

Particle Detectors

Lecture 13

26/04/17

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Misure di posizione e ionizzazione

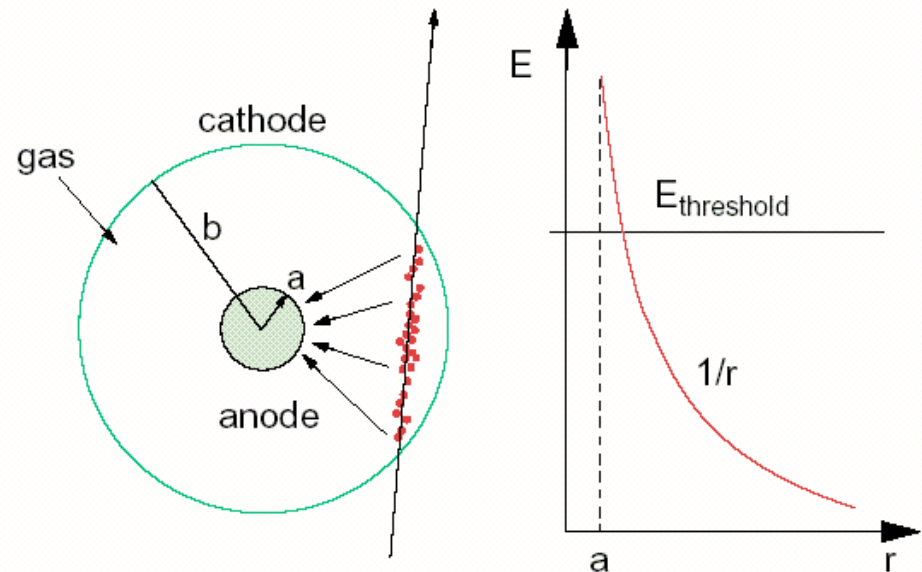
Gli elettroni e gli ioni primari e secondari prodotti per ionizzazione sono pochi (~ 100 in un cm di gas)



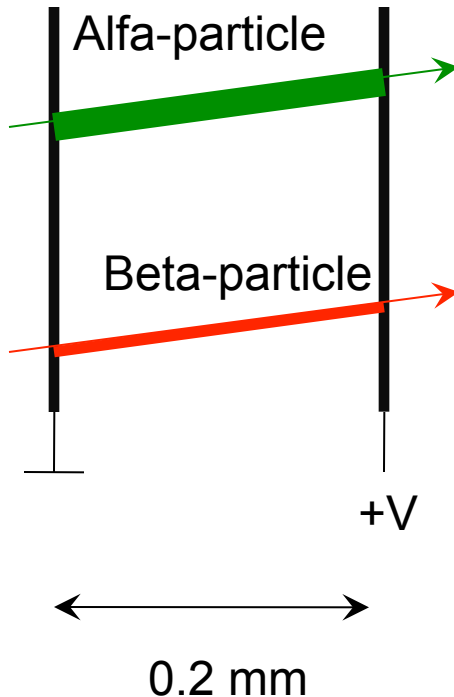
- ❖ Evitare il più possibile la ricombinazione (evitare il più possibile la presenza di gas elettronegativi quale Acqua ed Ossigeno)
- ❖ Aumentare il campo E per sfruttare la moltiplicazione a valanga delle coppie e-ione, alto guadagno.

Avalanche Multiplication

- The trick is to use avalanche multiplication of ionisation in the gas. This can be achieved by accelerating the primary ionisation electrons in an electric field to the point where they can also cause ionisation, as we will see in the next slides
- The number of ion pairs is controlled by the applied voltage.
- An electric field varying with the distance from electrodes is needed.



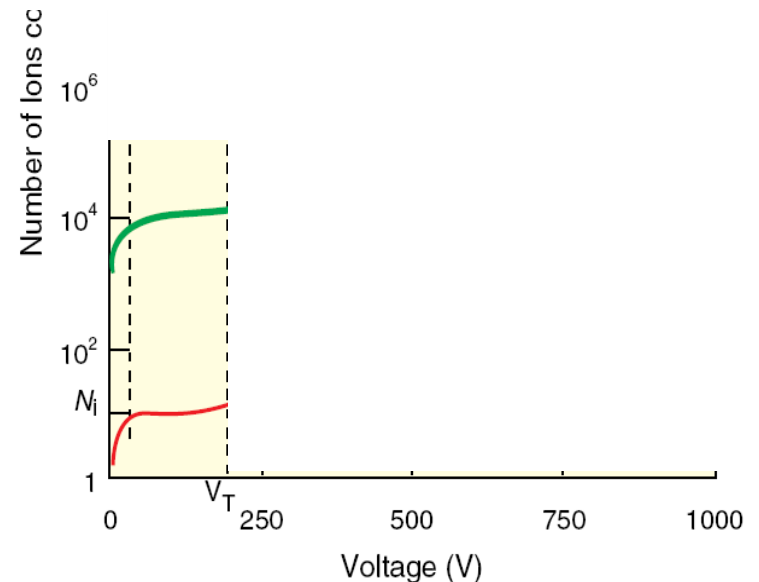
Gas amplification



Il numero di coppie raccolte dipende dal campo E nel gas: A valori bassi del campo, la ricombinazione elimina una frazione consistente delle coppie prima che giungano agli elettrodi.

Al crescere del campo, si arriva a raccogliere tutta la carica (al netto delle perdite) generata. In queste condizioni, l'aumento del campo non provoca più un aumento della carica raccolta: **c'è un plateau nella curva Q vs $V \rightarrow$ variazioni della tensione provocano minime variazioni nella raccolta di carica**

“Ionisation” mode of operation, i.e. no amplification yet...
used in the so called **ionisation chambers**.



Camere a ionizzazione

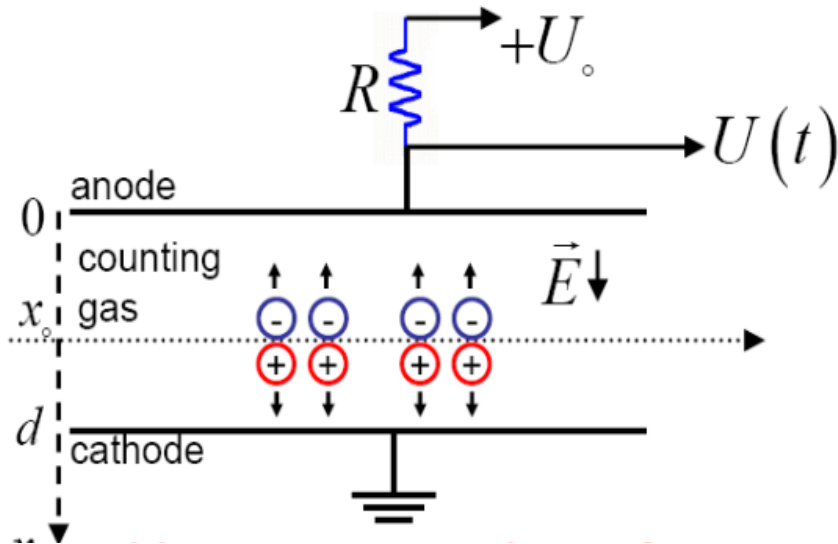
Una camera a ionizzazione è un apparato che misura la perdita di energia per ionizzazione di una particella carica o la perdita di energia di un fotone (effetto fotoelettrico, compton o produzione di coppie).

In linea di principio il materiale attraversato dalla particella può essere un gas (e.g. Argon) oppure un liquido (e.g. calorimetri ad argon o kripton o xenon liquido) od un solido (camere a ionizzazione a stato solido).

Non c'è alcuna moltiplicazione delle coppie ione-elettrone primarie e secondarie.

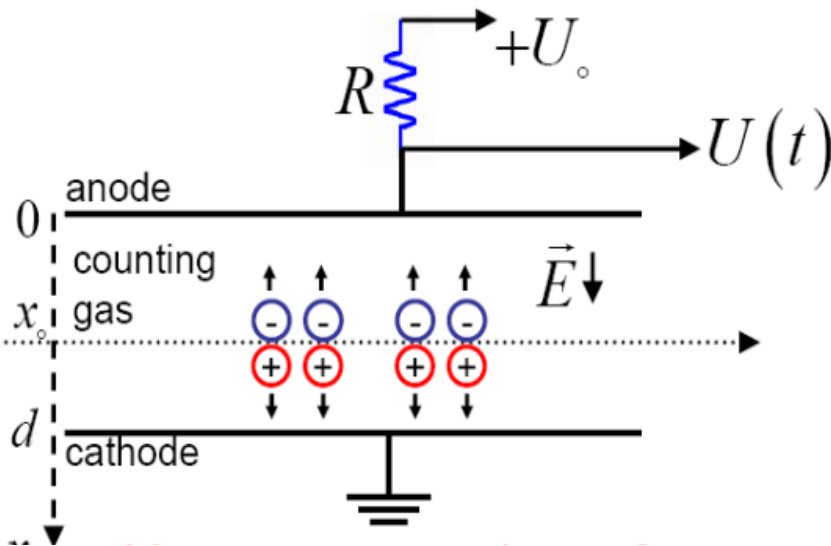
Camere a ionizzazione

Nel caso più semplice una camera ad ionizzazione consiste in un sistema di elettrodi paralleli. Un voltaggio applicato fra gli elettrodi produce un campo elettrico omogeneo (lontano dai bordi). Gli elettrodi sono montati in una scatola a tenuta riempita con gas o con liquido o con solido.



R (di solito molto grande) serve a disaccoppiare l'anodo dalla sorgente di tensione V_o , il rivelatore ha una capacità C

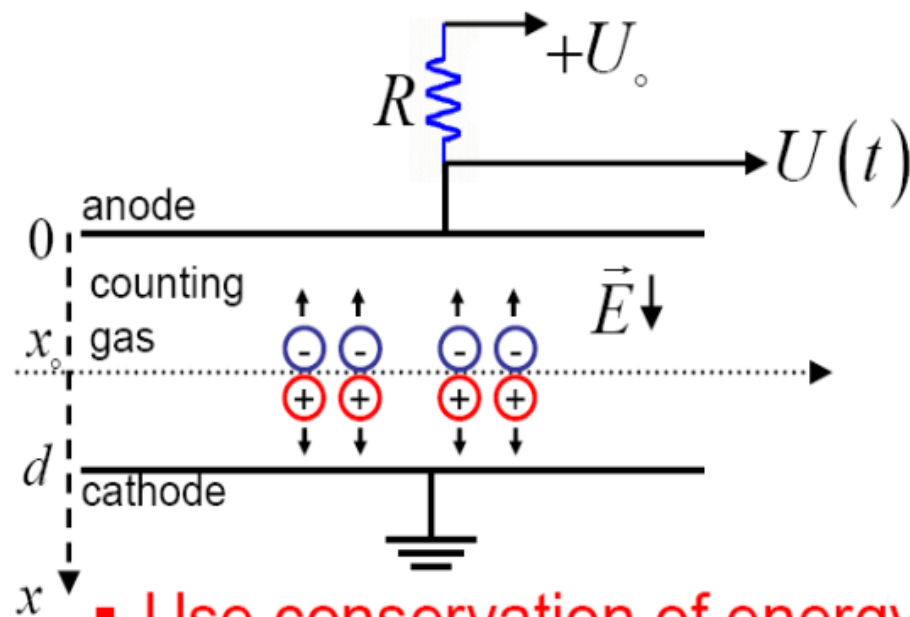
Camere a ionizzazione



- The parallel electrodes of the ionisation chamber, acting as a capacitor with capacitance C , are initially charged to the voltage U_0 .
- To simplify the consideration let us assume that the load resistor R is very large so that the capacitor can be considered to be independent (the voltage of the capacitor can then vary, it is not kept fixed by the voltage source), that is assume that the e^- s and ions collection times are long before the power supply can recharge the plates back to U_0 (RC_d very large) the charge on each plate will be diminished by $N|e|$,
- Suppose N charge-carrier pairs are produced along the particle track at a distance x_0 from the anode.
- **The drifting charge carriers induce an electric charge on the electrodes which leads to certain change of the voltage, ΔU .**
- Thereby the stored energy $(1/2)CV_0^2$ will be reduced to $(1/2)CV^2(t)$ because of the work done by the field to move the charge carriers

Camere a ionizzazione

■ Parallel electrodes



$$|\vec{E}| = \frac{U(t)}{d} \quad C = \frac{\epsilon A}{d}$$

$$\int_0^{t_d^-} v^- dt = x_0$$

$$\int_0^{t_d^+} v^+ dt = d - x_0$$

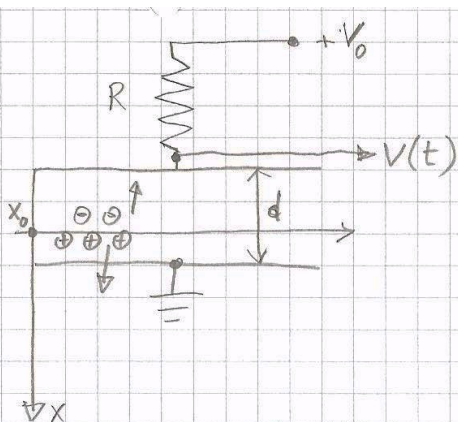
$$v^+ < v^- \quad \text{drift velocities depend on the electric field}$$

■ Use conservation of energy in C

$$\frac{1}{2} C U_0^2 = \frac{1}{2} C U^2(t) + \text{work done by } \vec{E} \text{ on drifting charges}$$

obtain
$$-\frac{dU(t)}{dt} = \frac{Ne}{Cd} [v^- + v^+] \quad \text{holds for } v = v(|\vec{E}|(t))$$

Camere a ionizzazione



Dalla cons. dell'energia

$$\frac{1}{2} C V(t)^2 = \frac{1}{2} C V_0^2 - N \int_{x_0}^x q E_x dx \Rightarrow \frac{1}{2} C (V(t)^2 - V_0^2) = -Nq E_x (x - x_0)$$

$$\frac{1}{2} C (V(t) + V_0)(V(t) - V_0) = -Nq E_x (x - x_0) \rightarrow$$

$$C V_0 \Delta V \approx -Nq \frac{V_0}{d} (x - x_0)$$

$$\Rightarrow \Delta V = -\frac{Nq}{Cd} (x - x_0) \text{ con } x - x_0 = v_d t \Rightarrow$$

$$\Delta V = -\frac{Nq}{Cd} v_d t \Rightarrow \begin{cases} \Delta V^+ = -\frac{Ne}{Cd} v^+ t, & t < t_d^+ \\ \Delta V^- = -\frac{N(-e)(-v^-)t}{Cd} = -\frac{Ne v^- t}{Cd}, & t < t_d^- \end{cases}$$

$$\Delta V(t) = \Delta V^+(t) + \Delta V^-(t)$$

$$\Delta V^+(t_d^+) = -\frac{Ne v^+ t_d^+}{Cd} = -\frac{Ne(d - x_0)}{Cd}$$

$$\Delta V^-(t_d^-) = -\frac{Ne v^- t_d^-}{Cd} = -\frac{Ne x_0}{Cd}$$

La caduta di pot. per $t = t_d^+$ è

$$\Delta V_{Max} = -\frac{Ne(d - x_0 + x_0)}{Cd} = -\frac{Ne}{C}$$

La carica raccolta sugli elettrodi è $Q = C \Delta V_{max} = -Ne$

la corrente nella camera $i = \frac{dQ}{dt} = C \frac{dV}{dt} = -\frac{eN}{d} (v^+ + v^-)$

• Il segnale $V(t) \approx V_0$ **dV piccolo!**

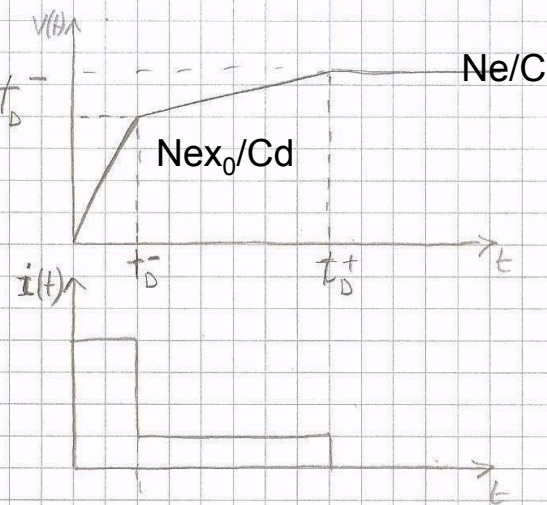
$$e V(t) - V_0 = \Delta V = V_R$$

$$V(t) + V_0 \approx 2V_0$$

$$E_x = V_0/d$$

NB: $v^- \gg v^+$

C è la capacità della camera + quelle parassite

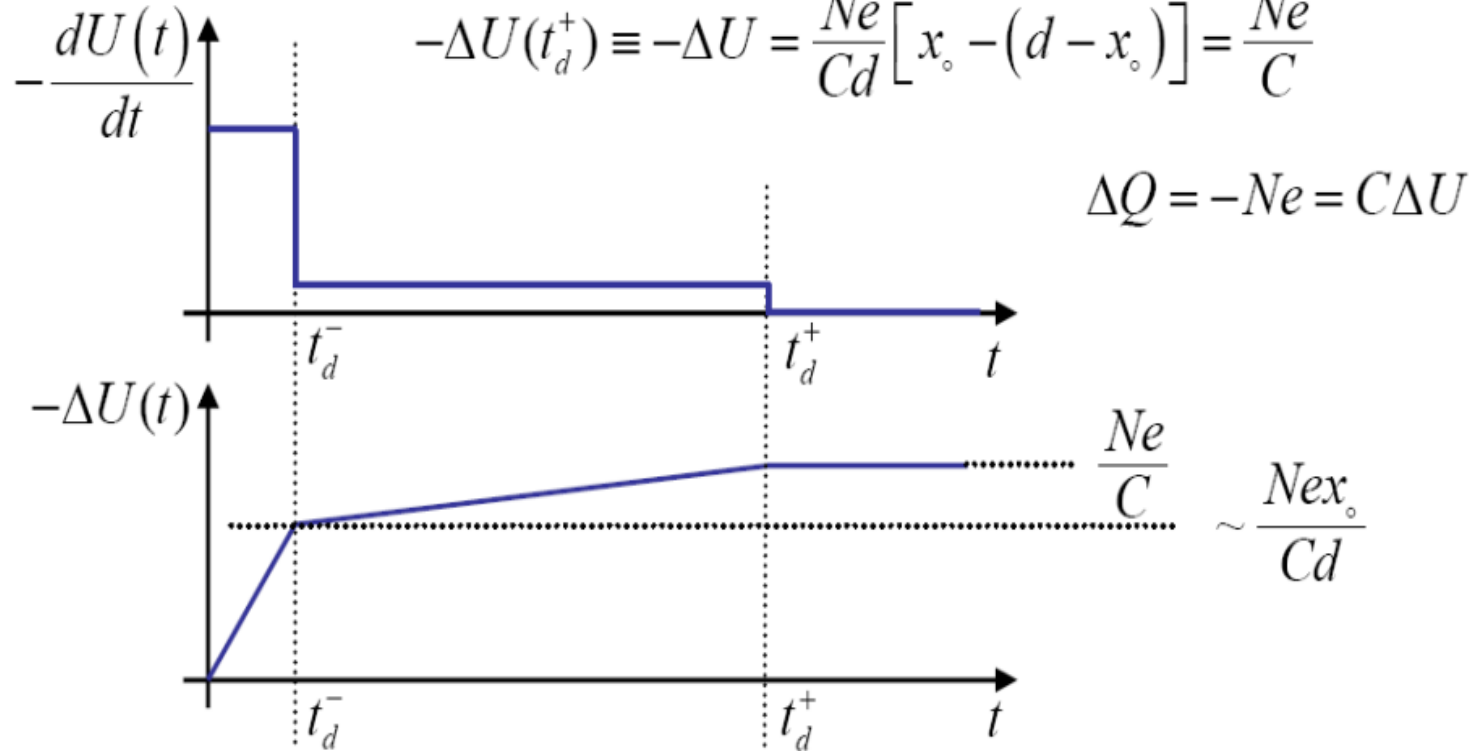


Ionization chambers

■ Signal

$$\Delta U(t) = U(t) - U_0 = \Delta U^-(t) + \Delta U^+(t)$$

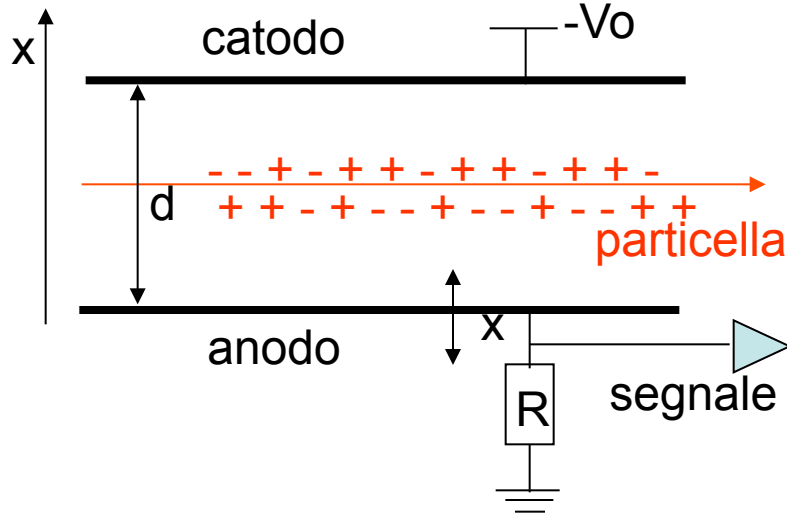
$$-\Delta U(t_d^+) \equiv -\Delta U = \frac{Ne}{Cd} [x_0 - (d - x_0)] = \frac{Ne}{C}$$



$$\frac{\Delta U^+}{\Delta U^-} = \frac{d - x_0}{x_0} \quad \text{If } x_0 = \frac{1}{2}d \quad \text{then } \frac{\Delta U^+}{\Delta U^-} = 1$$

Camere a ionizzazione

Si può ottenere lo stesso risultato osservando la variazione di energia potenziale delle particelle in moto fra le piastre. Il risultato è lo stesso (ovviamente). In questo caso la carica raccolta è quella che si muove, nel caso precedente la carica raccolta è indotta sulla armature dal moto delle cariche fra le piastre.



Il campo elettrico nella camera è costante $E = V_0/d$

carica q a distanza x dall'anodo $\rightarrow U = qV(x)$
 se la carica si sposta di $dx \rightarrow$

$$\Delta U = qV(x+dx) - qV(x) = -qEdx$$

La variazione di energia potenziale ΔU deve essere compensata dal lavoro del generatore
 $V_0 idt = V_0 dQ \rightarrow$

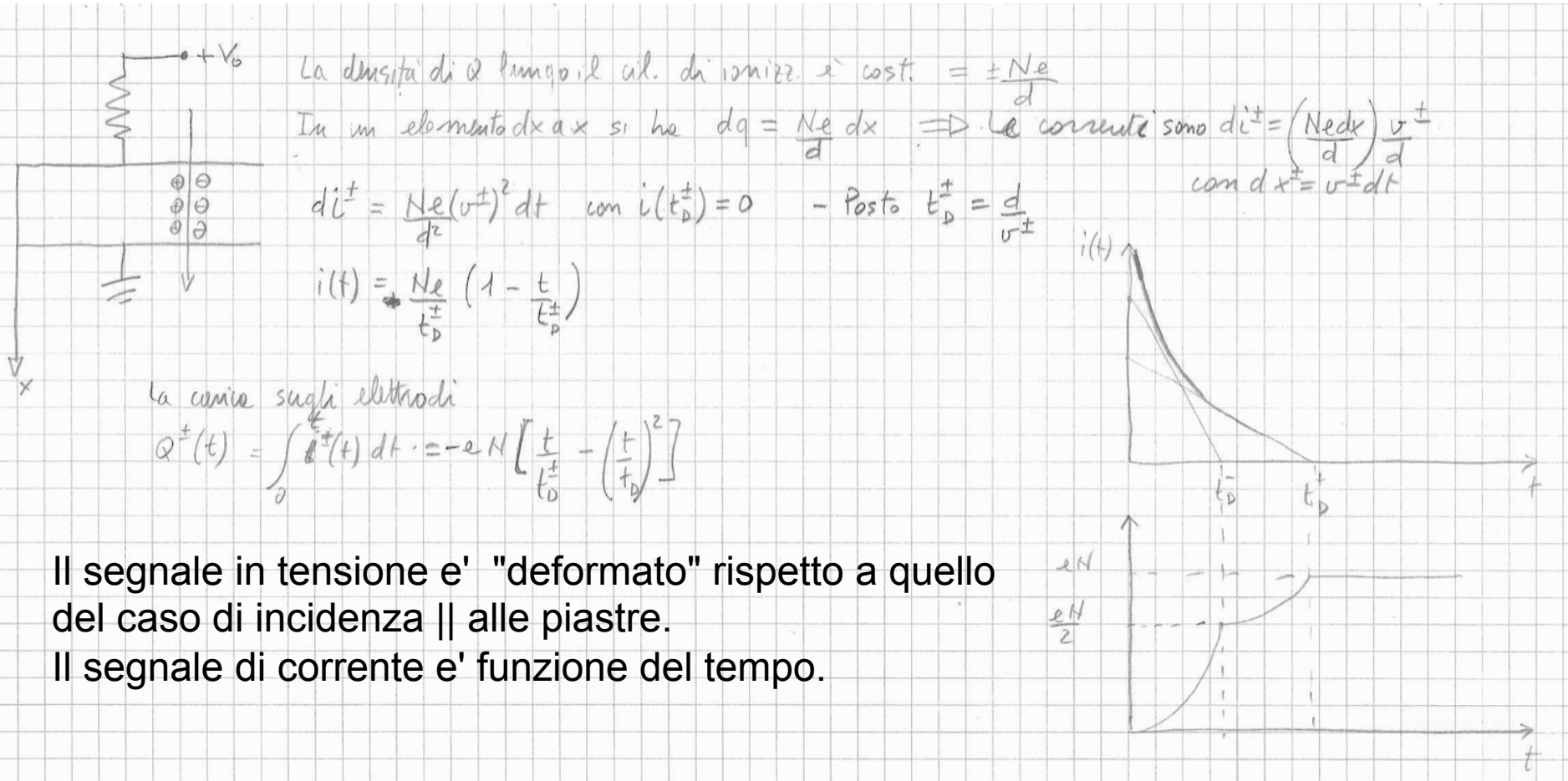
$$-qEdx = V_0 idt \text{ ed } E = V_0/d$$

$$\rightarrow i = -q(v/d)$$

i è dunque il segnale in corrente.

Camere a ionizzazione

Se la particella attraversa la camera in direzione perpendicolare alle piastre:



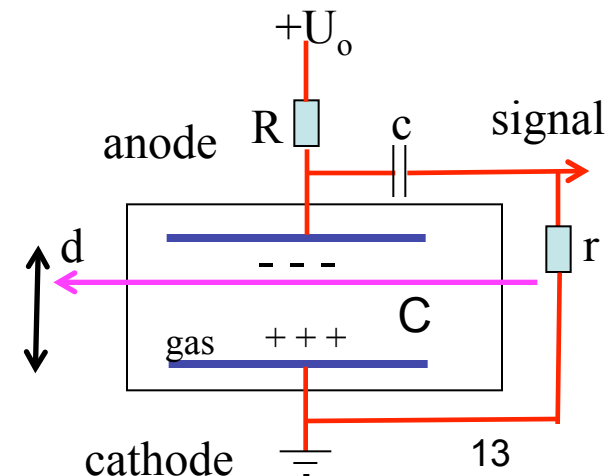
Camere a ionizzazione

- These considerations are only true if the charging resistor is infinitely large or, more precisely, if $t_D^\pm \ll RC$, time constant of the apparatus (C is the total capacitance: camera+circuit+parasitic elements).
- When $RC < \infty$, in this case one obtains

$$\Delta U^+ = -\frac{Ne}{d}v^+R(1 - e^{-\Delta t/RC}) ,$$

$$\Delta U^- = -\frac{N(-e)}{d}(-v^-)R(1 - e^{-\Delta t/RC}) .$$

NB: this R corresponds to r in figure



Camere a ionizzazione

- In practical cases RC is usually large compared to t_- , but smaller than t_+ , $t_- \ll RC \ll t_+$. In this case one obtains

$$\Delta U = -\frac{Ne}{Cd}x_0 - \frac{Ne}{d}v^+R(1 - e^{-\Delta t/RC})$$

- For electric field strengths of 500V/cm and typical drift velocities of $v_- = 5$ cm/ μ s, collection times for electrons of 2 μ s and for ions of about 2 ms are obtained for a drift path of 10 cm. If the time constant $RC \gg 2$ ms, the signal amplitude is independent of x_0 .
- For many applications this is much too long. If one restricts oneself to the measurement of the electron component, which can be done by differentiating the signal, the total amplitude will not only be smaller, but also depend on the point in which the ionisation is produced.

Camere a ionizzazione

Perciò se introduciamo un $R'C'$ all'ingresso dell'amplificatore tale che:

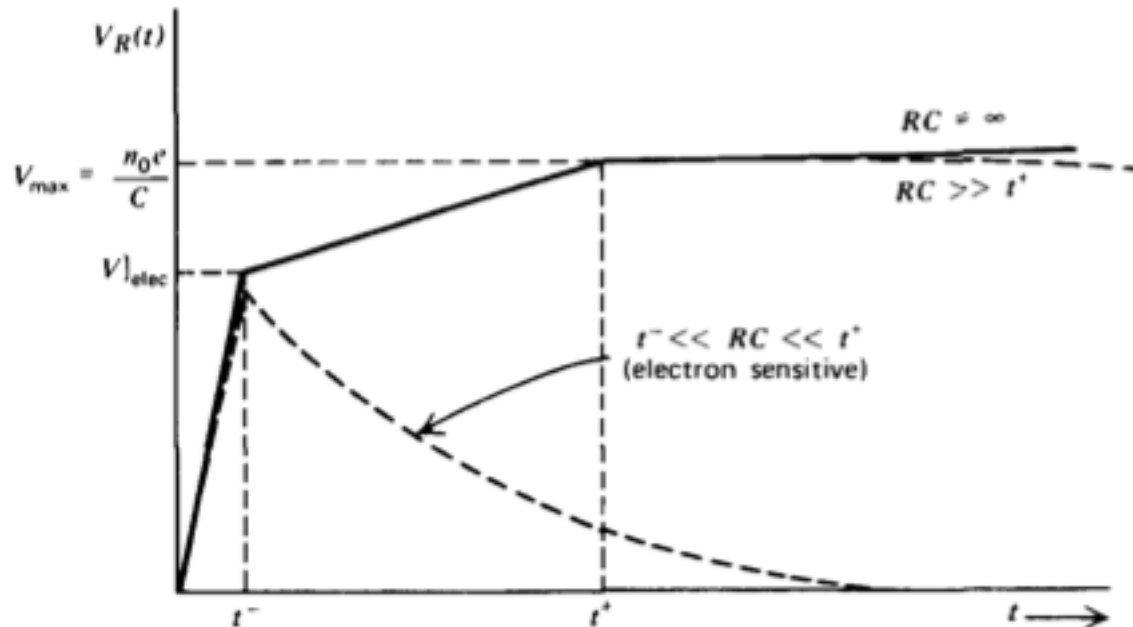
$$\Delta t^- \ll R'C' \ll \Delta t^+$$

Avremo un segnale in tensione essenzialmente dovuto solo agli elettroni.

Camera a ionizzazione

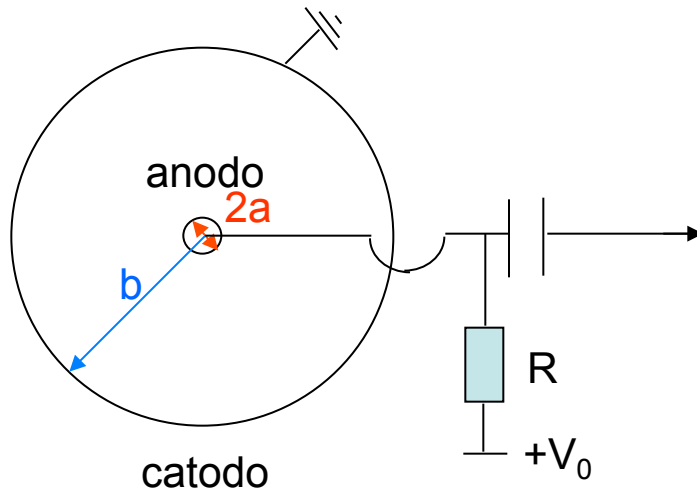
- In electron-sensitive operation, however, the portion of the pulse derived above that corresponds to drift of the ions is almost entirely lost by choosing a collection time constant that is much shorter than the ion collection time. The pulse that remains then reflects only the drift of the electrons and will have an amplitude $V_{\max} = (eN/C)(x/d)$

NB: in this case, the signal amplitude is reduced and depends on the distance from electrodes



Camere a ionizzazione

- Nelle situazioni reali, la camera a ionizzazione e' usualmente realizzata da un filo sottile tenuto a tensione V rispetto alle pareti del contenitore a massa.

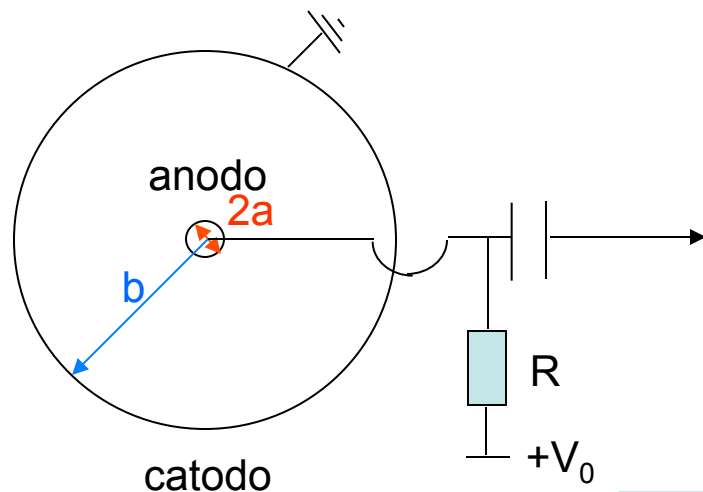


Camere ad ionizzazione

Il potenziale della camera cilindrica può essere ricavato dall'equazione di Laplace $\nabla^2 V = 0 \rightarrow$

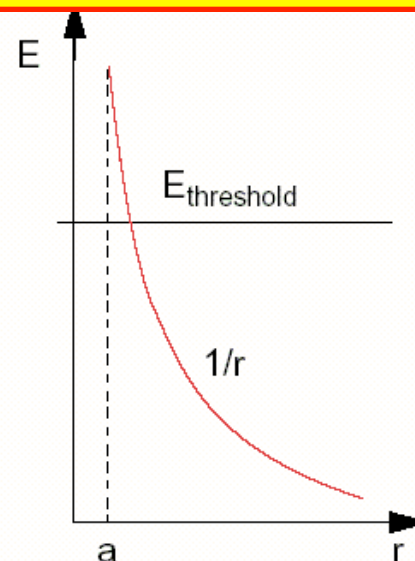
$$V(r) = -\frac{CV_0}{2\pi\epsilon L} \ln(r/a) \quad E(r) = \frac{CV_0}{2\pi\epsilon L} \frac{1}{r} \quad C = \frac{2\pi\epsilon L}{\ln(b/a)}$$

r è la distanza radiale dal filo (di raggio a), V_0 il potenziale applicato al filo, ϵ la costante dielettrica del gas e $C = (2\pi\epsilon / \ln(b/a))$ la capacità per unità di lunghezza del condensatore cilindrico.



Note: I put the L dependence in E and C .
Leo's C is my C/L .

$$|\vec{E}(r)| = \frac{V_0}{r} \frac{1}{\ln(b/a)}$$



Il **campo elettrico** va come $1/r$.

La velocità di deriva non è più costante, ma la **diffusione** è, in buona approssimazione, **costante**.

Camere a ionizzazione

In questa configurazione (**cilindrica**) il tempo di deriva degli elettroni è :

$$\Delta t^- = \int_r^a \frac{dr}{v^-(r)} = - \int_r^a \frac{dr}{\mu^- E} = \frac{\ln b/a}{2\mu^- V_0} (r^2 - a^2)$$

In practical cases μ does depend on the field strength, so that the v_D of electrons is not a linear function of the field E . For this reason this is only a rough approximation.

L'impulso in tensione generato dal moto degli elettroni può essere ricavato dalla conservazione dell'energia (l lunghezza del cilindro):

$$\Delta V^- = \frac{-Ne}{l \cdot \ln b/a} \ln r/a$$

Analogamente per gli ioni.

$$\Delta V^+ = \frac{Ne}{l \cdot \ln b/a} \ln b/r$$

Se $b \gg a$ il contributo degli elettroni è dominante:

$$\frac{\Delta V^+}{\Delta V^-} = \frac{\ln b/r}{\ln r/a}$$

e.g. $b/a=10^3$ e $r=b/2 \rightarrow$

$$\frac{\Delta V^+}{\Delta V^-} = \frac{\ln 2}{\ln 500} \sim 0.1$$

Camere a ionizzazione

- The pulse duration from ionisation chambers varies in a wide range depending on the gas mixtures (e.g., 80% Ar and 20% CF₄ provides very fast pulses, ≈ 35 ns) . The length of a tube ionisation chamber is almost unlimited, for example, a detector in the form of a gas dielectric cable with a length of 3500 m was used as a beam-loss monitor at SLAC
- For radiation-protection purposes ionisation chambers are frequently used in a current mode, rather than a pulse mode, for monitoring the personal radiation dose. These ionisation dosimeters usually consist of a cylindrical air capacitor. The capacitor is charged to a voltage U_0 . The charge carriers which are produced in the capacitor under the influence of radiation will drift to the electrodes and partially discharge the capacitor. The voltage reduction is a measure for the absorbed dose.

Camere ad ionizzazione

Camere ad ionizzazione con liquidi.

I liquidi hanno parecchi vantaggi rispetto ai gas quando usati per la misura di dE/dx o di E . La densità di un liquido è ~ 1000 volte superiore a quella del gas $\rightarrow$ anche dE/dx o il numero di ionizzazioni è ~ 1000 volte più grande.

L'energia necessaria per produrre una coppia ione-elettrone è $W_i(LAr)=24\text{eV}$, $W_i(LKr)=20.5\text{ eV}$ e $W_i(LXe)=16\text{ eV}$ $\rightarrow$ per 1 MeV di energia assorbita ci si attende $N \geq 4 \times 10^4$ elettroni $\rightarrow dN/N = \sigma(E)/E = N^{-1/2} < 10^{-2}$.

Elementi nobili liquidi sono usati quali calorimetri (quasi omogenei) sia elettromagnetici che adronici. Il problema maggiore sono le impurità elettronegative (essenzialmente ossigeno), ma è possibile raggiungere impurità non superiori a $0.2 \div 8\text{ ppm}$. Il cammino libero medio λ_t degli elettroni (prima che vengano catturati dalle impurità) è inversamente proporzionale alla concentrazione k delle impurità. Con basse concentrazioni di impurità, λ_t può essere qualche mm $\rightarrow$ camere ad ionizzazione con gap di qualche mm.

La mobilità μ_e in argon liquido (purificato) con un campo $E = 1\text{MV/m}$ è $\mu_e = 4 \times 10^{-3}\text{ m}^2/(\text{Vs})$ $\rightarrow v_D = 4 \times 10^3\text{ m/s}$ simile a quella in argon gassoso con un campo $E = 100\text{ KV/m}$.

In compenso la mobilità degli ioni nei liquidi è molto bassa $\rightarrow$ possiamo trascurare il moto degli ioni ancor più che nelle camere ad ionizzazione a gas.

Camere a ionizzazione

Abbiamo introdotto la camera ad ionizzazione con gas, essenzialmente per capire come si forma il segnale.

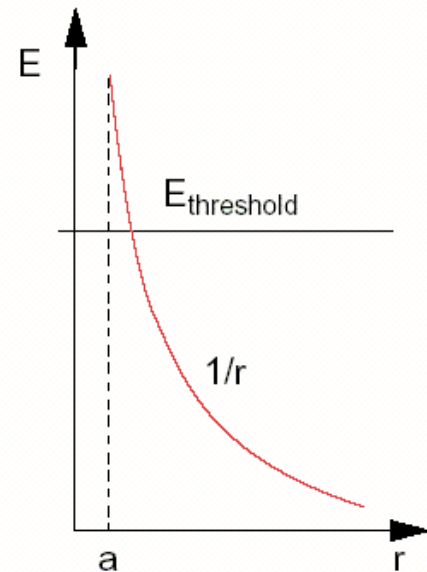
Il segnale, dovuto essenzialmente agli elettroni è comunque molto piccolo, in quanto poche sono le coppie prodotte.

Camere ad ionizzazione sono invece spesso usate con elementi liquidi nobili. (e.g. calorimetri a Argon, Krypton o Xenon liquidi) o in rivelatori a stato solido, silicio.

Avalanche Multiplication

- In ionisation chambers the primary ionisation produced by the incident particle is merely collected via the applied electric field.
- If, however, the field strength in some region of the counter volume is high, an electron can gain enough energy between two collisions to ionise another atom. Then the number of charge carriers increases. In cylindrical chambers the maximum field strength is around the thin-diameter anode wires due to the $1/r$ dependence of the electric field.
- The signal amplitude is increased by the gas amplification factor $A(E)$; therefore one gets

$$dU = A(E) (eN/C)$$



Avalanche Multiplication

- The energy gain between two collisions is $\Delta E_{\text{kin}} = eE\lambda_0$ assuming that the field strength E does not change over the mean free path length λ_0 .
- To consider the multiplication process let us take a simple model. **When the electron energy ΔE_{kin} at the collision is lower than a certain threshold, w_{ion} , the electron loses its energy without ionisation, while, when $\Delta E_{\text{kin}} > w_{\text{ion}}$, ionisation nearly always occurs.**
- The probability for an electron to pass the distance $\lambda > \lambda_{\text{ion}} = w_{\text{ion}}/(eE)$ without collision is $e^{-\lambda_{\text{ion}}/\lambda_0}$ (Poisson!) (NB: λ_{ion} depends on E).
- Since an electron experiences $1/\lambda_0$ collisions per unit length, the total number of ionisation acts per unit length – or the **first Townsend coefficient** – can be written as

$$\alpha = \frac{1}{\lambda_0} e^{-\lambda_{\text{ion}}/\lambda_0}$$

- Taking into account the inverse proportionality of λ_0 to the gas pressure p , this can be rewritten as

$$\frac{\alpha}{p} = a \cdot e^{\frac{b}{E/p}}$$

- where a and b are constants (i.e. not dependent on E and p). In spite of its simplicity, this model reasonably describes the observed dependence when a and b are determined from experiment.
- **In general $\alpha = \alpha(E)$ and if $E = E(r)$, $\alpha = \alpha(r)$**

Avalanche Multiplication

■ The change in the no. of electrons dn is: $dn = n\alpha(r)dr$

■ For a uniform field: $n = n_{prim} \exp(\alpha r)$

■ The first Townsend coefficient α depends on the field strength E and thereby on the position x in the gas counter. Therefore, more generally, it holds that

$$n = n_{prim} \exp \int_a^{r_c} \alpha(r) dr$$

■ where the gas amplification factor is given by $A = \exp \int_a^{r_c} \alpha(r) dr$

r_c = radius at which $E=E_c$ (critical value for which avalanche multiplication starts)

a = radius of anode

Avalanche Multiplication

- The proportional range of a counter is characterised by the fact that the gas amplification factor A takes a constant value. As a consequence, the measured signal is proportional to the produced ionisation.

$$\Delta U = A \text{ (Ne/C)}$$

- Gas amplification factors of up to 10^6 are possible in the proportional mode. Typical gas amplifications are rather in the range between 10^4 up to 10^5 .

$$A = \exp \int_a^{r_c} \alpha(r) dr$$

r_c = radius at which $E=E_c$ (critical value for which avalanche multiplication starts)

a = radius of anode

Avalanche Multiplication

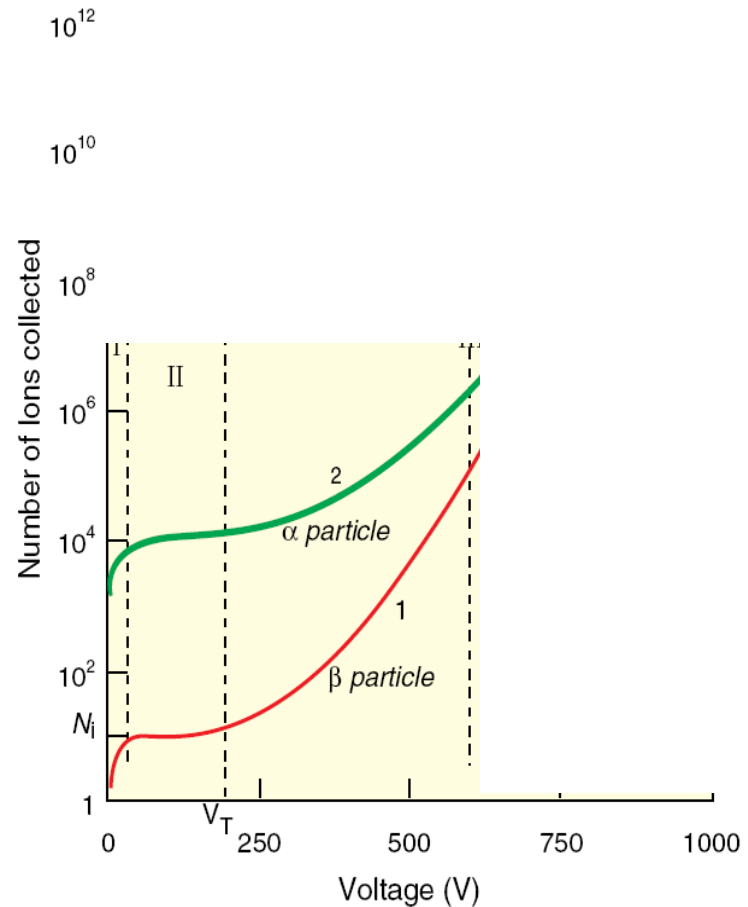
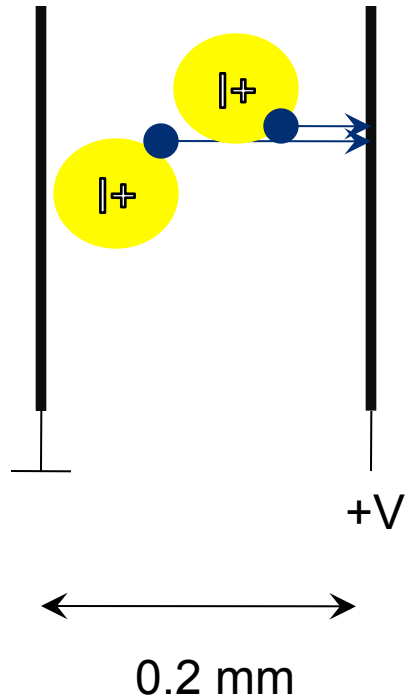
$$\frac{n}{n_{prim}} = \exp \int_a^{r_c} \alpha(r) dr$$

- This is a constant for a given detector, hence such a detector is called a **“proportional counter”**
 - Measured voltage pulse is proportional to the total primary ionization, which is in turn proportional to the total energy loss of the incident particle
 - Measured voltage pulse is also proportional to CV_0
- Some typical values:

r (μm)	E(kV/cm)	α (ion pairs/cm)	λ (μm)
10	200	4000	2.5
20	100	2000	5
100	20	80	125
200	10	~1	1000

- 50% (90%) of electrons are produced within 2.5 (10) μm of the sense wire

Gas amplification: capacitor with gas



“Proportional” mode, linear amplification...

$$\Delta U = A \text{ (Ne/C)}$$

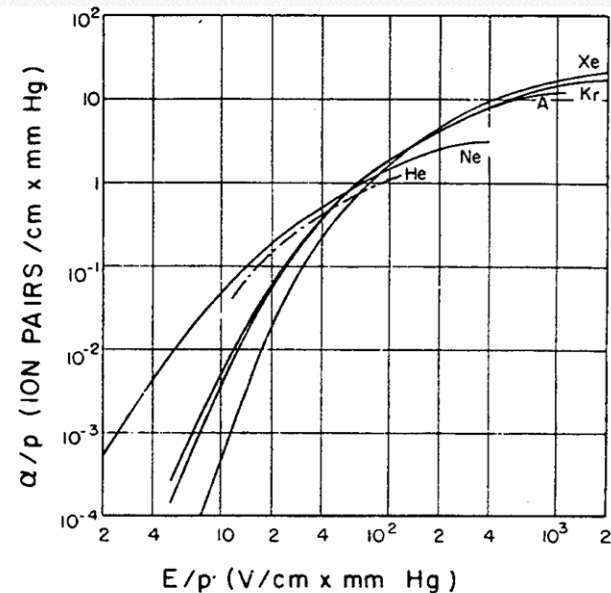
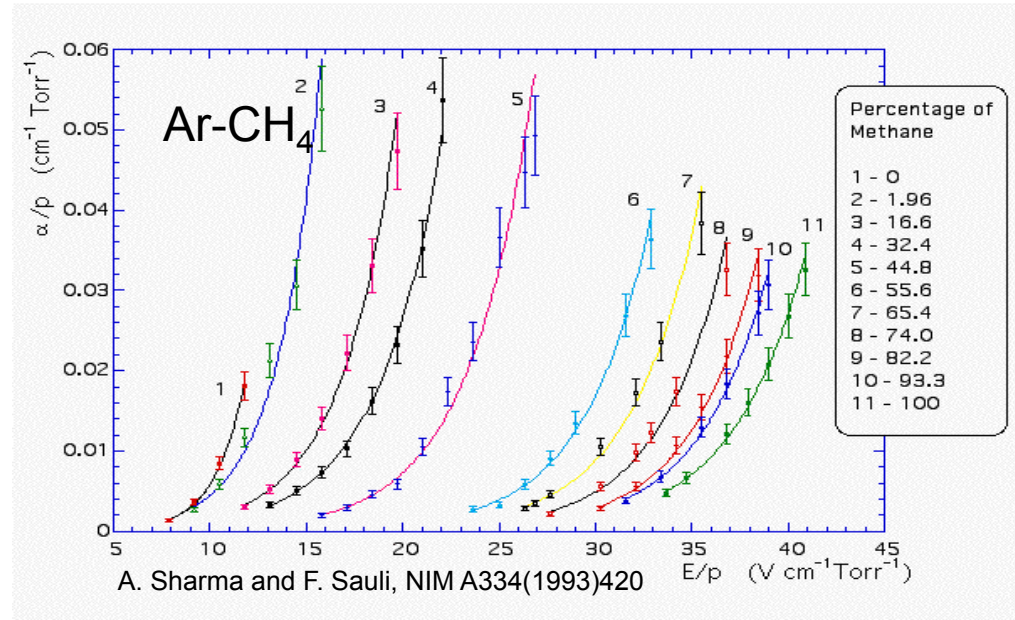
Single Wire Proportional Chamber

$$A = \exp \int_a^{r_c} \alpha(r) dr$$

$\alpha(E)$ is determined by the excitation and ionization cross sections of the electrons in the gas.

It depends also on various and complex energy transfer mechanisms between gas molecules. There is no fundamental expression for $\alpha(E) \rightarrow$ it has to be measured for every gas mixture.

Amplification factor or
Gain



Contatori proporzionali

$$n = n_0 e^{\alpha(E)x} \quad \text{or} \quad n = n_0 e^{\alpha(r)x} \quad \alpha: \text{First Townsend coefficient (e-ion pairs/cm)}$$

$$A = \frac{n}{n_0} = \exp \left[\int_a^{r_c} \alpha(r) dr \right] \quad \text{Gain} \quad \Delta U = A \text{ (Ne/C)}$$

If U_{th} is the threshold voltage for the onset of the proportional range, the gas amplification factor expressed by the detector parameters is

$$A = \exp \left\{ 2 \sqrt{\frac{k L C U_0 r_i}{2 \pi \epsilon_0}} \left[\sqrt{\frac{U_0}{U_{th}}} - 1 \right] \right\}$$

in the case $U_0 \gg U_{th}$ Eq. (5.34) simplifies to $A \approx k e^{U/U_{ref}} \quad A \approx k e^{C V_0}$

where k is a "magic" constant dependent on the gas

Limitazioni alla moltiplicazione

La presenza di gas **elettronegativi** (Fluoro, Ossigeno ...) può limitare lo sviluppo di una valanga attraverso la cattura di elettroni liberi:

$$-\frac{dn}{dx} = \eta(x)x$$

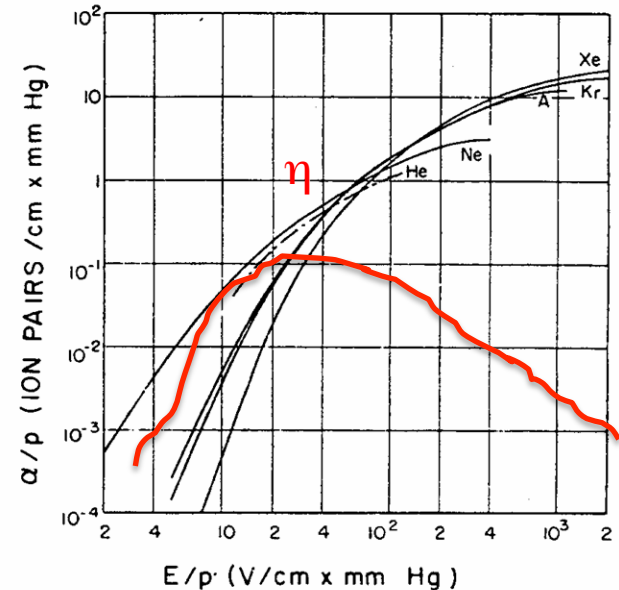
η coefficiente di attachment.

Il guadagno effettivo diventa quindi:

$$M = \exp \left[\int (\alpha(x) - \eta(x)) dx \right]$$

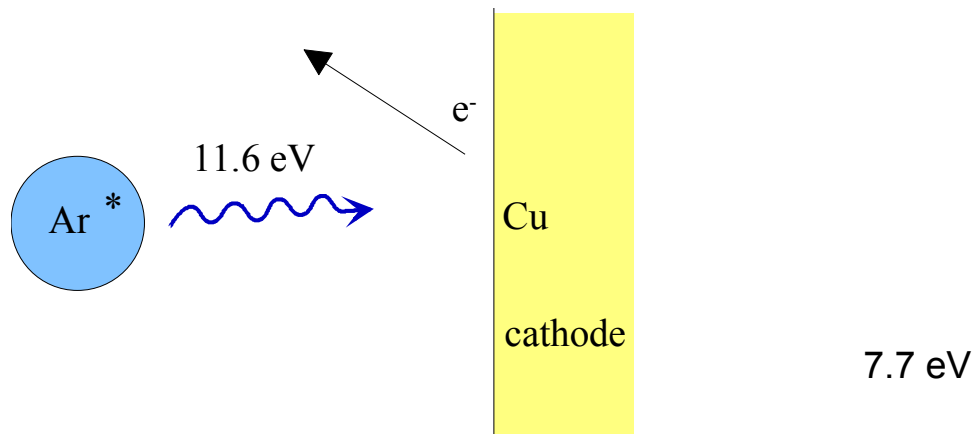
Inoltre se la densità di carica cresce molto si può avere la formazione di scariche nel gas.

Uno studio fenomenologico ha portato al limite di **Raether** per cui le scariche si innescano se in una valanga sono presenti più di $10^7 \div 10^8$ elettroni.



Photoemission

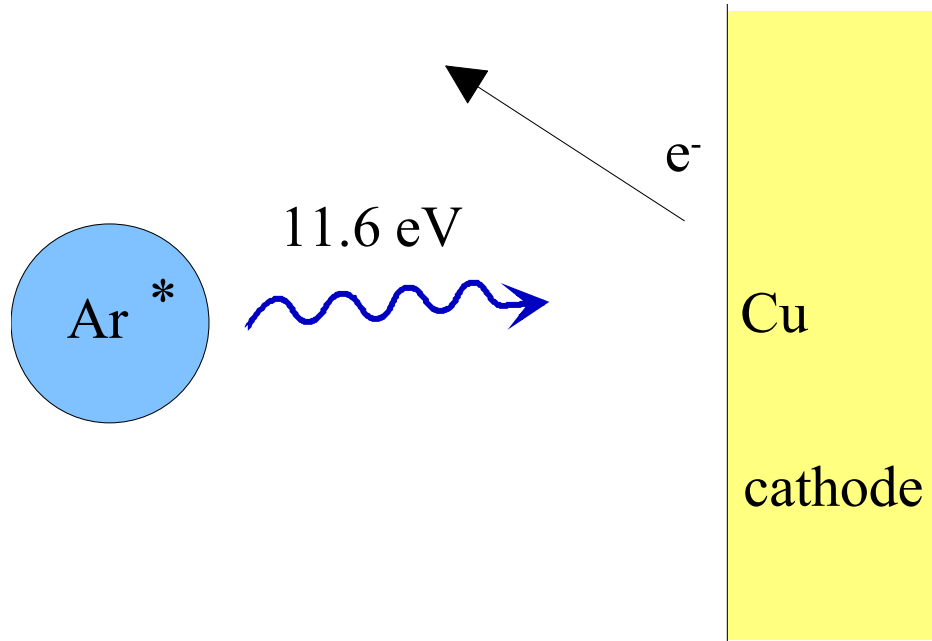
- At high field collisions of electrons with atoms or molecules can cause not only ionisation but also excitation.
- De-excitation is often followed by photon emission. The previous considerations are only true as long as photons produced in the course of the avalanche development are of no importance.
- These photons, however, will produce further electrons by the photoelectric effect in the gas or at the counter wall, which affect the avalanche development. Apart from gas-amplified primary electrons, secondary avalanches initiated by the photoelectric processes must also be taken into account.



Photoemission

In the avalanche process molecules of the gas can be brought to excited states.

De-excitation of noble gases only via emission of photons;
e.g. 11.6 eV for Ar.
This is above ionization threshold of metals;
e.g. Cu 7.7 eV.



When gain exceeds about 10^8 - new avalanches $\rightarrow$ increase of the discharge current

- In the first generation, N_0 primary electrons are produced by the ionising particle. These N_0 electrons are gas amplified by a factor A . If γ is the probability that one photoelectron per electron is produced in the avalanche, an additional number of $\gamma(N_0A)$ photoelectrons is produced via photoprocesses. These, however, are again gas amplified so that in the second generation $(\gamma N_0A) \cdot A = \gamma N_0A^2$ gas-amplified photoelectrons are created, which again create $(\gamma N_0A^2)\gamma$ further photoelectrons in the gas amplification process, which again are gas amplified themselves. The gas amplification A_γ under inclusion of photons, therefore, is obtained from

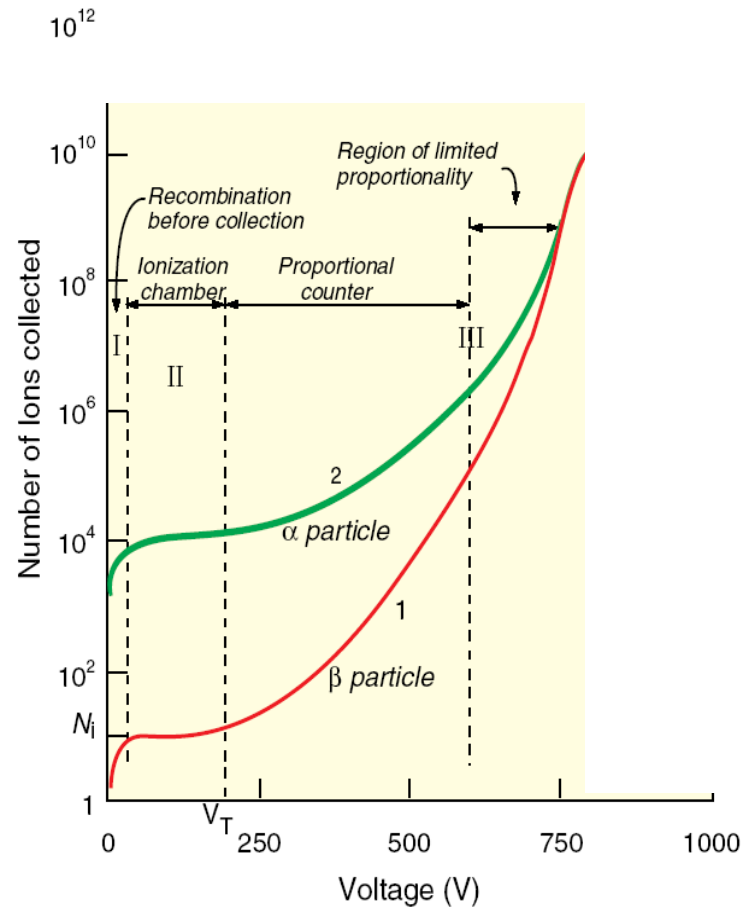
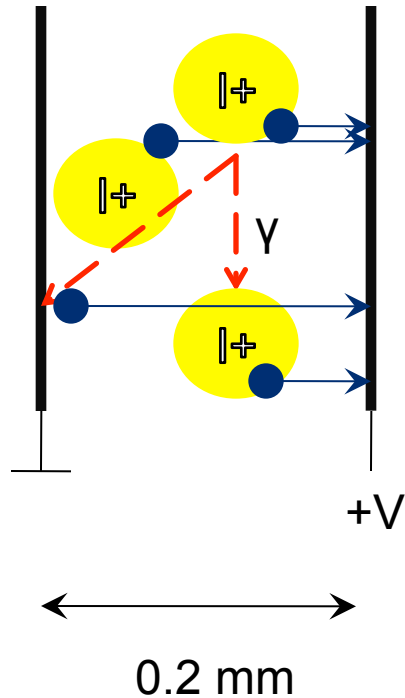
$$\begin{aligned}
 N_0A_\gamma &= N_0A + N_0A^2\gamma + N_0A^3\gamma^2 + \dots \\
 &= N_0A \cdot \sum_{k=0}^{\infty} (A\gamma)^k = \frac{N_0A}{1 - \gamma A} \quad \text{therefore } A_\gamma = A/(1 - \gamma A)
 \end{aligned}$$

The factor γ , which determines the gas amplification under inclusion of photons, is also called the second Townsend coefficient.

As the number of produced charges increases, they begin to have an effect on the external applied field and saturation effects occur. For $\gamma A \rightarrow 1$ the signal amplitude will be independent of the primary ionisation. The proportional or, rather, the saturated proportional region is limited by gas amplification factors around $A_\gamma = 10^8$.

Gas amplification: capacitor with gas

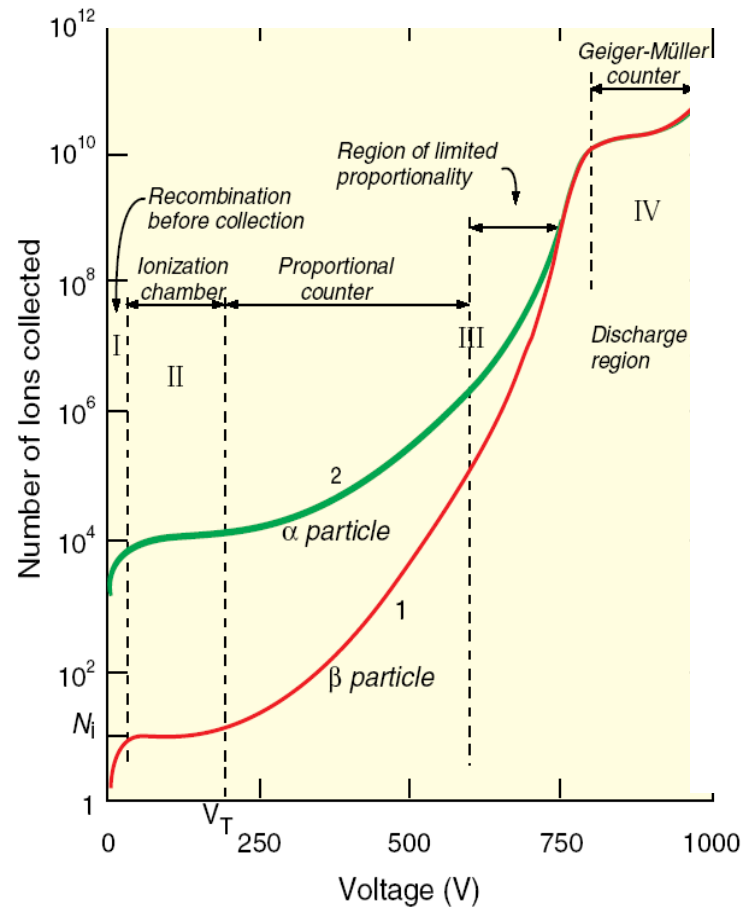
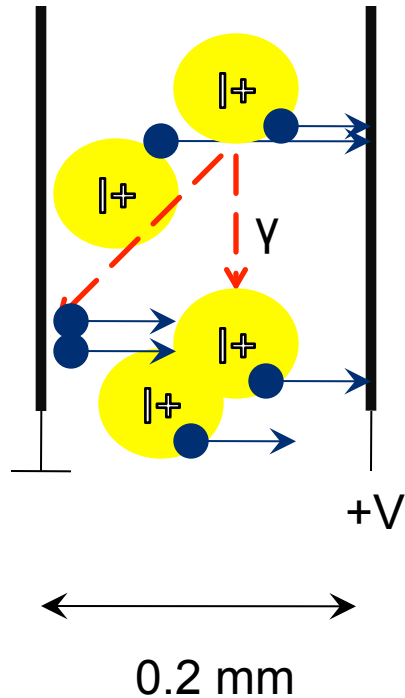
Photoemission starts...



“Saturated” mode, logarithmic amplification, saturation...

Gas amplification: capacitor with gas

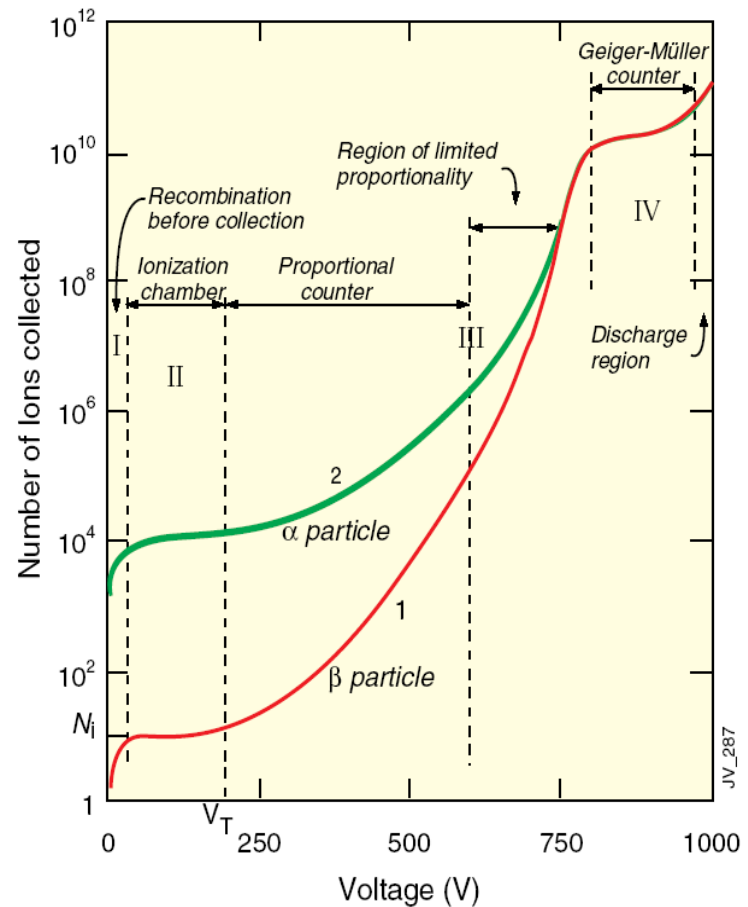
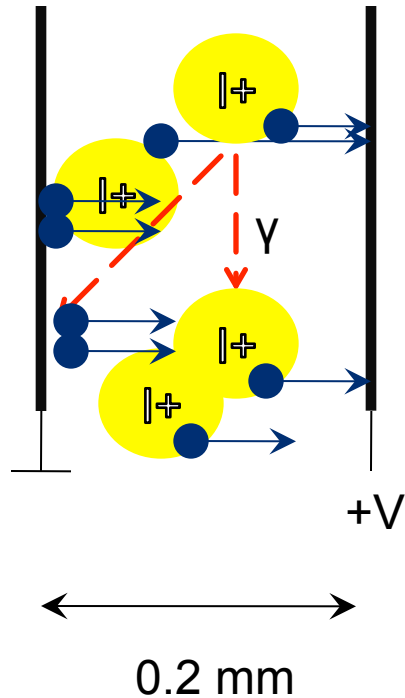
Strong photoemission...



“Geiger” mode, only counting is possible, info about primary ionization is lost!

Gas amplification: capacitor with gas

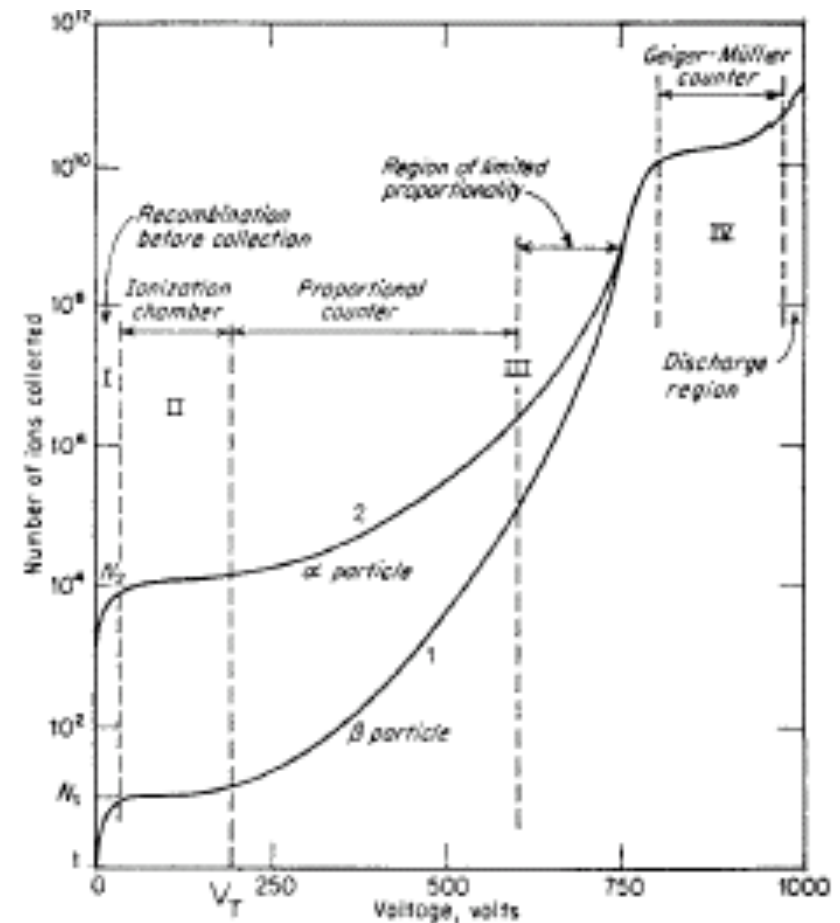
Strong photoemission, ion impact ionisation...



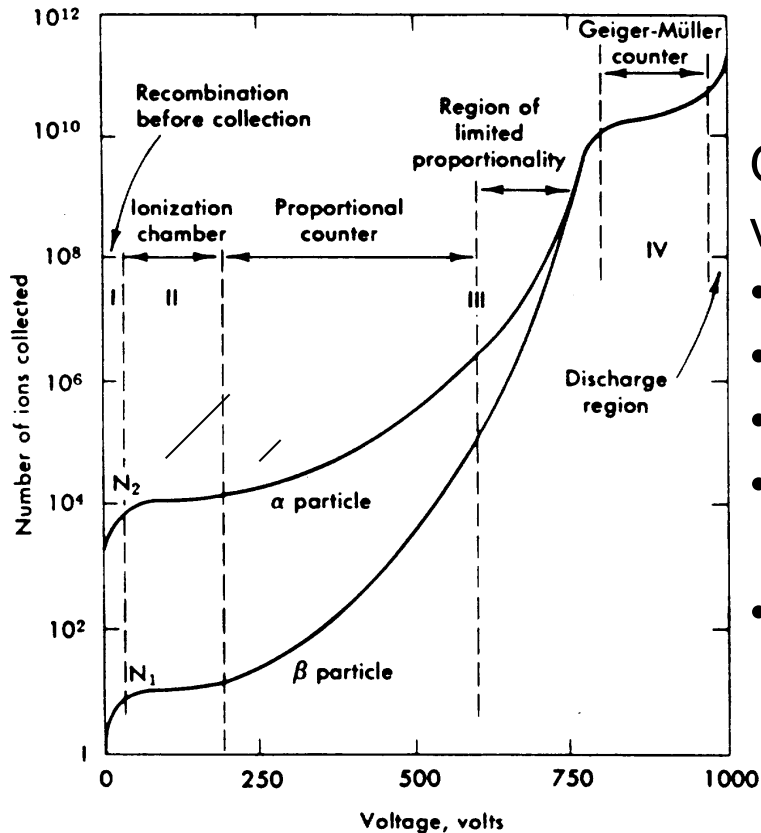
“Saturated” mode, logarithmic amplification, saturation...

Misure di posizione e ionizzazione

- **ionizzazione**: si raccoglie tutta la carica nessuna moltiplicazione delle coppie ione-elettrone.
- **proporzionale**: presente una moltiplicazione a valanga. Il segnale dell'apparato è proporzionale alla ionizzazione → misura di dE/dx e guadagno 10^4 - 10^6
- **proporzionale limitato → saturazione → streamer**. Forte emissione di fotoni, moltiplicazioni a valanga secondarie, alti guadagni (10^{10}) → elettronica semplice.
- **geiger**: grossa fotoemissione, il filo anodico è tutto coinvolto, regime di scarica eliminata abbassando HV. Necessari forti moderatori.



Wire Chamber Operation



Operating characteristics depend on the applied voltage (E-field)

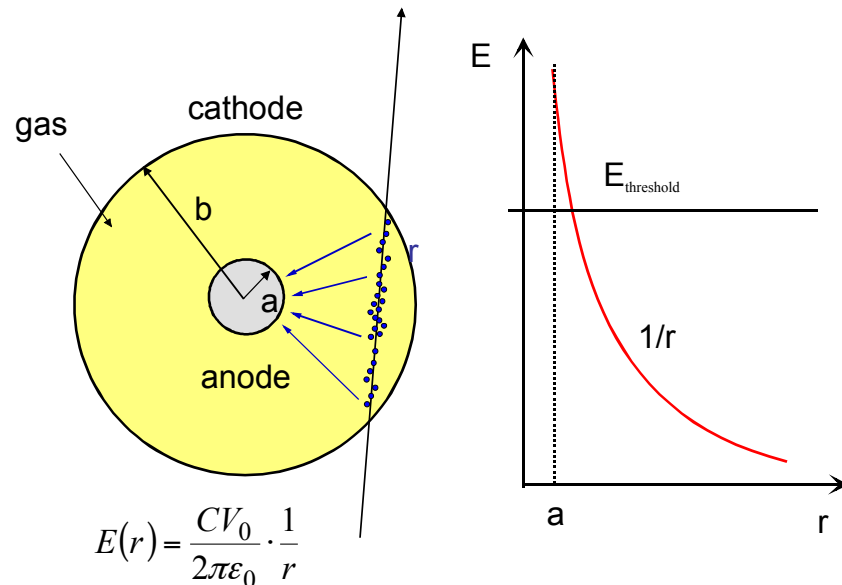
- recombination: no signals
- ionization: signals, but no gas gain
- proportional: “big” signals due to gas gain
- limited proportionality: very large signals, almost independent of initial ionisation
- Geiger-Muller: gas gain so large it produces sparks (discharge)

- Have to run in ionisation or proportional mode to measure dE/dX of incident particles.
- In limited proportionality and geiger mode only counting is possible.
- In both cases, a position measurement is also possible (if we know the "position" of the detector).

Contatori proporzionali

Il contatore proporzionale cilindrico.

Essenzialmente identico alla camera ad ionizzazione cilindrica ma il segnale è dato dal moto degli ioni positivi invece che dal moto degli elettroni.



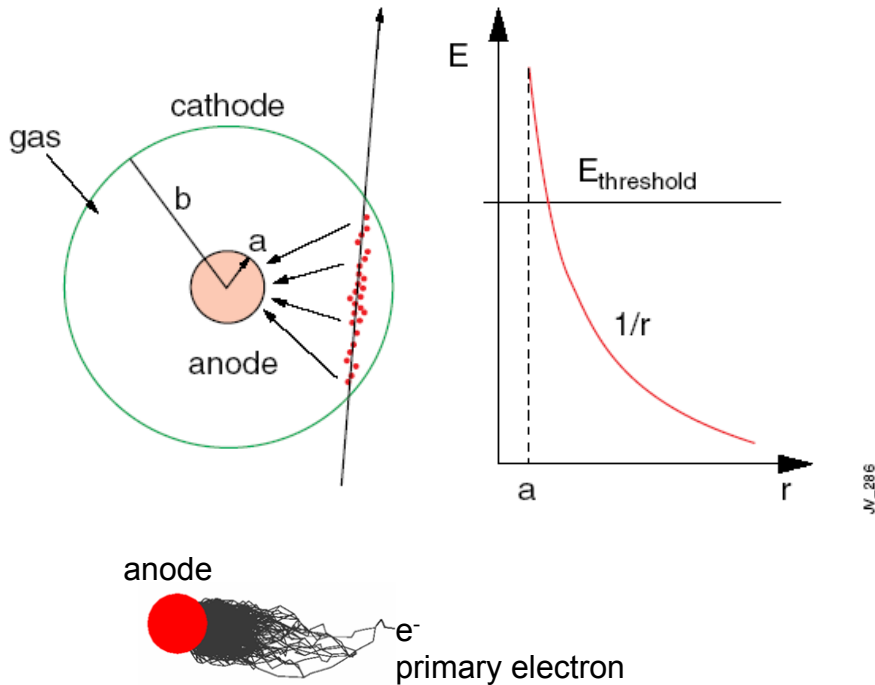
$$E(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \frac{1}{r}$$

$$V(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \ln \frac{r}{a} \quad C = \text{capacitance / unit length} \quad (C = 2\pi\epsilon / \ln(b/a))$$

Gli e driftano verso l'anodo dove il campo è sufficientemente alto (alcuni KV/cm), ed acquistano abbastanza energia da moltiplicarsi.

$$n = n_0 e^{\int_{x_1}^{x_2} \alpha(x) dx}$$

Single Wire Proportional Chamber



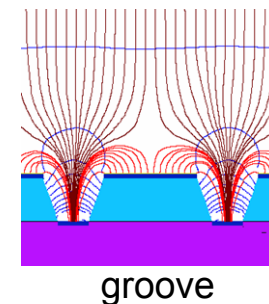
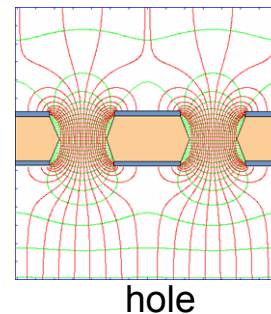
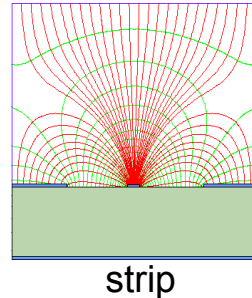
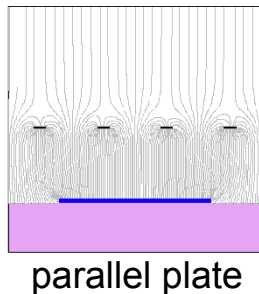
Electrons liberated by ionization drift towards the anode wire.

Electrical field close to the wire (typical wire $\varnothing$ ~few tens of μm) is sufficiently high for electrons (above 10 kV/cm) to gain enough energy to ionize further $\rightarrow$ **avalanche** – exponential increase of number of electron ion pairs
- **the proportional operation mode**.

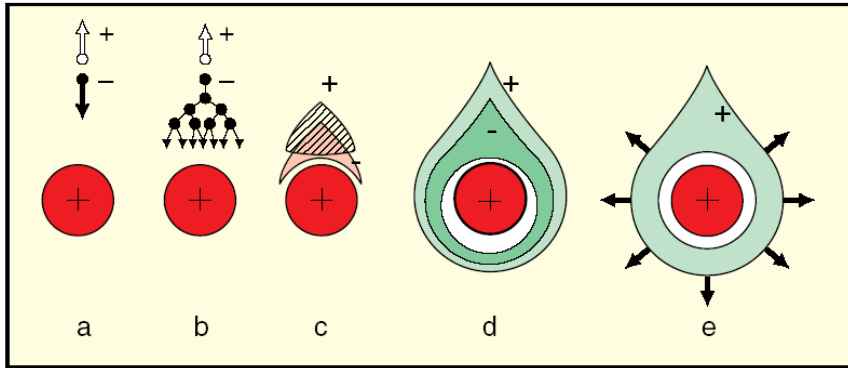
$$E(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \frac{1}{r} \quad M = \frac{n}{n_0} = \exp\left[\int_a^{r_c} \alpha(r) dr\right]$$

$$V(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \ln \frac{r}{a} \quad C - \text{capacitance/unit length}$$

Cylindrical geometry is not the only one able to generate strong electric field:



SWPC – Signal Formation

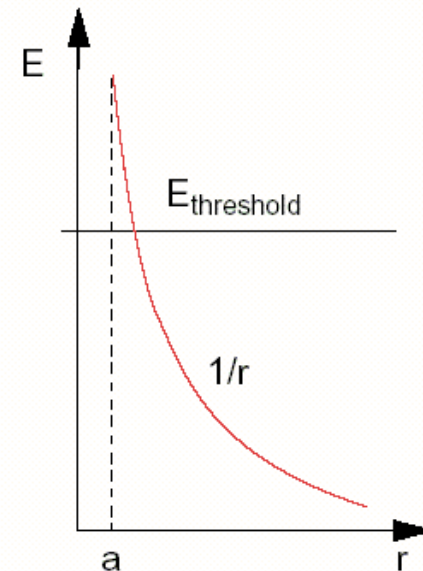


Electrons collected by the anode wire i.e. dr is very small (few μm) – almost no induction signal

Ions have to drift back to cathode i.e. dr is large (few mm). Signal duration limited by total ion drift time.

Need electronic signal differentiation to limit dead time.

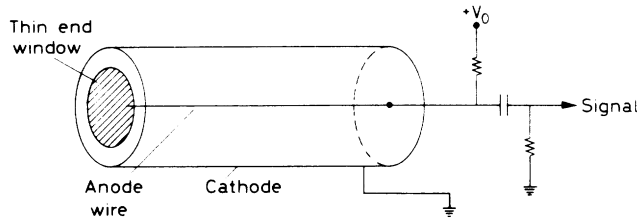
Avalanche formation within a few wire radii and within $t < 1$ ns, where The electric field goes above threshold for avalanche formation. Signal induction both on anode and cathode due to moving charges (both electrons and ions).



t (ns)

Pulse Formation in a Cylindrical Wire Chamber

In a cylindrical chamber the electric potential, E-field and capacitance are given by:



$$V(r) = -\frac{CV_0}{2\pi\epsilon L} \ln(r/a) \quad E(r) = \frac{CV_0}{2\pi\epsilon L} \frac{1}{r} \quad C = \frac{2\pi\epsilon L}{\ln(b/a)}$$

Note: I put the L dependence in φ , E, and C. Leo's C is my C/L.

wire radius= a, tube radius=b, length of tube= L

The potential energy stored in the electric field is $W = 1/2 CV_0^2$.

Assume a charged particle goes through the cylinder and ionizes the gas.

As a charge, q, moves a distance dr there is a change in the potential energy (dW):

$$dW = q \frac{d\varphi(r)}{dr} dr \quad \text{and} \quad dW = CV_0 dV \Rightarrow dV = \frac{q}{CV_0} \frac{d\varphi(r)}{dr} dr$$

The total induced voltage from electrons produced at r' is:

$$V^- = \frac{-q}{CV_0} \int_{a+r'}^a \frac{d\varphi(r)}{dr} dr = \frac{-q}{CV_0} \int_{a+r'}^a \frac{(-CV_0)}{2\pi\epsilon L} \frac{dr}{r} = \frac{(-q)}{2\pi\epsilon L} \ln \frac{a+r'}{a}$$

The total induced voltage from positive ions produced at r' is:

$$V^+ = \frac{+q}{CV_0} \int_{a+r'}^b \frac{d\varphi(r)}{dr} dr = \frac{+q}{CV_0} \int_{a+r'}^b \frac{(-CV_0)}{2\pi\epsilon L} \frac{dr}{r} = \frac{-q}{2\pi\epsilon L} \ln \frac{b}{a+r'}$$

Note: the total induced voltage is: $\Delta V = V^+ + V^- = -q/C$

Pulse formation in a cylindrical wire chamber

Note: the positive ions and electrons do not contribute equally to the ΔV if there is multiplication in the gas. Since the avalanche takes place near the wire ($r' = 1\text{-}2\mu\text{m}$) and the electrons are attracted to the wire the positive ions travel a much greater distance.

For typical values of a ($10\mu\text{m}$) and b (1cm) we find:

$$\frac{V^+}{V^-} = \frac{\ln \frac{b}{a+r'}}{\ln \frac{a+r'}{a}} \approx \frac{\ln 10^3}{\ln(11/10)} \approx 75$$

So in proportional chambers the amplified signal comes almost entirely from ions drift

Pulse formation in a cylindrical wire chamber

We can find the voltage vs time by looking at $V(t)$ for the positive ions:

$$V(t) = V^+(t) = \int_{r(0)=a}^{r(t)} \frac{dV(r)}{dr} dr = \frac{-q}{2\pi\epsilon L} \ln \frac{r(t)}{a}$$

The problem now is to find $r(t)$.

By definition, the mobility, μ , of a gas is the ratio of its drift velocity to electric field.

$$\mu \equiv v / E(r) = \frac{1}{E(r)} \frac{dr}{dt}$$

For cylindrical geometry we have:

$$\frac{dr}{dt} = \mu E(t) = \mu \frac{CV_0}{2\pi\epsilon L} \frac{1}{r} \Rightarrow r dr = \mu \frac{CV_0}{2\pi\epsilon L} dt$$

Pulse Formation in a Cylindrical Wire Chamber

From previous page we had: $rdr = \mu \frac{CV_0}{2\pi\epsilon L} dt$

$$\int_{r(0)=a}^{r(t)} r dr = \mu \frac{CV_0}{2\pi\epsilon L} \int_0^t dt \Rightarrow r(t) = \left(a^2 + \frac{\mu CV_0}{\pi\epsilon L} t \right)^{1/2}$$

$$V(t) = \frac{-q}{2\pi\epsilon L} \ln \frac{r(t)}{a} = \frac{-q}{4\pi\epsilon L} \ln \left(1 + \frac{\mu CV_0}{\pi\epsilon L a^2} t \right) = \frac{-q}{4\pi\epsilon L} \ln \left(1 + \frac{t}{t_0} \right) \quad \text{With: } t_0 = \frac{a^2 \ln(b/a)}{2\mu V_0}$$

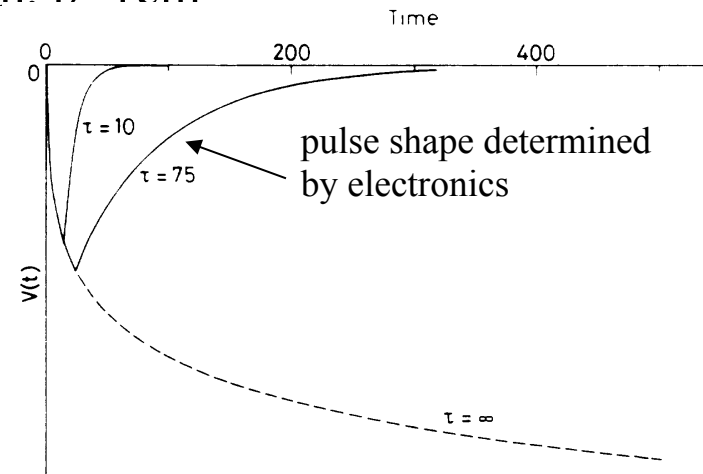
The total drift time is: $T = \frac{t_0}{a^2} (b^2 - a^2) \approx \frac{b^2}{a^2} t_0$

Typical gas mobilities are $\mu = 1-2 \text{ cm}^2\text{s}^{-1}\text{V}^{-1}$.

Example: Let $\mu = 1.5 \text{ cm}^2\text{s}^{-1}\text{V}^{-1}$, $V = 1500\text{V}$, $a = 10\mu\text{m}$, $b = 1\text{cm}$

then: $t_0 = 1.5 \times 10^{-9} \text{ s}$ and $T = 1.5 \times 10^{-3} \text{ s}$.

t	$\ln[1+t/t_0]$
0	0
t_0	0.69
$10t_0$	2.4
$10^2 t_0$	4.6
$10^3 t_0$	6.9
T	13.8

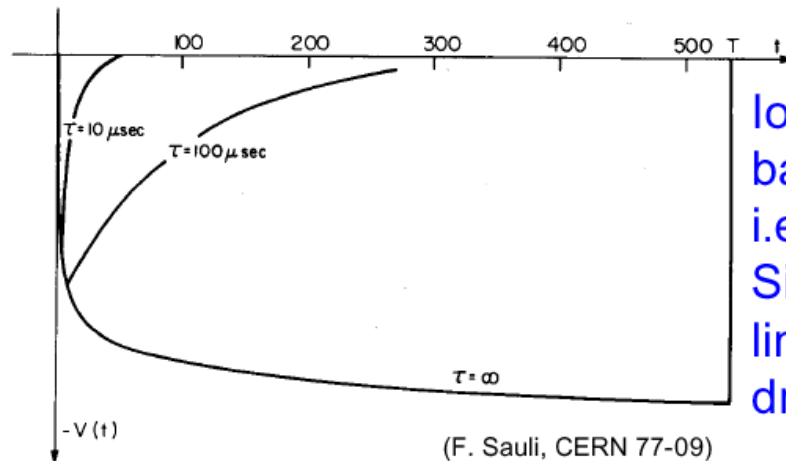


Time development of voltage pulse

Ionisation Detectors

In modern fast ionisation detectors the electrons are used (faster) as well as the beginning of the ion signal.

For example if we use 5% of the signal with a gain 10^4 we still have a healthy signal compared to the noise – and we can operate mostly with the fast part of the signal (electrons) and differentiate away the tails



Ions have to drift back to cathode, i.e. dr is big. Signal duration limited by total ion drift time !

Need electronic signal differentiation to limit dead time.