Particle Detectors

Lecture 10
18/04/18

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Pair Production

The total pair production cross section is obtained integrating over the the energy fraction

- In the Born approximation (which is not very accurate for low energy or high Z) one finds

No screening $(\xi >> 1)$ and $m_e c^2 << h\nu << 137 m_e c^2 Z^{-1/3}$

$$\sigma_{pair} = 4Z^2 \alpha r_e^2 \left[\frac{7}{9} \left(\ln \frac{2h\nu}{m_e c^2} \right) - \frac{109}{54} \right]$$

NB: ξ = screening parameter

Complete screening $(\xi \to 0)$ and $h\nu >> 137 m_e c^2 Z^{-1/3}$

$$\sigma_{pair} = 4Z^2 \alpha r_e^2 \left[\frac{7}{9} \left(\ln 183 Z^{-1/3} \right) \right] - \frac{1}{54}$$

ie high energy

Interazioni dei γ

- In caso di schermatura completa ($\Gamma \sim 0$) la σ espressa in funzione dell'energia cinetica del positrone e normalizzata a $\sigma_o = \alpha Z^2 r_e^2$ è quasi uniforme → I positroni sono prodotti con tutte le energie possibili → gli e^+ ed e^- della coppia non hanno la stessa energia.
- La σ dipende dal materiale come Z^2, $\sigma_o = \alpha Z^2 r_e^2 \sim (10^{-4}\ \text{barn}) \times Z^2$
- Ad alta energia e^+ ed e^- tendono ad essere prodotti a piccolo angolo rispetto alla direzione di volo del γ.

$$<\theta> = mc^2/E \qquad (E = E_+ \text{ od } E_-)$$

Interazioni dei γ

Trascurando 1/54 ottengo: $$\sigma_{pair} \approx 4\alpha r_e^2 Z^2 \cdot \left[\frac{7}{9}\ln\frac{183}{Z^{1/3}}\right]$$

D'altra parte, la lunghezza di radiazione e' (cfr. bremsstrahlung)

$$L_0 = \frac{A}{4\alpha N_A \rho Z_2^2 r_e^2 \ln\frac{183}{Z_2^{1/3}}} \qquad (cm)$$

$$X_0 = L_0\rho = \frac{A}{4\alpha N_A Z_2^2 r_e^2 \ln\frac{183}{Z_2^{1/3}}} \qquad (g/cm^2)$$

Percio' $$\sigma_{pair} = 4\alpha r_e^2 Z^2 \frac{N_A}{A}\left[\frac{7}{9}\ln\frac{183}{Z^{1/3}}\right] = \frac{7}{9}\frac{A}{N_A}\frac{1}{X_0}$$

→La sezione d'urto e' inversamente proporzionale alla lunghezza di radiazione **espressa in g/cm²**

NB: il numero atomico A e' espresso in kg/mol!

Cfr. lunghezza di radiazione

Ricordiamo che la perdita di energia per un e- che attraversa un mezzo con n_a atomi/cm^3, cioe' con $n_a = \rho N_A/A$, nel caso di schermatura completa:

$$-\left.\frac{dE}{dx}\right|_{Br} = \int_0^{k_{\max}} k n_a \cdot \frac{d\sigma}{dk} dk = 4\rho N_o \frac{Z_2^2}{A} r_e^2 \alpha E_i \ln\frac{183}{Z_2^{1/3}} \qquad \text{ovvero}$$

$$-\frac{dE}{dx} = \frac{E}{L_0}$$

con

$$L_0 = \frac{A}{4\alpha N_o \rho Z_2^2 r_e^2 \ln\frac{183}{Z_2^{1/3}}} \qquad (cm)$$

n_a =numero di atomi per cm^3.

N_0 è il numero di Avogadro

A è il peso atomico

r è la densità del materiale

$X_0 = L_0\rho$ e' la lunghezza di radiazione espressa in g/cm^2

X_0 e' inversamente proporzionale a Z^2 e NON dipende dalla densita' se espresso in g/cm^2

Interazioni dei γ

- Per definizione, il n.ro di particelle –dN rimosse da un fascio a causa di un processo con sez. d'urto σ in un tratto dx e':
- $-dN(x) = N(x)n\sigma dx = N(N_A\rho\sigma/A)dx = N(N_A\sigma/A)dX$ con $dX = \rho dx$, cioe' espresso in g/cm^2.
- La quantita' $\lambda = A/N_A\sigma$ e' la lunghezza di attenuazione

ed e' tale che $N(X) = N(0)e^{-X/\lambda}$

λ dipende dalla sezione d'urto del processo in gioco:

$\sigma = \sigma_T = \sigma_{ph} + \sigma_{Comp} + \sigma_{pair}$ e quindi dall'energia del fotone.

NB: λ e' misurata in g/cm^2!

γ interactions

Photon pair conversion probability

Cross-section independent of photon energy (once well above threshold), ~ Z^2

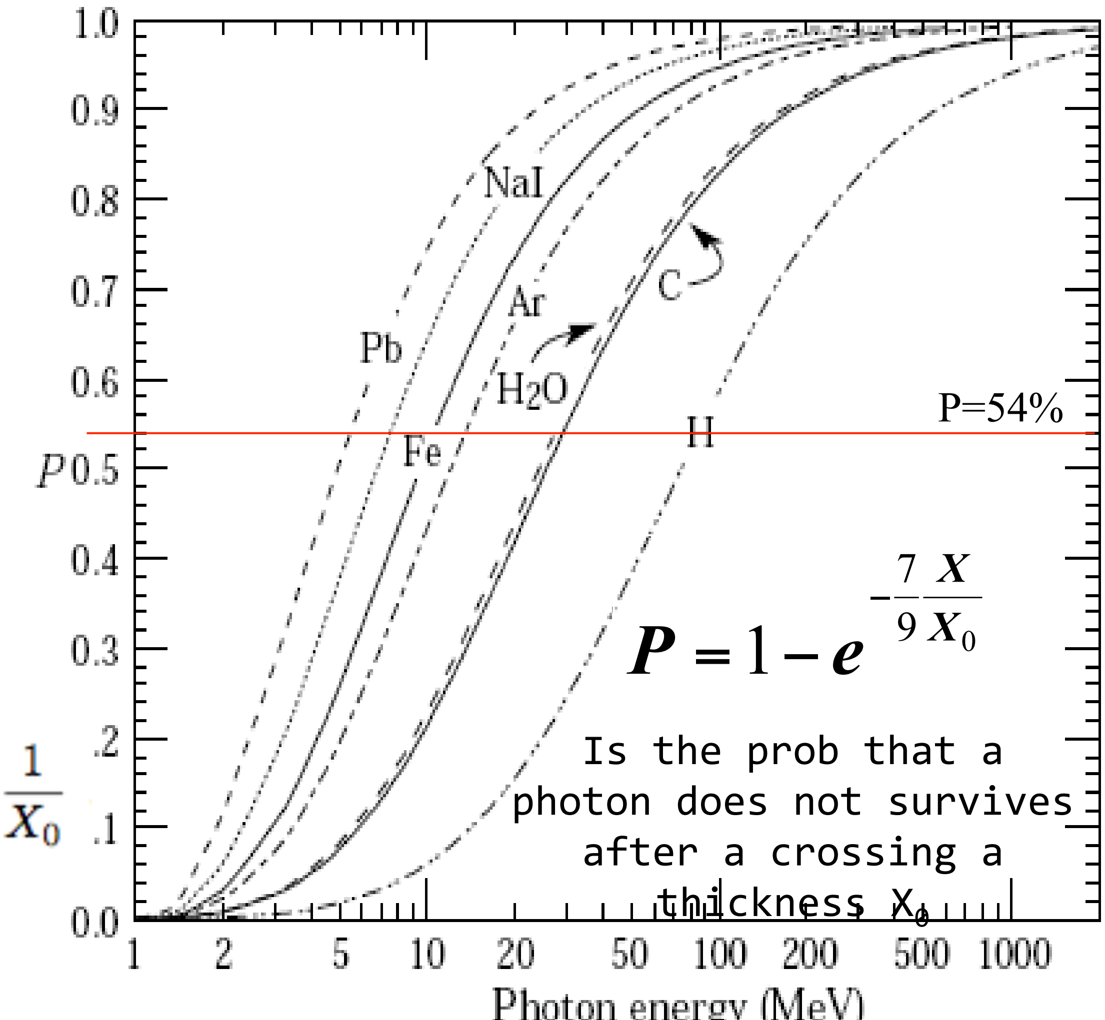
$$\sigma^n_{pair} \approx r_e^2 4\alpha Z^2 \left[\frac{7}{9}\ln\left(\frac{183}{Z^{1/3}}\right)\right]$$

$$\sigma_{\rm pair} \approx \frac{7}{9} 4\alpha\, r_e^2 Z^2 \ln\frac{183}{Z^{1/3}} \approx \frac{7}{9}\cdot\frac{A}{N_A}\cdot\frac{1}{X_0}$$

→ The attenuation length is

$\lambda = A/N_a\sigma$ con $\sigma = \sigma_{pair}$

$\lambda = (9/7)X_0$



$$P = 1 - e^{-\frac{7}{9}\frac{X}{X_0}}$$

Is the prob that a photon does not survives after a crossing a thickness X_0

http://pdg.lbl.gov

Interazioni dei γ

- A causa del bremsstrahlung, un fascio di $e^{\pm}$ di energia iniziale E_0, dopo un tratto x di materiale ha un'energia:

$$E = E_0 e^{-x/X_0}$$

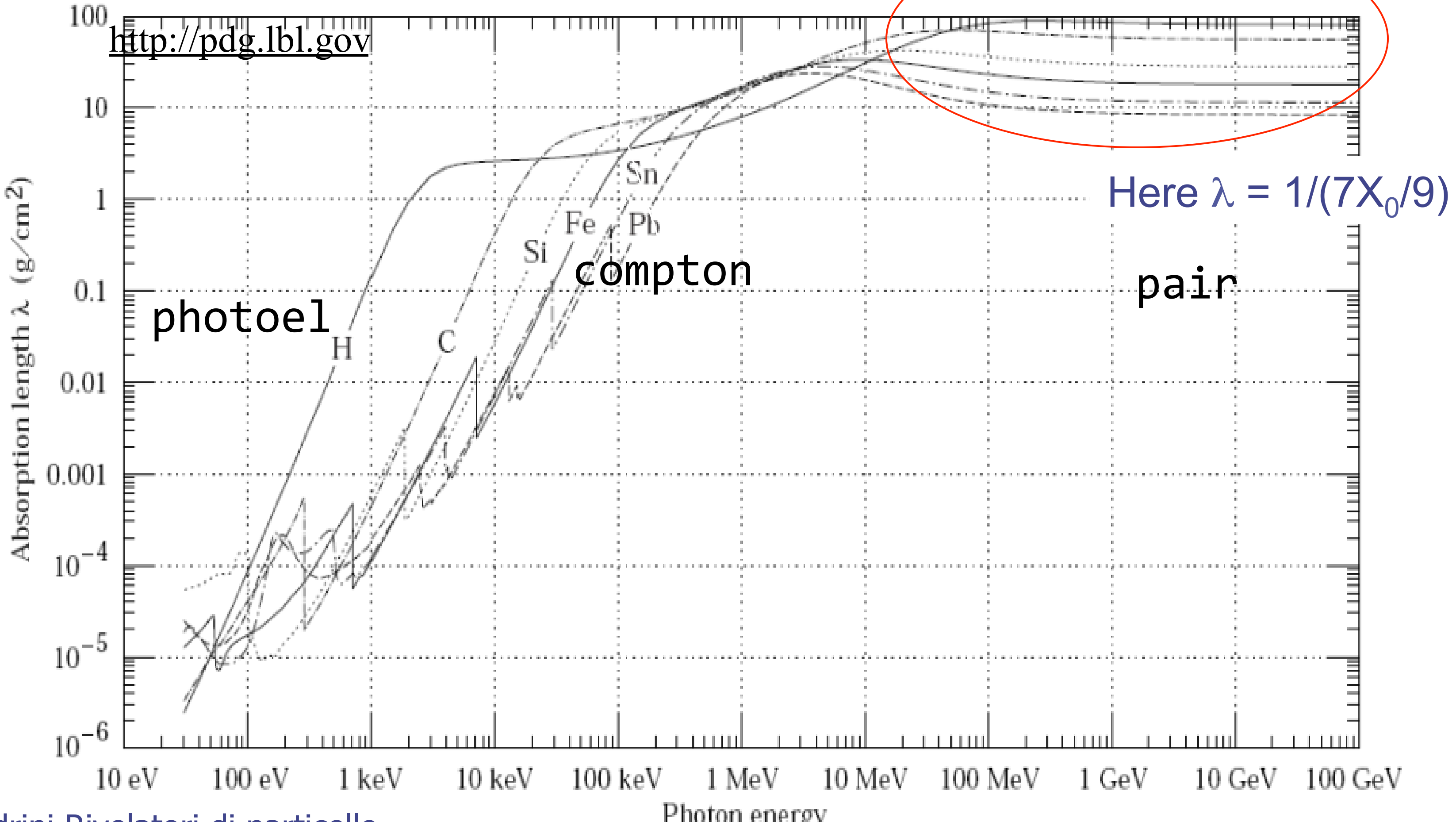
- In un mezzo omogeneo di lunghezza di radiazione X_0, a causa della produzione di coppie, l'intensità di un fascio monocromatico di γ, diminuisce dopo un tratto x di materiale:

$$I = I_0 e^{-(7/9)x/X_0}$$

NB: nel primo caso il numero di e- e' costante, cambia l'energia delle particelle, non il loro numero. Nel secondo, l'energia dei fotoni rimane la stessa ma scompaiono particelle

Photon absorbtion lengths

Photon attenuation length for different elemental absorbers **versus photon energy**



Pair Production

- Notes
  - $\sigma_{pair} \sim Z^2$
  - Above some photon energy (say > 1 GeV), σ_{pair} becomes a constant
  - In order to account for pair production from the Coulomb field of atomic electrons, Z^2 is replaced by $Z(Z+1)$ approximately since the cross section is smaller by a factor of Z
  - Usually we don't distinguish between the source of the field

Pair Production

- Notes
  - In the case of the nuclear field and for large photon energies, the mean scattering angle of the electron and positron is

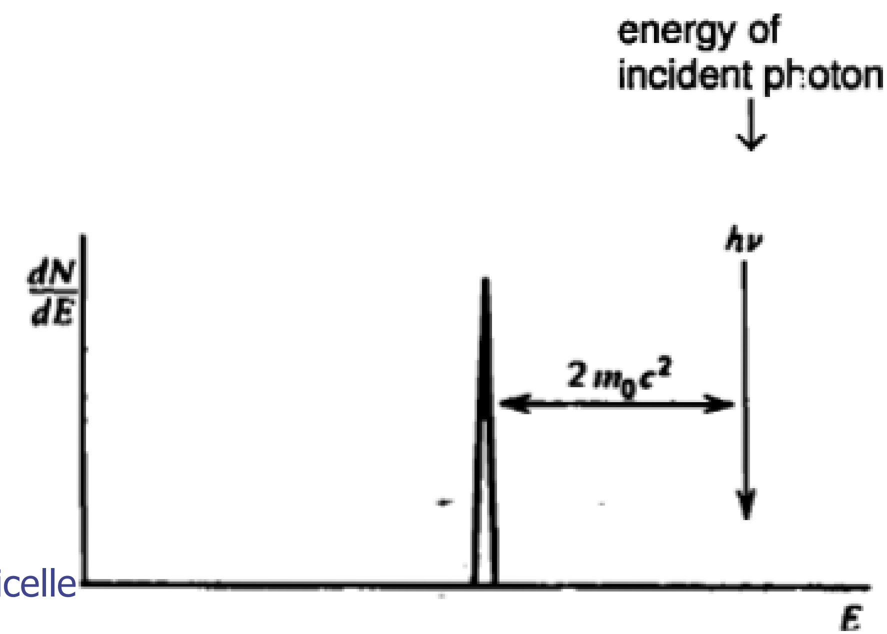
$$\bar{\theta} \approx \frac{m_e c^2}{\bar{T}}$$

$$\bar{T} = \frac{h\nu - 1.022}{2}$$

$$\text{For } h\nu = 5MeV \rightarrow \bar{T} \approx 2MeV \text{ and } \bar{\theta} \approx 15^\circ$$

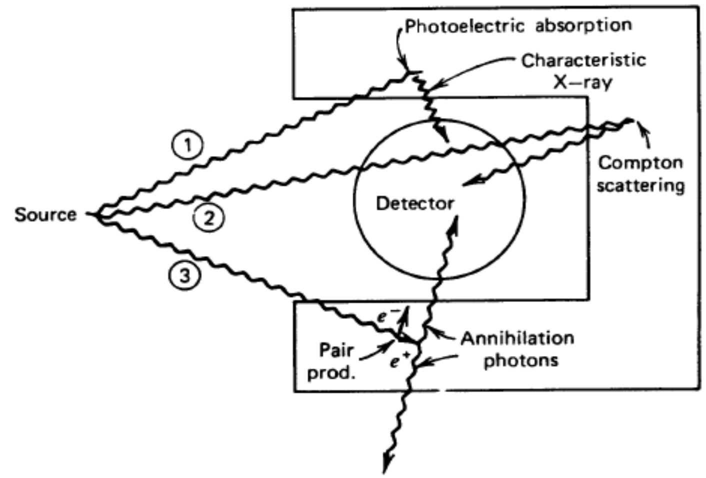
Pair Production

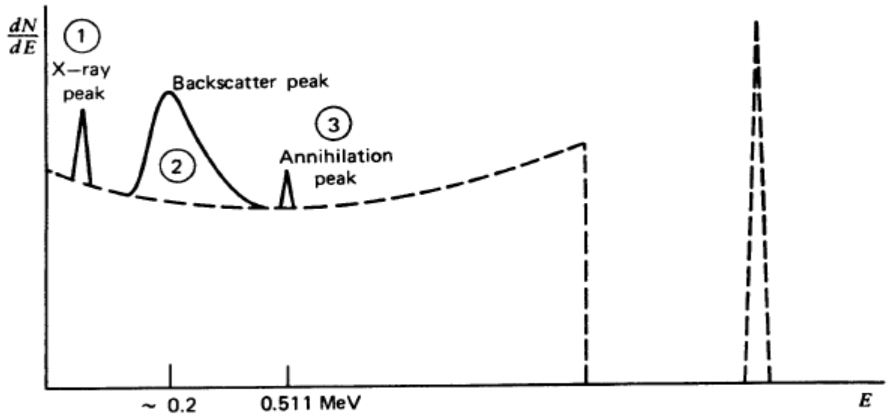
- $2m_e$ (1.022 MeV) of the photon's energy goes into creating the electron and positron
- The electron will typically be absorbed in a detector
- The positron will typically annihilate with an electron producing two annihilation photons of energy m_e (0.511 MeV) each
- If these photons are not absorbed in the detector then the pair production energy spectrum will look like



Scattering

d) Effect of Surrounding Material





(1) x-ray from photoelectric absorption in external material

(2) Compton backscatter peak (see next page)

(3) Absorption of one 511 keV annihilation photon from pair-production in external material.

Interazioni dei γ

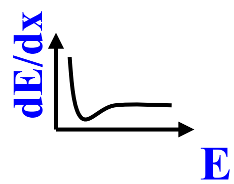
Sia l'effetto fotoelettrico, che quello Compton e la produzione di coppie sono dei processi **distruttivi.**

- **Effetto fotoelettrico:** assorbe γ ed emette e^- (riveliamo l'e-)
- **Effetto Compton:** cambia la ν dei γ (trasferimento di energia al mezzo) (riveliamo l'e- di rinculo)
- **Produzione di coppie:** trasforma i γ in e^+ ed e^- (riveliamo e+ ed e-)

Interactions of Particles with Matter

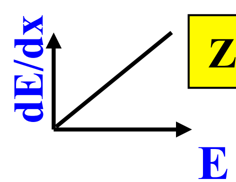
charged	γ

- Ionisation



Z

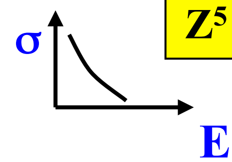
- Bremsstrahlung



$Z(Z+1)$

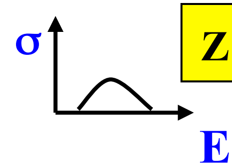
+ cerenkov and transition radiation ($\sim Z^2$)

- P.e. effect



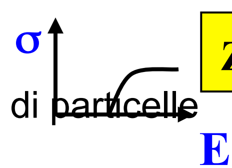
Z^5

- Comp. effect



Z

- Pair production

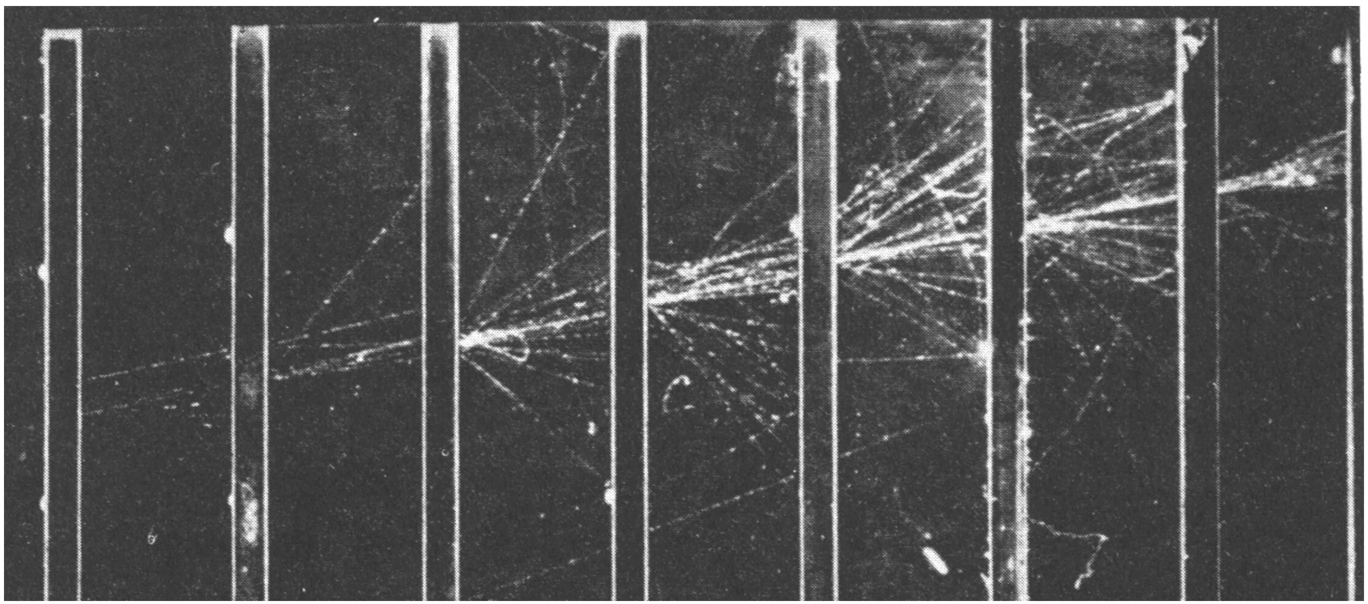


$Z(Z+1)$

Sciami elettromagnetici

A questo punto e' abbastanza facile capire cosa succede se si ha un fascio di $e^{\pm}$ o di γ che attraversa del materiale:

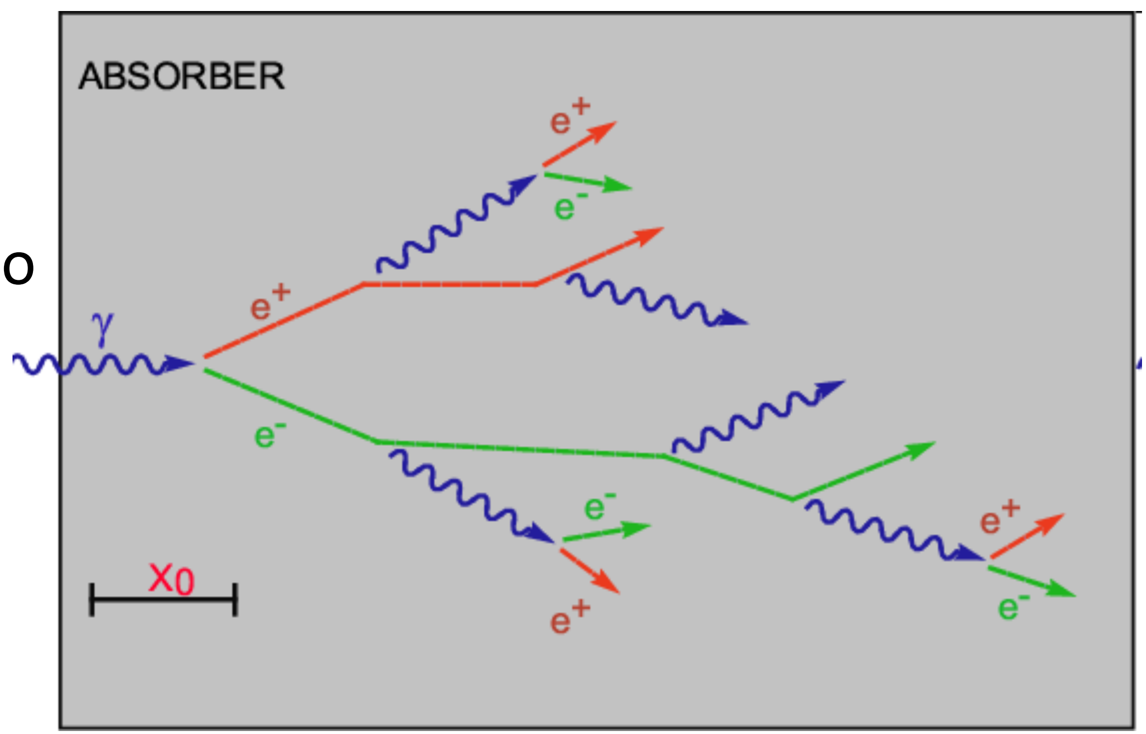
Produzione di una cascata di particelle



Electron shower in a cloud chamber with lead absorbers

Electromagnetic showers

- Cascade of pair production and bremsstrahlung is known as an electromagnetic shower
- number of low-energy photons (or electrons) produced is proportional to initial energy of electron or gamma
- Energy collected in each of $e^{\pm}$ and γ is also proportional to initial energy

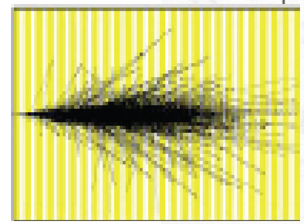


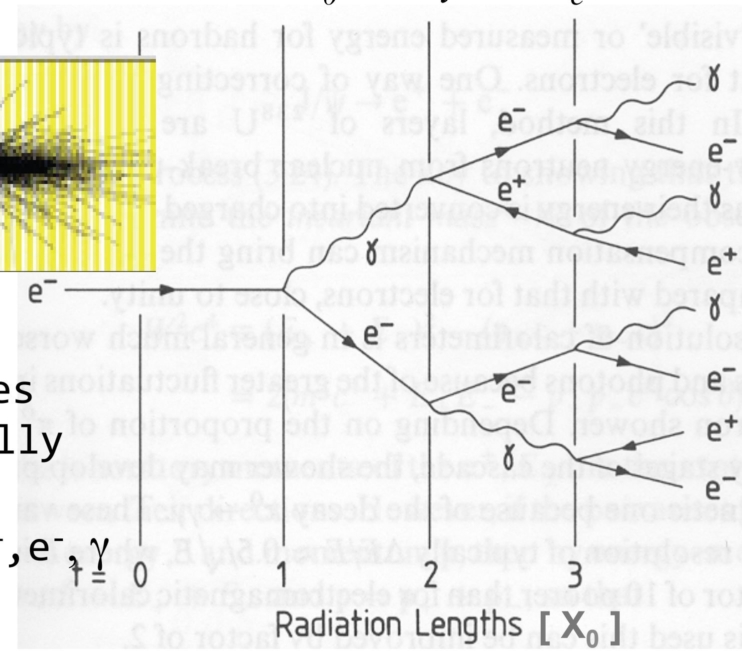
Electromagnetic Shower Development

A simple shower model

Two dimensionless variables: $t=x/X_0$ and $y=E/E_c$ drive shower development

Shower development:





Start with an electron with $E_0 >> E_c$

→ After $1X_0$: $1\ e^-$ and $1\ \gamma$, each with $E_0/2$

→ After $2X_0$: $2\ e^-$, $1\ e^+$ and $1\ \gamma$, each with $E_0/4$

→ After tX_0 :

$$N(t) = 2^t = e^{t \ln 2}$$
$$E(t) = E_0 / 2^t$$

→ Number of particles increases exponentially with t

→ equal number of e^+, e^-, γ

$$t(E') = \frac{\ln(E_0/E')}{\ln 2}$$
$$N(E > E') = \frac{1}{\ln 2}\frac{E_0}{E'}$$

→ Depth at which the energy of a shower particle equals some value E'

→ Number of particles in the shower with energy > E'

Maximum number of particles reached at $E = E_c$

Ec is the critical energy →

$$t_{\max} = \frac{\ln(E_0/E_c)}{\ln 2}$$
$$N_{\max} = e^{t_{\max} \ln 2} = E_0/E_c$$

When the particle E < Ec, absorption processes like ionisation for electrons and Compton and photoelectric effects for photons start to dominate.

Energy deposition

- During the shower development, the energy deposit is mainly due to charged e+ and e- with $E \gg E_c$.
- They are MIPs, so the energy loss by ionization stays constant $\approx (dE/dX)_{mip} = X_0 (dE/dt)_{mip}$, $t = X/X_0$
- The energy deposit is the sum of energy loss by each charged particle: $(dE/dt)_{tot} = (1/2)N(t)\,(dE/dt)_{mip}$ (since in the toy model we have 50% of e+e- and 50% photons)
- So dE/dt grows as N(t) and is max when N(t) is max

$$(dE/dt)_{max} = (1/2)N_{max}(dE/dt)_{mip}$$
$$= (1/2)(E_0/E_c)(dE/dt)_{mip}$$

at the position $t_{max} = \ln(E_0/E_c)/\ln 2$

Electromagnetic Shower Development

Let us take as an example the shower in a CsI (ρ = 4.53 g/cm^3, X0 = 8.39 g/cm^2 or 1.85 cm) crystal detector initiated by a 1 GeV photon. Using the value $E_c \approx 10$ MeV, the number of particles in the shower maximum $N_{max} = E_0/E_c = 100$ and for the depth of the shower maximum at $\approx 6.6\ X_0$.

$$t_{\max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

$$N_{\max} = e^{t_{\max}\ln 2} = E_0/E_c$$

Note that tmax depends only on the ratio $Y = E/E_c$. The only dependence on the material properties is in $E_c = 580/Z$ MeV

Electromagnetic Shower Development

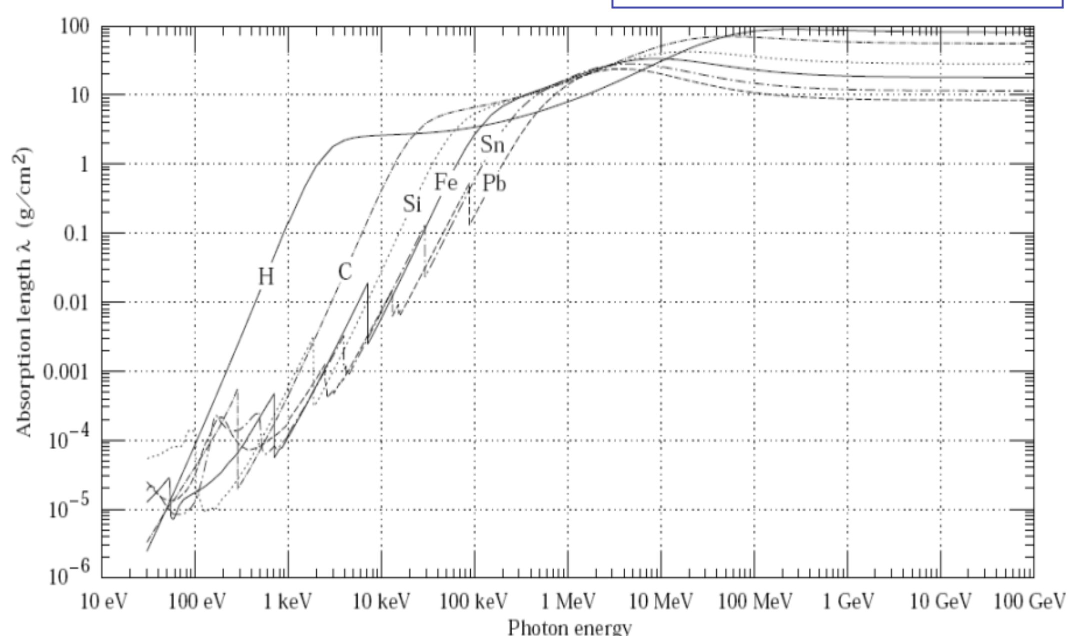
After the shower maximum, particle production stops and electrons and positrons having an energy below E_c will stop in a layer of $\sim 1X_0$ by ionization loss.

The remaining photons ($\sim N_{max}/2$) of the same energy have to be absorbed by p.e. and/or compton (since pair prod does not occur)

$$t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

$$N_{max} = e^{t_{max}\ln 2} = E_0/E_c$$

To absorb the 95% of ~1÷10 MeV photons we need
$x/\lambda = \ln(I_0/I) = \ln(1/0.05) = 3$
Example: let take Si.
At ~1 MeV, λ is ~20 g/cm^2, so we need 60 g/cm2 of material after shower max
The radiation length of Si is 20.82 g/cm^2 (or 9.36 cm) ≈ 1 X0.
So using Si, we need at least additional 3 X0 to absorb the 95% of the energy of the initial particle.

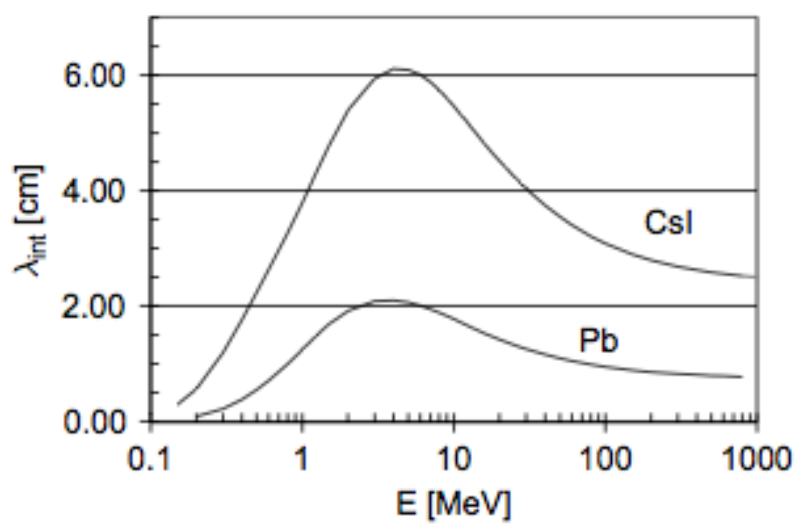


Electromagnetic Shower Development

$$t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

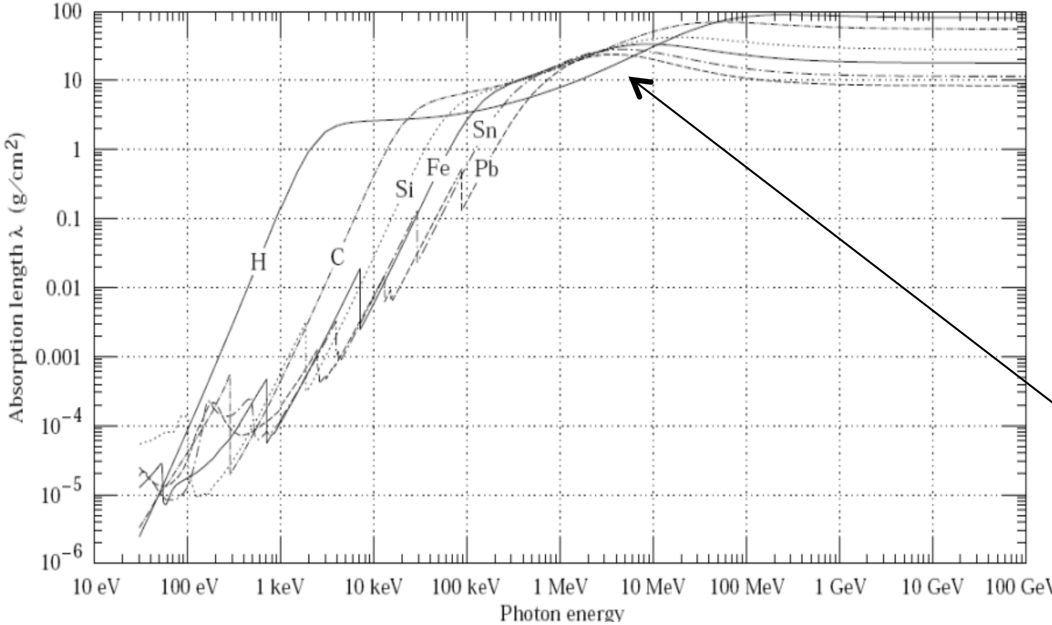
$$N_{max} = e^{t_{max}\ln 2} = E_0/E_c$$

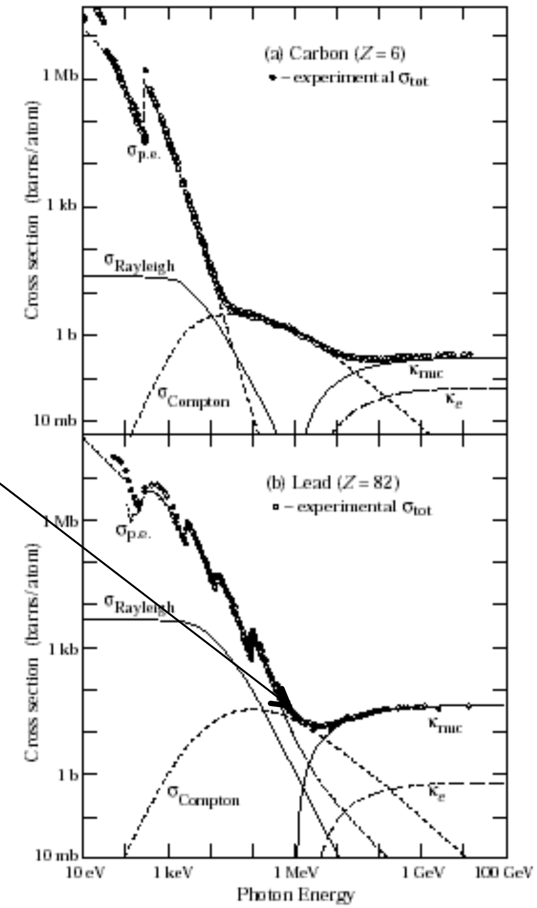
For CsI, λ_{int} has broad maximum between 1 MeV÷10 MeV where it amounts to about $3X_0$.
The energy of photons in the shower maximum close to E_c is just in this range.
Thus, to absorb 95% of γ produced in the shower maximum, an additional $9X_0$ of material is necessary which implies that the thickness of a calorimeter with high shower containment should be at least $16X_0$.



2. Photon interaction length in lead and CsI [4].

Broad maxima in λ correspond to the minima of cross section





→ The attenuation length is $\lambda = A/N_a\sigma$

Electromagnetic Shower Development

This very simple model already correctly describes the most important qualitative characteristics of electromagnetic showers:

(i) To absorb most of the energy of the incident photon the total detector thickness should be more than 10–15X0.

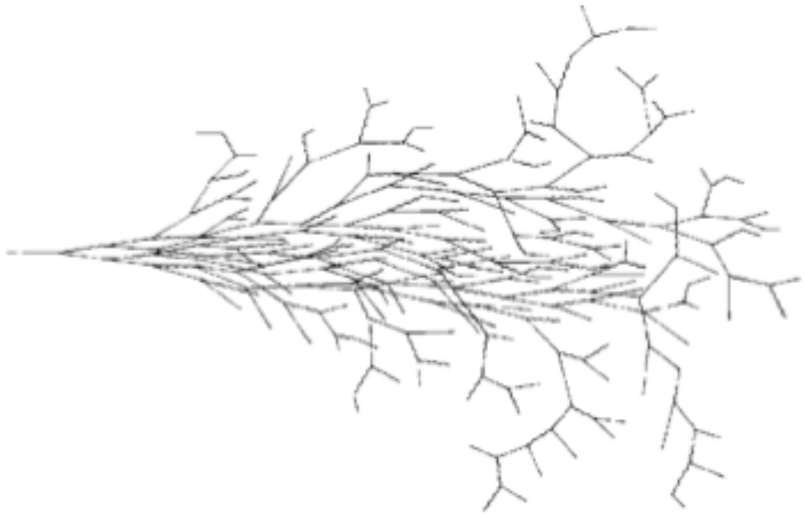
(ii) The position of the shower maximum increases slowly with energy. Hence, the thickness of the calorimeter should increase as the logarithm of the E.

(iii)The energy deposition in an absorber is a result of the ionisation losses of electrons and positrons. Since the $(dE/dx)_{ion}$ value for relativistic electrons is almost energy independent (MIP!), the amount of E deposited in a thin layer of absorber is ∝ to the number of electrons and positrons crossing this layer.

(iv) The energy leakage is caused mostly by soft photons escaping the calorimeter at the sides (lateral leakage) or at the back (rear leakage).

Electromagnetic Shower Development

In real life, the shower development is much more complicated.



(A) $$\frac{dE}{dt} = E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)} ,$$

where $\Gamma(a)$ is Euler's Γ function, defined by

$$\Gamma(g) = \int_0^\infty e^{-x} x^{g-1}\, dx .$$

The gamma function has the property

$$\Gamma(g+1) = g\,\Gamma(g) .$$

At present, an accurate description is obtained from Monte Carlo simulations.
The longitudinal distribution of the energy deposition in electromagnetic cascades is reasonably described by a gamma distribution (A), where a and b are model parameters.

The maximum t_{max} occurs at (a−1)/b.
Fits to shower profiles were made in elements ranging from C to U, at energies from 1 GeV to 100 GeV. The energy deposition profiles are well described by Eq. (A) with $\mathbf{t_{max}}$ **=** **(a-1)/b = ln(E_0/E_c) + $C_{\gamma e}$ (B)** with $C_{\gamma e}$ = 0.5 for γ induced, -0.5 for electron induced shower.
To use Eq. (A, one finds (a−1)/b from Eq. (B) then finds a either by assuming b≈0.5 or by finding a more accurate value from tabulated values

Fitted b values

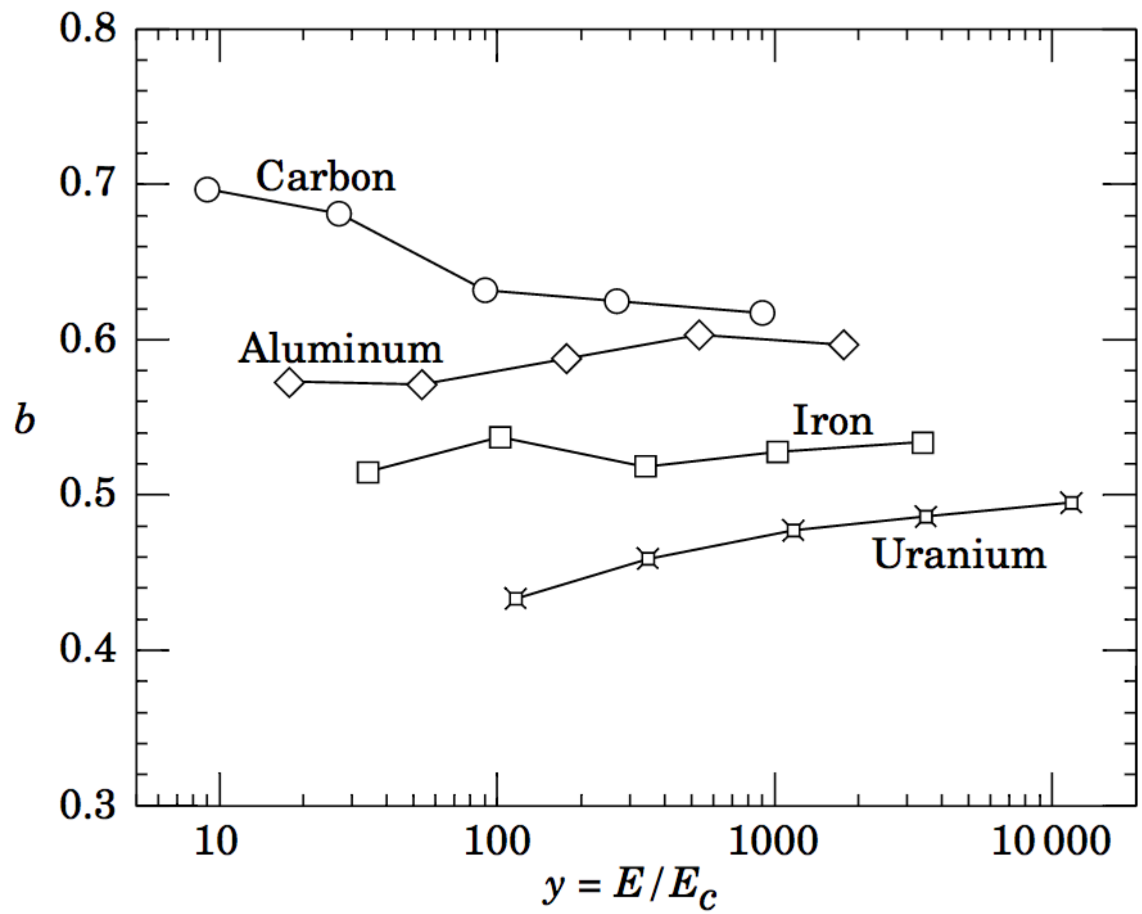


Figure 32.21: Fitted values of the scale factor b for energy deposition profiles obtained with EGS4 for a variety of elements for incident electrons with $1 \leq E_0 \leq 100$ GeV. Values obtained for incident photons are essentially the same.

Electromagnetic Shower Development

Simulation of the energy deposit in copper as a function of the shower depth for incident electrons shows the logarithmic dependence of t_{max} with E.

EGS4* (electron-gamma shower simulation)

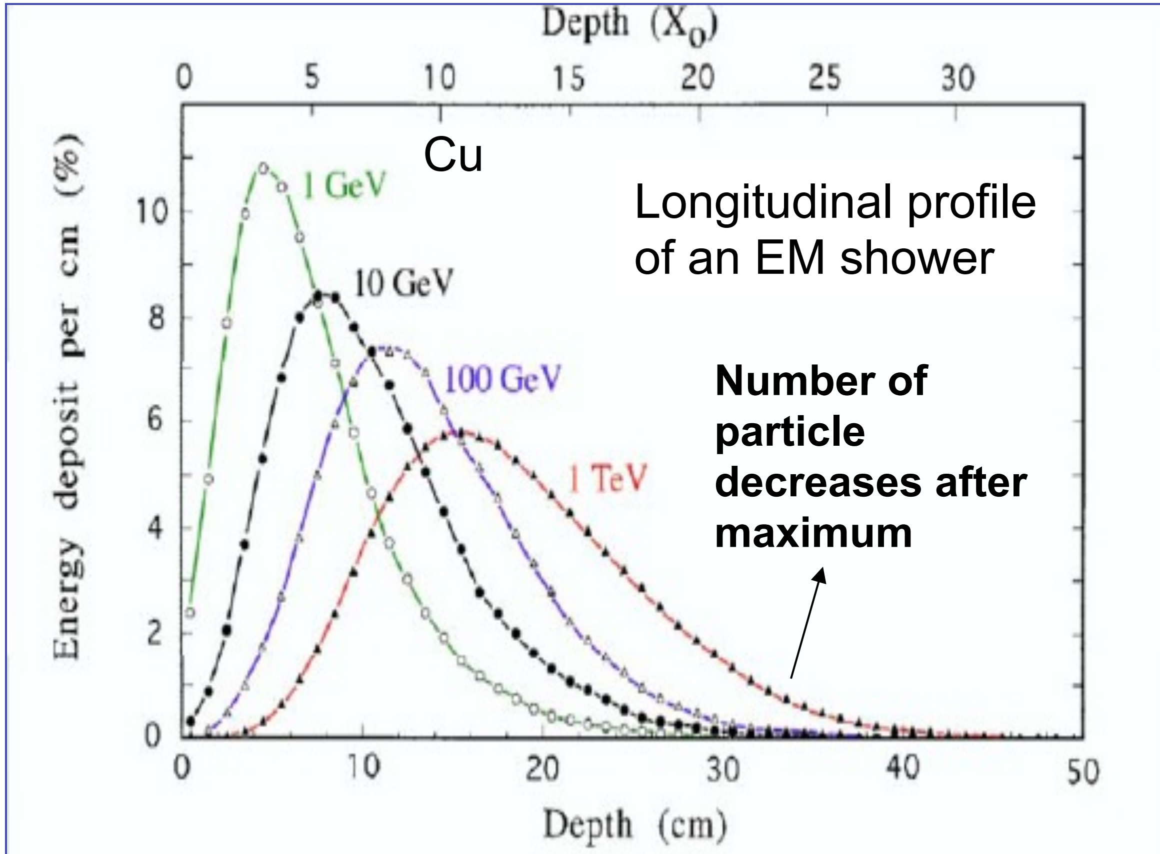


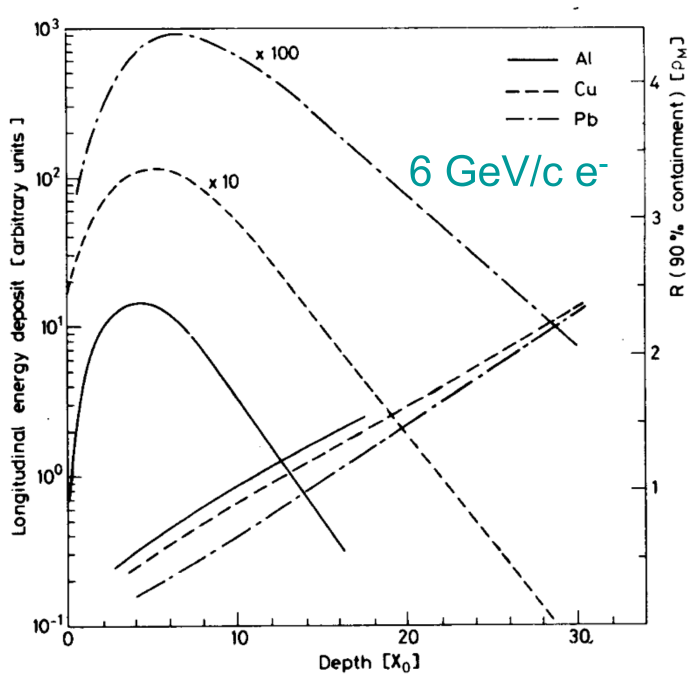
FIG. 2.9. The energy deposit as a function of depth, for 1, 10, 100 and 1000 GeV electron showers developing in a block of copper. In order to compare the energy deposit profiles, the integrals of these curves have been normalized to the same value. The vertical scale gives the energy deposit per cm of copper, as a percentage of the energy of the showering particle. Results of EGS4 calculations.

*EGS4 is a Monte Carlo code for doing simulations of the transport of electrons and photons in arbitrary geometries and materials.

Sciami elettromagnetici

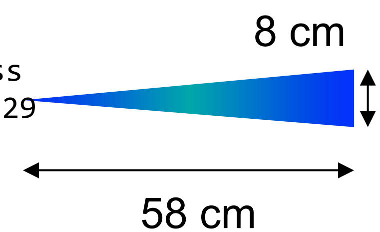
Lo sviluppo trasversale dello sciame non e' tanto dovuto agli angoli di emissione di γ od $e^{\pm}$ (entrambi molto piccoli) quanto allo scattering multiplo. 95% dello sciame e' in un cilindro di raggio $2R_M$

$$R_M = \frac{21 MeV}{E_c} X_0 \; [gr/cm^2]$$ Raggio di Molière



Sviluppo longitudinale e trasversale dello sciame scalano con X_0, R_M

Example: E_0 = 100 GeV in lead glass
Ec=11.8 MeV → tmax ≈ 13, t95% ≈ 29
X_0 ≈ 2 cm, RM = 1.8·X_0 ≈ 3.6 cm



(C. Fabjan, T. Ludlam, CERN-EP/82-37)

Sciami elettromagnetici

- <u>Sviluppo longitudinale</u> dello sciame $$\boxed{\frac{dE}{dt} \propto t^{\alpha} e^{-t}}$$

  Massimo dello sciame $$\boxed{t_{\max} = \ln\frac{E_0}{E_c} + C_{\gamma e}}$$

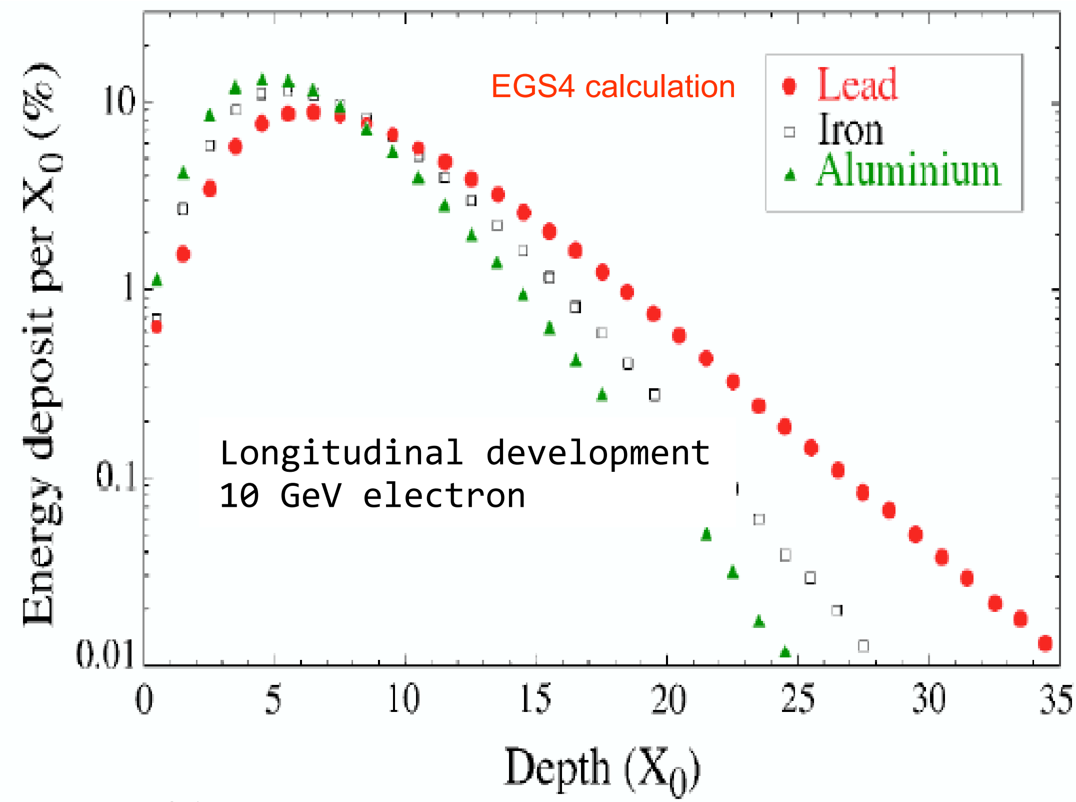
  Contenimento longitudinale $$\boxed{t_{95\%} \approx t_{\max} + 0.08Z + 9.6}$$

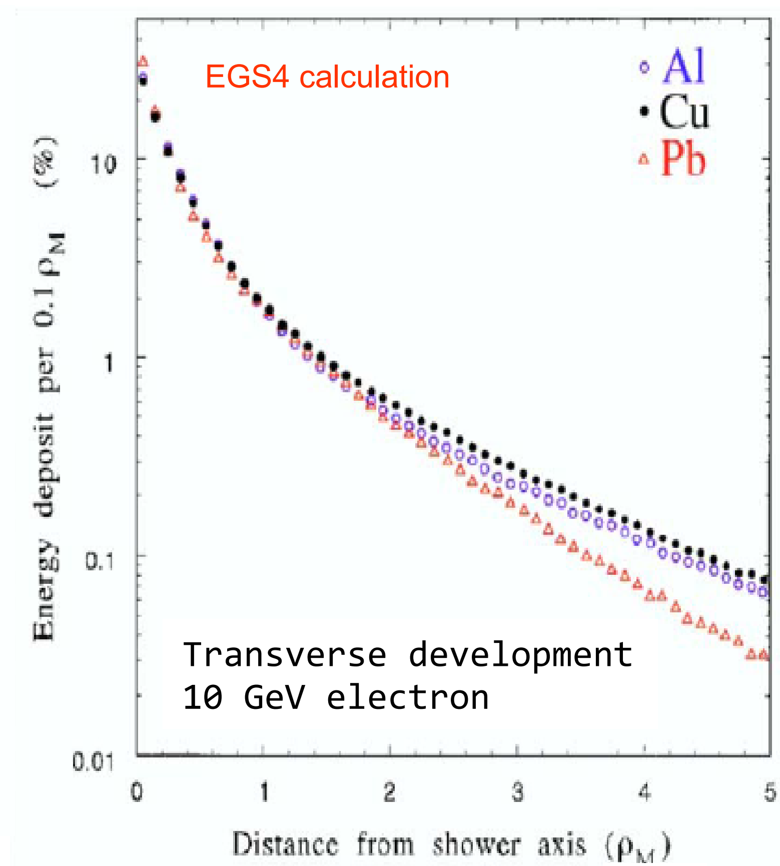
  Le dimensioni longitudinali dello sciame crescono solo logaritmicamente con E_0

Electromagnetic Shower Development

Shower profile

the longitudinal and transverse developments to scale with X_0





$$R_M = \frac{21MeV}{E_c} X_0 \left[gr/cm^2\right]$$

R_M

$X_0 \propto A/Z^2\ ,\quad E_c \propto 1/Z \Rightarrow \quad R_M \propto A/Z$ → $\boldsymbol{R_M}$ less dependent on Z than $\boldsymbol{X_0}$:

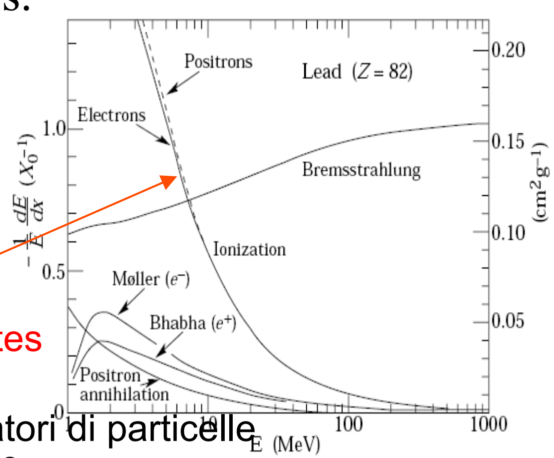
Electromagnetic Shower Development

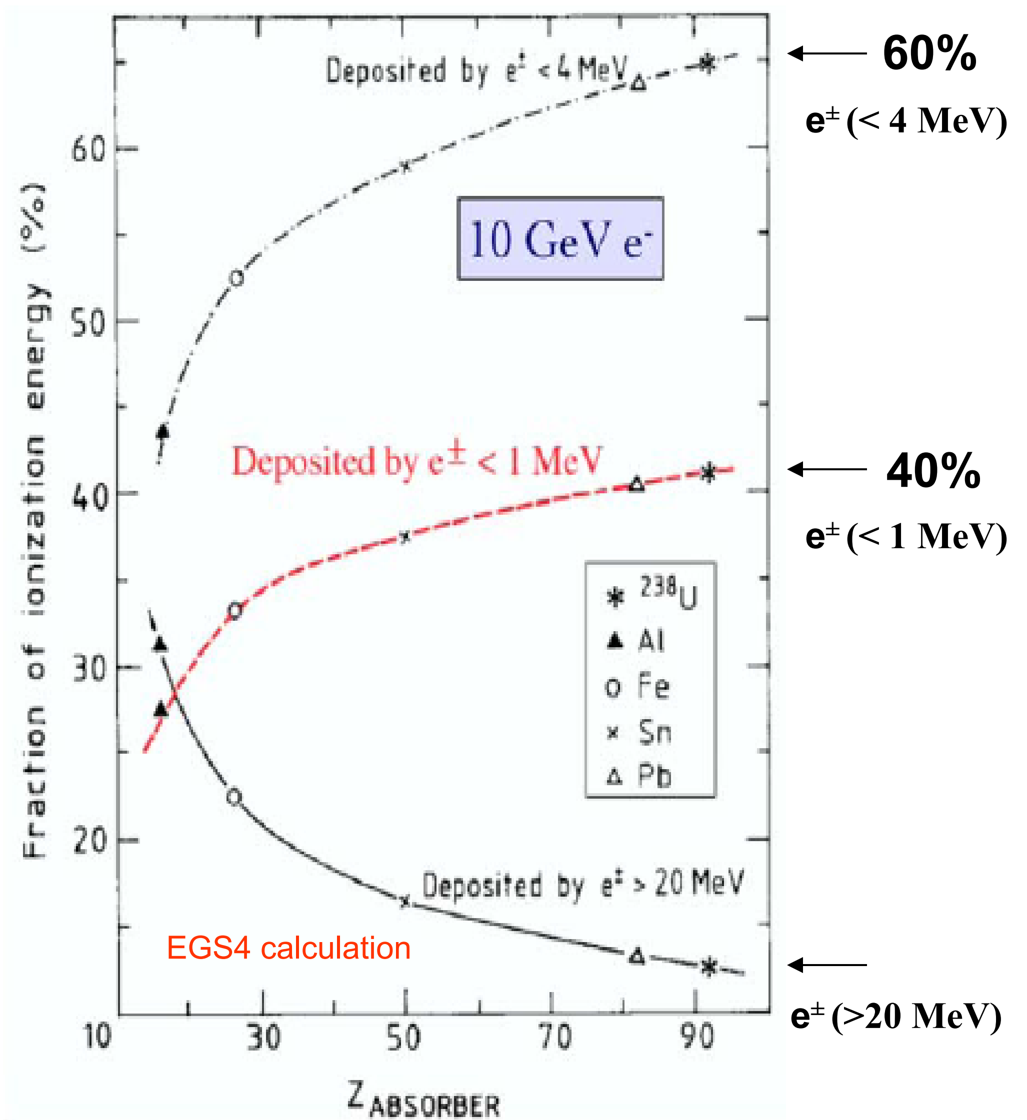
Energy deposition

The fate of a shower is to develop, reach a maximum, and then decrease in number of particles once $E_0 < E_c$

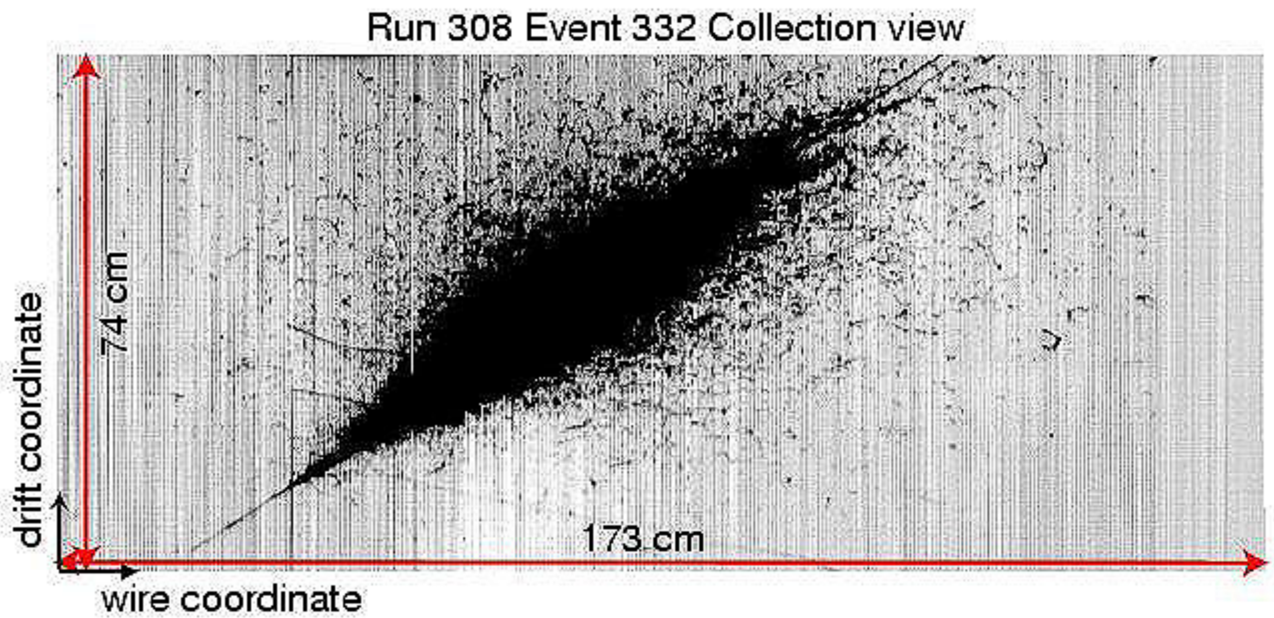
Given that several processes compete for energy deposition at low energies, it is important to understand the fate of the particles in a shower.

→ Most of energy deposition is by low energy $e^{\pm}$' s.

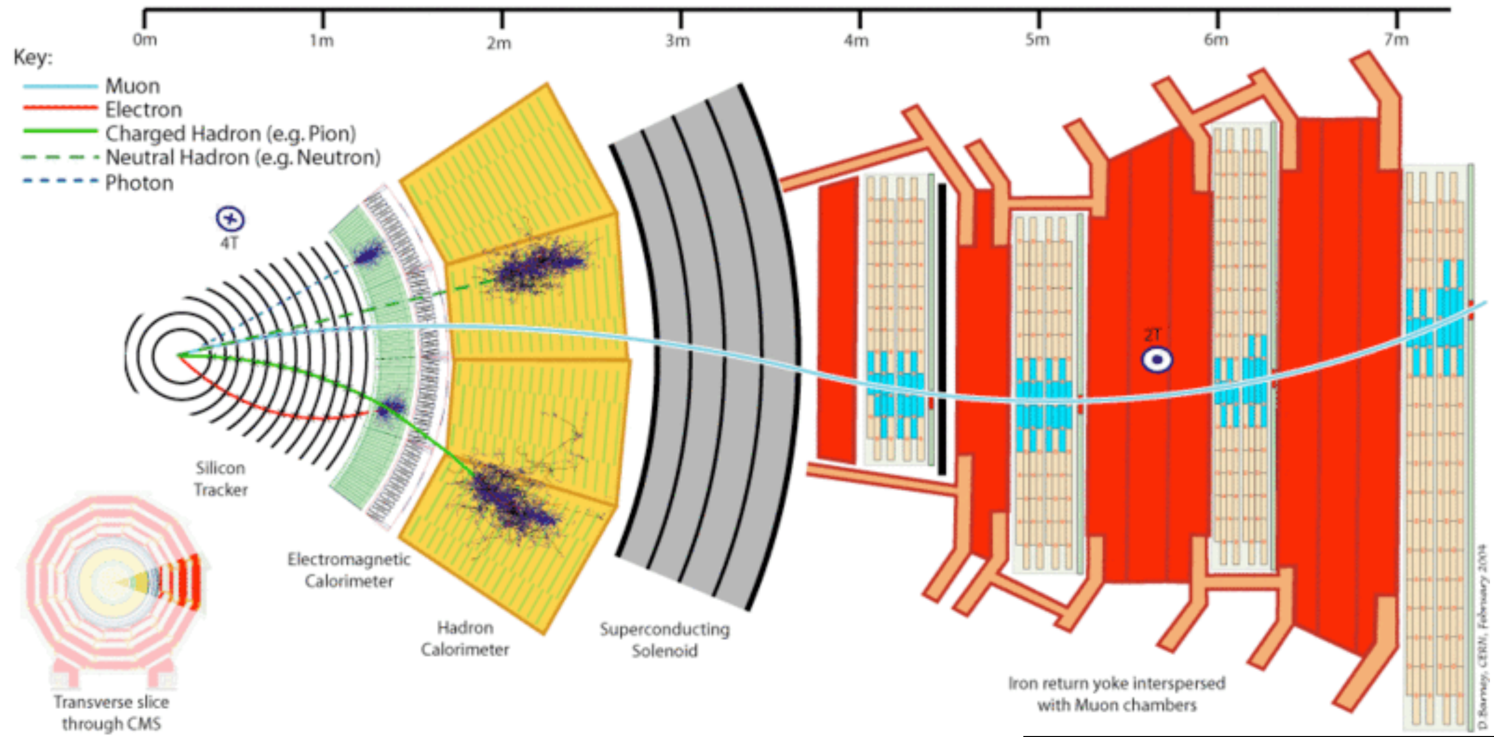




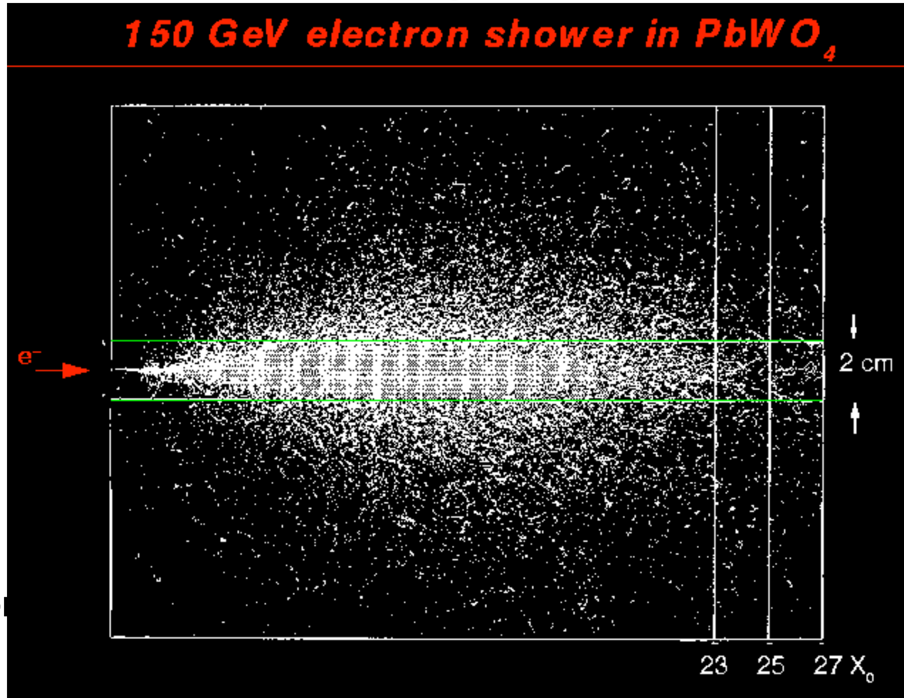
Shower



- ICARUS, liquid argon drift chamber (misura la ionizzazione)
- Quando si forma lo sciame, la particella iniziale idealmente deposita tutta la sua energia nel materiale e viene o assorbita o distrutta (nella pratica ci sono sempre "perdite" dovute a particelle che sfuggono dal volume attivo)
- Se si misura l'energia depositata, possiamo conoscere quella della particella iniziale
- Applicazione tipica nei calorimetri



Cms elm calorimeter



Sciami elettromagnetici

Risoluzione in energia

Per misurare l'energia si contano le tracce cariche

$$\boxed{N_{totale} \propto \frac{E_0}{E_c}}$$

$$\Longrightarrow$$

$$\boxed{\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}}}$$

La risoluzione relativa in energia migliora crescendo E_0

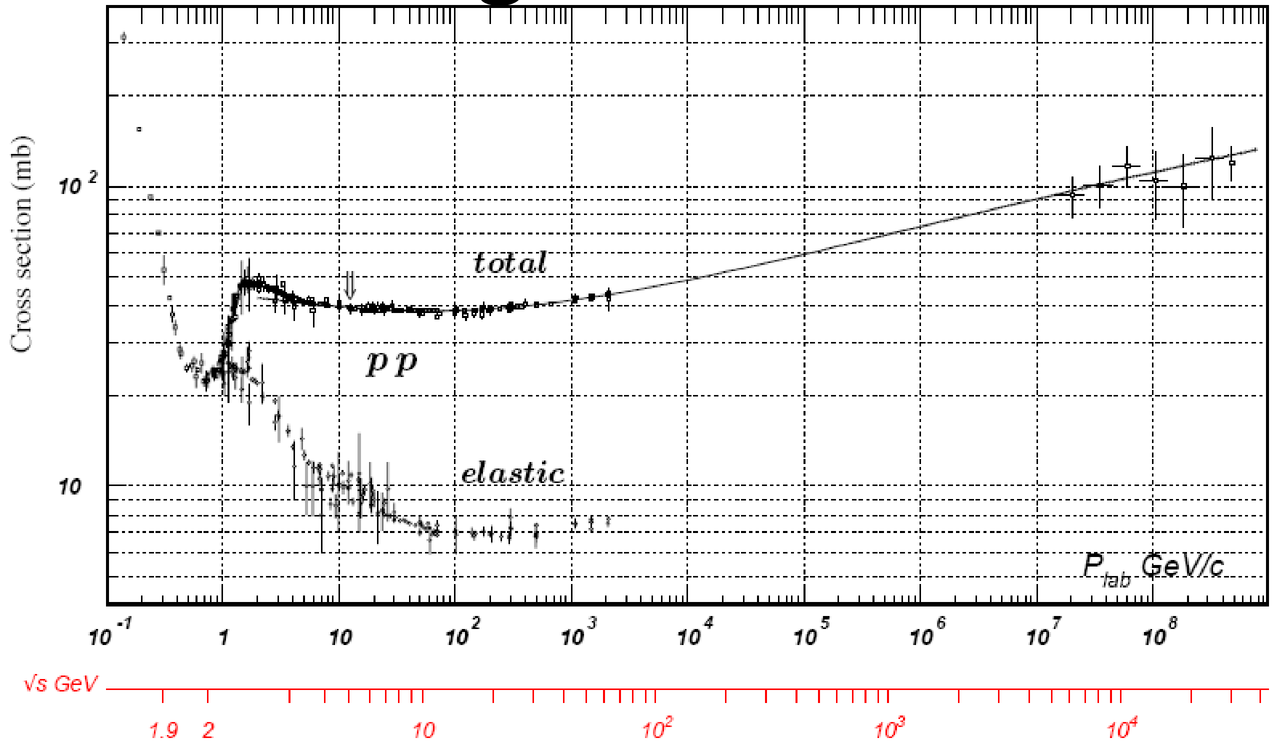
Anche la risoluzione spaziale ed angolare scalano con $E^{-1/2}$

Strong Interactions

Apart from the electromagnetic interactions of charged particles strong interactions may also play a role for particle detection. This is true for all the hadrons (particles sensitive to the strong nuclear force), charged and neutral.
Charged hadrons lose energy also by the elm processes.
Neutral hadrons DO NOT interact electromagnetically. So nuclear interactions are the only ones they can interact with materials

In this case, we are dealing mostly with inelastic processes, where secondary strongly interacting particles are produced in the collision, which in turn may produce new particles that undergo to nuclear and elm interactions, decay, emit radiation, producing new particles, …

Strong Interactions



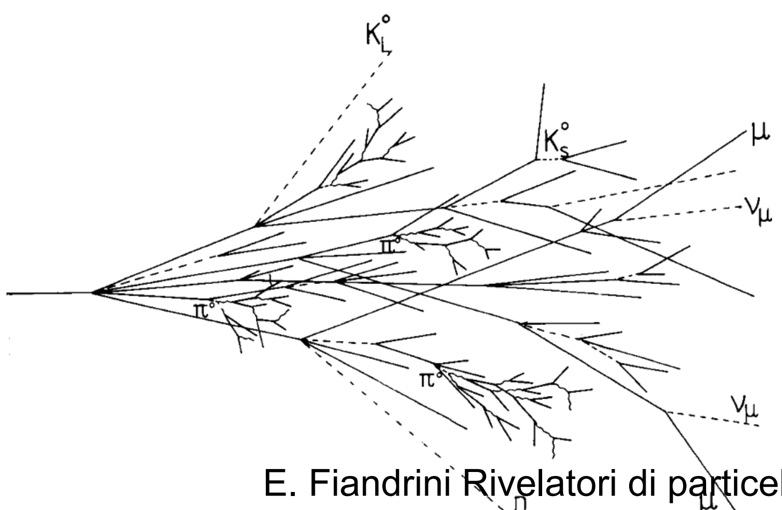
The total cross section for proton–proton scattering can be approximated by a constant value of 50 mb (1 mb = 10^{-27} cm^2) for energies ranging from 2 GeV to 100 TeV. Both the elastic and inelastic part of the cross section show a rather strong energy dependence at low energies, due to the formation of resonances (J/Ψ,Y,ρ D,...:
The elastic part of the cross section → 0 at high energy and the inelastic part is dominant $\sigma_{tot} = \sigma_{el} + \sigma_{inel}$

Hadronic showers

When a strongly interacting particle above 5 GeV enters matter, both inelastic and elastic scattering between particles and nucleons occur.

Secondary hadrons → examples: π and K mesons, p and n.
Energy from primary goes to secondary, then tertiary, etc.

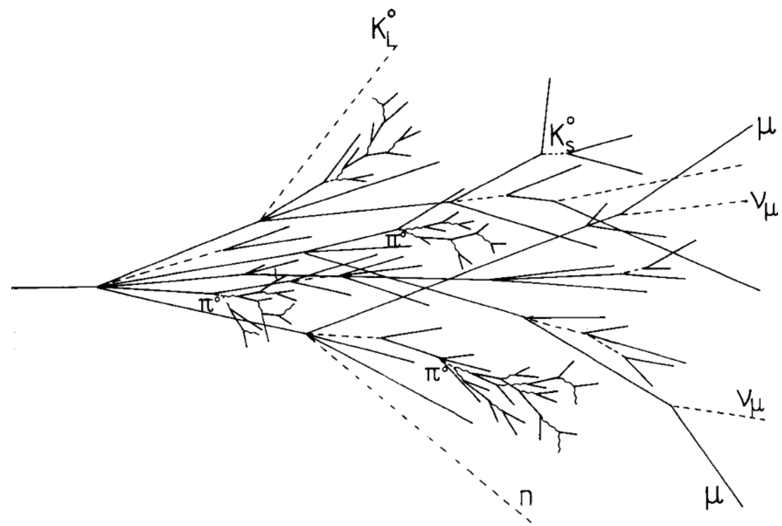
Cascade only ceases when hadron energies small enough to stop by ionization energy loss or nuclear absorption.



(Grupen)

hadronic showers

Many processes involved → much more complicated compared to elm showers.



(Grupen)

Longitudinal development is much longer than for elm showers.

While the lateral structure of electron showers is mainly determined by multiple scattering, in hadron cascades it is caused by large transverse momentum transfers in nuclear interactions, $cP_T \sim hc/R_n \sim 350$ MeV.

Interazioni nucleari (forti)

In analogia alla lunghezza di interazione elettromagnetica possiamo definire una lunghezza di assorbimento –per reazioni anelastiche-

$$\lambda_a = A/(N_a \sigma_{inel})$$

Ed una lunghezza di interazione totale che comprende anche quelle elastiche

$$\lambda_I = A/(N_a \sigma_{total})$$

NB: e' tutto espresso in g/cm^2

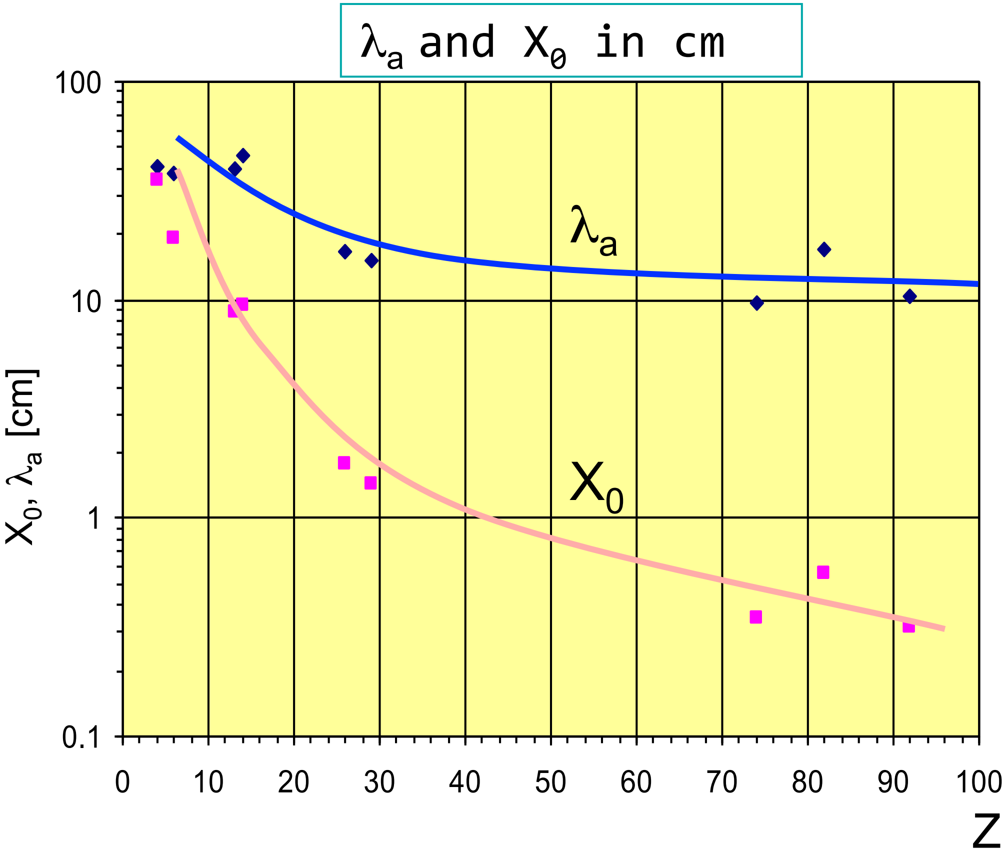
Ad alte energie (>qualche GeV) la sezione d'urto anelastica dipende poco dall'energia e dal tipo di particella incidente (p, π, K….) e domina il processo. Puo' essere parametrizzata come

$\sigma_{inel} \approx \sigma_0 A^{2/3}$ con $\sigma_0 \approx 35$ mb sez. d'urto pp

Quindi $\lambda_a = (A^{1/3}/N_A)\sigma_0$

Interazioni nucleari (forti)

Per Z>6 $\lambda_a > X_0$



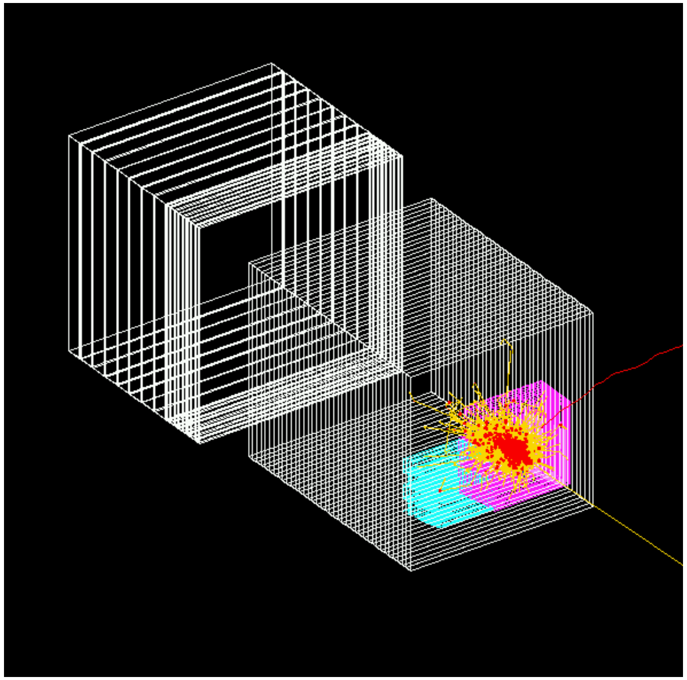
Le due grandezze sono quelle che fissano le dimensioni longitudinali degli sciami: quelli adronici sono piu' "lunghi" di quelli elettromagnetici

Interazioni nucleari (forti)

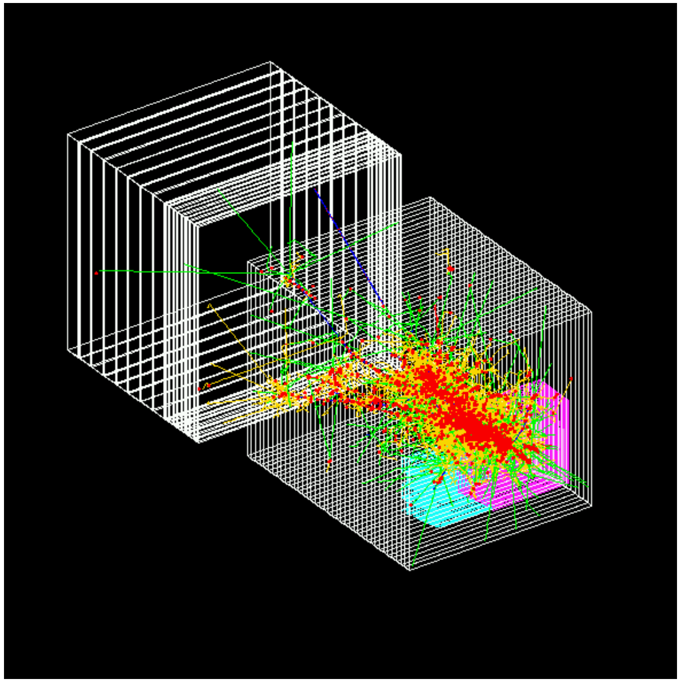
Material	Z	A	ρ [g/cm^3]	X_0 [g/cm^2]	λ_a [g/cm^2]
Hydrogen (gas)	1	1.01	0.0899 (g/l)	63	50.8
Helium (gas)	2	4.00	0.1786 (g/l)	94	65.1
Beryllium	4	9.01	1.848	65.19	75.2
Carbon	6	12.01	2.265	43	86.3
Nitrogen (gas)	7	14.01	1.25 (g/l)	38	87.8
Oxygen (gas)	8	16.00	1.428 (g/l)	34	91.0
Aluminium	13	26.98	2.7	24	106.4
Silicon	14	28.09	2.33	22	106.0
Iron	26	55.85	7.87	13.9	131.9
Copper	29	63.55	8.96	12.9	134.9
Tungsten	74	183.85	19.3	6.8	185.0
Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

Comparison Elm Shower - Hadronic Shower

elm. Shower



hadronic Shower



Characterized by Radiation Length: $X_0 \propto \frac{A}{Z^2}$

Characterized by Interaction Length:

$$\lambda_{\text{int}} \propto A^{1/3}$$

$$\frac{\lambda_{\text{int}}}{X_0} = \frac{A^{1/3} Z^2}{A} \propto \frac{Z^2}{A^{2/3}} = \left(\frac{Z}{A}\right)^{2/3} Z^{4/3} \Rightarrow$$

$\text{Size}_{\text{HadrShowers}} >> \text{Size}_{\text{elm. Showers}}$

Since Z/A is of the order of 0.5-1