Particle Detectors

Lecture 8
29/04/17

a.a. 2016-2017

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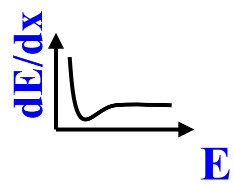
Interactions of Particles with Matter

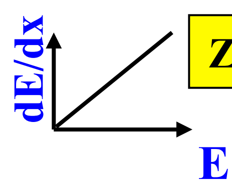
charged	γ

- Ionisation



Z

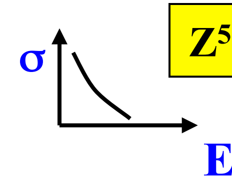
- Bremsstrahlung



Z(Z+1)

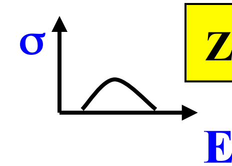
+ cerenkov and transition radiation ($\sim Z^2$)

- P.e. effect



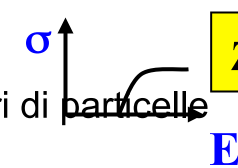
Z^5

- Comp. effect



Z

- Pair production

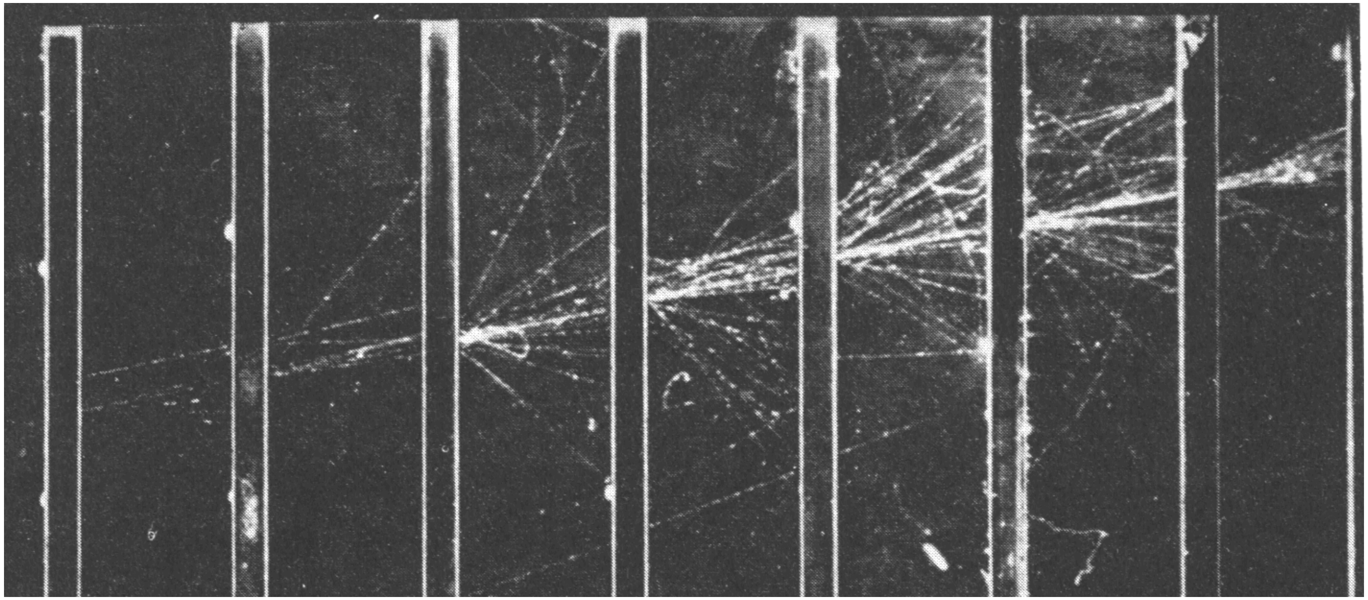


Z(Z+1)

Sciami elettromagnetici

A questo punto e' abbastanza facile capire cosa succede se si ha un fascio di $e^{\pm}$ o di γ che attraversa del materiale:

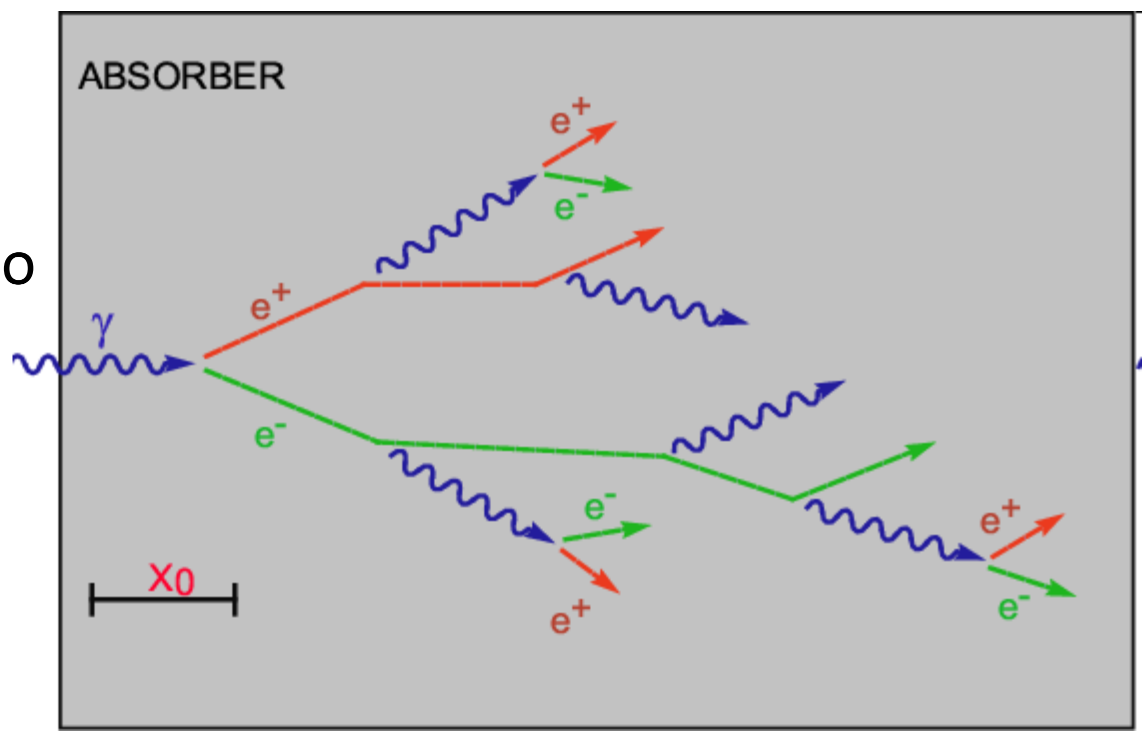
Produzione di una cascata di particelle



Electron shower in a cloud chamber with lead absorbers

Electromagnetic showers

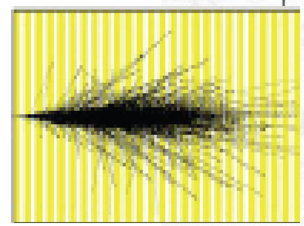
- Cascade of pair production and bremsstrahlung is known as an electromagnetic shower
- number of low-energy photons (or electrons) produced is proportional to initial energy of electron or gamma
- Energy collected in each of $e^{\pm}$ and γ is also proportional to initial energy

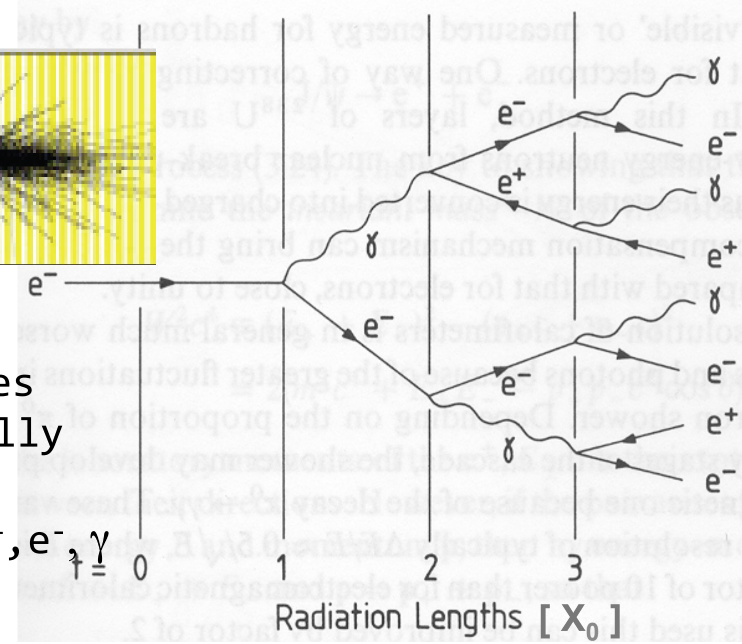


Electromagnetic Shower Development

[A simple shower model](#)

Two dimensionless variables: $t=x/X_0$ and $y=E/E_c$ drive shower development





Shower development:

Start with an electron with $E_0 \gg E_c$

→ After $1X_0$: 1 e^- and 1 γ, each with $E_0/2$

→ After $2X_0$: 2 e^-, 1 e^+ and 1 γ, each with $E_0/4$

→ After tX_0 :

$$N(t) = 2^t = e^{t\ln 2}$$

$$E(t) = E_0 / 2^t$$

→ Number of particles increases exponentially with t

→ equal number of e^+, e^-, γ

$$t(E') = \frac{\ln(E_0/E')}{\ln 2}$$

$$N(E > E') = \frac{1}{\ln 2}\frac{E_0}{E'}$$

→ Depth at which the energy of a shower particle equals some value E'

→ Number of particles in the shower with energy > E'

Maximum number of particles reached at $E = E_c$

Ec is the critical energy →

$$t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

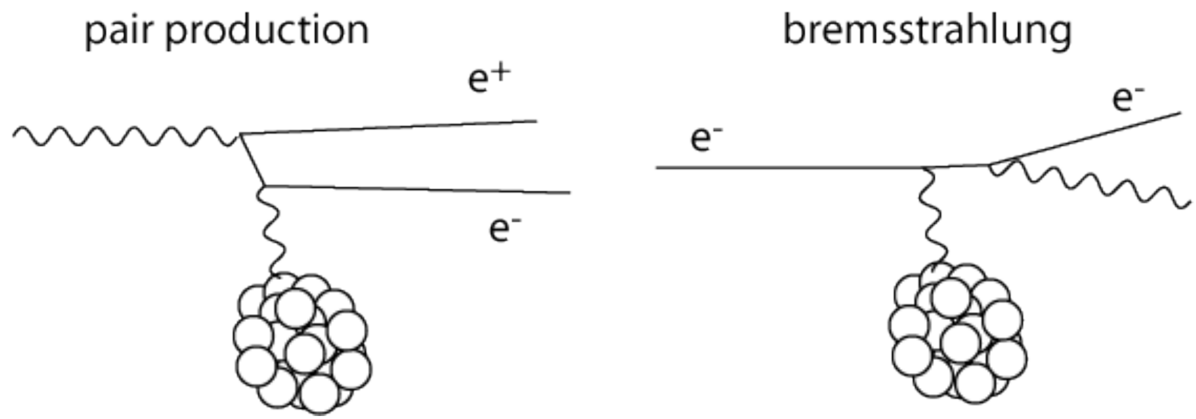
$$N_{max} = e^{t_{max}\ln 2} = E_0/E_c$$

When the particle E < Ec, absorption processes like ionisation for electrons and Compton and photoelectric effects for photons start to dominate.

Electron and γ Interactions

At E > 10 MeV, interactions of γs and e^-s in matter is dominated by e^+e^- pair production and Bremsstrahlung.

At lower energies, Ionization becomes important.



The ratio of the energy loss for these processes is:

$$R = \left(\frac{dE}{dx}\right)_{Brem} \bigg/ \left(\frac{dE}{dx}\right)_{ion} \sim \frac{ZE}{580\,MeV}$$

Critical Energy:

When energy loss due to Brem and energy loss due to ionization are =.

$$E_c = \frac{580\,MeV}{Z}$$

Energy deposition

- During the shower development, the energy deposit is mainly due to charged e+ and e- with $E \gg E_c$.
- They are MIPs, so the energy loss by ionization stays constant $\approx (dE/dX)_{mip} = X_0 (dE/dt)_{mip}$, $t = X/X_0$
- The energy deposit is the sum of energy loss by each charged particle: $(dE/dt)_{tot} = (1/2) N(t)\ (dE/dt)_{mip}$ (since in the toy model we have 50% of e+e- and 50% photons)
- So dE/dt grows as N(t) and is max when N(t) is max

$$(dE/dt)_{max} = (1/2) N_{max}(t) (dE/dt)_{mip}$$
$$= (1/2)(E_0/E_c)(dE/dt)_{mip}$$

at the position $t_{max} = \ln(E_0/E_c)/\ln 2$

Electromagnetic Shower Development

Let us take as an example the shower in a CsI (ρ = 4.53 g/cm^3, X0 = 8.39 g/cm^2 or 1.85 cm) crystal detector initiated by a 1 GeV photon. Using the value $E_c \approx 10$ MeV, the number of particles in the shower maximum $N_{max} = E_0/E_c = 100$ and for the depth of the shower maximum at $\approx 6.6\ X_0$.

$$\boxed{\begin{aligned} t_{\max} &= \frac{\ln(E_0/E_c)}{\ln 2} \\ N_{\max} &= e^{t_{\max}\ln 2} = E_0/E_c \end{aligned}}$$

Note that tmax dependes only on the ratio $Y = E/E_c$. The only dependence on the material properties is in $E_c = 580/Z$ MeV

Electromagnetic Shower Development

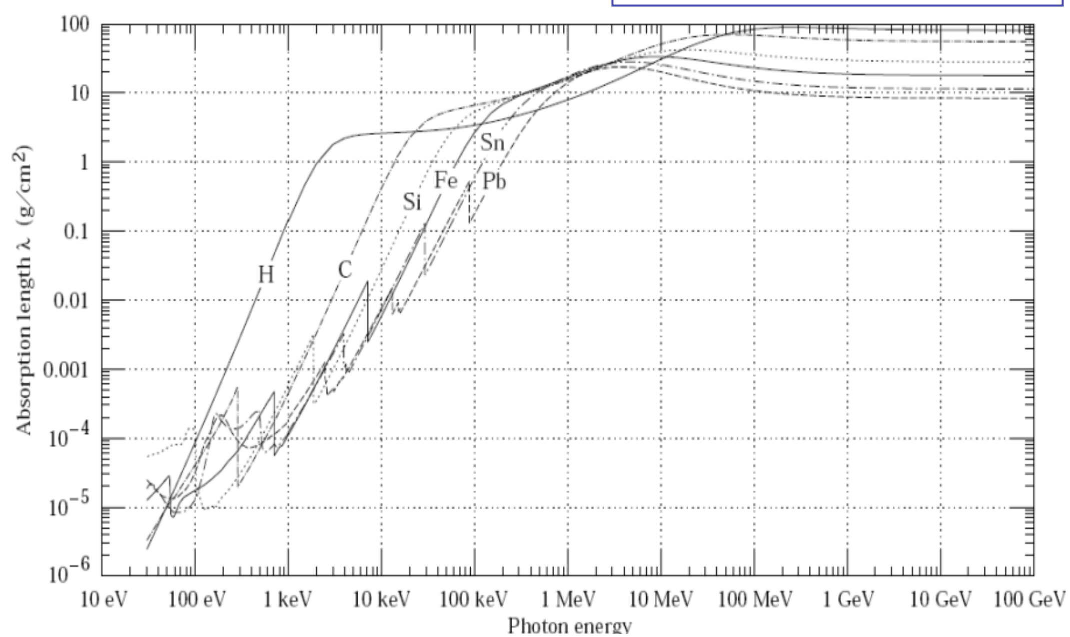
After the shower maximum, particle production stops and electrons and positrons having an energy below E_c will stop in a layer of $\sim 1X_0$ by ionization loss.

The remaining photons ($\sim N_{max}/2$) of the same energy have to absorbed by p.e. and/or compton (since pair prod does not occur)

$$t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

$$N_{max} = e^{t_{max}\ln 2} = E_0/E_c$$

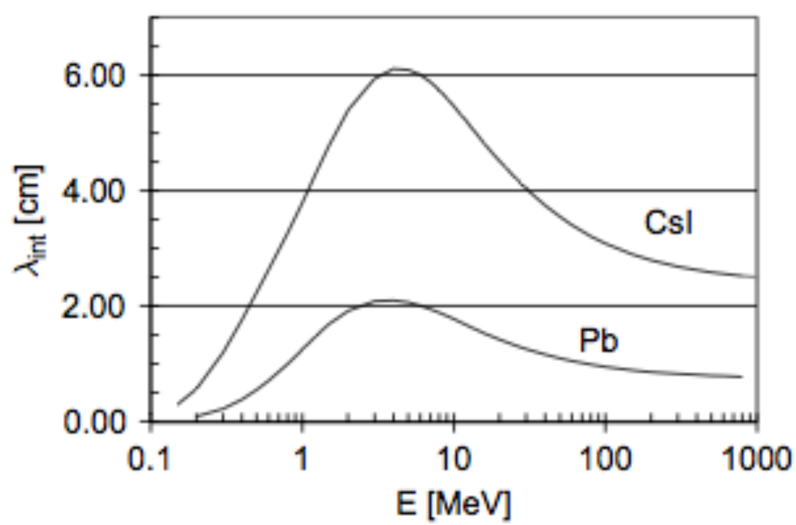
To absorb the 95% of ~1÷10 MeV photons we need
$x/\lambda = \ln(I_0/I) = \ln(1/0.05) = 3$
Example: let take Si.
At ~1 MeV, λ is ~20 g/cm^2, so we need 60 g/cm2 of material after shower max
The radiation length of Si is 20.82 g/cm^2 (or 9.36 cm) ≈ 1 X0.
So using Si, we need at least additional 3 X0 to absorb the 95% of the energy of the initial particle.



Electromagnetic Shower Development

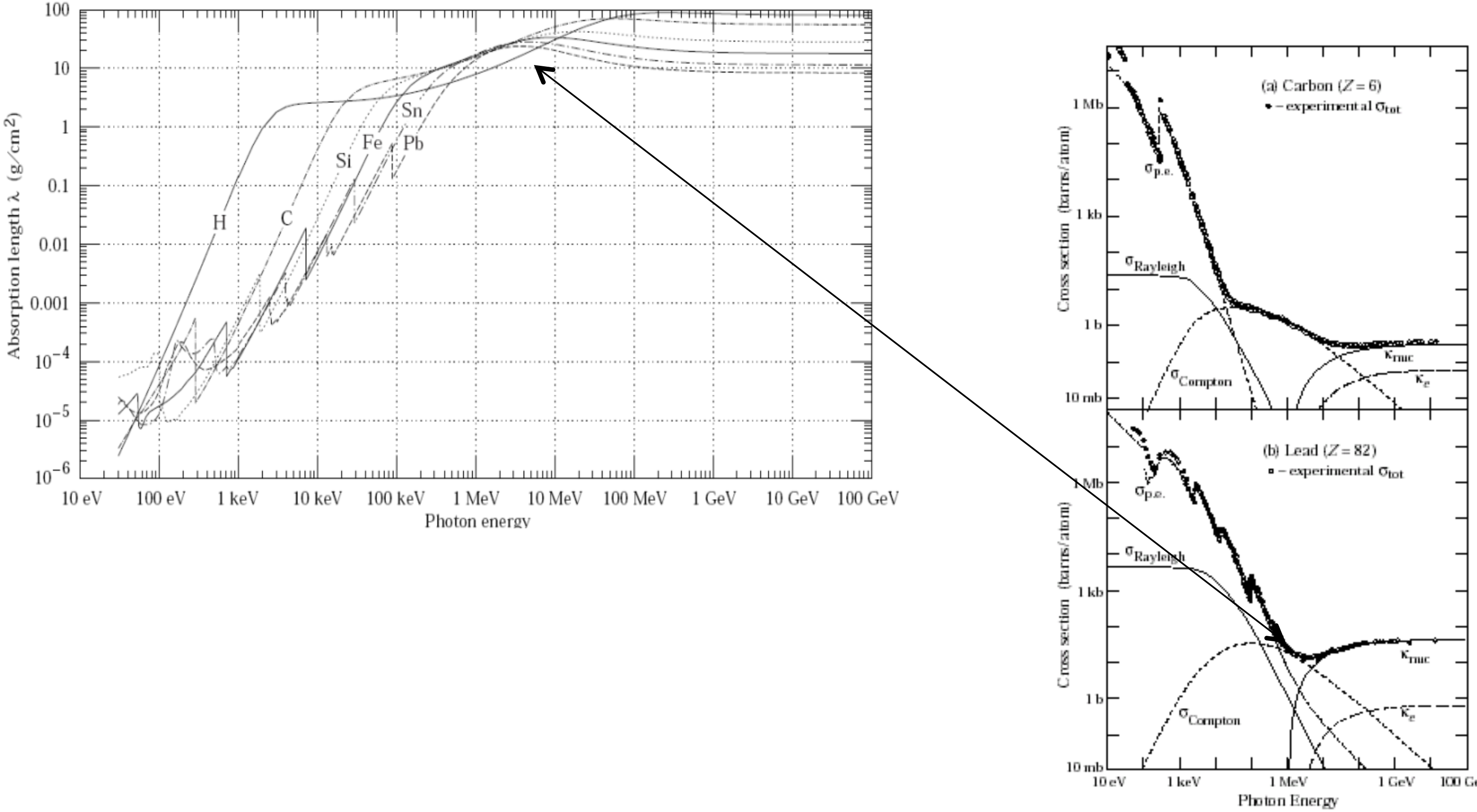
$$t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$$
$$N_{max} = e^{t_{max}\ln 2} = E_0/E_c$$

For CsI, λ_{int} has broad maximum between 1 MeV÷10 MeV where it amounts to about $3X_0$.
The energy of photons in the shower maximum close to E_c is just in this range.
Thus, to absorb 95% of γ produced in the shower maximum, an additional $9X_0$ of material is necessary which implies that the thickness of a calorimeter with high shower containment should be at least $16X_0$.



2. Photon interaction length in lead and CsI [4].

Broad maxima in λ correspond to the minima of cross section



Electromagnetic Shower Development

This very simple model already correctly describes the most important qualitative characteristics of electromagnetic showers:

(i) To absorb most of the energy of the incident photon the total detector thickness should be more than 10–15X0.

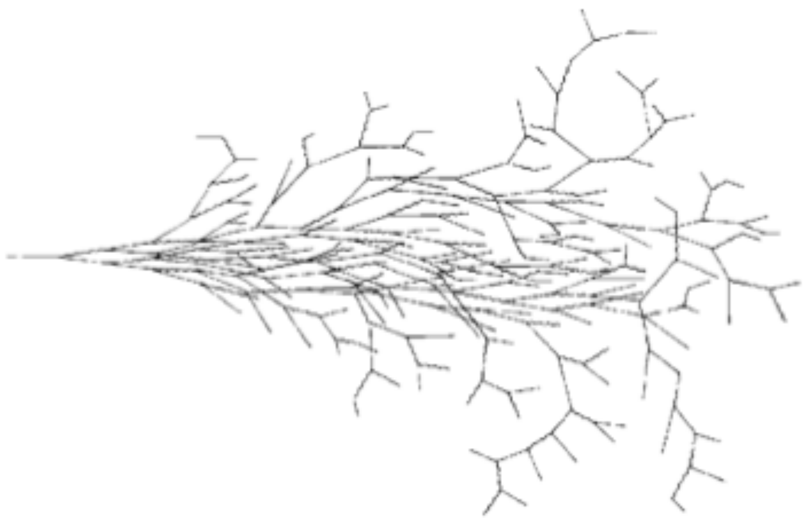
(ii) The position of the shower maximum increases slowly with energy. Hence, the thickness of the calorimeter should increase as the logarithm of the E.

(iii)The energy deposition in an absorber is a result of the ionisation losses of electrons and positrons. Since the $(dE/dx)_{ion}$ value for relativistic electrons is almost energy independent (MIP!), the amount of E deposited in a thin layer of absorber is ∝ to the number of electrons and positrons crossing this layer.

(iv) The energy leakage is caused mostly by soft photons escaping the calorimeter at the sides (lateral leakage) or at the back (rear leakage).

Electromagnetic Shower Development

In real life, the shower development is much more complicated.



(A) $$\frac{dE}{dt} = E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)} ,$$

where $\Gamma(a)$ is Euler's Γ function, defined by

$$\Gamma(g) = \int_0^\infty e^{-x} x^{g-1}\, dx .$$

The gamma function has the property

$$\Gamma(g+1) = g\,\Gamma(g) .$$

At present, an accurate description is obtained from Monte Carlo simulations.
The longitudinal distribution of the energy deposition in electromagnetic cascades is reasonably described by a gamma distribution (A), where a and b are model parameters.

The maximum t_{max} occurs at (a−1)/b. Fits to shower profiles were made in elements ranging from C to U, at energies from 1 GeV to 100 GeV. The energy deposition profiles are well described by Eq. (A) with $\mathbf{t_{max}} = \mathbf{(a-1)/b = \ln(E_0/E_c) + C_{\gamma e}}$ **(B)** with $C_{\gamma e}$ = 0.5 for γ induced, -0.5 for electron induced shower.
To use Eq. (A, one finds (a−1)/b from Eq. (B) then finds a either by assuming b≈0.5 or by finding a more accurate value from tabulated values

Fitted b values

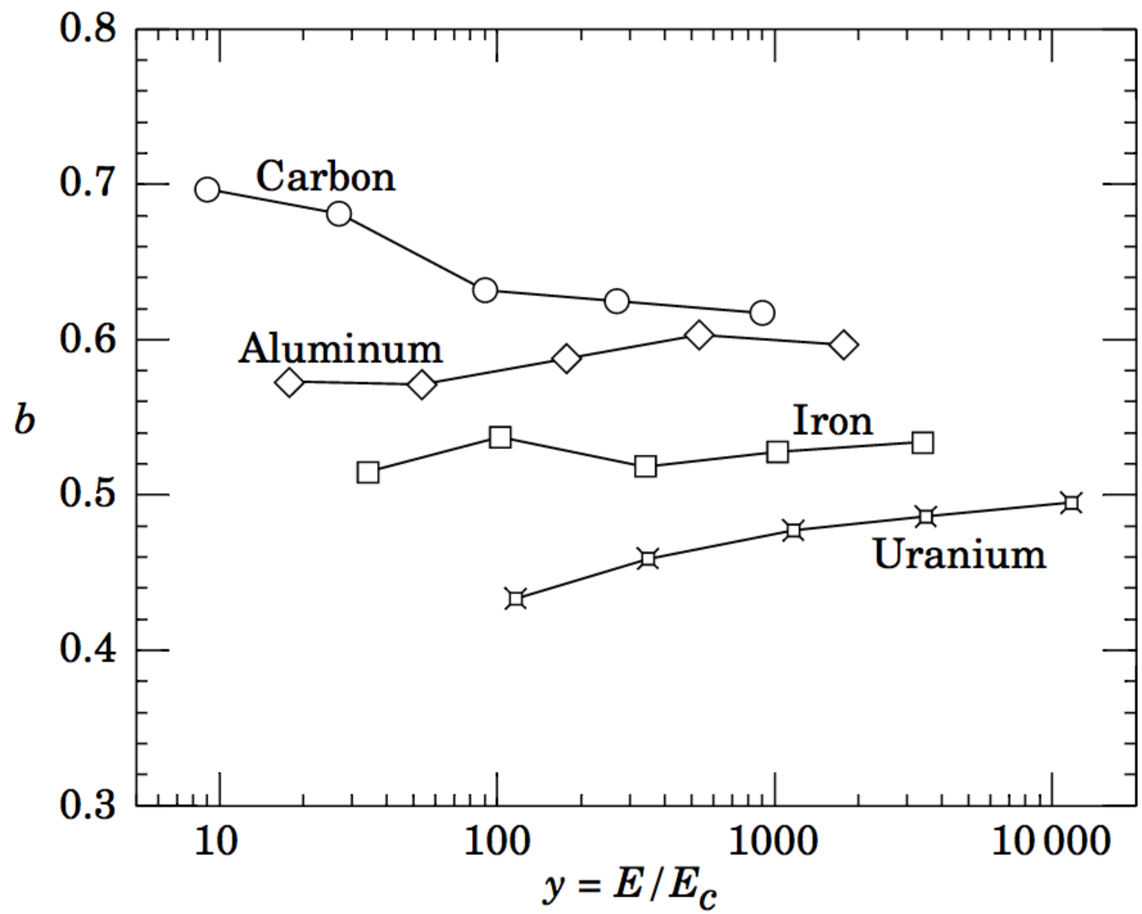


Figure 32.21: Fitted values of the scale factor b for energy deposition profiles obtained with EGS4 for a variety of elements for incident electrons with $1 \leq E_0 \leq 100$ GeV. Values obtained for incident photons are essentially the same.

Electromagnetic Shower Development

Simulation of the energy deposit in copper as a function of the shower depth for incident electrons shows the logarithmic dependence of t_{max} with E.

EGS4* (electron-gamma shower simulation)

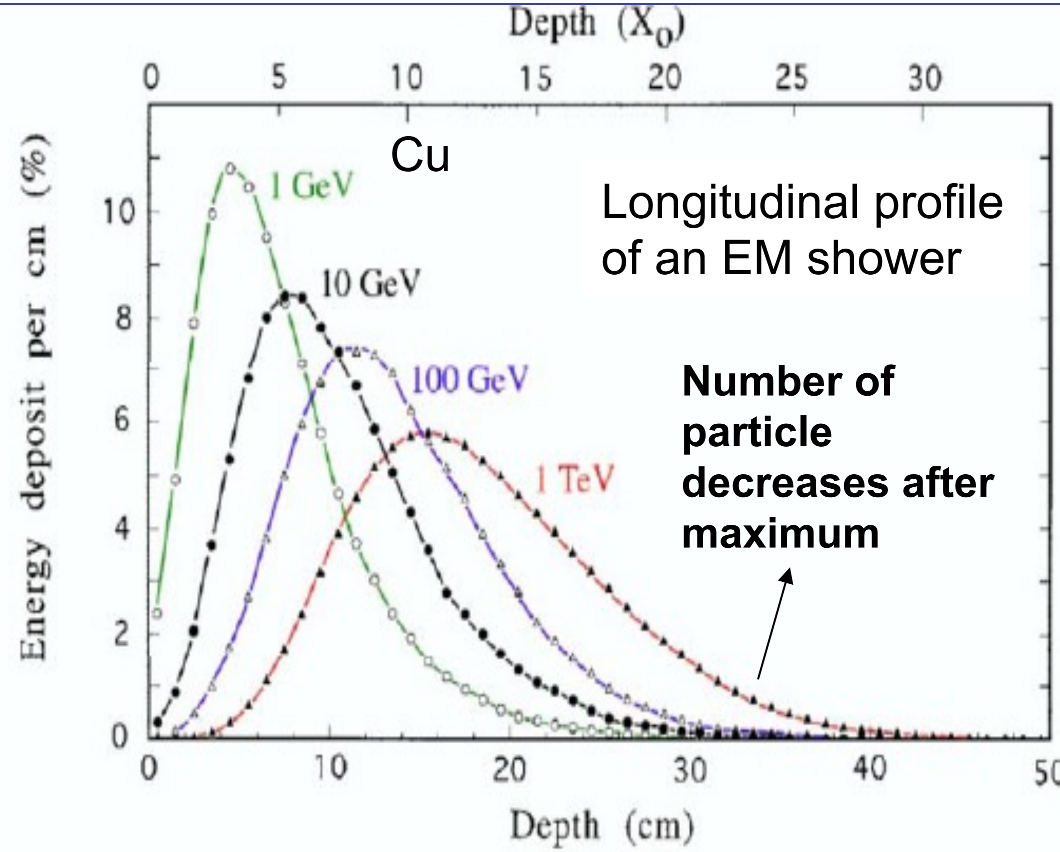


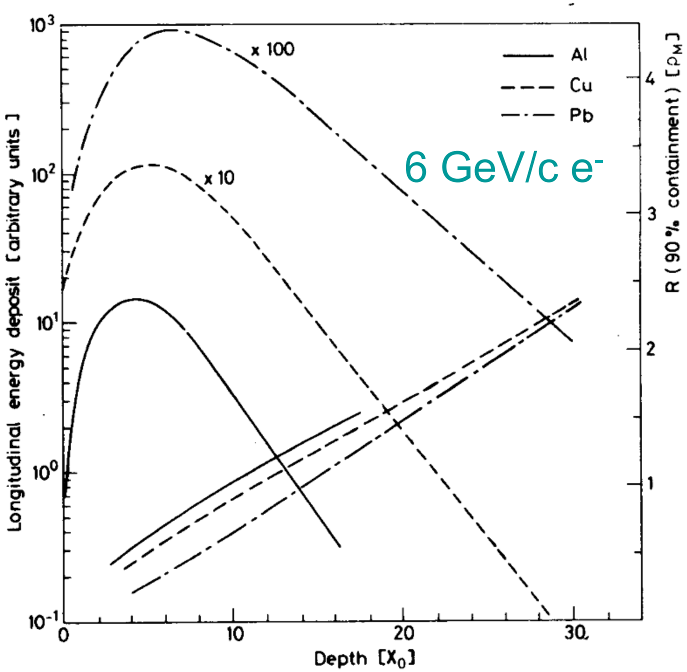
FIG. 2.9. The energy deposit as a function of depth, for 1, 10, 100 and 1000 GeV electron showers developing in a block of copper. In order to compare the energy deposit profiles, the integrals of these curves have been normalized to the same value. The vertical scale gives the energy deposit per cm of copper, as a percentage of the energy of the showering particle. Results of EGS4 calculations.

*EGS4 is a Monte Carlo code for doing simulations of the transport of electrons and photons in arbitrary geometries and materials.

Sciami elettromagnetici

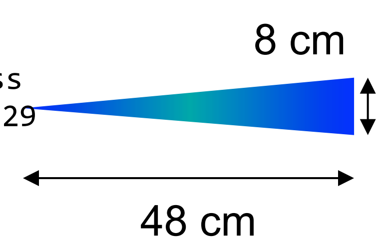
Lo sviluppo trasversale dello sciame non e' tanto dovuto agli angoli di emissione di γ od $e^{\pm}$ (entrambi molto piccoli) quanto allo scattering multiplo. 95% dello sciame e' in un cilindro di raggio $2R_M$

$$R_M = \frac{21 MeV}{E_c} X_0 \left[gr/cm^2\right]$$ Raggio di Molière



Sviluppo longitudinale e trasversale dello sciame scalano con X_0, R_M

Example: E_0 = 100 GeV in lead glass
E_c=11.8 MeV → tmax ≈ 13, t95% ≈ 29
X_0 ≈ 2 cm, R_M = 1.8·X_0 ≈ 3.6 cm



(C. Fabjan, T. Ludlam, CERN-EP/82-37)

Sciami elettromagnetici

- <u>Sviluppo longitudinale</u> dello sciame

$$\boxed{\frac{dE}{dt} \propto t^{\alpha} e^{-t}}$$

Massimo dello sciame

$$\boxed{t_{\max} = \ln\frac{E_0}{E_c} + C_{\gamma e}}$$

Contenimento longitudinale

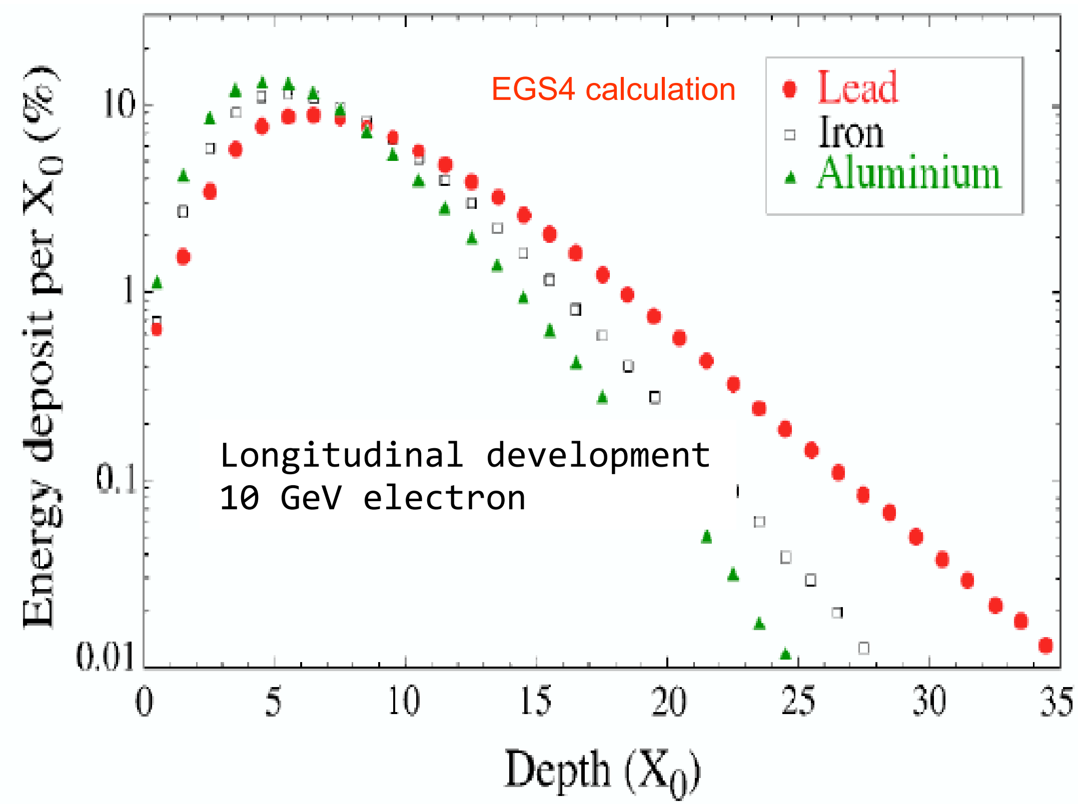
$$\boxed{t_{95\%} \approx t_{\max} + 0.08Z + 9.6}$$

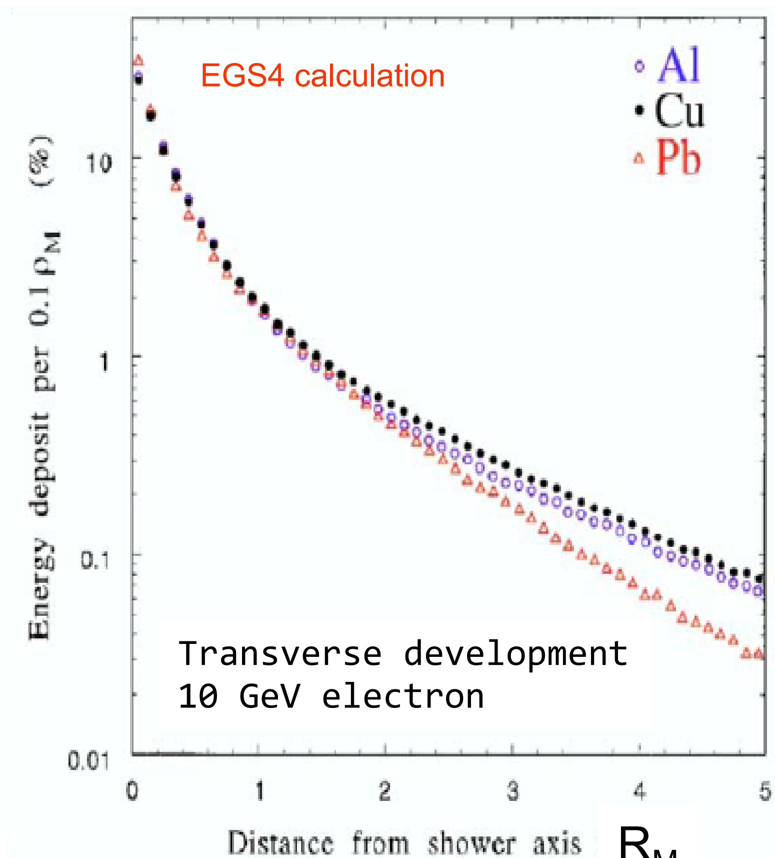
Le dimensioni longitudinali dello sciame crescono solo logaritmicamente con E_0

Electromagnetic Shower Development

Shower profile

the longitudinal and transverse developments to scale with X_0





$X_0 \propto A/Z^2$, $E_c \propto 1/Z \Rightarrow R_M \propto A/Z$ → $\boldsymbol{R_M}$ less dependent on Z than $\boldsymbol{X_0}$:

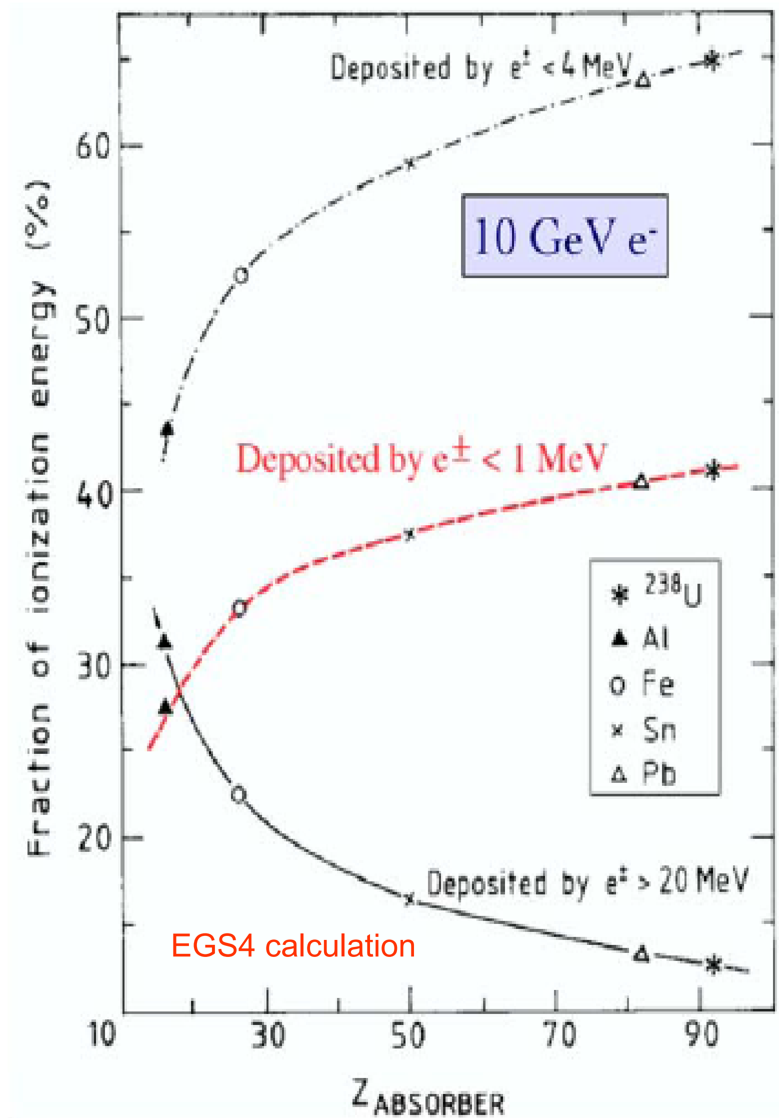
Electromagnetic Shower Development

Energy deposition

The fate of a shower is to develop, reach a maximum, and then decrease in number of particles once $E_0 < E_c$

Given that several processes compete for energy deposition at low energies, it is important to understand the fate of the particles in a shower.

→ Most of energy deposition is by low energy $e^{\pm}$' s.

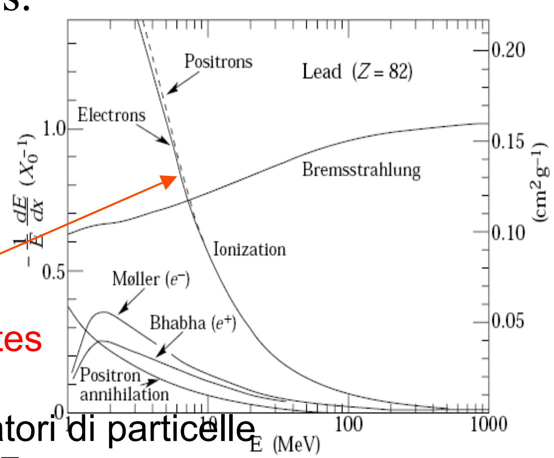


60% $e^{\pm}$ (< 4 MeV)

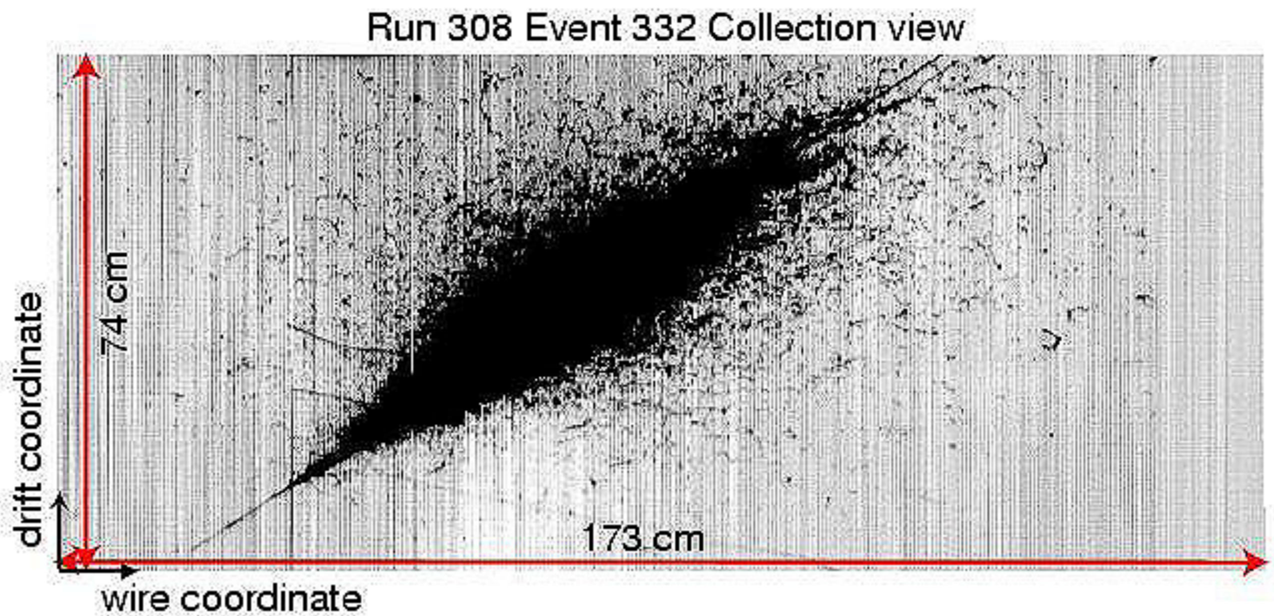
40% $e^{\pm}$ (< 1 MeV)

$e^{\pm}$ (>20 MeV)

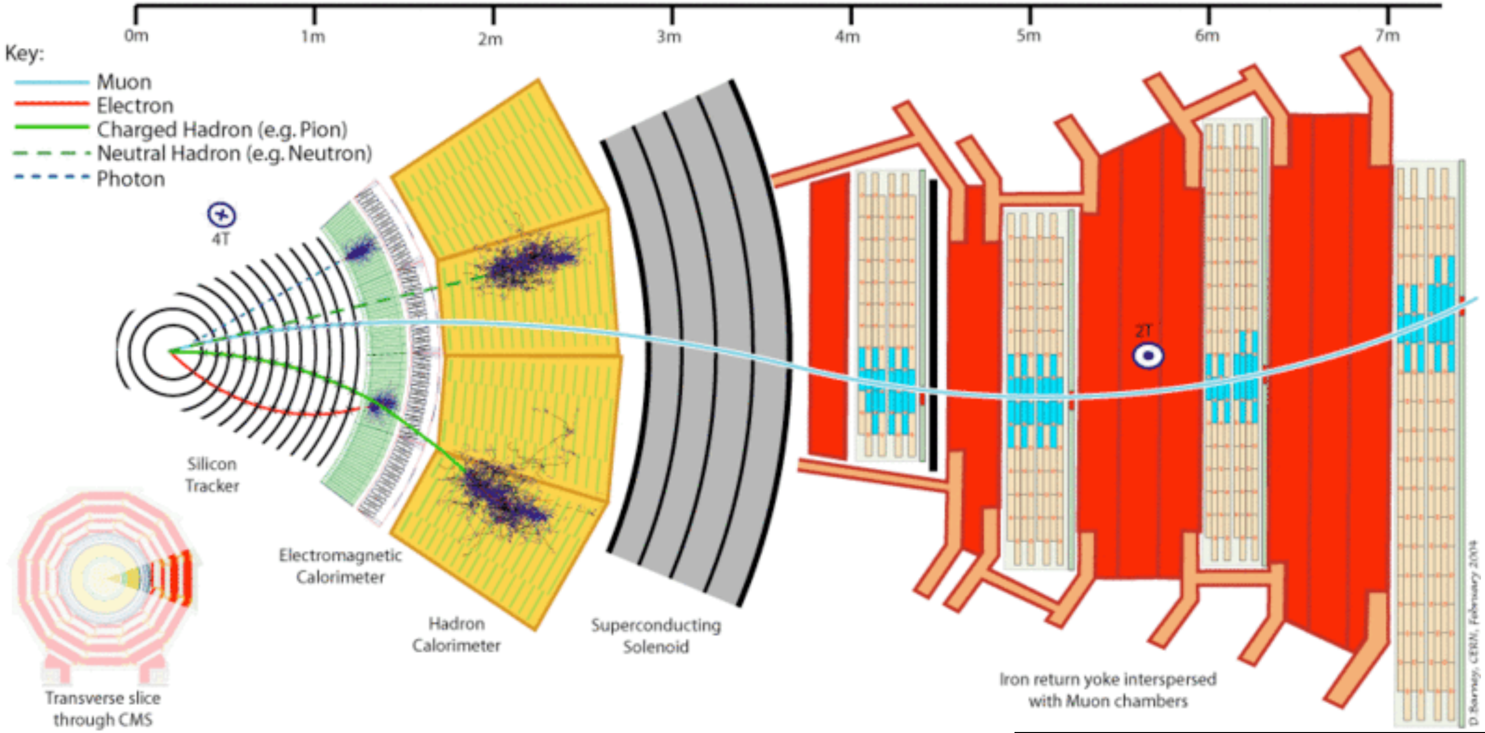
Ionization dominates



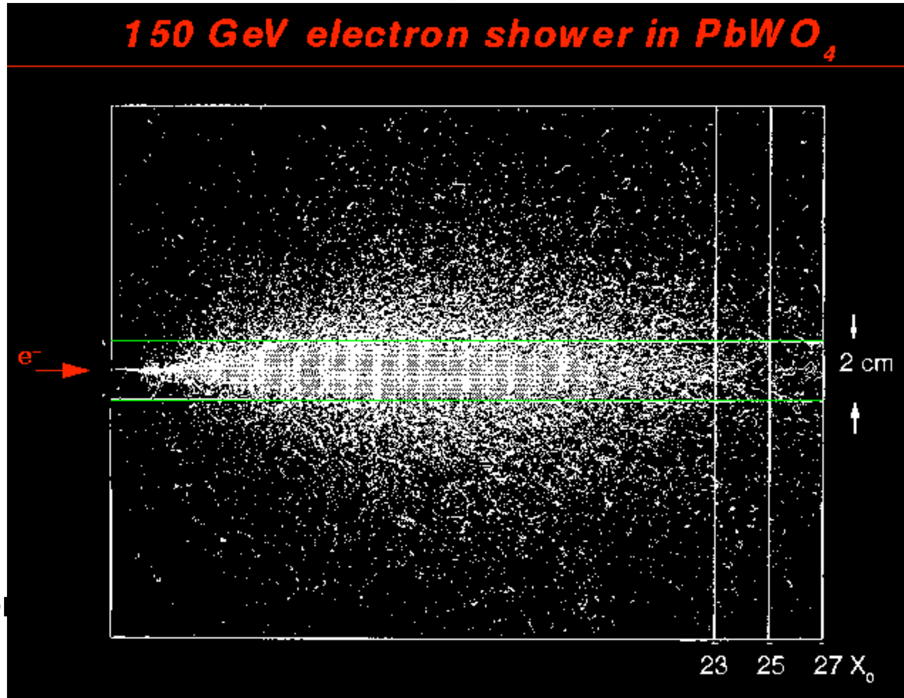
Shower



- ICARUS, liquid argon drift chamber (misura la ionizzazione)
- Quando si forma lo sciame, la particella iniziale idealmente deposita tutta la sua energia nel materiale e viene o assorbita o distrutta (nella pratica ci sono sempre "perdite" dovute a particelle che sfuggono dal volume attivo)
- Se si misura l'energia depositata, possiamo conoscere quella della particella iniziale
- Applicazione tipica nei calorimetri



Cms elm calorimeter



Sciami elettromagnetici

❖ Risoluzione in energia

Per misurare l'energia si contano le tracce cariche

$$\boxed{N_{totale} \propto \frac{E_0}{E_c}}$$

$$\Rightarrow$$

$$\boxed{\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}}}$$

La risoluzione relativa in energia migliora crescendo E_0

Anche la risoluzione spaziale ed angolare scalano con $E^{-1/2}$