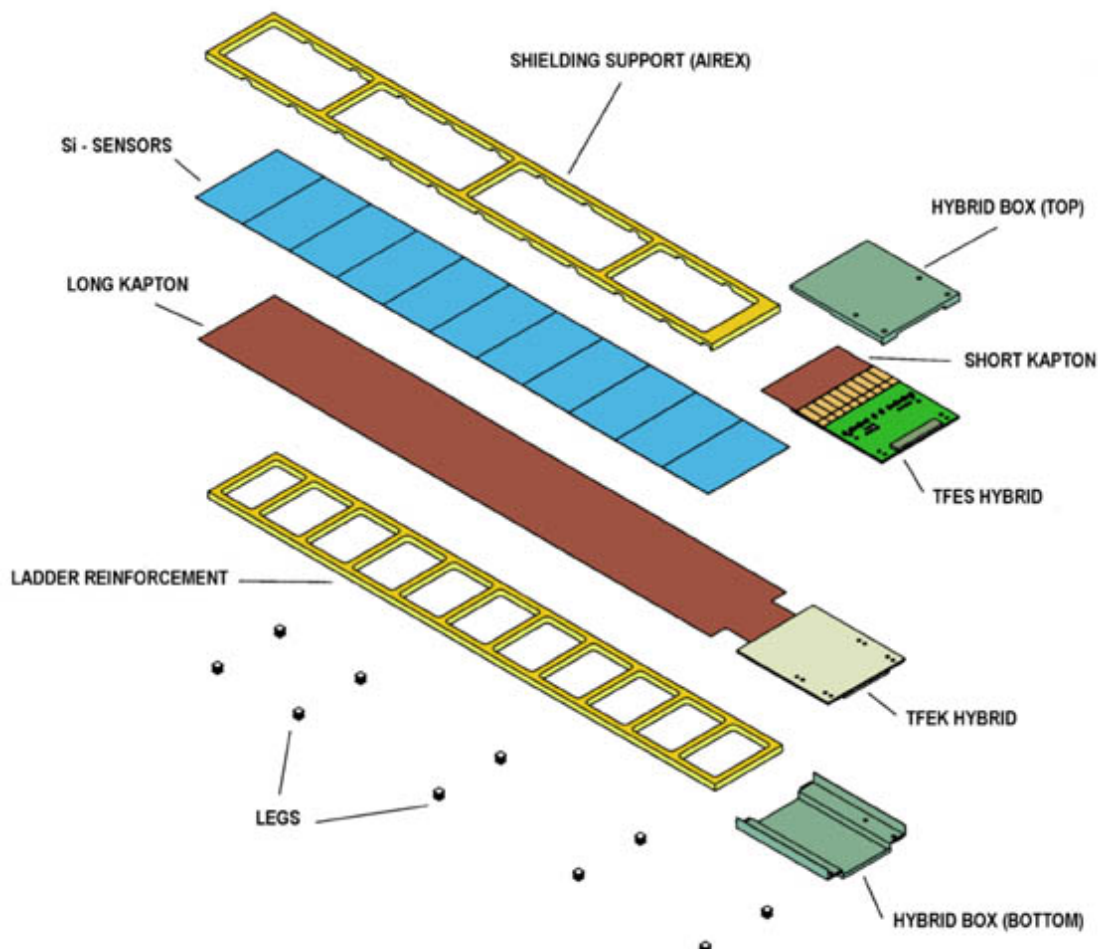




The charge collection is made via the interstrip capacitive charge sharing

To read out all the strips would result in a too much high number of FE electronics channels → the read out pitch is arranged so that a floating strips is left between two readout strips on both sides → On p side the RO pitch is **110 μm → 642 RO chann./ladder**
On N side the RO is **208 μm → 384 RO chann./ladder**
for 196608 RO channels

Silicon Tracker Ladder



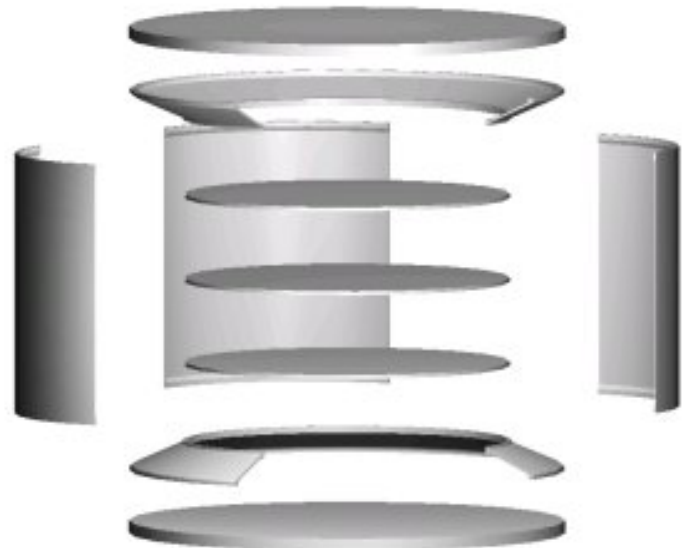
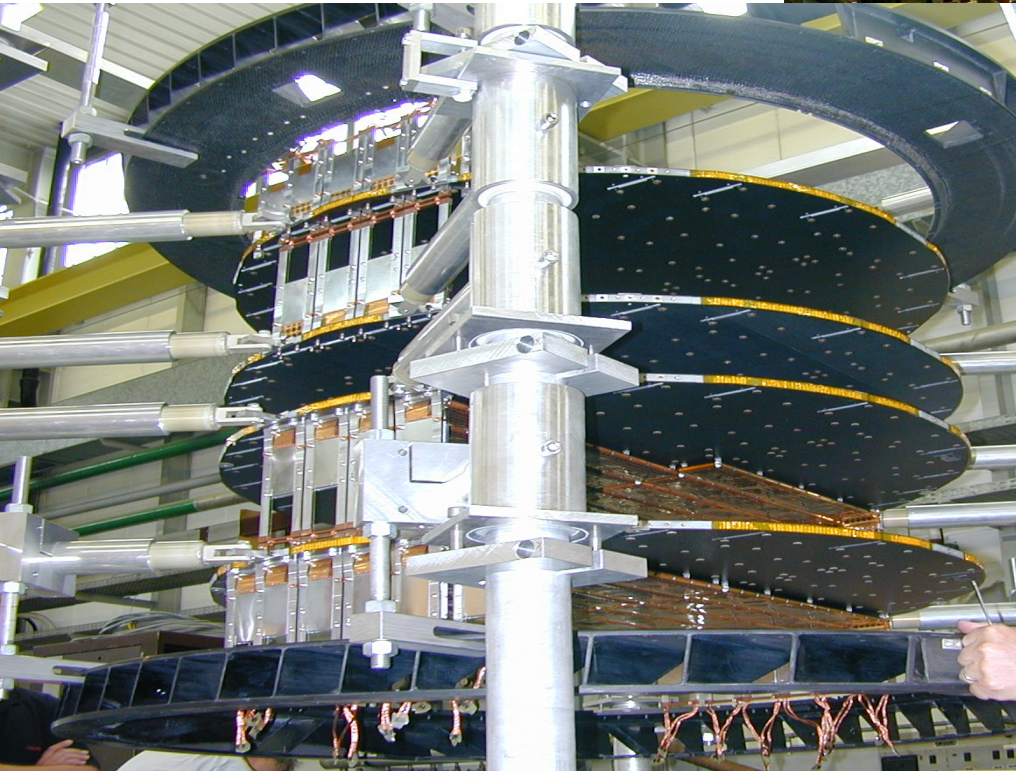
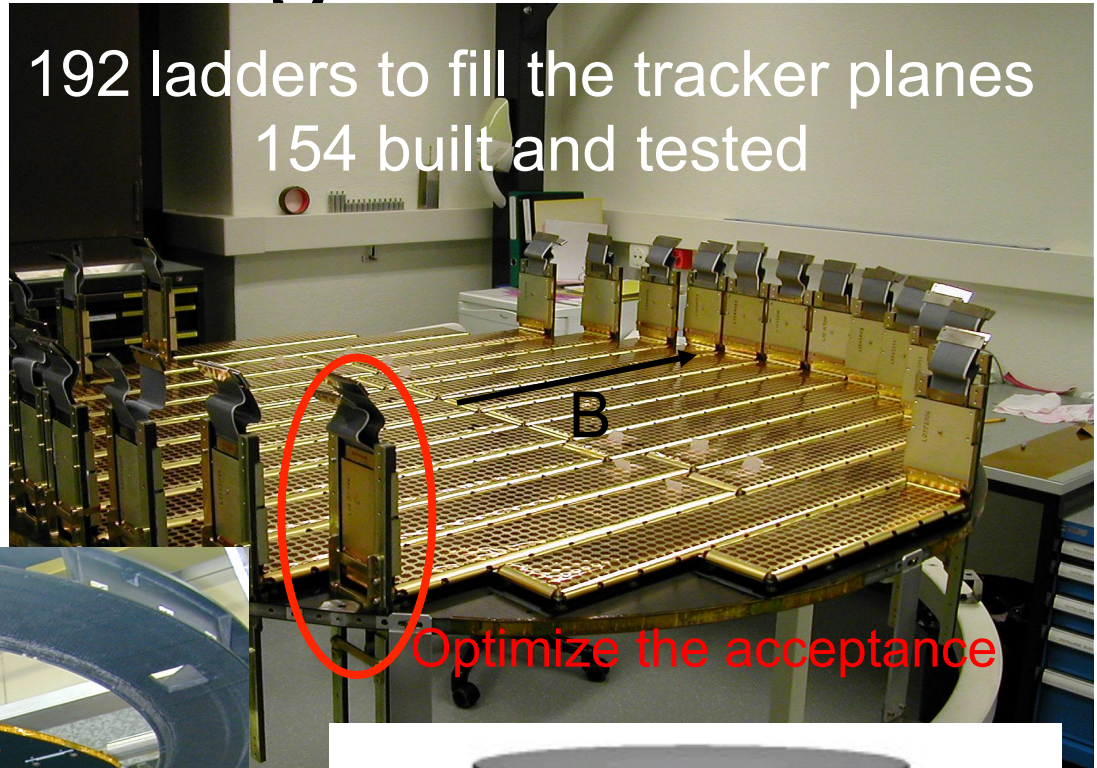
Ladder structural stability is ensured by a carbon fiber foam support structure

To fit the circular magnet bore, the ladders have length from 7 to 15 sensors with 8200- 16500 bonds/ladder

Plane integration

The planes are made a aluminium honeycomb structure enveloped in a carbon fiber foil. This ensures great stability against vibrations and G-forces during the launch

192 ladders to fill the tracker planes
154 built and tested



Rivelatori a stato solido

- ★ **The signal** generated in a silicon detector depends essentially only on the thickness of the depletion zone and on the dE/dx of the particle.
- ★ **The noise** in a silicon detector system depends on various parameters: geometry of the detector, the biasing scheme, the readout electronics, etc.
- ★ Noise is typically given as “equivalent noise charge” ENC. This is the noise at the input of the amplifier in elementary charges.

The Charge Signal

■ **Collected Charge for a Minimum Ionizing Particle (MIP)** $N = (dE/dx)(x/w)$

- **Mean energy loss**

$$dE/dx (\text{Si}) = 3.88 \text{ MeV/cm}$$

⇒ 116 keV for 300 μm thickness

- **Most probable energy loss**

$$\approx 0.7 \times \text{mean}$$

⇒ 81 keV

- **3.6 eV to create an e-h pair**

⇒ 72 e-h / μm (most probable)

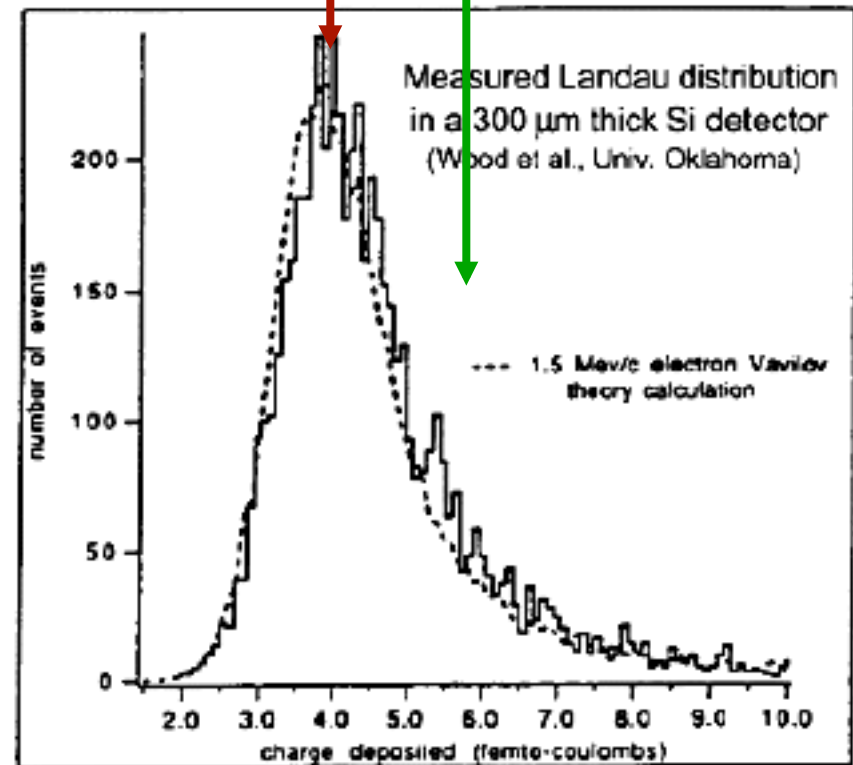
⇒ 108 e-h / μm (mean)

- **Most probable charge (300 μm)**

≈ 22500 e ≈ 3.6 fC

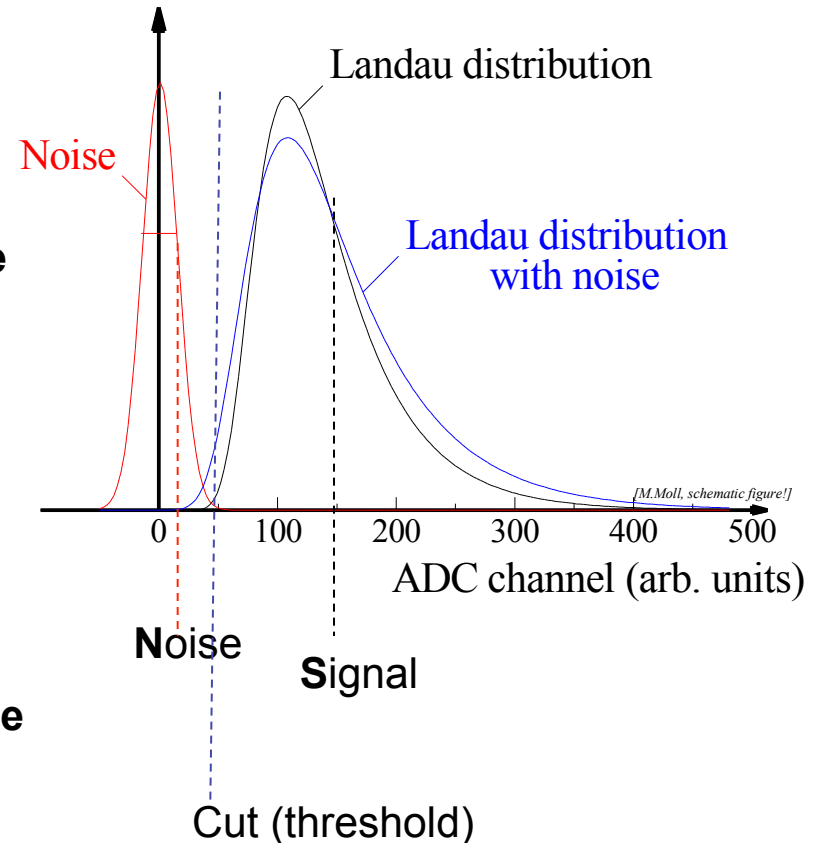
Most probable charge ≈ 0.7 × mean

Mean charge



Signal to noise ratio (S/N)

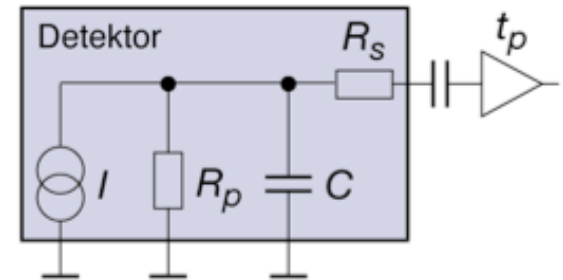
- **Landau distribution** has a low energy tail
 - becomes even lower by noise broadening
- **Good hits selected by requiring $N_{\text{ADC}} > \text{noise}$ tail**
 - If cut too high \Rightarrow efficiency loss
 - If cut too low \Rightarrow noise occupancy
- **Figure of Merit: Signal-to-Noise Ratio S/N**
- **Typical values $>10-15$, people get nervous below 10.**
 - Radiation damage severely degrades the S/N.



Rivelatori a semiconduttore

The most important noise contributions are:

1. Leakage current (ENC_I)
2. Detector capacity (ENC_C)
3. Det. parallel resistor (ENC_{R_p})
4. Det. series resistor (ENC_{R_s})



Alternate circuit diagram of a silicon detector.

The overall noise is the quadratic sum of all contributions:

$$ENC = \sqrt{ENC_C^2 + ENC_I^2 + ENC_{R_p}^2 + ENC_{R_s}^2}$$

Shot noise

- ★ The detector leakage current comes from thermally generated electron holes pairs within the depletion region. These charges are separated by the electric field and generate the leakage current. The fluctuations of this current are the source of noise.

In a typical detector system (good detector quality, no irradiation damage) the leakage current noise is usually negligible.

Assuming an amplifier with an integration time (“peaking time”) t_p followed by a CR-RC filter the noise contribution by the leakage current I can be written as:

$$\text{ENC}_I = \frac{e}{2} \sqrt{\frac{I t_p}{e}}$$

e Euler number (2.718...)
 e ... Electron charge

Using the physical constants, the leakage current in units of nA and the integration time in μs the formula can be simplified to:

$$\text{ENC}_I \approx 107 \sqrt{I t_p} \quad [I \text{ in nA, } t_p \text{ in } \mu\text{s}]$$

To minimize this noise contribution the detector should be of high quality with small leakage current and the integration time should be short.

Capacitance noise

The detector capacity at the input of a charge sensitive amplifier is usually the dominant noise source in the detector system.

This noise term can be written as:

$$\text{ENC}_C = a + b \cdot C$$

The parameter a and b are given by the design of the (pre)-amplifier. C is the detector capacitance at the input of the amplifier channel.

Typical values are (amplifier with $\sim 1 \mu\text{s}$ integration time):

$$a \approx 160 \text{ e und } b \approx 12 \text{ e/pF}$$

To reduce this noise component segmented detectors with short strip or pixel structures are preferred.

Parallel resistor noise

The parallel resistor R_p in the alternate circuit diagram is the bias resistor. The noise term can be written as:

$$\text{ENC}_{R_p} = \frac{e}{e} \sqrt{\frac{kTt_p}{2R_p}}$$

e Euler number (2.718...)
 e ... Electron charge

Assuming a temperature of $T=300\text{K}$, t_p in μs and R_p in $\text{M}\Omega$ the formula can be simplified to:

$$\text{ENC}_{R_p} \approx 772 \sqrt{\frac{t_p}{R_p}} \quad [R_p \text{ in } \text{M}\Omega, t_p \text{ in } \mu\text{s}]$$

To achieve low noise the parallel (bias) resistor should be large!

However the value is limited by the production process and the voltage drop across the resistor (high in irradiated detectors).

Serie resistor noise

The series resistor R_s in the alternate circuit diagram is given by the resistance of the connection between strips and amplifier input (e.g. aluminum readout lines, hybrid connections, etc.). It can be written as:

$$\text{ENC}_{R_s} \approx 0.395 C \sqrt{\frac{R_s}{t_p}}$$

C ... Detector capacity on pF
 t_p ... Integration time in μs
 R_s ... Series resistor in Ω

Note that, in this noise contribution t_p is inverse, hence a long t_p reduces the noise. The detector capacitance is again responsible for larger noise.

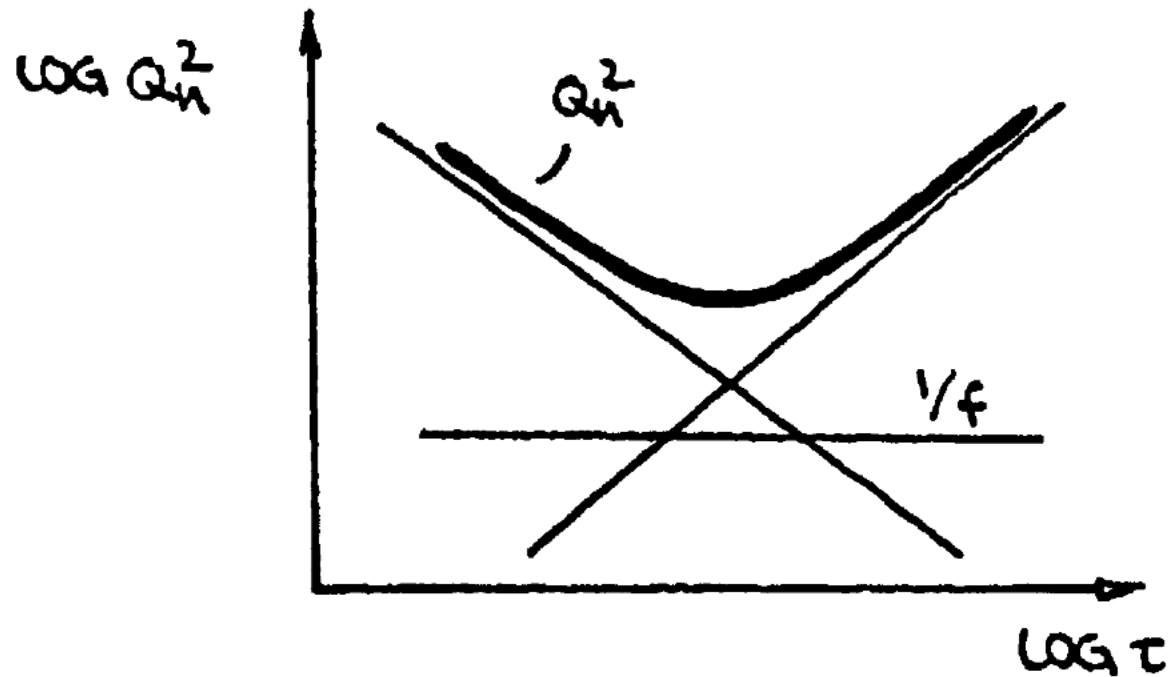
To avoid excess noise the aluminum lines should have low resistance (e.g. thick aluminum layer) and all other connections as short as possible.

Q_n assumes a minimum when the current and voltage noise contributions are equal.

dominated by

voltage

current noise



To achieve a high signal to noise ratio in a silicon detector system the following conditions are important:

- ★ Low detector capacity (i.e. small element size)
- ★ Low leakage current
- ★ Large bias resistor
- ★ Short and low resistance connection to the amplifier
- ★ Usually long integration time

Obviously some of the conditions are contradictory. Detector and front end electronics have to be designed as one system. The optimal design depends on the application.

DELPHI Microvertex:

- ★ readout chip (MX6):
 $a = 325 \text{ e}$, $b = 23 \text{ e/pF}$, $t_p = 1.8 \mu\text{s}$
- ★ 2 detectors in series each 6 cm long strips, $C = 9 \text{ pF}$
→ $\text{ENC}_C = 532 \text{ e}$
- ★ typ. leakage current/strip: $I \approx 0.3 \text{ nA}$
→ $\text{ENC}_I = 78 \text{ e}$
- ★ bias resistor $R_p = 36 \text{ M}\Omega$
→ $\text{ENC}_{R_p} = 169 \text{ e}$
- ★ series resistor $= 25 \Omega$
→ $\text{ENC}_{R_s} = 13 \text{ e}$
- **Total noise: $\text{ENC} = 564 \text{ e}$ (SNR 40:1)**

CMS Tracker:

- ★ readout chip (APV25, deconvolution):
 $a = 400 \text{ e}$, $b = 60 \text{ e/pF}$, $t_p = 50 \text{ ns}$
- ★ 2 detectors in series each 10 cm long strips, $C = 18 \text{ pF}$
→ $\text{ENC}_C = 1480 \text{ e}$
- ★ max. leakage current/strip: $I \approx 100 \text{ nA}$
→ $\text{ENC}_I = 103 \text{ e}$
- ★ bias resistor $R_p = 1.5 \text{ M}\Omega$
→ $\text{ENC}_{R_p} = 60 \text{ e}$
- ★ series resistor $= 50 \Omega$
→ $\text{ENC}_{R_s} = 345 \text{ e}$
- **Total noise: $\text{ENC} = 1524 \text{ e}$ (SNR 15:1)**

Calculated for the signal of a minimum ionizing particle (mip) of 22500 e.

Si detectors: typical noise performance

- Example of noise

- Some typical values for LEP silicon strip modules (OPAL):
 - $ENC = 500 + 15 \cdot C_d$
 - Typical strip capacitance is about 1.5pF/cm, strip length of 18cm so $C_d=27\text{pF}$

so ENC = 900e. Remember S=22500e

$$\Rightarrow S/N \approx 25/1$$

- Some typical values for LHC silicon strip modules

- $ENC = 425 + 64 \cdot C_d$
- Typical strip capacitance is about 1.2pF/cm, strip length of 12cm so $C_d=14\text{pF}$

so ENC = 1300e

$$\Rightarrow S/N \approx 17/1$$

Capacitive term is much worse for LHC in large part due to very fast shaping time needed (bunch crossing of 25ns vs 22μs for LEP)

Risoluzione spaziale

The position resolution – the main parameter of a position detector – depends on various factors, some due to physics constraints and some due to the design of the system (external parameters).

★ Physics processes:

- Statistical fluctuations of the energy loss
- Diffusion of charge carriers

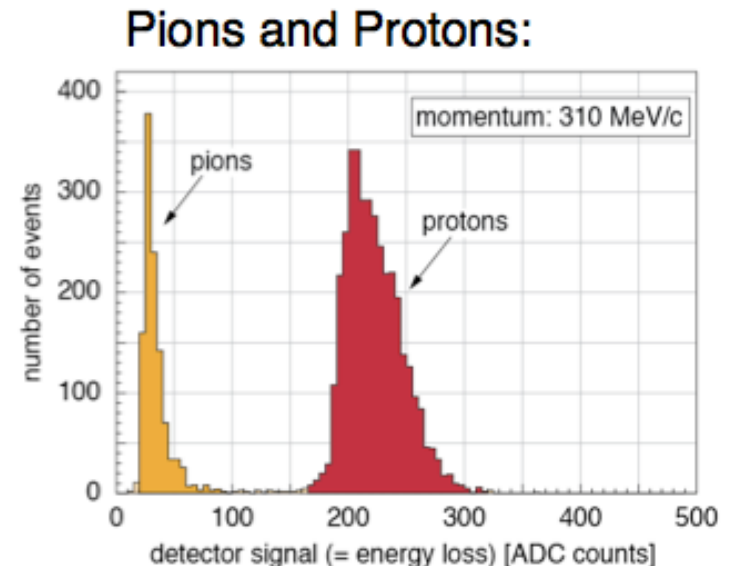
★ External parameter:

- Binary readout (thresh hold counter) or read out of analogue signal value
- Distance between strips (strip pitch)
- Signal to noise ratio

Risoluzione spaziale

- ★ Silicon position detectors are thin (300–500 μm) and absorb only a small fraction of the total energy of through going particles.
- ★ The energy loss dE/dx follows a Landau distribution, an asymmetric probability function with a long “tail” to large energy deposits.
- ★ Example of a mip measured in a 300 μm thick silicon detector:

- Most probable energy loss
(Maximum of the distribution):
78 keV in 300 μm $\rightarrow \approx 72$ e^-h^+ pairs per μm
- Mean of the energy loss:
116 keV in 300 μm $\rightarrow \approx 108$ e^-h^+ pairs per μm



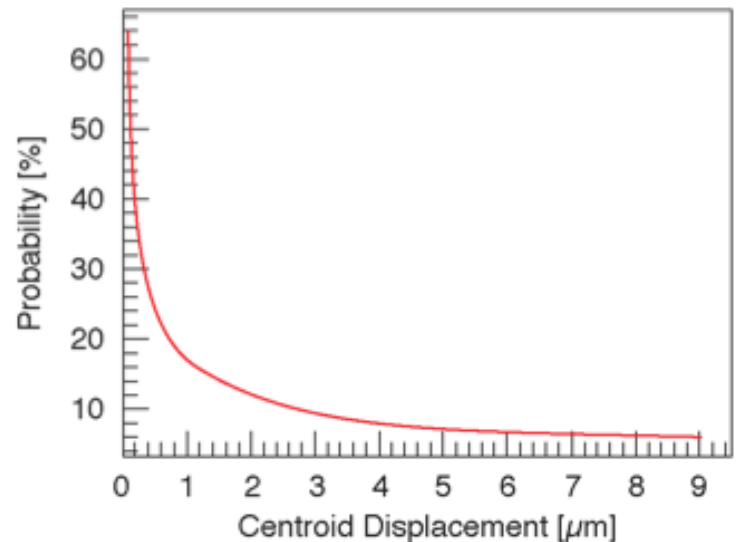
Risoluzione spaziale

Long tail in energy loss distribution is due to δ -electrons.

δ -electrons have a high energy (keV) and are produced by rare, hard collisions between incident particle and electrons from the detector material.

- ★ The probability to produce a δ -electrons is small.
- ★ δ -electrons have a long track length in the detector material and may produce e^+h^- pairs along the track.
- Dislocate the measured track
- Measurement errors in the order of μm unavoidable

Displacement probability (calculation) of the charge center of gravity due to δ -electrons:



A. Peisert, *Silicon Microstrip Detectors*,
DELPHI 92-143 MVX 2, CERN, 1992

Ionization cylinder

Ground

Intrinsic collection time is O(tens of ns)

t=5 ns

Time evolution of current

300 um Silicon layer

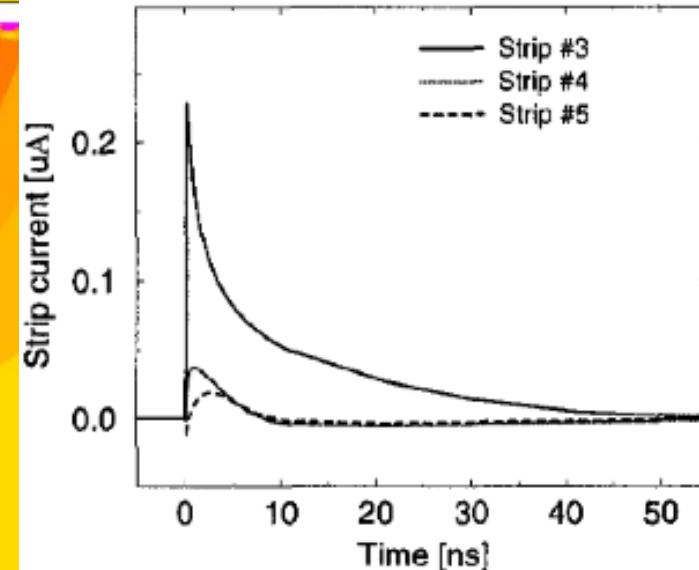
t=0

V_{bias}

Il raggio del cilindro di ionizzazione e' fissato essenzialmente dal range dei raggi δ della ionizzazione secondaria

t=12 ns

t=30 ns



T-j
A/cm2

+4.683e+01
+1.301e-01
+3.613e-04
+1.003e-06
+2.787e-09

T-j
A/cm2

+6.753e+00
+1.774e-02
+4.658e-05
+1.223e-07
+3.213e-10

T-j
A/cm2

+3.014e+00
+1.028e-02
+3.508e-05
+1.197e-07
+4.084e-10

T-j
A/cm2

+6.233e-01
+1.677e-03
+4.510e-06
+1.213e-08
+3.263e-11

Equazione di diffusione

The convection–diffusion equation can be derived in a straightforward way from the continuity equation, which states that the rate of change dn/dt for a scalar quantity in a differential control volume is given by flow and diffusion into and out of that part of the system along with any generation or destruction inside the control volume:

$$\frac{\partial n}{\partial t} + \nabla \cdot \vec{j} = R,$$

where \vec{j} is the total flux and R is a net volumetric source for n . There are two sources of flux in this situation. First, **diffusive flux** arises due to diffusion. This is typically approximated by Fick's first law:

$$\vec{j}_{\text{diffusion}} = -D \nabla n$$

i.e., the flux of the diffusing material (relative to the bulk motion) in any part of the system is proportional to the local concentration gradient. Second, when there is overall convection or flow, there is an associated flux called **advective flux**: $\vec{j}_{\text{advective}} = \vec{v} n$

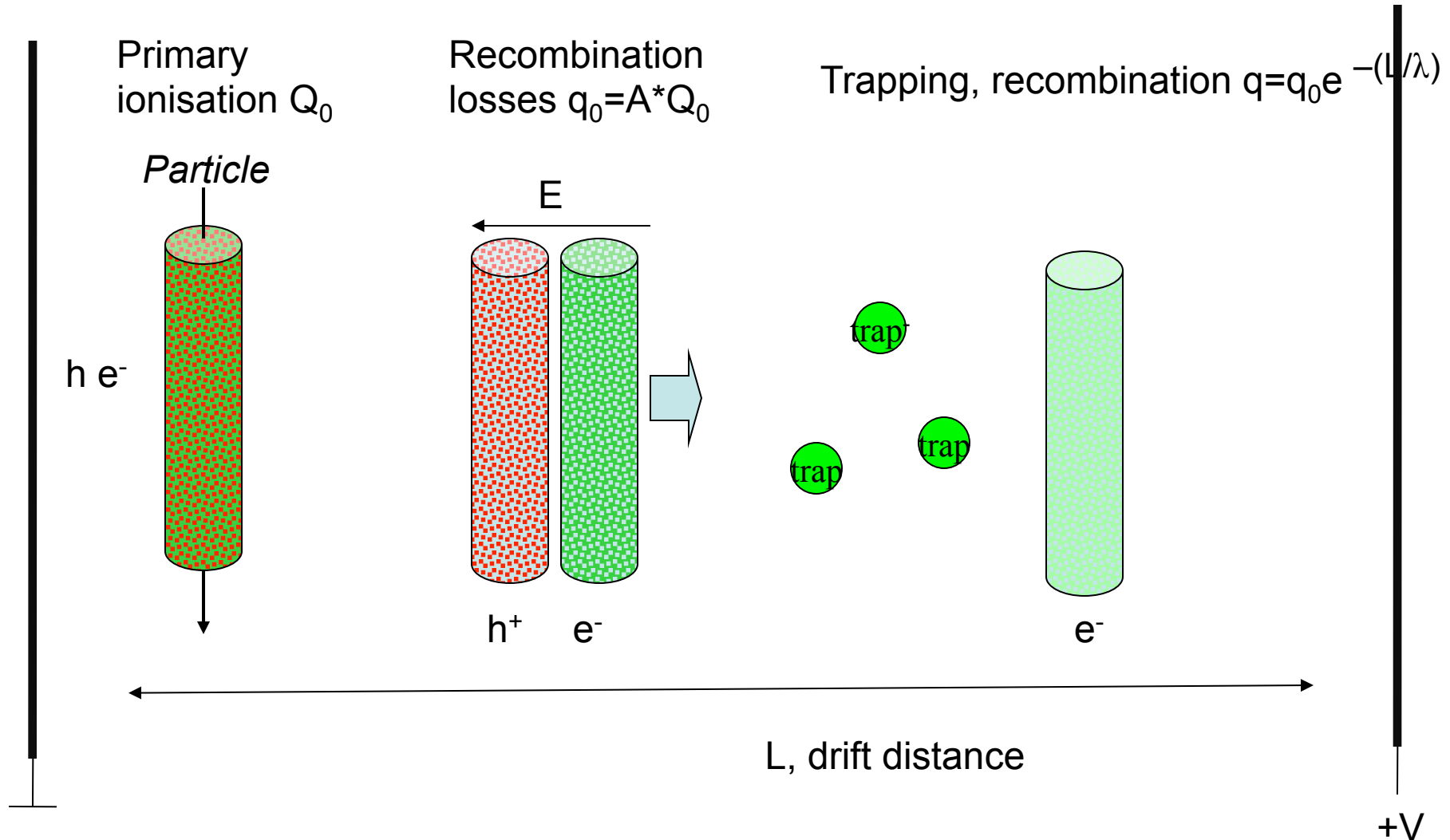
The total flux is given by the sum of these two: $\vec{j} = \vec{j}_{\text{diffusion}} + \vec{j}_{\text{advective}} = -D \nabla n + \vec{v} n$

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

As for gas: initial volume recomb and Q loss during their travel to electrodes

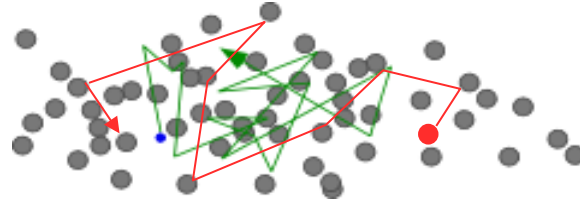
Response to a primary ionization

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

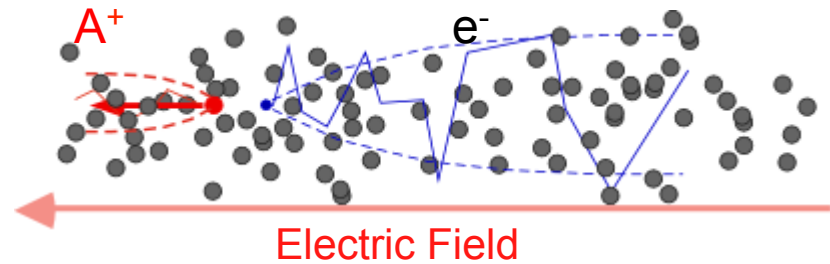


Drift and Diffusion in Presence of E field

$E=0$ thermal diffusion $\langle v \rangle_t = 0$



$E>0$ charge transport and diffusion $\langle v \rangle_t = v_D$



The solution of diffusion-transport eqn is then

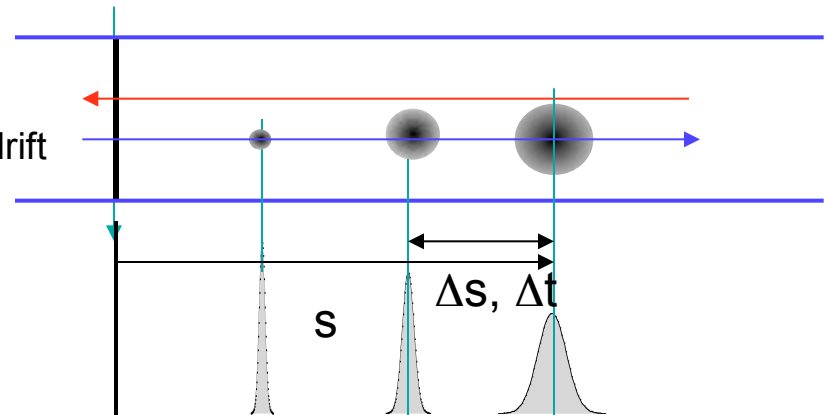
$$N(x,t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{(x-v_D t)^2}{4Dt}}$$

Electron swarm drift

Drift velocity

$$v_D = \mu E$$

Diffusion



drift + diffusion motion: the average position of the charge swarm moves as $x = vt$, while the width increases in time

Charge Collection time and diffusion

■ Charge Collection time

- Drift velocity of charge carriers $v \approx \mu E$, so drift time, $t_d = d/v = d/\mu E$

Typical values: $d=300 \mu\text{m}$, $E= 2.5 \text{ kV/cm}$,
with $\mu_e= 1350 \text{ cm}^2 / \text{V}\cdot\text{s}$ and $\mu_h= 450 \text{ cm}^2 / \text{V}\cdot\text{s}$

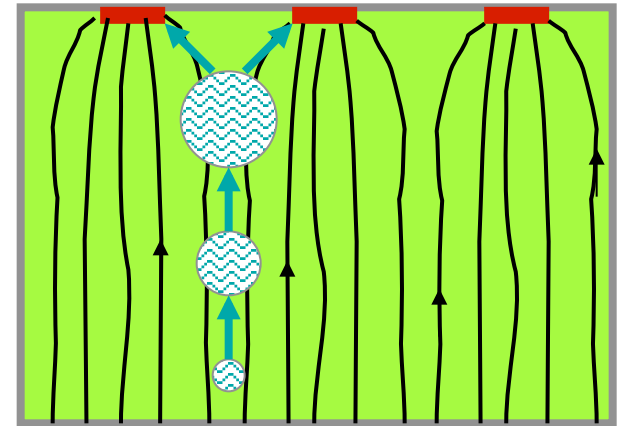
$$\Rightarrow t_d(e)= 9\text{ns} , t_d(h)= 27\text{ns}$$

■ Diffusion

- Diffusion of charge “cloud” caused by scattering of drifting charge carriers, radius of distribution after time t_d :

$$\sigma = \sqrt{2Dt_d} \text{ with diffusion constant } D = \mu kT/q$$

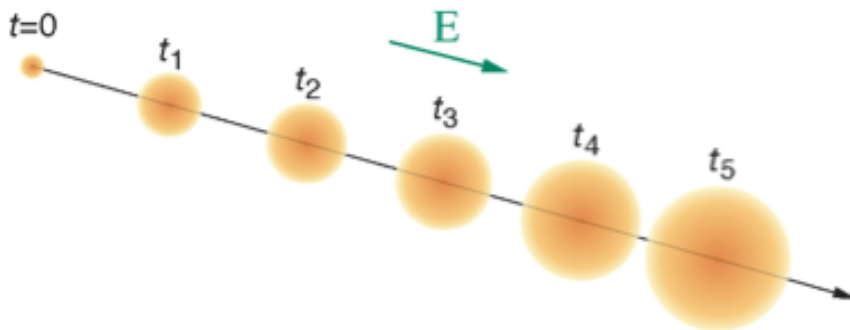
- Same radius for e and h since $t_d \propto 1/\mu$
Typical charge radius: $\sigma \approx 6\mu\text{m}$, could exploit this to get better position resolution due to charge sharing between adjacent strips (using centroid finding), but need to keep drift times long (low field).



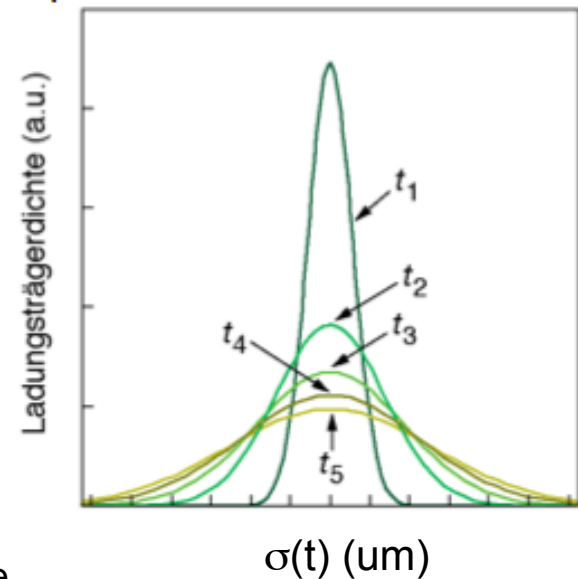
Risoluzione spaziale

- ★ h^+ created close to the anode (i.e. the n^+ backplane) and e^- created close to the cathode (i.e. the p^+ strips or pixels) have the longest drift path. As a consequence the diffusion acts much longer on them compared to $e^- h^+$ with short track paths.
- The signal measured comes from many overlapping Gaussian distributions.

Drift and diffusion acts on charge carriers:



Charge density distribution for 5 equidistant time intervals:



Risoluzione spaziale

- ★ Diffusion widens the charge cloud. However, this has a positive effect on the position resolution!
 - charge is distributed over more than one strip, with interpolation (calculation of the charge center of gravity) a better position measurement is achievable.
- ★ This is only possible if analogue read out of the signal is implemented.
- ★ Interpolation is more precise the larger the signal to noise ratio is.
 - Strip pitch and signal to noise ratio determine the position resolution.
- ★ Larger charge sharing can also be achieved by tilting the detector.

Risoluzione spaziale

★ Threshold readout (one strip signal):

→ position: $x = \text{strip position}$

→ resolution:

$$\sigma_x \approx \frac{p}{\sqrt{12}}$$

p ... distance between strips
(readout pitch)

x ... position of particle track

★ charge center of gravity (signal on two strips):

→ position:

$$x = x_1 + \frac{h_2}{h_1 + h_2} (x_2 - x_1) = \frac{h_1 x_1 + h_2 x_2}{h_1 + h_2}$$

x_1, x_2 ... position of 1st and 2nd strip

h_1, h_2 ... signal on 1st and 2nd strip

→ resolution:

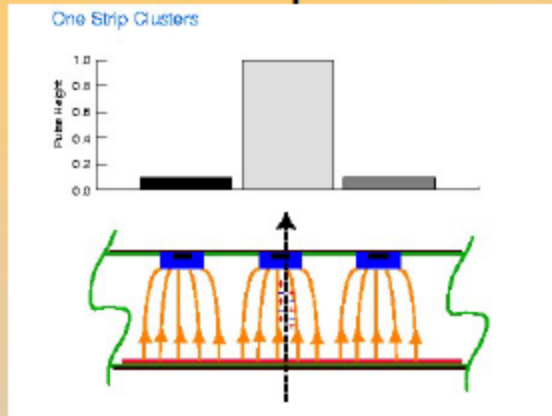
$$\sigma_x \propto \frac{p}{SNR}$$

SNR ... signal to noise ratio

Si strip detectors: spatial resolution

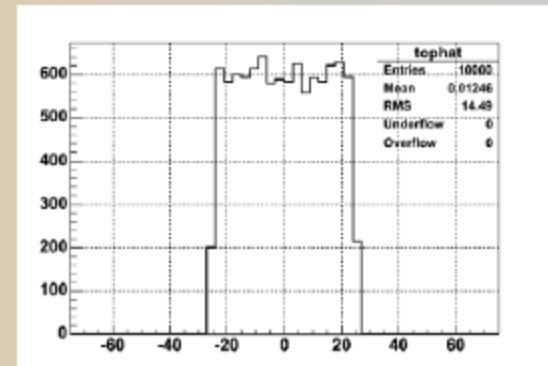
Resolution is the spread of the reconstructed position minus the true position

For one strip clusters

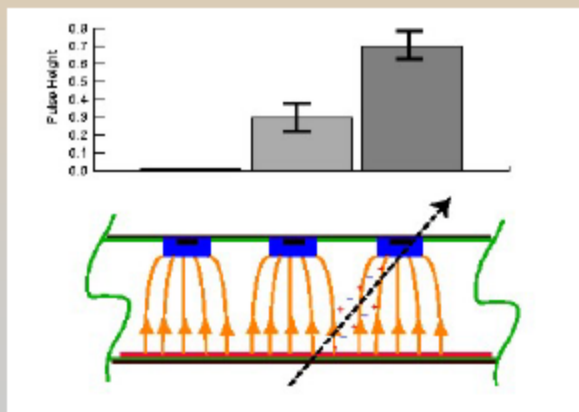


$$\sigma = \frac{\text{pitch}}{\sqrt{12}}$$

"top hat" residuals

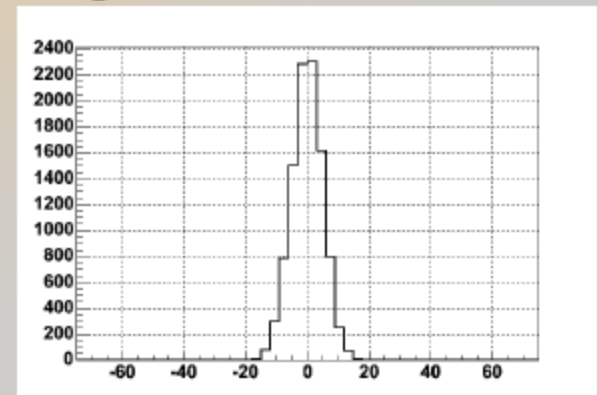


For two strip clusters



$$\sigma \approx \frac{\text{pitch}}{1.5 * (S/N)}$$

"gaussian" residuals



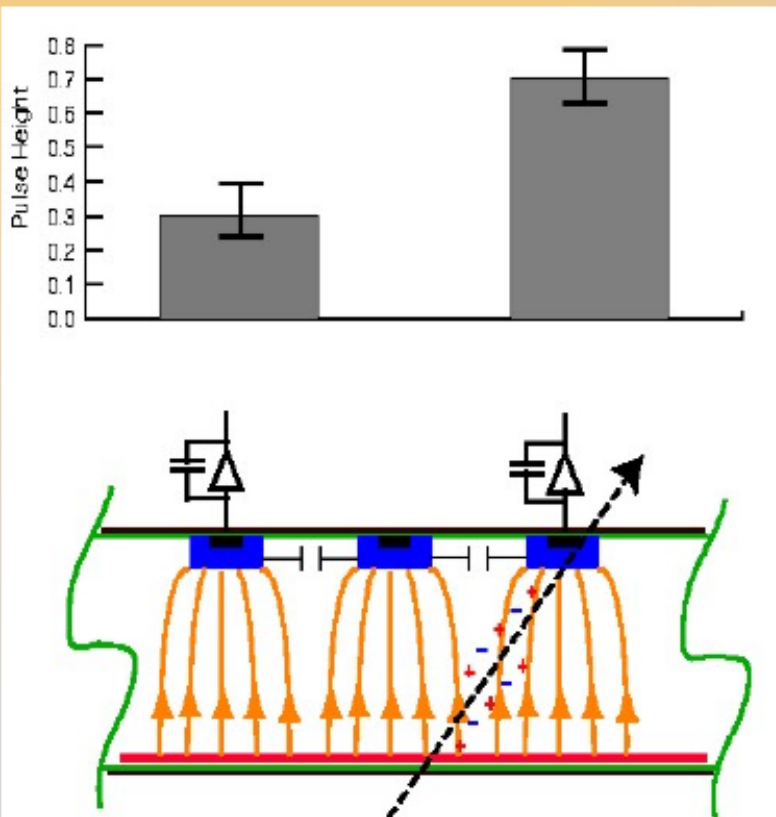
Risoluzione spaziale

- ★ The strip pitch determines to a large extent the position resolution. With small strip pitch a better position resolution is achievable.
But the signal-to-noise ratio plays an essential role
 - small strip pitch requires large number of electronic channels
 - cost increase
 - power dissipation increase

- ★ A possible solution is the implementation of intermediate strips. These are strips not connected to the readout electronics located between readout strips.
The signal from these intermediate strips is transferred by capacitive coupling to the readout strips.
 - more hits with signals on more than one strip
 - Improved resolution with smaller number of readout channels.

Si strip detectors: spatial resolution

Fine pitch is good... but there is a price to pay! \$\$\$\$
The floating strip solution can help



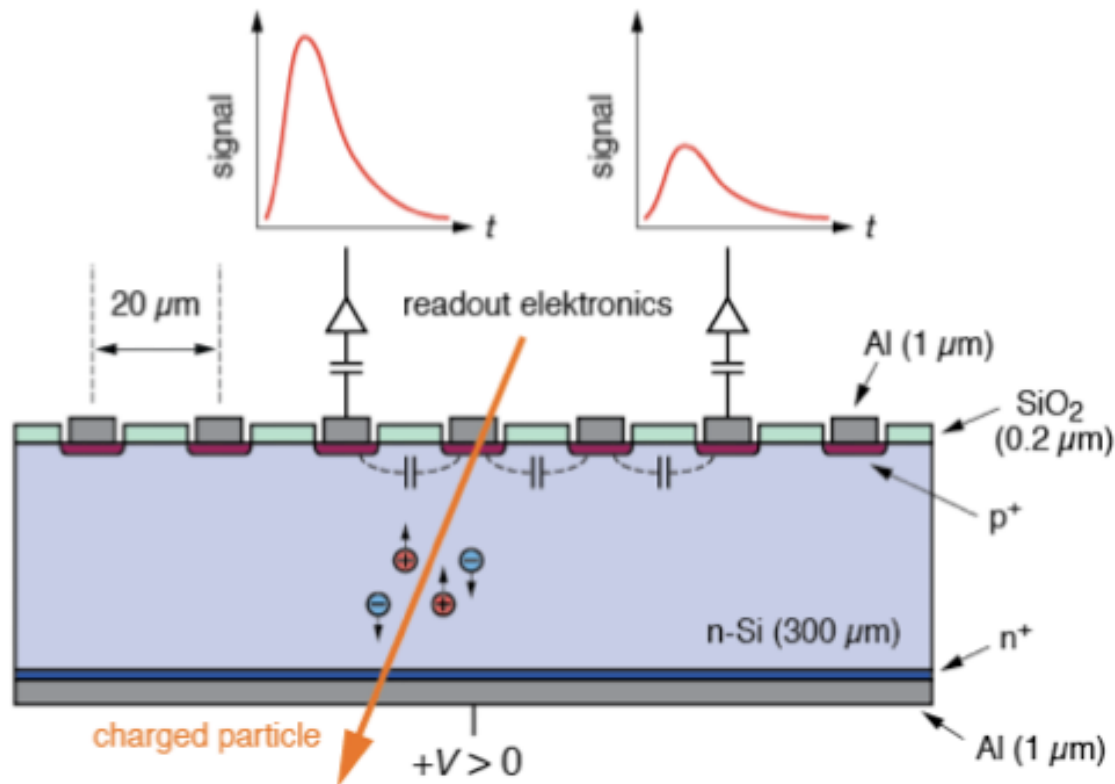
➤ The charge is shared to the neighboring strips via capacitive coupling. We don't have to read out every strip but we still get great resolution

➤ This is a very popular solution. ALEPH for instance obtain $\sigma \approx 12 \mu\text{m}$ using a readout pitch of $100 \mu\text{m}$ and an implant pitch of $25 \mu\text{m}$

➤ But you can't have everything for nothing! You can lose charge from the floating strips to the backplane, so you must start with a good signal to noise

Risoluzione spaziale

Scheme of a detector with two intermediate strips. Only every 3rd strip is connected to an electronics channel. The charge from the intermediate strips is capacitive coupled to the neighbor strips.

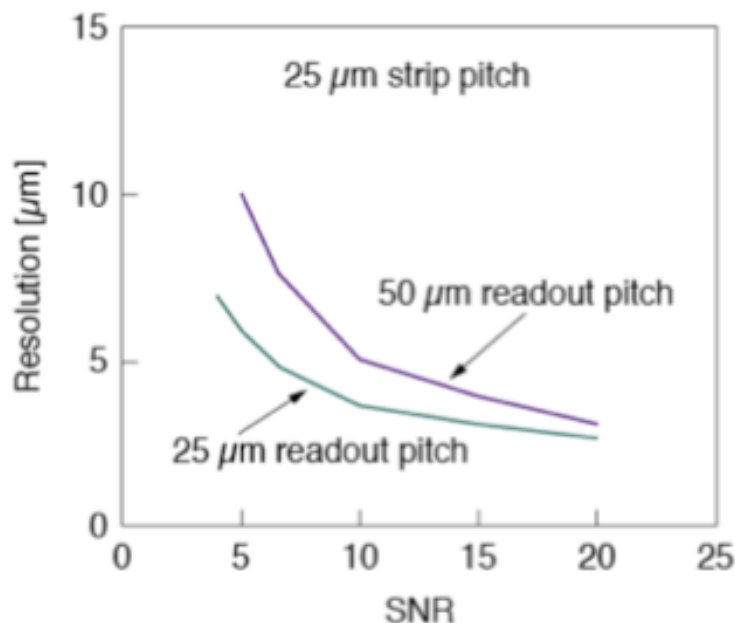


Risoluzione spaziale

Example of a detector with strip pitch of $25\ \mu\text{m}$ and analogue readout. The position resolution is plotted as a function of the SNR.

Bottom curve: every strip is connected to the readout electronics

Top curve: every 2nd strip is connected, one intermediate strip



To benefit from intermediate strips large SNR is required!

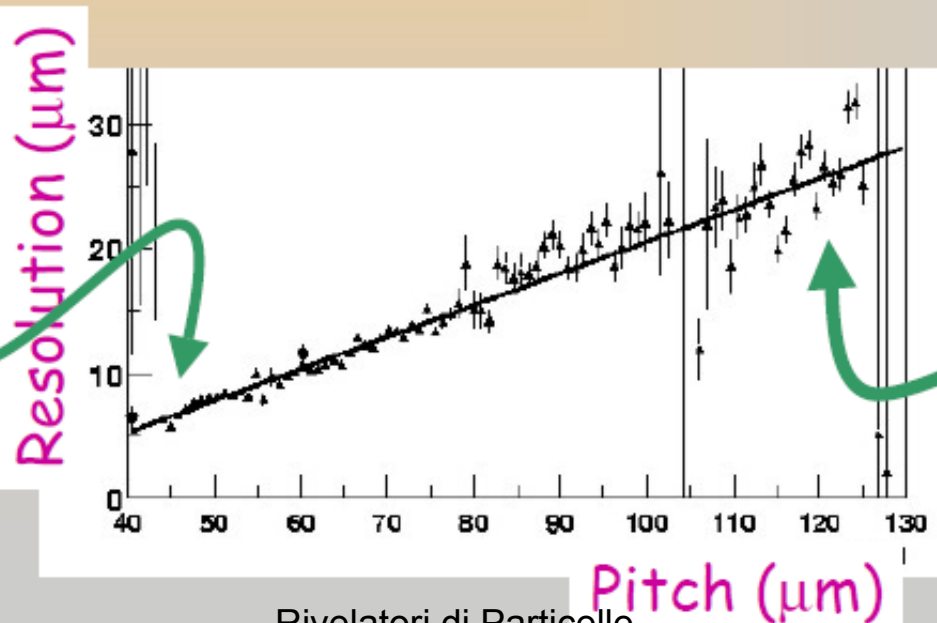
Si strip detectors: spatial resolution

In real life, position resolution is degraded by many factors

- relationship of strip pitch and diffusion width (typically 25-150 μm and 5-10 μm)
- Statistical fluctuations on the energy deposition

Typical real life values for a 300 μm thick sensor with $S/N=20$

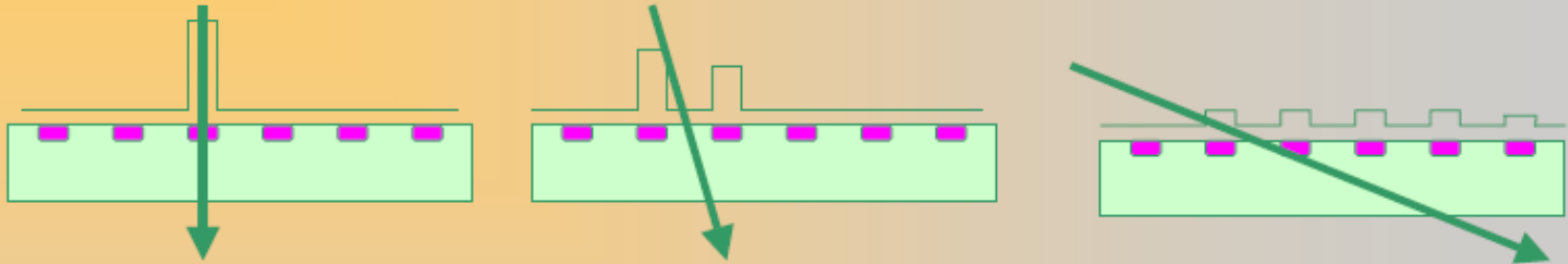
Here charge sharing dominates



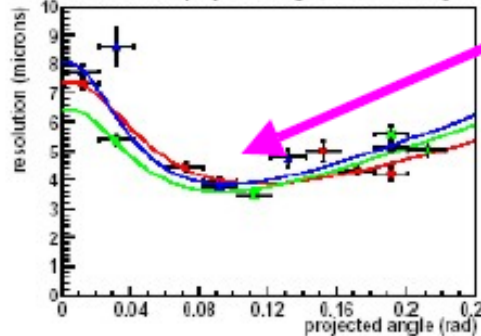
Here single strips dominate

Si strip detectors: spatial resolution

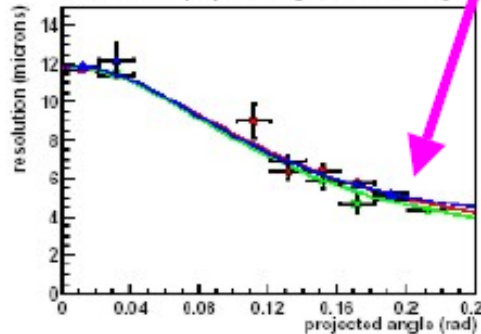
There is also a strong dependence on the track incidence angle



resolution vs projected angle, 40 micron region



resolution vs projected angle, 60 micron region



At small angles you win

At large angles you lose
(but a good clustering
algorithm can help)

Optimum is at

$$\tan^{-1} \frac{\text{pitch}}{\text{width}}$$

