Particle Detectors

Lecture 14
27/04/16

a.a. 2015-2016

Emanuele Fiandrini

Ionizzazione dei gas

Diffusione e deriva in presenza di campo elettrico.

A volte invece di pressione e temperatura, si usa la densita' del mezzo

$$\langle v_D \rangle = a\tau/2 = \frac{e\tau}{2m}E = \frac{e}{2m}\left(\frac{\langle L \rangle}{v_t}\right)E$$

$$\langle L \rangle^{-1} = N_A \frac{\rho\sigma}{A}$$

$$\langle v_D \rangle \sim \left(\frac{eE}{2m}\right)\frac{A/N_o\rho\sigma}{v_t} = \frac{eA}{2N_A\sigma\sqrt{3KTm}}\left(\frac{E}{\rho}\right)$$

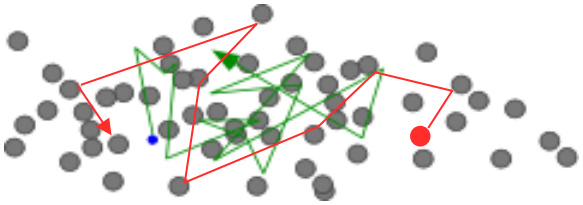
N_A = nr. di Avogadro, A peso atomico in kg/mol

$$\langle v_D \rangle = \frac{e}{2\sigma}\left(\frac{KT}{m}\right)^{1/2}\left(\frac{E}{p}\right)$$

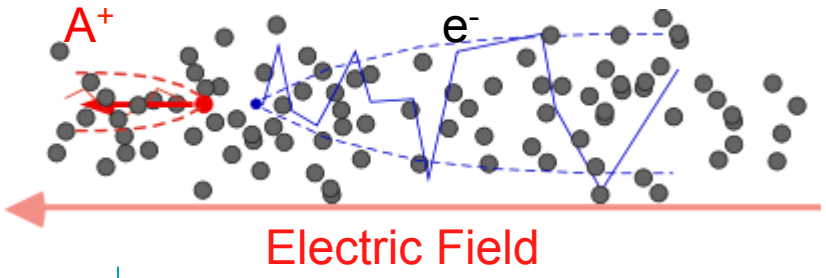
La velocita' di drift e' direttamente proporzionale al libero cammino medio e quindi inversamente proporzionale alla densita' e pressione

Drift and Diffusion in Presence of E field

E=0 thermal diffusion $\langle v \rangle_t = 0$

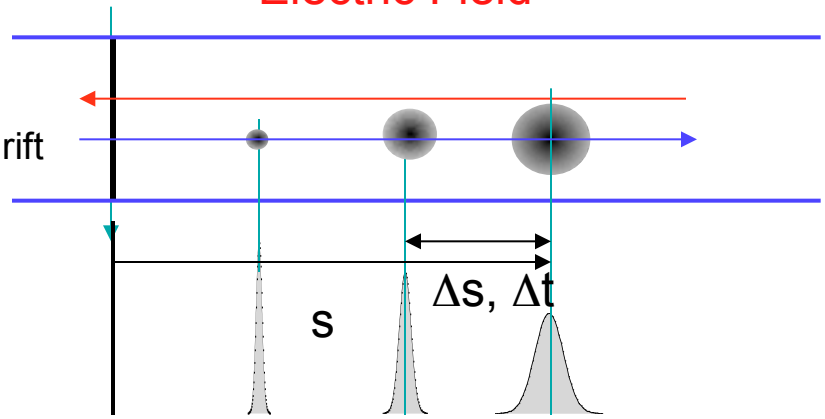


E>0 charge transport and diffusion $\langle v \rangle_t = v_D$



The solution of diffusion-transport eqn is then

$$N(x,t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{(x-v_D t)^2}{4Dt}}$$



drift + diffusion motion: the average position of the charge swarm moves as x = vt, while the width increases in time

Drift and Diffusion in Presence of E field

- As the electrons are drifting through the gas under the influence of the electric field, they will to first approximation follow the path of the electric field line that passes through their point of origin (but in the reverse direction to the electric field vector).
- Random diffusion of the electrons will still be taking place, however, causing each individual electron to follow a slightly different path. For strong electric fields, the increased average energy given the electron in the direction of the field results in different values of the diffusion coefficient D for the directions that are parallel to or transverse to the field. The same happens for magnetic fields. Over the few μseconds that are typically required for the electrons to reach a collecting electrode, the diffusion in either direction might be of the order of a millimeter or less.
- This charge spreading does play a potential role in limiting the position resolution attainable in "position- sensitive" gas detectors that deduce the location of the ionizing event through the position of arrival of the electrons at the anode, or through measuring the electron drift time.

Ionizzazione dei gas

Diffusione in presenza di campo elettrico E.

Abbiamo:

$$\sigma(x) \sim \sqrt{2Dt} \sim \sqrt{2v_t \langle L \rangle \, x / \langle v_D \rangle}$$

ma: $v_t \sim \sqrt{\frac{3KT}{m}}$ $\qquad \langle v_D \rangle \sim a\tau \sim \frac{eE}{m}\left(\frac{\langle L \rangle}{v_t}\right)$

➔

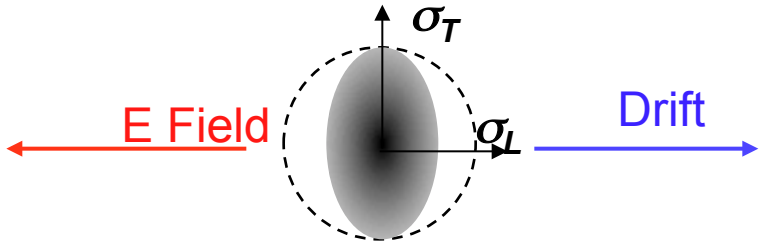
$$\sigma_x \approx \sqrt{\left(\frac{2KT}{eE}\right) \cdot x} \approx \sqrt{\frac{2KT}{eV_0} \cdot x}$$

L'energia termica KT è in competizione con l'energia del campo elettrico eV_0.

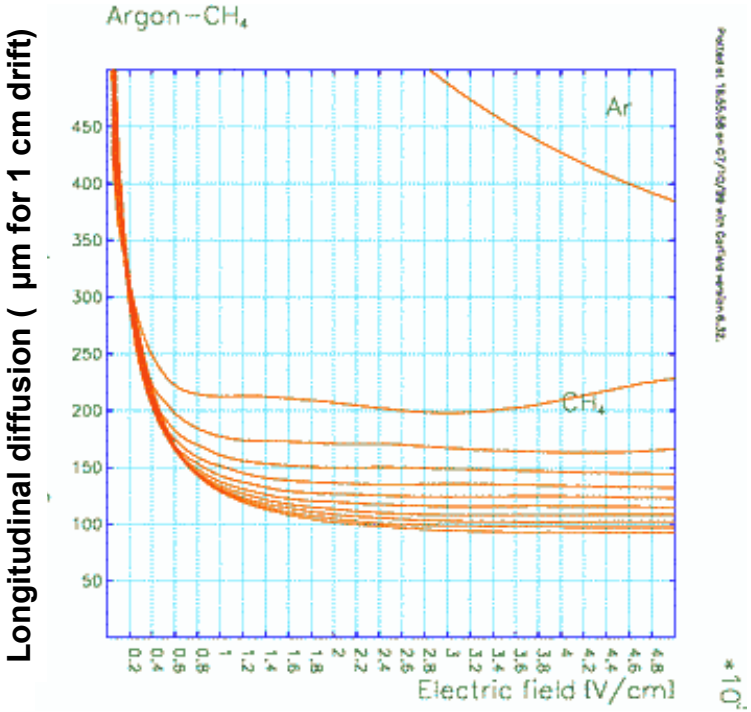
$\sigma_x \sim 100\ \mu$ con un campo elettrico di 1 KV/cm^2 e una distanza di deriva di 1 cm.

➔ limitazione intrinseca delle camere a deriva.

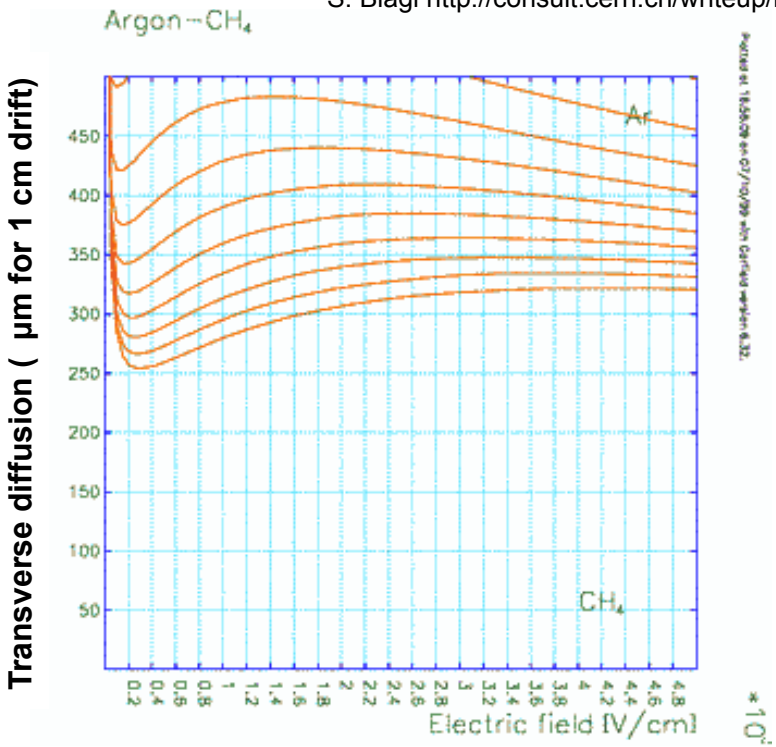
Diffusion Electric Anisotropy



S. Biagi http://consult.cern.ch/writeup/magboltz/



E (V/cm)



E(V/cm)

Nella direzione del campo (longitudinale) il coefficiente di diffusione risulta < di quella nella direzione trasversa → diffusione anisotropa.

Ionizzazione dei gas

Mobilità.

La velocità di deriva è proporzionale ad E ed inversamente proporzionale alla sezione d'urto di collisione ed alla velocità termica ➔ ci attendiamo quindi che la velocità di deriva per campo elettrico unitario diviso per la pressione o la densità sia costante, cioè mobilità costante.

La mobilità μ e' definita come la $\langle v_D \rangle$ per unita' di campo elettrico E/p ridotto.

$$\langle v_D \rangle = \frac{e}{2\sigma}\left(\frac{KT}{m}\right)^{1/2}\left(\frac{E}{p}\right)$$

$$\mu = \frac{\langle v_D \rangle}{E/p}$$

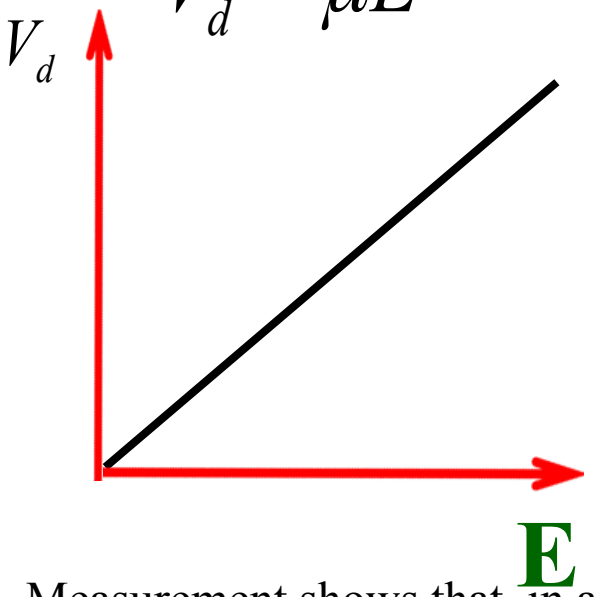
$$\mu^{ion} = \frac{e\tau}{m}$$

$$\mu = \frac{e}{2\sigma}\left(\frac{KT}{m}\right)^{1/2}$$

Ionizzazione dei gas

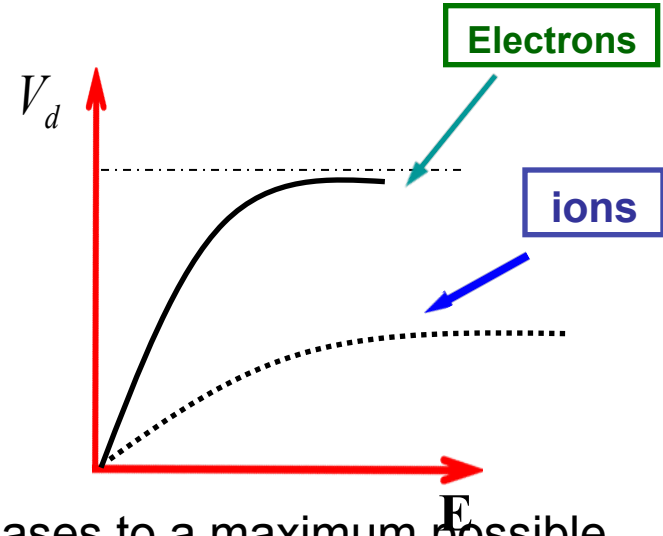
- The mobility μ tends to remain fairly constant over wide ranges of electric field and gas pressure and does not differ greatly for either positive or negative ions in the same gas. Typical values are between 1 - 1.5 X 10^{-4} m^2 atm / V s for detector gases of medium atomic number. Therefore, at 1 atm pressure, a typical electric field of 10^4 V/ m will result in a v_D ~ 1m/s.
- Ion transit times over typical detector dimensions of a centimeter will therefore be approximately 10 ms. By most standards, this is a very loooong time.
- Free electrons behave quite differently. Their much lower mass allows a greater acceleration between encounters with neutral gas molecules, and the value of the mobility is typically 1000x than that for ions. Typical collection times for electrons are therefore of the order of microseconds, rather than milliseconds.

$$V_d = \mu E$$



Obviously, this says that the **V_d vs E curve** looks qualitatively like:

Measurement shows that, in almost all materials, at high enough **E**, the **V_d vs E curve** looks qualitatively like:



As the electric field is increased the carrier velocity increases to a maximum possible value, the *saturation velocity* v_{sat}. For example, the value of v_{sat} is on the order of 1×10^7 cm/s for electrons in gas. This velocity is a characteristic of the material. This is quite important in achieving linear space-time correlations, since it minimizes the change in drift velocity as the e- approach the sense wire.

- Saturation occurs since at high fields, new scatterings processes play a role.
- By definition, mobility is is $\mu = dV_D/dE$.
- The main factor determining drift velocity is the scattering time, i.e. how long the carrier is accelerated by the electric field until it scatters (collides) with something that changes its direction and/or energy.
- $\mu = e\tau/m$ with $\tau = 1/nv_t\sigma$
- The x-section is addictive: $\sigma = \sigma_1 + \sigma_2 + \ldots \rightarrow 1/\tau = 1/\tau_1 + 1/\tau_2 + \ldots \rightarrow 1/\mu = 1/\mu_1 + 1/\mu_2 + \ldots$
- The fastest process (ie the shortest interaction time) dominates → μ decreases (e- interacts more frequently) → dv/dE decreases, v saturates.
- The most important sources of scattering are impurity scattering and acoustic and optical phonon scattering: at high fields, carriers are accelerated enough to gain sufficient kinetic energy between collisions to emit an optical phonon, and they do so very quickly, before being accelerated once again. The velocity that the electron reaches before emitting a phonon is $mv^2/2 = hw_{phonon}$ where $\omega_{phonon(opt.)}$ is the optical-phonon angular frequency and m the carrier mass in the direction of the electric field. The typical value of $E_{phonon\ (opt.)}$ is 0.06 eV. The saturation velocity is only one-half of v_{emit}, because the electron starts at zero velocity and accelerates up to v_{emit} in each cycle.
- At any T>0 K, the vibrating atoms create pressure (acoustic) waves in the crystal, which are termed phonons. Like electrons, phonons can be considered to be particles. A phonon can interact (collide) with an electron (or hole if in semiconductors) and scatter it. At higher temperature, there are more phonons, and thus increased phonon scattering, which tends to reduce mobility.

DRIFT OF IONS

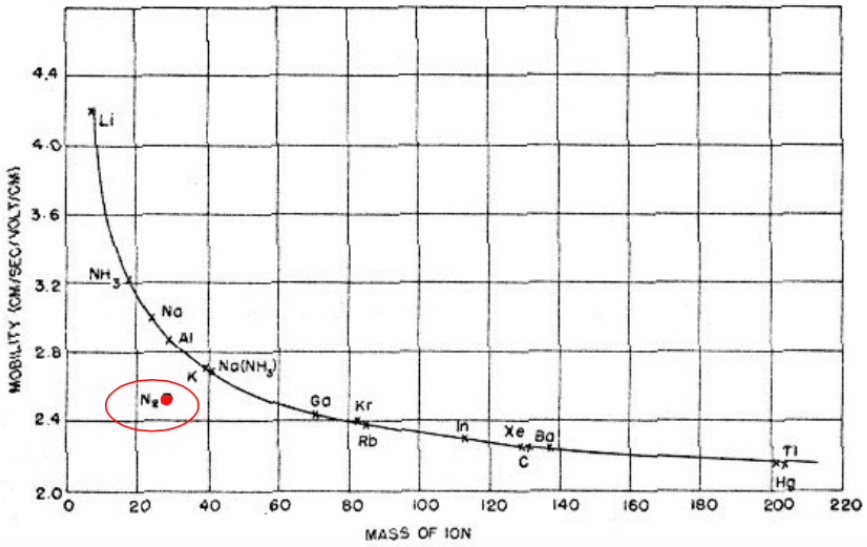
MOBILITY: RATIO OF VELOCITY AND FIELD $\mu^+ = \frac{w^+}{E}$

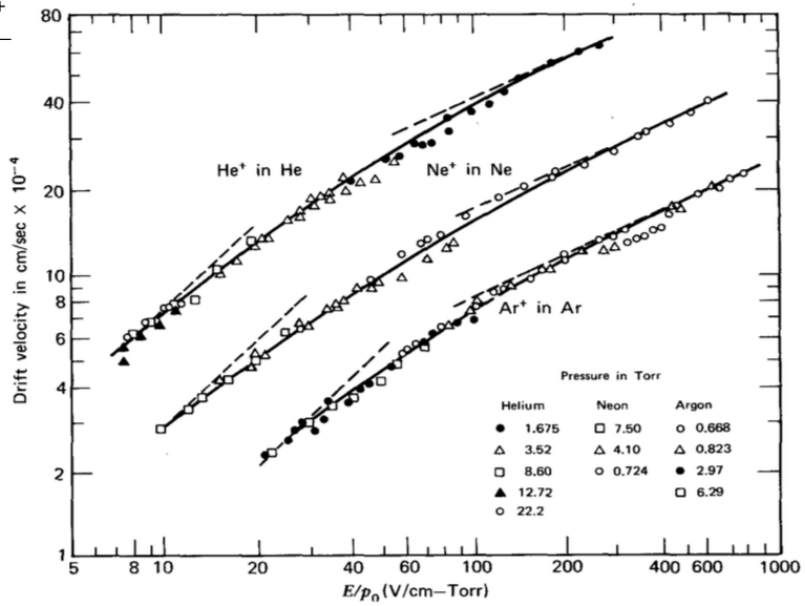
IONS MOBILITY (NTP: 300 K, 760 mm Hg)

GAS	ION	$\mu^+(cm^2s^{-1}V^{-1})$
He	He^+	10.2
Ar	Ar^+	1.7
CH_4	CH_4^+	2.26
Ar	CH_4^+	1.87
CO_2	CO_2^+	1.09

Ar-CH_4, E=1kV cm^{-1} w^+= 1.87 cm ms^{-1}

IONS MOBILITY IN NITROGEN:





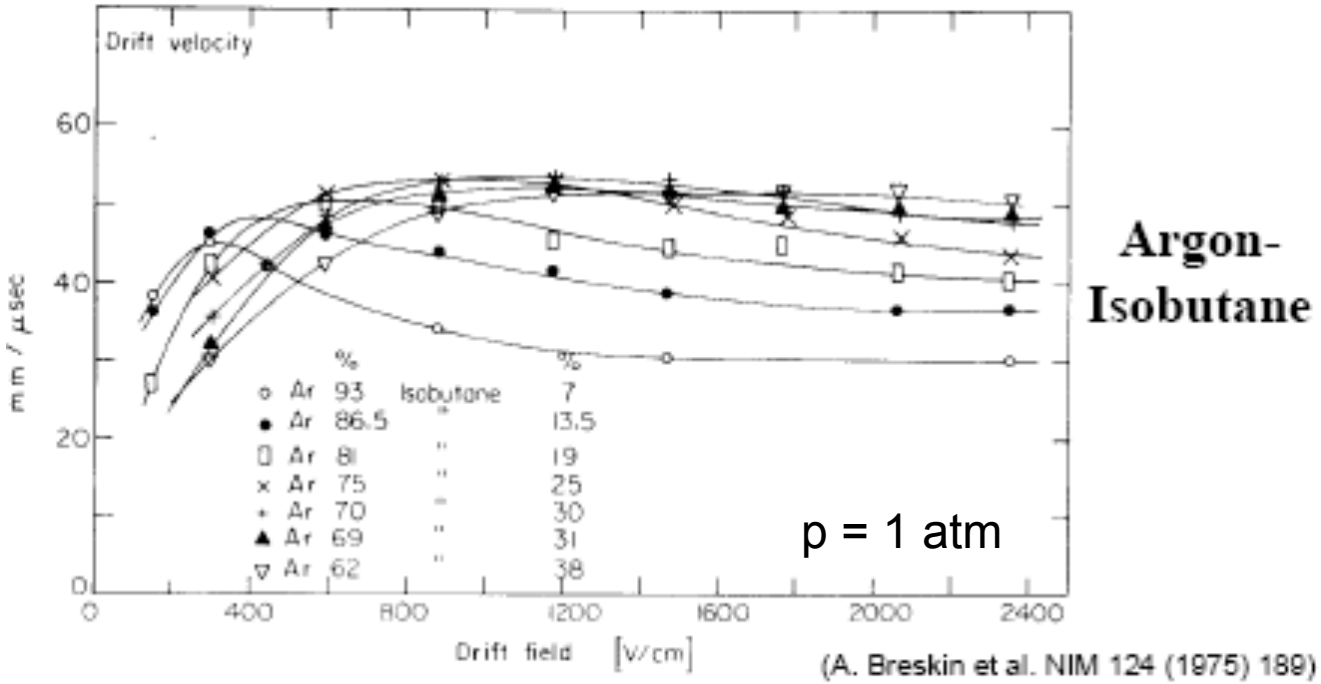
S. C. Brown
Basic Data in Plasma Physics (Wiley, New York 1959)

- Ion drift velocities << electron v_d $w^-/w^+ \approx 1000$
- Mobility μ (velocity / applied field) independent of field, inversely proportional to density
- Cloud of accumulated ions can change electrostatics, affect gain, distort field
- Issue mostly for high-rate detectors (large ionization density)

Ionizzazione dei gas

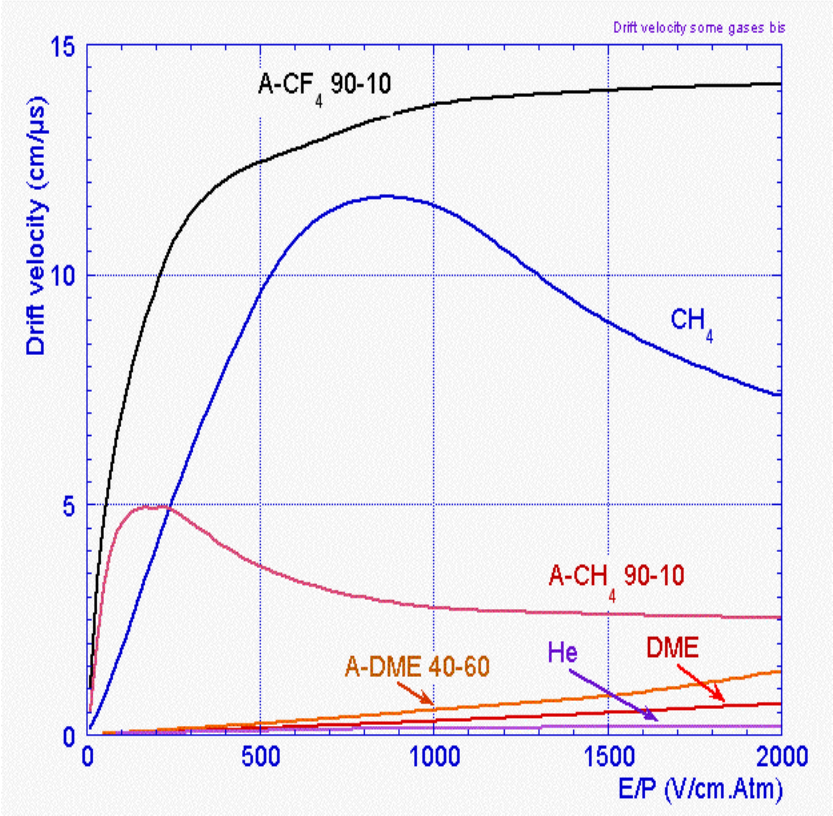
In figura sono indicate alcune velocità di deriva di elettroni per diversi campi elettrici e diversi miscugli di gas.

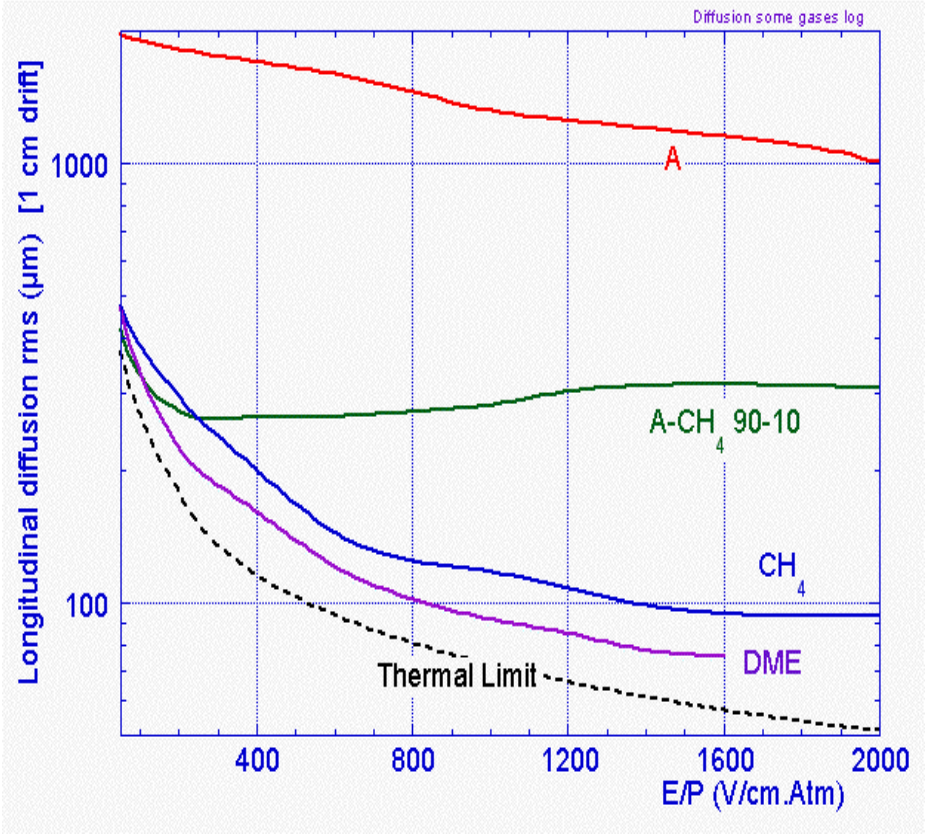
Con 75% argon e 25% isobutano e campi di 800-1000 V/ cm si ha $\langle v_D \rangle \sim$ 50 mm/μs



Drift and Diffusion of Electrons in Gases

Large range of drift velocity and diffusion:





Drift in Presence of E and B Fields

Equation of motion of free charge carriers in presence of E and B fields:

$$m\frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v}\times\vec{B}) + \vec{Q}(t)$$ where $\vec{Q}(t)$ stochastic force resulting from collisions

Time averaged solutions with assumptions: $\vec{v}_D = \langle\vec{v}\rangle = const.$; $<\vec{Q}(t)> = \frac{m}{\tau}\vec{v}_D$ friction force

$$\left\langle\frac{d\vec{v}}{dt}\right\rangle = 0 = e\vec{E} + e(\vec{v}_D\times\vec{B}) - \frac{m}{\tau}\vec{v}_D$$ τ mean time between collisions

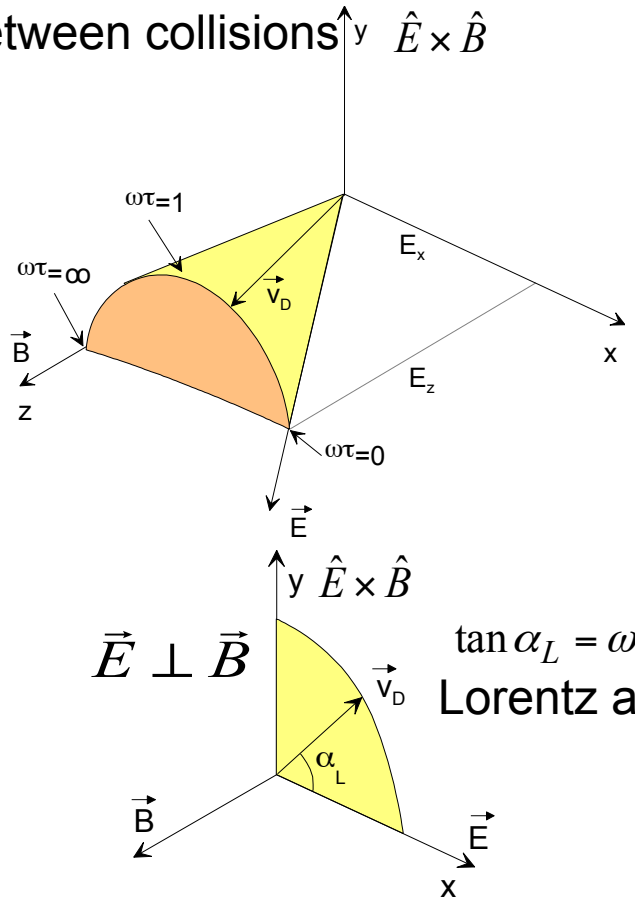
$$\vec{v}_D = \frac{\mu|\vec{E}|}{1+\omega^2\tau^2}\left[\hat{E} + \omega\tau(\hat{E}\times\hat{B}) + \omega^2\tau^2(\hat{E}\cdot\hat{B})\hat{B}\right]$$

$\mu = \frac{e\tau}{m}$ mobility $\omega = \frac{eB}{m}$ cyclotron frequency

$B=0 \quad \rightarrow \quad \vec{v}_D^{\,B} = \vec{v}_D^{\,0} = \mu\vec{E}$

$\vec{E} \parallel \vec{B} \quad \rightarrow \quad v_D^B = v_D^0$

$\vec{E} \perp \vec{B} \quad \rightarrow \quad v_D^B = \frac{E}{B}\frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$



$\tan\alpha_L = \omega\tau$ Lorentz angle

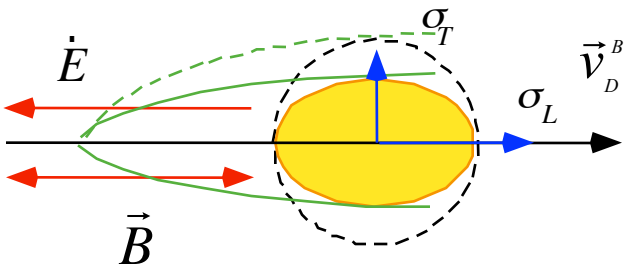
In general drift velocity has 3 components:

$\omega\tau << 1$ particles follow E-field $\quad \parallel \vec{E};\parallel \vec{B};\parallel \vec{E}\times\vec{B}$

$\omega\tau >> 1$ particles follow B-field

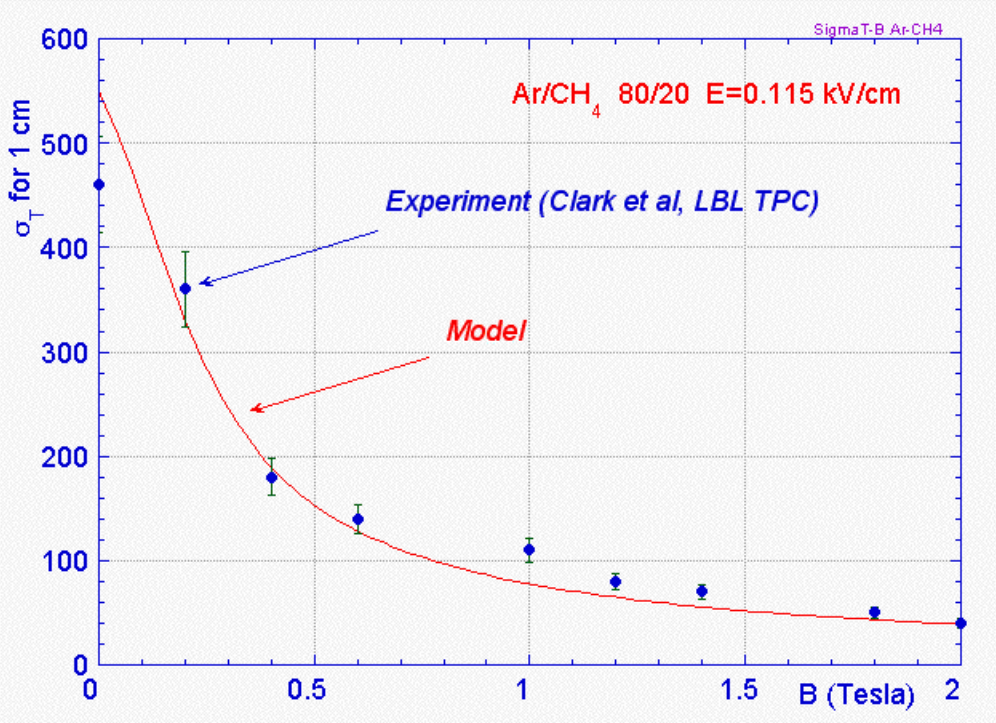
Diffusion Magnetic Anisotropy

$\vec{E} \parallel \vec{B}$



$$\sigma_L = \sigma_0$$

$$\sigma_T = \frac{\sigma_0}{\sqrt{1+\omega^2\tau^2}}$$



Ionizzazione dei gas

Ricombinazione ed ioni negativi.

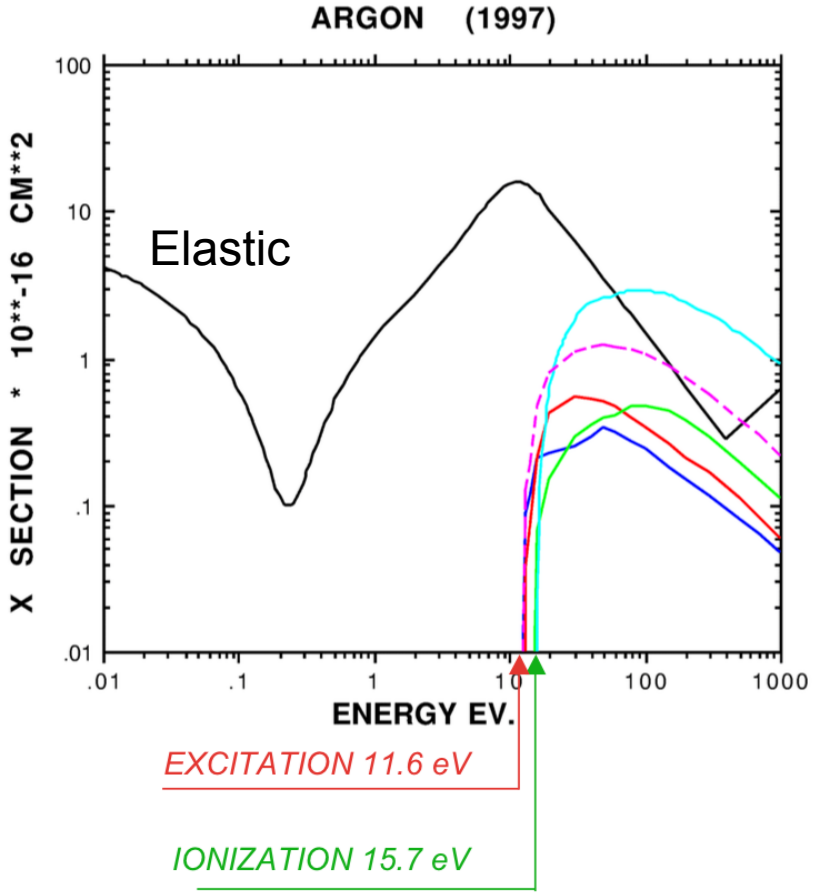
Siccome abbiamo poche coppie ione-elettrone –dell'ordine di O(100)!- è bene che non se ne perdano.

Ma ci sono numerosi processi di cattura elettronica e ricombinazione ionica che portano in genere ad una perdita di carica

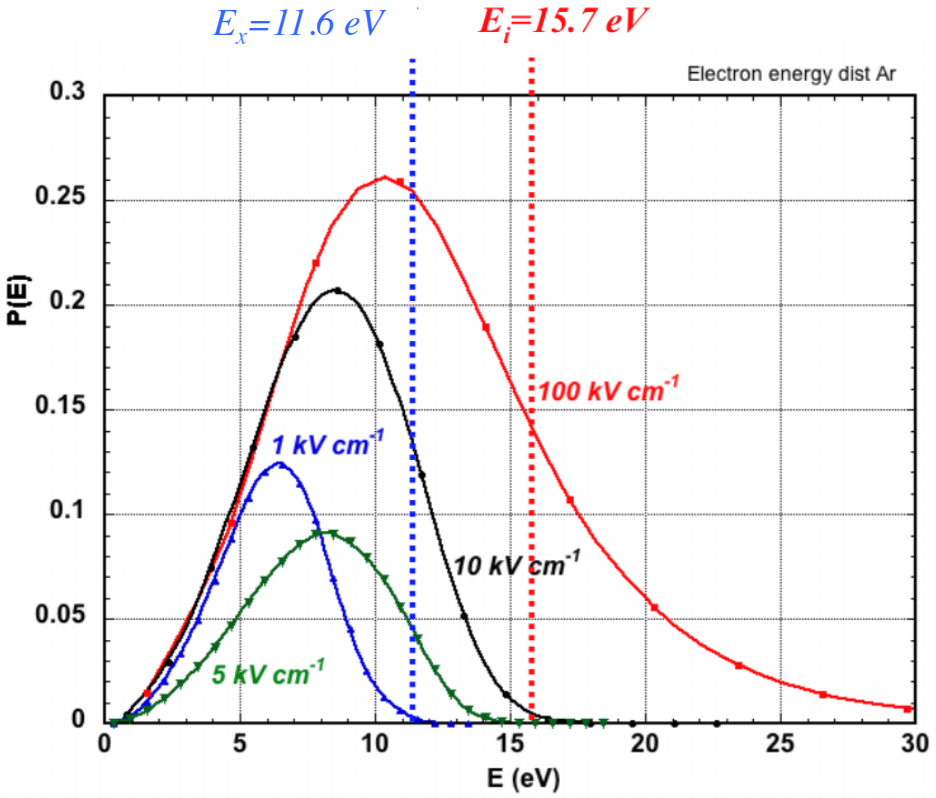
MAIN ELECTRON-MOLECULE INELASTIC PROCESSES:

1)	$A+e$	$\Rightarrow$ $A^+ + e + e$	Ionisation by electronic impact.
2)	$A+e$	$\Rightarrow$ $A^* + e$	Excitation by electronic impact.
3)	A^*+e	$\Rightarrow$ $A+e$	Deexcitation by electronic collision.
4)	$A+h\nu$	$\Rightarrow$ A^*	Photo-excitation (absorption of light).
5)	A^*	$\Rightarrow$ $A+h\nu$	Photo-emission (radiative deexcitation).
6)	$A+h\nu$	$\Rightarrow$ $A^+ + e$	Photoionisation.
7)	$A^+ + e$	$\Rightarrow$ $A+h\nu$	Radiative recombination.
8)	$A^+ + B + e$	$\Rightarrow$ $A+B$	Three body recombination.
9)	A^*+B	$\Rightarrow$ $A+B^*$	Collisional deexcitation.
10)	A^*+B	$\Rightarrow$ $A+B^+ + e$	Penning effect.
11)	$A^+ + B$	$\Rightarrow$ $A+B^+$	Charge exchange.
12)	$A^+ + B$	$\Rightarrow$ $A^+ + B^+ + e$	Ionisation by ionic impact.
13)	$A+B$	$\Rightarrow$ A^*+B	Excitation by atomic impact.
14)	$A+B$	$\Rightarrow$ $A^+ + B + e$	Ionisation by atomic impact.
15)	$A+e$	$\Rightarrow$ A^-	Formation of negative ions.
16)	A^-	$\Rightarrow$ $A+e$	Electrons release by negative ions.
17)	$A^{**}+A$	$\Rightarrow$ $A_2^+ + e$	Associative ionisation.
18)	$A^+ + 2A$	$\Rightarrow$ $A_2^+ + A$	Molecular ion formation.
19)	A^*+A+A	$\Rightarrow$ $A_2^* + A$	Excimer formation.
20)	A_2^*	$\Rightarrow$ $A+A+h\nu$	Radiative excimer dissociation.
21)	$(XY)^*$	$\Rightarrow$ $X+Y^*$	Dissociation.
22)	$(XY)^+ + e$	$\Rightarrow$ $X+Y^*$	Recombinational dissociation

ELECTRON CROSS SECTIONS IN ARGON:



ELECTRONS ENERGY DISTRIBUTION IN ARGON AT INCREASING FIELDS:



Ionizzazione dei gas

- ***Charge transfer collisions*** can occur when a positive ion encounters another neutral gas molecule. In such a collision an electron is transferred from the neutral molecule to the ion, thereby reversing the roles of each.
- This charge transfer is particularly significant in gas mixtures containing several different molecular species.There will then be a tendency to transfer the net positive charge to the gas with the lowest ionization energy because energy is liberated in collisions which leave that species as the positive ion.

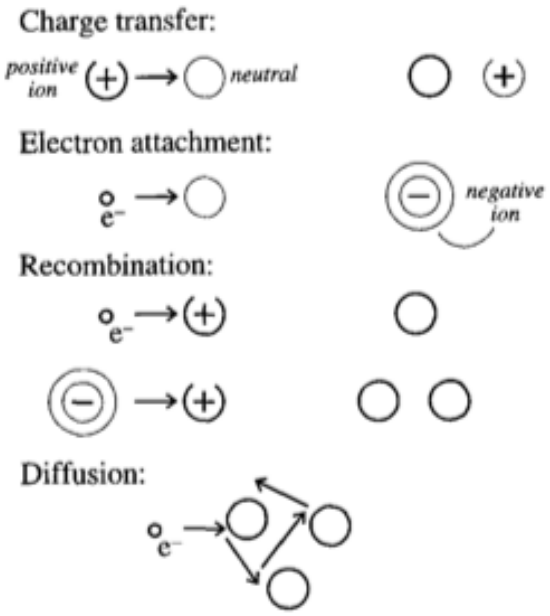


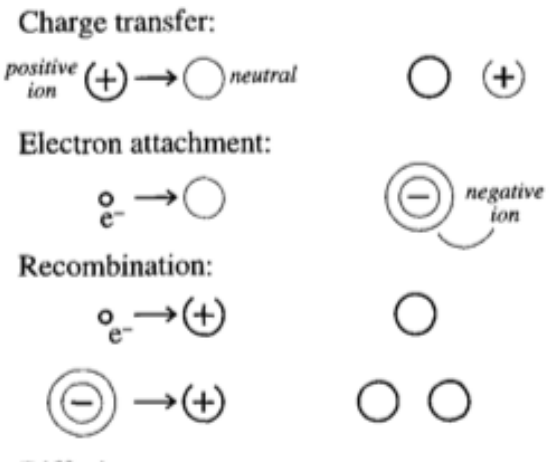
Figure 5.1 Shown are some types of interactions of charged species in a gas that can influence the behavior of gas-filled detectors. In the first four illustrations, the interacting species are on the left and the products of the interaction are on the right. Neutral atoms or molecules are shown as simple circles.

Ionizzazione dei gas

Atomi elettronegativi.

Atomi elettronegativi possono catturare un e^- formando degli ioni negativi. Questi atomi hanno il livello più esterno quasi pieno per cui l'aggiunta di un e^- risulta in un rilascio di energia.

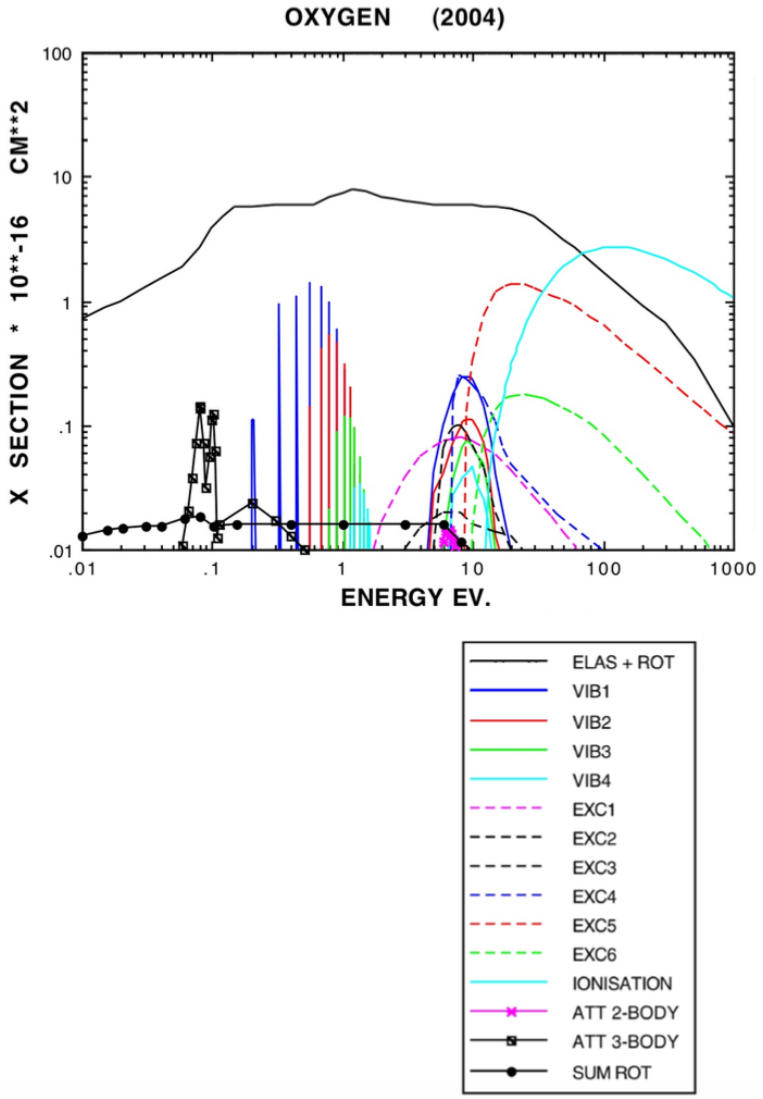
L'energia rilasciata è nota come affinità elettronica. La presenza di atomi elettronegativi diminuisce l'efficienza di collezione ione-elettrone mangiandosi gli elettroni prima che questi possano raggiungere gli elettrodi di rivelazione.



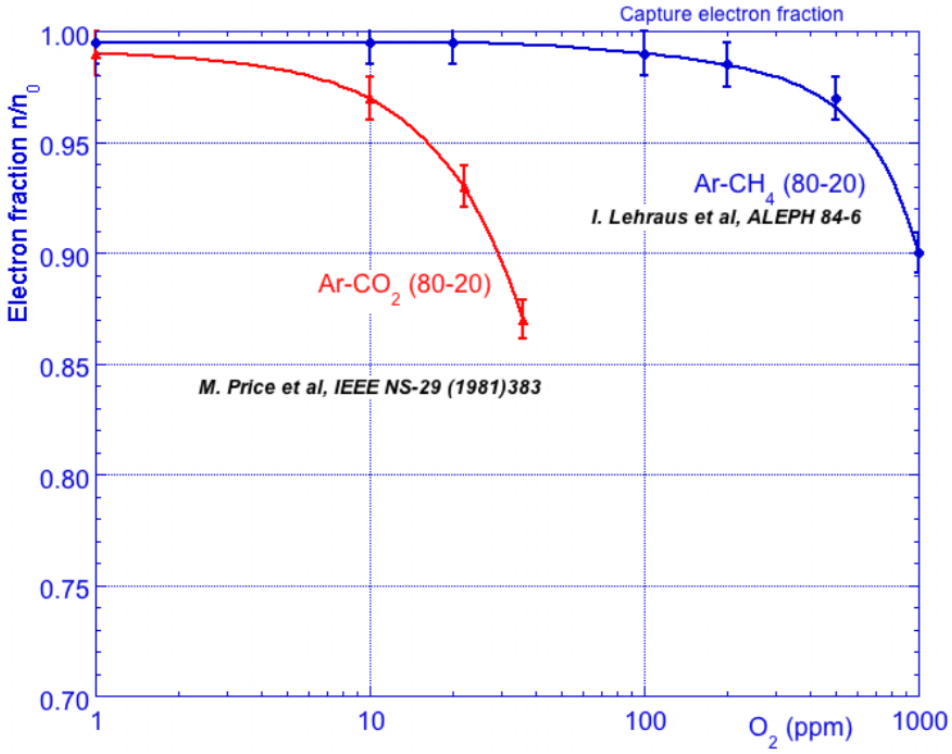
Gas elettonegativi : O_2, H_2O, CO_2, SF_6 (➔ filtri per H_2O e O_2).

Hanno invece affinità elettronica negativa He, Ne, Ar, Xe, Kr (gas nobili).

ATTACHMENT CROSS SECTION OF OXYGEN:



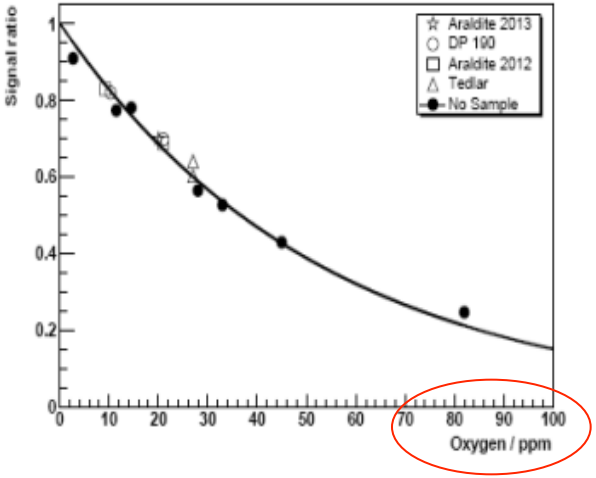
ELECTRONS SURVIVING AFTER 20 CM DRIFT (E = 200 V/cm):



Attachment, recombination

- Electronegative molecules (e.g., O_2, H_2O, CF_4) capture electrons to form negative ions; reduces signal, impacts measurement of ionization energy loss (dE/dx)

- Effect highly sensitive to contaminant concentration, differs in different primary gas mixtures



- Recombination most likely in regions of low **E** field

Ionizzazione dei gas

- Collisions between positive ions and free electrons may result in ***recombination*** in which the electron is captured by the positive ion and returns it to a state of charge neutrality.
- Alternatively, the positive ion may undergo a collision with a negative ion in which the extra electron is transferred to the positive ion and both ions are neutralized. In either case, the charge represented by the original pair is lost and cannot contribute further to the signal in detectors based on collection of the ionization charge.

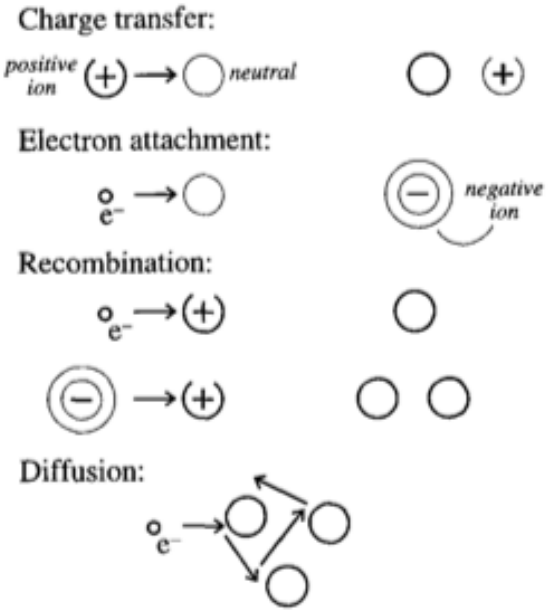


Figure 5.1 Shown are some types of interactions of charged species in a gas that can influence the behavior of gas-filled detectors. In the first four illustrations, the interacting species are on the left and the products of the interaction are on the right. Neutral atoms or molecules are shown as simple circles.

1) $X^+ + e^- \rightarrow X + h\nu$ ricombination and emission of 1 γ

2) $X^- + Y^+ \rightarrow XY + h\nu$ molecular ions

Ionizzazione dei gas

- There are two general types of recombination loss: *columnar* recombination and *volume* recombination. The first type (sometimes also called *initial* recombination) arises from the fact that ion pairs are first formed in a column along the track of the ionizing particle. The local density of ion pairs is therefore high along the track until the ion pairs are caused to drift or diffuse away from their point of formation. Columnar recombination is most severe for densely ionizing particles such as incident heavy nuclei with high Z compared with fast electrons that deposit their energy over a much longer track. This loss mechanism is dependent only on the local conditions along individual tracks and does not depend on the rate at which such tracks are formed within the detector volume.
- In contrast, volume recombination is due to encounters between ions and/or electrons after they have left the immediate location of the track. Impurities are important and since many tracks are typically formed over the time it takes for ions to drift to the collecting electrodes, it is possible for ions and/or electrons from independent tracks to collide and recombine. Volume recombination therefore increases in importance with irradiation rate.
- Charge separation and collection should be as rapid as possible in order to minimize recombination, and high electric fields are indicated for this purpose.

Ionizzazione dei gas

Let do some more quantitative evaluation.

The ionization generates n+ = n- ions and e-. The pairs will interact with the charges present in the environment.

Given a specie of charge density $n^{\pm}$, since the collision frequency is proportional to the product of the concentrations of the species involved, the recombination rate can be written as

$dn^-/dt = - b_e n^- N^+$, where N+ is the local ion density

$dn^+/dt = - b_A n^+ N^*$, where N* is any ion/molecule/electron that can interact with another ion

The recombination coefficient b is normally orders of magnitude larger between positive ions and negative ions compared with that between positive ions and free electrons: $b_A >> b_e$.

In gases that readily form negative ions through electron attachment, virtually all the recombination takes place between positive and negative ions.

Ionizzazione dei gas

The initial recombination is due to the pairs before they get separated by the electric field.
In this case N+ = n+, N* = n- and n+ = n- so that
$dn^+/dt = dn^-/dt = dn/dt = -b_e n^2$ ➔ $n=n_0/(1+b_e n_0 t)$
n_0 is the concentration at t=0.

If R is the radius of the ionisation cylinder, the time needed for e-ion separation is $t_s \sim R/v_D$ →

nb: v_D is the electron drift speed since $\mu_e >> \mu_{ion}$

$n(t_s)=n_0/(1+b_e n_0 R/v_D)$ → this is the charge that has not recombined and migrates to the electrodes

Introducing a "density parameter" $n^* = v_D/b_e R$

$n=n_0/(1+n_0/n^*) = n^* n_0/(n^*+n_0)$ → $1/n = 1/n_0 + 1/n^*$ → $1/n - 1/n_0 = 1/n^*$
→$(n_0/n - 1) = n_0/n^*$

If $n^* >> n_0$, $n_0/n \approx 1$
If $n^* << n_0$, $n_0/n \approx n^* << n_0$ → if $v_D << b_e R$, the charge recombines before separation

In the limit $n^* >> n_0$, $n \approx n_0(1-n_0/n^*) \equiv A(n_0)\, n_0$

Ionizzazione dei gas

Now in the volume recombination, the ions N+ and N* are those present in the gas as impurities with a constant concentration, that the drifting charges encounter during the propagation to the electrodes.

$dn^-/dt = - b_e n^- N^+$

$dn^+/dt =- b_A n^+ N^*$ (N* is any ion that can interact with another ion)

We can define an average interaction time $\tau = 1/bN$. Dimensionally $b = \sigma v$ = interaction volume per unit of time

$dn^-/dt = -n^-/\tau_e$

$dn^+/dt =- n^+/\tau_+$

Since $d/dt = v(d/dx)$ and the velocity is the drift speed v_D, we have $\lambda = v_D\tau = v_D/bN$

$dn^-/dx = -n^-/\lambda_e$

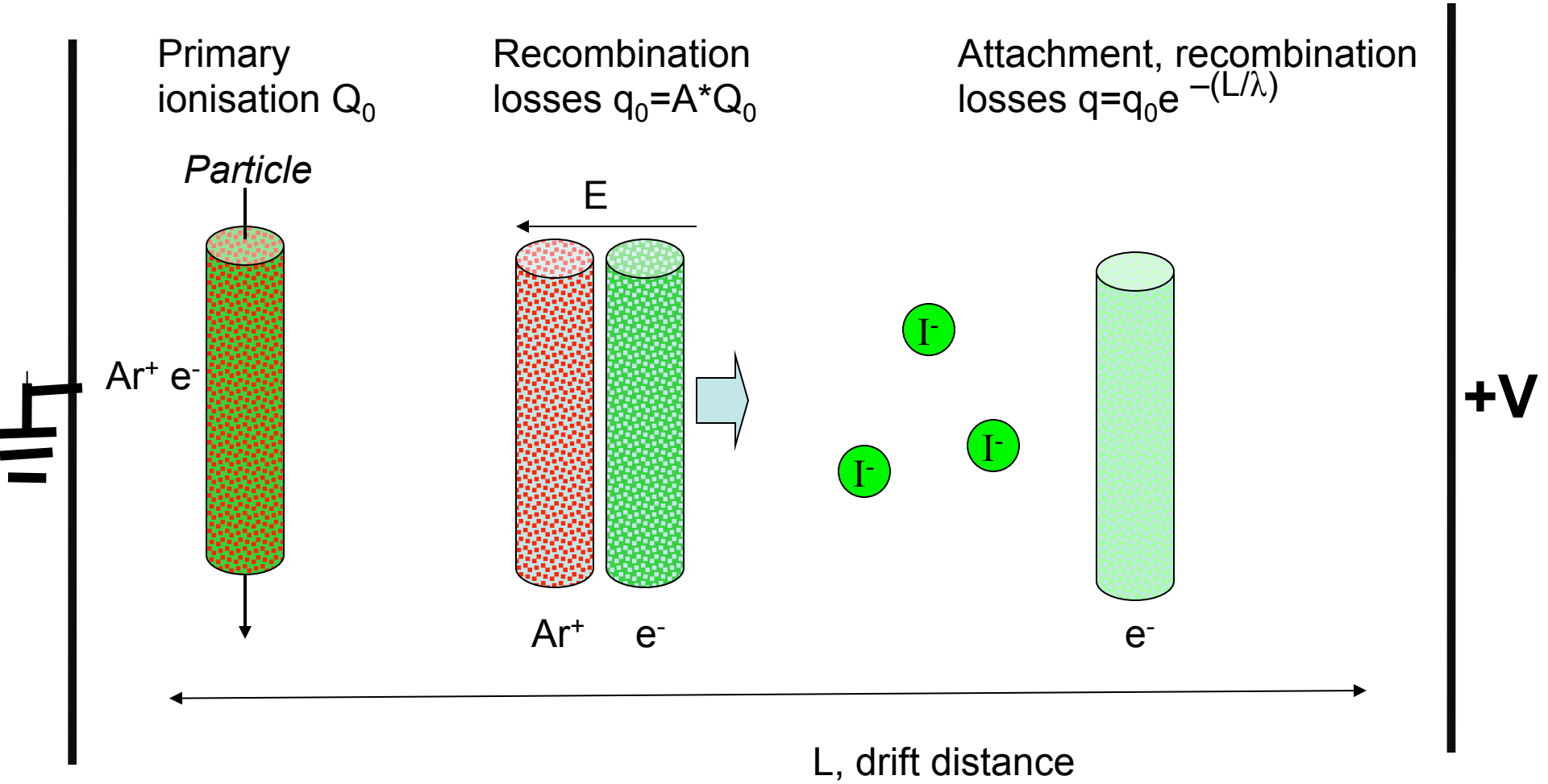
$dn^+/dx =-n^+/\lambda_+$

Therefore $n(x) = n(0)\exp(-x/\lambda)$

Columnar and volume recombination

Response to a primary ionization

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(-D \nabla n + \vec{v} n \right) = R.$$

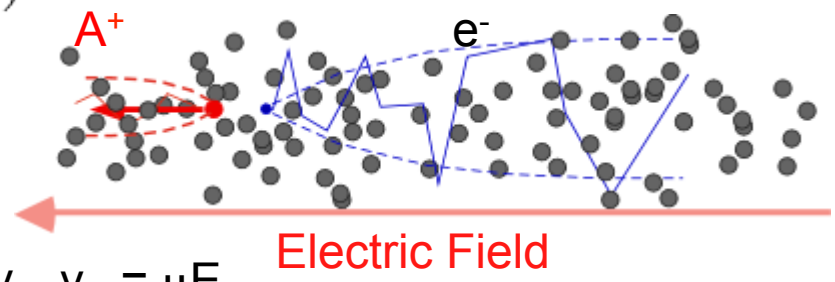


Losses are described in the source term R. The initial charge distribution is given as boundary condition.

Drift and Diffusion in Presence of E field

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(-D \nabla n + \vec{v} n \right) = R.$$

E>0 charge transport and diffusion

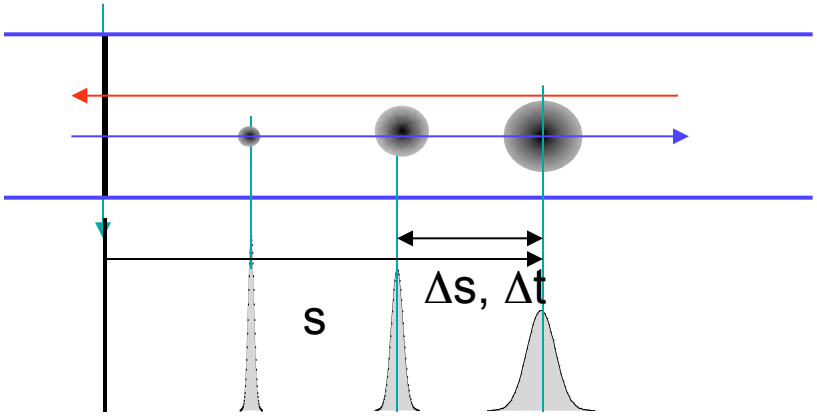


Charges move with average Drift velocity $v_D = \mu E$

While charge distribution width increases with time $\sigma \sim \sqrt{Dt}$ Diffusion

Ie the net motion is drift + diffusion: the average position of the charge swarm moves as $x = v_D t$, while the width increases in time

And some charge is lost by recombination, capture, etc. processes both in columnar and volume recombination



Misure di posizione e ionizzazione

Gli elettroni e gli ioni primari e secondari prodotti per ionizzazione sono pochi (~100 in un cm di gas)

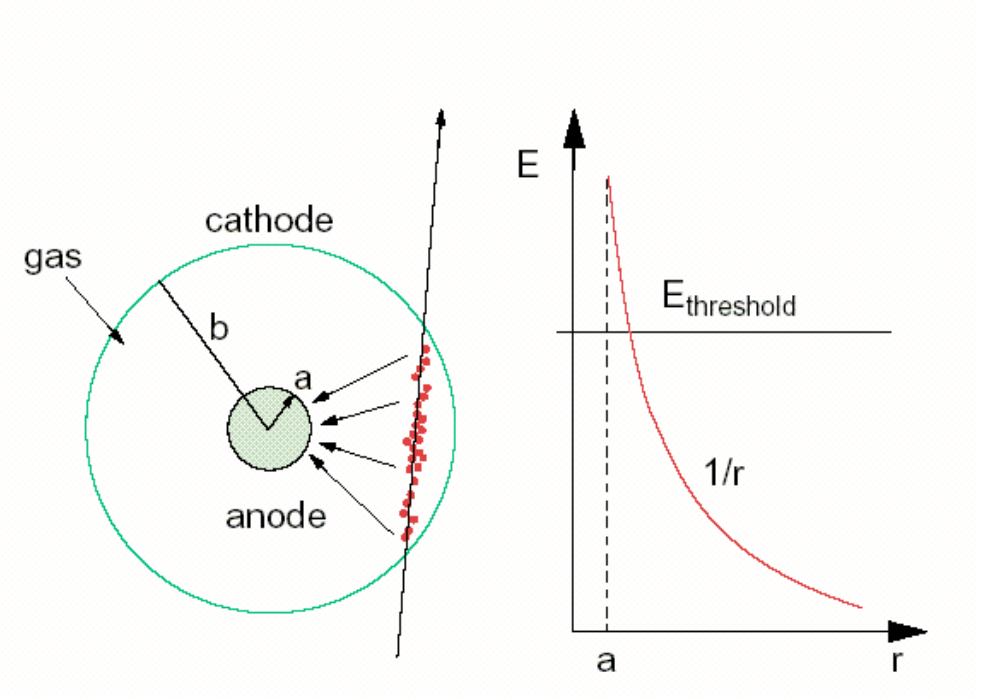
➔

- Evitare il più possibile la ricombinazione (evitare il più possibile la presenza di gas elettronegativi quale Acqua ed Ossigeno)
- Aumentare il campo E per sfruttare la moltiplicazione a valanga delle coppie e-ione, alto guadagno.

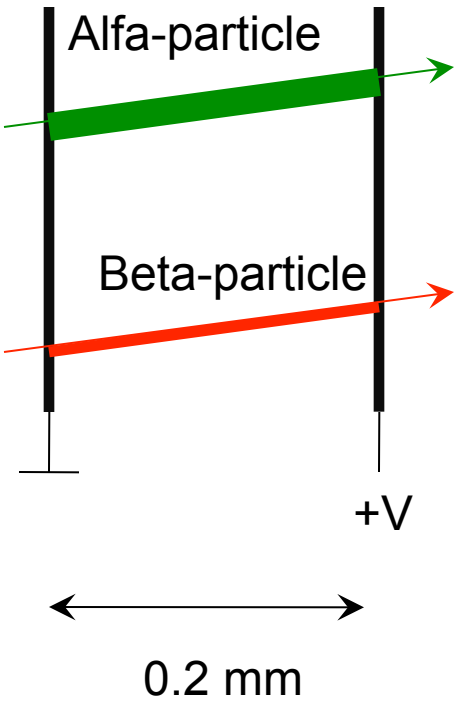
Avalanche Multiplication

- The trick is to use avalanche multiplication of ionisation in the gas. This can be achieved by accelerating the primary ionisation electrons in an electric field to the point where they can also cause ionisation, as we will see in the next slides

  - The number of ion pairs is controlled by the applied voltage.
  - An electric field varying with the distance form electrodes is needed.

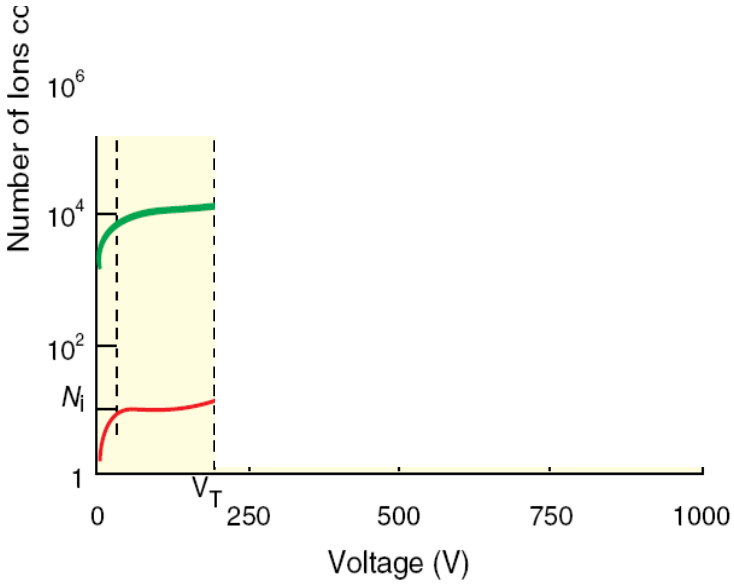


Gas amplification



Il numero di coppie raccolte dipende dal campo E nel gas:
A valori bassi del campo, la ricombinazione elimina una frazione consistente delle coppie prima che giungano agli elettrodi.
Al crescere del campo, si arriva a raccogliere tutta la carica (al netto delle perdite) generata. In queste condizioni, l'aumento del campo non provoca piu' un aumento della carica raccolta: c'e' un plateau nella curva Q vs V → variazioni della tensione provocano minime variazioni nella raccolta di carica

“Ionisation” mode of operation, i.e. no amplification yet…
used in the so called ionisation chambers.



Camere a ionizzazione

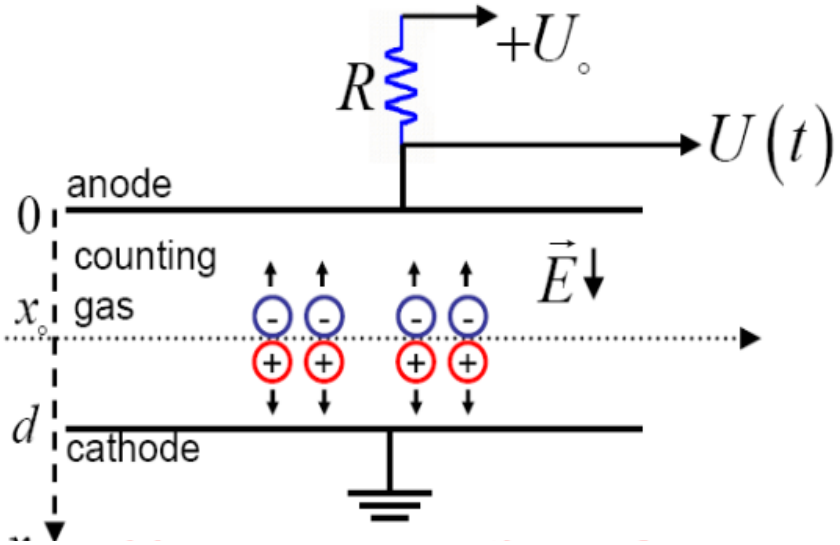
Una camera a ionizzazione è un apparato che misura la perdita di energia per ionizzazione di una particella carica o la perdita di energia di un fotone (effetto fotoelettrico, compton o produzione di coppie).

In linea di principio il materiale attraversato dalla particella può essere un gas (e.g. Argon) oppure un liquido (e.g. calorimetri ad argon o kripton o xenon liquido) od un solido (camere a ionizzazione a stato solido).

Non c'è alcuna moltiplicazione delle coppie ione-elettrone primarie e secondarie.

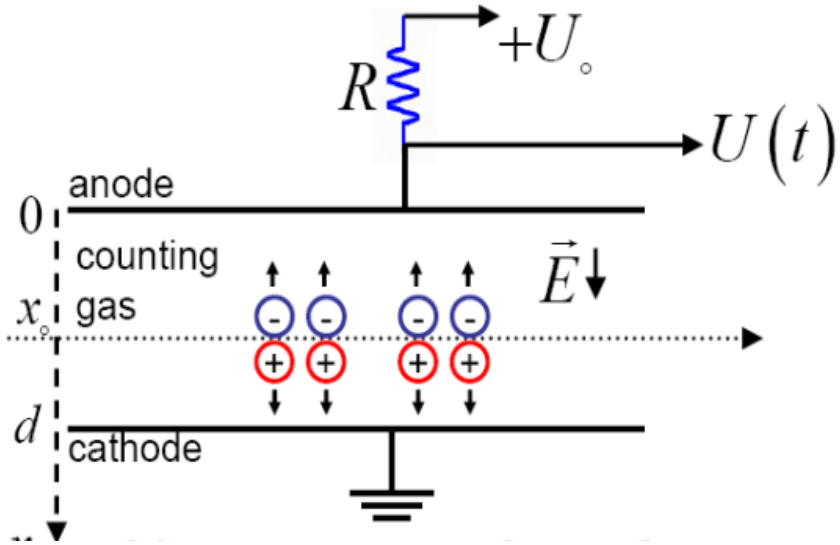
Camere a ionizzazione

Nel caso più semplice una camera ad ionizzazione consiste in un sistema di elettrodi paralleli. Un voltaggio applicato fra gli elettrodi produce un campo elettrico omogeneo (lontano dai bordi). Gli elettrodi sono montati in una scatola a tenuta riempita con gas o con liquido o con solido.



R (di solito molto grande) serve a disaccoppiare l'anodo dalla sorgente di tensione V_o, il rivelatore ha una capacita' C

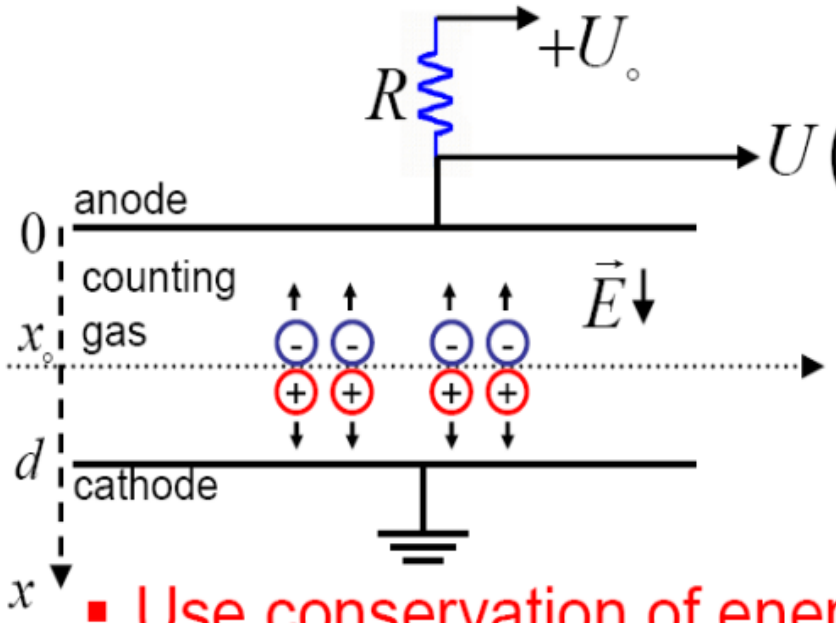
Camere a ionizzazione



- The parallel electrodes of the ionisation chamber, acting as a capacitor with capacitance C, are initially charged to the voltage U_0.
- To simplify the consideration let us assume that the load resistor R is very large so that the capacitor can be considered to be independent (the voltage of the capacitor can then vary, it is not kept fixed by the voltage source), that is ssume that the e^-'s and ions collection times are long before the power supply can recharge the plates back to U_o (RC_d very large) the charge on each plate will be diminished by N|e|,
- Suppose N charge-carrier pairs are produced along the particle track at a distance x_0 from the anode.
- The drifting charge carriers induce an electric charge on the electrodes which leads to certain change of the voltage, ΔU.
- Thereby the stored energy $(1/2)CV_0^2$ will be reduced to $(1/2)CV^2(t)$ because of the work done by the field to move the charge carriers

Camere a ionizzazione

Parallel electrodes



$$\left|\vec{E}\right| = \frac{U(t)}{d} \qquad C = \frac{\varepsilon A}{d}$$

$$\int_0^{t_d^-} v^- dt = x_\circ$$

$$\int_0^{t_d^+} v^+ dt = d - x_\circ$$

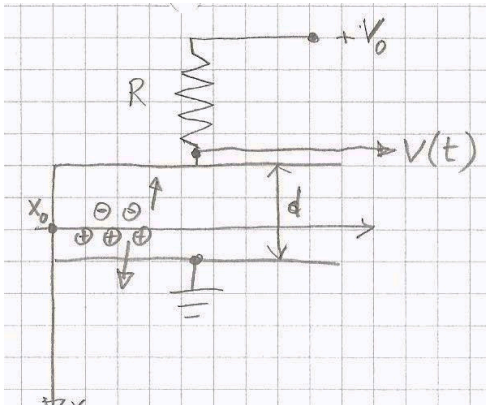
$v^+ < v^-$ drift velocities depend on the electric field

- Use conservation of energy in C

$\frac{1}{2}CU_\circ^2 = \frac{1}{2}CU^2(t)$ + work done by $\vec{E}$ on drifting charges

obtain $$-\frac{dU(t)}{dt} = \frac{Ne}{Cd}\left[v^- + v^+\right]$$ holds for $v = v\left(\left|\vec{E}\right|(t)\right)$

Camere a ionizzazione



Dalla cons. dell'energia

$$\frac{1}{2}CV^2(t) = \frac{1}{2}CV_0^2 - N\int_{x_0}^{x} qE_x\,dx \quad\Rightarrow\quad \frac{1}{2}C\left(V(t)^2 - V_0^2\right) = -NqE_x(x-x_0)$$

$$\frac{1}{2}C\left(V(t)+V_0\right)\left(V(t)-V_0\right) = -NqE_x(x-x_0)$$

- Il segnale $V(t) \approx V_0$ dV piccolo!
  e $V(t) - V_0 = \Delta V = V_R$
  $V(t) + V_0 \simeq 2V_0$
  $E_x = V_0/d$
  NB: $v^- >> v^+$

C è la capacità della camera + quelle parassite

$$CV_0\,\Delta V \approx -Nq\frac{V_0}{d}(x-x_0)$$

$$\Rightarrow \Delta V = -\frac{Nq}{Cd}(x-x_0) \quad \text{con} \quad x - x_0 = v_D t \Rightarrow$$

$$\Delta V = -\frac{Nq}{Cd}v_D t \Rightarrow \begin{cases} \Delta V^+ = -\dfrac{Ne}{Cd}v^+ t, & t < t_D^+ \\ \Delta V^- = -\dfrac{N}{Cd}(-e)(-v^-)t = -\dfrac{Ne}{Cd}v^- t, & t < t_D^- \end{cases}$$

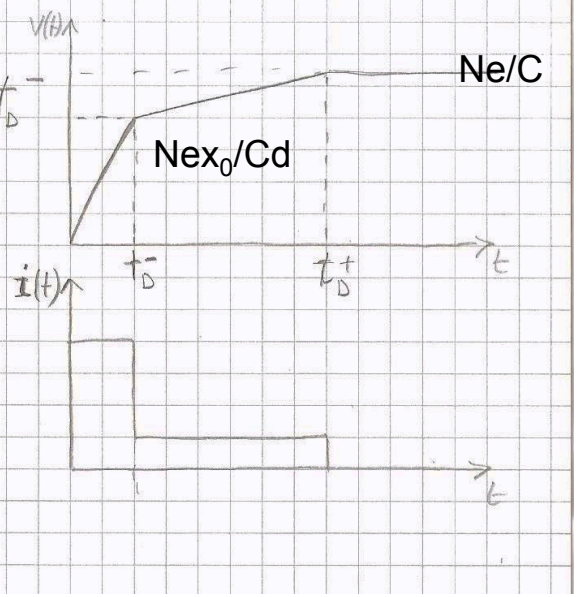
$$\Delta V(t) = \Delta V^+(t) + \Delta V^-(t)$$

$$\Delta V^+(t_D^+) = -\frac{Ne}{Cd}v^+ t_D^+ = -\frac{Ne}{Cd}(d - x_0)$$

La caduta di pot. per $t = t_D^+$ è

$$\Delta V_{Max} = -\frac{Ne}{Cd}(d - x_0 + x_0) = -\frac{Ne}{C}$$

$$\Delta V^-(t_D^-) = -\frac{Ne}{Cd}v^- t_D^- = -\frac{Ne}{Cd}x_0$$



La carica raccolta sugli elettrodi è $Q = C\Delta V_{max} = -Ne$

La corrente nella camera $i = \dfrac{dQ}{dt} = C\dfrac{dV}{dt} = -\dfrac{eN}{d}(v^+ + v^-)$

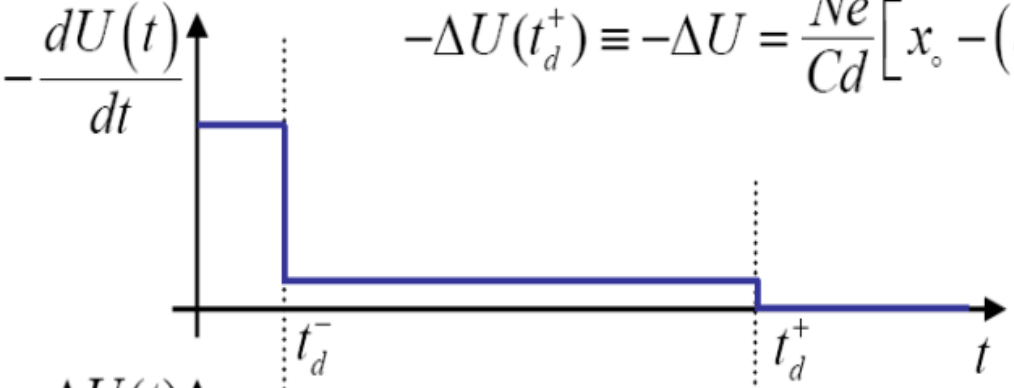
Ionization chambers

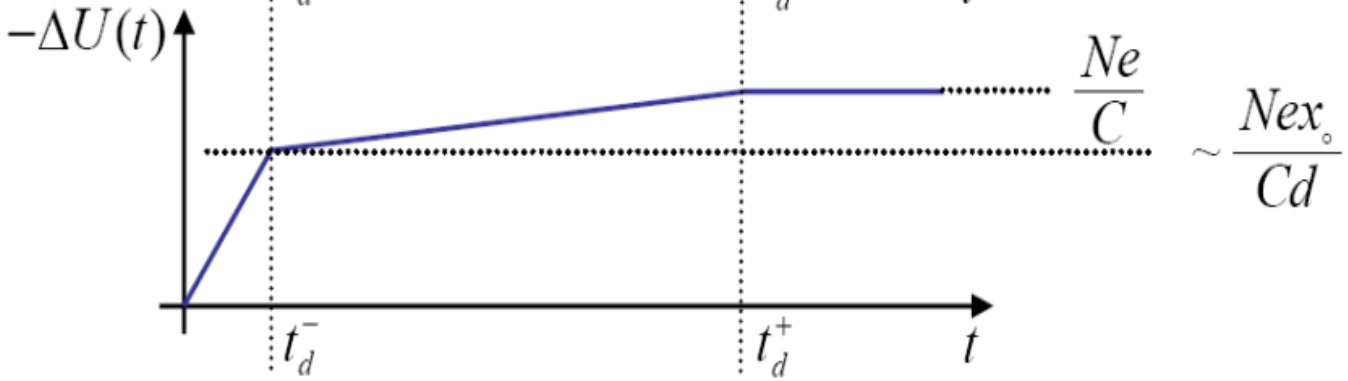
Signal

$$\Delta U(t) = U(t) - U_\circ = \Delta U^-(t) + \Delta U^+(t)$$

$$-\Delta U(t_d^+) \equiv -\Delta U = \frac{Ne}{Cd}\left[x_\circ - (d - x_\circ)\right] = \frac{Ne}{C}$$

$$\Delta Q = -Ne = C\Delta U$$



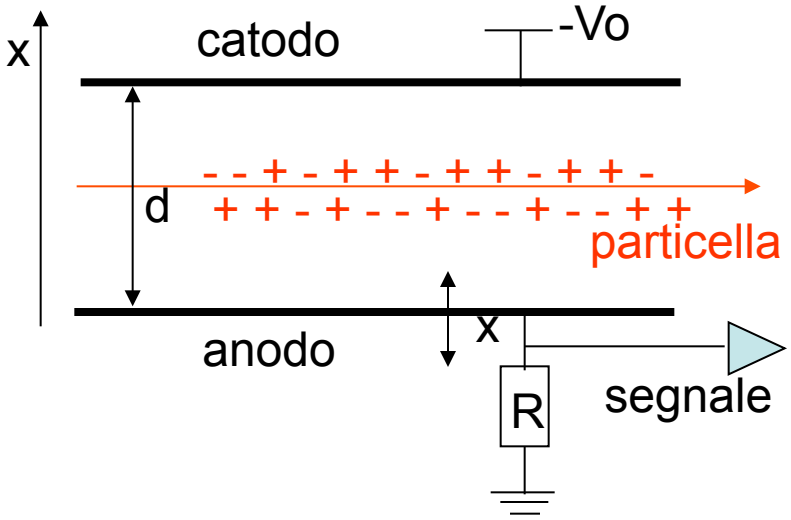


$$\sim \frac{Nex_\circ}{Cd}$$

$$\frac{\Delta U^+}{\Delta U^-} = \frac{d - x_\circ}{x_\circ} \qquad \text{If } x_\circ = \tfrac{1}{2}d \quad \text{then} \quad \frac{\Delta U^+}{\Delta U^-} = 1$$

Camere a ionizzazione

Si puo' ottenere lo stesso risultato osservando la variazione di energia potenziale delle particelle in moto fra le piastre. Il risultato e' lo stesso (ovviamente). In questo caso la carica raccolta e' quella che si muove, nel caso precedente la carica raccolta e' indotta sulla armature dal moto delle cariche fra le piastre.



Il campo elettrico nella camera è costante $E=V_0/d$

carica q a distanza x dall'anodo → $U=qV(x)$
se la carica si sposta di dx →

$$\Delta U=qV(x+dx)-qV(x)= -qEdx$$

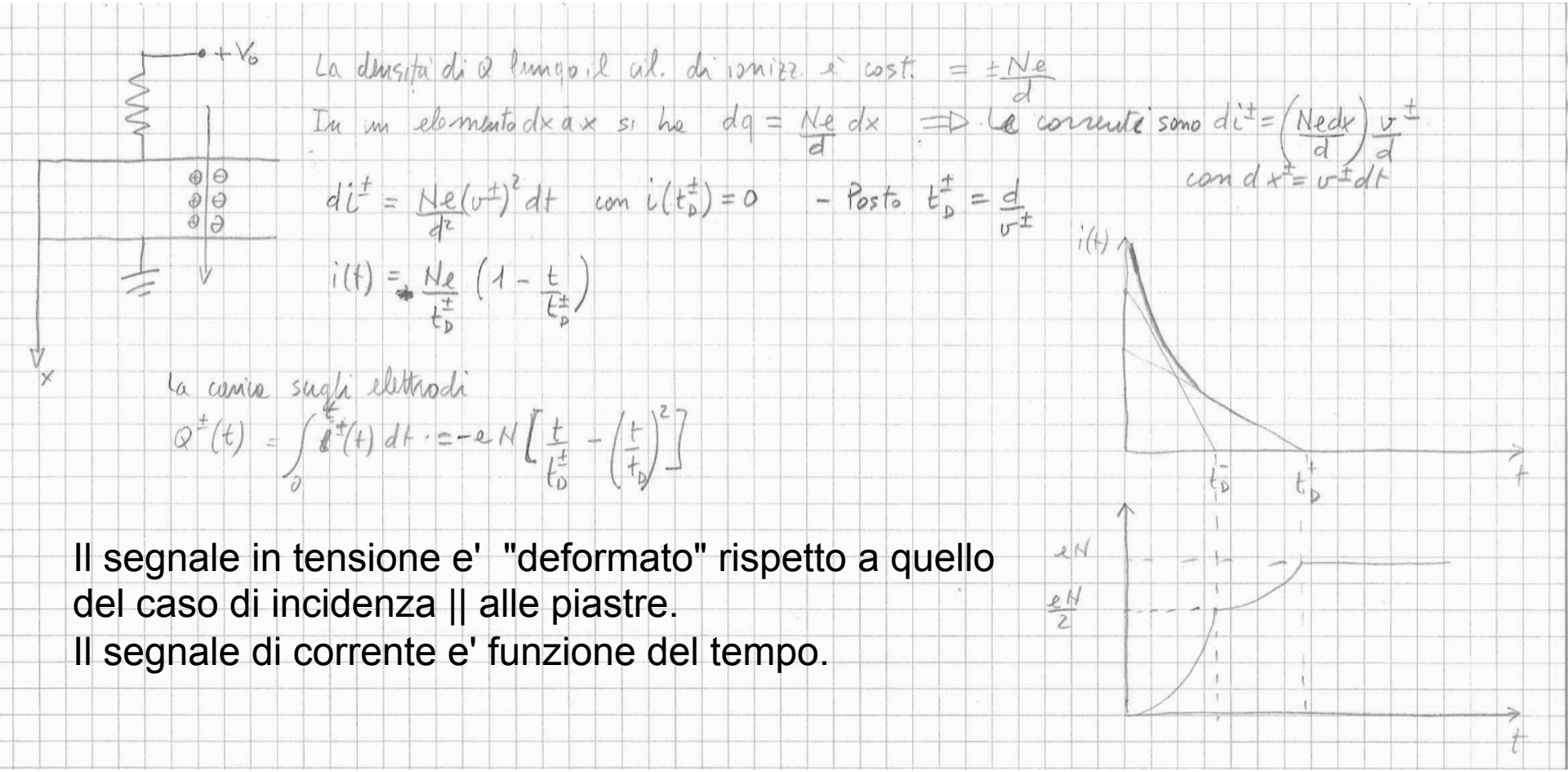
La variazione di energia potenziale ΔU deve essere compensata dal lavoro del generatore $V_0 idt=V_0 dQ$ ➔

$-qEdx=V_0 idt$ ed $E = V_0/d$
➔ $i=-q(v/d)$

i è dunque il segnale in corrente.

Camere a ionizzazione

Se la particella attraversa la camera in direzione perpendicolare alle piastre:



Il segnale in tensione e' "deformato" rispetto a quello del caso di incidenza || alle piastre.
Il segnale di corrente e' funzione del tempo.

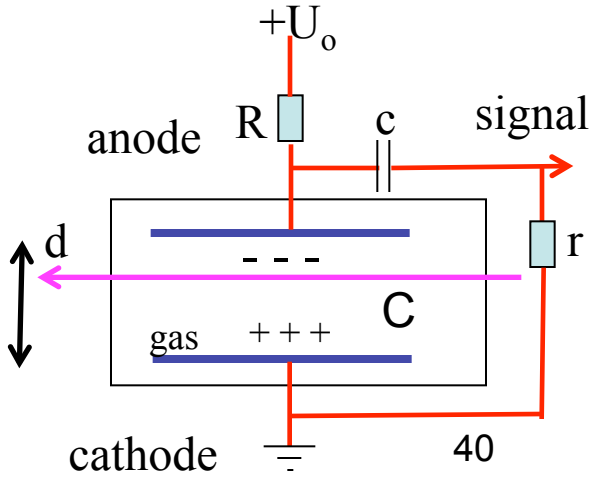
Camere a ionizzazione

- These considerations are only true if the charging resistor is infinitely large or, more precisely, if $t^{\pm}_D$ << RC, time constant of the apparatus (C is the total capacitance: camera+circuit+parasitic elements).
- When RC < ∞, in this case one obtains

$$\Delta U^{+} = -\frac{Ne}{d} v^{+} R(1 - \mathrm{e}^{-\Delta t/RC}) \ ,$$

$$\Delta U^{-} = -\frac{N(-e)}{d} (-v^{-}) R(1 - \mathrm{e}^{-\Delta t/RC}) \ .$$

NB: this R corresponds to r in figure



Camere a ionizzazione

- In practical cases RC is usually large compared to t_-, but smaller than t_+, $t_- << RC << t_+$. In this case one obtains

$$\Delta U = -\frac{Ne}{Cd}x_0 - \frac{Ne}{d}v^+ R(1 - \mathrm{e}^{-\Delta t/RC})$$

- For electric field strengths of 500V/cm and typical drift velocities of $v- = 5$ cm/μs, collection times for electrons of 2 μs and for ions of about 2 ms are obtained for a drift path of 10 cm. If the time constant $RC \gg 2$ms, the signal amplitude is independent of x0.
- For many applications this is much too long. If one restricts oneself to the measurement of the electron component, which can be done by differentiating the signal, the total amplitude will not only be smaller, but also depend on the point in which the ionisation is produced.

Camere a ionizzazione

Percio' se introduciamo un R'C' all'ingresso dell'amplificatore tale che:

$$\Delta t^{-} << R'C' << \Delta t^{+}$$

Avremo un segnale in tensione essenzialmente dovuto solo agli elettroni.

Camere a ionizzazione

- In electron-sensitive operation,however,the portion of the pulse derived above that corresponds to drift of the ions is almost entirely lost by choosing a collection time constant that is much shorter than the ion collection time. The pulse that remains then reflects only the drift of the electrons and will have an amplitude Vmax = (eN/C)(x/d)

NB: in this case, the signal amplitude is reduced and depends on the distance from electrodes