

Particle Detectors

Lecture 5

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Emissione Cherenkov e di radiazione di transizione

- Per rivelare e identificare particelle cariche due processi di emissione di radiazione elettromagnetica sono importanti in **mezzi diversi dal vuoto**:
 - emissione cherenkov
 - emissione di radiazione di transizione

(Cenni di) Emissione di radiazione elm nella materia condensata

- Per discutere i processi di emissione Cherenkov e di radiazione di transizione e' necessario fare alcuni brevi richiami di elettrodinamica classica

Equazioni di Maxwell (unita' Gauss)

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (1) \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad (3)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \quad (2) \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \quad (4)$$

Da (1), si puo' definire un potenziale vettore **A** t.c. $\vec{B} = \vec{\nabla} \times \vec{A}$

Da (4) segue che esiste un potenziale scalare Φ t.c. $\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$

Da (3) e (2) si ottengono le equazioni dei potenziali

$$\vec{\nabla}^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} + \frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t}) = -4\pi\rho$$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} - \vec{\nabla} (\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t}) = -\frac{4\pi}{c} \vec{j}$$

Potenziali elm (1)

$$\vec{\nabla}^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} + \frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t}) = -4\pi\rho$$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} - \vec{\nabla} (\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t}) = -\frac{4\pi}{c} \vec{j}$$

Grazie all'invarianza di gauge ($\mathbf{A} \rightarrow \mathbf{A} + \nabla\chi$, $\Phi \rightarrow \Phi - \partial\chi/\partial t \rightarrow$ **stessi campi $\mathbf{E}$, $\mathbf{B}$**) si possono scegliere i potenziali in modo da "semplificare" le equazioni:

Gauge di Lorentz $\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0$

Gauge di Coulomb $\vec{\nabla} \cdot \vec{A} = 0$

In questo gauge, $\vec{\nabla}^2 \Phi = -4\pi\rho$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\frac{4\pi}{c} \vec{j}_t$$

Il potenziale scalare e' "statico", mentre quello vettore si propaga come un'onda la cui sorgente e' la componente trasversa della corrente

- Per gli immancabili entusiasti: le soluzioni...

Potenziali elm: soluzioni

Gauge di Lorentz $\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0$

$$\vec{\nabla}^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi \rho$$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\frac{4\pi}{c} \vec{j}$$

4 Equazioni (3 $\mathbf{A}$, 1 ϕ) d'onda non omogenee disaccoppiate di forma

$$\vec{\nabla}^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -4\pi f(\vec{x}, t) \quad f(\vec{x}, t) = \rho, j_i/c$$

Soluzione:

soluzione dell'equ. omogenea associata + soluzione particolare della non omogenea

Soluzione omogenea: l'equazione omogenea e' quella delle onde $\rightarrow$

$$\vec{A} = A_{k1} \vec{\epsilon}_1 e^{i(\vec{k} \cdot \vec{x} - \omega t)} + A_{k2} \vec{\epsilon}_2 e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

Onda piana lungo $\mathbf{k}$ con vel. c

ϵ vettori di polarizzazione

$\Phi \equiv 0$ Nel nostro caso e' sufficiente solo $\mathbf{A}$, cioe' posso fare una ulteriore trasformazione (ristretta) di gauge t.c. $\phi' = 0$ (non $\mathbf{A}$ perche' c'e' solo una funzione scalare χ arbitraria che si puo' aggiungere

k ed ω sono legate dalla relazione di dispersione, necessaria affinche' A sia soluzione dell'equazione

$$k = \frac{\omega}{c}$$

Potenziali elm: soluzioni (2)

$$\vec{A} = \int [A_{k1} \vec{\epsilon}_1 e^{i(kx - \omega t)} + A_{k2} \vec{\epsilon}_2 e^{i(kx - \omega t)}] d^3k$$

La condizione di Lorentz ($\Phi = 0$)

$$\vec{B} = \vec{\nabla} \times \vec{A} = i\vec{k} \times \vec{A} \quad (1)$$

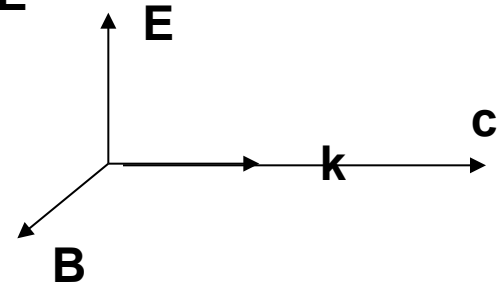
$$\vec{\nabla} \cdot \vec{A} = 0$$

$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} = ik\vec{A} \quad (2)$$

dice che $\vec{k} \cdot \vec{A} = 0$
 $\rightarrow \mathbf{k} \perp \mathbf{A}$

Da (1) e (2) segue che $|\mathbf{E}| = |\mathbf{B}|$, $\mathbf{E} \perp \mathbf{B}$, $\mathbf{k} \perp \mathbf{B}$ e $\mathbf{k} \perp \mathbf{E}$

Le onde elettromagnetiche sono trasversali



Potenziali elm: propagatore

$$\vec{\nabla}^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -4\pi f(\vec{x}, t) \quad (1)$$

La soluzione particolare si ottiene con il propagatore **G(x,t;x',t')**

$$\psi = \int G(\vec{x}, t; \vec{x}', t') f(\vec{x}', t') d^3 x' dt'$$

Facciamo le trasformate di Fourier nel tempo per eliminare la dipendenza esplicita da t della (1)

$$\psi(\vec{x}, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\psi}(\vec{x}, \omega) e^{-i\omega t} d\omega \quad f(\vec{x}, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{f}(\vec{x}, \omega) e^{-i\omega t} d\omega$$

$$\psi(\vec{x}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \psi(\vec{x}, t) e^{i\omega t} dt \quad f(\vec{x}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\vec{x}, t) e^{i\omega t} dt$$

Sostituisco in (1)

$$\vec{\nabla}^2 \psi = \int_{-\infty}^{+\infty} \vec{\nabla}^2 \tilde{\psi}(\vec{x}, \omega) e^{-i\omega t} d\omega$$

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = - \int_{-\infty}^{+\infty} \frac{\omega^2}{c^2} \tilde{\psi}(\vec{x}, \omega) e^{-i\omega t} d\omega$$



$$(\vec{\nabla}^2 + k^2) \tilde{\psi} = -4\pi \tilde{f}(\vec{x}, \omega)$$

Equazione di Helmholtz, per ciascun valore di ω , con $k=\omega/c$

Potenziali elm: propagatore (2)

$$(\nabla^2 + k^2)\tilde{\Psi} = -4\pi\tilde{f}(\vec{x}, \omega) \quad \text{Equ. ellittica derivate parziali che si riduce a quella di Poisson per } k=0$$

Posto $\tilde{\Psi}(\vec{x}, \omega) = \int \tilde{G}_k(\vec{x}, \vec{x}') \tilde{f}(\vec{x}', \omega) d^3x'$

L'equazione che $\tilde{G}_k(\vec{x}, \vec{x}')$ soddisfa e' $(\nabla_{\vec{x}}^2 + k^2)\tilde{G}_k(\vec{x}, \vec{x}') = -4\pi\delta(\vec{x} - \vec{x}')$

NB: l'operatore ∇ agisce sulle coordinate $\vec{x}$ non su $\vec{x}'$

Nel vuoto, senza superfici di confine essa puo' dipendere solo da $\mathbf{R} = \mathbf{x} - \mathbf{x}'$ e poiche' deve esserci simmetria sferica a causa della omogeneita' e simmetria dello spazio

$$\tilde{G}_k(\vec{x}, \vec{x}') = \tilde{G}_k(|\vec{x} - \vec{x}'|) = \tilde{G}_k(R)$$

In coord. sferiche $\nabla^2 = \frac{1}{R} \frac{\partial^2}{\partial R^2} R \quad \longrightarrow \quad \frac{1}{R} \frac{\partial^2}{\partial R^2} R \tilde{G}_k(R) + k^2 \tilde{G}_k(R) = -\delta(R)$

In tutti i punti tranne che in $R=0$, G soddisfa l'equazione

$$\frac{\partial^2}{\partial R^2} (R \tilde{G}_k) + k^2 (R \tilde{G}_k) = 0 \quad \longrightarrow \quad R \tilde{G}_k(R) = A e^{ikR} + B e^{-ikR}$$

Potenziali elm: propagatore (3)

$$\tilde{\Psi}(\vec{x}, \omega) = \int \tilde{G}_k(\vec{x}, \vec{x}', \omega) \tilde{f}(\vec{x}', \omega) d^3x' \quad (\nabla_{\vec{x}}^2 + k^2) \tilde{G}_k(\vec{x}, \vec{x}') = -4\pi \delta(\vec{x} - \vec{x}')$$

$$\frac{\partial^2}{\partial R^2} (R \tilde{G}_k) + k^2 (R \tilde{G}_k) = 0$$

R ≠ 0

$$R \tilde{G}_k(R) = A e^{ikR} + B e^{-ikR}$$

La δ influisce solo nel limite $R \rightarrow 0$, ma in questo limite, $kR \ll 1$ e l'equazione si riduce a quella dell'elettrostatica, per cui

$$\lim_{R \rightarrow 0} \tilde{G}_k(R) = \frac{1}{R}$$

e' la condizione di normalizzazione corretta

La soluzione e' quindi

$$\tilde{G}_k(R) = A \tilde{G}_k^{(+)}(R) + B \tilde{G}_k^{(-)}(R) \quad \text{con} \quad \tilde{G}_k^{\pm}(R) = \frac{e^{\pm ikR}}{R}, \quad \mathbf{A+B=1}$$

- = onda sferica che esce dall'origine, += onda sferica entrante

La scelta di A e B dipende dalle condizioni al contorno nel tempo per il problema fisico in questione: e', ovvio che se una sorgente e' "spenta" fino ad un tempo $t=0$, la funzione di Green corretta e' quella di un'onda sferica uscente

Ora dobbiamo trovare la dipendenza temporale delle funzioni di Green

Potenziali elm: propagatore (4)

$$\nabla^2 \Psi - \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = -4\pi f(\vec{x}, t) \quad (1)$$

$$\Psi = \int G(\vec{x}, t; \vec{x}', t') f(\vec{x}', t') d^3 x' dt'$$

Sostituendo Ψ in (1) si ottiene subito l'equazione soddisfatta da G

$$(\nabla_x^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) G = -4\pi \delta(\vec{x} - \vec{x}') \delta(t - t')$$

Posto $G(R, t, t') = \int \tilde{G}_k(R, t') e^{-i\omega t} d\omega$ e $\delta(t - t') = \int e^{-i\omega(t-t')} d\omega$ Si ha

$$(\nabla_x^2 + k^2) \tilde{G}_k(R, t') = -4\pi \delta(R) e^{i\omega t'}$$

E' la stessa equazione di prima con un fattore di fase 

$$\tilde{G}_k^\pm(R) = \frac{e^{\pm i k R}}{R} e^{i\omega t'}$$



$$G^\pm(R, t, t') = \int \frac{e^{\pm i k R}}{R} e^{i\omega t'} e^{-i\omega t} d\omega$$

$$G^\pm(R, t, t') = \frac{1}{R} \int e^{\pm i \frac{\omega R}{c}} e^{-i\omega(t-t')} d\omega$$



$$G^\pm(R, t, t') = \frac{1}{R} \int e^{-i\omega(t-t' \mp \frac{R}{c})} d\omega$$

$$= \frac{1}{R} \delta(t - t' \mp \frac{R}{c})$$

$\mathbf{R} = \mathbf{x} - \mathbf{x}'(t')$

E' il propagatore cercato: - = funzione ritardata, + funzione anticipata

La soluzione particolare e' quindi: $\Psi = \int \frac{1}{R} \delta(t' - (t \mp \frac{R}{c})) f(\vec{x}', t') d^3 x' dt'$

Potenziali elm: propagatore (5)

$$\nabla^2 \Psi - \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = -4\pi f(\vec{x}, t)$$

$$\Psi = \int G(\vec{x}, t; \vec{x}', t') f(\vec{x}', t') d^3 x' dt'$$

$$\vec{R}(t) = \vec{x} - \vec{x}'(t')$$

$$G^\pm(R, t, t') = \frac{1}{R} \delta(t - t' \mp \frac{R}{c})$$

$$\Psi = \int \frac{1}{R} \delta(t' - (t \mp \frac{R}{c})) f(\vec{x}', t') d^3 x' dt'$$

La funzione di Green ritardata mostra il comportamento causale associato ad una perturbazione ondulatoria: l'argomento della δ indica che l'effetto in un punto x all'istante t e' dovuto all'azione di una sorgente ad un istante precedente o ritardato $t'=t-R(t')/c$.

La differenza temporale $t - t' = R/c$ e' semplicemente interpretata come propagazione a velocita' c della perturbazione da un punto all'altro dello spazio

Potenziali ritardati (1)

$$\vec{A}(\vec{x}, t) = \int \frac{1}{cR} \delta(t' - (t - \frac{R}{c})) \vec{j}(\vec{x}', t') d^3x' dt'$$

$$\Phi(\vec{x}, t) = \int \frac{1}{R} \delta(t' - (t - \frac{R}{c})) \rho(\vec{x}', t') d^3x' dt'$$

Correnti e densità in x' contribuiscono a x all'istante t , solo agli istanti $t' = t - R/c$, cioè con un ritardo R/c dovuto alla velocità di propagazione finita delle onde elm

$$\vec{A}(\vec{x}, t) = \int \frac{1}{cR} \delta(t' - (t - \frac{R}{c})) \vec{j}(\vec{x}', t') d^3x' dt' \quad \Phi(\vec{x}, t) = \int \frac{1}{R} [\rho(\vec{x}', t')]_{rit} d^3x'$$

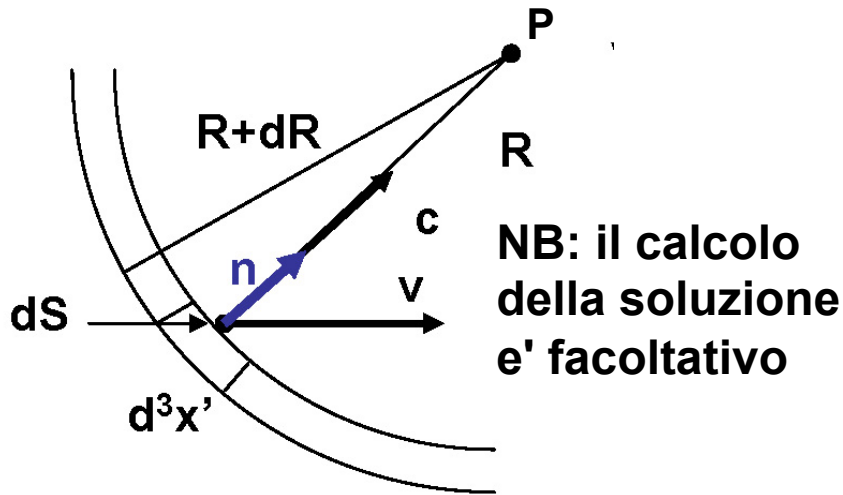
Dove con $[\mathbf{F}]_{rit} \equiv \mathbf{F}(\mathbf{r}', t - (\mathbf{x} - \mathbf{x}')/c)$, i.e. va usato il valore ritardato $t' = t - R(t')/c$

Gli integrandi non rappresentano (a parte il termine R al denominatore) la carica e la corrente nel volume, come nel caso statico, dato che gli elementi di carica in $d^3\mathbf{x}'$ contribuiscono a istanti t' diversi al potenziale in x , cioè l'integrale va valutato con $t' = t'(\mathbf{x}')$ mentre la carica totale nel volume V è pari all'integrale con $t' = \text{cost}$.

Bisogna esprimere $[\dots]_{rit}$ in funzione della carica TOTALE contenuta nel volume che contribuisce al potenziale in P all'istante t :

per questo occorre tenere conto del moto delle cariche

Potenziali ritardati (2): Lienard e Wiechert



Per farlo consideriamo una sfera centrata in P che si contrae con velocità c e che raggiunge P proprio all'istante t , l'equazione oraria della sfera è

$$R(t') = c(t - t')$$

L'istante in cui i punti della distribuzione di carica in d^3x' contribuiscono al punto P all'istante t sono quelli che vengono raggiunti dalla sfera quando essa attraversa l'elemento d^3x' , cioè $t' = t - R/c$. Parte della carica sfugge dall'elemento di volume $d^3x' = dr \phi dS$ nel tempo impiegato dalla sfera a spazzarlo e va sottratta a quella presente a t'

Il tempo impiegato dalla sfera a spazzare il volume fra R e $R + dR$ è $dt' = dR/c$

La carica che sfugge dalla superficie nel tempo dt' è $dq' = \vec{j} \cdot \vec{n} dt' dS$ con $\vec{j} = \rho \vec{v}$

$$dq' = \rho \vec{v} \cdot \vec{n} dt' dS = \rho \vec{v} \cdot \vec{n} \frac{dR}{c} dS = \rho \vec{n} \cdot \frac{\vec{v}}{c} d^3x' \quad \text{All'istante } t' = t - R/c$$

Quindi la carica che contribuisce è

$$dq = \rho d^3x' - dq' = \rho d^3x' - \rho \vec{n} \cdot \frac{\vec{v}}{c} d^3x' = \rho d^3x' \left(1 - \vec{n} \cdot \frac{\vec{v}}{c} \right)$$

Potenziali ritardati (3): L&W

$$\vec{A}(\vec{x}, t) = \int \frac{1}{cR} [\vec{j}(\vec{x}', t')]_{rit} d^3x' \quad \Phi(\vec{x}, t) = \int \frac{1}{R} [\rho(\vec{x}', t')]_{rit} d^3x'$$

$$dq|_{t-r/c} = [1 - \frac{\vec{v}}{c} \cdot \vec{n}] \rho(\vec{x}', t')|_{t-r/c} d^3x'$$

Quindi
$$\rho(\vec{x}', t')|_{t-r/c} d^3x' = \frac{dq}{[1 - \frac{\vec{v}}{c} \cdot \vec{n}]}|_{t-r/c}$$

$$\Phi(\vec{x}, t) = \int \frac{dq}{[R(1 - \frac{\vec{v}}{c} \cdot \vec{n})]|_{t-r/c}} \quad \vec{A}(\vec{x}, t) = \int \frac{\vec{v}|_{t-r/c} dq}{[cR(1 - \frac{\vec{v}}{c} \cdot \vec{n})]|_{t-r/c}}$$

Sono i **potenziali di Lienard e Wiechert** per una distribuzione di cariche in moto arbitrario

Potenziali di Lienard e Wiechert (1)

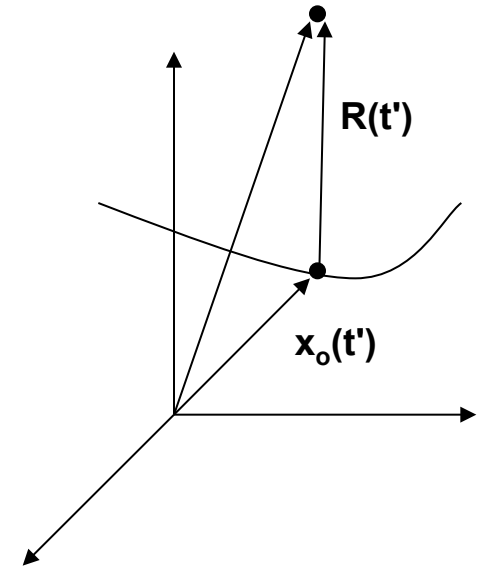
Nel caso di una particella puntiforme che si muove lungo una traiettoria

$\vec{x}(t) = \vec{x}_0(t)$ con velocita' $\vec{\dot{x}}(t) = \vec{\dot{x}}_0(t)$, le densita' di carica e corrente sono date da

$$\rho(\vec{x}, t) = q\delta(\vec{x} - \vec{x}_0(t)) \quad \vec{j}(\vec{x}, t) = q\vec{\dot{x}}\delta(\vec{x} - \vec{x}_0(t)) \quad \vec{R}(t) = \vec{x} - \vec{x}_0(t') \quad P$$

La funzione δ ha la proprieta' di localizzare carica e corrente e di dare le corrette carica e corrente totali

$$q = \int \rho(\vec{x}, t) d^3x \quad q\vec{v} = \int \vec{j}(\vec{x}, t) d^3x$$



Potenziali di Lienard e Wiechert (2)

I potenziali elm hanno la forma $\psi = \int \frac{1}{R} \delta(t' - t + \frac{R}{c}) f(\vec{x}', t') d^3x' dt'$

dove $\Psi = \Phi, \mathbf{A}_i$ e $\mathbf{f} = \rho, \mathbf{j}_i/c, i=1,2,3$

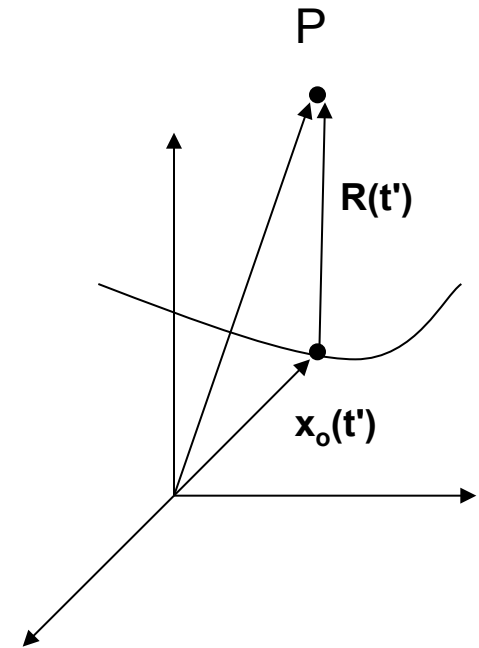
Possiamo usare la δ per effettuare l'integrazione nello spazio d^3x'

$$\rho(\vec{x}, t) = q\delta(\vec{x} - \vec{x}_0(t))$$

$$\vec{j}(\vec{x}, t) = q\vec{\dot{x}}\delta(\vec{x} - \vec{x}_0(t))$$

$$\Phi = q \int \frac{1}{|\vec{x} - \vec{x}_o(t')|} \delta(t' - t + \frac{|\vec{x} - \vec{x}_o(t')|}{c}) dt'$$

$$\vec{A}(\vec{x}, t) = \frac{q}{c} \int \frac{\vec{v}(t')}{|\vec{x} - \vec{x}_o(t')|} \delta(t' - t + \frac{|\vec{x} - \vec{x}_o(t')|}{c}) dt'$$



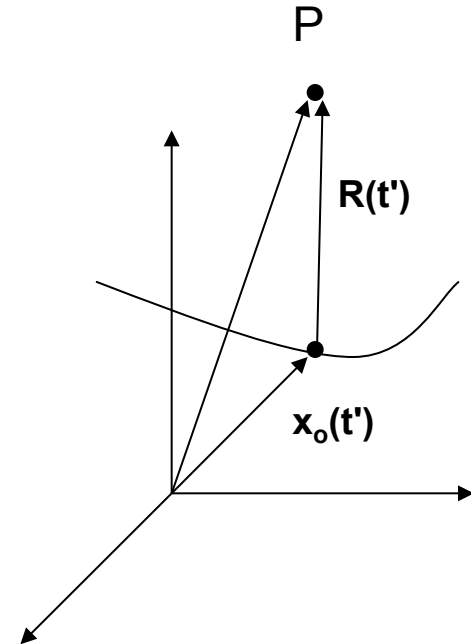
$$\vec{R}(t) = \vec{x} - \vec{x}_o(t')$$

Potenziali di Lienard e Wiechert (3)

$$\Phi = q \int \frac{1}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' \quad A(\vec{x}, t) = \frac{q}{c} \int \frac{\vec{v}(t')}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' \quad \vec{R}(t) = \vec{x} - \vec{x}_o(t')$$

Si possono semplificare ancora notando che l'argomento della δ si annulla quando $\mathbf{t}' = \mathbf{t}_{\text{rit}}$ dato da $\mathbf{c}(\mathbf{t} - \mathbf{t}_{\text{rit}}) = \mathbf{R}(\mathbf{t}_{\text{rit}})$, i.e. l'equazione oraria di una perturbazione che viaggia a vel. c (la sfera di prima)

Si noti che questa e' una relazione implicita in $\mathbf{t}_{\text{rit}}$ dato che $R = R(\mathbf{t}_{\text{rit}})$, mentre t rimane fissato, i.e. fissato l'istante di osservazione cerchiamo gli istanti precedenti connessi causalmente al campo in x all'istante t



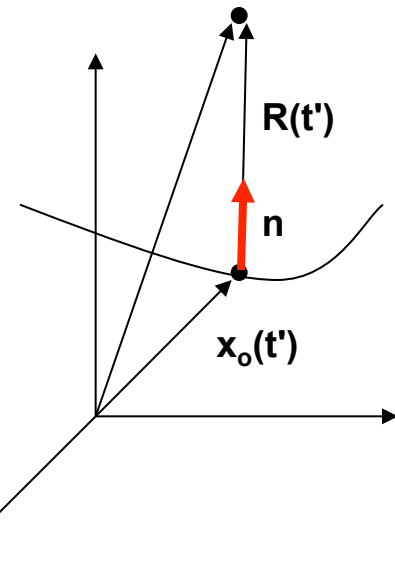
Potenziali di Lienard e Wiechert (4)

$$\Phi = q \int \frac{1}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' \quad A(\vec{x}, t) = \frac{q}{c} \int \frac{\vec{v}(t')}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' \quad \vec{R}(t) = \vec{x} - \vec{x}_o(t')$$

Cambiamo variabile $\mathbf{t''} = \mathbf{t' - t + R(t')/c}$ $\longrightarrow dt'' = dt' + \frac{\dot{R}(t')}{c} dt'$

P

Dobbiamo eliminare $\dot{R}(t')$ esprimendolo in funzione di $\mathbf{v}$ e della direzione di osservazione



Dall'identità $|R|^2 = \vec{R} \cdot \vec{R}$ $\longrightarrow$

$$\frac{d}{dt} |R|^2 = 2R(t') \dot{R}(t') \quad \frac{d}{dt} \vec{R}^2 = 2\vec{R} \cdot \dot{\vec{R}}$$

$\longrightarrow R(t') \dot{R}(t') = \vec{R} \cdot \dot{\vec{R}}$ ma $\dot{\vec{R}} = -\dot{\vec{x}}_o(t') = -\vec{v}(t')$

$\longrightarrow \vec{R} \cdot \dot{\vec{R}} = -\vec{R}(t') \cdot \vec{v}(t')$ $\longrightarrow R(t') \dot{R}(t') = -\vec{R}(t') \cdot \vec{v}(t')$

$\longrightarrow \dot{R}(t') = -\frac{\vec{R}(t') \cdot \vec{v}(t')}{R(t')}$ Posto $\vec{n}(t') = \frac{\vec{R}(t')}{R(t')}$ $\vec{R}(t') = -n(t') \cdot \vec{v}(t')$

$\longrightarrow dt'' = dt' + \frac{\dot{R}(t')}{c} dt' = dt' - \frac{\vec{n} \cdot \vec{v}}{c} dt' = dt' (1 - \frac{\vec{n} \cdot \vec{v}}{c})$

Potenziali di Lienard e Wiechert (5)

$$\Phi = q \int \frac{1}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' = q \int \frac{1}{R(t')} \delta(t'') \frac{dt''}{(1 - \frac{\vec{n} \cdot \vec{v}}{c})}$$

$$t'' = t' - t + R(t')/c$$

$$\text{i.e. } t' = t'(t'')$$

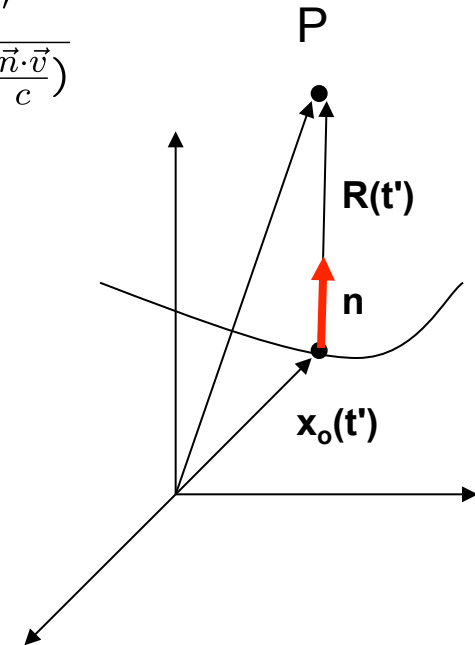
$$A(\vec{x}, t) = \frac{q}{c} \int \frac{\vec{v}(t')}{R(t')} \delta(t' - t + \frac{R(t')}{c}) dt' = \frac{q}{c} \int \frac{\vec{v}(t')}{R(t')} \delta(t'') \frac{dt''}{(1 - \frac{\vec{n} \cdot \vec{v}}{c})}$$

Integrando su t'' si ottiene quindi

$$\Phi = q \frac{1}{R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})}$$

$$t' = t - R/c$$

$$A(\vec{x}, t) = \frac{q}{c} \frac{\vec{v}(t')}{R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})}$$



NB: il calcolo e' facoltativo

Questi sono i potenziali di LW per una carica puntiforme

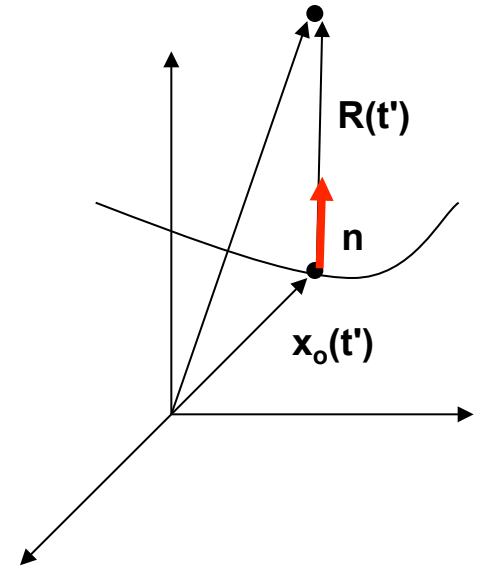
Potenziali di Lienard e Wiechert (6)

$$\Phi(\vec{x}, t) = q \frac{1}{R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})} \quad \vec{A}(\vec{x}, t) = \frac{q \vec{v}(t')}{c R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})} \quad t' = t - R/c$$

I potenziali differiscono da quelli statici in due aspetti:

► il fattore $k(t') = (1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})$

tende a concentrare i potenziali in uno stretto cono intorno alla direzione della velocità quando la velocità $\rightarrow c$



► tutte le quantità a destra vanno valutate al tempo ritardato. La conseguenza più importante è che la carica può irradiare. Se la dipendenza fosse solo in R , i potenziali decrescerebbero come $1/R$, così che differenziando si avrebbe una dipendenza $1/R^2$. La dipendenza implicita sulla posizione tramite il tempo ritardato $t' = t - R/c$ porta ad una dipendenza di tipo $1/R$ per una componente dei campi, il che implica il trasporto netto di energia a distanza ∞ , i.e. propagazione per onde

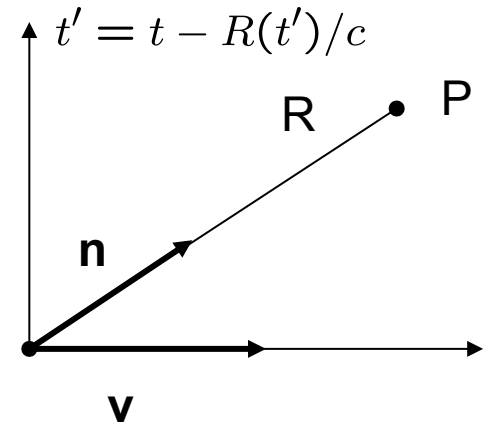
Campi di L&W (7): campi

$$\Phi(\vec{x}, t) = q \frac{1}{R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})} \quad \vec{A}(\vec{x}, t) = \frac{q \vec{v}(t')}{c R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})}$$

I campi sono $\vec{B} = \vec{\nabla} \times \vec{A}|_{rit}$ $\vec{E} = (-\vec{\nabla} \Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t})|_{rit}$

NB: il calcolo e' facoltativo

Le derivate sono calcolate al tempo t nella posizione $\mathbf{x}$, i potenziali vanno valutati al tempo ritardato $t' = t - R(t')/c$



$\mathbf{R}(t') = \mathbf{x} - \mathbf{x}_o(t')$ (1) $\rightarrow t'$ e' una funzione implicita delle coord. $\mathbf{x}, t \rightarrow$ quando si deriva rispetta a $\mathbf{x}$ e t bisogna fare una derivata di una funzione di funzione

P.es. Il campo elettrico:

$$\vec{E} = (-\vec{\nabla} \Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t})|_{rit} \equiv -\vec{\nabla}_x \Phi[t'(\vec{x}, t)] - \frac{1}{c} \frac{\partial \vec{A}[t'(\vec{x}, t)]}{\partial t} = -\frac{\partial \Phi}{\partial t'} \vec{\nabla}_x t'(\vec{x}, t) - \frac{1}{c} \frac{\partial \vec{A}}{\partial t'} \frac{\partial t'(\vec{x}, t)}{\partial t}$$

$$\frac{\partial t'}{\partial t} = 1 - \frac{\partial R}{\partial t'} \frac{\partial t'}{\partial t} \quad \Rightarrow \quad \frac{\partial t'}{\partial t} = \frac{1}{1 + \frac{\partial R}{\partial t'}}$$

$$\therefore dR/dt' \text{ si ricava notando che } |R|^2 = \vec{R} \cdot \vec{R} \quad \Rightarrow \quad R \frac{dR}{dt'} = \vec{R} \cdot \frac{d\vec{R}}{dt'} = -\vec{R} \cdot \vec{v} \quad \text{da (1)}$$

$$\Rightarrow \frac{dR}{dt'} = -\vec{R} \cdot \vec{v} / R \quad \Rightarrow \quad \frac{\partial t'}{\partial t} = \frac{1}{1 - \vec{R} \cdot \vec{v} / cR} = \frac{1}{1 - \vec{n} \cdot \vec{\beta}}$$

Campi di Lienard e Wiechert (8)

$$\Phi(\vec{x}, t) = q \frac{1}{R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})} \quad \vec{A}(\vec{x}, t) = \frac{q \vec{v}(t')}{c R(t')} \frac{1}{(1 - \frac{\vec{n}(t') \cdot \vec{v}(t')}{c})} \quad \vec{E} = -\frac{\partial \Phi}{\partial t'} \vec{\nabla}_x t'(\vec{x}, t) - \frac{1}{c} \frac{\partial \vec{A}}{\partial t'} \frac{\partial t'(\vec{x}, t)}{\partial t}$$

$$t' = t - R(t')/c \quad \frac{dR}{dt'} = -\vec{R} \cdot \vec{v}/R \quad \mathbf{R(t')} = \mathbf{x-x_o(t')} \quad (1)$$

Calcolo $\vec{\nabla}_x t'(\vec{x}, t)$

$$\vec{\nabla}_x t'(\vec{x}, t) = -\vec{\nabla}_x R(\vec{x}, t')/c = -\frac{\partial R(\vec{x}, t')}{c \partial t'} \vec{\nabla}_x t' - \vec{\nabla}_x R(\vec{x}, t') \quad \text{ma} \quad \vec{\nabla}_x R(\vec{x}, t') = \vec{R}/R$$



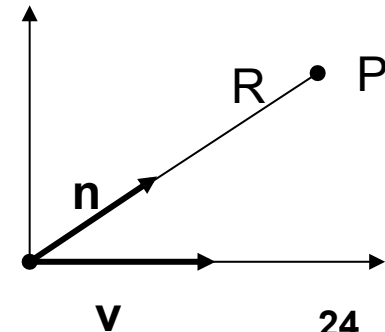
$$\vec{\nabla}_x t' = -\frac{\partial R}{c \partial t'} \vec{\nabla}_x t' - \vec{R}/R$$

Risolve in $\vec{\nabla}_x t'$

$$\vec{\nabla}_x t' = -\frac{\vec{R}/R}{1 + \frac{\partial R(\vec{x}, t')}{c \partial t'}} = -\frac{\vec{R}/R}{1 - \vec{R} \cdot \vec{v}/cR} = -\frac{\vec{n}}{1 - \vec{n} \cdot \vec{\beta}}$$

Le derivate dei potenziali rispetto a t' sono immediate. Si ottiene

$$\vec{E} = q \left[\frac{\vec{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \vec{n})^3 R^2} \right] + \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right] \quad \vec{B} = \vec{n} \times \vec{E}$$



(Elettrodin. Classica, Jackson par. 14.1, Landau, Teoria dei campi par 63)

Campi di Lienard e Wiechert (9)

$$\vec{E} = q \left[\frac{\vec{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \vec{n})^3 R^2} \right] + \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right]$$

$$\vec{B} = \vec{n} \times \vec{E}$$

Ci sono due componenti con caratteristiche diverse

Campo di velocita' $\vec{E}_v$

- ✓ $\sim R^{-2}$, si riduce a quello di Coulomb nel caso di $\beta = 0$,
- ✓ Il campo punta nella direzione della posizione attuale (i.e. all'istante t) della particella: infatti la distanza dal punto ritardato e' $\vec{n}c\tilde{t}$ dove $\tilde{t} = t - t_{rit}$

Nello stesso tempo, la particella si e' spostata di $\vec{\beta}c\tilde{t}$

La posizione della particella rispetto al punto di osserv. e'

$$(\vec{n} - \vec{\beta})c\tilde{t}$$

che e' appunto la direzione del campo di velocita'

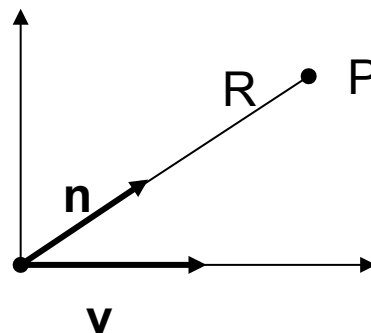
Campo di accelerazione $\vec{E}_a$

- $\sim R^{-1}$,
- Dipende da $\mathbf{a}$,
- Il campo $\vec{E}_a$ e' $\perp \mathbf{n}$, e $\mathbf{B} \perp \vec{E}, \mathbf{n}$
- Trasporta energia a tutte le distanze

A spanne (i.e. leading terms), il rapporto E_a/E_v e'

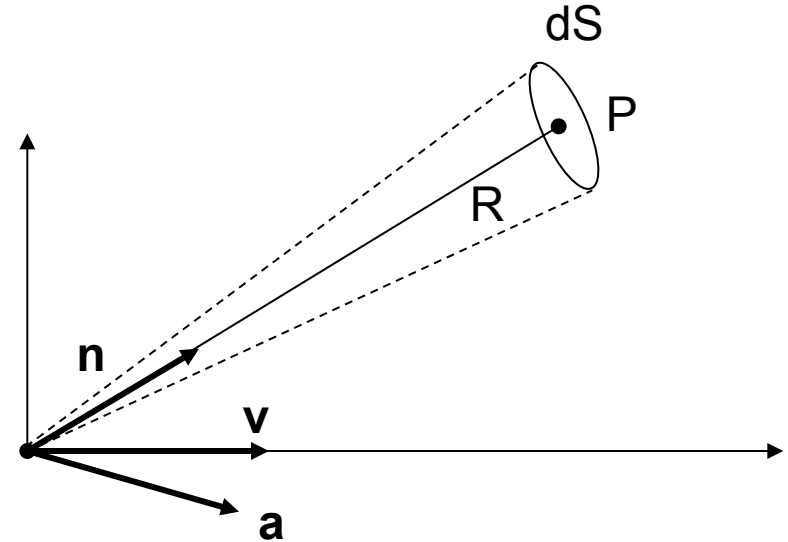
$$\frac{E_a}{E_v} \sim \frac{R\gamma^2 a}{c^2}$$

Il campo di radiazione domina a distanze t.c. $R \gg \frac{c^2}{\gamma^2 a}$



Potenza emessa

$$\left\{ \begin{array}{l} \vec{B} = \vec{n} \times \vec{E} \\ \vec{E} = \underbrace{q \left[\frac{\vec{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \vec{n})^3 R^2} \right]}_{\vec{E}_v} + \underbrace{\frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right]}_{\vec{E}_a} \end{array} \right.$$



Il trasporto di energia e' descritto dal vettore di Poynting

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B} \quad [\mathbf{ES^{-2}T^{-1}}]$$

La potenza emessa e' quindi $dP = \vec{S} \cdot \vec{n} dS = \vec{S} \cdot \vec{n} R^2 d\Omega$

$$\longrightarrow \frac{dP}{d\Omega} = R^2 \vec{S} \cdot \vec{n}$$

E' la distribuzione angolare della potenza emessa

Potenza emessa (2)

$$\vec{B} = \vec{n} \times \vec{E} \quad \vec{E} = q \left[\frac{\vec{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \vec{n})^3 R^2} \right] + \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right]$$

I campi sono tali che $|\vec{B}| = |\vec{E}|$ $\vec{B} \perp \vec{E}$ $\vec{B}, \vec{E} \perp \vec{n}$ 

$$\vec{S} = \frac{c}{4\pi} E^2 \vec{n} \quad \text{blue arrow} \quad \frac{dP}{d\Omega} = \frac{cE^2}{4\pi} R^2$$

Per il campo di velocita' $E_v (\sim R^{-2})$

$(\frac{dP}{d\Omega})_v \propto R^{-2} \rightarrow$ la potenza va a zero all'infinito $\rightarrow$ non si ha trasporto netto di energia

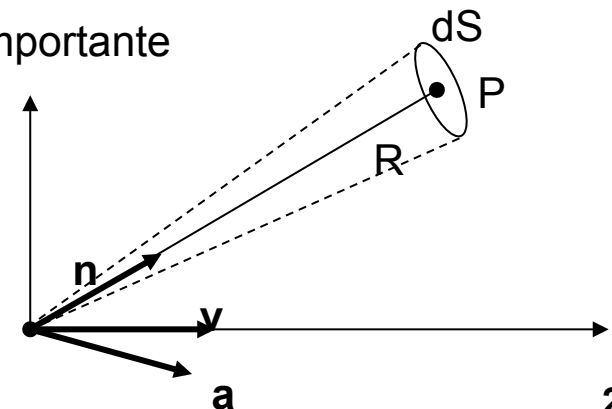
Per il campo di radiazione $E_a (\sim R^{-1})$

$(\frac{dP}{d\Omega})_a \propto \text{cost.} \rightarrow$ la potenza rimane finita per ogni $R \rightarrow$ si ha trasporto netto di energia sotto forma di onde

Per l'emissione a grandi distanze dalla sorgente $R \gg \frac{c^2}{\gamma^2 a}$ e' importante solo il campo di accelerazione

$$\vec{E}_a = \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right]$$

NB: i campi di LW sono validi per tutte le velocita' e accelerazioni



Potenza: Larmor

$$\vec{E}_a = \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})^3 R} \right]$$

$$\frac{dP}{d\Omega} = \frac{cE^2}{4\pi} R^2$$

Nel sistema di quiete istantanea della carica (o per particelle in moto non relativistico) $\beta = 0$ ma $d\vec{\beta}/dt \neq 0$

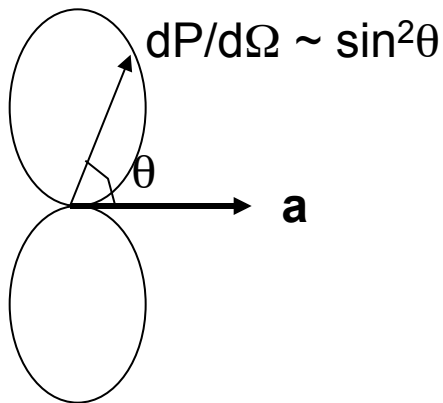
$$\vec{E}_a = \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} \times \dot{\vec{\beta}})}{R} \right]$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} |\vec{n} \times (\vec{n} \times \dot{\vec{\beta}})|^2$$

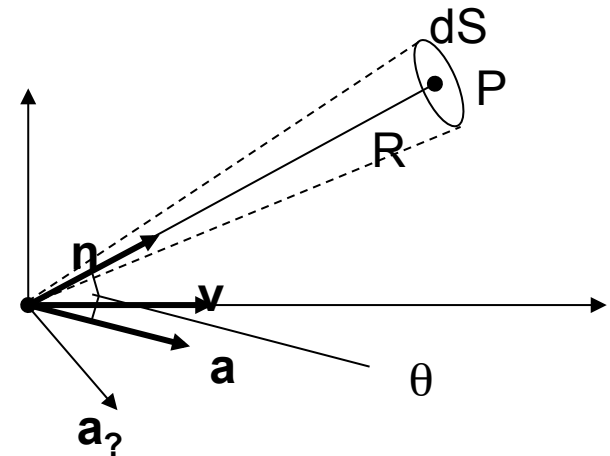
Sviluppo il doppio prodotto vettoriale $\vec{n} \times (\vec{n} \times \dot{\vec{\beta}}) = (\vec{n} \cdot \dot{\vec{\beta}})\vec{n} - \dot{\vec{\beta}} = -\dot{\vec{\beta}}_{\perp n}$

$$|\dot{\vec{\beta}}_{\perp n}| = \dot{\beta} \sin \theta$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \dot{\beta}^2 \sin^2 \theta = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta$$



- E' la distribuzione angolare di potenza emessa nel riferimento in cui $\mathbf{v}_{\text{ist}} = \mathbf{0}$:
- diagramma polare di tipo dipolare
 - Emissione nulla lungo $\mathbf{a}$
 - Max a $\pi/2$ rispetto ad $\mathbf{a}$

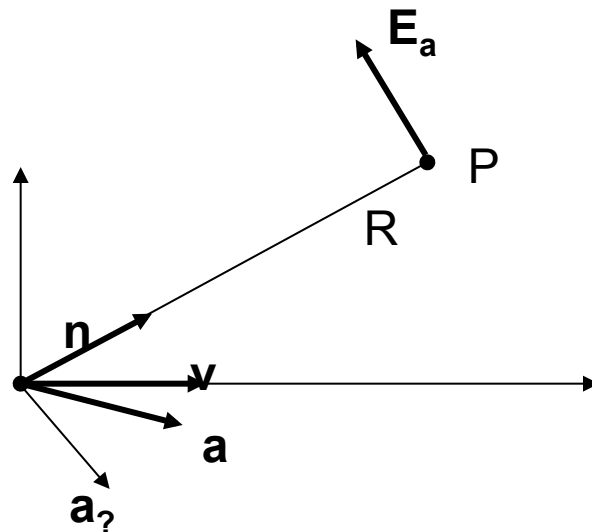
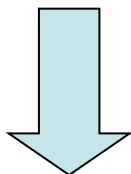


Larmor (2): polarizzazione

Nel sistema di quiete istantanea
della carica ($\beta = 0$)

$$\vec{E}_a = \frac{q}{c} \left[\frac{\vec{n} \times (\vec{n} \times \dot{\vec{\beta}})}{R} \right] \propto -a_{\perp} \vec{n}$$

Il campo $\mathbf{E}$ oscilla nel piano $\perp \mathbf{n}$ con la
direzione della proiezione di $\mathbf{a}$ su tale
piano ($\mathbf{a}_{\perp}$)



La radiazione emessa e' polarizzata, se la direzione dell'accelerazione e' costante

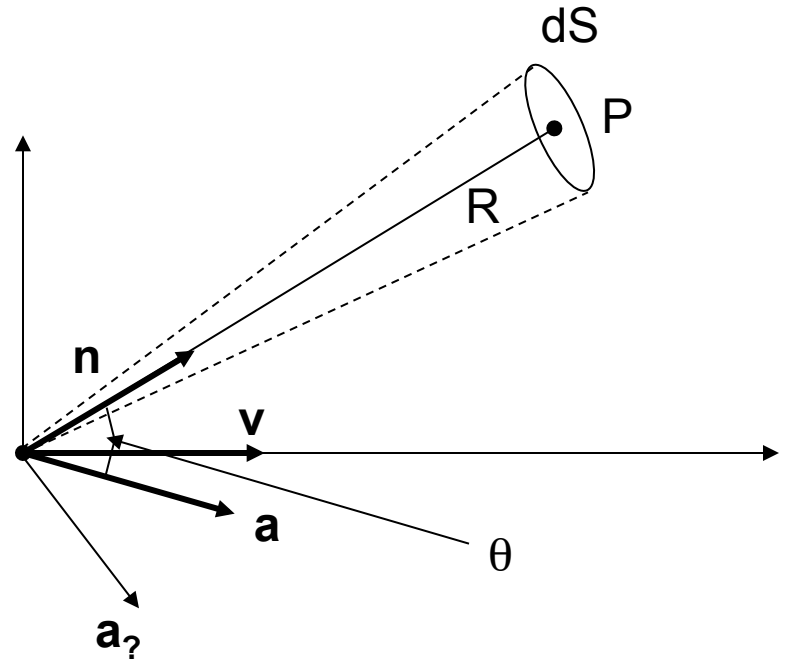
Larmor (3): potenza totale

La potenza totale emessa si ottiene integrando sull'angolo solido $\frac{dE}{dt} = \int \frac{dP}{d\Omega} d\Omega$

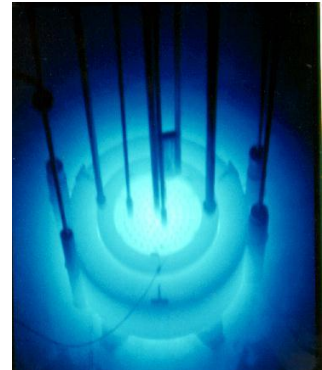
$$\begin{aligned} \frac{dE}{dt} &= \frac{q^2}{4\pi c^3} a^2 \int \sin^2 \theta d\Omega = \frac{q^2}{4\pi c^3} a^2 \int_{-1}^{+1} (1 - \cos^2 \theta) 2\pi d(\cos \theta) \\ &= \frac{q^2}{4\pi c^3} a^2 2\pi (4/3) = \frac{2q^2}{3c^3} a^2 \end{aligned}$$

Formula di Larmor per la potenza emessa da una carica accelerata in moto non relativistico.

E' valida in tutti i casi in cui $\mathbf{v} \ll \mathbf{c}$



Emissione Cerenkov

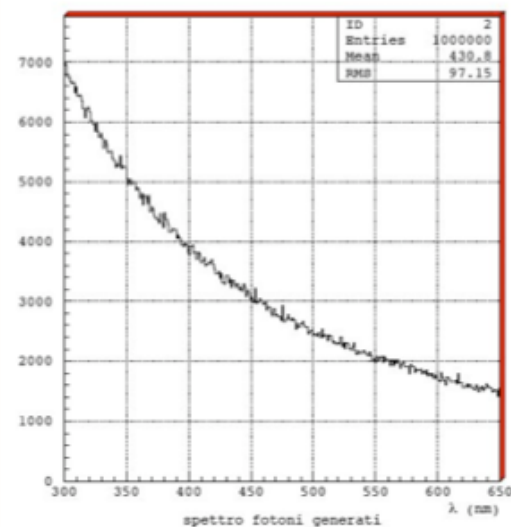
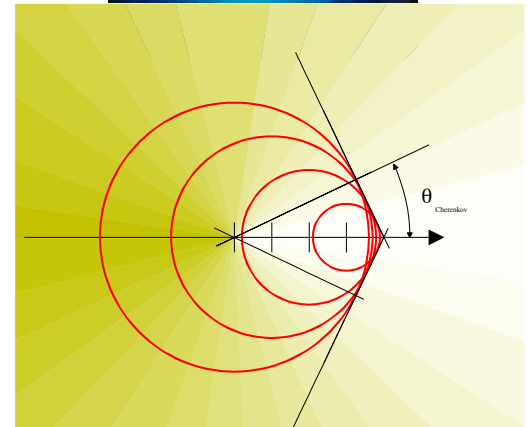


I fatti sperimentali:

□ in un materiale con indice di rifrazione n , una particella carica emette fotoni se la sua velocità βc è maggiore della velocità di fase locale della luce $\beta c = v > c/n$

□ l'emissione avviene lungo una direzione ben precisa rispetto alla linea di volo della particella nel mezzo, $\cos\theta = 1/\beta n$

□ ha una distribuzione spettrale caratteristica



Emissione Cerenkov

Come fa una particella che viaggia a velocità costante a emettere radiazione?

Intuitivamente la particella carica polarizza gli atomi lungo la sua traiettoria.

I dipoli sono dipendenti dal tempo (causa del moto della particella nel mezzo) ed emettono radiazione.

Se $v < c/n$ la distribuzione dei dipoli intorno alla posizione della particella è simmetrica $\rightarrow$ il dipolo totale è nullo.

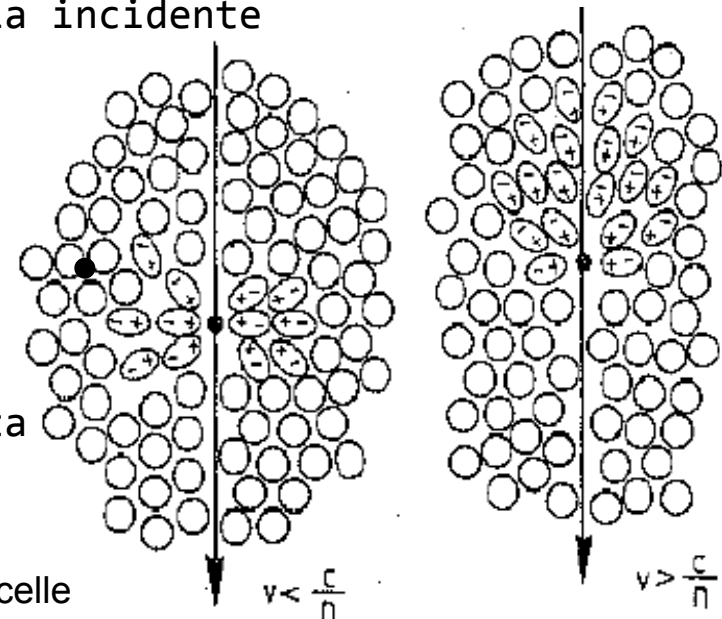
Se $v > c/n$ la distribuzione è asimmetrica nella direzione avanti/indietro $\rightarrow$ il momento di dipolo netto non è nullo e irraggia.

L'energia irraggiata è fornita dalla particella incidente.

È un effetto dei campi elettrici della particella incidente sugli atomi del mezzo, lo stesso che dà luogo all'effetto densità nella Bethe Bloch

L'emissione avviene solo nella direzione in cui i campi di radiazione dei dipoli sono coerenti.

L'emissione Cherenkov avviene insieme alla perdita di E per collisioni.

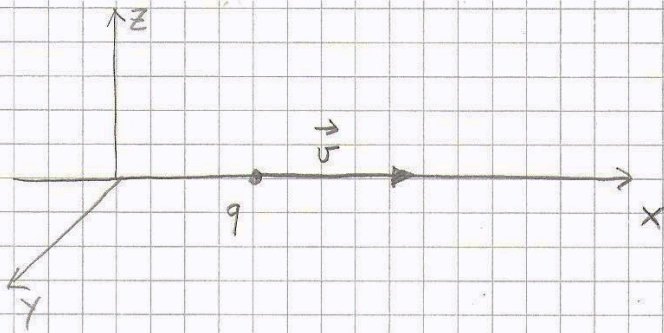


Meccanismo di emissione

Per gli incurabili maniaci dei conti, una trattazione completa (classica) dell'effetto Cerenkov in **Jackson: Classical Electrodynamics** cap 13 e paragrafi 13.4 e 13.5

Per quelli almeno un po' entusiasti, una semplice derivazione si trova in "**High Energy Astrophysics**", Vol.1 Longair (disponibile in formato pdf)

Emissione cherenkov: meccanismo di emissione



- PART CON VEL. COST. LUNGO x $\vec{v} = \text{cost.}$

$\Rightarrow$ CORRENTE $\vec{J}(\vec{x}, t) = q \vec{v} \delta(y) \delta(z) \delta(x - vt)$

- MEZZO POLARIZZABILE MA NON MAGNETICO

(Di Fourier)

$\epsilon \neq \epsilon_0 \quad \mu = \mu_0$

DECOMPOS. IN CORRENTI OSCILLANTI A FREQ. ω

$\vec{J}(\vec{x}, t) = \frac{1}{(2\pi)^{1/2}} \int \vec{J}(\omega, \vec{r}) e^{-i\omega t} d\omega$ CON $\vec{J}(\omega, \vec{r}) = \frac{1}{(2\pi)^{1/2}} \int \vec{J}(\vec{r}, t) e^{i\omega t} dt$

CORRISPONDE A RAPPRESENTARE IL MOTO DELLA PART CON UNA DISTRIB. LINEARE DI CORRENTI OSCILLANTI DI AMPIEZZA (1)

$= \frac{q}{(2\pi)^{1/2}} \delta(y) \delta(z) e^{i\omega x/v} \hat{u}_x \quad (1)$

Emissione cherenkov: meccanismo di emissione

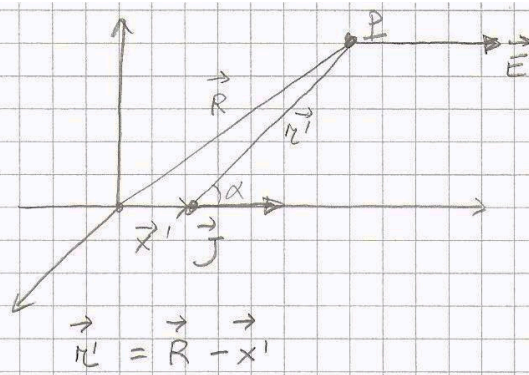
LA SOLUZ. STANDARD E'

(ps: la δ dirac in t' e' sottintesa)

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{x}', t - |\vec{r}'|/c)}{|\vec{r}'|} d^3x' \equiv \frac{\mu_0}{4\pi} \int \frac{[\vec{j}]}{|\vec{r}'|} d^3x'$$

$$\vec{E}(\vec{r}, t) = -\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0}{4\pi} \int \frac{[\dot{\vec{j}}]}{|\vec{r}'|} d^3x'$$

NB: [...] significa valutare al tempo ritardato



Nel limite di campo lontano, la componente del campo di radiazione E_r e' perpendicolare alla direzione di osservazione che risulta

$$\rightarrow (\sim \parallel \vec{r})$$

$$\vec{E}_n = \vec{E} \times \vec{n} \Rightarrow |E_n| = \frac{\mu_0 \sin \theta}{4\pi} \left| \int d^3x' \frac{[\dot{\vec{j}}]}{|\vec{r}'|} \right|$$

LA POT. EMESSA E' $P = \int_S \vec{S} \cdot d\vec{A}$

$S = v u_E$ dove v e' la vel. Di propagazione dell'onda e u e' la densita' di energia. In un mezzo polarizzabile, ma non magnetico $v = c/n$ e $u_E = \epsilon E^2$. Quindi $S = c\epsilon E^2/n$. Ma $n^2 = \epsilon \rightarrow S = ncE^2$

$$= \int_{\Omega} nc E_n^2 R^2 d\Omega = \frac{n\epsilon_0 \mu_0^2 c}{16\pi^2} \int_{\Omega} R^2 \sin^2 \theta \left| \int d^3x' \frac{[\dot{\vec{j}}]}{|\vec{r}'|} \right|^2 d\Omega$$

SE $|R| \gg L$, DIM DELLA REGIONE CHE EMETTE

NOTO CHE $\epsilon_0 \mu_0^2 = \frac{1}{c^4}$

$$P \approx \frac{n}{16\pi^2 \epsilon_0 c^3} \int_{\Omega} \sin^2 \theta \left| \int d^3x' [\dot{\vec{j}}] \right|^2 d\Omega$$

$$|\vec{r}'|^2 = |\vec{r}|^2 + |\vec{x}|^2 - 2|\vec{x}||\vec{r}|\cos\alpha \approx |\vec{r}|^2 (1 - 2\frac{|\vec{x}|}{|\vec{r}|}\cos\alpha) \approx |\vec{r}|^2$$

Emissione cherenkov: meccanismo di emissione

$$L' \text{ ENERGIA TOTALE EMESSA E' } U(\vec{R}, t) = \int_{-\infty}^{+\infty} P dt = \int_{-\infty}^{+\infty} dt \int_{\Omega} d\Omega \sin^2 \theta \left| \int d^3x' [\vec{J}] \right|^2 \cdot \left(\frac{m}{16\pi\epsilon_0 c^3} \right)$$

CON IL TEOR DI PARSEVAL PASSO DA $t \rightarrow \omega$ $\left(\int_{-\infty}^{+\infty} dt |f(t)|^2 = 2 \int_0^{\infty} d\omega |f(\omega)|^2 \right)$

$$U(\vec{R}, \omega) = \frac{2\pi}{16\pi^2\epsilon_0 c^3} \int_{\Omega} d\Omega \sin^2 \theta \int d\omega \underbrace{\left| \int d^3x' [\vec{J}(\omega, \vec{x}')] \right|^2}_{B(\omega)}$$

LA POT EMESSA DIPENDE DALLA VAR DELLA CORR. RITARDATA

I CAMPI IN P ALL'IST. t SONO LA SOMMA DEI CONTRIBUTI DEGLI ELEM. DI CORR.

$[\vec{J}(\omega, \vec{x}')] d^3x'$, CIOE' DI ELEM. IN PUNTI $\vec{x}'$ DIVERSI A IST. $t' = t - |\vec{r}'|/c$ DIVERSI

CIASCUN CONTRIBUTO ARRIVA IN P all'ist. t DA CAMPI EMESSI A $\vec{r}(t')$ DA P

IL CONTRIB. IN P A t DA $\vec{x}'$ A t' VIENE DA $\vec{J}(\omega, \vec{x}') e^{-i\omega t'} = \vec{J}(\omega, \vec{x}') e^{-i\omega(t - r'/c)} = \vec{J}(\omega, \vec{x}') e^{+i\omega r'/c} e^{-i\omega t}$
 $\omega/c = k, k \parallel \vec{r}' \Rightarrow \omega r'/c = \vec{k} \cdot \vec{r}'$
 IN P C'E UNA FASE

INOLTRE $\frac{\partial}{\partial t} \left\{ \vec{J}(\omega, \vec{x}') e^{i\omega t'} \right\} = i\omega \vec{J}(\omega, \vec{x}') e^{i\omega t'}$ (cfr. $t' = t - r'/c$) $\Rightarrow [\vec{J}(\omega, \vec{x}')] = i\omega [\vec{J}(\omega, \vec{x}')] e^{i\omega r'/c}$

$$\text{QUINDI } B(\omega) = \omega \left| \int d^3x' [\vec{J}(\omega, \vec{x}') e^{i\vec{k} \cdot \vec{r}'}] \right|^2 = \omega \left| \int d^3x' \left[\frac{q}{(2\pi)^{3/2}} \delta(y) \delta(z) e^{i(\omega x'/c + \vec{k} \cdot \vec{r}')} \right] \right|^2$$

$$= \frac{q\omega}{(2\pi)^{3/2}} \left| \int dx' \left[e^{i(\omega x'/c + \vec{k} \cdot \vec{r}')} \right] \right|^2$$

DOVE $\vec{k} = \frac{\omega}{c} \hat{m}$

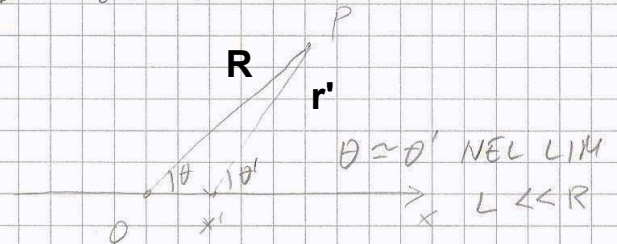
Emissione cherenkov: meccanismo di emissione

$$B(\omega) = \frac{q\omega}{(2\pi)^{1/2}} \left| \int dx' \left[e^{i(\omega x'/v + \vec{k} \cdot \vec{r}')}] \right] \right|$$

$$\vec{r}' = \vec{R} - \vec{x}' \Rightarrow \vec{k} \cdot \vec{r}' = \vec{k} \cdot \vec{R} - \vec{k} \cdot \vec{x}'$$

$$= \frac{q\omega}{(2\pi)^{1/2}} \left| e^{i\vec{k} \cdot \vec{R}} \int dx' \left[e^{i(\omega x'/v + kx' \cos\theta)} \right] \right|$$

=



$$= \frac{q\omega}{(2\pi)^{1/2}} \left| \int dx' \left[e^{ix'(\frac{\omega}{v} + k \cos\theta)} \right] \right|$$

PERCIO'

$$(2) U = \frac{n}{8\pi^2 \epsilon_0 c^3} \int d\Omega \sin^2\theta \int d\omega \frac{q^2 \omega^2}{(2\pi)} \left| \int dx' \left[e^{ix'(\frac{\omega}{v} + k \cos\theta)} \right] \right|^2 \quad \text{E' L'ENERGIA TOTALE CHE ARRIVA IN P}$$

La chiave per l'emissione e' nell'integrale dentro | ... |: se l'integrando oscilla, l'integrale e' nullo

- SE LA FASE E' T.C. $\neq 0 \Rightarrow$ L'INTEGRALE $= 0$ NON C'E' ENERGIA NETTA
 $= 0$ " " $\neq 0$ C'E' E NETTA

Emissione cherenkov: meccanismo di emissione

CON $k = \frac{\omega}{c}$ SI HA $\omega \left(\frac{1}{v} + \frac{1}{c} \cos \theta \right) = \frac{\omega}{c} \left(\frac{c}{v} + \cos \theta \right)$ NELL'EXP

• NEL VUOTO $\frac{c}{v} > 1, \forall v \Rightarrow \frac{c}{v} + \cos \theta > 0, \forall \theta \Rightarrow$ NO EMISSIONE

• IN UN MEZZO $c' = \frac{c}{n} < c \Rightarrow \frac{c'}{v} = \frac{c}{nv} < 1$ PER $v > \frac{c}{n}$

ALLORA QUANDO $\frac{c}{v} + \cos \theta = 0$, L'EXP NON OSCILLA PIU' $\Rightarrow$

LUNGO LA DIR $\cos \theta = -\frac{c}{v} = -\frac{1}{n\beta}$ C'E' EMISSIONE NETTA!
(I.E. I CAMPI SONO COERENTI)

$$\text{DA (2) SI HA } \frac{dU}{d\omega} = \frac{m\omega^2 q^2}{16\pi^3 \epsilon_0 c^3} \int_{\Omega} d\Omega \sin^2 \theta \left| \int dx' [e^{ikx'(\cos \theta + \frac{c}{v})}] \right|^2$$

POICHE' SI HA EMISSIONE SOLO QUANDO $\alpha = k(\cos \theta + \frac{c}{v}) \approx 0 \Rightarrow$

$$\sin^2 \theta \approx \sin^2 \theta_c = 1 - \cos^2 \theta = 1 - \frac{1}{n^2 \beta^2} \text{ SI PUO' RITENERE } \approx \text{ COSTANTE}$$

$$\frac{dU}{d\omega} = \frac{mq^2 \omega^2}{16\pi^3 \epsilon_0 c^3} \left(1 - \frac{1}{n^2 \beta^2}\right) \int_{\Omega} d\Omega \left| \int dx' [e^{i\alpha x'}] \right|^2, \alpha \approx 0$$

L'integrale formalmente va $-\infty$ a $+\infty$; per eseguirlo prendiamo un L finito (eventualmente facciamo il limite a ∞)

$$\int_{-L}^L dx' e^{i\alpha x'} = \frac{e^{i\alpha L} - e^{-i\alpha L}}{i\alpha} = \frac{2 \sin \alpha L}{\alpha} \Rightarrow \frac{dU}{d\omega} = \frac{mq^2 \omega^2}{16\pi^3 \epsilon_0 c^3} \left(1 - \frac{1}{n^2 \beta^2}\right) \int_{\Omega} \frac{4 \sin^2 \alpha L}{\alpha^2} 2\pi d\cos \theta$$

Emissione cherenkov: meccanismo di emissione

$$\frac{dU}{d\omega} = \frac{n^2 q^2 \omega^2}{2\pi^2 \epsilon_0 c^3} \left(1 - \frac{1}{n^2 \beta^2}\right) \underbrace{\int \frac{\sin^2 \alpha L}{\alpha^2} d\cos\theta}_C$$

$$\alpha = k(\cos\theta + \frac{c}{v}) \Rightarrow d\alpha = k d\cos\theta$$

$$C = L \int \frac{L d\alpha}{k} \frac{\sin^2 \alpha L}{\alpha^2 L^2} = \frac{L}{k} \int_{-\infty}^{+\infty} du \frac{\sin^2 u}{u^2} = \frac{\pi L}{k} = \frac{\pi L c}{n \omega}$$

$$\frac{dU}{d\omega} = \frac{q^2 \omega}{2\pi \epsilon_0 c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) L$$

$\Rightarrow$ POICHE' LA REGIONE DI EMISSIONE E' LUNGA $2L$ SI HA $\frac{dU}{d\omega dx} = \frac{1}{2L} \frac{dU}{d\omega}$

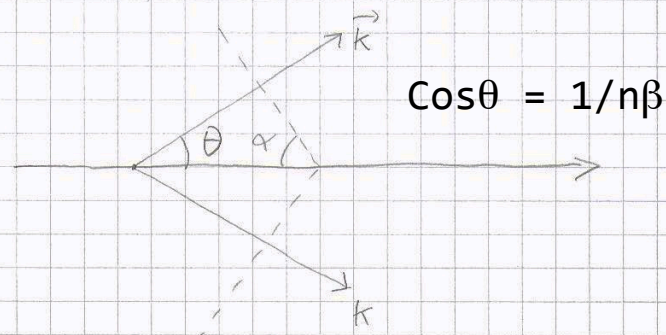
$$\frac{dU}{d\omega dx} = \frac{q^2 \omega}{4\pi \epsilon_0 c^2} \left(1 - \frac{1}{n^2 \beta^2}\right)$$

E' L'ENERGIA EMESSA PER UNITA' DI LUNGHEZZA PERCORSO NEL MATERIALE, ALLA FREQ. ω NELLA DIR θ (RISP. ALLA LINEA DI VOLO DELLA PART.)

LA POTENZA ALLA FREQ. ω : $dx = v dt \Rightarrow$

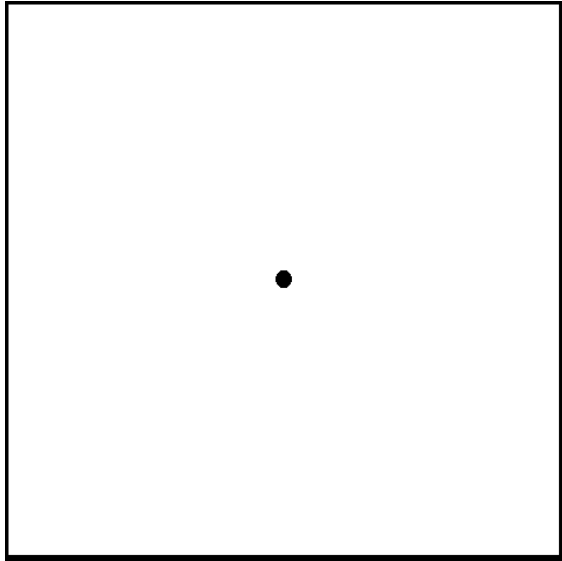
$$\frac{dP}{d\omega} = \frac{q^2 \omega v}{4\pi \epsilon_0 c^2} \left(1 - \frac{1}{n^2 \beta^2}\right)$$

NB: poiche' la potenza e' positiva, non c'e' emissione quando $1 - 1/\beta^2 n^2 < 0$

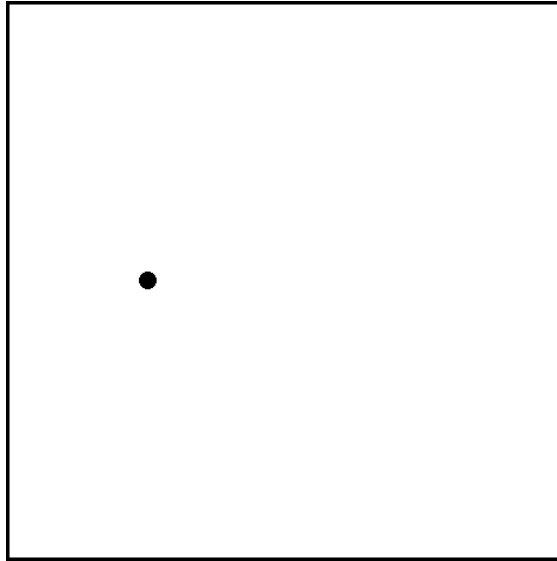


Radiazione Cherenkov

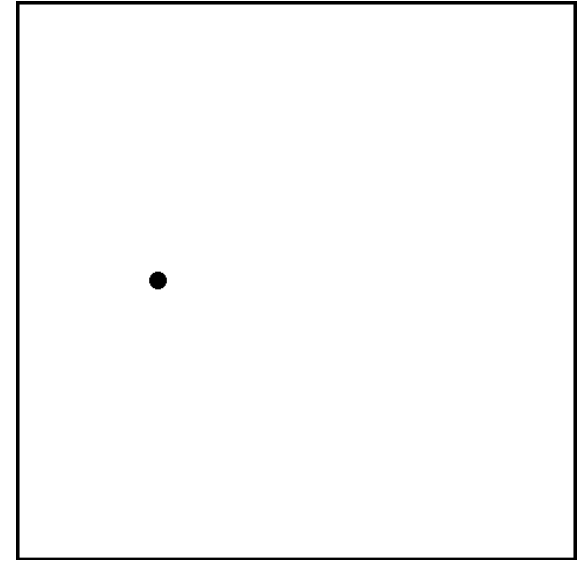
- Carica in moto nella materia



a riposo



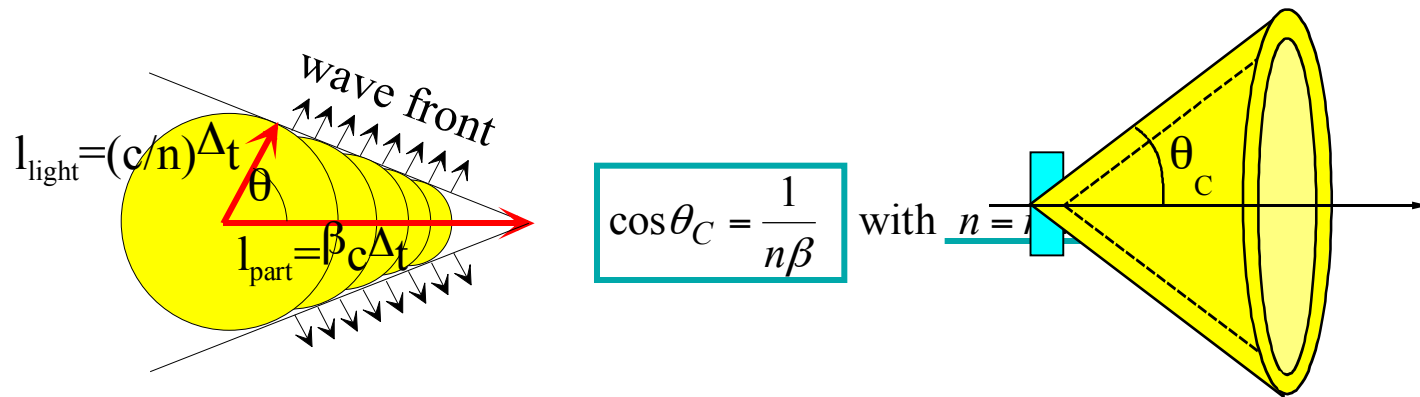
lenta



veloce

Radiazione Cherenkov

Il fronte d'onda emesso a θ_c e' analogo (sebbene di origine completamente diversa) al fronte d'onda di shock che si forma dietro ad aerei in moto supersonico o all'onda di prua (bow shock) di una nave in acqua a velocita' $>$ di quella delle onde di superficie (si usa in tutti i casi il principio di Huygens per costruire il fronte d'onda, che non dipende dal processo che da luogo all'emissione delle onde che si sovrappongono)



Due caratteristiche:

1) Esiste una velocita' di soglia $\beta_s = 1/n \rightarrow \theta_c \sim 0$ emissione collineare

2) Esiste un angolo massimo quando $\beta=1$ $\theta_{max} = \arccos(1/n)$

$\cos(\theta) = 1/\beta n$ e' valida solo per un radiatore infinito, e' comunque una buona approssimazione ogni qualvolta il radiatore e' lungo $L \gg \lambda$ essendo λ la lunghezza d'onda della luce emessa

Radiazione Cherenkov

L'energia totale persa si ottiene integrando sull'intervallo di frequenza

$$-\frac{dE}{dx} = z^2 \alpha \frac{\hbar}{c} \int \omega \left(1 - \frac{1}{\beta^2 n^2(\omega)} \right) d\omega$$

NB: e' la stessa ricavata prima, con $q = ze$ e $\alpha = e^2/4\pi\epsilon_0\hbar c$

L' energia persa per radiazione Cerenkov cresce con β . Comunque anche con $\beta \rightarrow 1$ è molto piccola.

Molto piu' piccola di quella persa per collisione (Bethe Block), al massimo 1% $\rightarrow$ pochi fotoni emessi.

•	medium	n	$\theta_{\max}(\beta=1)$	$N_{\text{ph}} (\text{eV}^{-1} \text{ cm}^{-1})$
	air	1.000283	1.36	0.208
	isobutane	1.00127	2.89	0.941
	water	1.33	41.2	160.8
	quartz	1.46	46.7	196.4

Number of photons from Cerenkov Radiation

energy loss per wavelength (λ) per dx for charge $z=1$, $\beta n > 1$:

$$\frac{dE}{dx d\lambda} = \frac{2\pi\alpha E}{\lambda^2} \left[1 - \frac{1}{\beta^2 n(\lambda)^2} \right] \quad \text{nb: } E = h\omega \text{ is the photon energy}$$

We can rewrite the above in terms of the number of photons (N) using:
 $E = h\omega/2\pi$ and $dN/dx = (dE/dx)/E$

$$\frac{dE}{dx d\lambda} = \frac{2\pi\alpha E}{\lambda^2} \left[1 - \frac{1}{\beta^2 n(\lambda)^2} \right] \Rightarrow \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left[1 - \frac{1}{\beta^2 n(\lambda)^2} \right]$$

We can simplify by considering a region where $n(\lambda)$ is a constant $= n$:

$$1 - \frac{1}{\beta^2 n(\lambda)^2} = 1 - \cos^2 \theta = \sin^2 \theta \Rightarrow \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left[1 - \frac{1}{\beta^2 n(\lambda)^2} \right] = \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \sin^2 \theta$$

We can calculate the number of photons/dx by integrating over the wavelengths that can be detected by our phototube (λ_1, λ_2):

$$\frac{dN}{dx} = 2\pi\alpha \sin^2 \theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} = 2\pi\alpha \sin^2 \theta \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right]$$

Note: if we are using a phototube with a photocathode efficiency $e(\lambda)$ that varies as a function of λ then we have:

$$\frac{dN}{dx} = 2\pi\alpha \sin^2 \theta \int_{\lambda_1}^{\lambda_2} \frac{e(\lambda) d\lambda}{\lambda^2}$$

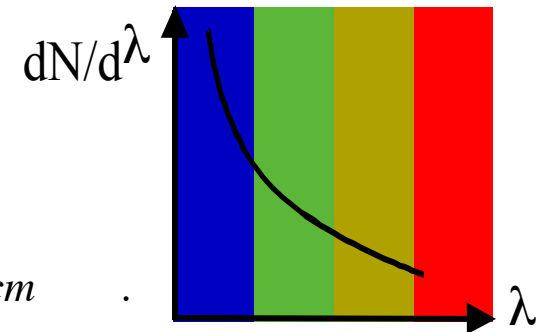
Number of photons from Cerenkov Radiation

Numero di fotoni emessi per unita' di lunghezza e intervallo unitario di lunghezza d'onda.

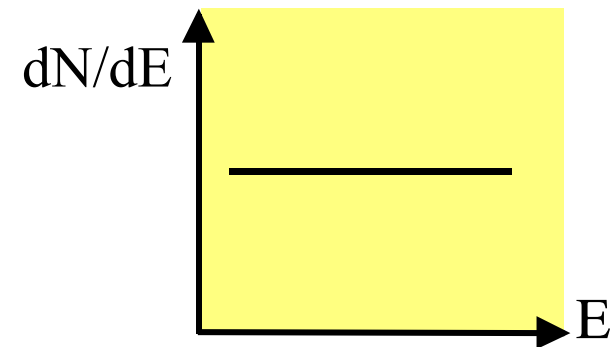
Osserviamo che decresce al crescere della λ

$$\frac{d^2N}{dx d\lambda} = \frac{2\pi z^2 \alpha}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2} \right) = \frac{2\pi z^2 \alpha}{\lambda^2} \sin^2 \theta_c$$

$$\frac{d^2N}{dx d\lambda} \propto \frac{1}{\lambda^2} \quad \text{with} \quad \lambda = \frac{c}{\nu} = \frac{hc}{E} \quad \frac{d^2N}{dx dE} = \frac{z^2 \alpha}{\hbar c} \sin^2 \theta_c^2 \approx 390 z^2 \sin^2 \theta_c^2 \quad ph / eV cm$$



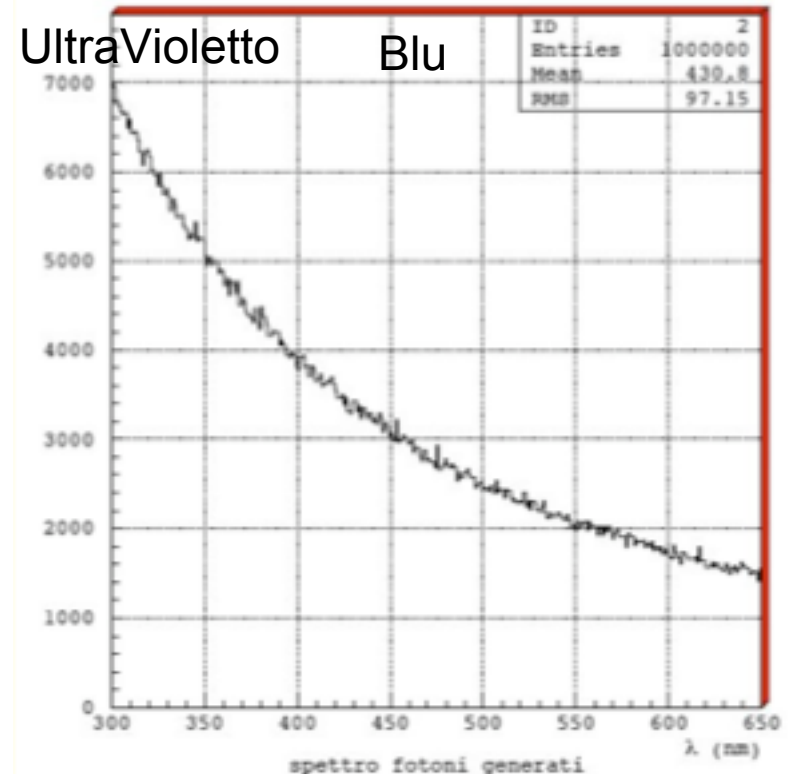
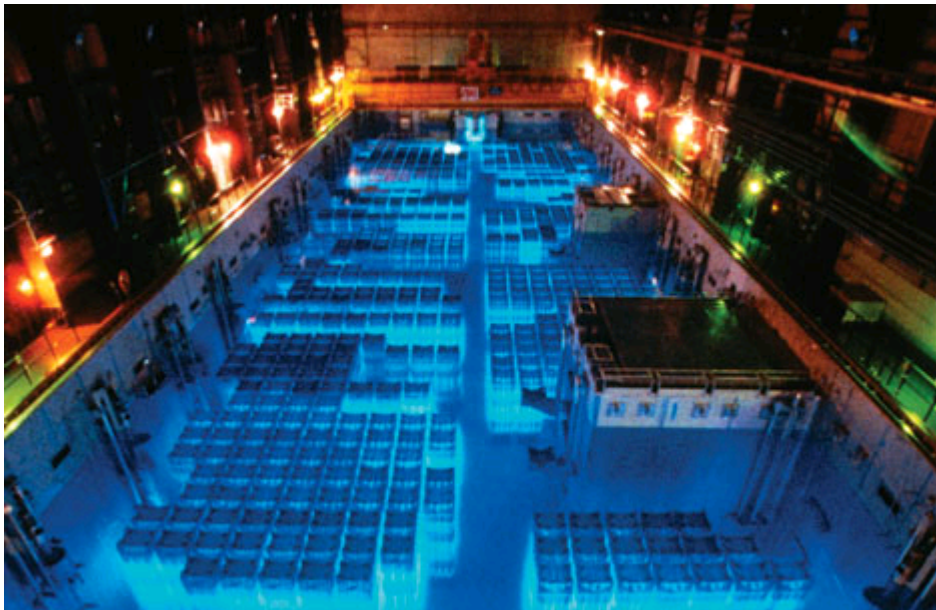
Il numero di fotoni emessi per unita' di Lunghezza non dipende dall'energia del fotone



Blue Light

- More photons are produced at short wavelengths.
- Enhanced visible light at the blue end of the spectrum.
 - Characteristic glow from a reactor

$$\frac{\partial N}{\partial \lambda} = 2\pi\alpha L \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{1}{\lambda^2}$$



Number of photons from Cerenkov Radiation

For a typical phototube the range of wavelength (λ_1, λ_2) is (350nm, 500nm).

$$\frac{dN}{dx} = 2\pi\alpha \sin^2 \theta \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] = 2\pi\alpha \sin^2 \theta \left[\frac{1}{3.5} - \frac{1}{5} \right] \frac{10^5}{\text{cm}} = 390 \sin^2 \theta \text{ photons/cm}$$

We can simplify using:

$z=1$

$$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{1}{\beta^2 n^2} = \frac{(\beta n + 1)(\beta n - 1)}{\beta^2 n^2} \Rightarrow_{\beta \rightarrow 1} \frac{(n+1)(n-1)}{n^2}$$

For a highly relativistic particle going through a gas the above reduces to:

$$\sin^2 \theta = \frac{(\beta n + 1)(\beta n - 1)}{\beta^2 n^2} \Rightarrow_{\beta \rightarrow 1} \frac{(n+1)(n-1)}{n^2} \Rightarrow_{\beta \rightarrow 1, \text{gas}} 2(n-1)$$

$$\frac{dN}{dx} = 780(n-1) \text{ photons/cm}$$

GAS

For He we find: 2-3 photons/meter (not a lot!)

For CO₂ we find: ~33 photons/meter

For H₂O we find: ~34000 photons/meter

Photons are preferentially emitted at small λ 's (blue)

For most Cerenkov counters the photon yield is limited (small) due to space limitations, the index of refraction of the medium, and the phototube quantum efficiency.

Radiazione Cherenkov

La luce emessa e rivelabile e' poca.

- Se consideriamo un radiatore spesso 1 cm, un angolo di emissione $\theta_c = 30^\circ$ ed una finestra sensibile del rivelatore di fotoni $\Delta E = E_2 - E_1 = 1$ eV ed una particella di carica 1.

$$\frac{dN}{dE dx} = \frac{z^2 \alpha}{\hbar c} \sin^2 \vartheta_c$$

$$\Rightarrow N_{ph} = 370 \cdot \sin^2 \vartheta_c \cdot L \cdot \Delta E = 370 \times 0.25 = 92.5$$

Considerando inoltre che l'efficienza quantica tipica di un fotomoltiplicatore e' ~20% $\rightarrow$

$N_{pe}=18 \rightarrow$ fluttuazioni alla Poisson

Number of photons from Cerenkov Radiation

medium	n	$\theta_{\max} (\beta=1)$	$N_{\text{ph}} (\text{eV}^{-1} \text{cm}^{-1})$
air	1.000283	1.36	0.208
isobutane	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

To obtain the correct number of photons produced in a Cherenkov counter, must be integrated over the region for which $\beta \cdot n(\lambda) > 1$.

Also the response function of the light collection system must be taken into account to obtain the number of photons arriving at the photon detector.

All transparent materials are candidates for Cherenkov radiators. In particular, Cherenkov radiation is emitted in all scintillators and in the light guides which are used for the readout.

The scintillation light, however, is approximately 100 times more intense than the Cherenkov light.

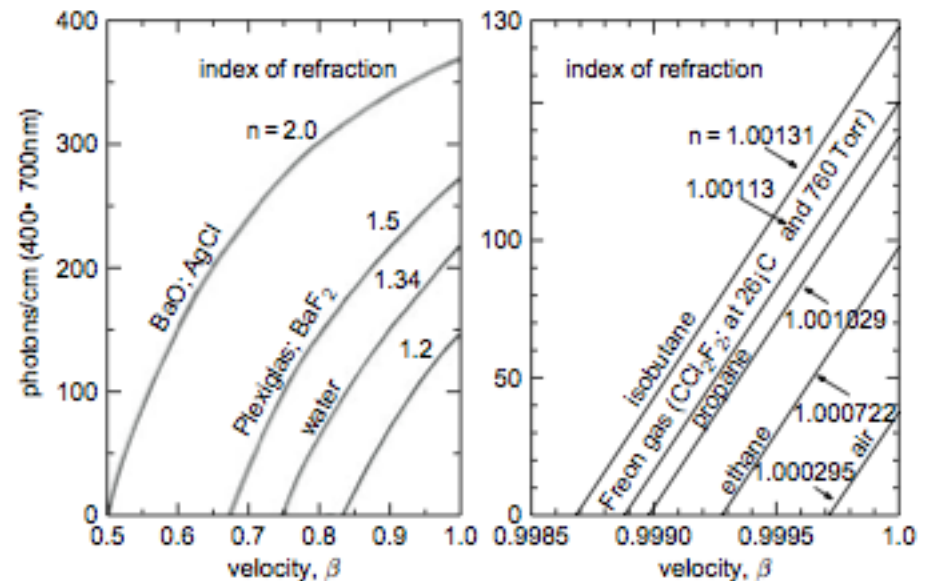
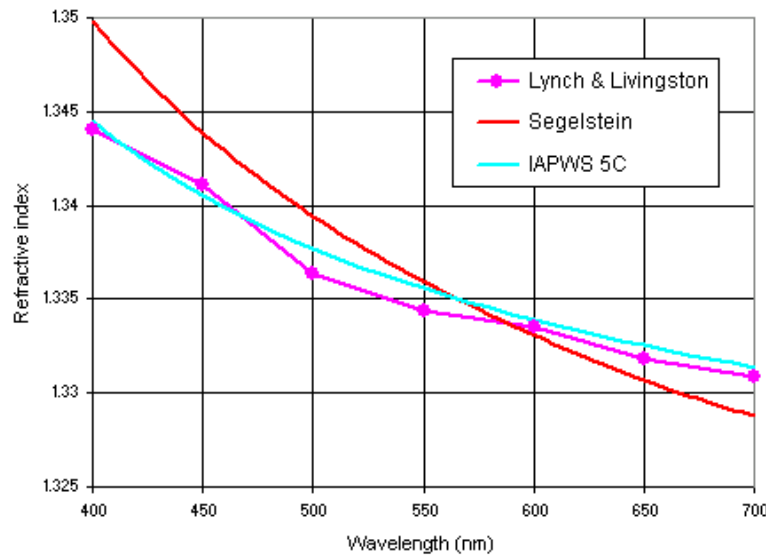


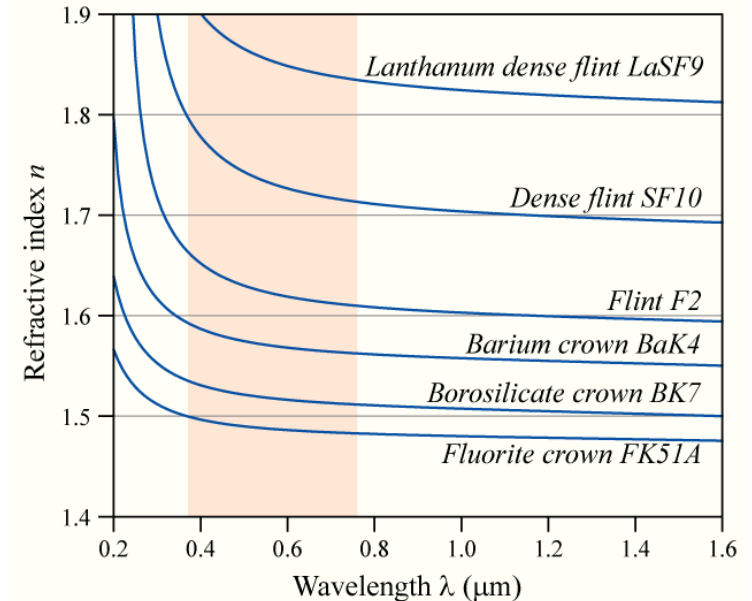
Fig. 5.40. Number of produced Cherenkov photons per unit path length for various materials as a function of the particle velocity [142].

Frequency Dependence

- The refractive index varies with wavelength.



water



glass

Therefore, an integration over the frequency/energy/wavelength range of photomultiplier is actually needed to obtain the the number of photons from emission (the so called photon yield

$$\frac{dN}{dx} = 2\pi\alpha \int_{\lambda_1}^{\lambda_2} \frac{e(\lambda)d\lambda}{\lambda^2} \left(1 - \frac{1}{n^2(\lambda)\beta^2}\right)$$

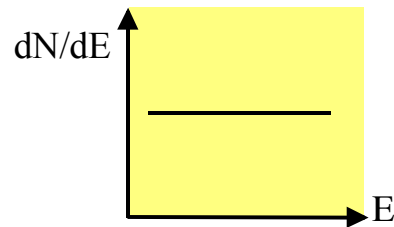
Radiazione Cherenkov

L'indice di rifrazione dipende dalla frequenza, cioè dell'energia, $n=n(E)$ e $dn/dE \neq 0$

→ errore cromatico

$$\sigma_{\vartheta}^c = \frac{1}{n \cdot \operatorname{tg} \vartheta} \sigma_n = \frac{1}{n \cdot \operatorname{tg} \vartheta} \frac{dn}{dE} \sigma_E$$

σ_E e' connesso all'intervallo ΔE considerato.



Assumendo distribuzione piatta in energia

$$\rightarrow \sigma_E = \Delta E / (12)^{1/2}$$

L'errore cromatico puo' essere ridotto riducendo ΔE e quindi il numero di fotoni emessi o con correzioni ottiche complicate

Radiazione Cherenkov

Le due caratteristiche importanti dell'emissione cherenkov sono:

- 1) Esiste una soglia per emissione di luce Cerenkov $\beta n > 1$
- 2) La luce e' emessa ad un angolo particolare $\sin\theta_c = 1/\beta n$
- 3) Il numero di fotoni emessi dipende dalla carica

$$\frac{d^2N}{dx dE} = \frac{z^2 \alpha}{\hbar c} \sin^2\theta_c \approx 390 z^2 \sin^2\theta_c \text{ ph / eVcm}$$

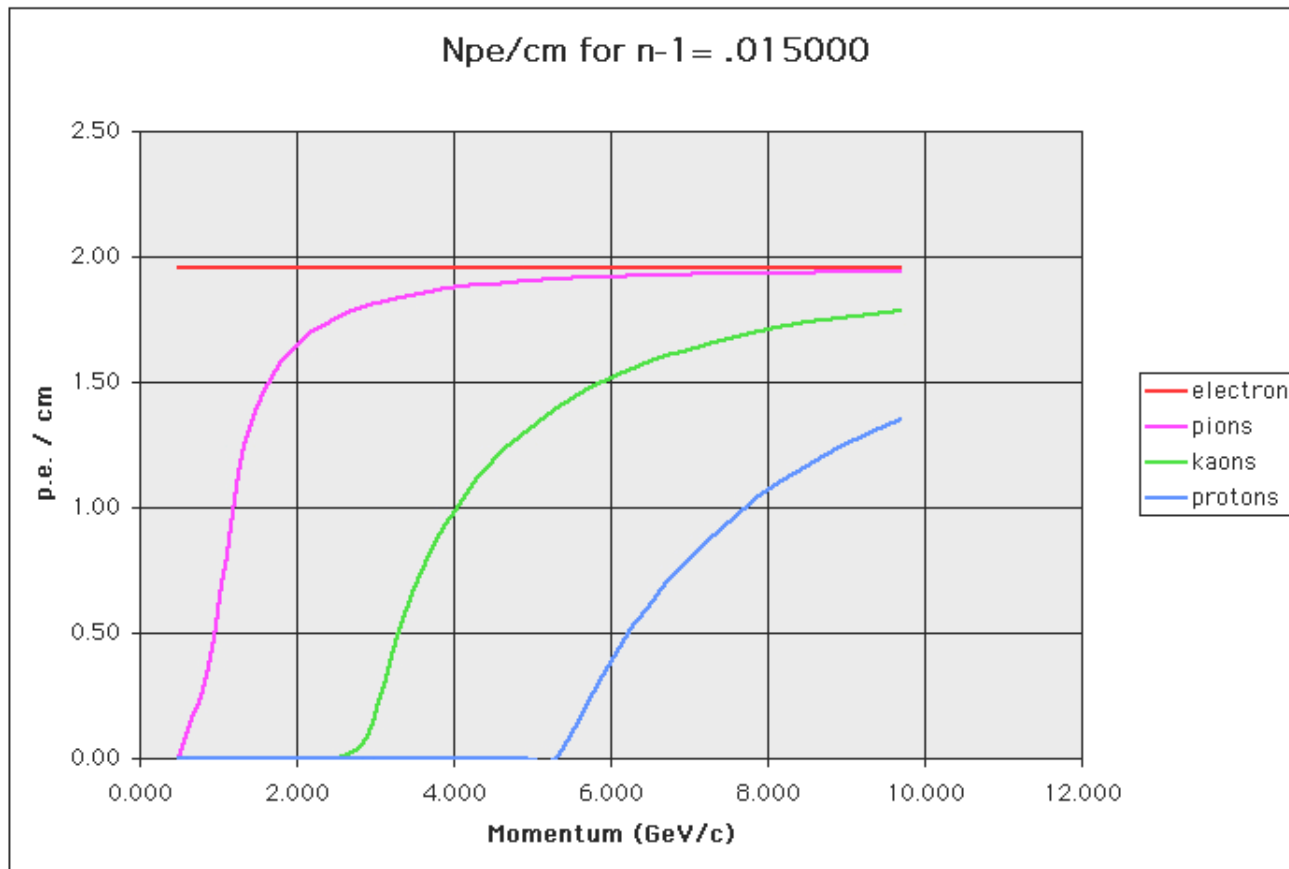
→ Facile utilizzare l'effetto Cerenkov per identificare le particelle.

Con 1) posso sfruttare la soglia → Cerenkov a soglia.

Con 2) e 3) misurare l'angolo e PID → DISC, RICH etc.

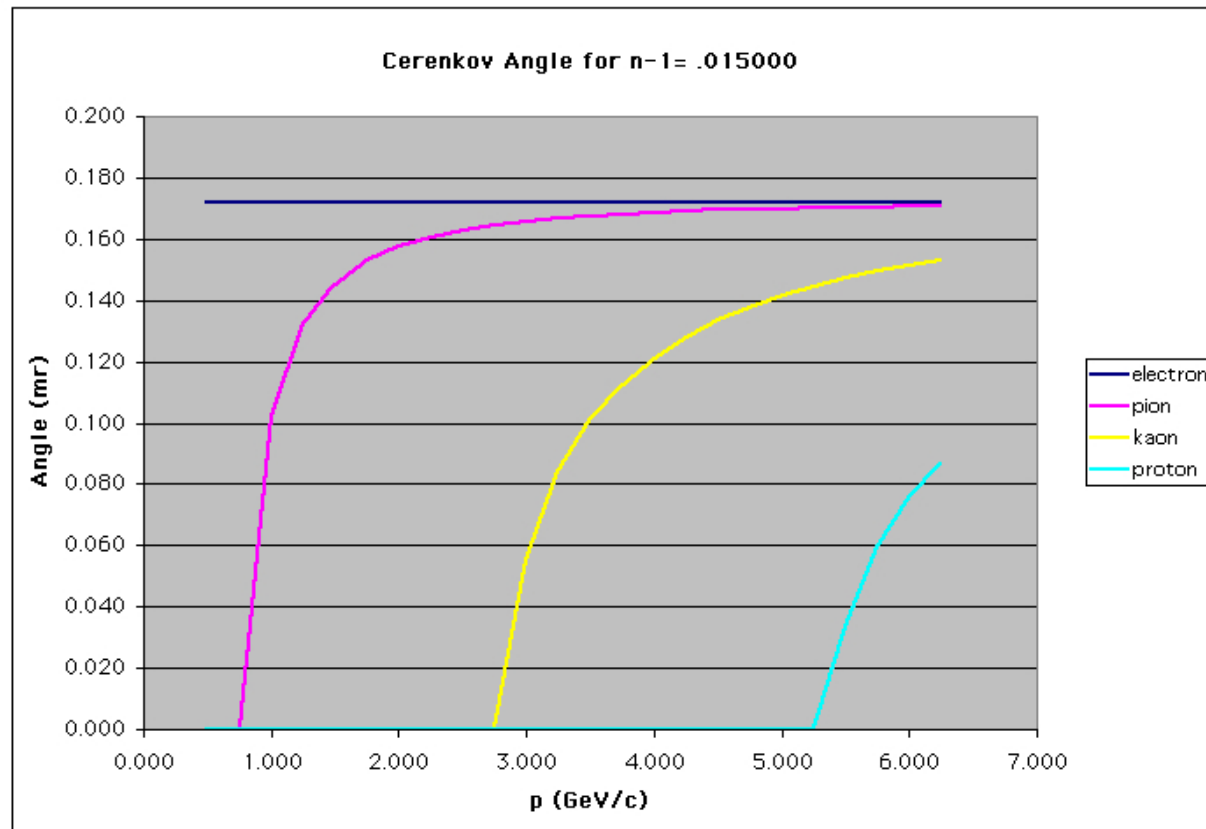
Cherenkov: Particle Identification

Threshold Cerenkov Counter. # Photons vs. Momentum.



Cherenkov: Particle Identification

Cerenkov Counter. Light Emission Angle vs. Particle Momentum.



Different Cherenkov Detectors

■ Threshold Detectors

- Yes/No on whether the speed is $\beta > 1/n$

■ Differential Detectors

- makes use of the angle of the Cerenkov radiation
- $\beta_{\max} > \beta > \beta_{\min}$

■ Ring-Imaging Detectors

- makes use of the “cone” of light
- Measure β

Each of the above counters is designed to work in a certain momentum range.

Threshold Momentum for Cerenkov Radiation

Example: Threshold momentum for Cerenkov light:

$$\beta_t = \frac{1}{n} \quad \gamma_t = \frac{1}{\sqrt{1-\beta_t^2}} = \frac{n}{\sqrt{n^2-1}} = \frac{1}{\beta_t \sqrt{n^2-1}}$$

$$\beta_t \gamma_t = \frac{1}{\sqrt{n^2-1}} = \frac{1}{\sqrt{(n-1)(n+1)}}$$

Medium	$\delta=n-1$	γ_t
Helium	3.3×10^{-5}	123
CO ₂	4.3×10^{-4}	34
H ₂ O	0.33	1.52
Glass	0.46-0.75	1.37-1.22

For gases it is convenient to let $\delta=n-1$. Then we have:

$$\beta_t \gamma_t = \frac{1}{\sqrt{\delta(\delta+2)}}$$

The momentum (p_t) at which we get Cerenkov radiation is:

$$p_t = m \beta_t \gamma_t = \frac{m}{\sqrt{\delta(\delta+2)}}$$

For a gas $\delta+2 \approx 2$ so the threshold momentum can be approximated by:

$$p_t = m \beta_t \gamma_t = \frac{m}{\sqrt{2\delta}}$$

For helium $\delta=3.3 \times 10^{-5}$ so we find the following thresholds:

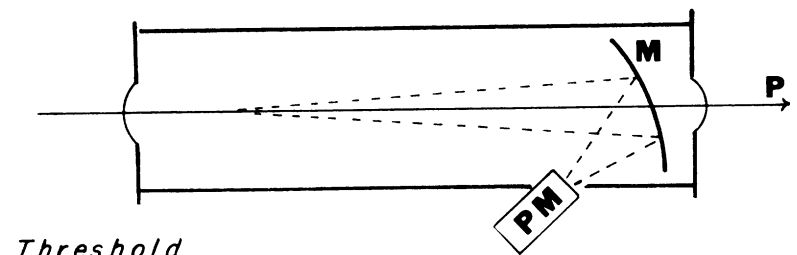
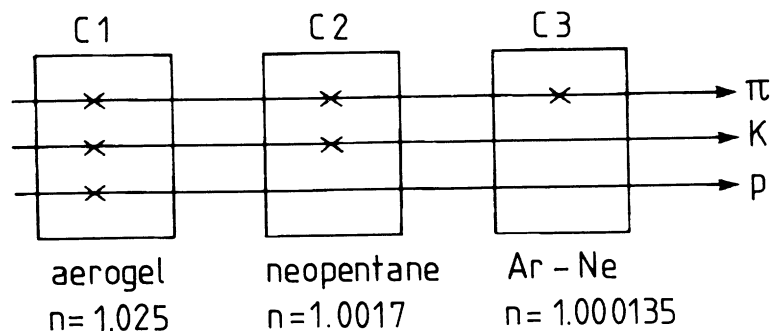
electrons	63 MeV/c	kaons	61 GeV/c
pions	17 GeV/c	protons	115 GeV/c

Table 5.4. *Compilation of Cherenkov radiators [1, 143]. The index of refraction for gases is for 0 °C and 1 atm (STP). Solid sodium is transparent for wavelengths below 2000 Å [144, 145]*

Material	$n - 1$	β threshold	γ threshold
Solid sodium	3.22	0.24	1.029
Diamond	1.42	0.41	1.10
Flint glass (SFS1)	0.92	0.52	1.17
Lead fluoride	0.80	0.55	1.20
Aluminium oxide	0.76	0.57	1.22
Lead glass	0.67	0.60	1.25
Polystyrene	0.60	0.63	1.28
Plexiglas (Lucite)	0.48	0.66	1.33
Borosilicate glass (Pyrex)	0.47	0.68	1.36
Lithium fluoride	0.39	0.72	1.44
Water	0.33	0.75	1.52
Liquid nitrogen	0.205	0.83	1.79
Silica aerogel	0.007–0.13	0.993–0.884	8.46–2.13
Pentane (STP)	$1.7 \cdot 10^{-3}$	0.9983	17.2
CO ₂ (STP)	$4.3 \cdot 10^{-4}$	0.9996	34.1
Air (STP)	$2.93 \cdot 10^{-4}$	0.9997	41.2
H ₂ (STP)	$1.4 \cdot 10^{-4}$	0.99986	59.8
He (STP)	$3.3 \cdot 10^{-5}$	0.99997	123

Types of Cerenkov: Threshold counter

- Identify the particle(s) which give off light.
- Can be used to separate electrons from heavier particles (π , K, p) since electrons will give off light at a much lower momentum (e.g. 63 MeV/c vs 17 GeV/c for He)
- Problems with device:
 - ✓ above a certain momentum several particles will give light.
 - ✓ usually threshold counters use gas which implies low light levels (n-1 small) low light levels leads to inefficiency, e.g. $\langle n_g \rangle = 3$, the prob. of zero photons: $P(0) = e^{-3} = 5\%$
 - ✓ Phototubes must be shielded from magnetic fields above a few tenths of a gauss.



Threshold Cherenkov Detector

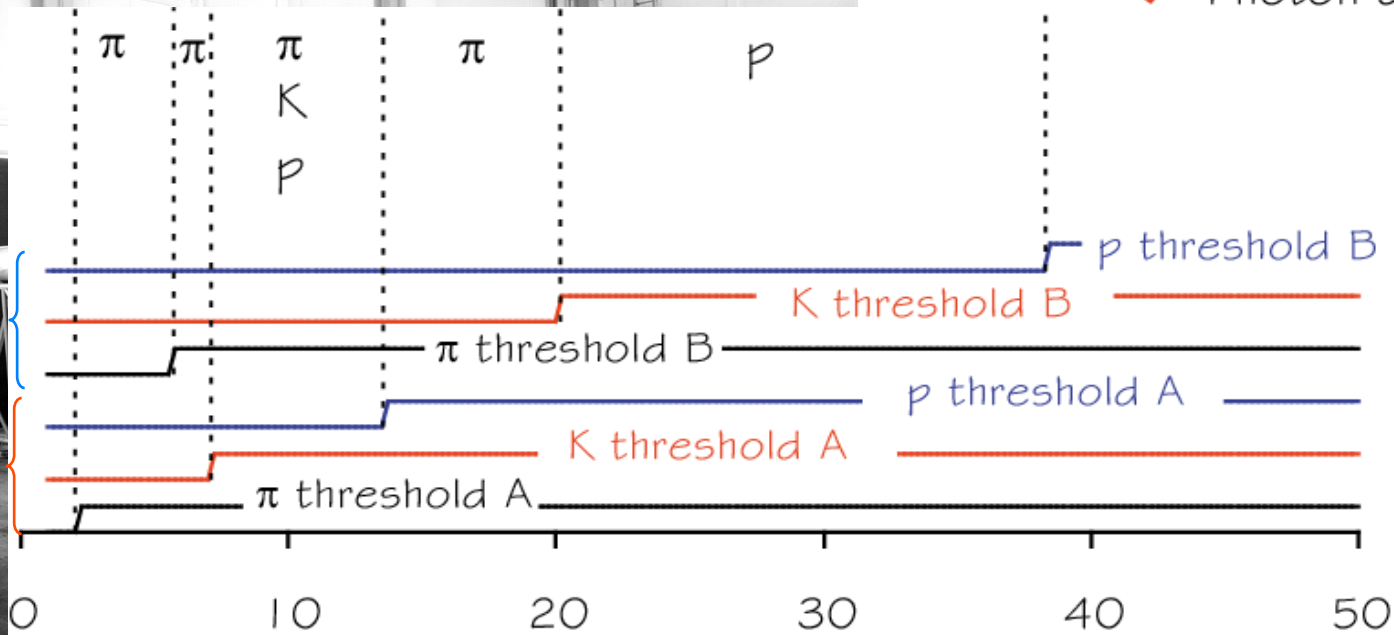
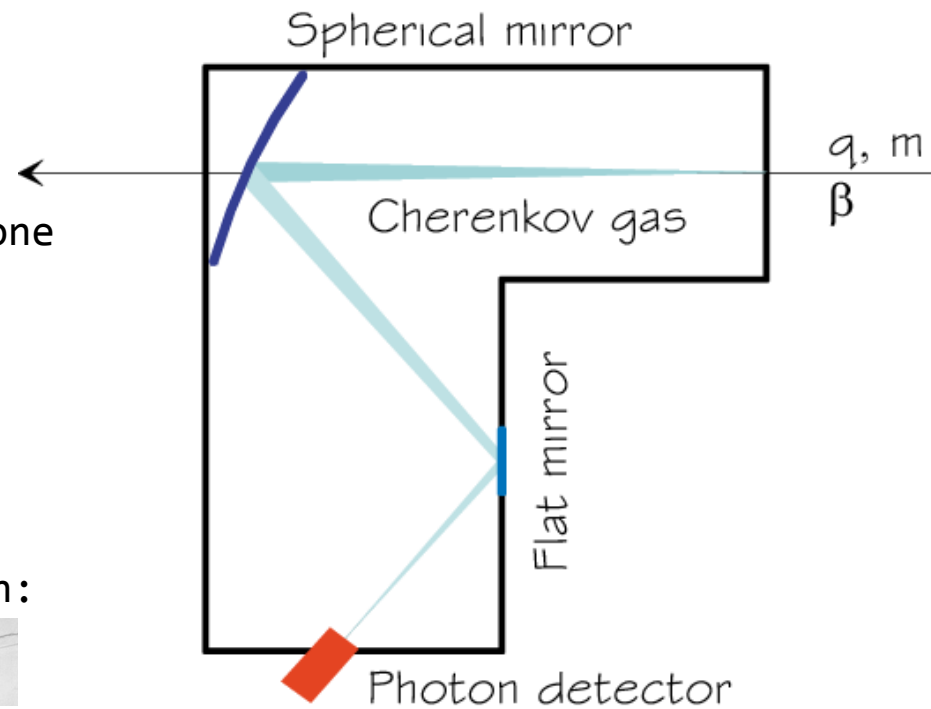
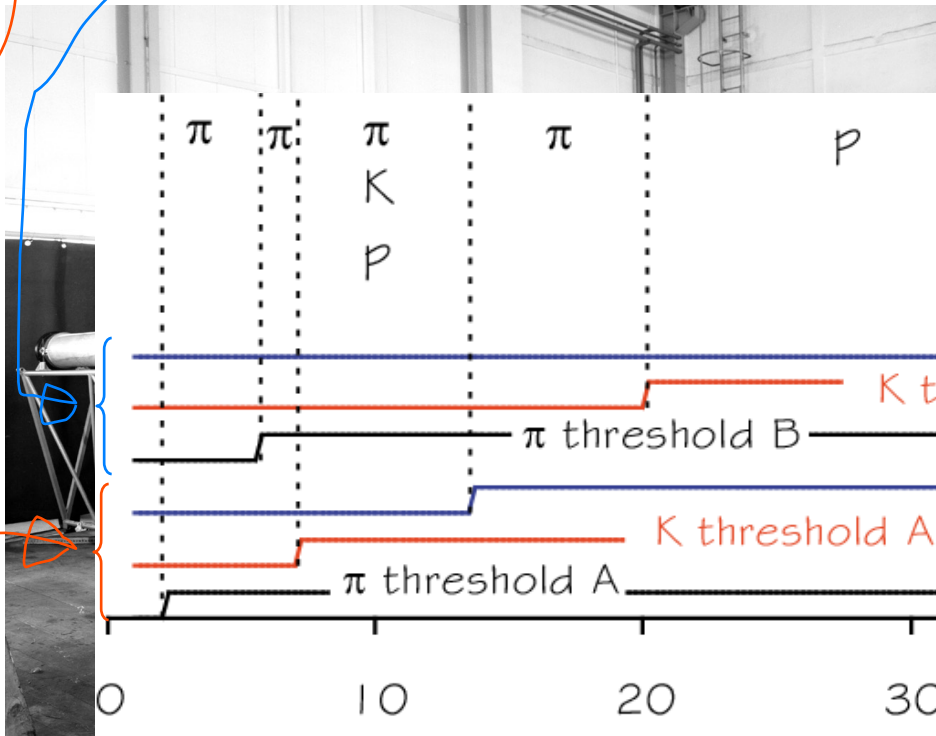
To get a wider momentum range for particle identification, use more than one radiator.

Assume

A radiator: $n=1.0024$

B radiator: $n=1.0003$

Positive particle identification:



A practical example

$$\cos \theta = \frac{1}{\beta n}$$

$$m = \frac{p}{\beta \gamma}$$

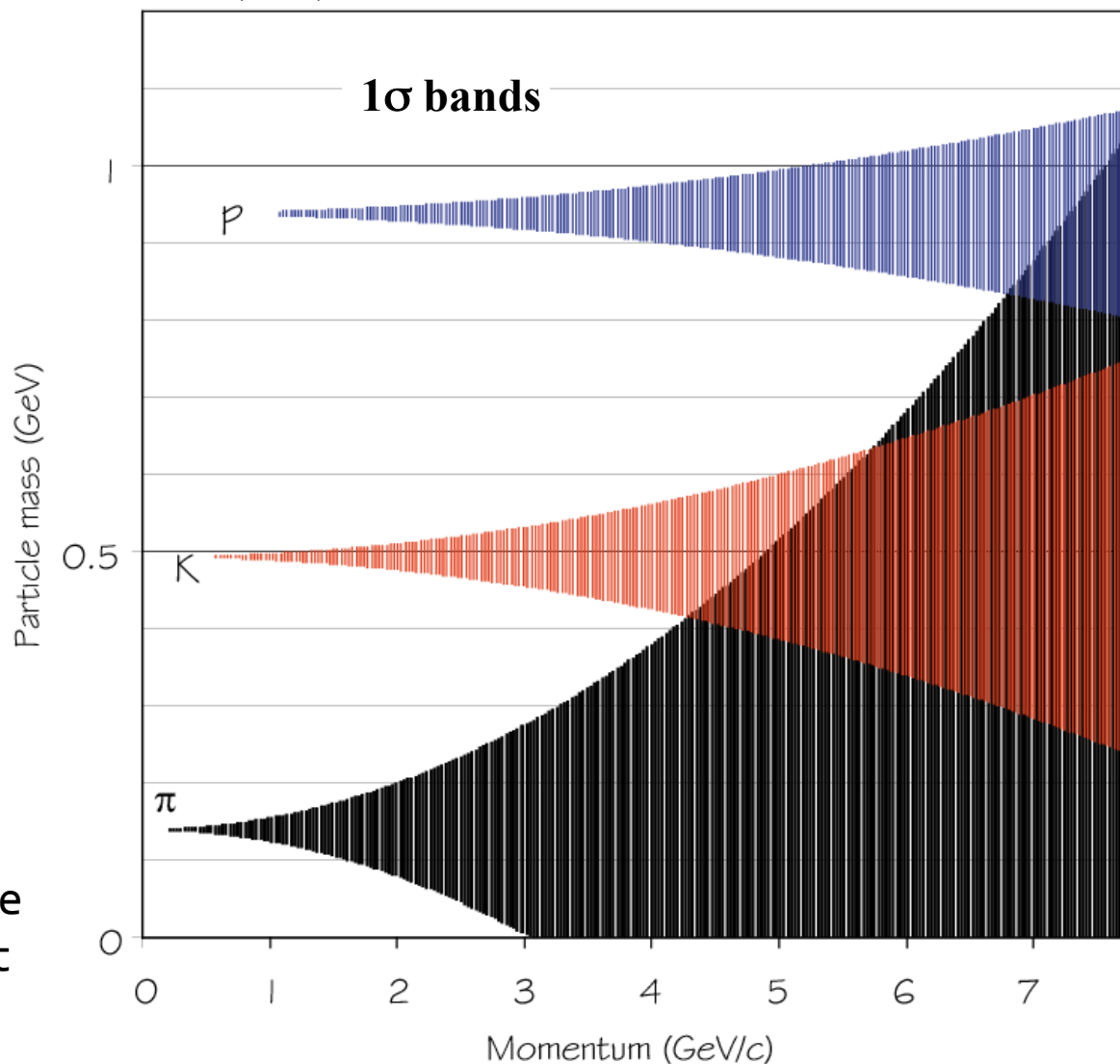
$$\frac{\Delta m}{m} = \sqrt{\left(\frac{\Delta p}{p}\right)^2 + \left(\gamma^2 \cdot \tan \theta \cdot \Delta \theta\right)^2}$$

set :
 $n=1.333$ (H_2O)

$$\rightarrow \begin{cases} \theta_{\max} = 41.4^\circ \\ \beta_{\min} = 0.75 \end{cases}$$

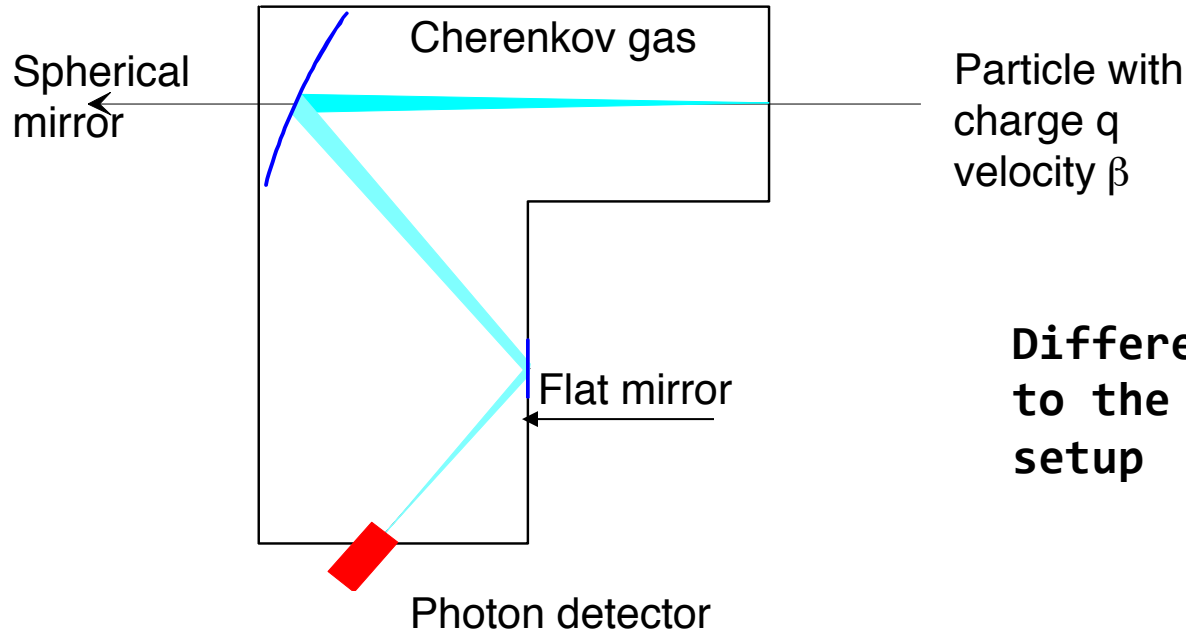
$\Delta p/p$ 5×10^{-4}
 $\Delta \theta$ 15 mrad
 l 1 cm

Error on mass depends critically as γ^2 on the measurement of β , that is on θ measurement

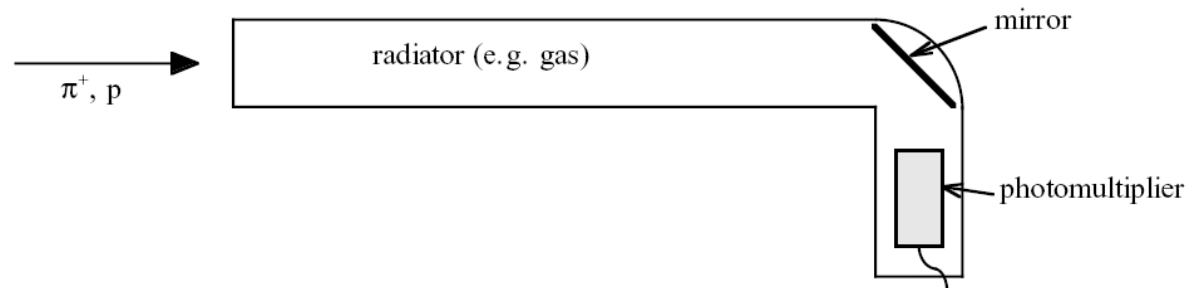


Cherenkov

Threshold Cherenkov Counter, chose suitable medium (n)



Different geometries according to the particular experimental setup



To get a better particle identification, use more than one radiator.

All transparent materials are candidates for Cherenkov radiators. In particular, Cherenkov radiation is emitted in all scintillators and in the light guides which are used for the readout.

The threshold counter has only one adjustable parameter, the refractive index of the radiator. To control the threshold you must control the refractive index (e.g., vary the pressure in a gas filled counter). In fig. are shown the appropriate refractive indices for pions and kaons of varying momentum.

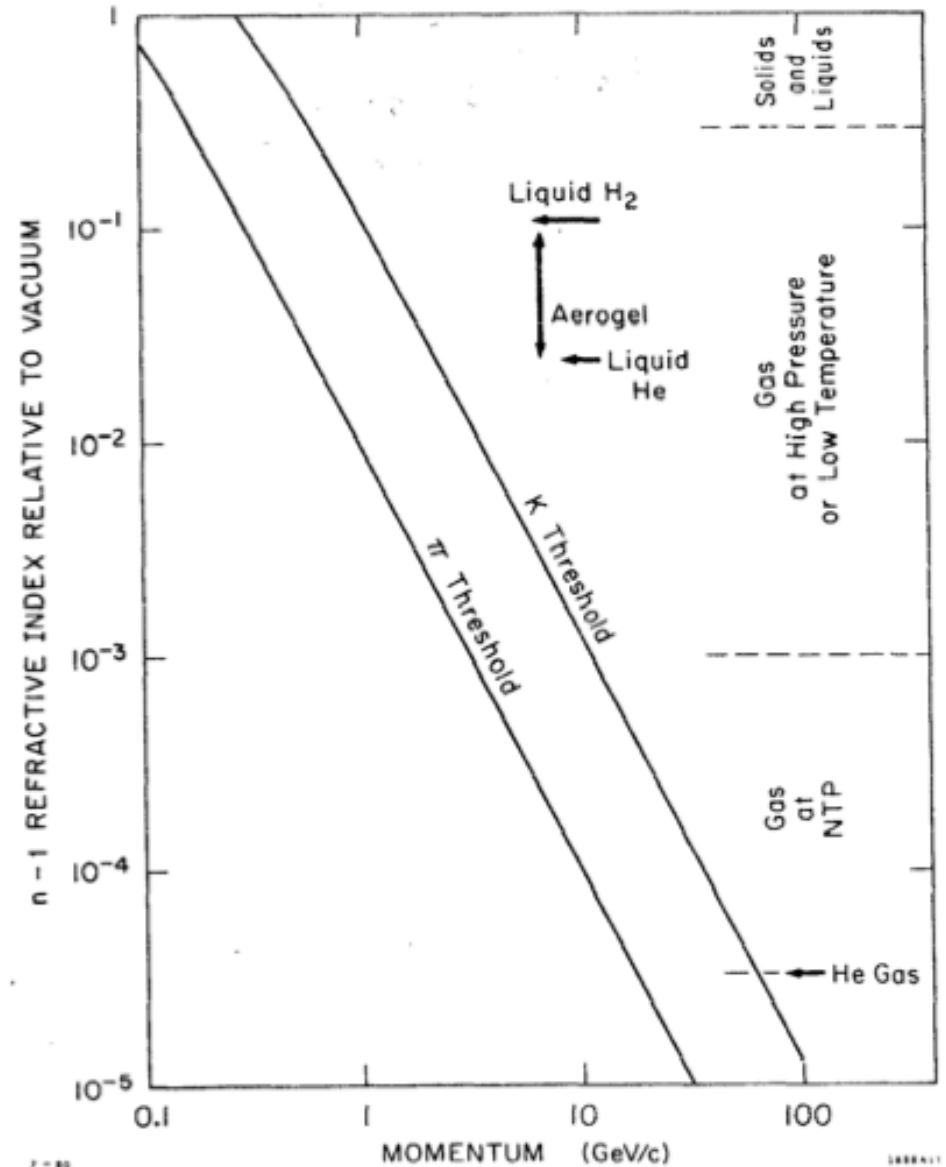


Fig. 4. The refractive index required so that of pions or kaons of given momentum be at threshold for Čerenkov radiation.

Types of Cerenkov Counters

Differential Cerenkov Counter:

Makes use of the angle of Cerenkov radiation and only samples light at certain angles.

For fixed momentum $\cos\theta$ is a function of mass:

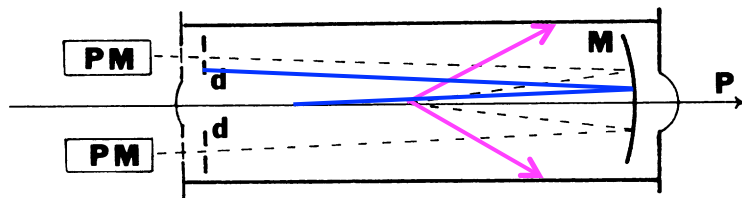
$$\cos\theta = \frac{1}{n\beta} = \frac{1}{n(p/E)} = \frac{\sqrt{m^2 + p^2}}{np}$$

Differential cerenkov counters typically on work over a fixed momentum range (good for beam monitors, e.g. measure π or K content of beam).

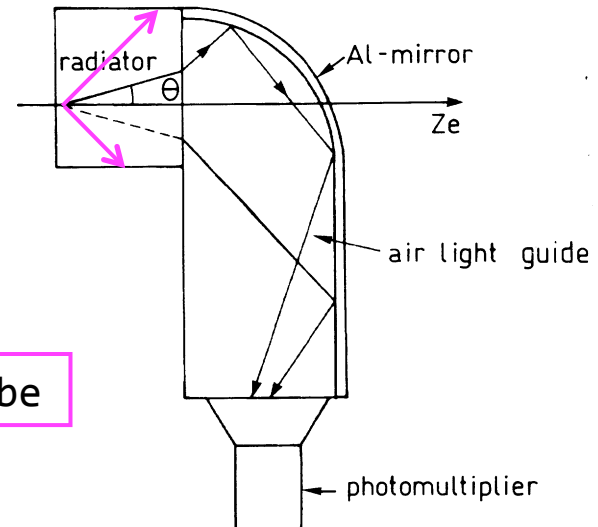
Problems with differential Cerenkov counters:

Optics are usually complicated.

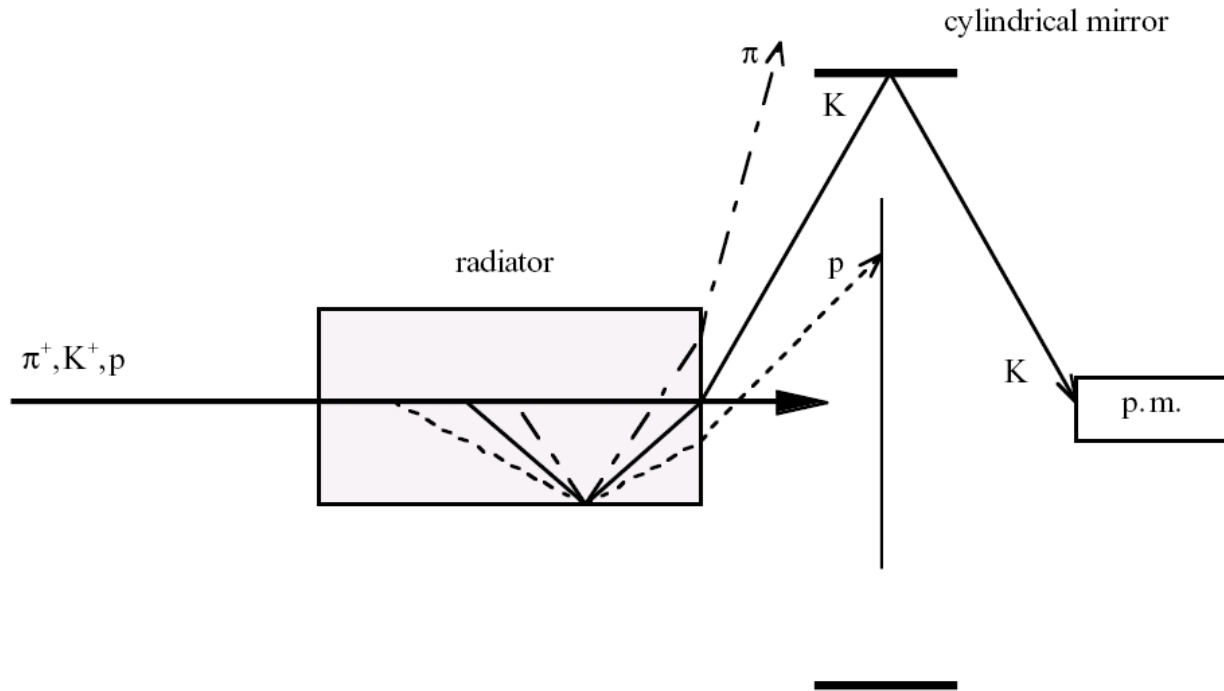
Have problems in magnetic fields since phototubes must be shielded from B-fields above a few tenths of a gauss.



Not all light will make it to phototube



Differential Detectors



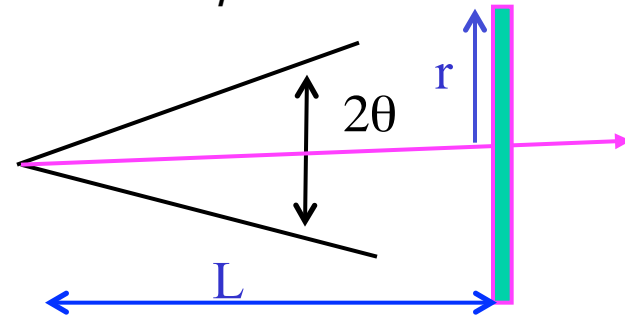
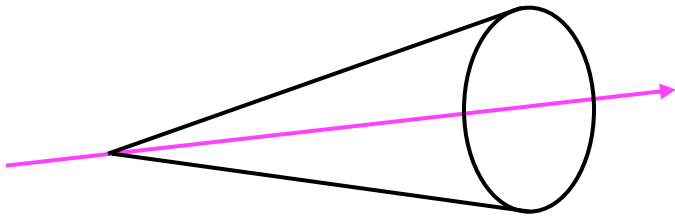
- Will reflect light onto PMT for certain angles only $\Leftrightarrow \beta$ Selection

Ring Imaging Cherenkov Counters (RICH)

RICH counters measure the cone angle θ_c of the Cerenkov light by projecting it on a photon-sensitive plane.

By measuring θ_c , one measures β .

The $\frac{1}{2}$ angle (θ) of the cone is given by: $\cos\theta = \frac{1}{n\beta}$



The radius of the cone is: $r = L \tan\theta$, with L the distance to the where the ring is imaged.

Since one is not only interested in detecting photons, but also in measuring their coordinates, a position-sensitive detector is necessary.

Problems with RICH:

- optics can be very complicated (projections are not usually circles)
- readout system very complicated (e.g. wire chamber readout, 10^5 - 10^6 channels)
- photon yield usually small (10-20), only a few points on “circle”

Ring Imaging Detectors

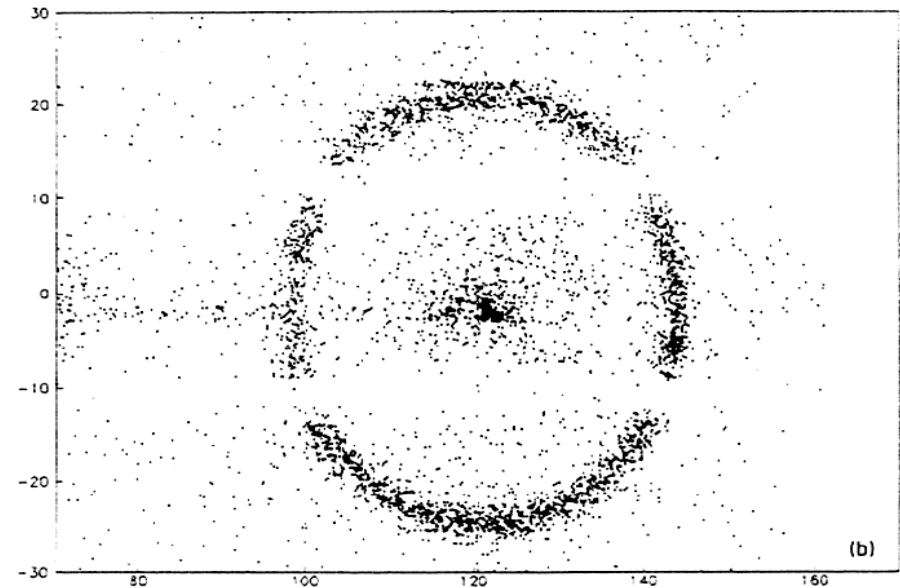
If the momentum of the charged particle is already known, e.g. by magnetic deflection, then the particle can be identified (i.e. its mass m determined) from the size of the Cherenkov ring, r . By use of the relationship $\beta = p/E$ the mass m can be determined.

$$\cos\theta = \frac{1}{n\beta} \Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{m^2 + p^2}}{np}\right)$$

For a particle with $p=1\text{GeV}/c$, $z = 1$, $L=1\text{ m}$, and LiF as the medium ($n=1.392$) we find:

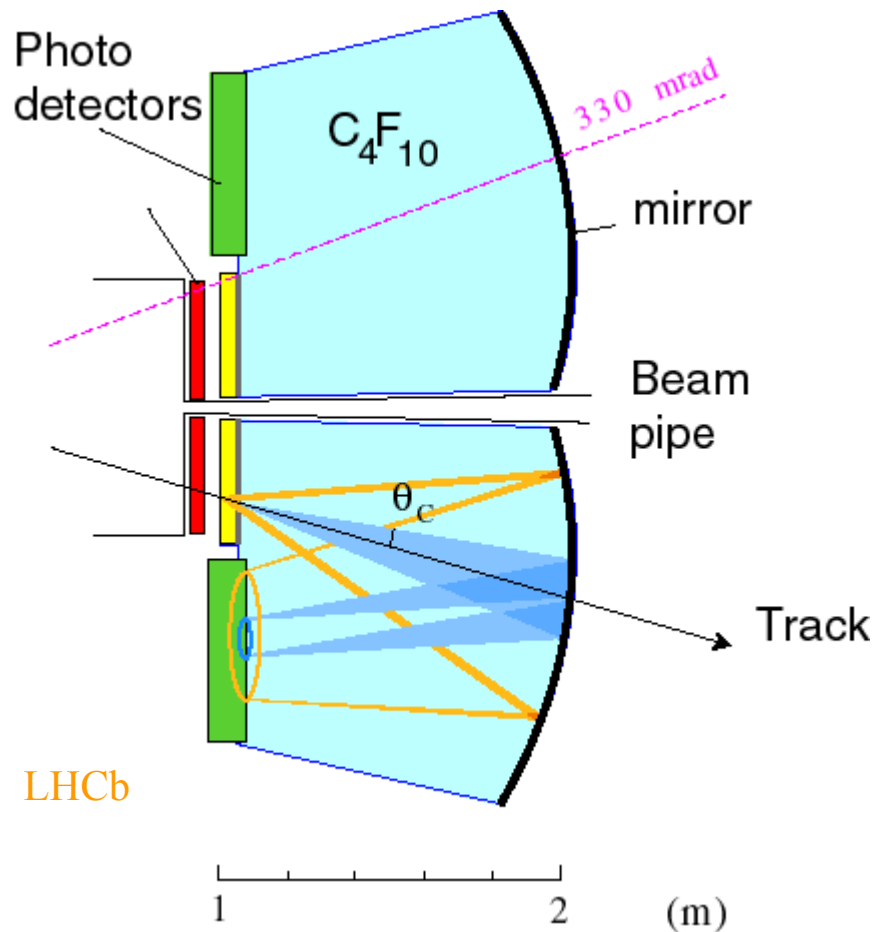
	$\theta(\text{deg})$	$r(\text{m})$
π	43.5	0.95
K	36.7	0.75
P	9.95	0.18

Great $\pi/K/p$ separation!
(until cherenkov angle does not saturate...)



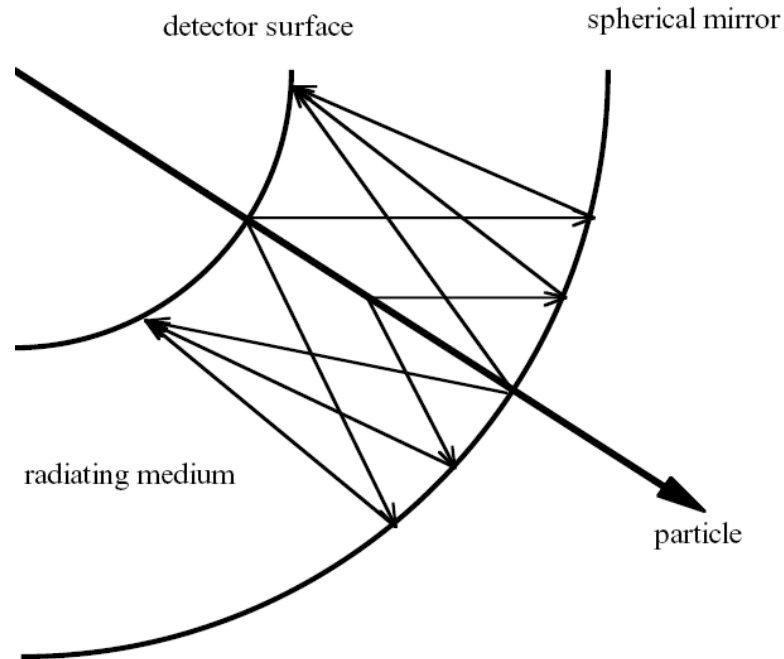
Thus by measuring p and r we can identify what type of particle we have.

Ring Imaging Detectors



- In accelerator experiments the photodetectors are usually placed as far as possible away from beam.
- Light is reflected by mirrors on the photosensitive detectors
 - Center sets the particle position
 - Radius sets the speed

Ring Imaging Detectors



For example a spherical mirror of radius R_S , with centre of curvature coincident with the interaction point (IP), projects the cone of Cherenkov light produced in the radiator onto a ring on the surface of a spherical detector of radius R_D .

The radiator fills the volume between the spherical surfaces with radii R_S and R_D . In general, one takes $R_D = R_S/2$, since the focal length f of a spherical mirror is $R_S/2$.

Because all photons are emitted at same angle θ_c with respect to the particle trajectory pointing away from the IP, all of them will be focussed to the thin detector ring on the inner sphere. The radius of the Cherenkov ring on the detector surface is

$$r = f \cdot \theta_c = R_S \cdot \theta_c / 2$$

The measurement of r allows one to determine the particle velocity

$$\cos \theta_c = 1/n\beta \rightarrow \beta = 1/ n \cos (2r/R_S)$$

Ring Imaging Detectors

The most crucial aspect of RICH counters is the detection of Cherenkov photons with high efficiency on usually large detector surface, since few photons are usually emitted and ever fewer are collected.

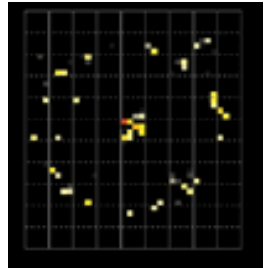
Better Cherenkov rings are obtained from fast heavy ions, because the number of produced photons is proportional to the square of the projectile charge.

$$-\frac{dE}{dx} = z^2 \alpha \frac{\hbar}{c} \omega \left(1 - \frac{1}{\beta^2 n^2(\omega)} \right) d\omega$$

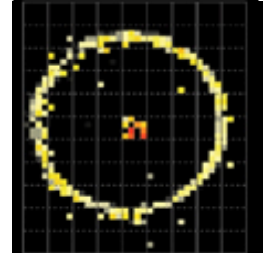
The z^2 dependence can be used to measure the absolute charge of incident particle if signal amplitude is measured (ie the # of photons emitted).

This usually done in astroparticle experiments to measure cosmic ray nuclear abundances or in heavy ion collisions

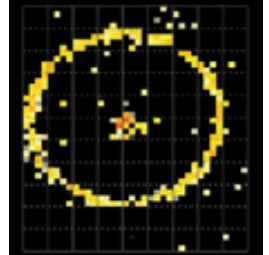
He



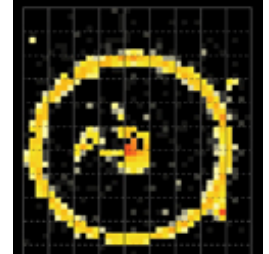
Li



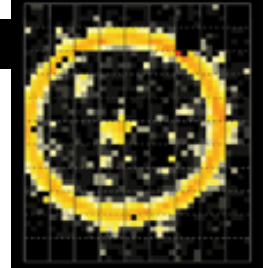
C



O



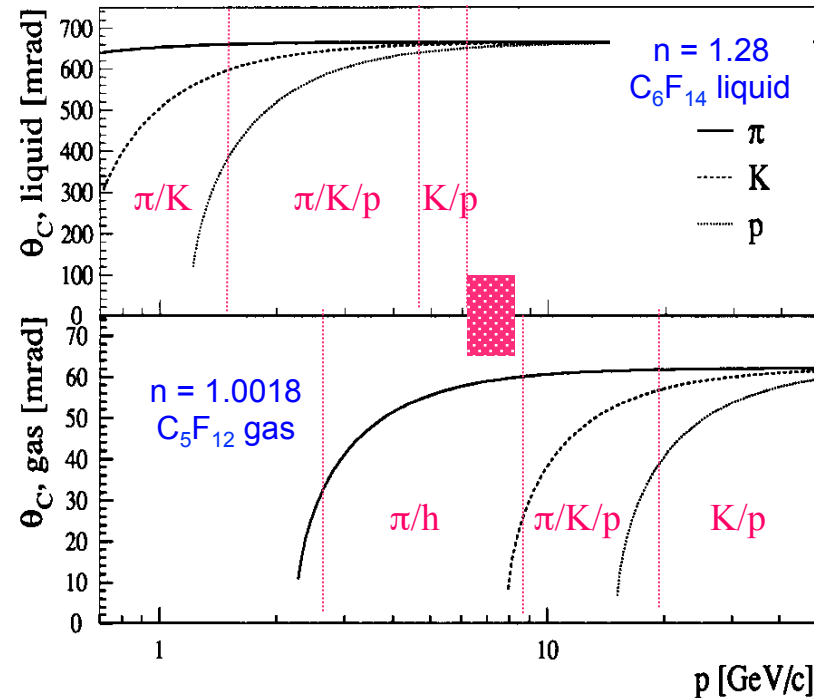
Ca



Ring Imaging Detectors

Particle Identification in DELPHI at LEP I and LEP II

→ $0.7 \leq p \leq 45 \text{ GeV/c}$
 → $15 \leq \theta \leq 165 \text{ mrad}$

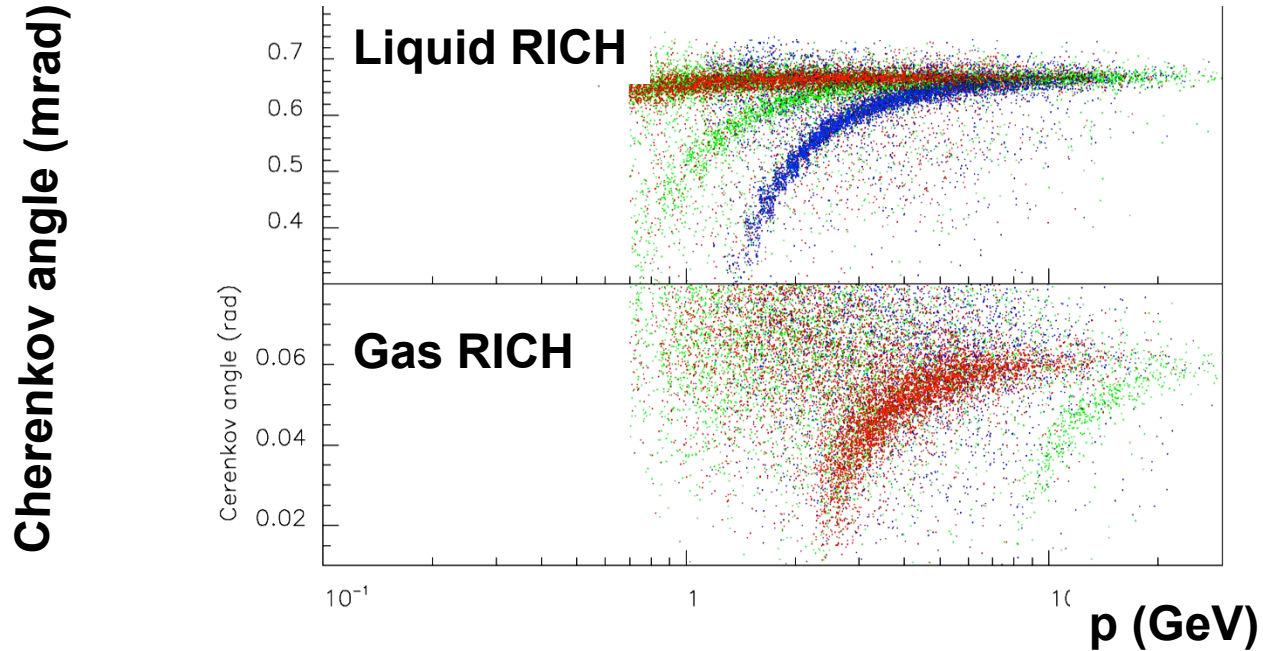


Liquid RICH

Gas RICH

2 radiators + 1 photodetector

Ring Imaging Detectors



From data

p from Λ

K from Φ D^*

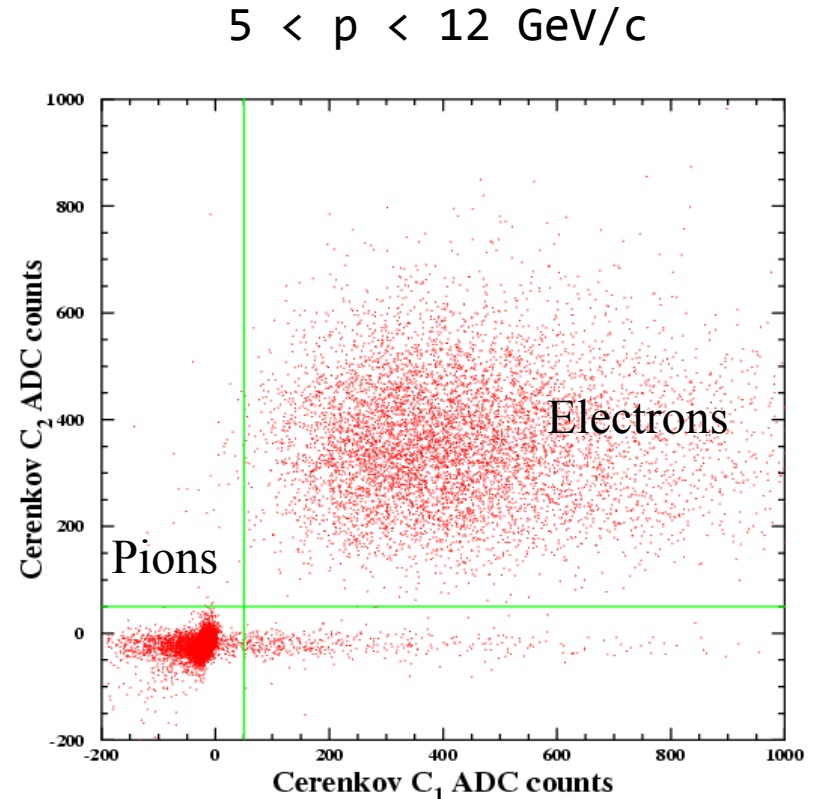
π from K^0

Particle ID

- Momentum and speed differ based on the mass of the particle. The nbr. of photons emitted goes as

$$N \approx 1 - \frac{1}{n^2 \beta^2}$$
$$= 1 - \frac{1}{n^2} \cdot \left(1 + m^2/p^2\right)$$

- Beam magnets can select a fixed momentum.
- Cherenkov counters can identify particles by mass.



Pions start emission at 17
GeV/c (say, 20 GeV/c)

NA62 in 1 minuto

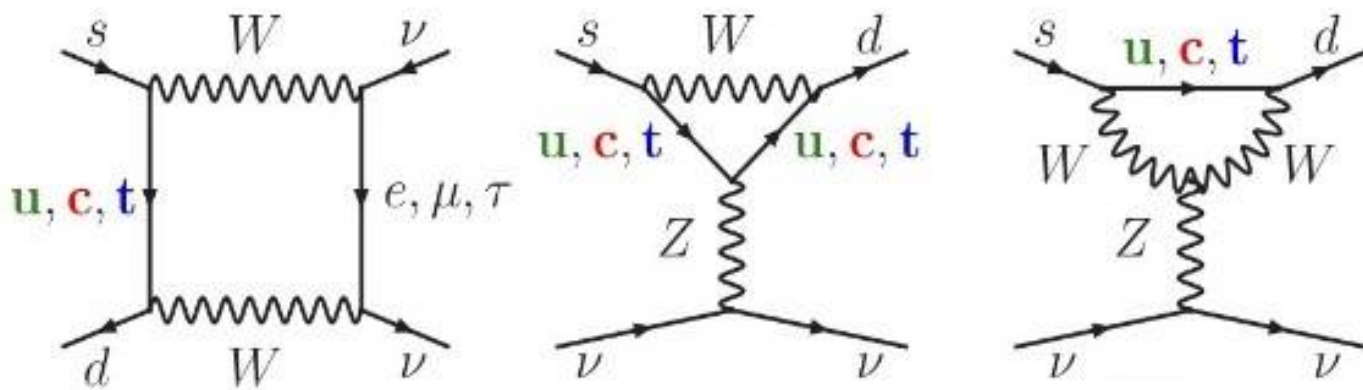
Misura di ~ 100 eventi del decadimento ultra-raro $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$$BR_{\text{exp}} = (1.73^{+1.15}_{-1.05}) \cdot 10^{-10}$$

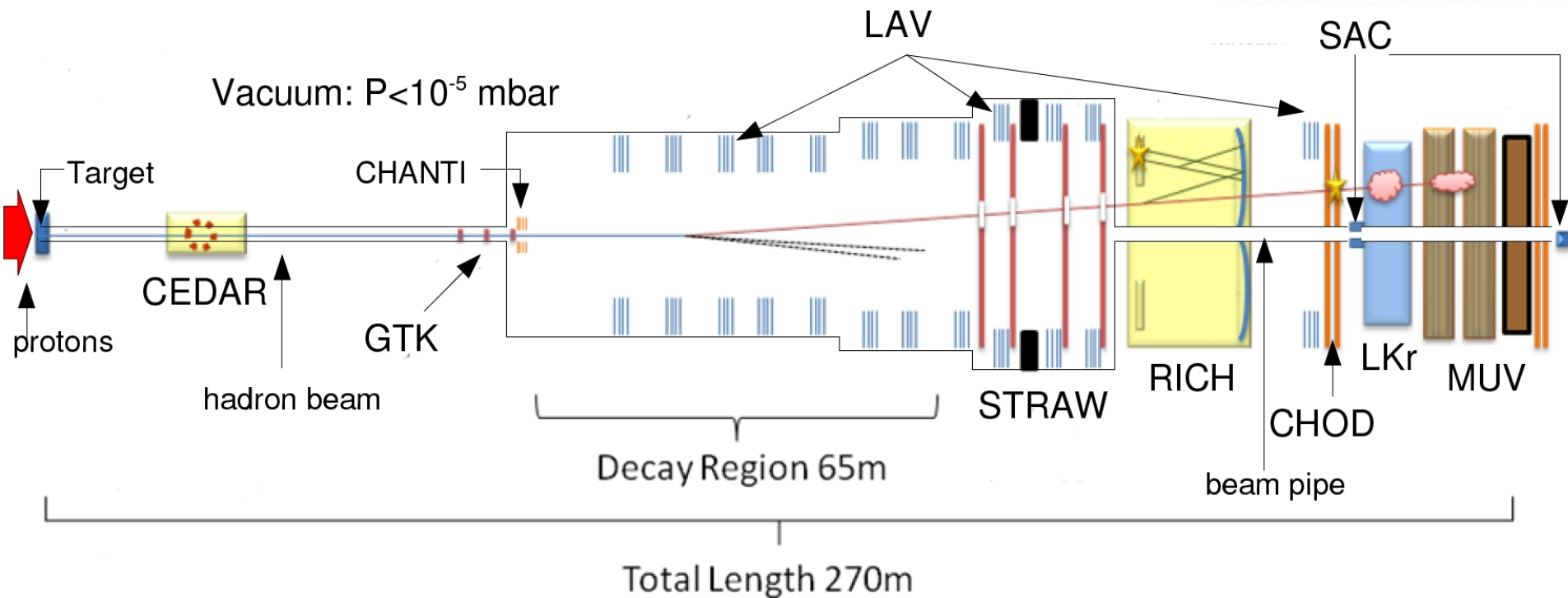
$$BR_{\text{th}} = (0.78 \pm 0.08) \cdot 10^{-10}$$

Estrema precisione, estrema sensibilità a NP

Nuova tecnica: decadimento in volo ad alta energia



Schema del rivelatore



- Decadimenti di Kaoni in volo da un fascio “non separato” a **75 GeV/c**, prodotto da un fascio di protoni a 400 GeV/c estratto dall'SPS contro un bersaglio fisso di berillio (fascio a **~800 MHz**, **~6% kaons**)
- Le particelle non decadute viaggiano nel tubo a vuoto centrale
- Goal: misura di **O(100)** eventi in 2 anni di presa dati riducendo l'errore sistematico fino al livello di **qualche %**

Background suppression

Signal: $BR_{TH}(K^+ \rightarrow \pi^+ \nu \nu) = (0.85 \pm 0.07) \times 10^{-10}$ SM@NLO

❖ Main background:

$K^+ \rightarrow \mu^+ \nu$ ($K_{\mu 2}$) BR = 63.4%

❖ Rejection factor at least 10^{-12}

❖ Kinematics : $\sim 10^{-5} \rightarrow$ Straw tracker + GTK

❖ Veto for muons $\sim 10^{-5} \rightarrow$ Muon Veto

❖ Particle Identification:

μ suppression $< 5 \times 10^{-3}$



RICH

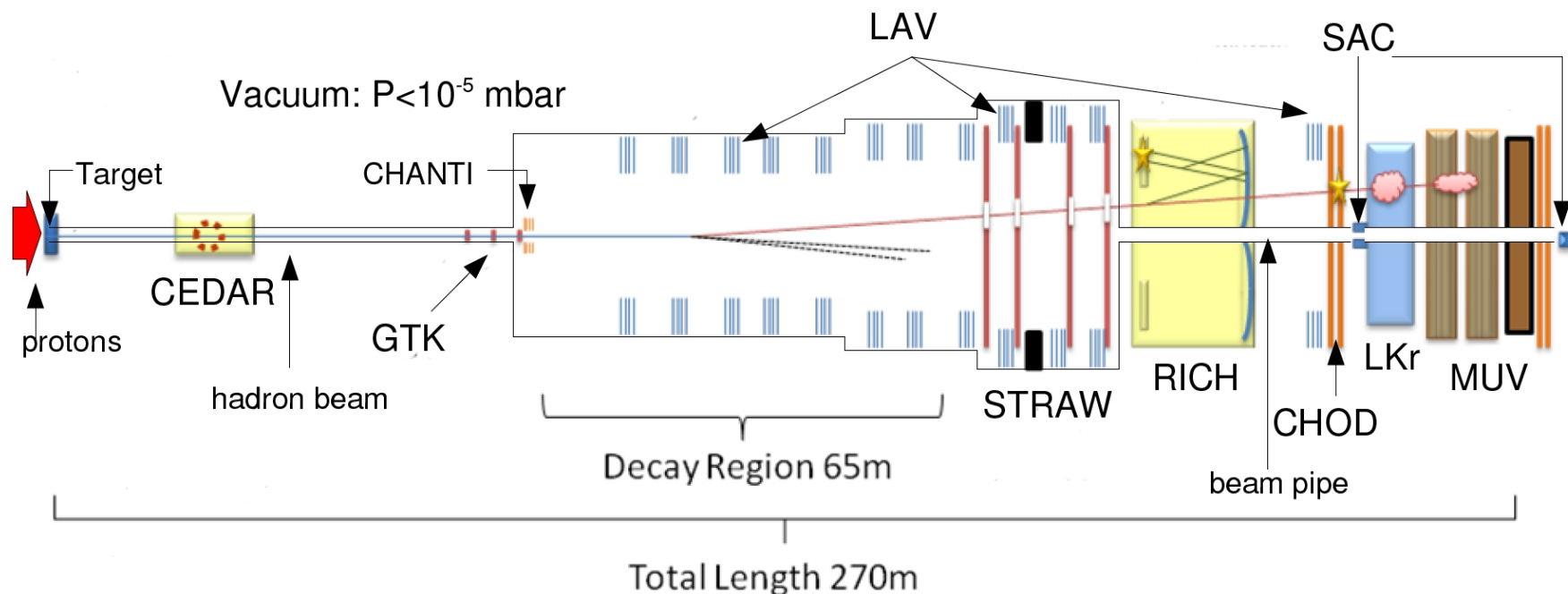


❖ Precise K- π timing

time resolution: ≤ 100 ps

✗ Impossibile visualizzare l'immagine. La memoria del computer potrebbe essere insufficiente per aprire l'immagine oppure l'immagine potrebbe essere danneggiata. Riavviare il computer e aprire di nuovo il file. Se viene visualizzata di nuovo la x rossa, potrebbe essere necessario eliminare l'immagine e inserirla di nuovo.

Decay	BR
$\mu^+ \nu$ ($K_{\mu 2}$)	63.5%
$\pi^+ \pi^0$ ($K_{\pi 2}$)	20.7%
$\pi^+ \pi^+ \pi^-$	5.6%
$\pi^0 e^+ \nu$ ($K_{e 3}$)	5.1%
$\pi^0 \mu^+ \nu$ ($K_{\mu 3}$)	3.3%



The kaons are produced by protons from the CERN SPS hitting a fixed target. Down-stream of the target, an achromatic system selects charged particles with 75 GeV/c momentum. These particles (of those about 6% are kaons) are tagged and measured by a silicon pixel detector, working at a rate of 1 GHz (GigaTracker). The kaons decay then in flight in a 80 m long vacuum vessel. The pion track is measured in a spectrometer, made from 4 straw chambers, two before and two after a dipole magnet. To reduce multiple scattering, the straw tubes are operated in vacuum. Background arises from mainly two decay channels: $K^+ \rightarrow \pi^+ \pi^0$ and $K^+ \rightarrow \mu^+ \nu$, which together are responsible for about 87% of all kaon decays. For suppression of the $\pi^+ \pi^0$ decay, several sets of photon veto-counters reject all events with additional photons. A large angle veto (LAV), made from several rings of lead glass scintillator around the vacuum vessel, detects photons which leave the detector on the outside. The main photon rejection is performed by a liquid-krypton calorimeter (LKr), already used in the NA48 experiment, with excellent energy resolution and granularity. Finally, calorimeters at the end of the beam pipe detect photons which are emitted along the beam line. The rejection of $K^+ \rightarrow \mu^+ \nu$ decays is partly done kinematically by the momentum measurement in the straw chambers. However, to achieve the required rejection power of about 10^{11} , we need a good muon-pion identification in addition. This is performed by a RICH detector, the LKr calorimeter and a new muon veto detector, which measures the shower shape of pion and muon showers in matter.

Mauro Piccini – INFN

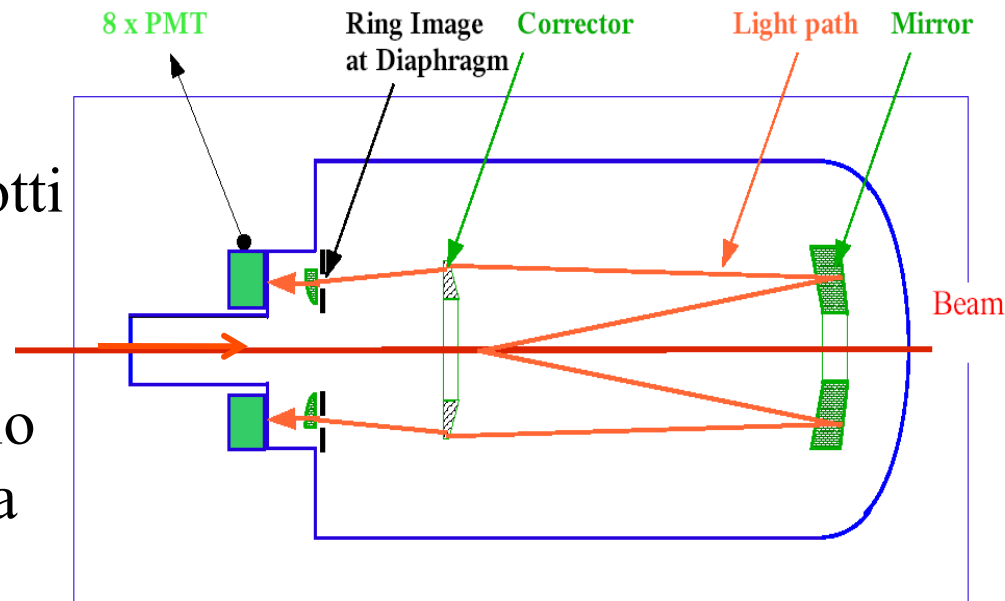
Perugia

E. Fiandrini Rivelatori di Particelle

PID dei K del fascio



Scopo: Identificazione del Kaone nel fascio non separato per associarlo temporalmente ai prodotti di decadimento rivelati a valle. Questo permette di rilasciare le condizioni sullo scattering multiplo nel gas residuo presente nella zona di decadimento (è sufficiente ottenere un vuoto a livello di 10^{-5} mbar).



Tecnica: Rivelatore Cherenkov differenziale (il radiatore e H_2)

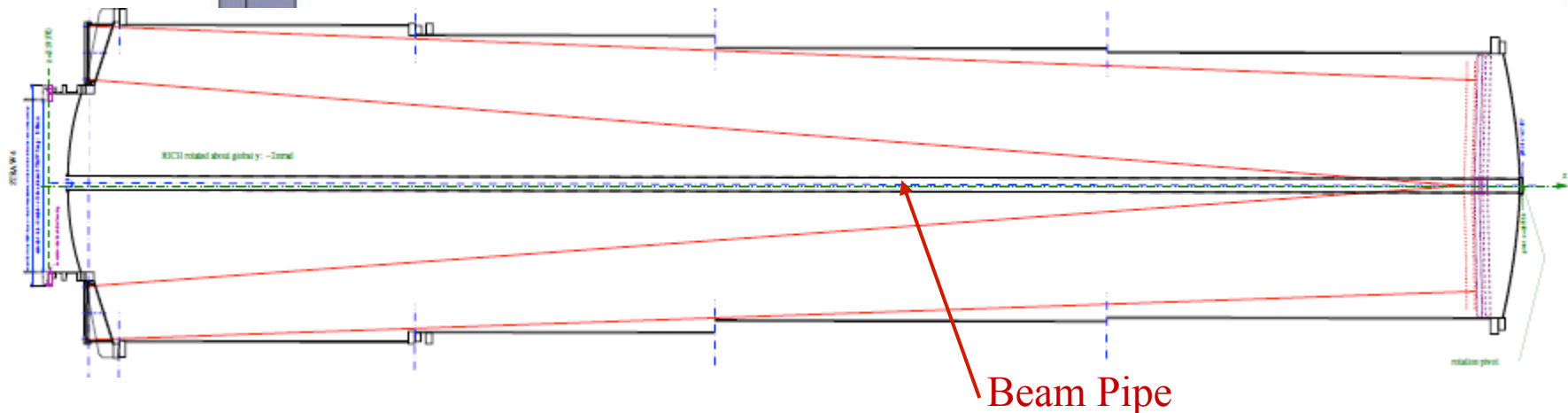
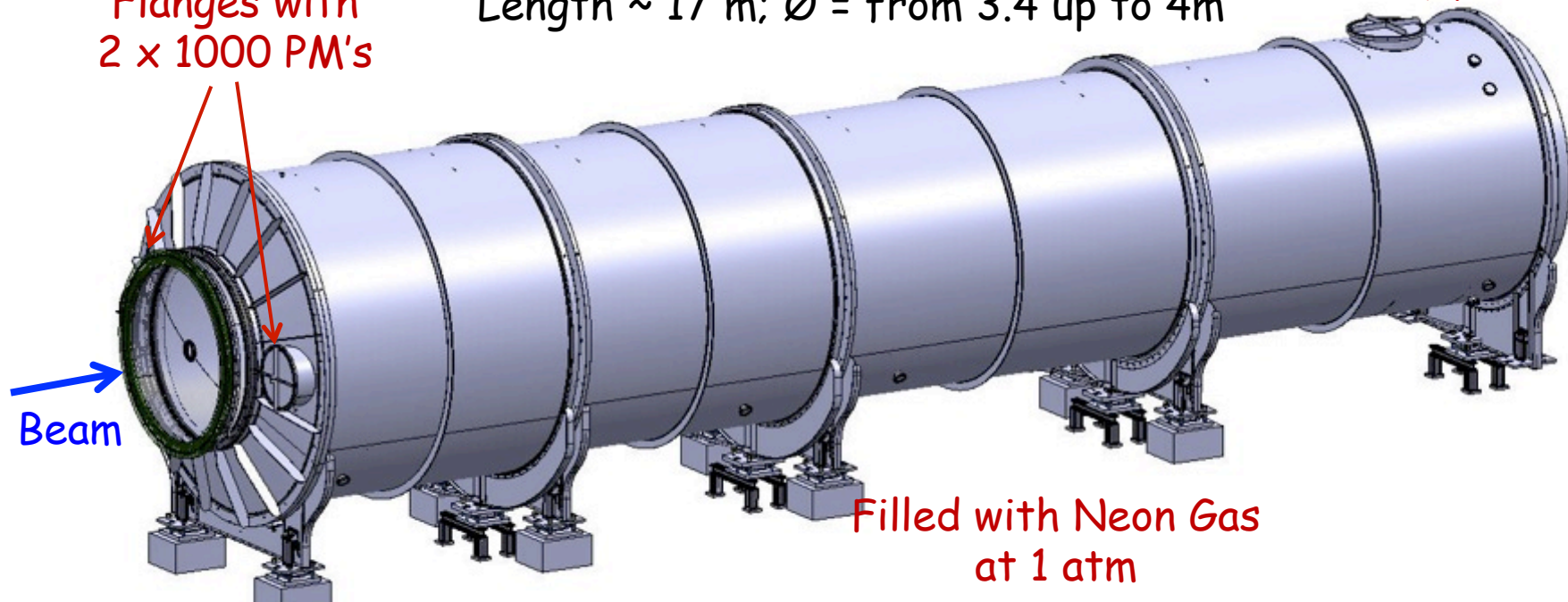
- Riutilizzabile un vecchio rivelatore costruito al CERN negli anni '70
- Nuovo readout (fotomoltiplicatori e elettronica di lettura)
- Nuovo sistema di specchi deflettori per diminuire il rate sul singolo canale in lettura

Il RICH di NA62

Flanges with
2 x 1000 PM's

Length ~ 17 m; $\varnothing$ = from 3.4 up to 4m

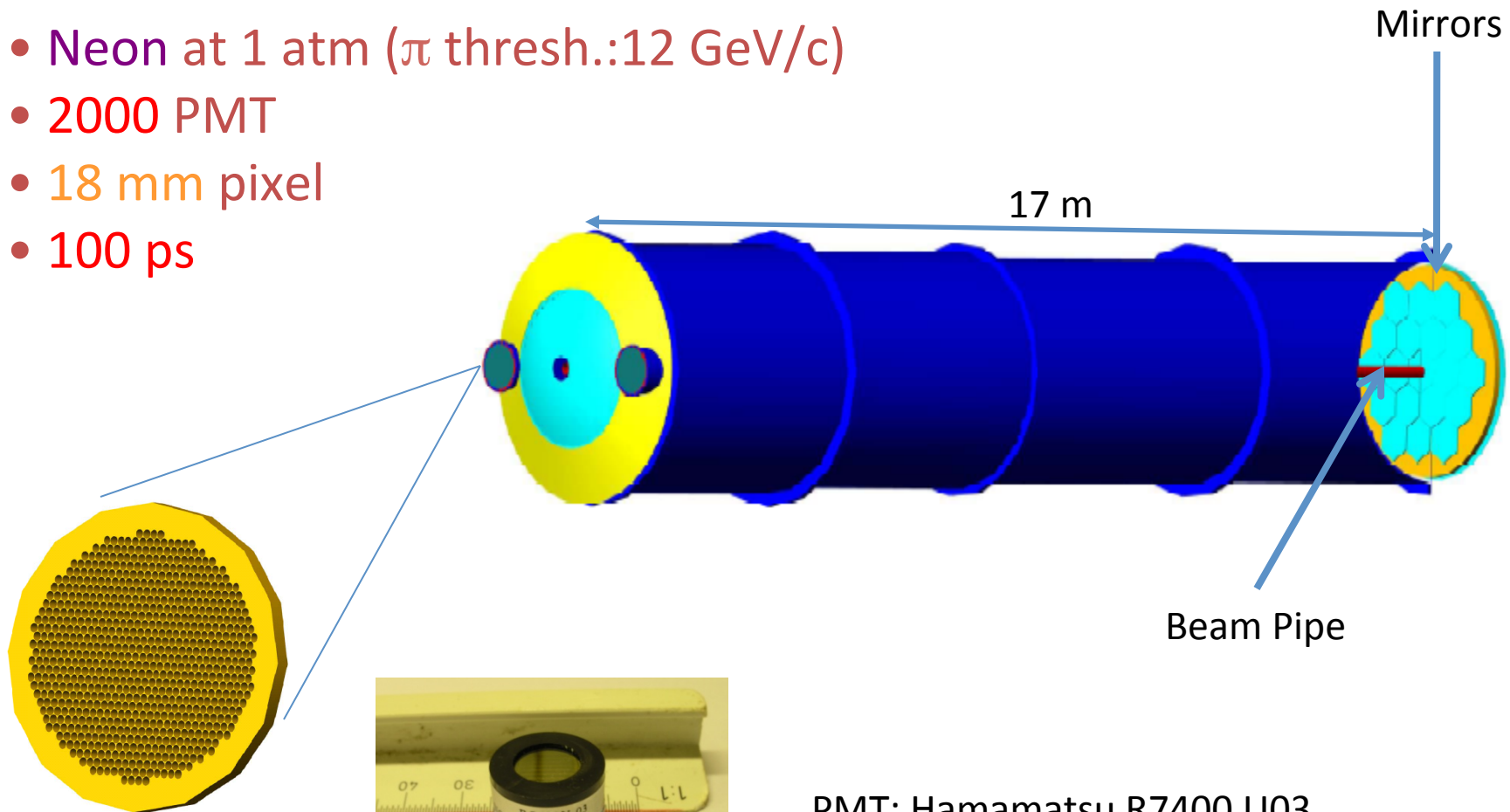
Mirror mosaic



The NA62 RICH

3σ π - μ separation (15-35 GeV/c)

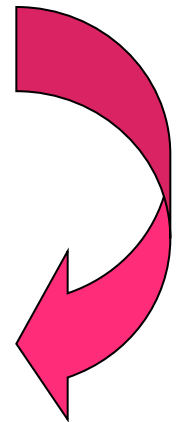
- Neon at 1 atm (π thresh.: 12 GeV/c)
- 2000 PMT
- 18 mm pixel
- 100 ps



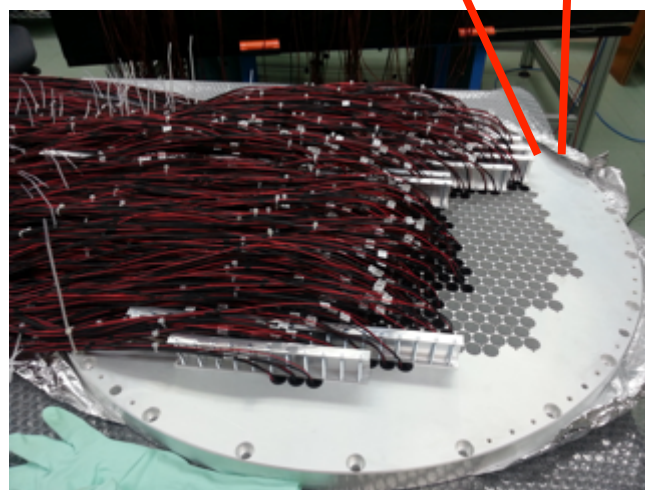
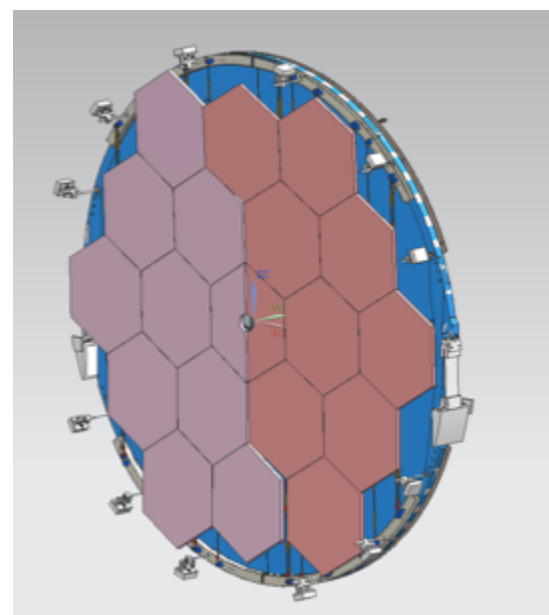
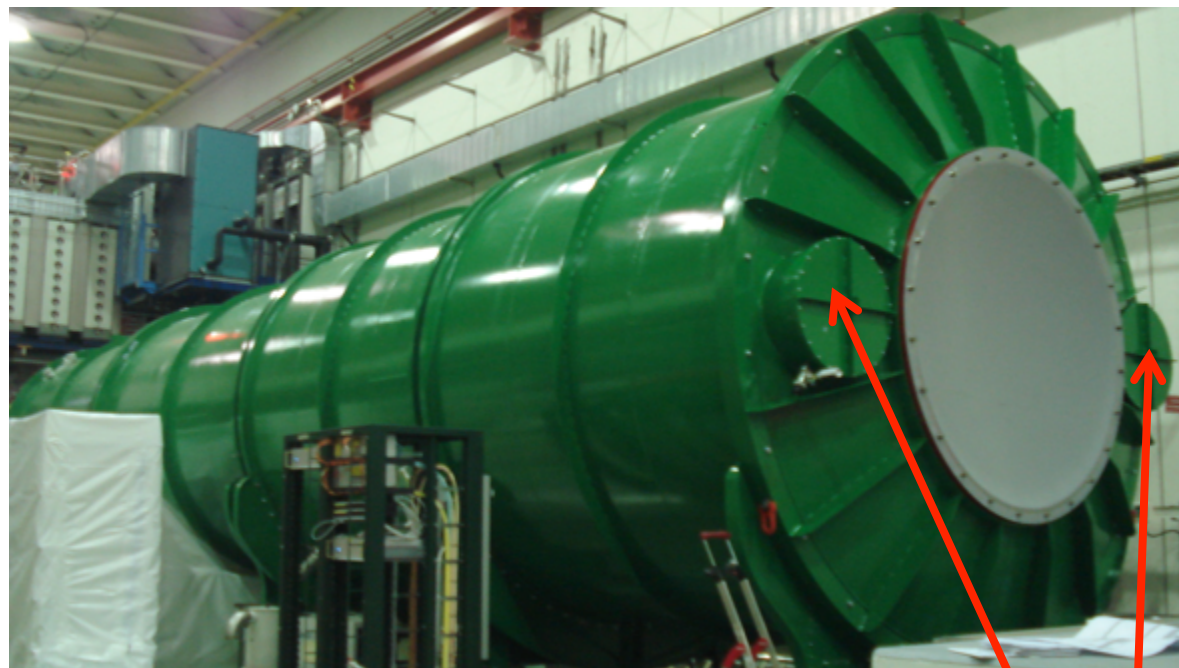
PMT: Hamamatsu R7400 U03

- Separate π - μ in $15 < p < 35$ GeV/c with a muon suppression factor better than 5×10^{-3}
- Measure pion crossing time with a resolution < 100 ps
- Provide a L0 trigger for charged tracks

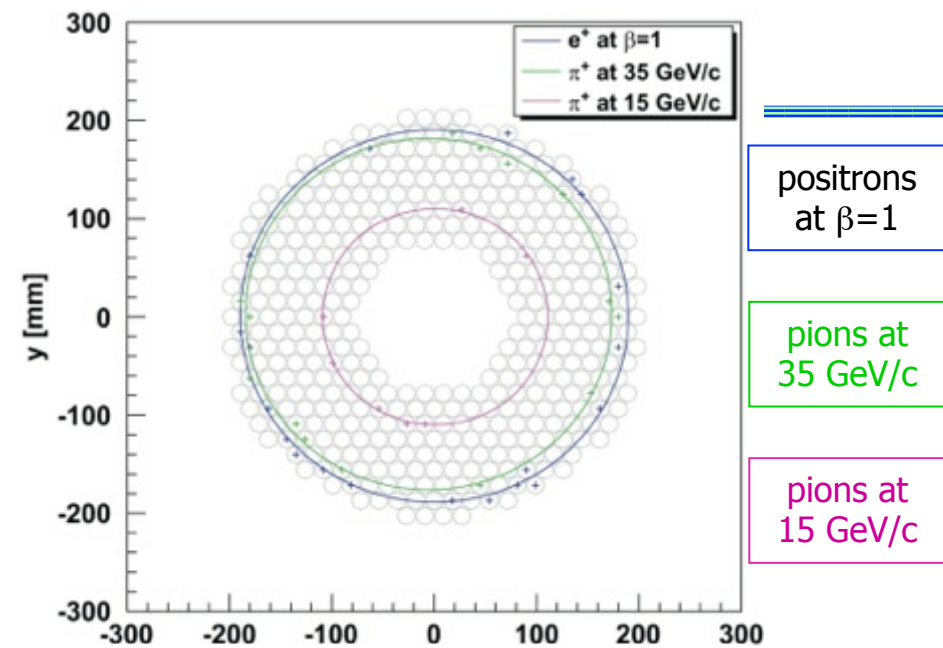
Radiator: Neon gas at atmospheric pressure



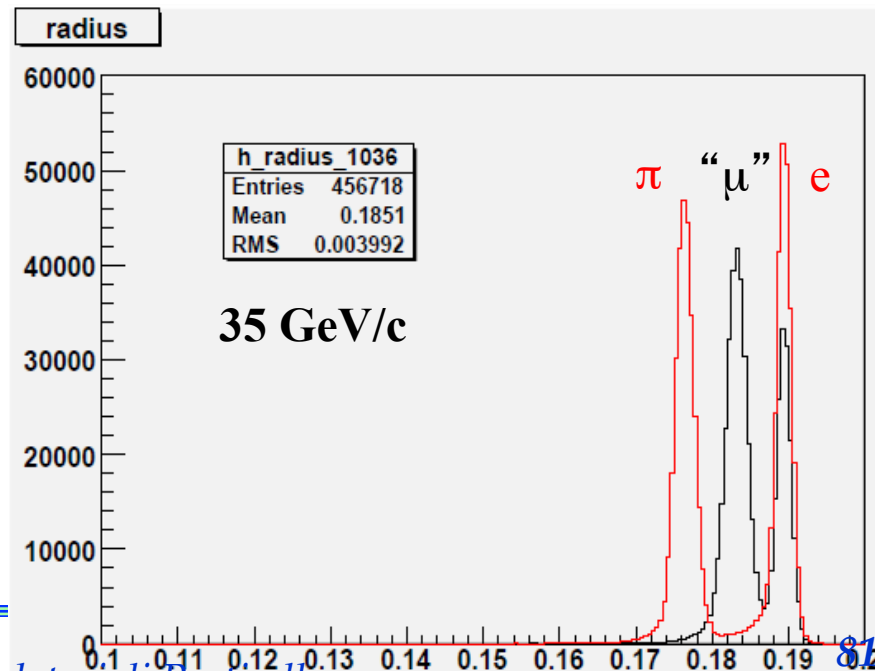
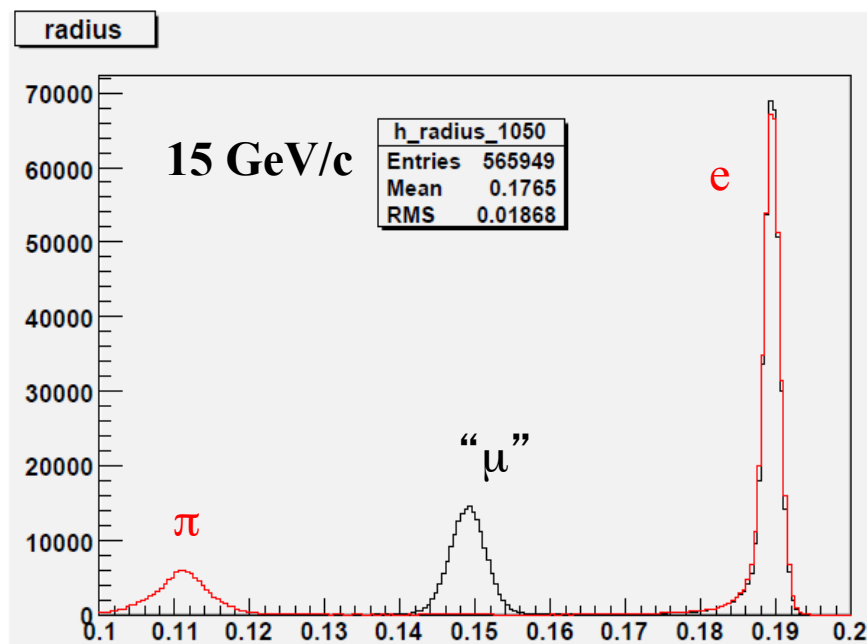
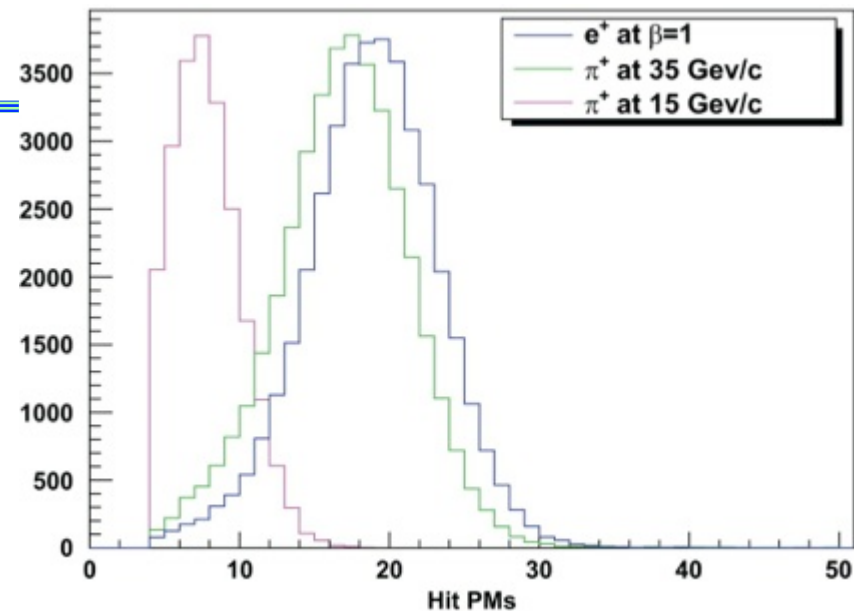
- ★ full efficiency requires $\rightarrow p_{\text{threshold}} = m / \sqrt{(n^2-1)} = 12.5$ GeV/c for π
- ★ $(n-1) = 62.8 \cdot 10^{-6}$ at $\lambda=300$ nm (small dispersion)
- ★ appropriate refractive index at atmospheric pressure (non-flammable)
- ★ good light transparency in visible and near-UV, low chromatic dispersion
- ★ low atomic number $\rightarrow$ small X_0
- ★ $\theta_{C \text{ max}} = 11.2$ mrad



E. Fiandrini Rivelatori di Particelle

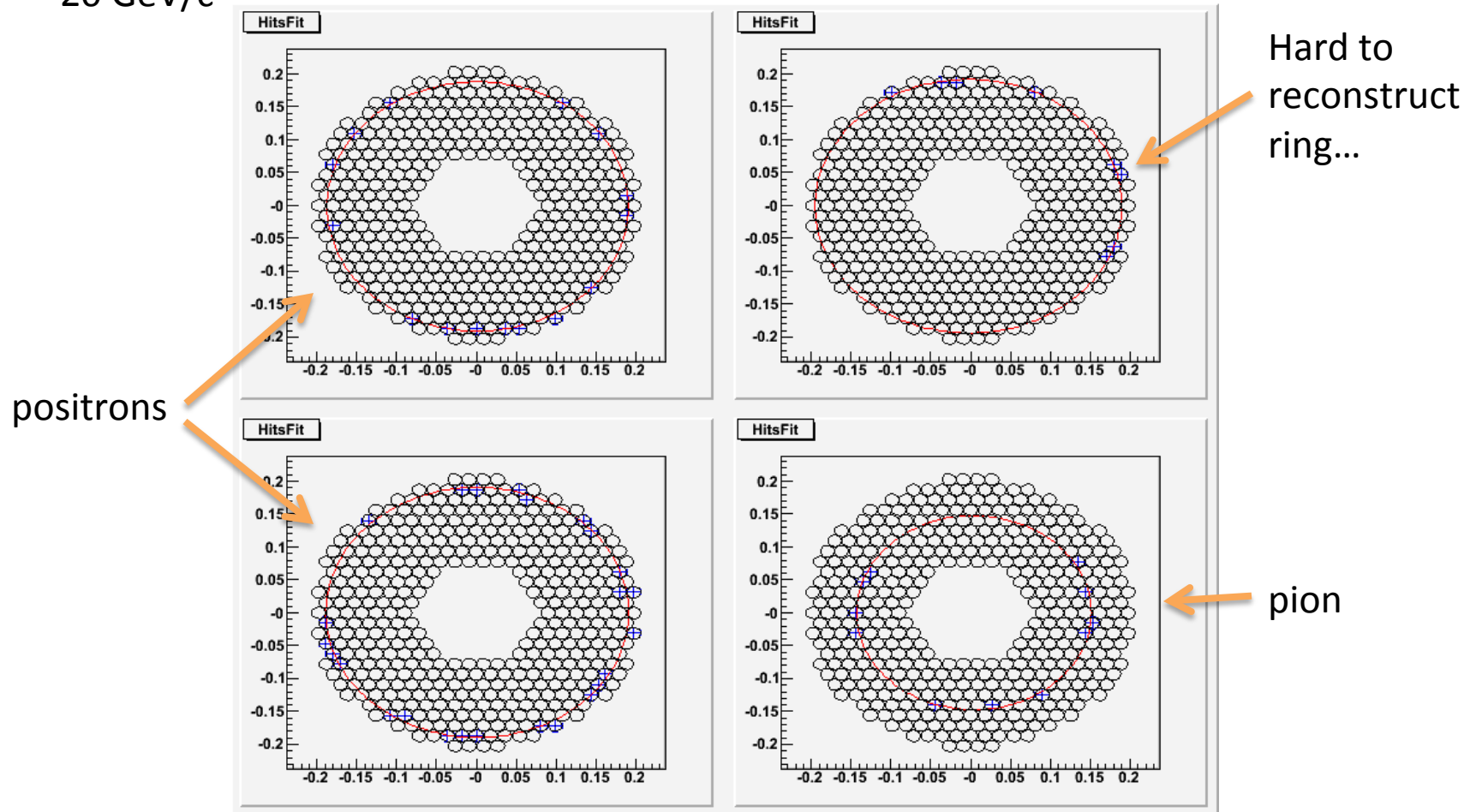


Typical data events with fitted rings



RICH-400: fitted rings

20 GeV/c

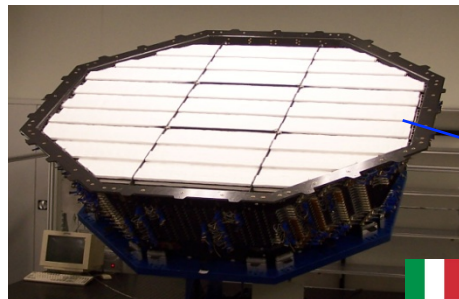


Few photons, no complete rings, difficult to fit, ie bigger uncertainties

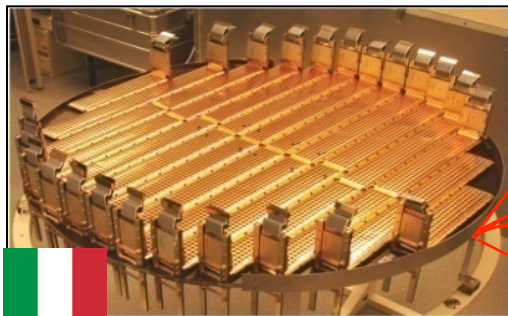


AMS-02

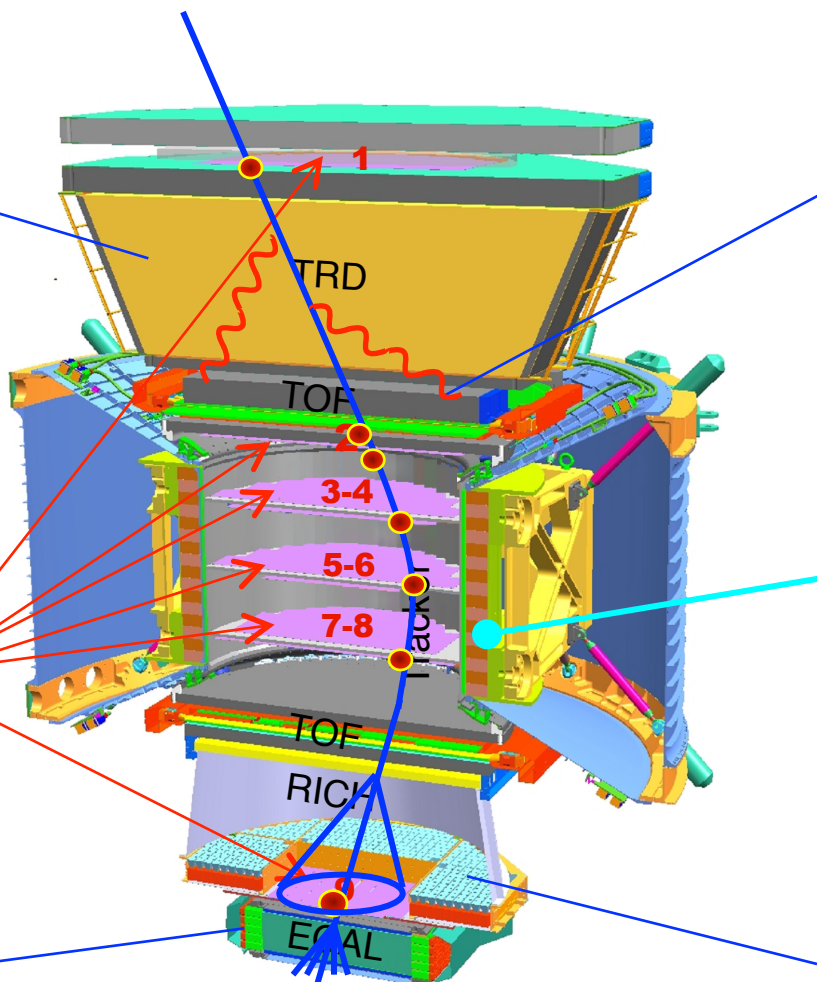
TRD ($|Q|$, e/p)



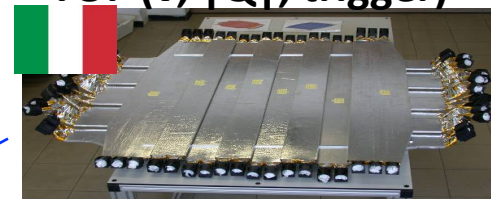
Silicon Tracker ($\pm Q, E$)



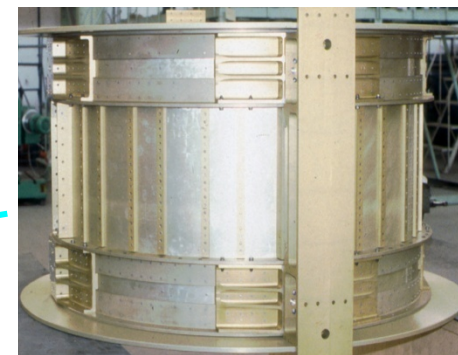
ECAL (E , e/p)



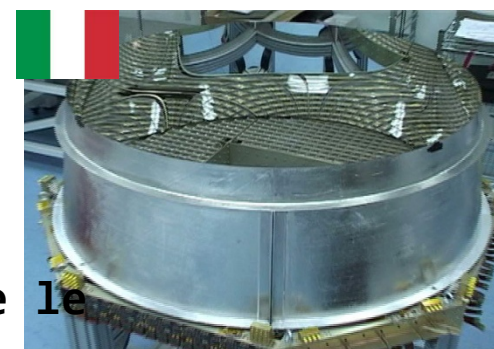
TOF (v , $|Q|$, trigger)



Permanent Magnet



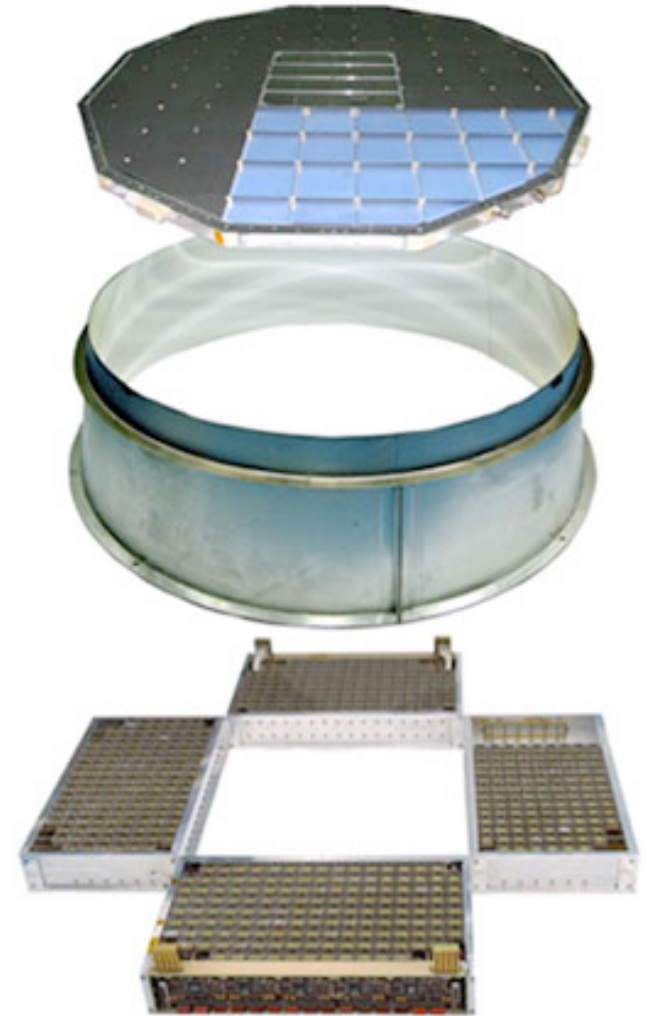
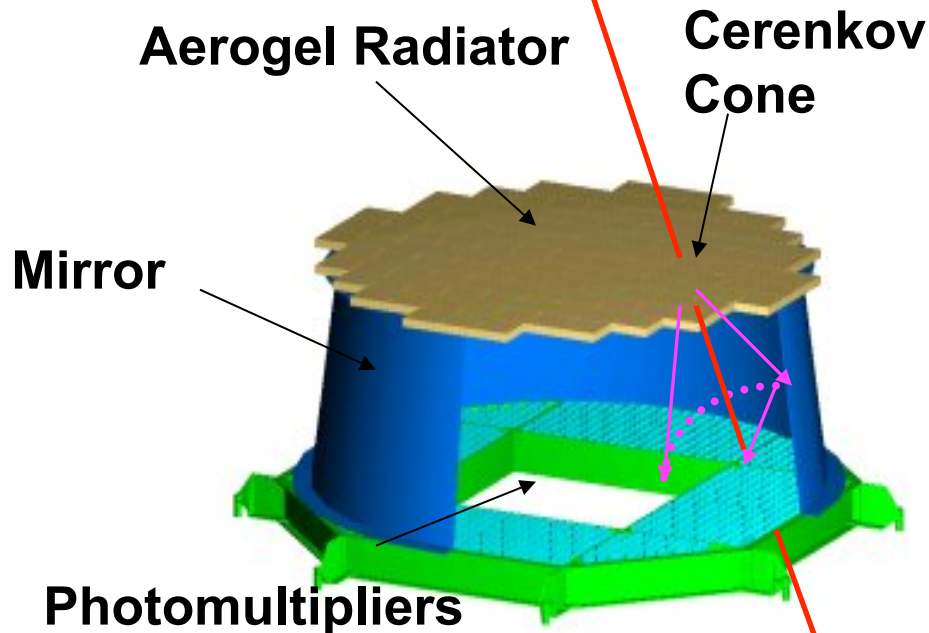
RICH (v , $|Q|$)



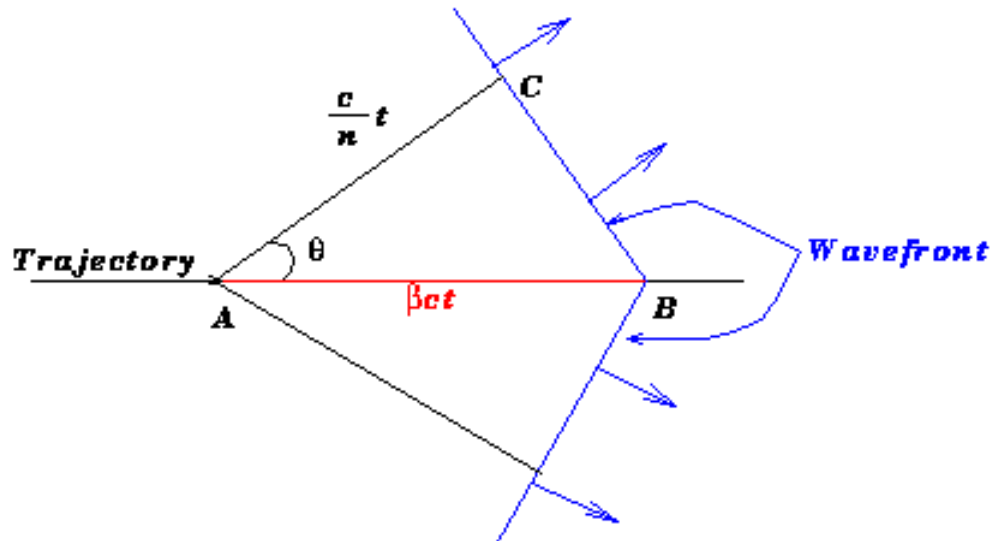
Occorre misurare Q , E , v
 Note v ed E , si ricava m
 Misure multiple per minimizzare le
 incertezze sperimentali

AMS Ring Imaging Cerenkov Counter

- Accurate Velocity Measurements → Isotopic Separation.
- $|Q|$ measurements
- $\Delta\beta/\beta \sim 0.1\%$
- Additional Particle Identification capability



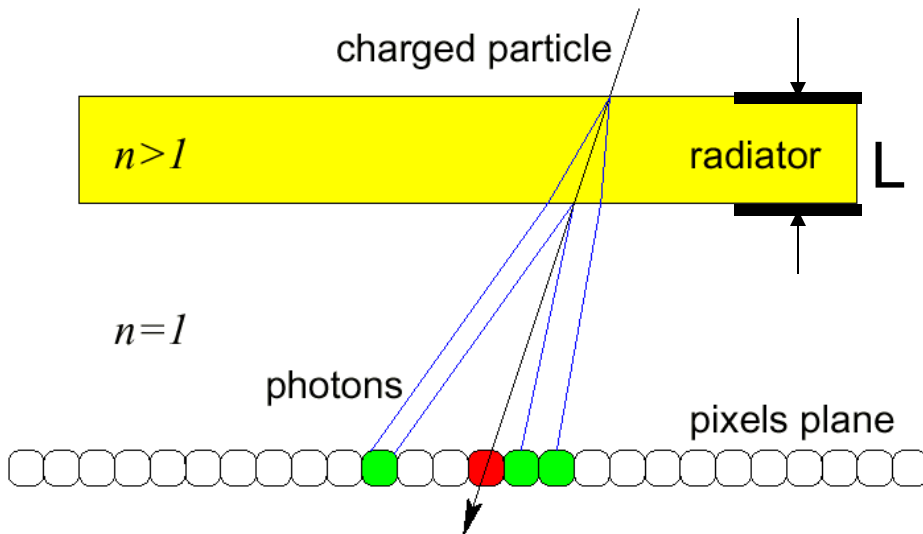
Operation Principle



βc = speed of particle

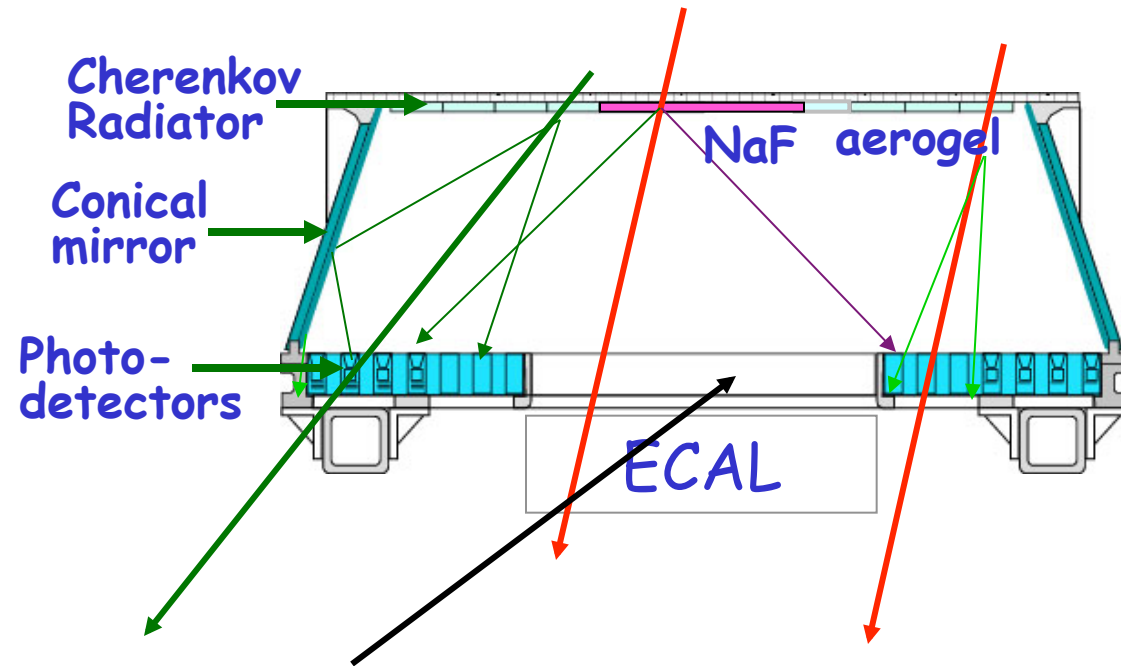
$\frac{c}{n}$ = speed of radiation

The number of emitted photons
is $\propto L Z^2 \langle \sin^2 \theta_c \rangle$



These can be detected with a ph
sensitive detector, e.g. PMT

RICH

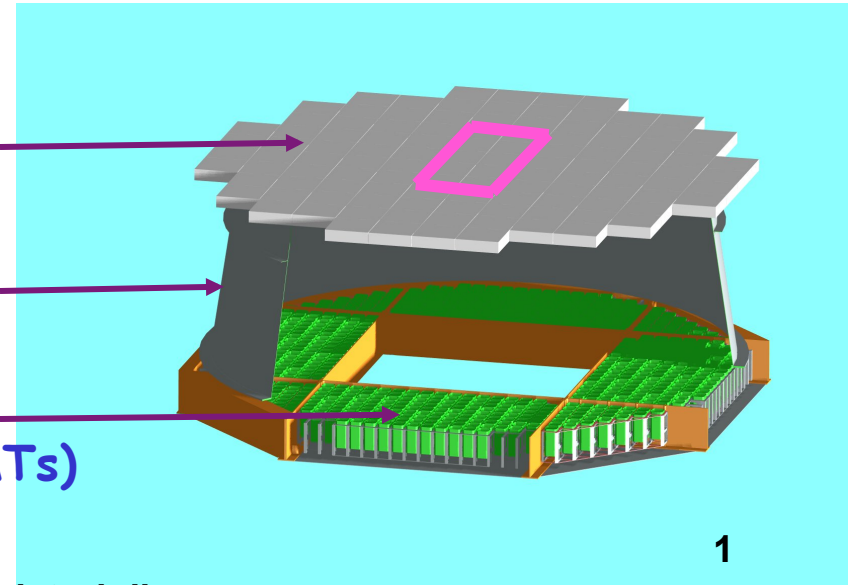


RICH is made by a radiator plane and a photodetector plane.

The conical mirror increases the acceptance for particles incident at large angles for which the Cerenkov photons would miss the detection plane

Because of the ECAL below RICH, the central part of the detector cannot be filled with to avoid material in front of the ECAL

Radiator(s)
Conical mirror
Photodetector plane (680 PMTs)



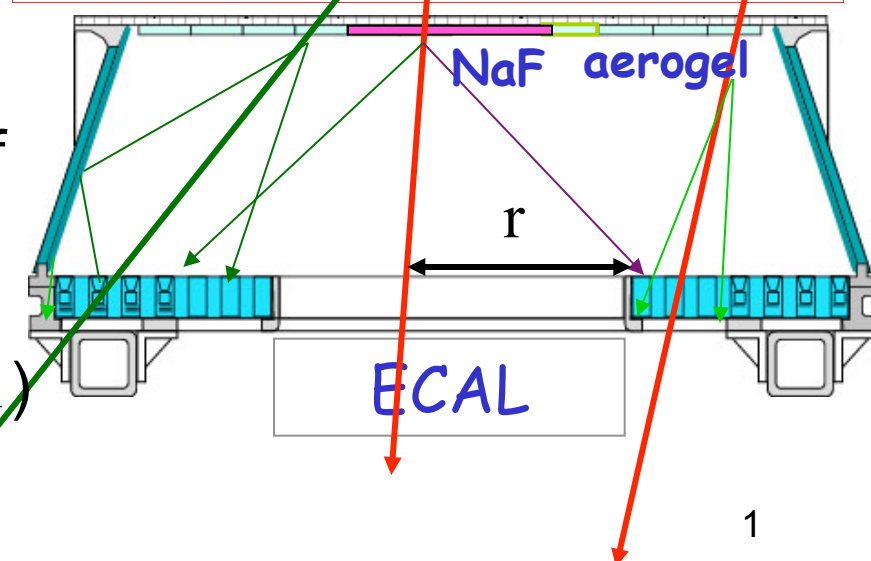
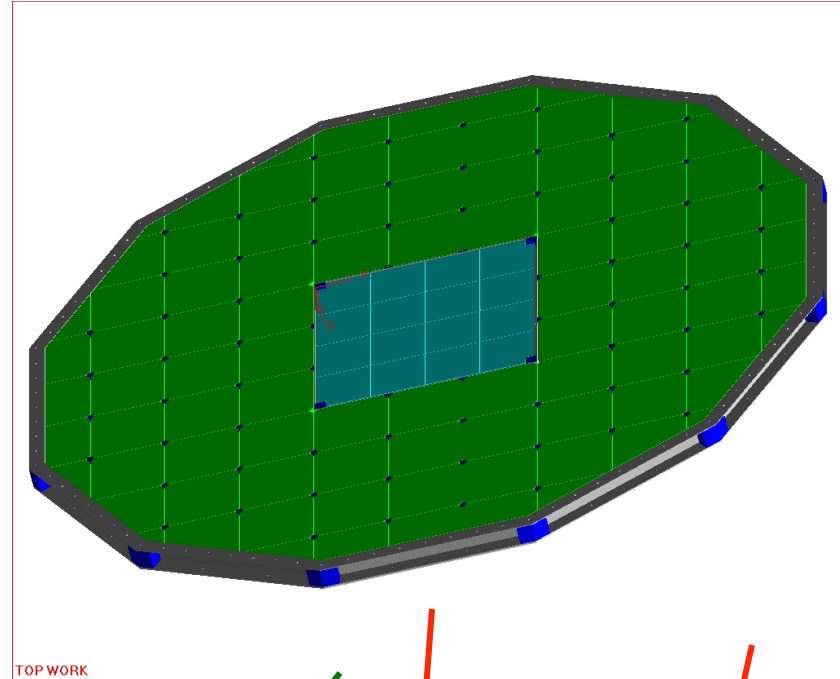
Radiator plane

The radiator plane is made of two different materials:

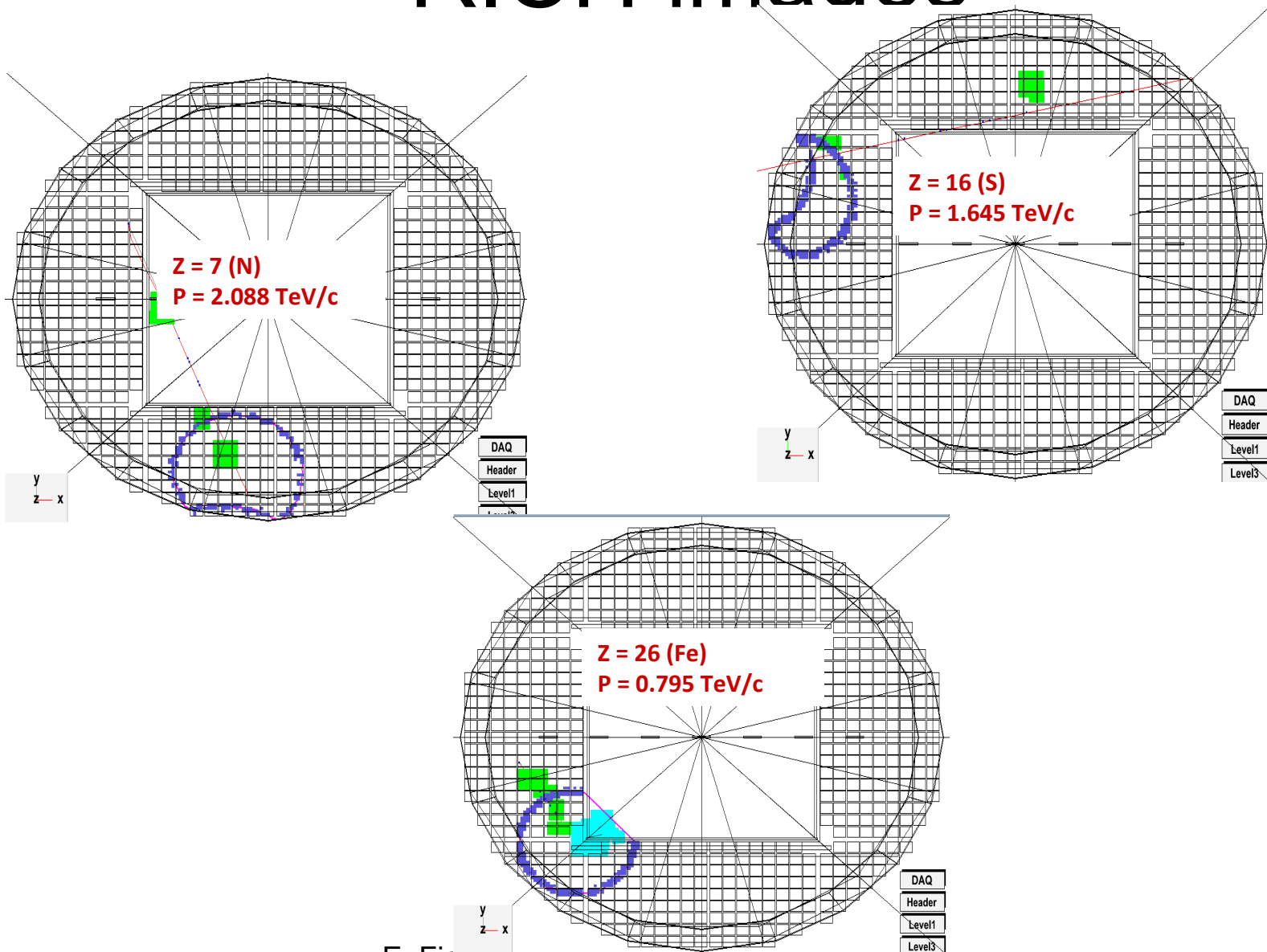
- the central part, above the ECAL hole, is a 5 mm layer of **sodium fluoride NaF** with large refractive index, $n=1.33$
- the outer part is a 30 mm layer of **aerogel** with refractive index close to 1, $n=1.03-1.05$

Particles hitting the NaF, will emit radiation at large angles because of $n_{\text{NaF}} \rightarrow$ it will miss the hole (e.g. for $\beta=1$, $\theta_c=41^\circ$ and $r=L \tan \theta_c=42$ cm)

When hit the aerogel, θ_c (14° @ $\beta=1$) is small and again the radiation will not enter the hole



RICH images

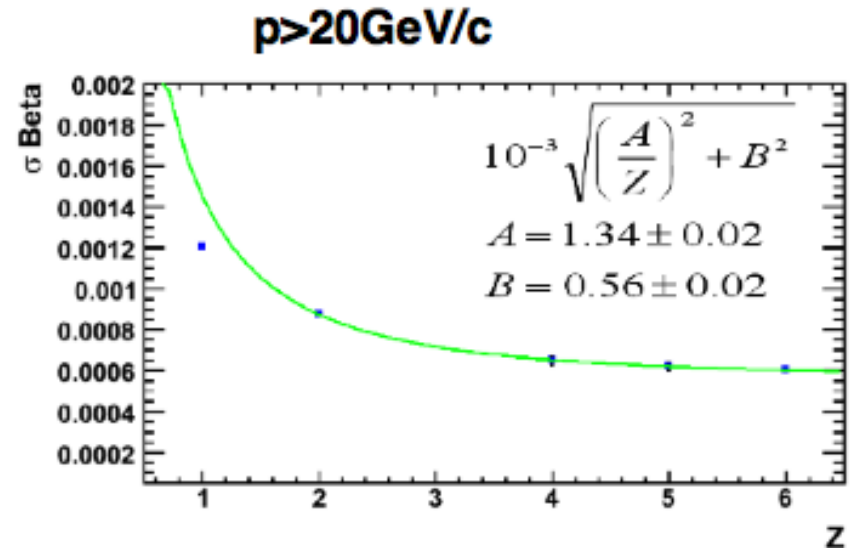
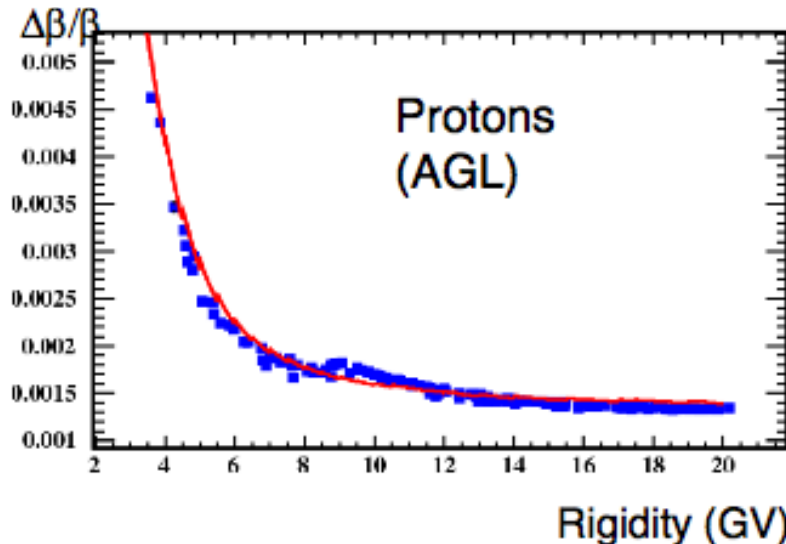


Rich performances

From “rings” radius it is possible to measure β

From the signal intensity it is possible to measure $|Z|$

If the particle does not arrive perpendicular to the radiator plane, the cone projection on the photon detector plane is not a circle, so it is needed to know the particle direction (with an other detector).

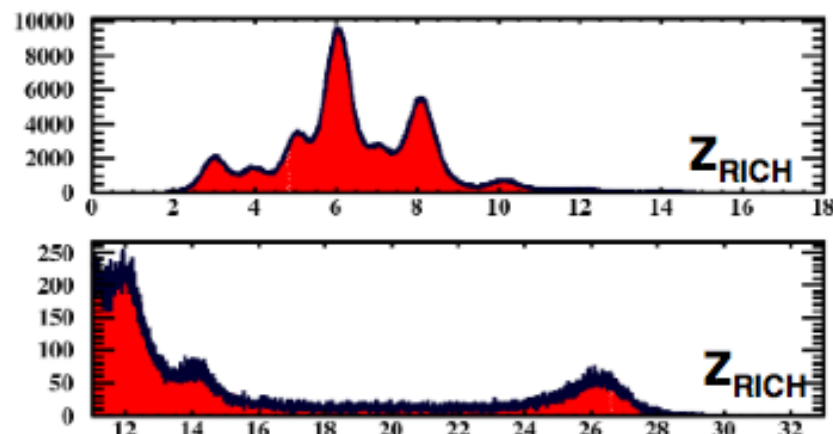


RICH Z measurement (F. Barão & R. Pereira work)

Z uncertainties:

- ✓ statistical
- ✓ systematics from non-uniformities:
 - ✓ radiator: *n*, thickness, clarity, ...
(data base with values)
 - ✓ detection: LG, PMT, temperature effects

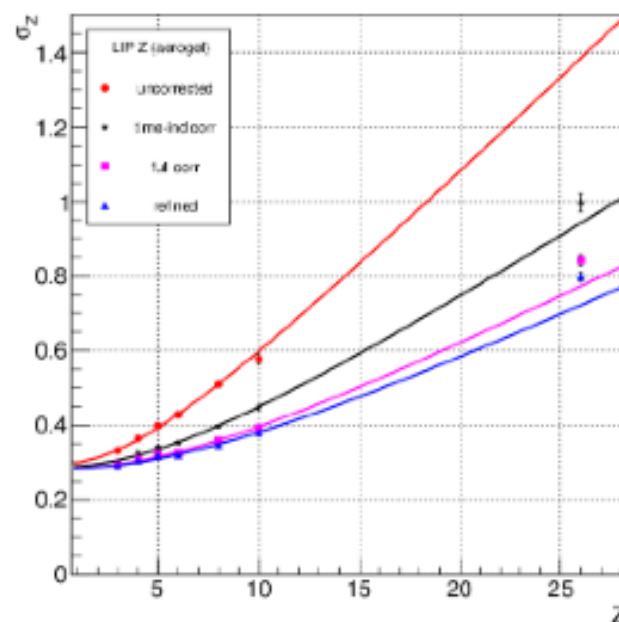
$$\Delta Z = \frac{1}{2} \sqrt{\frac{1 + \sigma_{p.e}^2}{N_0} + Z^2 \left(\frac{\Delta \epsilon}{\epsilon}\right)^2}$$



Charge corrections applied:

- ✓ PMT gain
- ✓ Cell efficiency (light transmission in LG, PMT/pixel quantum eff) **Time independent**
- ✓ Temperature effects (gains and eff) **Time dependent**

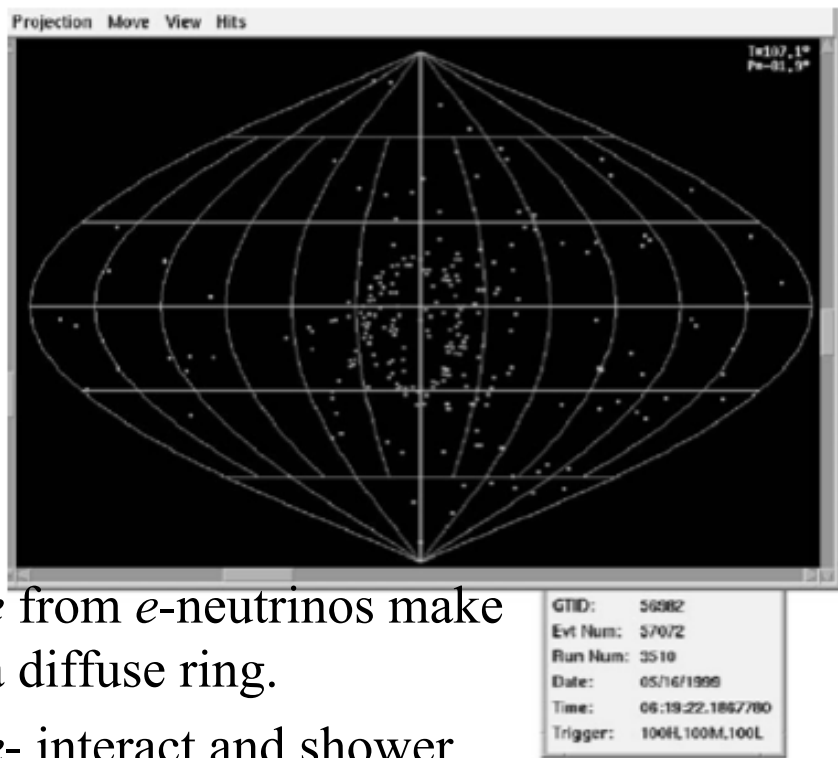
	uncorr	time-ind corr	+time-dep corr	final
σ stat	0.295	0.288	0.283	0.283
σ syst	5.20%	3.45%	2.77%	2.56%



Neutrino ID

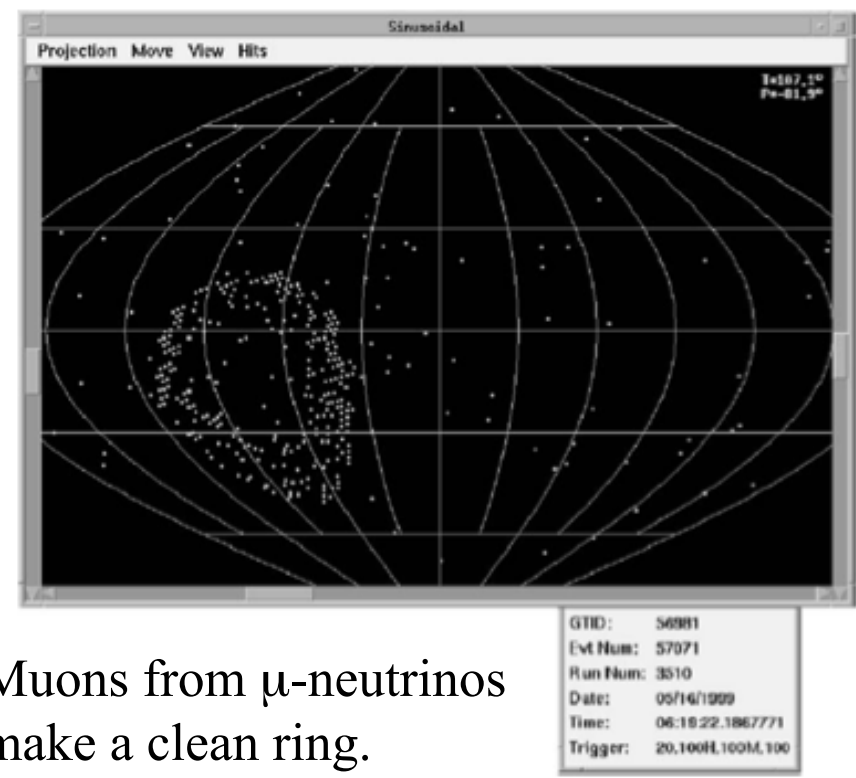
Another example of particle identification by Cherenkov rings comes from neutrino physics.

An important aspect of atmospheric neutrino studies is the correct identification of neutrino-induced muons and electrons. Figures 9.13 and 9.14 show a neutrino-induced event ($\nu\mu + N \rightarrow \mu^- + X$)



e^- from e^- -neutrinos make a diffuse ring.
 e^- interact and shower

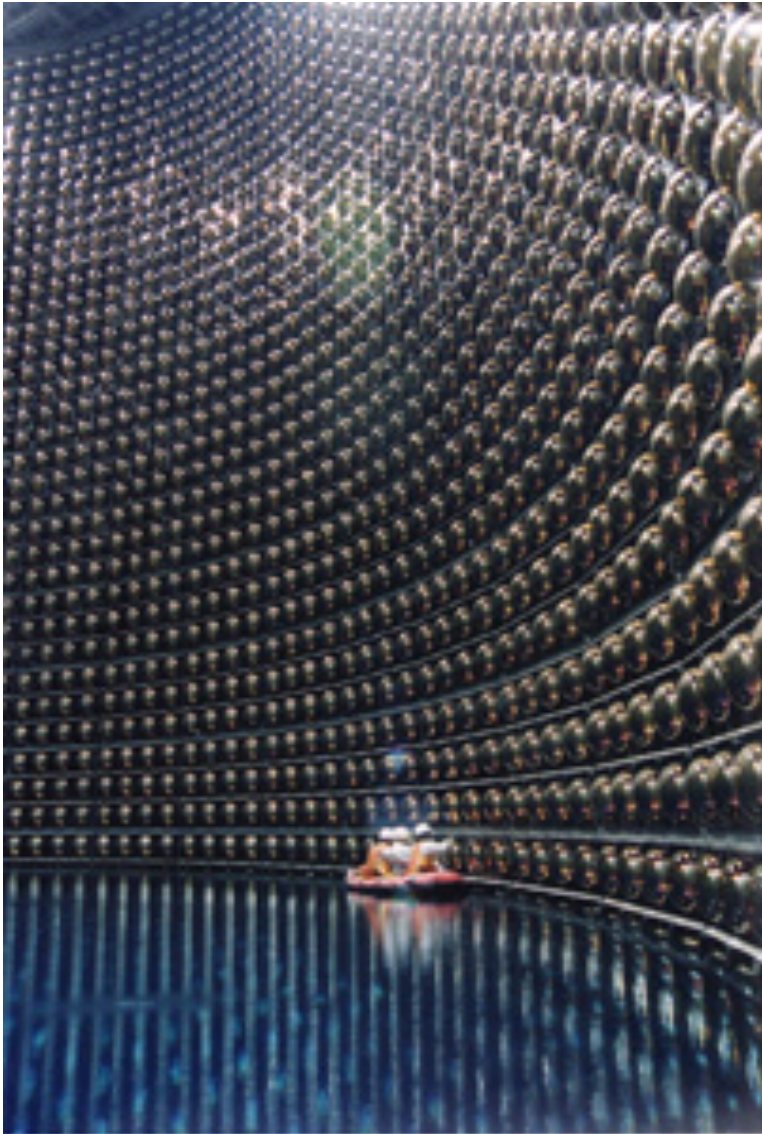
Fig. 9.14. Cherenkov ring produced by an electron from muon decay, where the muon was created by a muon neutrino [53].



Muons from μ -neutrinos make a clean ring.

Fig. 9.13. Neutrino-induced muon in the SNO experiment [53].

with subsequent ($0.9 \mu\text{s}$ later) decay $\mu^- \rightarrow e^- + \nu_e + \nu_\mu$ in the SNO experiment which contains a spherical vessel with 1000 tons of heavy water viewed by 10 000 PMTs. The particle-identification capability of large volume neutrino detectors is clearly seen.



Neutrino ID

- Cherenkov imaging is used in neutrino detectors.
 - Underground observatories

Radiazione di transizione

C'e' un altro fenomeno di emissione importante per la rivelazione delle particelle, che non e' dovuto all'accelerazione della particella ma alla sua interazione con il mezzo: la radiazione di transizione

Transition radiation

- **Transition radiation** is the radiation from a uniformly moving charge in a medium with a variable dielectric constant. Typically, it is produced by relativistic charged particles when they cross the interface of two media of different dielectric constants
- It is analogous to the radiation from a particle moving non-uniformly. In both cases the radiation is related to the phase velocity of the electromagnetic waves in the given medium and the velocity of the charged particle.
- The emission depends on the time variation of $\beta = nv/c$ (n =refractive index)
- The difference is that in the former case the phase velocity of the wave changes, while in the latter particle velocity changes. (In this sense, TR and bremsstrahlung have the same origin.)
- The emitted radiation is the homogeneous difference between the two inhomogeneous solutions of Maxwell's equations of the electric and magnetic fields of the moving particle in each medium separately. In other words, since the electric field of the particle is different in each medium, the particle has to "shake off" the difference when it crosses the boundary.

Radiazione di transizione

La radiazione di transizione è emessa quando una particella carica attraversa la superficie di discontinuità tra due mezzi con indici di rifrazione diversi, e.g. alla superficie di separazione fra il vuoto e un dielettrico.

L'emissione è legata alla polarizzazione del mezzo indotta dai campi della particella incidente: i campi variabili della particella incidente inducono una polarizzazione $P(x,t)$ dipendente dal tempo.

La polarizzazione emette radiazione.

I campi irraggiati si sommano coerentemente vicino alla traiettoria della particella e per un certo spessore nel mezzo, dando luogo ad una radiazione di transizione con una distribuzione angolare e un'intensità caratteristiche

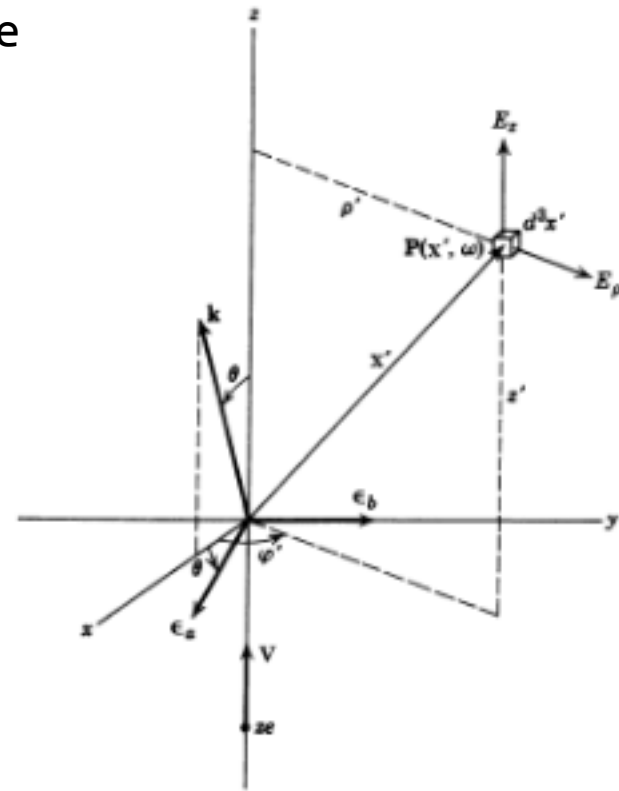


Figure 13.9 A charged particle of charge ze and velocity v is normally incident along the z axis on a uniform semi-infinite dielectric medium occupying the half-space $z > 0$. The transition radiation is observed at angle θ with respect to the direction of motion of the particle, as specified by the wave vector $\mathbf{k}$ and associated polarization vectors $\mathbf{E}_\rho$ and $\mathbf{E}_z$.

- When a charged particle crosses the boundary between two different media, radiation is emitted because the coulomb field of the particle has to readjust itself.
- The fields E_1, H_1 in medium 1 and E_2, H_2 in medium 2, given by the inhomogeneous Maxwell eqns does not match at the interface. In order to satisfy the continuity equations, a solution of the homogeneous eqns. must be added on both sides, that satisfies wave eqn. This is the transition radiation

Radiazione di transizione

Per gli immancabili entusiasti dei calcoli, una trattazione della radiazione di transizione si trova in Jackson : Classical Electrodynamics cap 14 paragrafo 9.

Un trattamento relativistico e' esposto in:
G.Garibian, Sov. Phys. JETP63 (1958) 1079

Qui faremo una trattazione moooolto qualitativa del processo di emissione, basata su quella del Jackson, Elettrodinamica classica (...ma molto piu' semplice e naive di quella)

La polarizzazione del campo a frequenze sopra il visibile (in cui $\epsilon \approx 1$) puo' essere scritta come

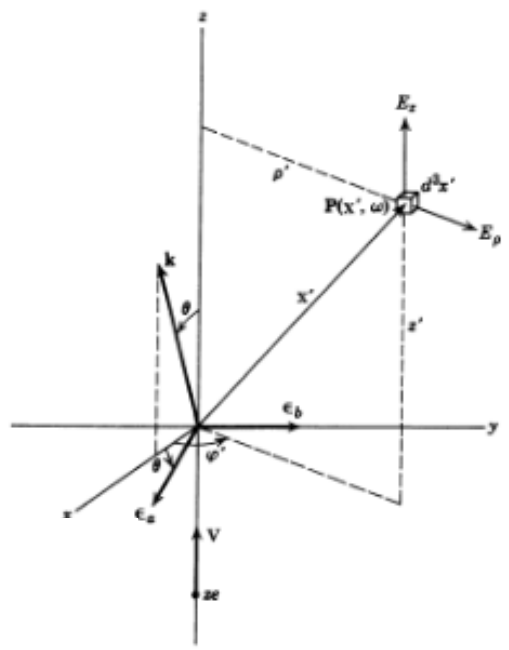
$$\mathbf{P}(\mathbf{x}', \omega) \approx \frac{\epsilon(\omega) - 1}{4\pi} \mathbf{E}_i(\mathbf{x}', \omega) \tag{9}$$

Il campo irraggiato puo' essere ottenuto nell'approssimazione di dipolo

$$\begin{aligned} d\mathbf{E}_{rad} &= \frac{e^{ikR}}{R} (\mathbf{k} \times \mathbf{P}) \times \mathbf{k} \, d^3x' \\ \mathbf{E}_{rad} &= \frac{e^{ikr}}{r} \cdot \frac{\epsilon(\omega) - 1}{4\pi} k^2 \int_{z' > 0} (\hat{\mathbf{k}} \times \mathbf{E}_i) \times \hat{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{x}'} d^3x' \\ &= \frac{e^{ikr}}{r} \cdot \frac{-\omega_p^2}{4\pi c^2} \int_{z' > 0} (\hat{\mathbf{k}} \times \mathbf{E}_i) \times \hat{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{x}'} d^3x' \end{aligned} \tag{10}$$

Se procediamo come nel caso dell'emissione cerenkov la distribuzione della radiazione emessa e'

$$\frac{d^2I}{d\omega d\Omega} = \frac{c}{32\pi^3} \left(\frac{\omega_p}{c}\right)^4 \left| \int_{z' > 0} (\hat{\mathbf{k}} \times \mathbf{E}_i) \times \hat{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{x}'} d^3x' \right|^2 \tag{11}$$



A charged particle of charge ze and velocity $\mathbf{v}$ is normally incident along the z -axis into a uniform semi-infinite dielectric medium occupying the half-space $z > 0$. Radiation is observed at angle θ with respect to the direction of motion, as specified by the wave vector $\mathbf{k}$ and associated polarization vectors $\mathbf{E}_{\parallel}$ and $\mathbf{E}_{\perp}$.

ω_p = frequenza di plasma del mezzo (cfr. poche slide piu' avanti)

Radiazione di transizione

Le caratteristiche principali della RT si possono comprendere qualitativamente senza fare calcoli complicati. Consideriamo il caso di una particella ultrarelativistica con $\gamma \gg 1$ che arriva lungo l'asse z dal vuoto ($z < 0$) e penetra in un mezzo con indice di rifrazione n ($z > 0$), la superficie a $z = 0$ e' quella di separazione dei due mezzi.

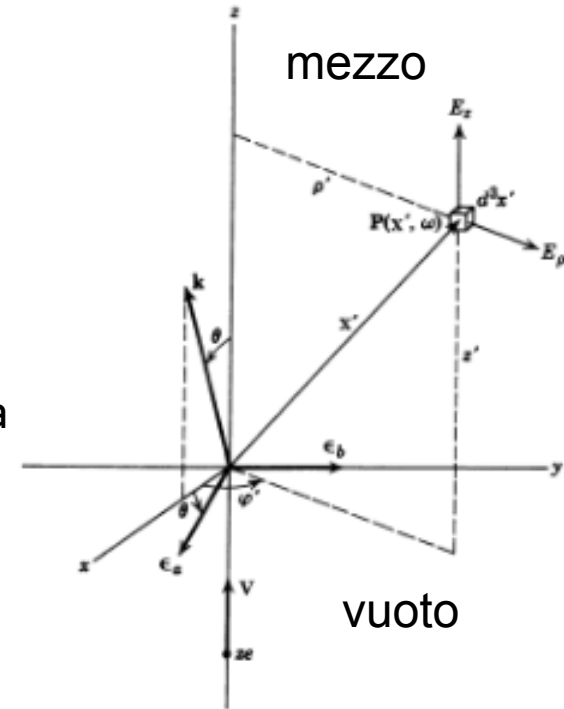


Figure 13.9 A charged particle of charge ze and velocity $\mathbf{v}$ is normally incident along the z axis on a uniform semi-infinite dielectric medium occupying the half-space $z > 0$. The transition radiation is observed at angle θ with respect to the direction of motion of the particle, as specified by the wave vector $\mathbf{k}$ and associated polarization vectors ϵ_a and ϵ_b .

Radiazione di transizione

- a) La distribuzione angolare della radiazione emessa e' confinata in un cono t.c. $\gamma\theta \sim 1$
- b) La radiazione di TR e' il risultato dell'interferenza tra il campo della particella incidente e l'onda irraggiata localmente a causa della polarizzazione dipendente dal tempo del mezzo.

Se decomponiamo la corrente in comp. Di fourier, la componente alla freq. ω del campo generato dalla particella ha una componente la cui ampiezza lungo z oscilla come (cfr. cherenkov)

$$\sim \exp(i\omega z/v)$$

La polarizzazione del mezzo emette onde con un'ampiezza che va come

$$\sim \exp(-ikz) = \exp(-i\omega n z/c)$$

Dato che

$$\mathbf{P}(\omega, \mathbf{x}') \propto \mathbf{E}_i(\omega, \mathbf{x}'),$$

l'onda emessa va come

$$\sim \exp(i\omega z/v) \exp(-in\omega z/c) \\ = \exp(i\omega z(1/v - n/c))$$

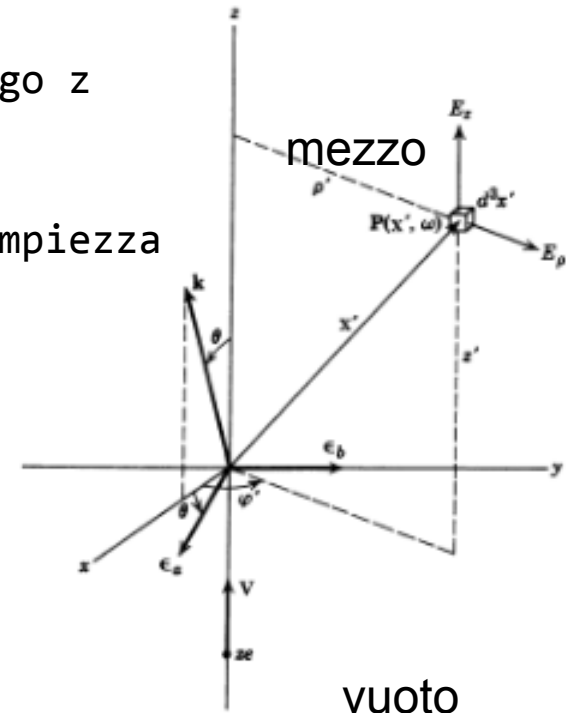


Figure 13.9 A charged particle of charge ze and velocity $\mathbf{v}$ is normally incident along the z axis on a uniform semi-infinite dielectric medium occupying the half-space $z > 0$. The transition radiation is observed at angle θ with respect to the direction of motion of the particle, as specified by the wave vector $\mathbf{k}$ and associated polarization vectors ϵ_0 and ϵ_z .

Radiazione di transizione

Lontano dall'interfaccia fra vuoto e mezzo posta a $z = 0$, ie per "grandi" z , la fase del campo oscilla rapidamente, l'onda pilota (del campo incidente) e quella della polarizzazione non sono coerenti
 $\rightarrow \langle |E| \rangle = 0$, cioe' non c'e' radiazione emessa.

L'unico contributo alla radiazione emessa viene dalla regione in cui la fase del campo non varia apprezzabilmente, cioe' entro uno spessore z t.c.

$$\omega z(1/v - n/c) \ll 1$$

Assumendo freq abbastanza alte (sopra la regione ottica, $\omega \gg \omega_p$), la costante dielettrica e puo' essere parametrizzata in funzione di ω come

$$\epsilon/\epsilon_0 \approx 1 - (\omega_p/\omega)^2$$

con ω_p frequenza di plasma del mezzo.

$\omega_p = (4\pi Ne^2/m)^{1/2} = 1.54 \times 10^4 N^{1/2} = 28.8(\rho Z/A)^{1/2} \text{ eV}$, con N densita' degli e- del mezzo e ρ densita' in gr/cm³ e Z/A e' il rapporto medio nr atomico e di massa del materiale. Nelle sostanze con densita' dell'ordine dell'unita' $\hbar\omega_p/2\pi \approx 20 \text{ eV}$, in aria 0.7 eV

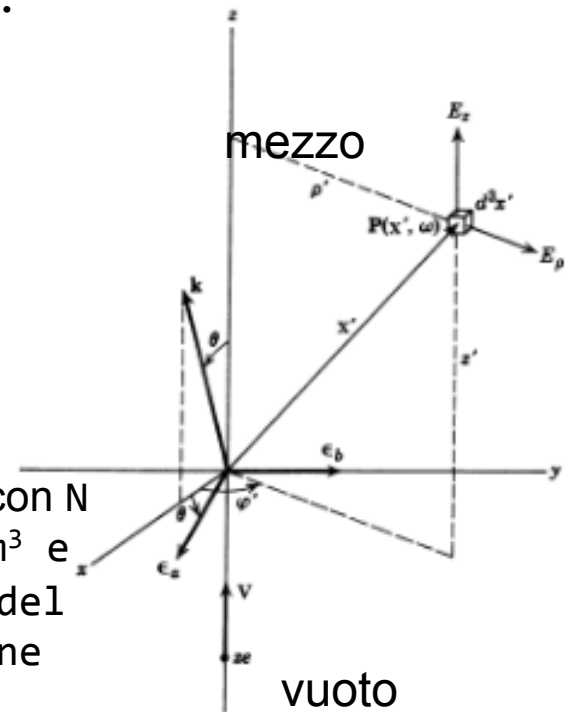


Figure 13.9 A charged particle of charge ze and velocity v is normally incident along the z axis on a uniform semi-infinite dielectric medium occupying the half-space $z > 0$. The transition radiation is observed at angle θ with respect to the direction of motion of the particle, as specified by the wave vector $\mathbf{k}$ and associated polarization $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.

Radiazione di transizione

$$\epsilon \approx 1 - (\omega_p/\omega)^2$$

Nel limite $\omega \gg \omega_p$ $n = (\epsilon)^{1/2} \approx 1 - (1/2)(\omega_p/\omega)^2$

Per $\beta \approx 1$, $\gamma \approx 1/[2(1 - \beta)] \rightarrow 1/\beta = 1 + 1/2\gamma^2$

Sostituisco in $\omega z(1/\beta - n/c) \ll 1$ e ho

$$z \sim 2\gamma/\omega_p(\nu + 1/\nu) \text{ con } \nu = \omega/\gamma\omega_p$$

La lunghezza di formazione D , cioè la profondità in z della regione da cui la radiazione viene emessa può essere definita come il max di $z(\nu)$ (cfr. Jackson).

Il max si ha quando $\nu = 1 \rightarrow$

$$D = z(1) = \gamma c/\omega_p$$

Per mezzi tipici e per particelle ultrarelativistiche $\gamma \sim 10^3$,
 $D \sim$ decine di microns

Radiazione di transizione

L'energia totale emessa nell'attraversare una singola interfaccia e'

$$I = \alpha \gamma \hbar \omega_p / 3(2\pi)$$

Qualitativamente il risultato puo' essere ottenuto nel modo seguente:
per un osservatore a riposo ad una distanza b dalla traiettoria della
particella, il valore max del campo elettrico e' $E = \gamma e/b^2$ e la durata
dell'interazione e' $\Delta t = b/\gamma c$

Percio' l'energia totale dell'emissione e'

$$I \sim E^2 \times \Delta V \sim E^2 b^2 c \Delta t = \gamma e^2/b$$

A spanne, il max dell'emissione si ha quando $b = b_{\min}$.

Occorre definire $b_{\min}$.

$b_{\min}$ e' la lunghezza minima che la particella deve percorrere
affinche' il mezzo reagisca al campo E .

L'interazione della particella non avviene con un atomo o un e-
singolo, ma con molti atomi vicini alla particella.

Il mezzo puo' essere pensato come un plasma.

Un plasma possiede una frequenza caratteristica ("naturale") che
corrisponde alla frequenza tipica di oscillazione elettrostatica in
risposta ad una piccola separazione spaziale di carica.

Frequenza di plasma

Per esempio consideriamo il caso in cui uno strato di cariche di un solo segno venga spostato dalla posizione quasi-neutra di un tratto infinitesimo δx . Sulla faccia dello strato si sviluppa una densità di carica $\sigma = en \delta x$ e una carica uguale e opposta si sviluppa sulla faccia opposta. In pratica si ha un condensatore a facce piane parallele, in cui si ha un campo

$$\mathbf{E} = -\sigma/\epsilon_0 = -e n \delta x/\epsilon_0$$

Se ora prendiamo una particella fra le due facce, l'equazione del moto dello spostamento δx ci dà

$$d(\delta x)/dt^2 = eE = -e^2 n \delta x/m\epsilon_0 = -\omega_p^2 \delta x$$

Il sistema oscilla con la frequenza di plasma.

In risposta ad una sollecitazione esterna, le oscillazioni di plasma verranno osservate solo se la sollecitazione agisce su periodi di tempo $> 1/\omega_p$ o su scale di lunghezza $> v/\omega_p$

Nel nostro caso perciò il mezzo non reagisce al campo della particella su scale di lunghezza inferiori a $b_{\min} = c/\omega_p$

Radiazione di transizione

$$I \sim \mathbf{E}^2 \times \Delta V \sim \mathbf{E}^2 b^2 c \Delta t = \gamma e^2 / b$$

Se poniamo $b = b_{\min}$ otteniamo

$$I \sim \gamma e^2 \omega_p / c = \gamma e^2 \hbar \omega_p / \hbar c = \gamma \alpha \hbar \omega_p$$

Lo spettro della radiazione che si ottiene dal calcolo esatto e'

$$\frac{dI}{d\nu} = \frac{z^2 e^2 \gamma \omega_p}{\pi c} \left[(1 + 2\nu^2) \ln\left(1 + \frac{1}{\nu^2}\right) - 2 \right]$$

Nel limite di piccoli e grandi ν

$$\frac{dI}{d\nu} = \frac{z^2 e^2 \gamma \omega_p}{\pi c} \begin{cases} 2 \ln(1/\nu), & \nu \ll 1 \\ \frac{1}{6\nu^4}, & \nu \gg 1 \end{cases}$$

Circa il 90% della radiazione emessa e' contenuto nell'intervallo di $0.1 < \nu < 1$. Per valori tipici $\hbar \omega_p \sim 20$ eV e $\gamma \sim 1000$, l'intervallo di energia dei fotoni emessi e' 2 - 20 keV, cioe' raggi X (molli). Al di sopra di $\nu = 1$ c'e' poca energia emessa

$$I = \int_0^\infty \frac{dI}{d\nu} d\nu = \frac{z^2 e^2 \gamma \omega_p}{3c} = \frac{z^2}{3(137)} \gamma \hbar \omega_p \quad (13.8)$$

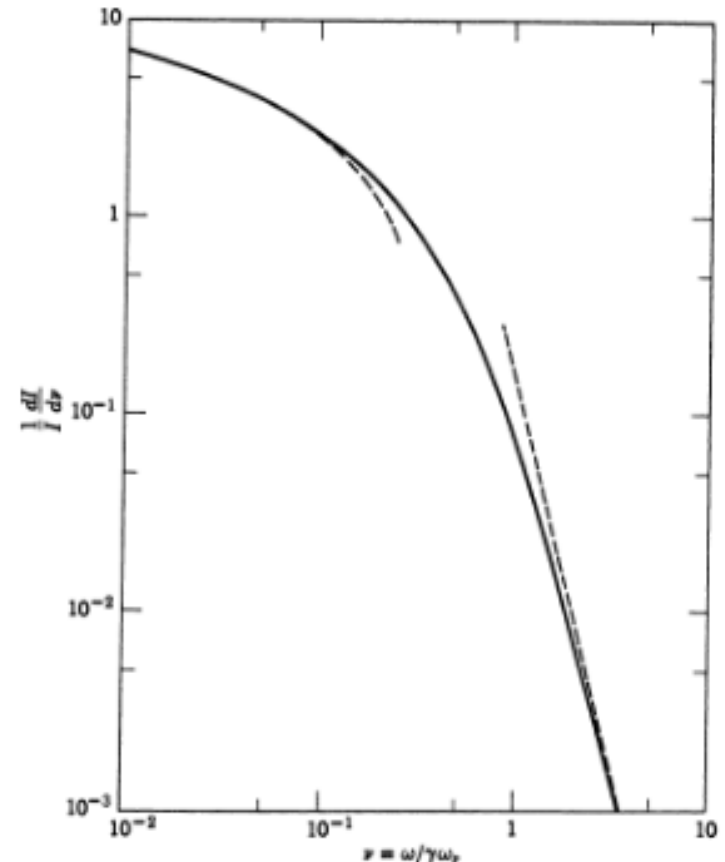


Figure 13.11 Normalized frequency distribution $(1/I)(dI/d\nu)$ of transition radiation as a function of $\nu = \omega/\gamma\omega_p$. The dashed curves are the two approximate expressions in (13.86).

Radiazione di transizione

- L'energia irradiata ad ogni superficie di separazione e' :

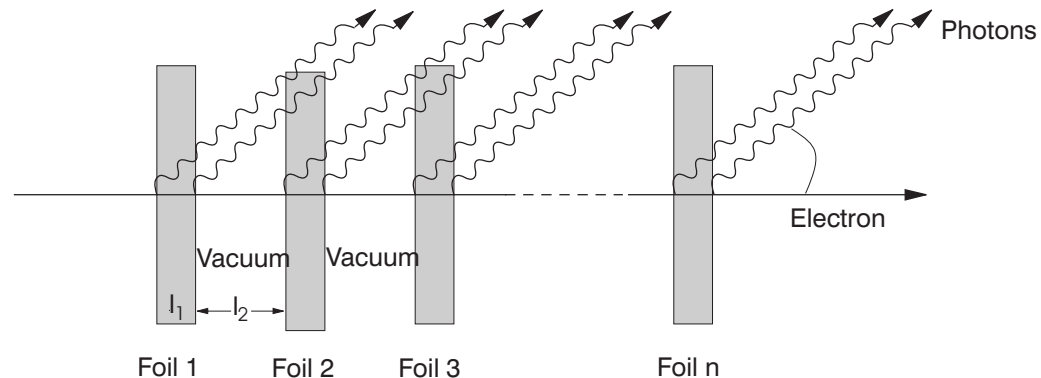
$$W = \frac{1}{3} \alpha \hbar \omega_p \gamma \quad W \propto \gamma = E / m$$

particelle della stessa energia ma massa molto diversa hanno γ molto diversi e quindi emissione diversa $\rightarrow$ separazione e/p , K/π , e/π ,...

- Il numero di fotoni emessi per superficie di separazione e' piccolo:

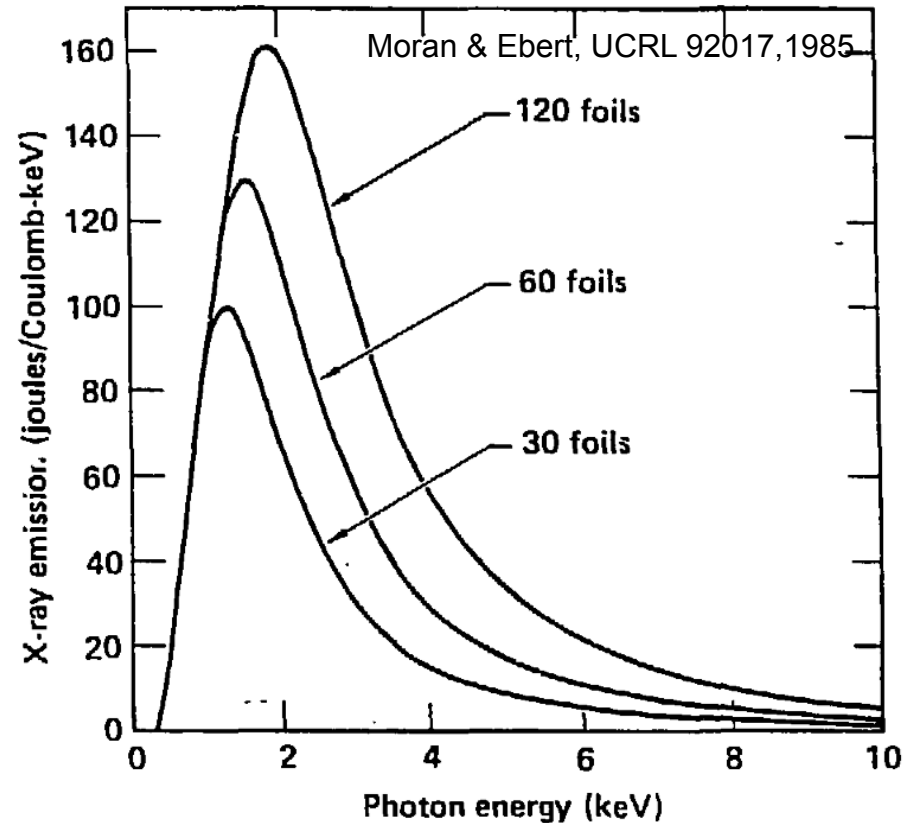
$$N_{ph} \approx \frac{W}{\hbar \omega} \propto \alpha \approx \frac{1}{137}$$

Si ha emissione apprezzabile solo per γ molto alti > 1000 e servono molte interfacce di separazione



Radiazione di transizione

The observed spectrum is modified for a stack of foils by the absorption characteristics of the foil material. The absorption length decreases rapidly as the photon energy decreases (ie in the soft x-ray range). When the x-ray absorption spectral variation is combined with the basic spectrum from a single foil, the net spectrum will often show a broad peak centered in the soft x-ray range. As the number of foils is increased the spectral peak will shift to higher energies, because the softer x-ray energies will be more strongly absorbed.



Radiazione di transizione

- ❑ I radiatori devono essere a basso Z

Bisogna evitare di riassorbire i fotoni emessi (vedi in seguito effetto fotoelettrico proporzionale a Z^5).

- ❑ Lo spessore dei radiatori deve essere $\geq$ della lunghezza di formazione D .

$$D = \frac{\gamma c}{\omega_p}$$

Per materiale tipo plastica $\omega_p \sim 30 \times 10^{15} \text{ s}^{-1}$ e se $\gamma \sim 1000$

$\rightarrow D \sim 10 \text{ } \mu\text{m}$, in aria $\sim \text{mm}$

Transition radiation

- Number of emitted photons / boundary is small

$$N_{ph} \approx \frac{W}{\hbar\omega} \propto \alpha \approx \frac{1}{137}$$

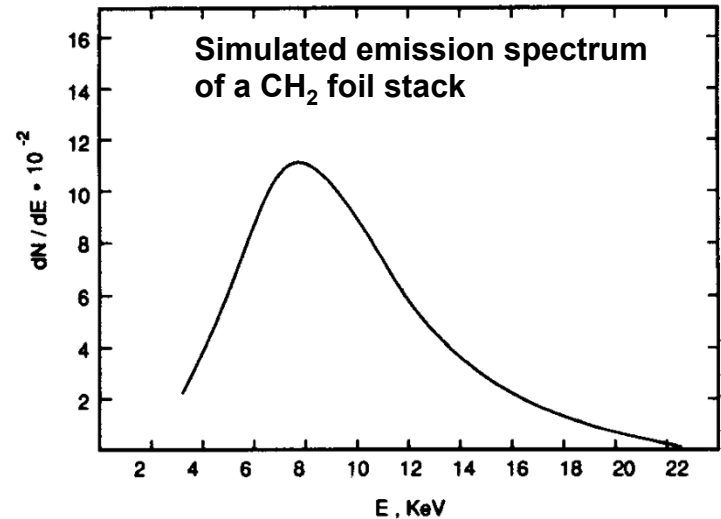
→ **Need many transitions** → build a stack of many thin foils with gas gaps

- Emission spectrum of TR = f(material, γ)

Typical energy: $\hbar\omega \approx \frac{1}{4}\hbar\omega_p\gamma$

→ photons in the keV range

- X-rays are emitted with a sharp maximum at small angles $\theta \propto 1/\gamma$
 → **TR stay close to track** and the particle loses energy by collisions (ie bethe bloch) → x-ray photons are not very well separated



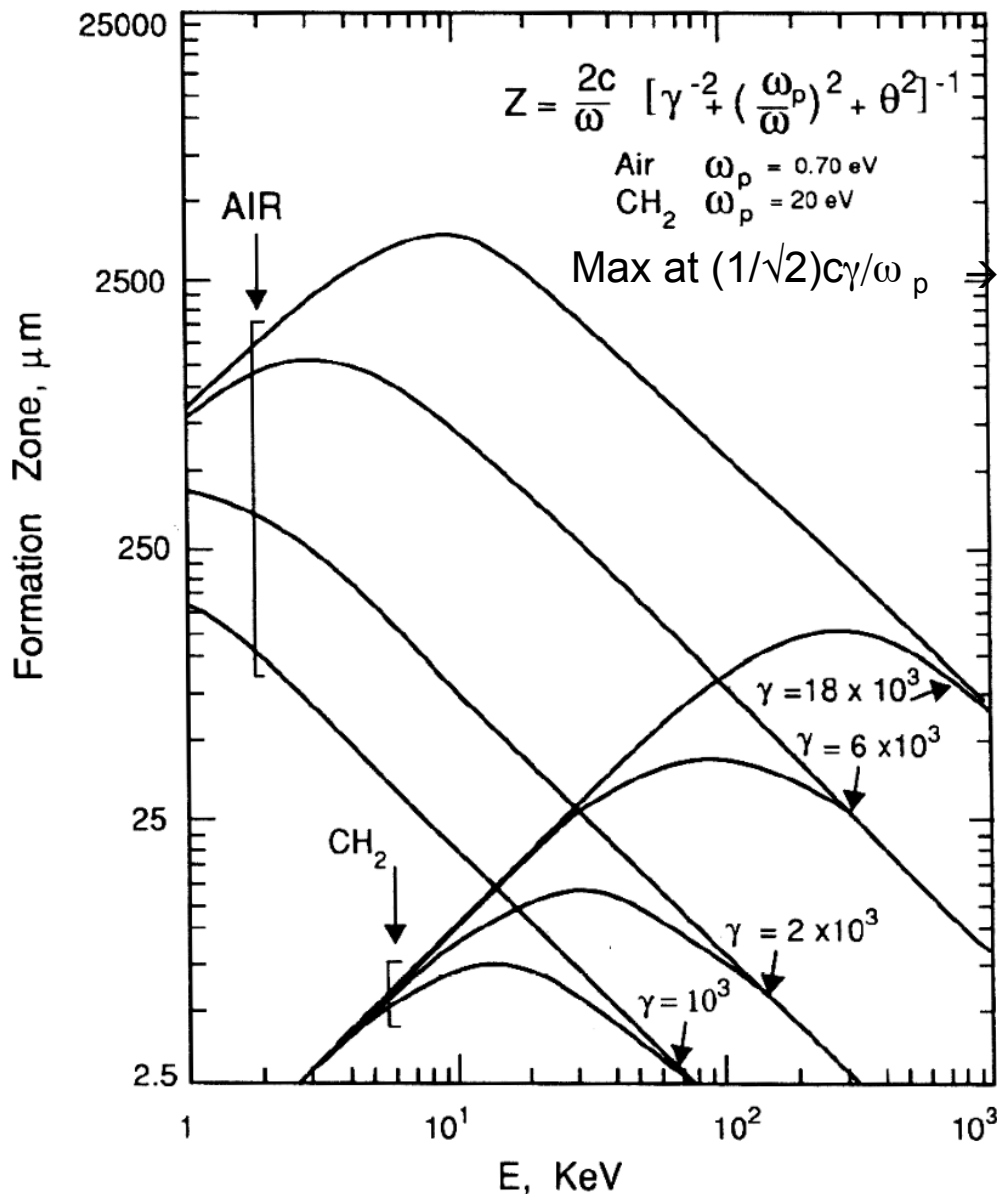
- Particle must traverse a minimum distance, the so-called **formation zone** Z_f , in order to efficiently emit TR.

$$Z_f = \frac{2c}{\omega(\gamma^{-2} + \theta^2 + \xi^2)}, \quad \xi = \omega_p / \omega$$

Z_f depends on the material (ω_p), TR frequency (ω) and on γ .

$Z_f(\text{air}) \sim \text{mm}$, $Z_f(\text{CH}_2) \sim 20 \mu\text{m}$ →

important consequences for design of TR radiator.



*Length of formation zone
for different γ and as
function of ω ($E = \omega$)*

NB: e' valore stimato con i "calcoli" di prima

High ω for CH₂:

$\gamma^{-2} \gg (\omega_p/\omega)^2$:

formation zone length

for air and CH₂ about equal,

but $\omega > \gamma\omega_p$, i.e. almost
no yield

ϕ in equations
corresponds
to θ in figure

From: B. Dolgoshein, "Transition Radiation
Detectors", NIM A326(1993)434-469

Radiazione di transizione: considerazioni per i rivelatori

Due to the dependence of TR on $\gamma = E/m$, it is evident that there is a wide momentum range (1–100 GeV/c) where electrons (resp. positrons) are the only particles producing transition radiation.

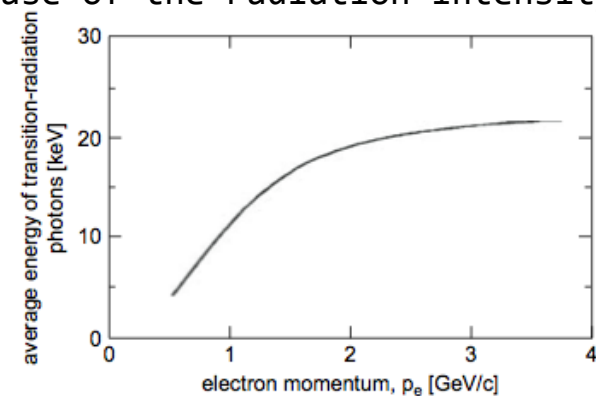
Kaons can also be separated from pions on the basis of TR in a certain momentum range (roughly 200–700 GeV/c) .

Protons start to emit at some 300 GeV, so e/p separation may be achieved up to hundreds of GeV (eg astroparticle experiments)

The attractive feature of transition radiation is that the energy radiated by TR photons increases with the Lorentz factor γ (i.e. the energy) of the particle, and is not proportional only to its velocity. Since most processes used for particle identification (energy loss by ionisation, time of flight, Cherenkov radiation, etc.) depend on the velocity, thereby representing only very moderate identification possibilities for relativistic particles ($\beta \rightarrow 1$), the γ -dependent effect of transition radiation is extremely valuable for particle identification at high energies.

An additional advantage is the fact that transition-radiation photons are emitted in the X-ray range. The increase of the radiated energy in transition radiation proportional to the Lorentz factor originates mainly from the increase of the average energy of X-ray photons and much less from the increase of the radiation intensity.

In Fig. the average energy of transition-radiation photons is shown in its dependence on the electron momentum for a typical radiator .



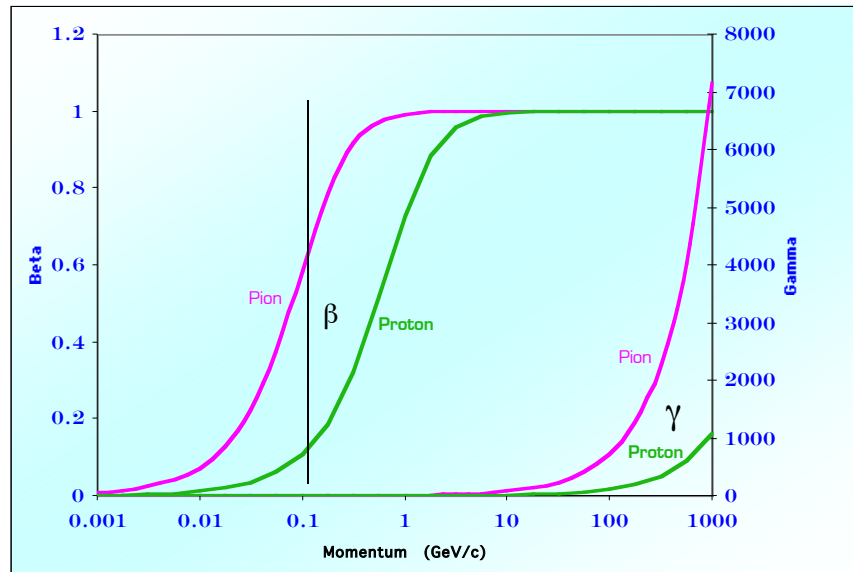
Particle Detectors: Particle Identification

Cerenkov and Transition Radiation Detectors are Used primarily for Particle Identification

- At fixed momentum, Heavy particles radiate less than Light particles.
- Further: angular distribution of radiation varies with particle speed.

Cerenkov
Counters –
sensitive to β

$$\beta = v/c$$
$$= p/E$$



Momentum (GeV/c)

E. Fiandrini Rivelatori di Particelle

1617

TRD Counters –
sensitive to γ

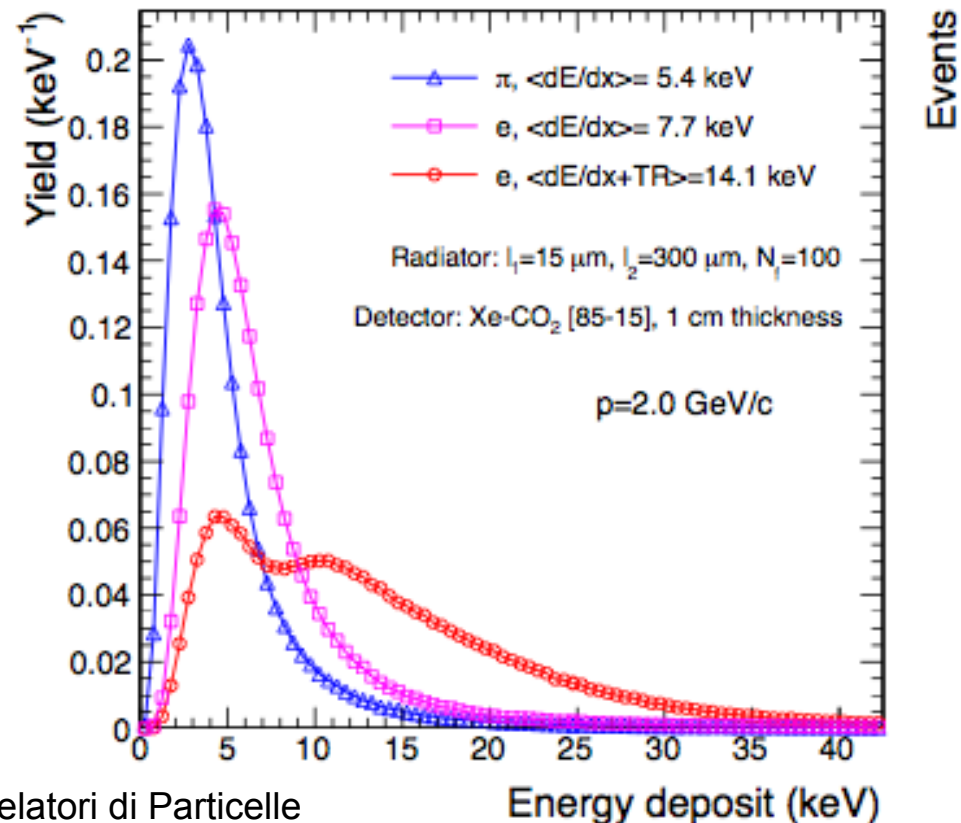
$$\gamma = (1 - \beta^2)^{-1/2}$$
$$= E/m$$

It is important to emphasize that, due to the very small TR emission angle, the TR signal generated in a detector is overlapping with the ionization due to the specific energy loss dE/dx and a knowledge (and proper simulation) of dE/dx is a necessity for the ultimate understanding and modeling of any TRD. For pions, the energy loss in the gas is close to a Landau distribution due to ionization. For electrons, it is the sum of the ionization energy loss and the signal produced by the absorption of the TR photons.

X rays are detected by photoelectric absorption in different material with high Z (photoel absoption $\sim Z^5$) where ionization occurs too, typical material Xe/CO2.

The typical layout of a TRD detector is a sequence of low Z radiators, where TR is emitted, alternated to high Z absorbers where X-rays are absorbed and ionization by the incident particle occurs.

It is evident that due to the large tails in the energy loss spectrum for pions, the detector has to have many layers. In case the full charge signal is available the discrimination is done using either a normal mean, a truncated mean (discarding the highest measured value of the detector sets) or, preferably, a likelihood method.



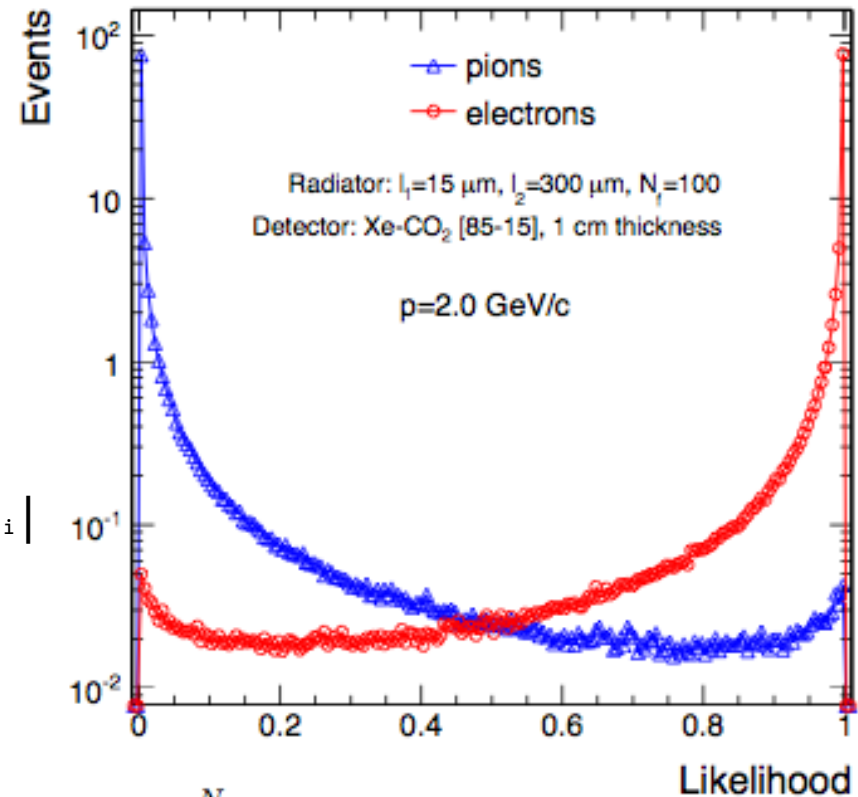
Radiazione di transizione

Likelihood distributions are constructed from the measured spectra of well identified particles (typically test beams). Taking these spectra (for each layer) as probability distributions for e^- ($P(E_i|e)$) and π ($P(E_i|\pi)$) to produce a signal of magnitude E_i , one constructs the likelihood (to be an e^-) as :

$$\text{likelihood} = \frac{P_e}{P_e + P_\pi}, \quad P_e = \prod_{i=1}^N P(E_i|e), \quad P_\pi = \prod_{i=1}^N P(E_i|\pi)$$

or, equivalently (also called log-likelihood) [51]:

$$\text{likelihood} = \sum_{i=1}^N \log \frac{P(E_i|e)}{P(E_i|\pi)}$$



Radiazione di transizione

The electron (or any other particle) identification performance of a TRD is quantified in terms of the pion efficiency at a given electron efficiency and is the fraction of pions wrongly identified as electrons.

The e- efficiency is the fraction of e- passing through the detector that are identified as electrons.

The pion efficiency depends strongly on the electron efficiency, which is a parameter that can be adjusted at the stage of the data analysis.

It is chosen such that the best compromise between electron efficiency and purity is reached. Usually 90% electron efficiency is the default value used to quote a TRD rejection power

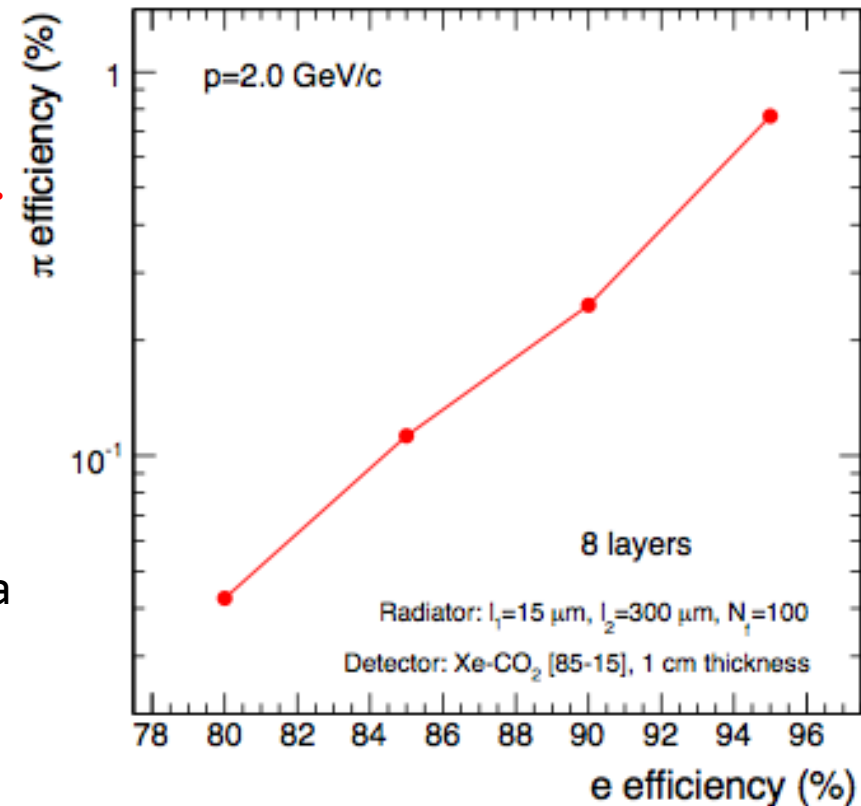


figure 7: Pion efficiency as a function of the electron efficiency (left panel).
number of layers (right panel).

Radiazione di transizione

Pion efficiency depends on momentum.
The rejection power improves dramatically between 0.5 and 1 GeV/c and slightly up to 2 GeV/c as a result of the onset of TR production in this momentum range.

Beyond 2 GeV/c TR yield saturates and the rejection power decreases as a consequence of the relativistic rise of the ionization energy loss of pions.

While in detail this behavior depends on the specific choice of the radiator, it is a generic feature of the TRD pion rejection capability.

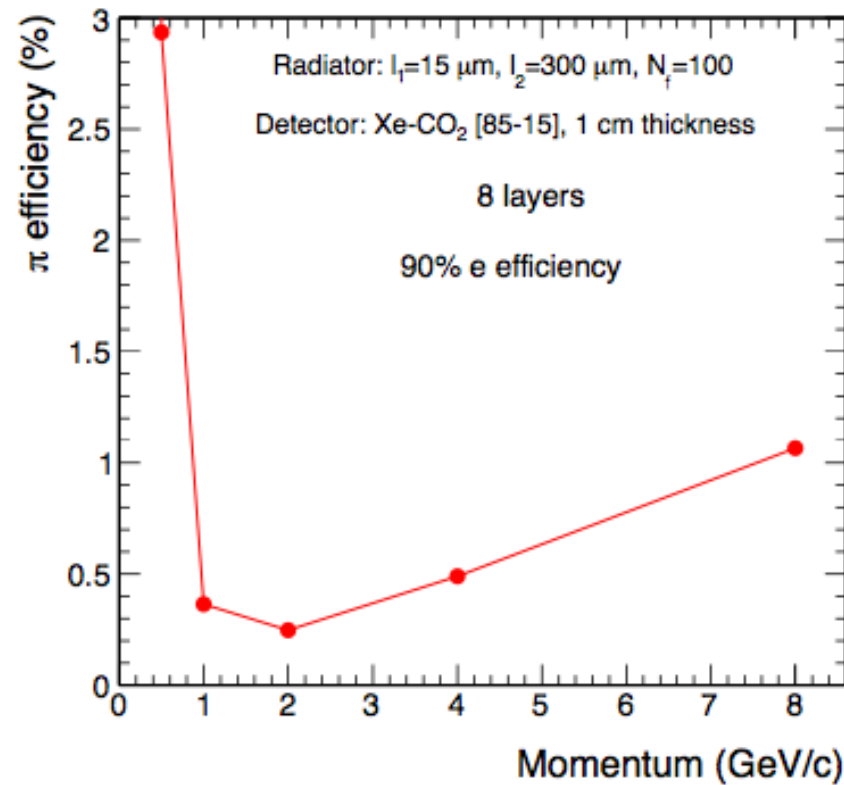


Figure 8: Pion efficiency as a function of momentum.

Radiazione di transizione: considerazioni per i rivelatori

The TRD is formed by a set of foils consisting of a material with an atomic number Z as low as possible. Because of the strong dependence of the photoabsorption cross section on Z ($\sigma_{\text{photo}} \propto Z^5$) the transition-radiation photons would otherwise not be able to escape from the radiator.

The transition-radiation photons have to be recorded in a detector with a high efficiency for X-ray photons. This requirement is fulfilled by a multiwire proportional chamber filled with krypton or xenon, i.e. gases with high atomic number for an effective absorption of X rays.

An obvious choice to detect transition radiation is a gaseous detector.

Affordability for large-area coverage, usually needed in (accelerator) experiments, is a major criterion.

In addition, a lightweight construction make gaseous detectors a widespread solution for TRDs (mandatory for space experiments) 117

Radiazione di transizione

The major factor in the performance of any TRD is its overall length. This is illustrated in Fig, which shows, for a variety of detectors, the pion efficiency at a fixed electron efficiency of 90% as a function of the overall detector length. The experimental data, covering a range of particle energies from a few GeV to 40 GeV, are rescaled to an energy of 10 GeV when possible.

Many recent TRDs combine particle identification with charged-track measurement in the same detector. This provides a powerful tool for electron identification even at very high particle densities.

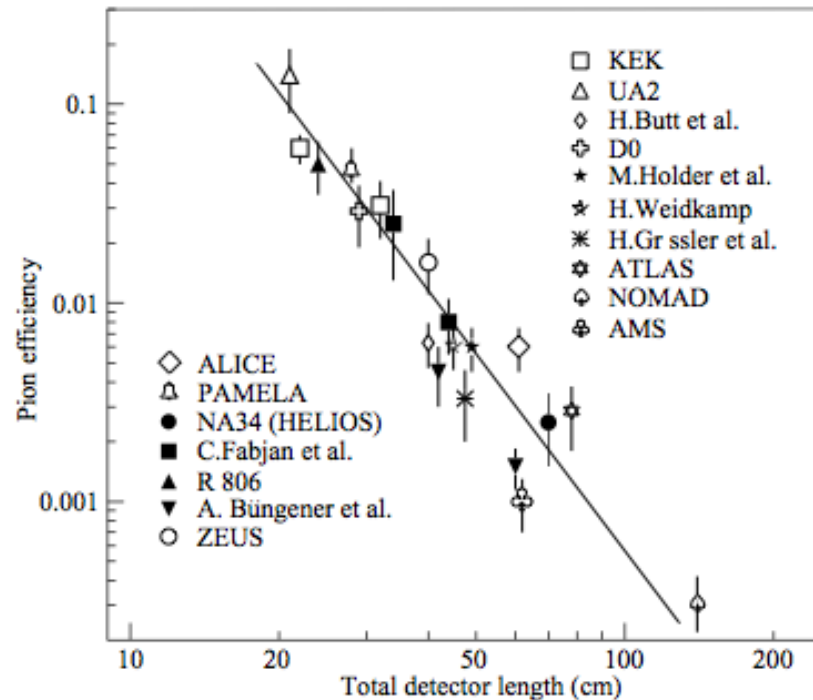


Figure 28.16: Pion efficiency measured (or predicted) for different TRDs as a function of the detector length for a fixed electron efficiency of 90%. The plot is taken from [102] with efficiencies of more recent detectors added [103].

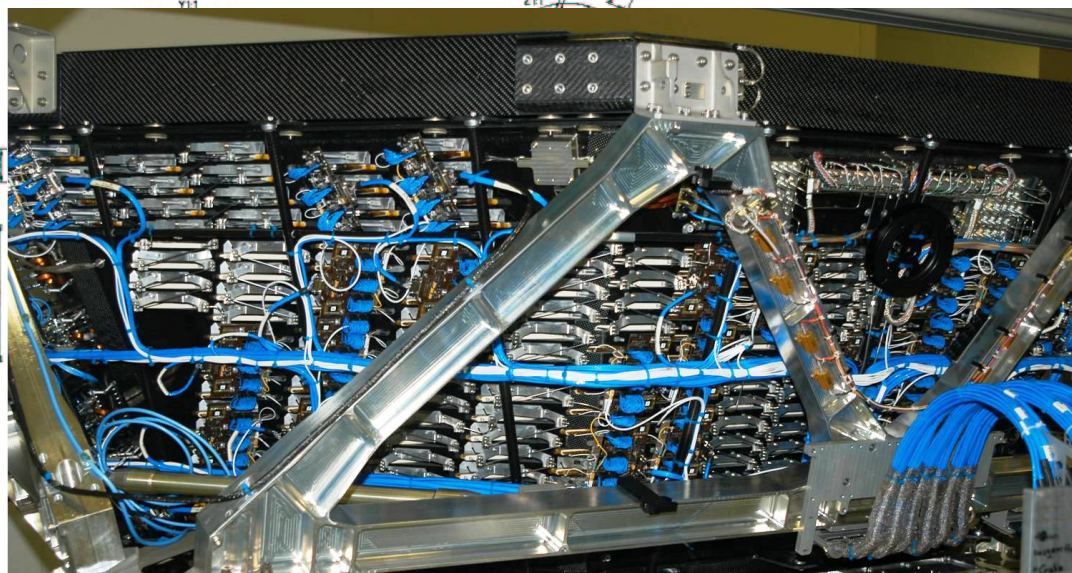
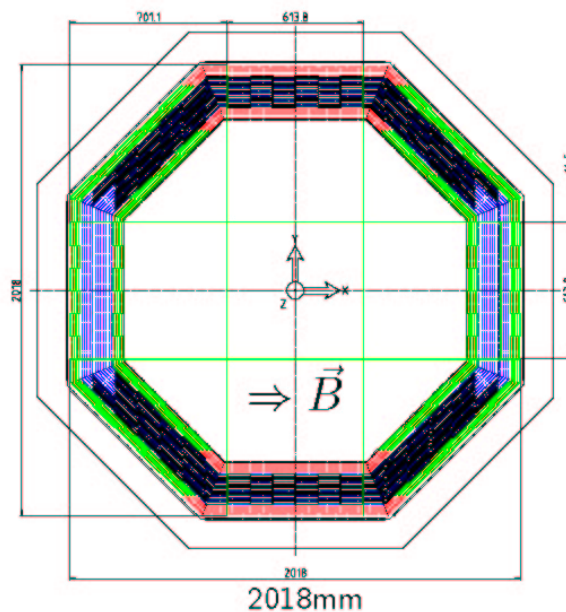
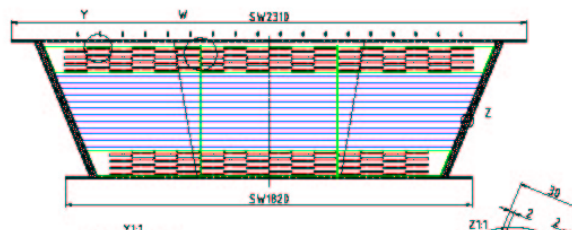
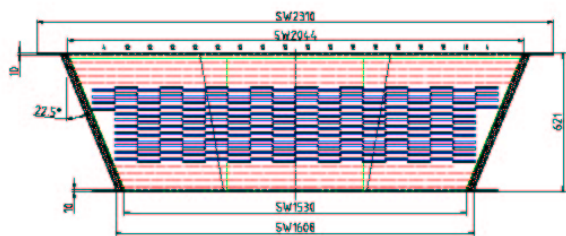
Radiazione di transizione

TRDs have been used and are currently being used or planned in a wide range of accelerator- based experiments, such as UA2 , ZEUS , NA31 , PHENIX , HE- LIOS , $D\bar{D}$, kTeV , H1 , WA89 , NOMAD, HERMES, HERA-B, ATLAS, ALICE, CBM and in astroparticle and cosmic- ray experiments: WIZARD, HEAT, MACRO, AMS, PAMELA, ACCESS.

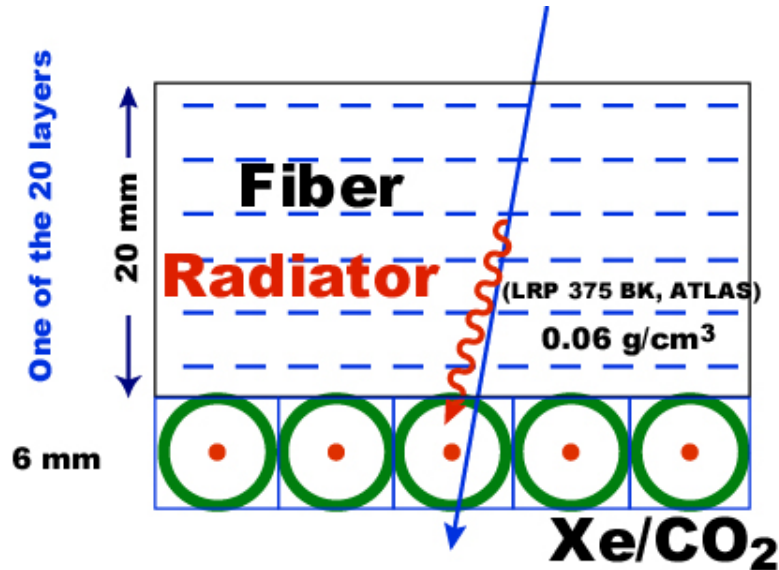
In these experiments the main purpose of the TRD is the discrimination of electrons from hadrons

AMS02 TRD

20 layers with 22mm radiator 6mm straw tube modules



TRD



A TRD is made usually of 2 components:

- Radiator where the photons are emitted
- photon and particle detectors where their energy deposit is measured

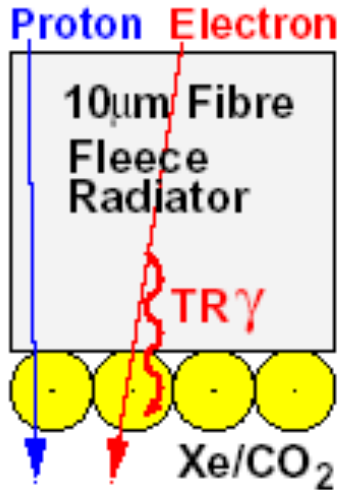
To increase the TR yield and minimize reabsorption of ph. in the radiator low, TRD are usually designed as a sandwich structure with alternating layers of radiators and photon detectors

Radiator: fleece fiber

Good photon (5–30KeV) detection: gas with high atomic number Z

The signal is the superposition of the dE/dX of the particle AND the deposited photon energy (if emitted)

TRD



- 20 layers 22 mm fleece-radiator
6mm straw-tubes Xe/CO₂(80/20)
- Tubewall 72µm Kapton-Al sandwich
- Wire 30µm W/au tensioned with 100g

**e⁻, e⁺ emit photons, protons do not in an energy interval
3-300 GeV**



- Pressure 1250 mbar
- Gas flow 1liter/h
- Gain 3000

- Modules 16 tubes
- 8 modules per gas-circuit

Event selection

- **Single track event.**

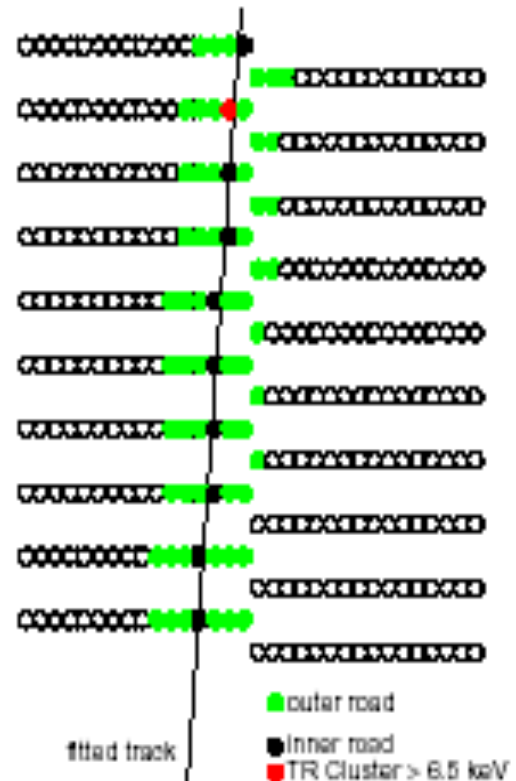
Secondary particles excluded.

Proton rejection

$$\frac{\text{Total number protons}}{\text{Selected protons}}$$

Electron efficiency

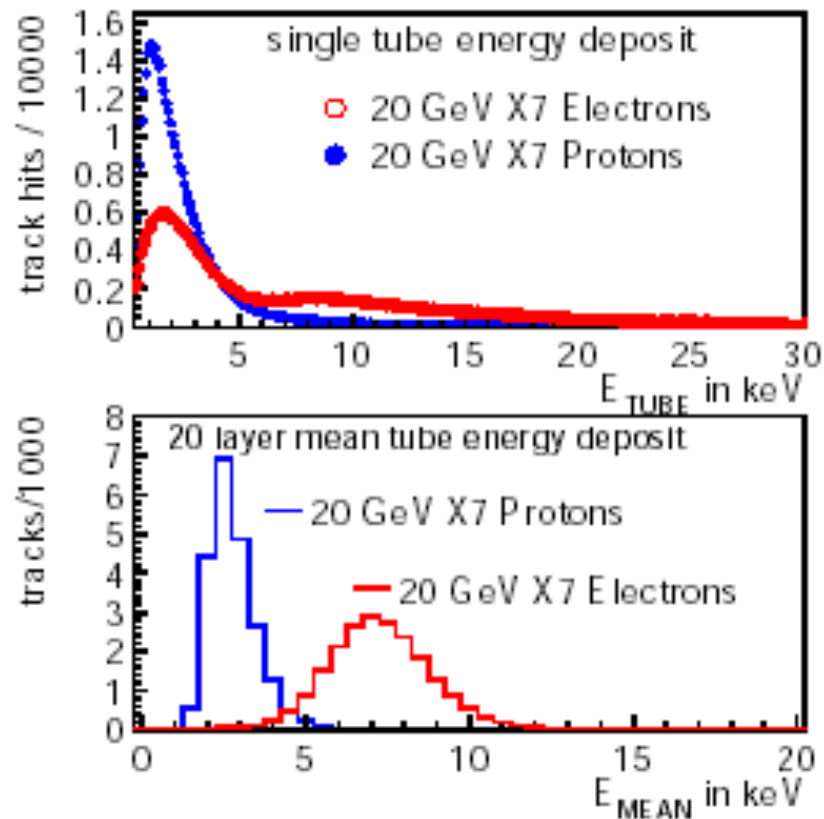
$$\frac{\text{Accepted electrons}}{\text{Total number of electrons}}$$



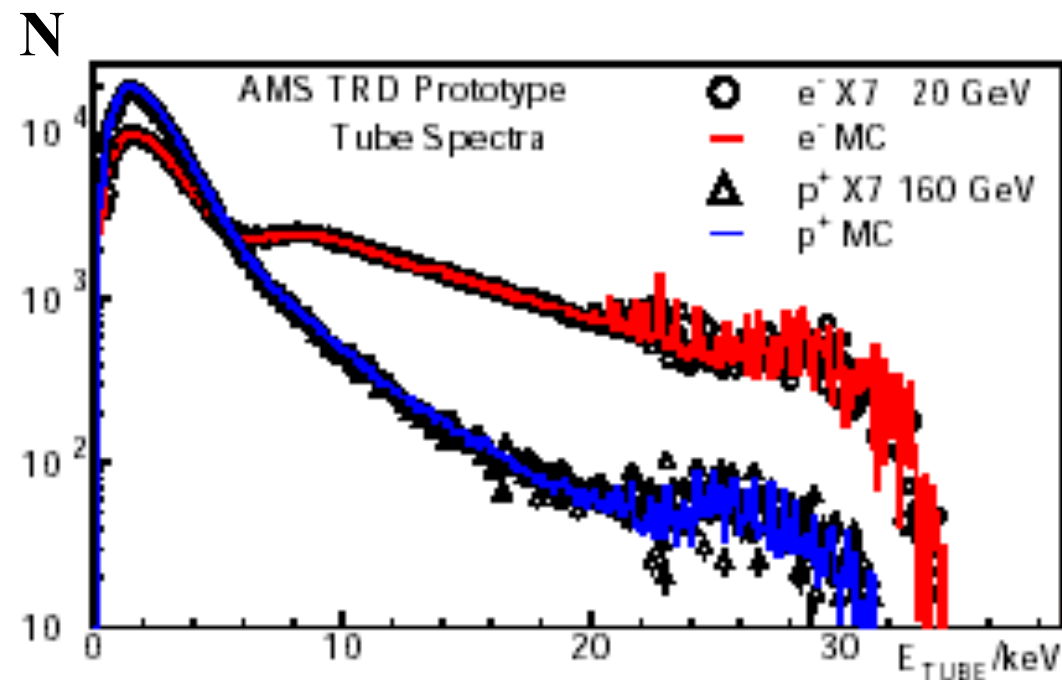
Measurement of proton rejection:

- **Cluster counting** Hit: $E_{\text{hits}} > 5-10 \text{ KeV}$, $N_{\text{hits}} > 6 \longrightarrow$ electron
 - **Likelihood method** energy deposit distribution normalized and for each hit $\longrightarrow P_{e,p}^i(E_i)$ prob. density function
- Combined probability $W_{e,p} = \prod_{i=1}^N P_{e,p}^i(E_i)$.

TRD beam test



TRD Beamtest Proton Rejection for 90% Electron efficiency



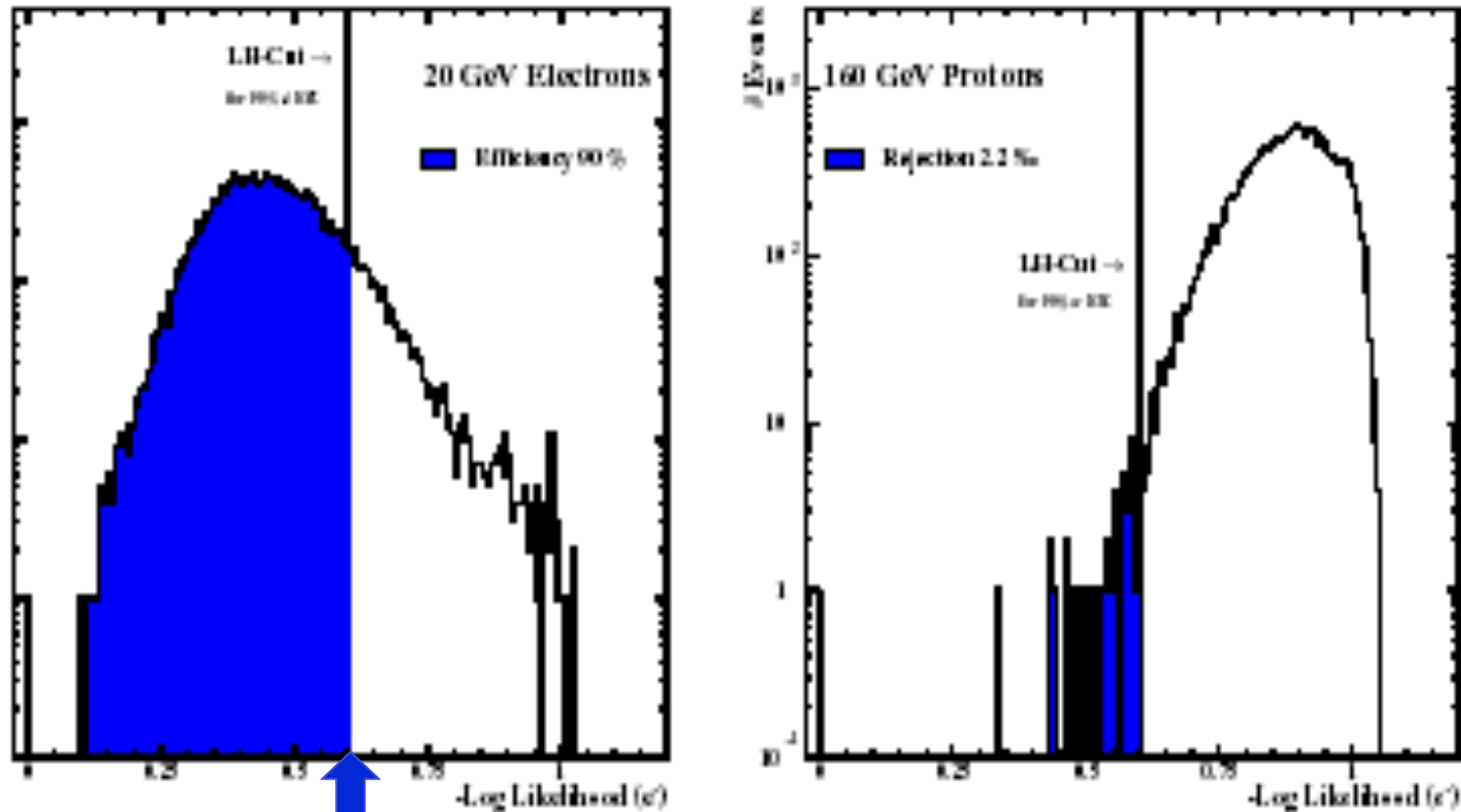
Single track preselection
with MC 90% efficiency
 p^+/e^- likelihood separation
with

$$\mathcal{L} = W_e / (W_e + W_p)$$

W_e, W_p
from single tube spectra

Likelihood cut

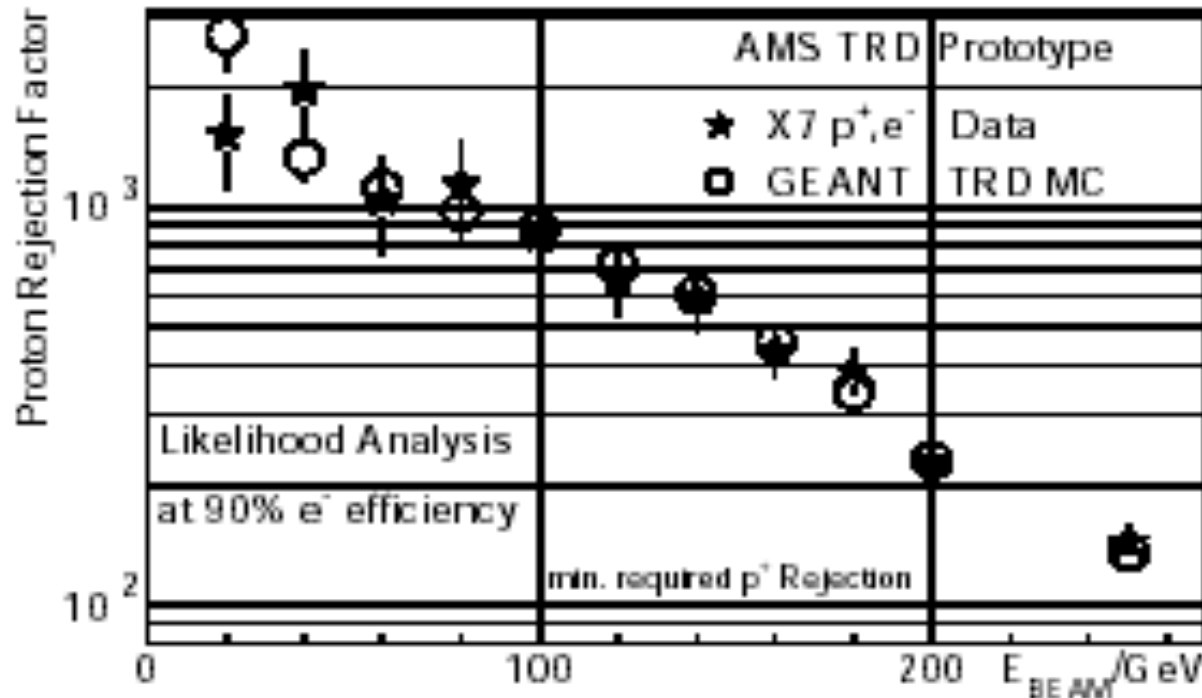
LH_{cut} Definition



Events with $\mathcal{L} = W_e / (W_e + W_p) < 0.6$ are light particles

TRD Testbeam

20 layer
prototype
tested with
 e^- , μ^- , π^+ , p^+



Proton rejection $>10^2$

reached up to 250GeV with 90% electron efficiency