

Particle Detectors

Lecture 14

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Ionizzazione dei gas

Cominciamo a descrivere il dettaglio del meccanismo di raccolta della carica creata dal passaggio di una particella.

- Una volta generata dal deposito di energia, la carica deve "viaggiare" nel mezzo fino agli elettrodi di raccolta.
- Durante la propagazione possono avvenire vari processi che influenzano la raccolta di carica, sia in termini di efficienza di raccolta ($Q_{\text{raccolta}}/Q_{\text{generata}}$) che di distribuzione spaziale e temporale del segnale → **risoluzione spaziale e temporale**.
- Tratteremo separatamente la ionizzazione in rivelatori a gas e quella in rivelatori a stato solido (ovvero silicio), con particolare attenzione alle problematiche relative alla precisione della misura di posizione, ovvero per applicazioni a misure di posizione, cioè dispositivi di tracciamento delle particelle. La trattazione formale è la stessa (cioè entrambi i tipi di rivelatori sono governati dallo stesso tipo di processi).

Ionizzazione dei gas

- ❑ The primary modes of interaction involve ionization and excitation of gas molecules along the particle track. Although the excited molecules can at times be used to derive an appropriate signal (as in the gas scintillators discussed), the majority of gas-filled detectors are based on sensing the direct ionization created by the passage of the radiation.
- ❑ The term ***ionization* chamber** has conventionally come to be used **exclusively for the type of detector in which ion pairs are collected from gases**. The corresponding process in solids is the collection of electron-hole pairs in the semiconductor detectors. Ion chambers in principle are the simplest of all gas-filled detectors.
- ❑ **Their normal operation is based on collection of all the charges created by direct ionization within the gas through the application of an electric field.**
- ❑ Ion chambers can be operated in current or pulse mode.

Ionizzazione dei gas

As a fast charged particle passes through a gas, the interaction creates both excited molecules and ionized molecules along its path: $x + A \rightarrow x + A^+ + e^-$, $A^{++} + e^- + e^-$,

- ❑ After a neutral molecule is ionized, **the resulting positive ion and free electron are called an *ion pair***, and it serves as the basic constituent of the electrical signal developed by the ion chamber. Ions can be formed either by direct interaction with the incident particle, or through a secondary process in which some of the particle energy is first transferred to an energetic electron or "delta ray", which in turn ionizes.
- ❑ The pairs, generated around the particle trajectory, form a ionisation cylinder of few tens of microns radius, which then evolves in time and space.
- ❑ Regardless of the detailed mechanisms involved, the practical quantity of interest is the total number of ion pairs created along the track of the radiation.

Ionization cylinder

Ground

Intrinsic collection time is O(tens of ns)

$t=5$ ns

Time evolution of current

300 um Silicon layer

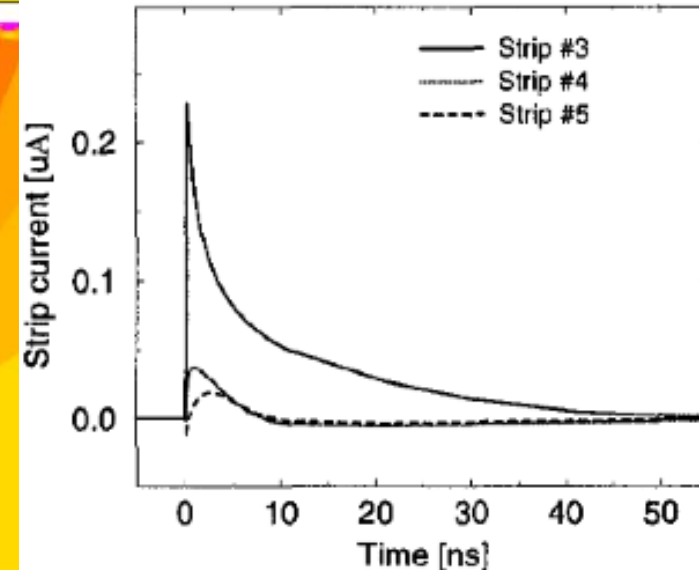
$t=0$

V_{bias}

Il raggio del cilindro di ionizzazione e' fissato essenzialmente dal range dei raggi δ della ionizzazione secondaria

$t=12$ ns

$t=30$ ns



Ionizzazione dei gas

Una particella carica (veloce) che attraversa un mezzo lascia una traccia di ionizzazione lungo la sua traiettoria (Bethe-Block):

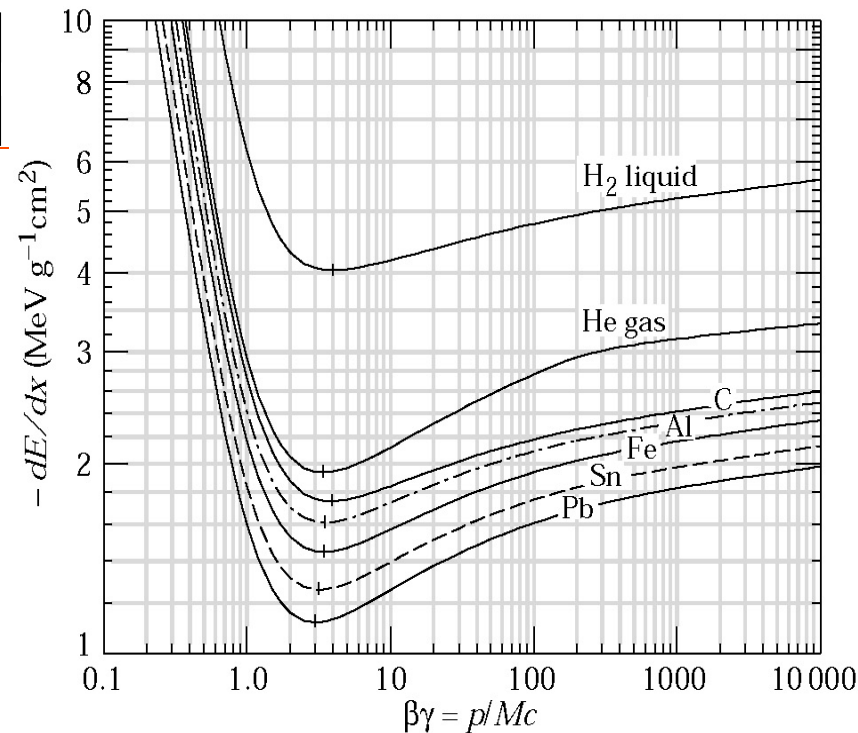
$$\frac{dE}{dx} = -KZ^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\gamma)}{2} \right]$$

Se il mezzo è molto sottile (in gr/cm²) i raggi δ molto energetici possono lasciare il mezzo → per alcune applicazioni si usa la perdita di **energia ristretta**.

$$\frac{dE}{dx} = -KZ^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2mc^2 \beta^2 \gamma^2 T_{\text{cut}}}{I^2} - \beta^2 - \frac{\delta(\gamma)}{2} \right]$$

dove T_{cut} non è più la massima energia trasferibile, ma è quella energia per cui il **range** dell'elettrone è **>** dello **spessore** del materiale.

Ricordiamo $T_{\max} = 2mc^2 \beta^2 \gamma^2$



Energy loss probability

- Trascurando lo spin dell'e- (ie scattering di Coulomb), la probabilita' di espellere un e- di energia fra E ed E + dE e' :

$$P(E)dE = k \frac{X}{E^2} dE \quad \text{where} \quad k = 2\pi N_A r_e^2 m_e c^2 Z^2 \frac{1}{A \beta^2} \quad (1)$$

Si ricordi la distribuzione del # di e- espulsi con E fra E ed E+dE in una collisione $n(E) = -\frac{dN}{dE dx} = \pi N_e \left(\frac{e^4 Z^2}{8\pi^2 \epsilon_0^2 m_e v^2} \right) \frac{1}{E^2}$

Infatti il # di interazioni e' $dN = n dV$.

dV e' il volume di interazione disponibile: per definizione di sezione d'urto, tutte gli atomi in $dV = L d\sigma$ interagiscono con la particella incidente, quindi $n L d\sigma$ e' il # di interazioni.

Poiche' osserviamo la distribuzione di energia, $d\sigma = (d\sigma/dE)dE \equiv P(E)dE$ e quindi la (1)

La distribuzioni esatta per elettroni e' $P(E)dE = k \frac{X}{E^2} (1 - \beta^2 \frac{E}{E_{\max}}) dE$

La distribuzioni esatta per particelle pesanti di spin ½ e'

$$P(E)dE = k \frac{X}{E^2} (1 - \beta^2 \frac{E}{E_{\max}} + \frac{1}{2} \frac{E}{E + mc^2}) dE$$

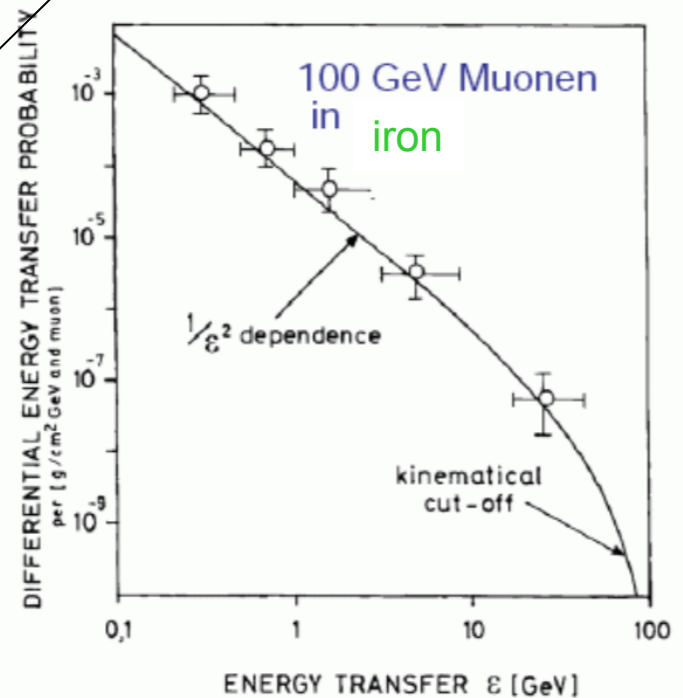
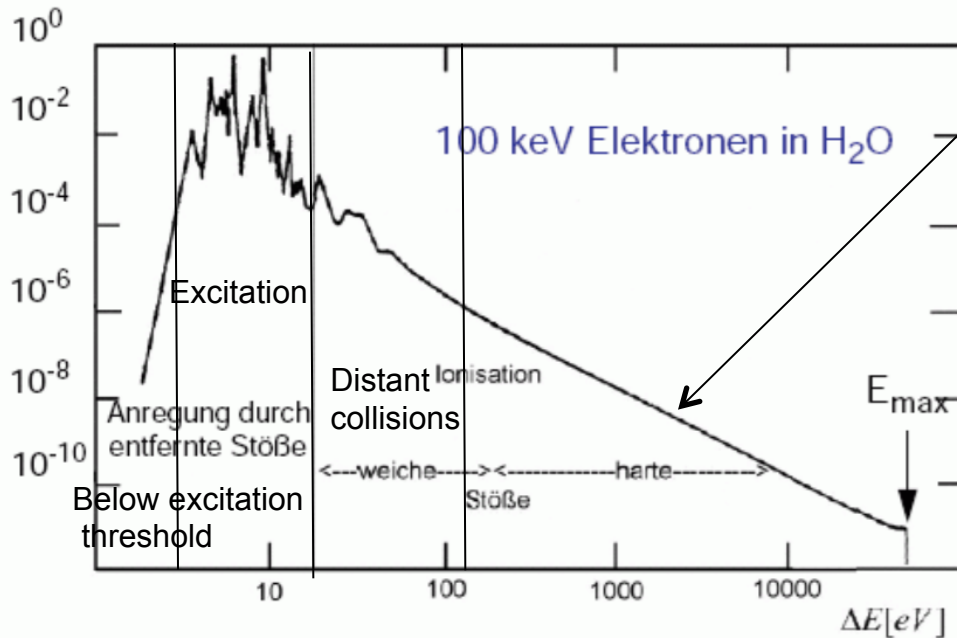
La probabilita' di emissione (cosi' come lo spettro) e' limitata dalla cinematica fra $E_{\min} < E < E_{\max}$

δ Electrons

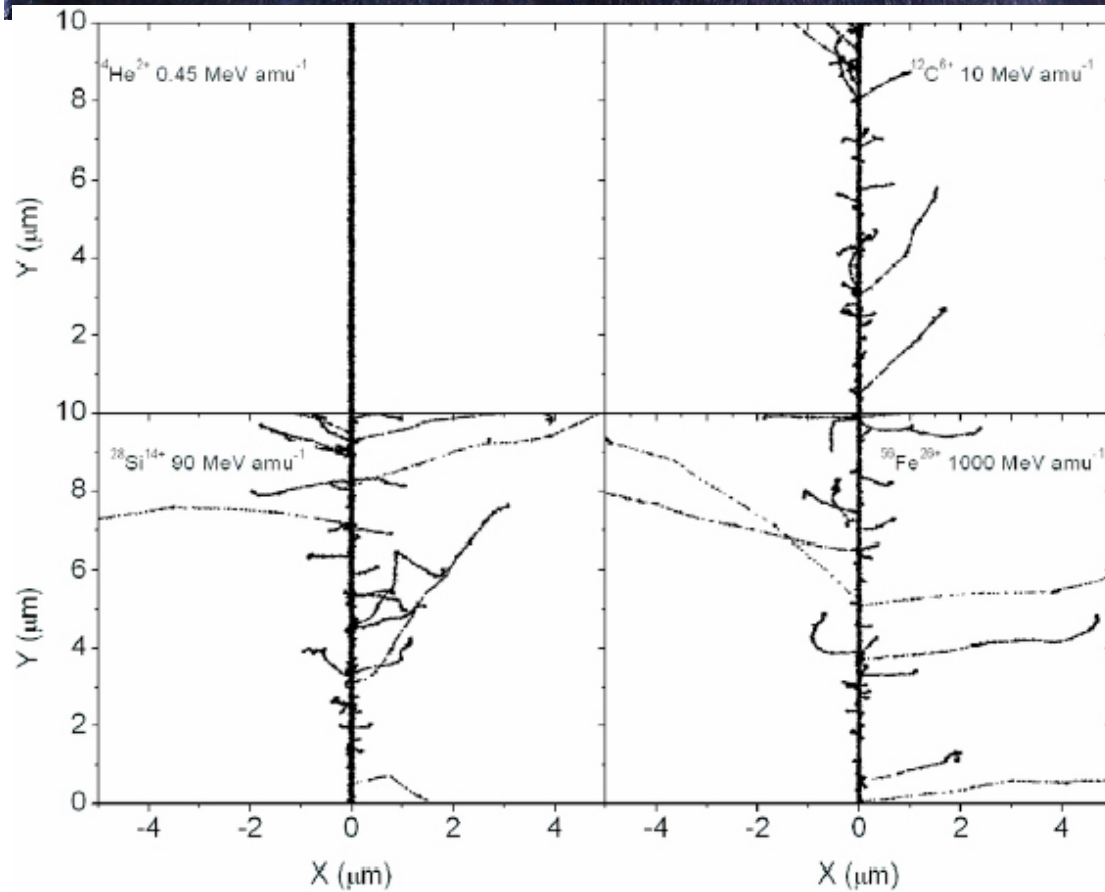
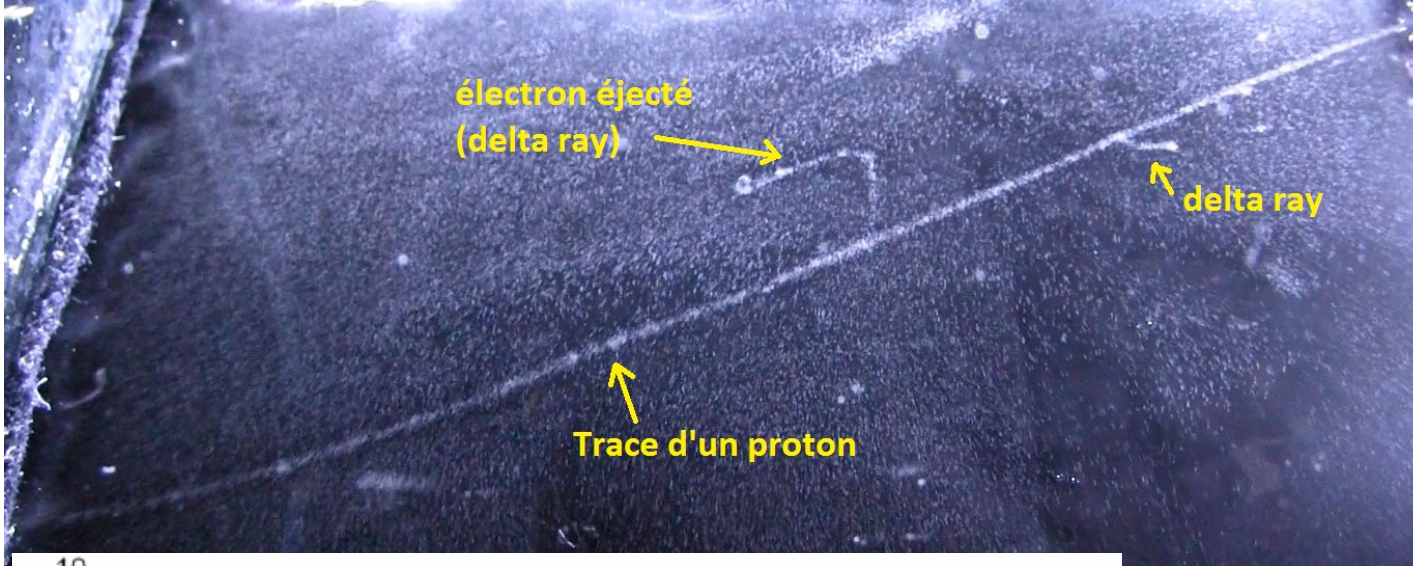
Energy distribution of secondary electrons:

$$F(T) = 1 - \beta^2 T / T_{max} \quad (\text{Spin } 0)$$

$$\frac{dN^2}{dT dx} = \frac{1}{2} k z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2}$$



Example: 500 MeV pion in 300 μm Si: 5% produce an electron with T > 166 keV
 important background in Cherenkov counter
 Number of δ-electrons proportional z²/β²



Delta rays can make additional secondary ionization trails that:

- may escape the active volume, so their energy can not be collected $\rightarrow$ potential bias to energy deposit
- confuse the main track, making difficult the pattern recognition, that is the task of assigning a energy deposit to a given track.

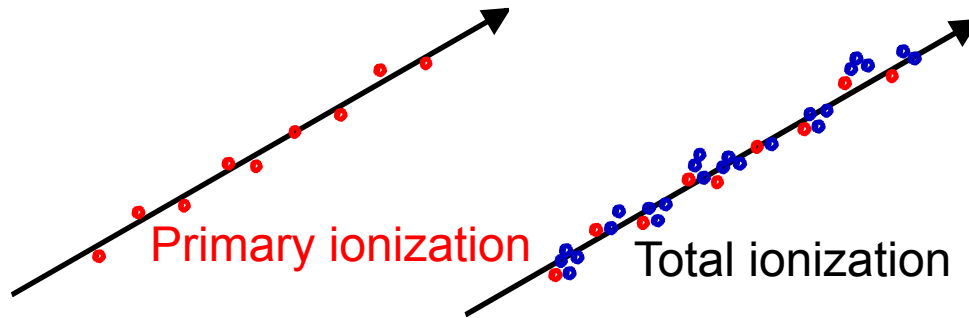
That's why we hate δ -rays

Ionizzazione dei gas

Meccanismi di ionizzazione

- **Primaria** uno o più elettroni sono strappati dall'atomo dalla particella incidente. (e.g. $\pi A \rightarrow \pi A^+ e^-$, $\pi A^{++} e^- e^- \dots$)
- **Secondaria istantanea**: gli elettroni sono strappati da atomi non colpiti dalla particella incidente. Questa ionizzazione è essenzialmente dovuta ai raggi δ prodotti dalla ionizzazione primaria. (il potenziale di ionizzazione medio è $\sim 10 - 20$ eV mentre la massima energia trasferibile all'atomo è $2mc^2\beta^2\gamma^2$ con una distribuzione di energia $\propto 1/E^2 \rightarrow$ è facile avere elettroni con energia cinetica dell'ordine dei KeV)
- **Secondaria ritardata** : gli elettroni sono emessi da stati intermedi eccitati A^* . Se il gas A contiene un altro gas B con potenziale di ionizzazione minore di quello di A posso avere $\pi A \rightarrow \pi A^*$ e $A^* B \rightarrow AB^+ e^-$. (effetto Penning)

Ionizzazione dei gas



Primary ionization

Total ionization

Fast charged particles ionize atoms of gas.
Often resulting **primary electron** will have enough kinetic energy ($dN/dE \propto 1/E^2$) to ionize other atoms.

$$n_{total} = \frac{\Delta E}{W_i} = \frac{\frac{dE}{dx} \Delta x}{W_i}$$

n_{total} - number of created electron-ion pairs
 ΔE = total energy loss

$$n_{total} \approx 3 \dots 4 \cdot n_{primary}$$

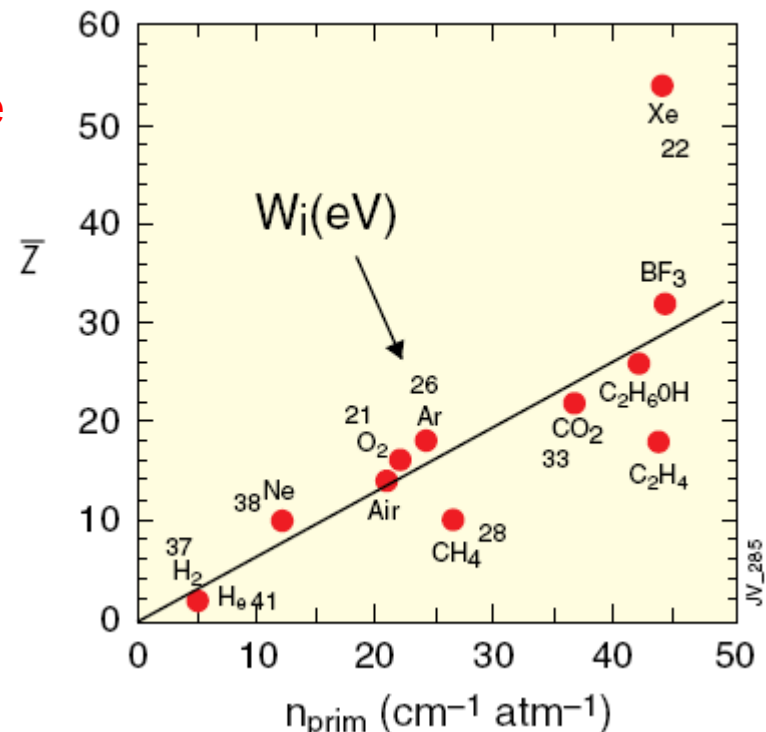
W_i = effective <energy loss>/pair

W_i - NOT the ionisation potential

$$W_i = N_i I_0 + E_{ex} N_{ex}$$

(I_0 = mean ion. Pot., E_{ex} = mean excitation energy)

Lohse and Witzeling, Instrumentation In High Energy Physics, World Scientific, 1992



Number of primary electron/ion pairs in frequently used gases for MIP.

Ionizzazione dei gas

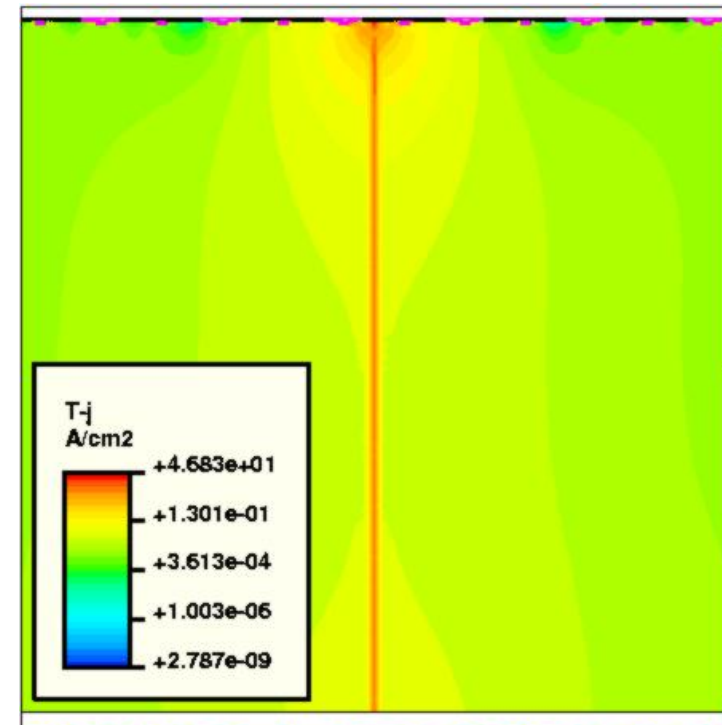
I raggi δ non troppo energetici ($E \ll E_{\max}$) sono emessi quasi ortogonali alla direzione di volo della particella incidente ($\cos^2\theta = E/E_{\max}$), ma... lo scattering multiplo randomizza la direzione del moto dei raggi δ , che diventa completamente casuale dopo pochi μm .

Il range R_T di un raggio δ di energia E può essere calcolato integrando la Bethe-Block sulla traiettoria, richiedendo che l'integrale sia $= E$, ma lo scattering multiplo rende complicato il calcolo.

- si usa un range empirico (ottenuto da misure di assorbimento). Andamento tipico: $R = AE^b$
- In Ar, per i primari $R_p = 0.71 E^{1.72}$ (E in MeV ed R_p in gcm^{-2}) valida fino a ~ 100 KeV

In argon un elettrone di 1 KeV si ferma in $\sim 50 \mu$, mentre uno di 10 KeV fa ~ 1.5 mm.

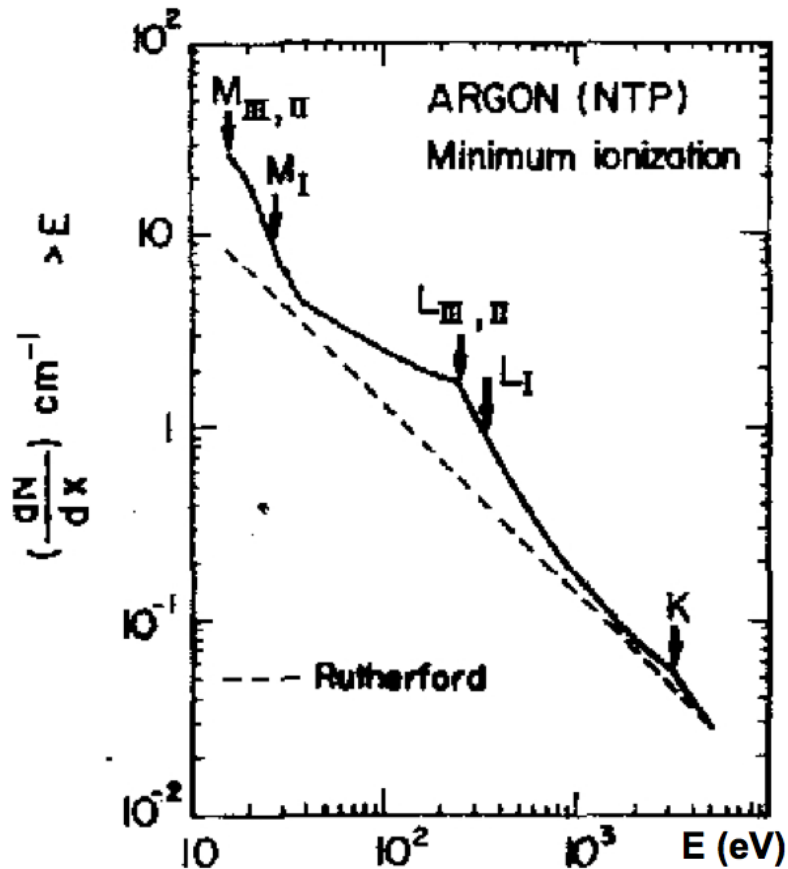
Il range medio dei delta fissa il raggio del cilindro di ionizzazione iniziale creato dalla particella incidente. Ai fini pratici può essere considerato unidimensionale (ma non sempre).



Cilindro di ionizzazione

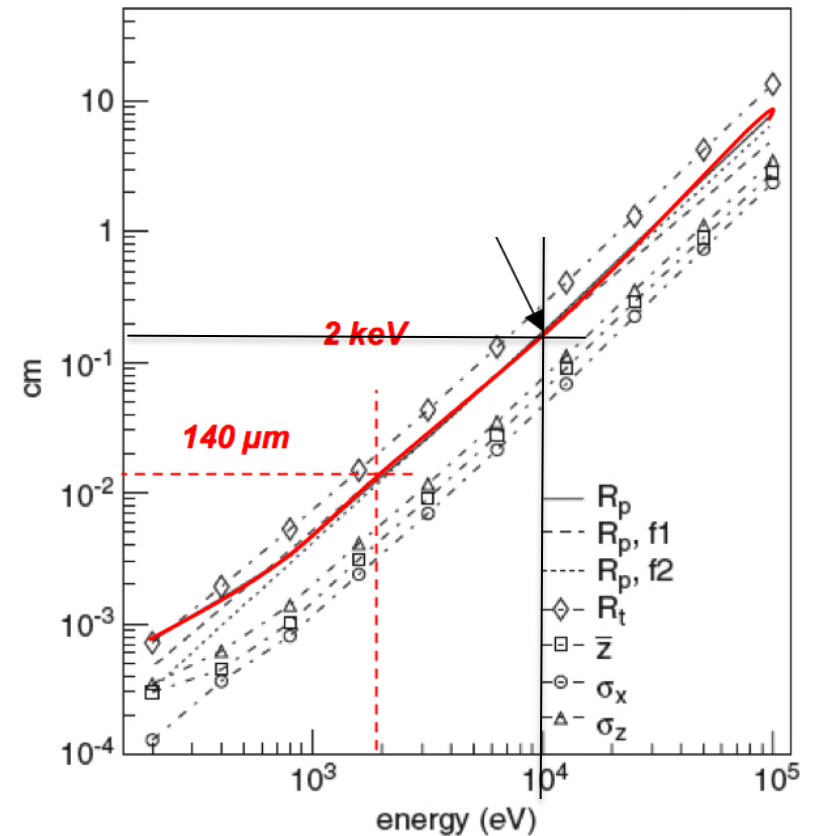
PROBABILITY AND RANGE OF DELTA ELECTRONS

PROBABILITY FOR AN ELECTRON OF ENERGY $> E$:



*F. Lapique and F. Piuz,
Nucl. instr. and Meth. 175(1980)297*

ELECTRON RANGE IN ARGON STP:



*I. B. Smirnov,
Nucl. Instr. and Meth. A554(2005)474*

Ionizzazione dei gas

At minimum, the particle should transfer $\delta E = E_{\text{ion}}$ of the gas molecule for the ionization process to occur. In most gases $E_{\text{ion}} = 10\text{-}25\text{ eV}$.

However, there are other mechanisms by which the incident particle may lose energy within the gas that do not create ions. Examples are excitation processes in which an electron may be elevated to a higher bound state in the molecule without being completely removed or atom excited as seen for the Fano factor.

Therefore, the average energy lost by the incident particle per ion pair formed (defined as the W-value) is always substantially greater than the ionization energy.

$$wN_i = N_i I_0 + E_{\text{ex}} N_{\text{ex}}$$

Empirical observations show that it is a remarkably constant parameter for many gases and different types of radiation.

A typical value is 25-35 eV/ion pair.

For an incident 1 MeV particle, if fully stopped in the gas, $N_{\text{pair}} \approx 30,000$ ion pairs.

Ionizzazione dei gas

Gas	Z	A	E_{ex} eV	E_i eV	I_0 eV	W_i eV	dE/dx MeV/g cm ⁻²	dE/dx KeV/cm	n_p i.p/cm	n_T i.p/cm
Ar	18	39.9	11.6	15.7	15.8	26	1.47	2.44	28	94
Kr	36	83.8	10.0	13.9	14.0	24	1.32	4.60	22	192
Xe	54	131.3	8.4	12.1	12.1	22	1.23	6.76	44	307
CO ₂	22	44	5.2	13.7	13.7	33	1.62	3.01	34	91
CH ₄	10	16		15.2	13.1	28	2.21	1.48	16	53
C ₄ H ₁₀	34	58		10.6	10.8	23	1.86	4.50	46	195

Dove: E_{ex} = energia minima di eccitazione; E_i = energia minima di ionizzazione;
 $I_0 = I/Z$ = potenziale efficace medio di ionizzazione per elettrone atomico;
 W_i = perdita di energia media per produrre una coppia ione-elettrone;
dE/dx = perdita di energia per particelle al minimo (MIP);
 n_p = numero di coppie primarie; n_T = numero totale di coppie.

Nel caso di composti e miscugli Z, A ed I sono valori medi.

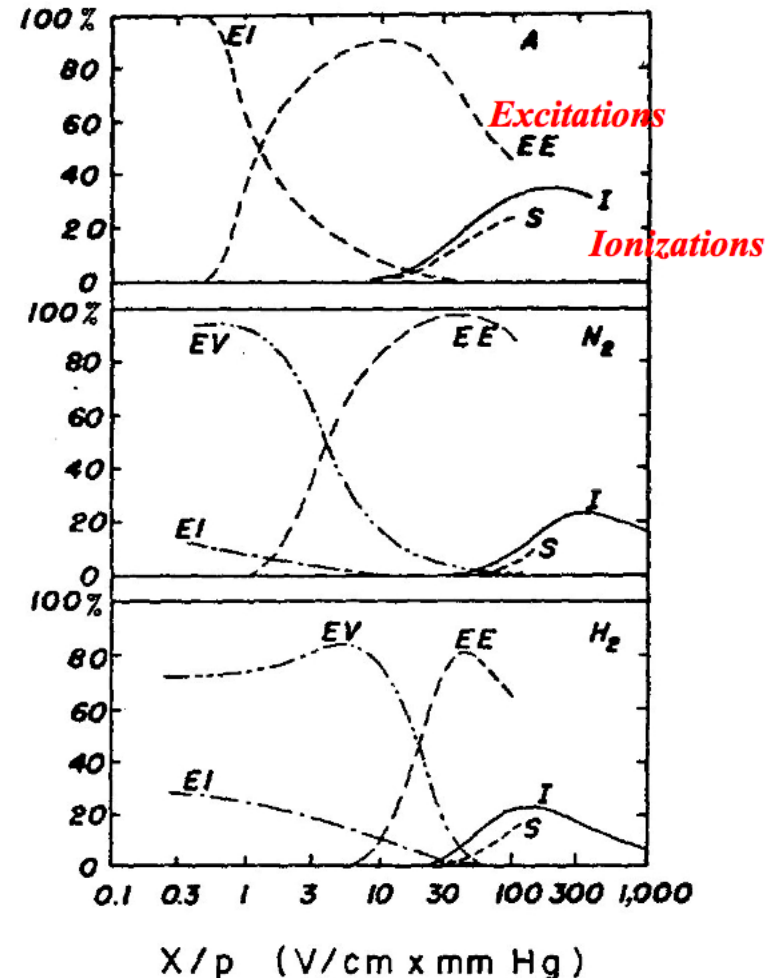
HIGH FIELD-INELASTIC COLLISIONS

MAIN ELECTRON-MOLECULE INELASTIC PROCESSES:

1) $A+e \Rightarrow A^++e+e$	Ionisation by electronic impact.
2) $A+e \Rightarrow A^++e$	Excitation by electronic impact.
3) $A^++e \Rightarrow A+e$	Deexcitation by electronic collision.
4) $A+h\nu \Rightarrow A^*$	Photo-excitation (absorption of light).
5) $A^* \Rightarrow A+h\nu$	Photo-emission (radiative deexcitation).
6) $A+h\nu \Rightarrow A^++e$	Photoionisation.
7) $A^++e \Rightarrow A+h\nu$	Radiative recombination.
8) $A^++B+e \Rightarrow A+B$	Three body recombination.
9) $A^++B \Rightarrow A+B^*$	Collisional deexcitation.
10) $A^++B \Rightarrow A+B^++e$	Penning effect.
11) $A^++B \Rightarrow A+B^+$	Charge exchange.
12) $A^++B \Rightarrow A^++B^++e$	Ionisation by ionic impact.
13) $A+B \Rightarrow A^++B$	Excitation by atomic impact.
14) $A+B \Rightarrow A^++B+e$	Ionisation by atomic impact.
15) $A+e \Rightarrow A^-$	Formation of negative ions.
16) $A^- \Rightarrow A+e$	Electrons release by negative ions.
17) $A^{++}+A \Rightarrow A_2^++e$	Associative ionisation.
18) $A^++2A \Rightarrow A_2^++A$	Molecular ion formation.
19) $A^++A+A \Rightarrow A_2^++A$	Excimer formation.
20) $A_2^* \Rightarrow A+A+h\nu$	Radiative excimer dissociation.
21) $(XY)^* \Rightarrow X+Y^*$	Dissociation.
22) $(XY)^++e \Rightarrow X+Y^*$	Recombinational dissociation.

J. Meek and J. D. Cragg, Electrical Breakdown of Gases (Clarendon Press, Oxford 1953)

ENERGY SHARING BETWEEN COLLISION PROCESSES:



L. B. Loeb, Basic Processes of Gaseous Electronics (UC Berkeley Press, 1961)

Ionizzazione dei gas

La particella incidente incontra gli atomi del gas e li ionizza. Gli scontri con gli atomi del gas sono casuali e caratterizzati da una lunghezza di interazione λ , dato dalla sezione d'urto di ionizzazione σ_i e dalla densità N di elettroni $\lambda = 1/(N\sigma_i)$

La prob che una particella subisca un'interazione in dl e' $dP = dl/\lambda$, quindi in un tratto dl , $-dN = N(l)dP$ e' il # di di particelle che hanno interagito in $dl \rightarrow N(l) = N(0)e^{-l/\lambda}$ e' il # di particelle che hanno viaggiato un tratto l senza interagire $\rightarrow P_{\text{surv}}(l) = N(l)/N(0)$ e' la prob che di arrivare alla profondita' l

Quindi la prob che una particella arrivi alla profondita' l senza interagire e che interagisca nel successivo dl e' $f(l)dl = P_{\text{surv}}(l) \times dP$

$$f(l)dl = \frac{1}{\lambda} \cdot e^{-l/\lambda} dl$$

Ionizzazione dei gas

La particella incidente incontra gli atomi del gas e li ionizza. Gli scontri con gli atomi del gas sono casuali e caratterizzati da una lunghezza di interazione λ , dato dalla sezione d'urto di ionizzazione σ_i e dalla densità N di elettroni.

$$\lambda = 1/(N\sigma_i)$$

Il numero di scontri in L ha una media L/λ e la distribuzione in frequenza è una Poissoniana (pochi scontri):

$$P(L/\lambda, k) = ((L/\lambda)^k)/k! e^{-L/\lambda} \quad (*)$$

→ la distribuzione di probabilità $f(l)dl$ dei cammini liberi medi è un esponenziale. Infatti la probabilità di non trovare atomi nel tratto l per la probabilità di uno scontro in dl è :

$$f(l)dl = P(l/\lambda, 0)P(dl/\lambda, 1) = \frac{1}{\lambda} \cdot e^{-l/\lambda} dl$$

La (*) fornisce anche la probabilità di avere 0 scontri in un percorso L

$$P(L/\lambda, 0) = e^{-L/\lambda}$$

Ionizzazione dei gas



Nella tabella abbiamo una collezione di misure eseguite con particelle veloci (è indicato il γ).

Se ho un contatore, sensibile nel tratto L, capace di contare anche un solo elettrone, posso misurare λ , contando tutte le volte che conto 0.

Naturalmente, per quanto perfetto sia tale contatore, bisogna applicare una correzione per gli elettroni sotto soglia.

gas	1cm/ λ (# collisioni)	γ
H ₂	5.32±0.06	4.0
He	5.02±0.06	4.0
Ne	12.4±0.13	4.0
Ar	27.8±0.3	4.0
Xe	44	4.0
N ₂	19.3	4.9
O ₂	22.2±2.3	4.3
Aria	18.5±1.3	3.5

Ionization of Gases

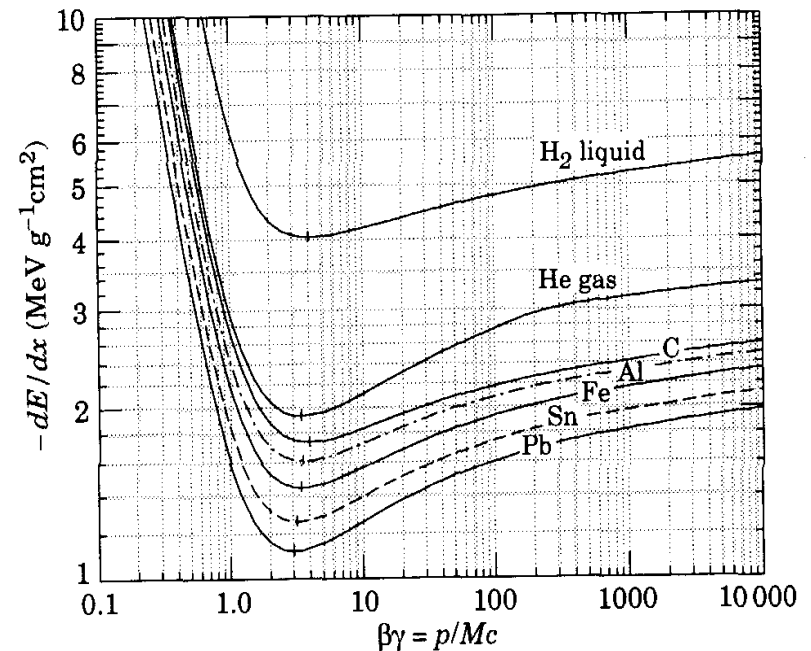
Example: Ar

Density $\sim 1.7 \text{ g/l}$

$\Delta_E = 1.8 \text{ MeV}/(\text{g}/\text{cm}^2) \sim 3 \text{ keV}/\text{cm}$

$W_i = 26 \text{ eV}/\text{ion}$

$n_{total} \sim 100 \text{ ions}/\text{cm}$ ($\sim 25 \text{ primary}$)



- We just saw the number of **primary** electron/ion pairs is approximately **Poisson** distributed.

$$P(m) = \frac{\bar{n}^m e^{-\bar{n}}}{m!} \quad \bar{n} = \frac{L}{\lambda} = LN\sigma_i$$

The detection efficiency is therefore limited to : $\varepsilon_{det} = 1 - P(0) = 1 - e^{-\bar{n}}$

For thin layers ε_{det} can be significantly lower than 1.

For example for 1 mm layer of Ar $n_{primary} = 2.5 \rightarrow \varepsilon_{det} = 0.92$.

Variation of the number of electron/pairs: $\sigma_n = \sqrt{\bar{n}}$ NOT exactly correct!

Ionization of Gases: second approximation

The number of electron/ion pairs is NOT **Poisson** distributed.

$$\sigma_n = \sqrt{F n};$$

F – Fano factor, related to the correlations in the ionization avalanche process (provided that the δ -electrons deposit all the energy).

Material dependent.

F ~ 1 for scintillators

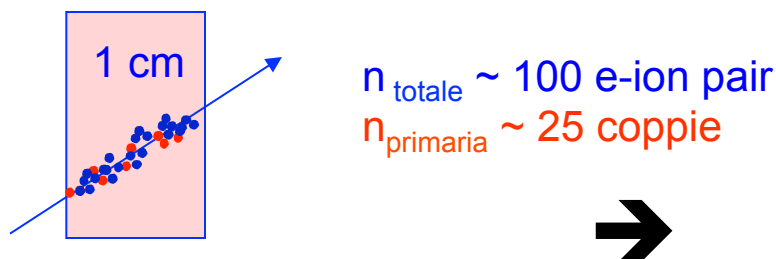
F ~ 0.2 – 0.8 for gas detectors

F ~ 0.12 for Silicon detectors

Ionizzazione dei gas

Commenti

- Assumiamo un apparato spesso 1 cm e riempito di Argon



- elettroni ioni primari fluttuano alla Poisson (poco), ma ... sono solo ~ 25 → contare il numero di cluster non è banale
- $n_{\text{totale}} \sim 100$ e code di Landau → difficile misurare dE/dx con una sola misura e strati sottili di gas e da questo dedurre (noto l'impulso) la massa della particella

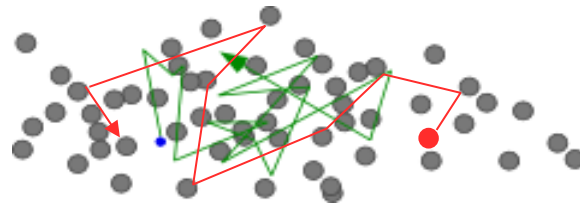
In ogni caso il rumore degli amplificatori è ~ 1000 e⁻ (ENC) → bisogna aumentare il numero di coppie ioni-elettroni.

Ionizzazione nei gas

- Dopo che e' stata generata, la carica si muove verso gli elettrodi di raccolta sotto l'azione del campo elettrico esterno applicato.
- Il moto avviene in un mezzo $\rightarrow$ ci sono urti in cui le cariche in moto perdono energia.
- Il risultato netto e' una velocita' di deriva costante nel tempo nella direzione del campo.
- Oltre al moto di deriva, c'e' anche un moto termico in tutte le direzioni, cioe' diffusione.
- Esistono processi che catturano le cariche prima che esse raggiungano gli elettrodi.
- L'evoluzione della nuvola di carica creata dalla ionizzazione a $t = 0$ e' descritta da un'equazione di trasporto "abbastanza" semplice.

Ionizzazione dei gas

- The neutral atoms or molecules of the gas are in constant thermal motion, characterized by a mean free path for typical gases under standard conditions of about 10^{-6} - 10^{-8} m.
- Positive ions or free electrons created within the gas also take part in the random thermal motion and therefore have some tendency to diffuse away from regions of high density.
- This diffusion process is much more pronounced for free electrons than for ions since their average thermal velocity is much greater.
- A point-like collection of free electrons will spread about the original point into a Gaussian spatial distribution whose width will increase with time.



Diffusion of Free Charges

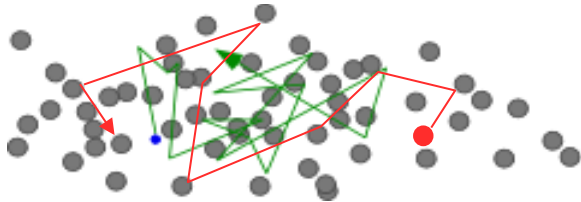
At the end, of the ionization stage (incident particle + A $\rightarrow$ A⁺e (primary)-, e⁻ + A $\rightarrow$ A⁺e-e⁻ (secondary), ... (terziary), ..., the N_i free pairs (e⁻ and ions) are in thermal equilibrium with the medium.

Free charges in a medium, regardless their origin, have a thermal random motion due to collisions with gas atoms and molecules.

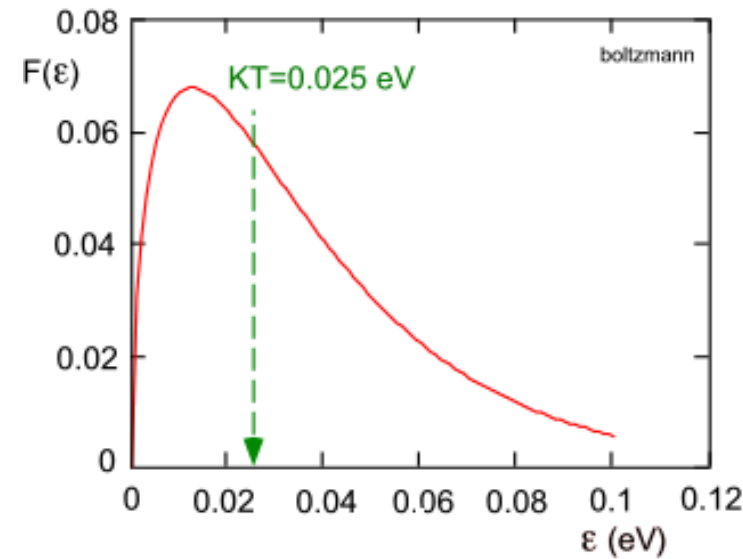
Maxwell - Boltzmann energy distribution:

$$F(\varepsilon) = \text{const} \sqrt{\varepsilon} e^{-\frac{\varepsilon}{kT}}$$

Average (thermal) energy:



$$\varepsilon_T = \frac{3}{2} kT \approx 0.040 \text{ eV}$$



La velocità termica media si ricava immediatamente dal principio di equipartizione dell'energia

$$(3/2)KT = \frac{1}{2}mv^2 \rightarrow v = (3kT/m)^{1/2} = c(3kT/mc^2)^{1/2}$$

Ioni ed elettroni hanno masse molto diverse: $m_e \ll m_{\text{ione}}$

velocità termica elettroni

$$v_t \sim 10^7 \text{ cm/s}$$

velocità termica ioni

$$v_t \sim 10^4 \text{ cm/s}$$

A T ambiente

Equazione di diffusione

The convection–diffusion equation can be derived in a straightforward way from the continuity equation or mass conservation law**, which states that the rate of change dn/dt for a scalar quantity in a differential control volume is given by flow and diffusion into and out of that part of the system along with any generation or destruction inside the control volume:

$$\frac{\partial n}{\partial t} + \nabla \cdot \vec{j} = R,$$

where $\vec{j}$ is the total flux and R is a net volumetric source for n . There are two sources of flux in this situation. First, **diffusive flux** arises due to diffusion. This is typically approximated by Fick's first law:

$$\vec{j}_{\text{diffusion}} = -D \nabla n$$

i.e., the flux of the diffusing material (relative to the bulk motion) in any part of the system is proportional to the local concentration gradient. Second, when there is overall convection or flow, there is an associated flux called **advective flux**:

$$\vec{j}_{\text{advective}} = \vec{v} n$$

The total flux is given by the sum of these two: $\vec{j} = \vec{j}_{\text{diffusion}} + \vec{j}_{\text{advective}} = -D \nabla n + \vec{v} n$

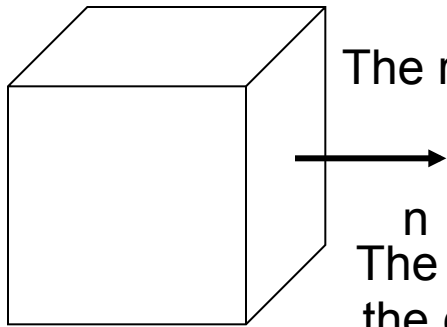
$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

****Mass conservation law**

The mass cannot be created nor destroyed (in non-relativistic classical dynamics)

Therefore in a volume V the mass can change only because some of it leaves or enters the volume or if there is some source feeding mass with rate Q

The amount of mass through $d\mathbf{A}$ per time unit is $dF = \mathbf{j} \cdot d\vec{A}$ Convention:
dF>0 if
outgoing



The rate of outgoing mass is $(\frac{dM}{dt})_{out} = \int_S \mathbf{j} \cdot d\vec{A}$

The outgoing flux must be balanced by the change of mass in the volume and the source $\frac{\partial M_V}{\partial t} = -(\frac{dM}{dt})_{out} + Q$

$$\frac{\partial M_V}{\partial t} = \frac{\partial}{\partial t} \left(\int_V \rho dV \right) \quad \frac{\partial}{\partial t} \left(\int_V \rho dV \right) = - \int_S \mathbf{j} \cdot d\vec{A} = - \int_V \nabla \cdot \mathbf{j} dV$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j} \quad + \int_V R dV$$

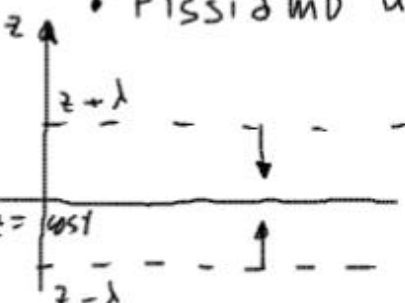
$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = R$$

R is the the rate/volume

Parentesi: moto diffusivo

- Il moto di diffusione si ha in presenza di gradienti di densità
 - Vale la legge di Fick: $\vec{J} = -D \vec{\nabla} n$
 - Il parametro D è il coeff. di diffusione: dipende dal mezzo in cui avviene la propagazione [$L^2 T^{-1}$]

Es. diffusione in un gas diluito

- Fissiamo un piano arbitrario nel gas. Il flusso netto è dato dalla somma dei flussi da sopra e da sotto il piano, J_+ e J_- .
$$J_{net} = J_+ + J_-$$
- Se l è il libero cammino medio, le particelle che attraversano il piano hanno subito l'ultimo urto in media a una dist. $\pm l$ dal piano, cioè a $z' = z \pm l$.
- Moto random $\Rightarrow$ il # di part in moto con vel. media $\langle v \rangle$ in una direzione è lo stesso in tutte le direzioni (isotropia) $\Rightarrow n_x = n_y = n_z = n/3$ si muovono con vel $\langle v \rangle$ lungo le 3 direzioni $\perp$ del rif. scelto.
- Di queste $1/2$ si muovono verso $+z$ e $1/2$ lungo $-z \Rightarrow$
$$J_{\pm} \approx J(z \pm l) = \pm \frac{\langle v \rangle n}{6} (z \pm l) \approx \pm \frac{\langle v \rangle}{6} \left[n(z) \pm \frac{\partial n}{\partial z} l \right]$$

• quindi. $J_{nel} = \frac{\langle v \rangle}{6} \left[-\frac{\partial n}{\partial z} \cdot l \right] - \frac{\langle v \rangle}{6} \left[\frac{\partial n}{\partial z} \cdot l \right] = -\frac{l \langle v \rangle}{3} \left(\frac{\partial n}{\partial z} \right)$

$\Rightarrow D = \frac{l \langle v \rangle}{3}$

• Se le part. diffondono in un mezzo di densità $N \Rightarrow l \sim \frac{1}{N \sigma_i}$

σ_i = sez. d'urto di collisione fra le part. che diffondono e quelle del mezzo

$\Rightarrow D = \frac{\langle v \rangle}{3 N \sigma_i}$

* $\langle v \rangle \ll c \Rightarrow v \sim \left(\frac{kT}{m} \right)^{1/2}$

$p = N kT \Rightarrow \frac{1}{m} = \frac{kT}{p} \Rightarrow D \propto \frac{(kT)^{3/2}}{3 \sigma_i p}$

* $\langle v \rangle \approx c \Rightarrow D \approx \frac{c}{3 N \sigma_i} \Rightarrow D \propto \frac{c (kT)}{3 \sigma_i p}$

* Se il moto avviene in un campo magnetico, la dist. alla quale la part. ha cambiato "molto" la direzione è il raggio di Larmor $\Rightarrow$

$l \sim r_L = \frac{cp}{e \hbar B} \approx \frac{E}{\hbar \omega_B} \quad (\text{se } v \approx c) \Rightarrow D \approx \frac{\lambda_L}{3} = \frac{c E}{3 \hbar \omega_B} \Rightarrow D \propto E$

In tal caso il coefficiente di diffusione dipende da E

Diffusione spaziale

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

No convezione (per ora)

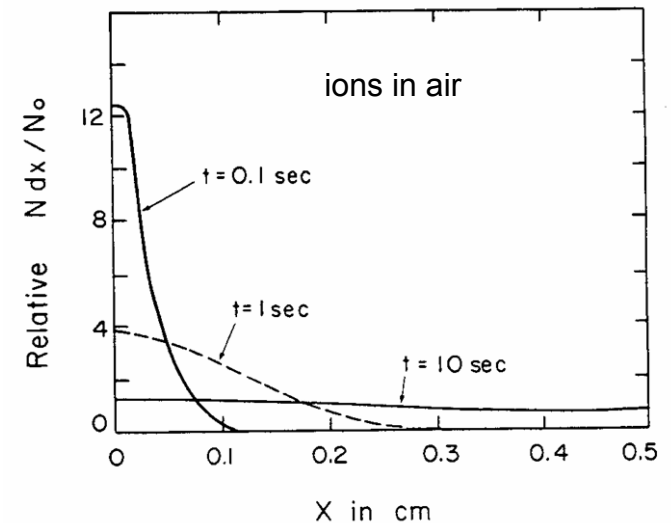
Se iniettiamo N particelle nell'origine con una distribuzione spaziale $N = N_0 \delta(x)$ a $t = 0$, l'evoluzione temporale e' data da

$$N(x, t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \quad (\text{calcoletto alla lavagna?})$$

Il bunch di particelle rimane in media fermo nell'origine, ma si allarga nel tempo con una distribuzione gaussiana di larghezza crescente

$$\sigma_x = \sqrt{2Dt}$$

Una frazione della carica N/N_0 , inizialmente tutta in $x = 0$, la si ritrova ad una distanza x dopo un tempo t



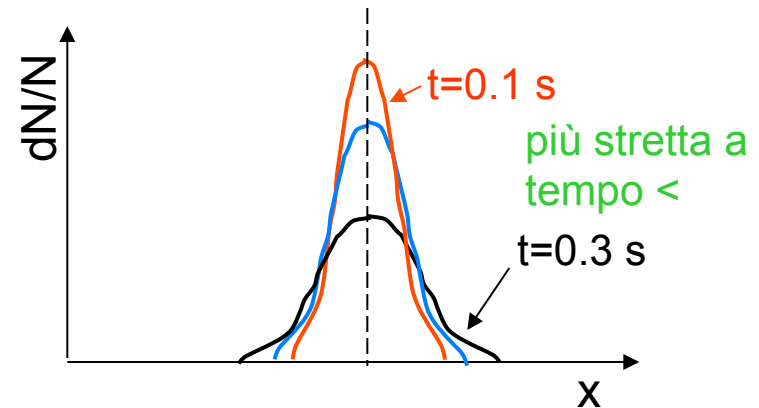
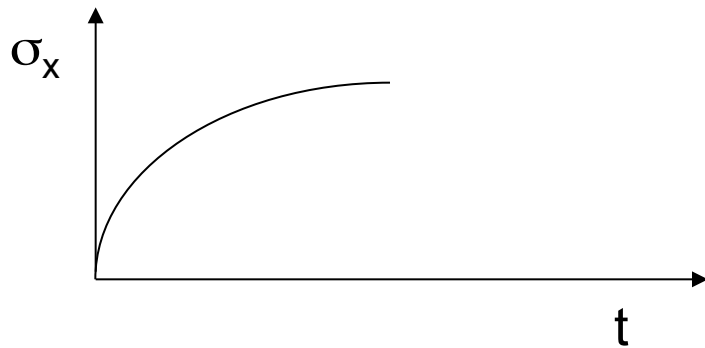
Ionizzazione dei gas

Varianza lineare

$$\sigma_x(t) = \sqrt{2Dt}$$

Varianza nel volume

$$\sigma_V(t) = \sqrt{3}\sigma_x = \sqrt{6Dt}$$



Ionizzazione dei gas

Diffusione e deriva in presenza di campo elettrico.

Se applico un campo elettrico alla velocità termica $v_t \approx \sqrt{\frac{3KT}{m}}$, si sommerà una velocità di deriva dovuta al campo **E**. Infatti fra una collisione e l'altra gli elettroni sono accelerati dal campo elettrico. Se $\langle L \rangle$ è il cammino libero medio e τ il tempo medio fra collisioni avremo (da $\mathbf{a} = e\mathbf{E}/m$):

$$\langle v_D \rangle = a\tau / 2 = \frac{e\tau}{2m} E = \frac{e}{2m} \left(\frac{\langle L \rangle}{v_t} \right) E$$

Ma $L = \frac{1}{N\sigma}$ quindi $\langle v_D \rangle = \frac{e}{2mv_t N\sigma} E$

In un gas, di solito non si misura la concentrazione ma la pressione e temperatura

$p = NkT \rightarrow L = \frac{kT}{p\sigma} \rightarrow \langle v_D \rangle = \frac{e}{2m} \frac{KT}{v_t \sigma} \left(\frac{E}{p} \right)$ Elimino anche v_t

$$\langle v_D \rangle = \frac{e}{2\sigma} \left(\frac{KT}{m} \right)^{1/2} \left(\frac{E}{p} \right)$$

La velocità di deriva dipende da:

- (E/p) , il campo elettrico per unità di pressione (V/m)/Pa, a T costante
- dal parametro $\mu = \frac{e}{2\sigma} \left(\frac{KT}{m} \right)^{1/2}$

Ionizzazione dei gas

Diffusione e deriva in presenza di campo elettrico.

A volte invece di pressione e temperatura, si usa la densita' del mezzo

$$\langle v_D \rangle = a\tau / 2 = \frac{e\tau}{2m} E = \frac{e}{2m} \left(\frac{\langle L \rangle}{v_t} \right) E$$

$$\langle L \rangle^{-1} = N_A \frac{\rho \sigma}{A}$$

$$\langle v_D \rangle \sim \left(\frac{eE}{2m} \right) \frac{A / N_o \rho \sigma}{v_t} = \frac{eA}{2N_A \sigma \sqrt{3KTm}} \left(\frac{E}{\rho} \right)$$

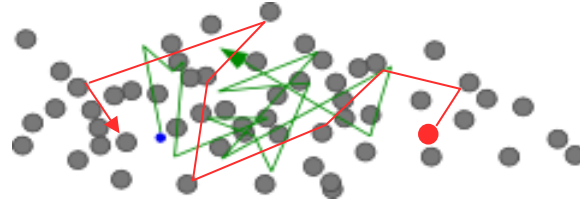
N_A = nr. di Avogadro, A peso atomico in kg/mol

$$\langle v_D \rangle = \frac{e}{2\sigma} \left(\frac{KT}{m} \right)^{1/2} \left(\frac{E}{p} \right)$$

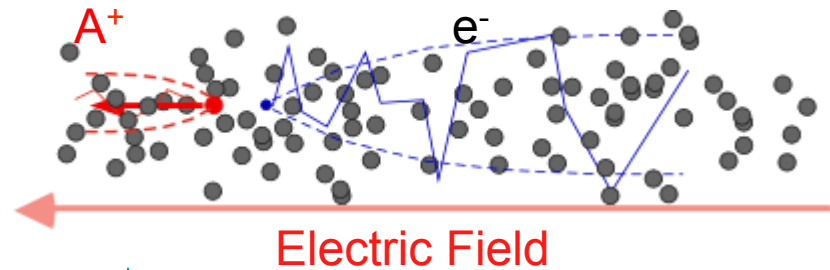
La velocita' di drift e' direttamente proporzionale al libero cammino medio e quindi inversamente proporzionale alla densita' e pressione

Drift and Diffusion in Presence of E field

$E=0$ thermal diffusion $\langle v \rangle_t = 0$



$E>0$ charge transport and diffusion $\langle v \rangle_t = v_D$



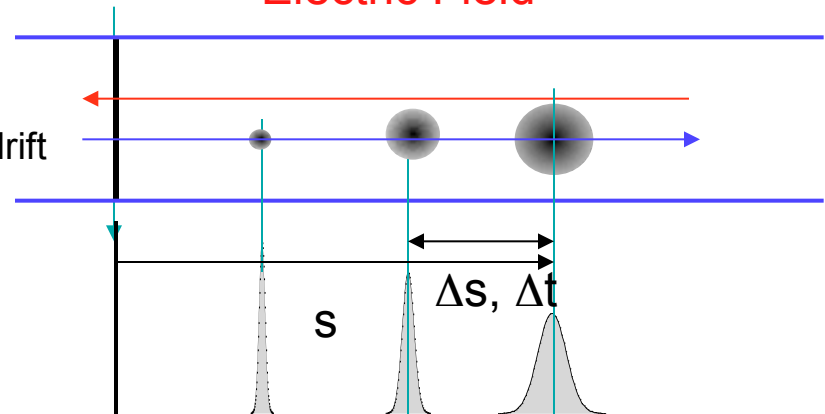
The solution of diffusion-transport eqn is then

$$N(x,t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{(x-v_D t)^2}{4Dt}}$$

Electron swarm drift

Drift velocity

Diffusion



drift + diffusion motion: the average position of the charge swarm moves as $x = vt$, while the width increases in time

Drift and Diffusion in Presence of E field

- As the electrons are drifting through the gas under the influence of the electric field, they will to first approximation follow the path of the electric field line that passes through their point of origin (but in the reverse direction to the electric field vector).
- Random diffusion of the electrons will still be taking place, however, causing each individual electron to follow a slightly different path. For strong electric fields, the increased average energy given the electron in the direction of the field results in different values of the diffusion coefficient D for the directions that are parallel to or transverse to the field. The same happens for magnetic fields. Over the few μ seconds that are typically required for the electrons to reach a collecting electrode, the diffusion in either direction might be of the order of a millimeter or less.
- This charge spreading does play a potential role in limiting the position resolution attainable in "position- sensitive" gas detectors that deduce the location of the ionizing event through the position of arrival of the electrons at the anode, or through measuring the electron drift time.

Ionizzazione dei gas

Diffusione in presenza di campo elettrico E.

Abbiamo:

$$\sigma(x) \sim \sqrt{2Dt} \sim \sqrt{2v_t \langle L \rangle \frac{x}{\langle v_D \rangle}}$$

$$\text{ma: } v_t \sim \sqrt{\frac{3KT}{m}} \quad \langle v_D \rangle \sim a\tau \sim \frac{eE}{m} \left(\frac{\langle L \rangle}{v_t} \right)$$



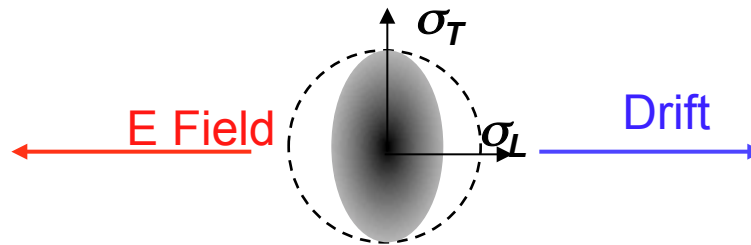
$$\sigma_x \approx \sqrt{\left(\frac{2KT}{eE} \right) \cdot x} \approx \sqrt{\frac{2KT}{eV_0}} \cdot x$$

L'energia termica KT è in competizione con l'energia del campo elettrico eV_0 .

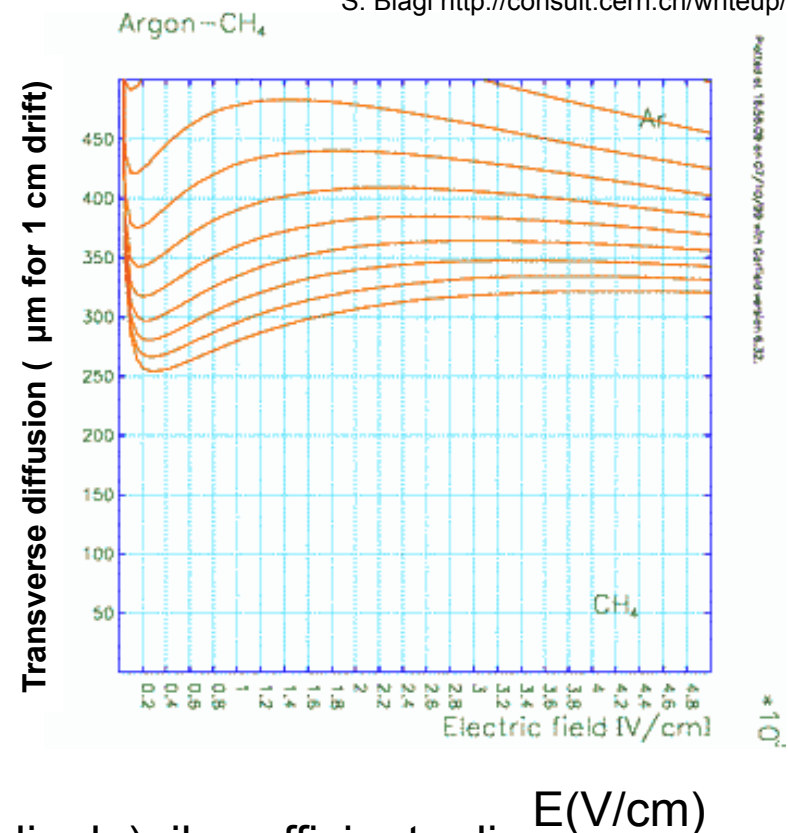
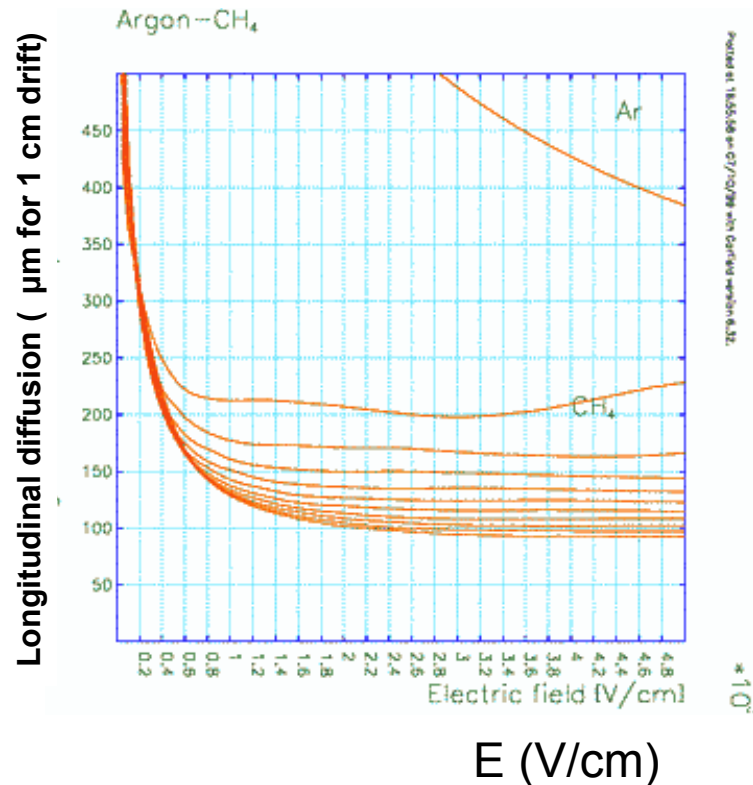
$\sigma_x \sim 100 \mu$ con un campo elettrico di 1 KV/cm^2 e una distanza di deriva di 1 cm .

→ limitazione intrinseca delle camere a deriva.

Diffusion Electric Anisotropy



S. Biagi <http://consult.cern.ch/writeup/magboltz/>



Nella direzione del campo (longitudinale) il coefficiente di diffusione risulta < di quella nella direzione trasversa → diffusione anisotropa.

Ionizzazione dei gas

Mobilità.

La **velocità di deriva** è proporzionale ad **E** ed inversamente proporzionale alla **sezione d'urto** di collisione ed alla **velocità termica** → ci attendiamo quindi che la velocità di deriva per campo elettrico unitario diviso per la pressione o la densità sia costante, cioè **mobilità** costante.

La mobilità μ e' definita come la $\langle v_D \rangle$ per unita' di campo elettrico **E/p** ridotto.

$$\langle v_D \rangle = \frac{e}{2\sigma} \left(\frac{KT}{m} \right)^{1/2} \left(\frac{E}{p} \right)$$

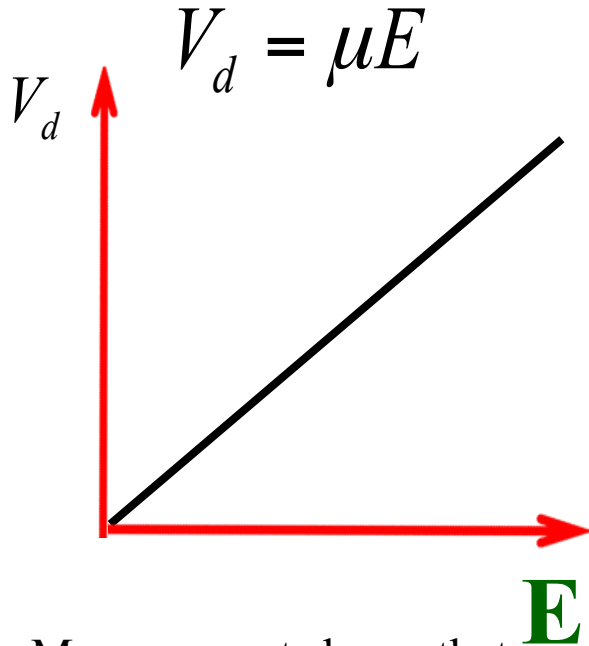
$$\mu = \frac{\langle v_D \rangle}{E/p}$$

$$\mu^{ion} = \frac{e\tau}{m}$$

$$\mu = \frac{e}{2\sigma} \left(\frac{KT}{m} \right)^{1/2}$$

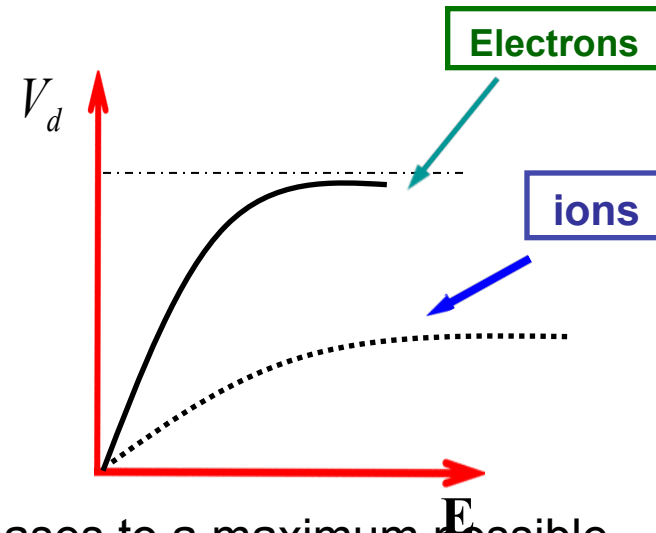
Ionizzazione dei gas

- The mobility μ tends to remain fairly constant over wide ranges of electric field and gas pressure and does not differ greatly for either positive or negative ions in the same gas. Typical values are between $1 - 1.5 \times 10^{-4} \text{ m}^2 \text{ atm} / \text{V s}$ for detector gases of medium atomic number. Therefore, at 1 atm pressure, a typical electric field of 10^4 V/m will result in a $v_D \sim 1 \text{ m/s}$.
- Ion transit times over typical detector dimensions of a centimeter will therefore be approximately 10 ms. By most standards, this is a very loooong time.
- Free electrons behave quite differently. Their much lower mass allows a greater acceleration between encounters with neutral gas molecules, and the value of the mobility is typically 1000x than that for ions. Typical collection times for electrons are therefore of the order of microseconds, rather than milliseconds.



Obviously, this says that the V_d vs E curve looks qualitatively like:

Measurement shows that, in almost all materials, at high enough E , the V_d vs E curve looks qualitatively like:



As the electric field is increased the carrier velocity increases to a maximum possible value, the *saturation velocity* v_{sat} . For example, the value of v_{sat} is on the order of 1×10^7 cm/s for electrons in gas. This velocity is a characteristic of the material. This is quite important in achieving linear space-time correlations, since it minimizes the change in drift velocity as the e^- approach the sense wire.

- Saturation occurs since at high fields, new scatterings processes play a role.
- By definition, mobility is $\mu = dV_D/dE$.
- The main factor determining drift velocity is the scattering time, i.e. how long the carrier is accelerated by the electric field until it scatters (collides) with something that changes its direction and/or energy.
- $\mu = e\tau/m$ with $\tau = 1/nv_t\sigma$
- The x-section is additive: $\sigma = \sigma_1 + \sigma_2 + \dots \rightarrow 1/\tau = 1/\tau_1 + 1/\tau_2 + \dots \rightarrow 1/\mu = 1/\mu_1 + 1/\mu_2 + \dots$
- The fastest process (ie the shortest interaction time) dominates $\rightarrow \mu$ decreases (e-interacts more frequently) $\rightarrow dv/dE$ decreases, v saturates.
- The most important sources of scattering are impurity scattering and acoustic and optical phonon scattering: at high fields, carriers are accelerated enough to gain sufficient kinetic energy between collisions to emit an optical phonon, and they do so very quickly, before being accelerated once again. The velocity that the electron reaches before emitting a phonon is $mv^2/2 = \hbar\omega_{\text{phonon}}$ where $\omega_{\text{phonon(opt.)}}$ is the optical-phonon angular frequency and m the carrier mass in the direction of the electric field. The typical value of $E_{\text{phonon (opt.)}}$ is 0.06 eV. The saturation velocity is only one-half of v_{emit} , because the electron starts at zero velocity and accelerates up to v_{emit} in each cycle.
- At any $T > 0$ K, the vibrating atoms create pressure (acoustic) waves in the crystal, which are termed phonons. Like electrons, phonons can be considered to be particles. A phonon can interact (collide) with an electron (or hole if in semiconductors) and scatter it. At higher temperature, there are more phonons, and thus increased phonon scattering, which tends to reduce mobility.

DRIFT OF IONS

MOBILITY: RATIO OF VELOCITY AND FIELD

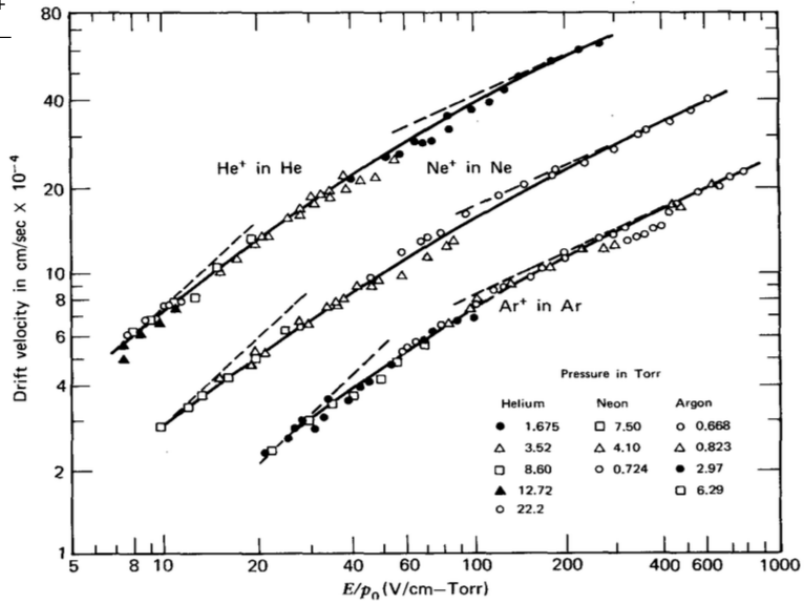
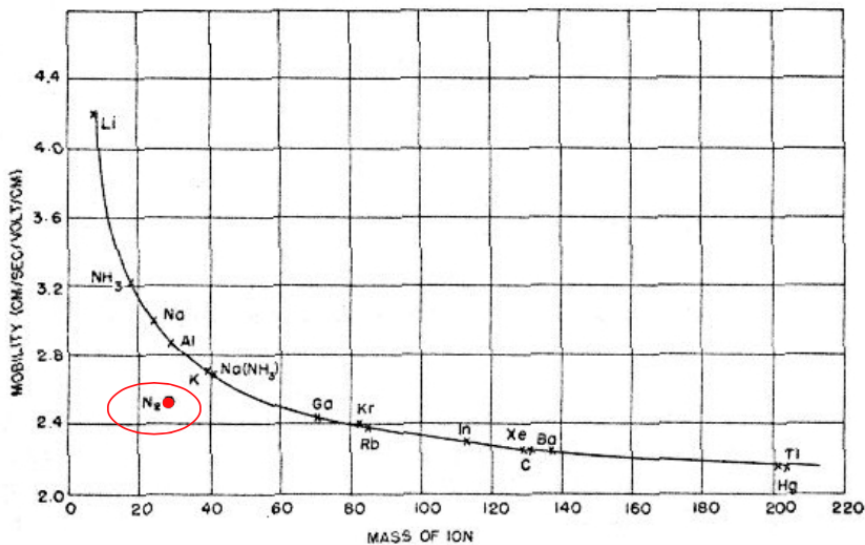
$$\mu^+ = \frac{w^+}{E}$$

IONS MOBILITY (NTP: 300 K, 760 mm Hg)

GAS	ION	$\mu^+(\text{cm}^2\text{s}^{-1}\text{V}^{-1})$
He	He ⁺	10.2
Ar	Ar ⁺	1.7
CH ₄	CH ₄ ⁺	2.26
Ar	CH ₄ ⁺	1.87
CO ₂	CO ₂ ⁺	1.09

Ar-CH₄, E=1kV cm⁻¹ w⁺= 1.87 cm ms⁻¹

IONS MOBILITY IN NITROGEN:



S. C. Brown

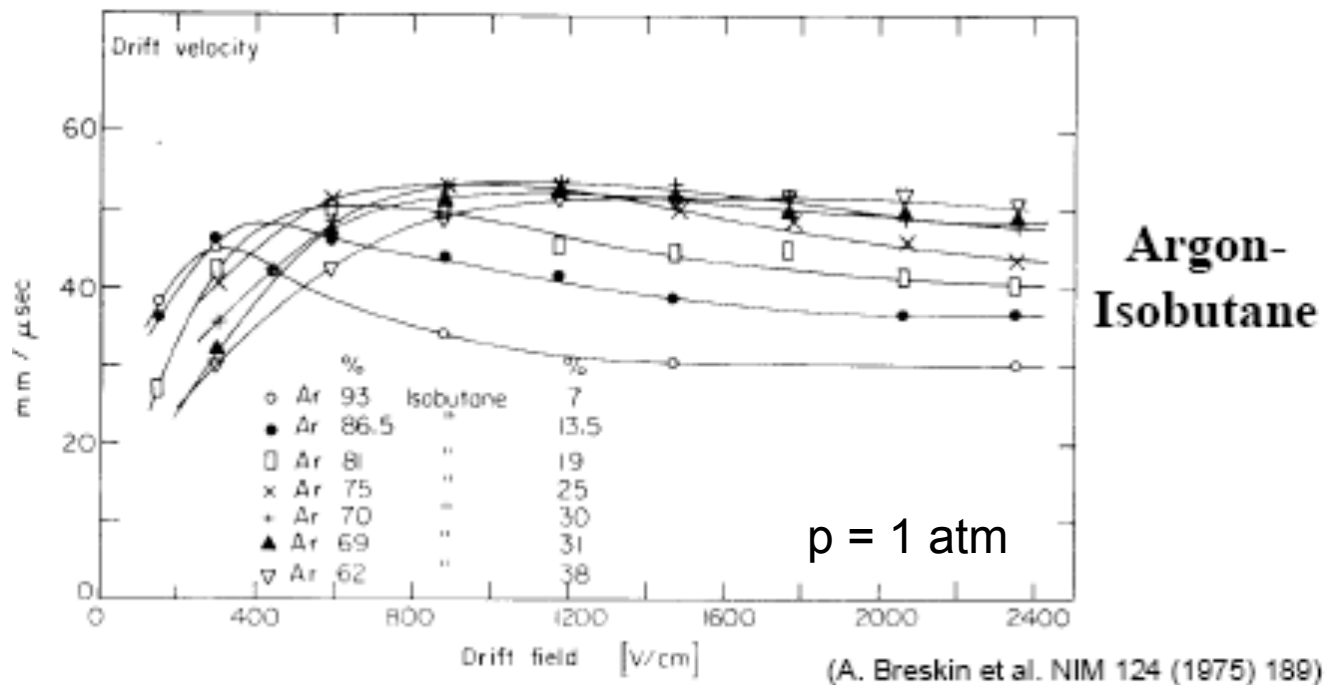
Basic Data in Plasma Physics (Wiley, New York 1959)

- Ion drift velocities $\ll$ electron v_d $w/w^+ \approx 1000$
- Mobility μ (velocity / applied field) independent of field, inversely proportional to density
- Cloud of accumulated ions can change electrostatics, affect gain, distort field
- Issue mostly for high-rate detectors (large ionization density)

Ionizzazione dei gas

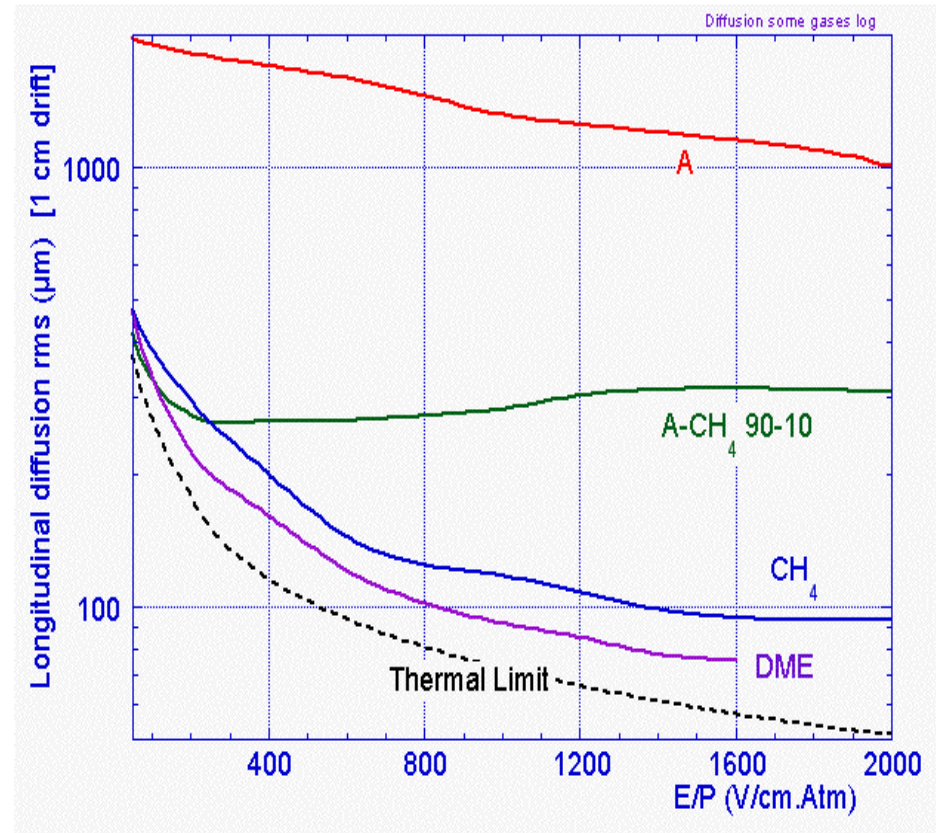
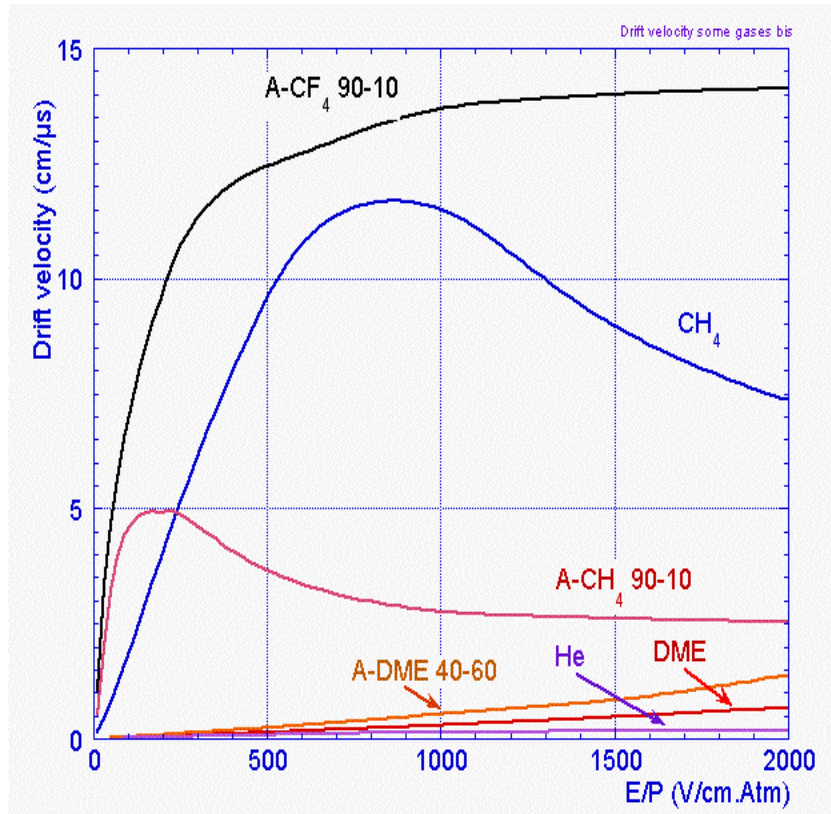
In figura sono indicate alcune velocità di deriva di elettroni per diversi campi elettrici e diversi miscugli di gas.

Con 75% argon e 25% isobutano e campi di 800-1000 V/ cm si ha $\langle v_D \rangle \sim 50 \text{ mm}/\mu\text{s}$



Drift and Diffusion of Electrons in Gases

Large range of drift velocity and diffusion:



Drift in Presence of E and B Fields

Equation of motion of free charge carriers in presence of E and B fields:

$$m \frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v} \times \vec{B}) + \vec{Q}(t) \quad \text{where } \vec{Q}(t) \text{ stochastic force resulting from collisions}$$

Time averaged solutions with assumptions: $\vec{v}_D = \langle \vec{v} \rangle = \text{const.}$; $\langle \vec{Q}(t) \rangle = \frac{m}{\tau} \vec{v}_D$ friction force

$$\left\langle \frac{d\vec{v}}{dt} \right\rangle = 0 = e\vec{E} + e(\vec{v}_D \times \vec{B}) - \frac{m}{\tau} \vec{v}_D \quad \tau \text{ mean time between collisions}$$

$$\vec{v}_D = \frac{\mu |\vec{E}|}{1 + \omega^2 \tau^2} \left[\hat{E} + \omega \tau (\hat{E} \times \hat{B}) + \omega^2 \tau^2 (\hat{E} \cdot \hat{B}) \hat{B} \right]$$

$$\mu = \frac{e\tau}{m} \text{ mobility} \quad \omega = \frac{eB}{m} \text{ cyclotron frequency}$$

$$B=0 \rightarrow \vec{v}_D^B = \vec{v}_D^0 = \mu \vec{E}$$

$$\vec{E} \parallel \vec{B} \rightarrow v_D^B = v_D^0$$

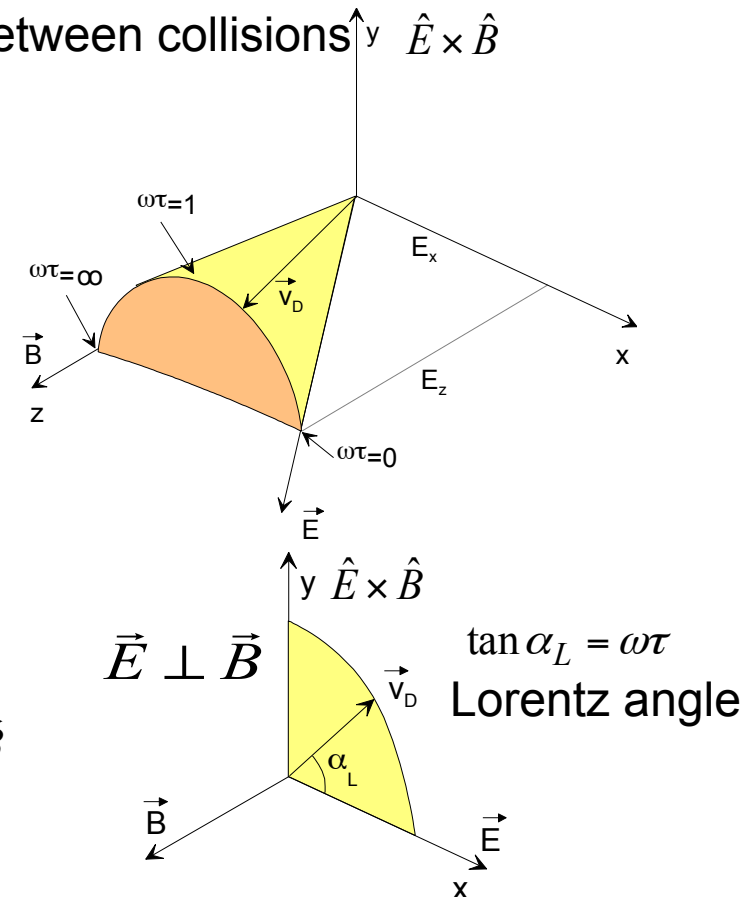
$$\vec{E} \perp \vec{B} \rightarrow v_D^B = \frac{E}{B} \frac{\omega \tau}{\sqrt{1 + \omega^2 \tau^2}}$$

In general drift velocity has 3 components:

$\omega \tau \ll 1$ particles follow E-field

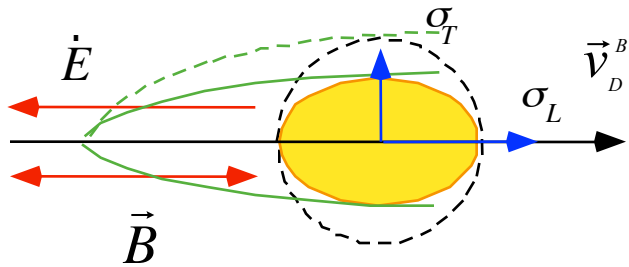
$$\parallel \vec{E}; \parallel \vec{B}; \parallel \vec{E} \times \vec{B}$$

$\omega \tau \gg 1$ particles follow B-field



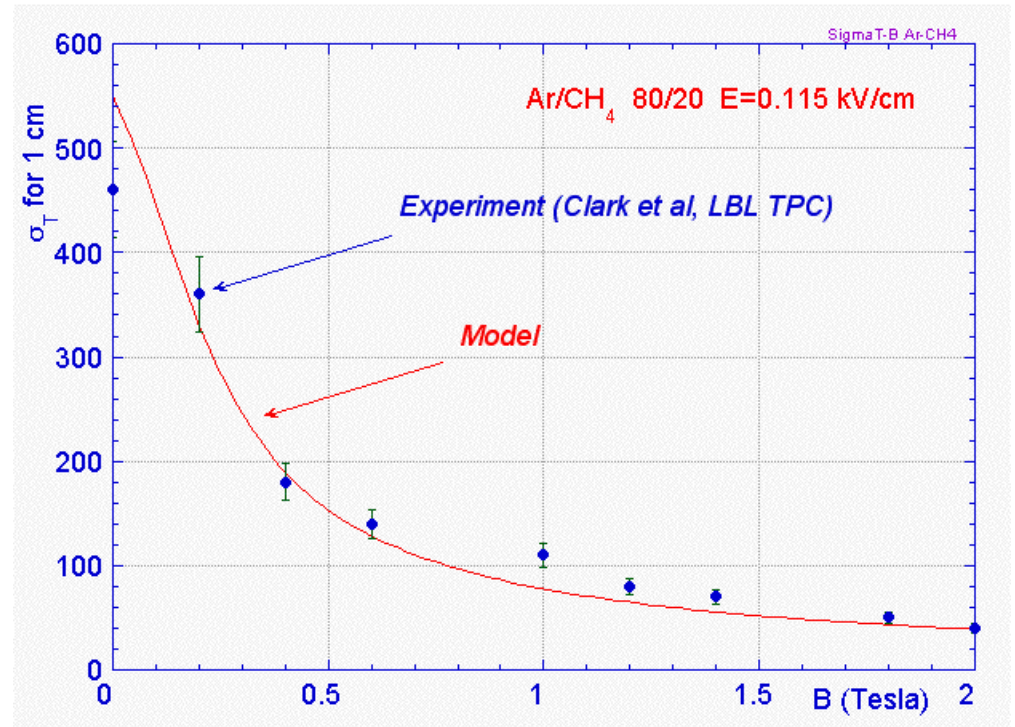
Diffusion Magnetic Anisotropy

$$\vec{E} \parallel \vec{B}$$



$$\sigma_L = \sigma_0$$

$$\sigma_T = \frac{\sigma_0}{\sqrt{1 + \omega^2 \tau^2}}$$



Ionizzazione dei gas

Ricombinazione ed ioni negativi.

Siccome abbiamo poche coppie ione-elettrone –**dell'ordine di $O(100)$** !- è bene che non se ne perdano.

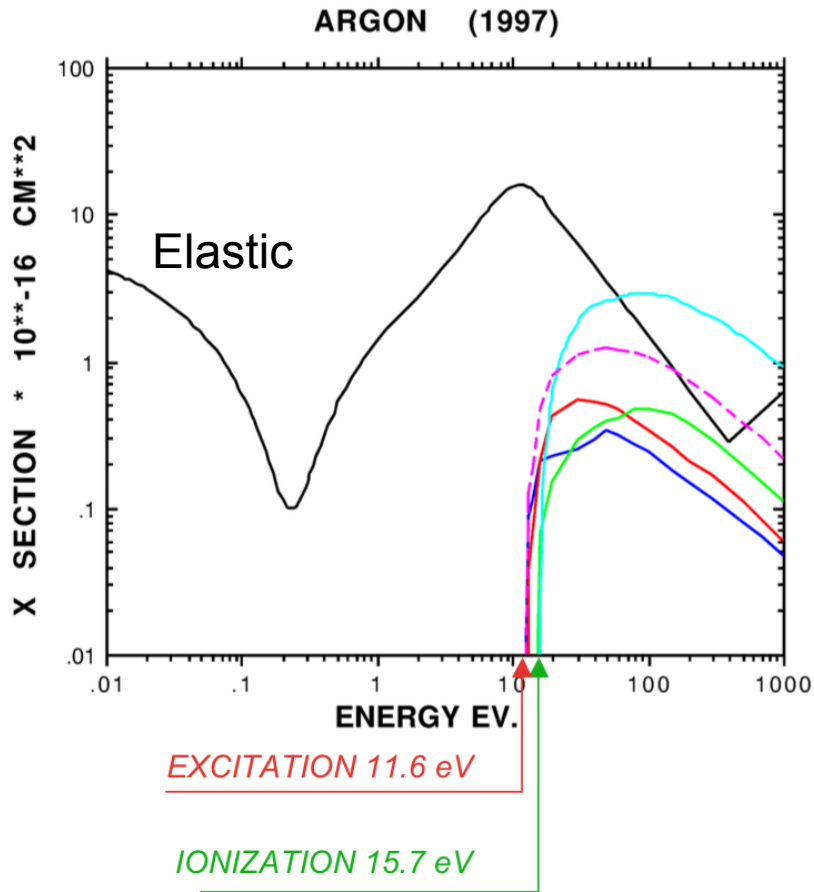
Ma ci sono numerosi processi di cattura elettronica e ricombinazione ionica che portano in genere ad una perdita di carica

MAIN ELECTRON-MOLECULE INELASTIC PROCESSES:

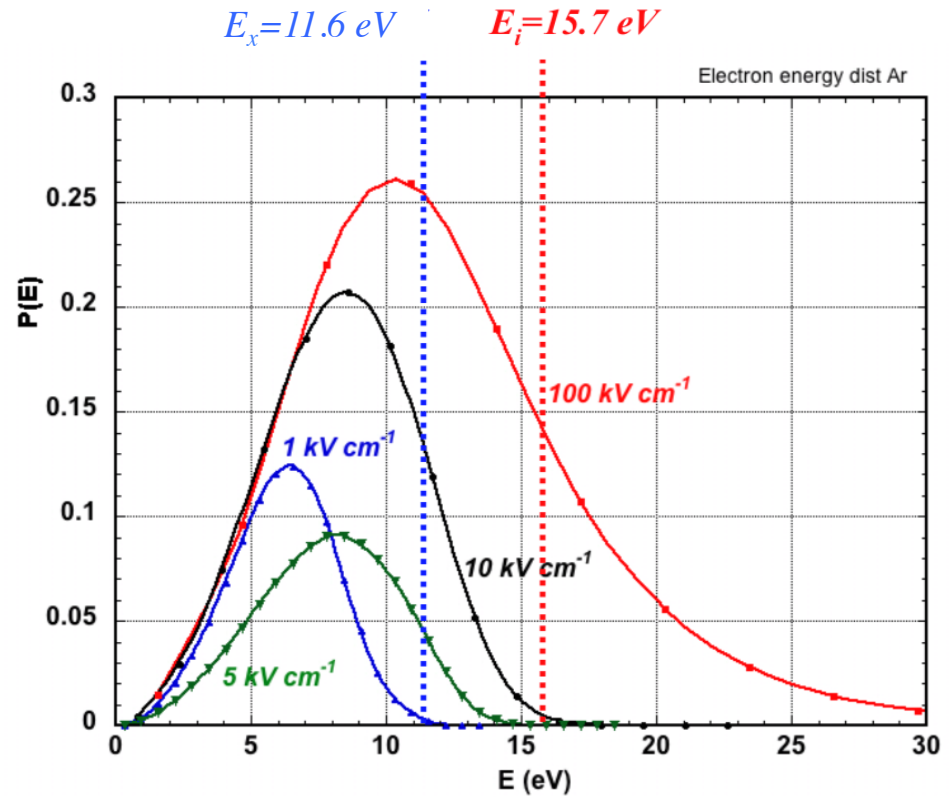
1) $A+e \Rightarrow A^++e+e$	Ionisation by electronic impact.
2) $A+e \Rightarrow A^*+e$	Excitation by electronic impact.
3) $A^*+e \Rightarrow A+e$	Deexcitation by electronic collision.
4) $A+h\nu \Rightarrow A^*$	Photo-excitation (absorption of light).
5) $A^* \Rightarrow A+h\nu$	Photo-emission (radiative deexcitation).
6) $A+h\nu \Rightarrow A^++e$	Photoionisation.
7) $A^++e \Rightarrow A+h\nu$	Radiative recombination.
8) $A^++B+e \Rightarrow A+B$	Three body recombination.
9) $A^++B \Rightarrow A+B^*$	Collisional deexcitation.
10) $A^++B \Rightarrow A+B^++e$	Penning effect.
11) $A^++B \Rightarrow A+B^+$	Charge exchange.
12) $A^++B \Rightarrow A^++B^++e$	Ionisation by ionic impact.
13) $A+B \Rightarrow A^++B$	Excitation by atomic impact.
14) $A+B \Rightarrow A^++B+e$	Ionisation by atomic impact.
15) $A+e \Rightarrow A^-$	Formation of negative ions.
16) $A^- \Rightarrow A+e$	Electrons release by negative ions.
17) $A^{**}+A \Rightarrow A_2^++e$	Associative ionisation.
18) $A^++2A \Rightarrow A_2^++A$	Molecular ion formation.
19) $A^++A+A \Rightarrow A_2^++A$	Excimer formation.
20) $A_2^* \Rightarrow A+A+h\nu$	Radiative excimer dissociation.
21) $(XY)^* \Rightarrow X+Y^*$	Dissociation.
22) $(XY)^++e \Rightarrow X+Y^*$	Recombinational dissociation

HIGH FIELD-INELASTIC COLLISIONS

ELECTRON CROSS SECTIONS IN ARGON:



ELECTRONS ENERGY DISTRIBUTION IN ARGON AT INCREASING FIELDS:



Ionizzazione dei gas

- **Charge transfer collisions** can occur when a positive ion encounters another neutral gas molecule. In such a collision an electron is transferred from the neutral molecule to the ion, thereby reversing the roles of each.
- This charge transfer is particularly significant in gas mixtures containing several different molecular species. There will then be a tendency to transfer the net positive charge to the gas with the lowest ionization energy because energy is liberated in collisions which leave that species as the positive ion.

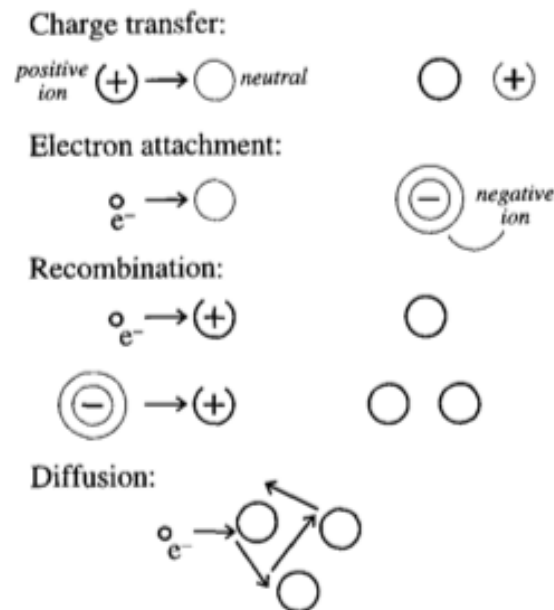


Figure 5.1 Shown are some types of interactions of charged species in a gas that can influence the behavior of gas-filled detectors. In the first four illustrations, the interacting species are on the left and the products of the interaction are on the right. Neutral atoms or molecules are shown as simple circles.

Ionizzazione dei gas

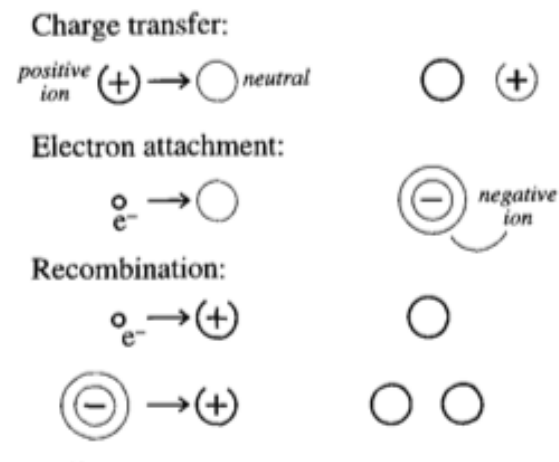
Atomi elettronegativi.

Atomi elettronegativi possono catturare un e^- formando degli ioni negativi. Questi atomi hanno il livello più esterno quasi pieno per cui l'aggiunta di un e^- risulta in un rilascio di energia.

L'energia rilasciata è nota come affinità elettronica. La presenza di atomi elettronegativi diminuisce l'efficienza di collezione ione-elettrone mangiandosi gli elettroni prima che questi possano raggiungere gli elettrodi di rivelazione.

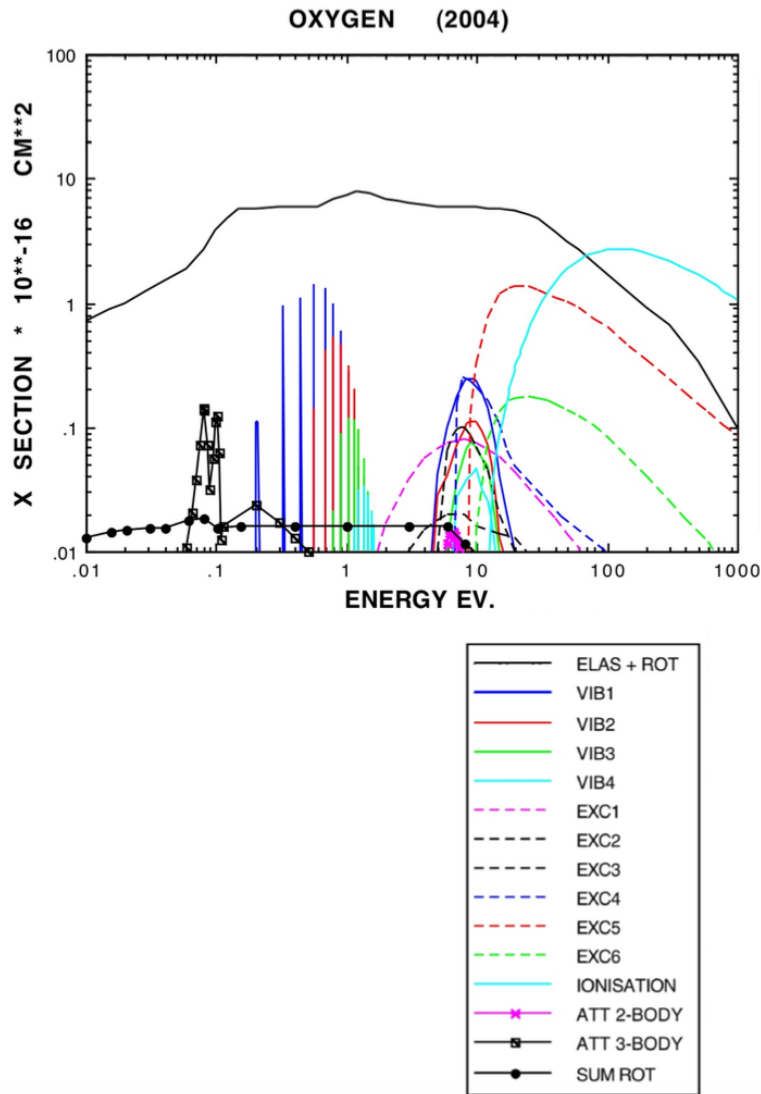
Gas elettonegativi : O_2 , H_2O , CO_2 , SF_6 (→ filtri per H_2O e O_2).

Hanno invece affinità elettronica negativa He, Ne, Ar, Xe, Kr (gas nobili).

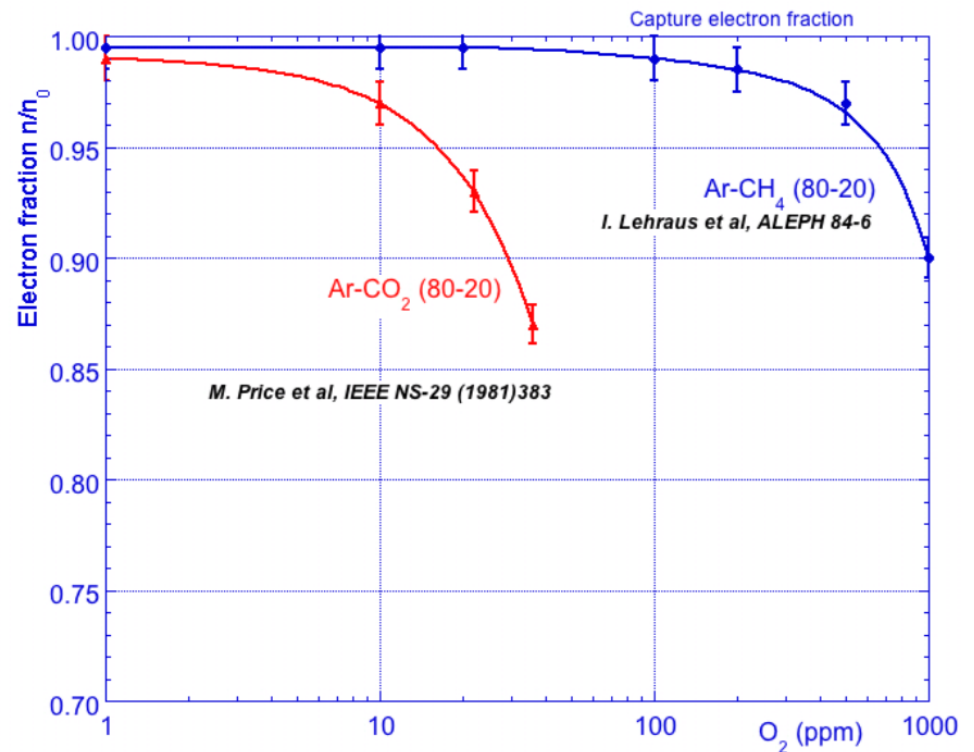


ELECTRON CAPTURE

ATTACHMENT CROSS SECTION OF OXYGEN:

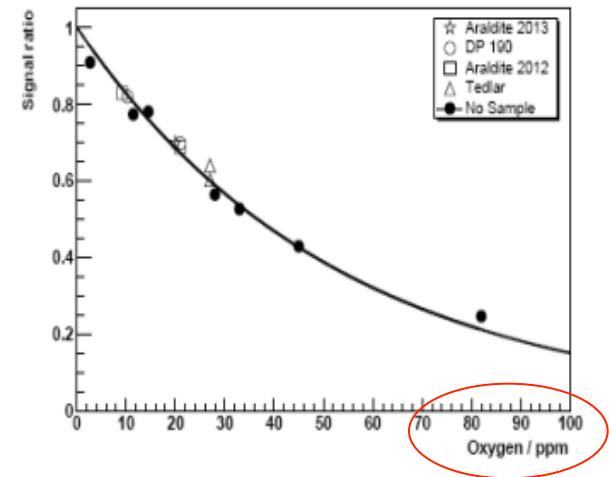


ELECTRONS SURVIVING AFTER 20 CM DRIFT (E = 200 V/cm):



Attachment, recombination

- Electronegative molecules (e.g., O_2 , H_2O , CF_4) capture electrons to form negative ions; reduces signal, impacts measurement of ionization energy loss (dE/dx)
- Effect highly sensitive to contaminant concentration, differs in different primary gas mixtures
- Recombination most likely in regions of low $\mathbf{E}$ field



Ionizzazione dei gas

- Collisions between positive ions and free electrons may result in **recombination** in which the electron is captured by the positive ion and returns it to a state of charge neutrality.
- Alternatively, the positive ion may undergo a collision with a negative ion in which the extra electron is transferred to the positive ion and both ions are neutralized. In either case, the charge represented by the original pair is lost and cannot contribute further to the signal in detectors based on collection of the ionization charge.

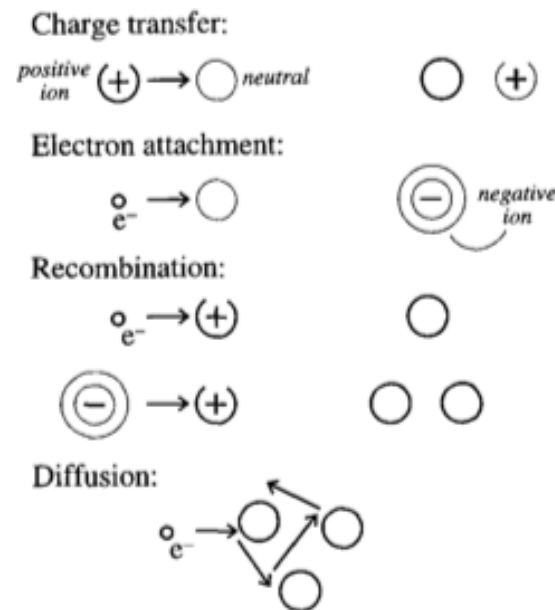


Figure 5.1 Shown are some types of interactions of charged species in a gas that can influence the behavior of gas-filled detectors. In the first four illustrations, the interacting species are on the left and the products of the interaction are on the right. Neutral atoms or molecules are shown as simple circles.



Ionizzazione dei gas

- There are two general types of recombination loss: *columnar* recombination and *volume* recombination. The first type (sometimes also called *initial* recombination) arises from the fact that ion pairs are first formed in a column along the track of the ionizing particle. The local density of ion pairs is therefore high along the track until the ion pairs are caused to drift or diffuse away from their point of formation. Columnar recombination is most severe for densely ionizing particles such as incident heavy nuclei with high Z compared with fast electrons that deposit their energy over a much longer track. This loss mechanism is dependent only on the local conditions along individual tracks and does not depend on the rate at which such tracks are formed within the detector volume.
- In contrast, volume recombination is due to encounters between ions and/or electrons after they have left the immediate location of the track. Impurities are important and since many tracks are typically formed over the time it takes for ions to drift to the collecting electrodes, it is possible for ions and/or electrons from independent tracks to collide and recombine. Volume recombination therefore increases in importance with irradiation rate.
- Charge separation and collection should be as rapid as possible in order to minimize recombination, and high electric fields are indicated for this purpose.

Ionizzazione dei gas

Let do some more quantitative evaluation.

The ionization generates $n^+ = n^-$ ions and e^- . The pairs will interact with the charges present in the environment.

Given a specie of charge density $n^\pm$, since the collision frequency is proportional to the product of the concentrations of the species involved, the recombination rate can be written as

$dn^-/dt = -b_e n^- N^+$, where N^+ is the local ion density

$dn^+/dt = -b_A n^+ N^*$, where N^* is any ion/molecule/electron that can interact with another ion

The recombination coefficient b is normally orders of magnitude larger between positive ions and negative ions compared with that between positive ions and free electrons: $b_A \gg b_e$.

In gases that readily form negative ions through electron attachment, virtually all the recombination takes place between positive and negative ions.

Ionizzazione dei gas

The initial recombination is due to the pairs before they get separated by the electric field.

In this case $N^+ = n^+$, $N^* = n^-$ and $n^+ = n^-$ so that

$$dn^+/dt = dn^-/dt = dn/dt = -b_e n^2 \rightarrow n = n_0 / (1 + b_e n_0 t)$$

n_0 is the concentration at $t=0$.

If R is the radius of the ionisation cylinder, the time needed for e-ion separation is $t_s \sim R/v_D \rightarrow$

nb: v_D is the electron drift speed since $\mu_e \gg \mu_{ion}$

$n(t_s) = n_0 / (1 + b_e n_0 R / v_D) \rightarrow$ this is the charge that has not recombined and migrates to the electrodes

Introducing a "density parameter" $n^* = v_D / b_e R$

$$n = n_0 / (1 + n_0 / n^*) = n^* n_0 / (n^* + n_0) \rightarrow 1/n = 1/n_0 + 1/n^* \rightarrow 1/n - 1/n_0 = 1/n^* \\ \rightarrow (n_0/n - 1) = n_0/n^*$$

If $n^* \gg n_0$, $n_0/n \approx 1$

If $n^* \ll n_0$, $n_0/n \approx n^* \ll n_0 \rightarrow$ if $v_D \ll b_e R$, the charge recombines before separation

In the limit $n^* \gg n_0$, $n \approx n_0(1 - n_0/n^*) \equiv A(n_0) n_0$

Ionizzazione dei gas

Now in the volume recombination, the ions N^+ and N^* are those present in the gas as impurities with a constant concentration, that the drifting charges encounter during the propagation to the electrodes.

$$dn^-/dt = -b_e n^- N^+$$

$$dn^+/dt = -b_A n^+ N^* \quad (N^* \text{ is any ion that can interact with another ion})$$

We can define an average interaction time $\tau = 1/bN$. Dimensionally $b = \sigma v =$ interaction volume per unit of time

$$dn^-/dt = -n^-/\tau_e$$

$$dn^+/dt = -n^+/\tau_+$$

Since $d/dt = v(d/dx)$ and the velocity is the drift speed v_D , we have $\lambda = v_D \tau = v_D/bN$

$$dn^-/dx = -n^-/\lambda_e$$

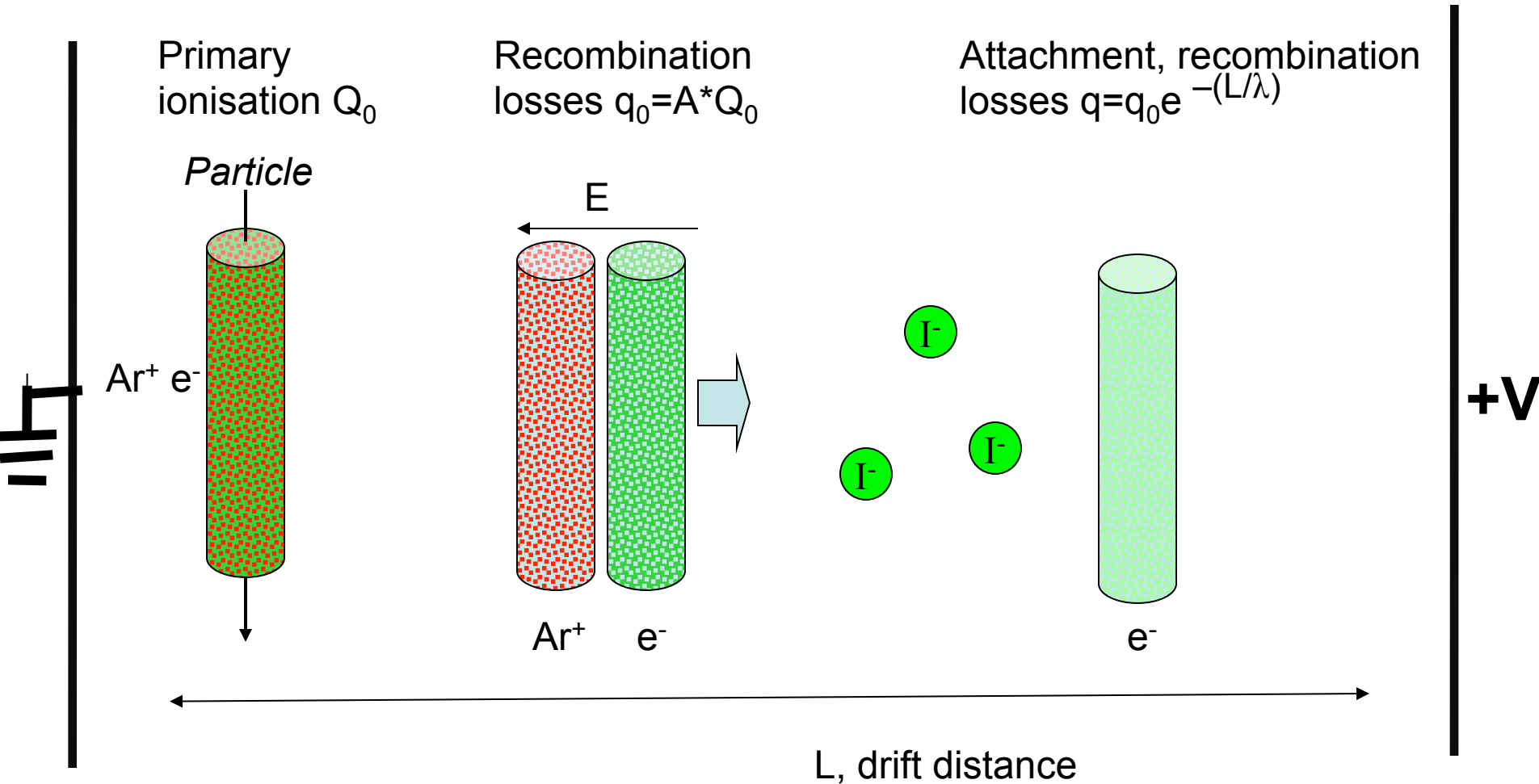
$$dn^+/dx = -n^+/\lambda_+$$

Therefore $n(x) = n(0)\exp(-x/\lambda)$

Columnar and volume recombination

Response to a primary ionization

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

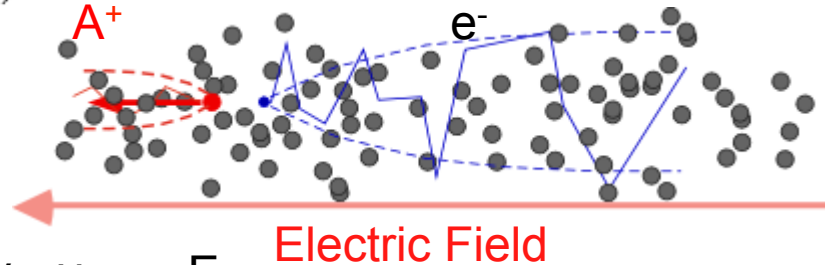


Losses are described in the source term R . The initial charge distribution is given as boundary condition.

Drift and Diffusion in Presence of E field

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

$E > 0$ charge transport and diffusion

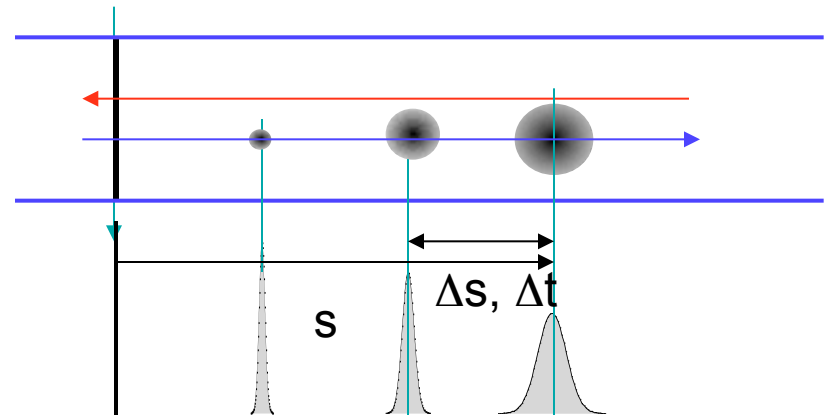


Charges move with average Drift velocity $v_D = \mu E$

While charge distribution width increases with time $\sigma \sim \sqrt{Dt}$ Diffusion

I.e. the net motion is drift + diffusion: the average position of the charge swarm moves as $x = v_D t$, while the width increases in time

And some charge is lost by recombination, capture, etc. processes both in columnar and volume recombination



Misure di posizione e ionizzazione

Gli elettroni e gli ioni primari e secondari prodotti per ionizzazione sono pochi (~ 100 in un cm di gas)



- ❖ Evitare il più possibile la ricombinazione (evitare il più possibile la presenza di gas elettronegativi quale Acqua ed Ossigeno)
- ❖ Aumentare il campo E per sfruttare la moltiplicazione a valanga delle coppie e-ione, alto guadagno.