Particle Detectors

Lecture 4
11/03/16

a.a. 2015-2016

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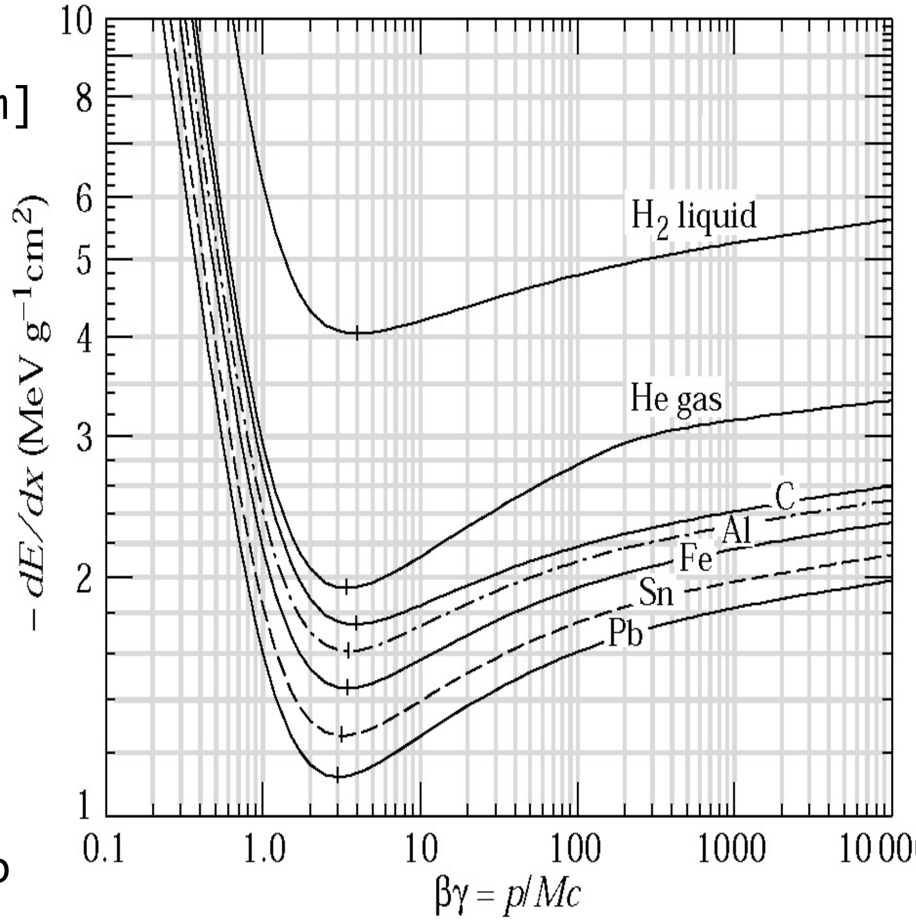
Ionizzazione: Bethe-Bloch

$$\frac{dE}{dx} = \frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 m_e v^2}[ln(\frac{2\gamma^2 m_e v^2}{I}) - \frac{v^2}{c^2}]$$ [E/cm]

Eliminiamo la densita' di elettroni. La densita' di numero di atomi del mezzo e' n = ρ/m_{mezzo}
La massa m_{mezzo} si ottiene dalla relazione $m_{mezz} = M_{mol}/N_A = A/N_A$, M_{mol} = massa molare (Kg/mol) = A kg/mol → $n_{mezzo} = \rho N_A/A$,
Il numero atomico e' Z quindi ci sono $N_e = Zn_{Be} = \rho N_A(Z/A)$ elettroni

$$\frac{dE}{dx} = \frac{N_A e^4 z^2}{4\pi\varepsilon_o m_e v^2}\frac{Z}{A}\rho\left\{\ln\left[\frac{2m_e c^2\beta^2\gamma^2}{I}\right] - \beta^2\right\}$$



La perdita dE non dipende SOLO dallo dallo spessore, ma dal prodotto **ρdx** (gcm^{-2}). Posto quindi X = ρx si ha

$$\frac{dE}{dX} = \frac{N_A e^4 z^2}{4\pi\varepsilon_o m_e v^2}\frac{Z}{A}\rho\left\{\ln\left[\frac{2m_e c^2\beta^2\gamma^2}{I}\right] - \beta^2\right\}$$ [E/(g/cm^{-2})]

Per cui dE/dX **NON** dipende dalla densita' del mezzo ed e' "universale". Le differenze osservate sono legate al potenziale di ionizzazione I, che varia da elemento ad elemento

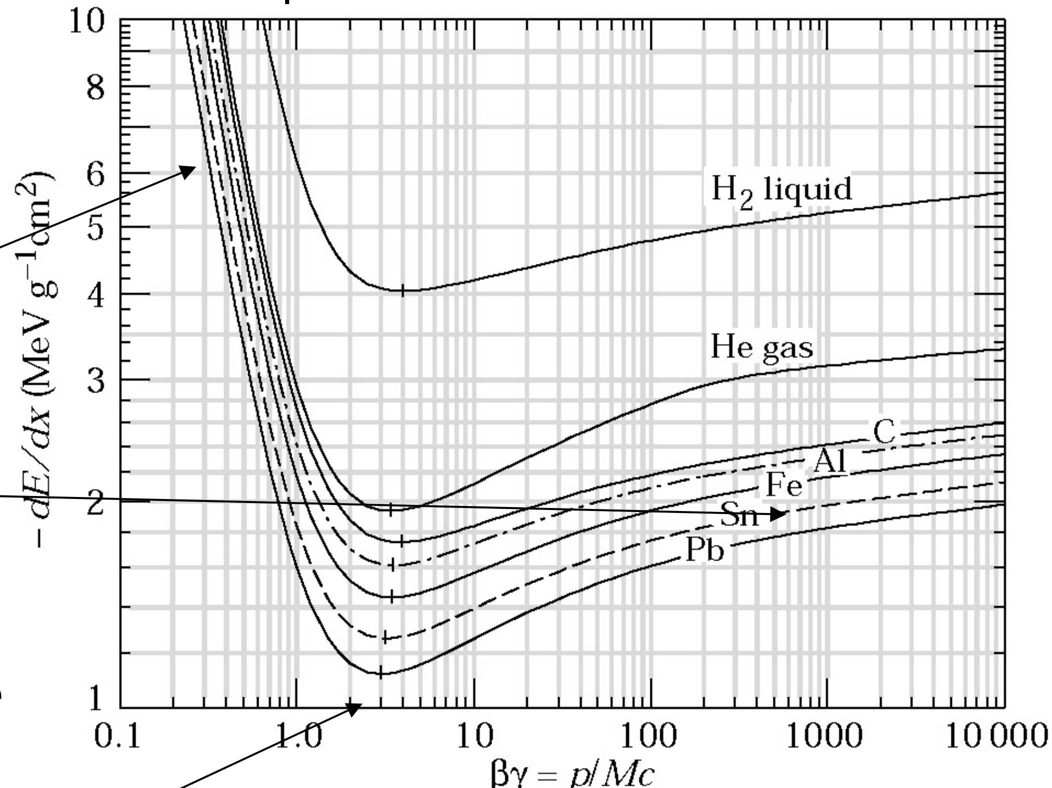
Material thickness

- Thickness of material (x) is usually measured in gm/cm^2
- Related to distance (y) traversed via density:
- $x = y\ \rho$
- Used because relevant issue in determining interaction in matter is how many scattering centres are traversed
- Benchmark values for traversing 1 cm of material:
  - Lead: $x = 11.4\ gm/cm^2$
  - Iron: $x = 7.9\ gm/cm^2$
  - Water: $x = 1.0\ gm/cm^2$
  - Liquid H_2 $x = 0.07\ gm/cm^2$
  - Air: $x \sim 0.001\ gm/cm^2$

Ionizzazione: Bethe-Bloch

$$\frac{dE}{d\xi} = K\frac{z^2 Z}{\beta^2 A}[ln(\frac{2m_e c^2\gamma^2\beta^2}{I}) - \beta^2]$$

- Non dipende dalla massa della particella incidente
- Dipende dal quadrato della carica della particella incidente



Misurando $\gamma = E/Mc^2$
e contemporaneamente
$dE/d\xi$ si misura z

A bassa energia $\propto \beta^{-2}$

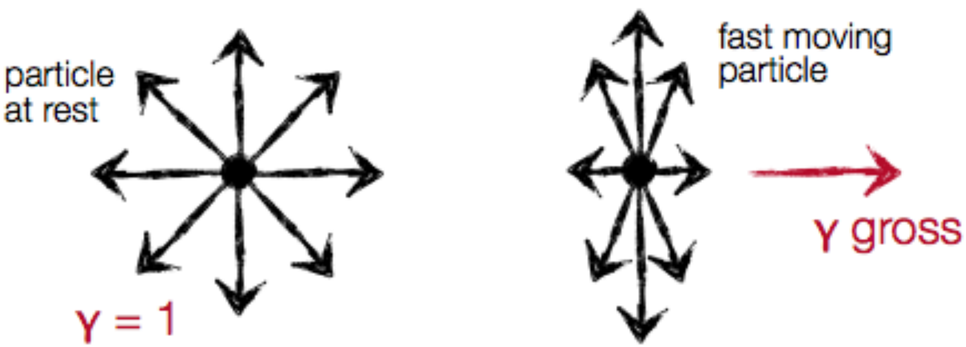
Ad alta energia $\propto \ln\gamma^2$

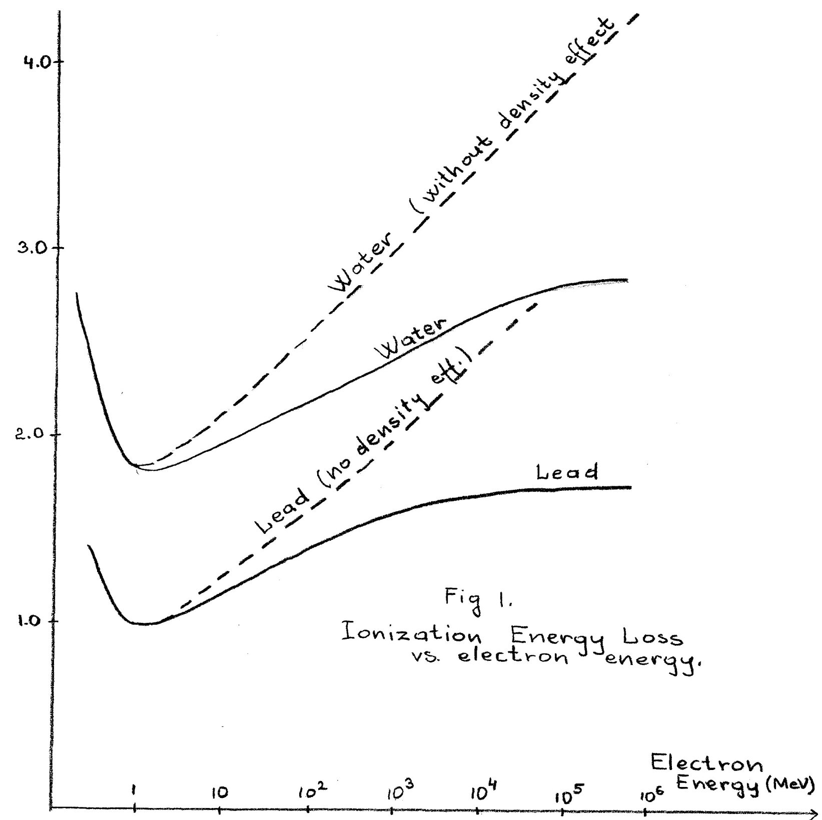
Minimo a $\beta\gamma \approx$ 2-3,
corrispondente a $E \approx Mc^2$, oltre
questo limite la particella e'
relativistica
(Minimum Ionizing Particle=MIP).

Relativistic rise for $\beta\gamma > 4$

Important at high energies. BB over-estimates the energy loss at high momentum of the incident particle.

High energy particle: transversal electric field increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$. Thus interaction cross section increases ...





Experimental curve increases less rapidly than ln γ^2

But rather as ~ln γ and then it reaches a plateau, named “Fermi plateau”

effetto densita'

High energy particle: transversal electric field increases
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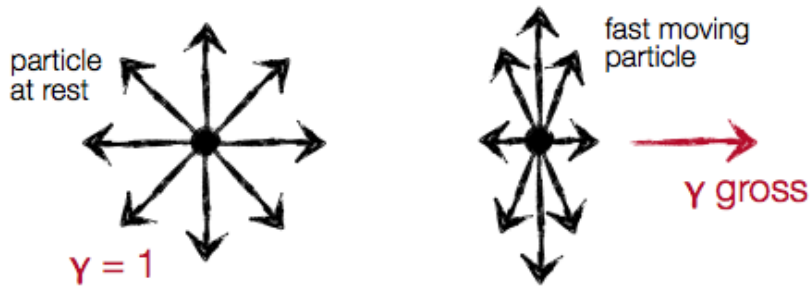
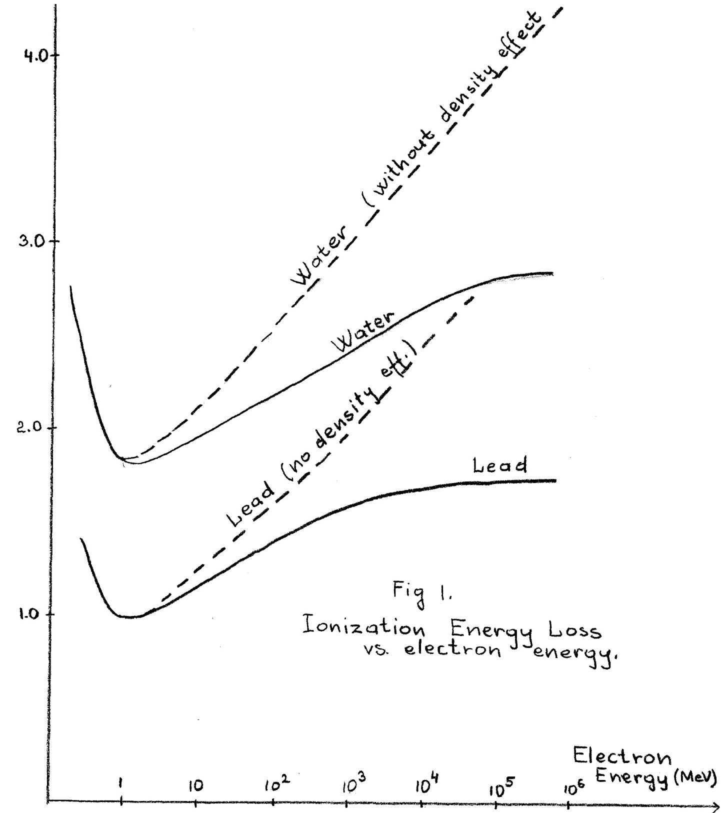
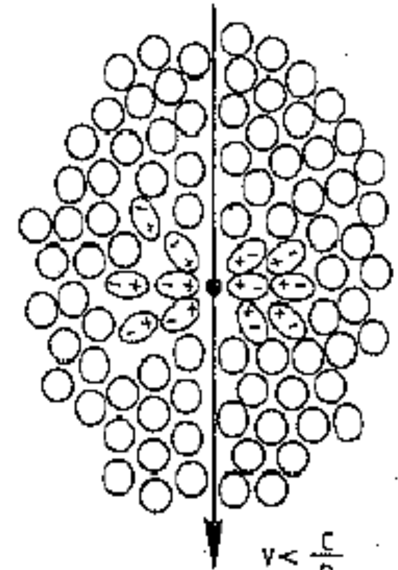


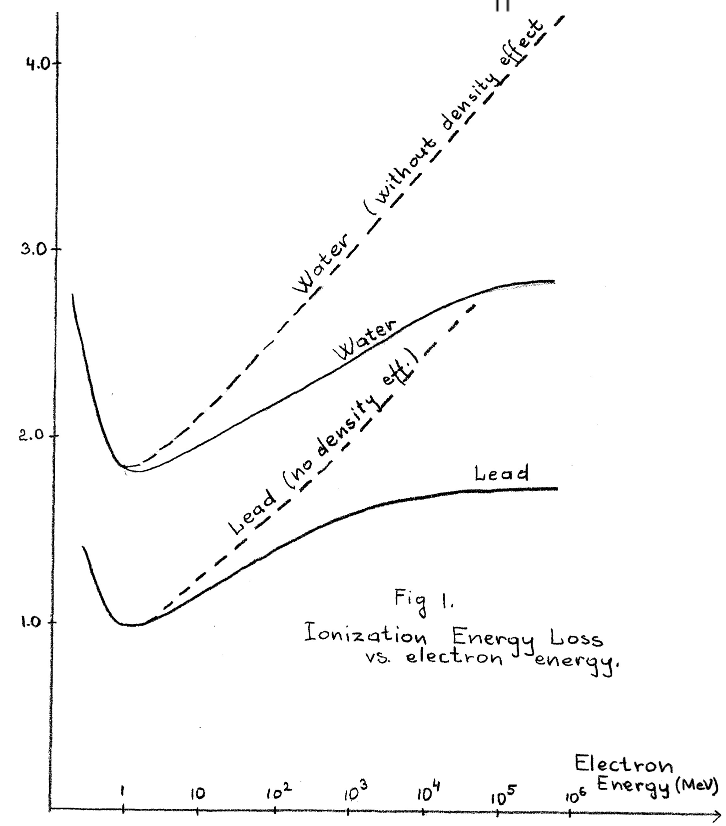


- per part relativistiche, la dE/dX tende al cosidetto plateau di Fermi, mentre la BB prevede che dE/dX cresca
- Per part incidenti ad alta E viene meno una delle ipotesi (sottintese) fatte per la BB, cioe' che fosse possibile calcolare l'effetto dei campi elm della part incidente su ciascun e- di ciascun atomo e quindi sommare in modo incoerente le E cedute a tutti gli e-di tutti gli atomi per i quali $b_{min} < b < b_{max}$

Effetto densita'

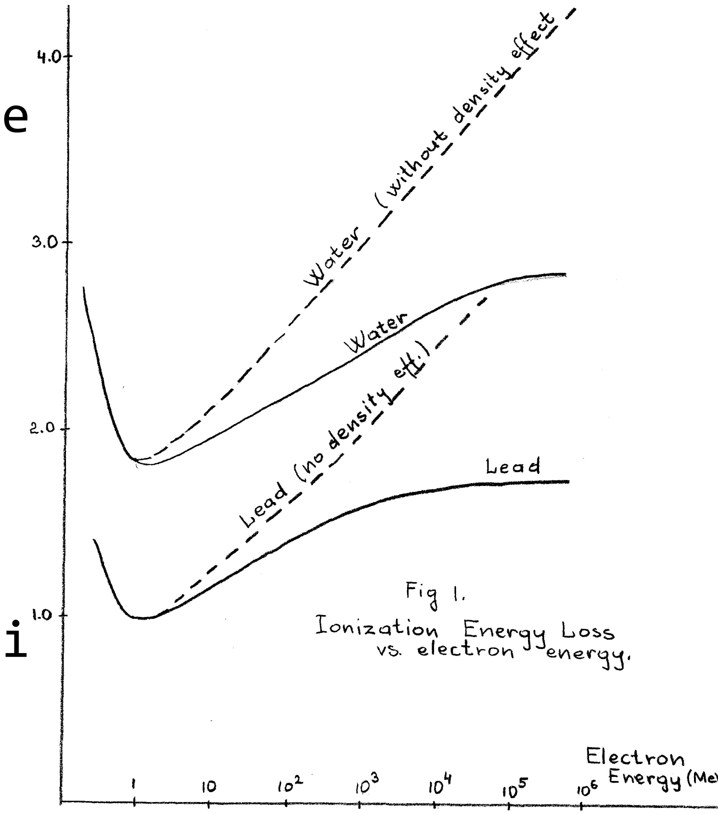
- Ora $b_{max}=\pi v\gamma/\omega_o$ >> R_{atomo}, specie quando γ>>1
- Percio' in un mezzo denso ci saranno molti atomi interposti fra la part incidente e l'atomo preso in considerazione, per b ~ bmax
- Questi atomi, perturbati dai campi della part incidente, produrranno campi perturbanti nella posizione occupata dall'atomo, modificando la risposta ai campi della part veloce
- In altre parole, nei mezzi densi la polarizzazione dielettrica del materiale altera i campi della particella veloce rispetto al valore che avrebbero nello spazio vuoto ai valori caratteristici dei campi macroscopici entro un dielettrico





Effetto densita'

- Nel calcolo delle energia trasferite negli urti lontani (b>>Ratomo) occorre tenere cont di tali modificazioni, dovute alla polarizzazione del mezzo: negli urti stretti (b<<Ratomo) la particella interagisce soltanto con atomo alla volta e si puo' usare l'approssimazione di e- liberi, senza polarizzazione
- Per urti lontani assumiamo che i campi nel mezzo si possano calcolare nell'approx del continuo, nella quale le proprieta' dielettriche si possono descrivere mediante una costante dielettrica $\varepsilon(\omega)$
- Calcolo complicato (i soliti entusiasti lo possono trovare in "Elettrodinamica classica", Jackson, 2° edition, cap. 13)



NB: la polarizzazione si oppone al campo perturbante, cioe' lo diminuisce

Correzioni di densita'

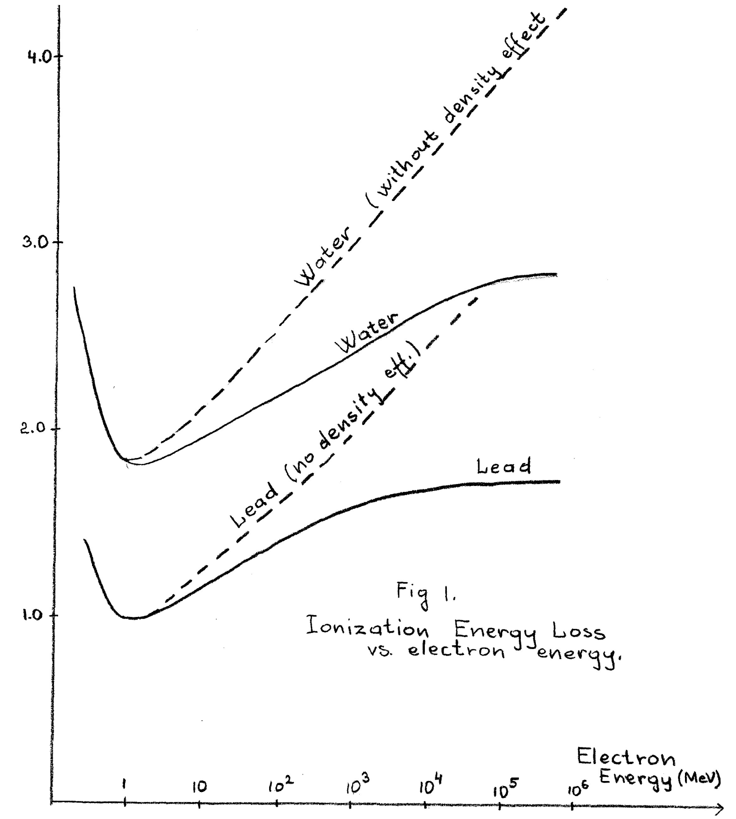
Fermi ha calcolato le perdite di energia dovute a urti lontani

$$\left(\frac{dE}{dX}\right)_{b>a} = \frac{(ze)^2\omega_p^2}{c^2}\ln\left(\frac{1.123c}{a\omega_p}\right)$$

con $\omega_p = (4\pi NZe^2/m)^{1/2}$ frequenza di plasma elettronico e a=raggio atomico

senza le correzioni di densita' si avrebbe

$$\left(\frac{dE}{dX}\right)_{b>a} = \frac{(ze)^2\omega_p^2}{c^2}\ln\left(\frac{1.123\gamma c}{a\langle\omega\rangle}\right)$$



- La perdita di energia non dipende piu' dai dettagli della struttura atomica attraverso $\langle\omega\rangle = 2\langle I\rangle/h$, ma solo dalla densita' di elettroni.
- Nel limite ultrarelativistico diventa costante.
- Due materiali con strutture atomiche molto diverse avranno la stessa perdita di energia per particelle ultrarelativistiche se le loro densita' sono tali che la densita' di elettroni e' la stessa.

Correzioni di densita'

Per mantenere la forma della bethe bloch, calcolata senza l'effetto di schermo, ed eliminare il raggio atomico a, si parametrizza l'effetto della polarizzazione con un parametro δ che e' pari alla differenza tra la perdita di energia calcolata con effetto di polarizzazione e quello che si ha senza la polarizzazione $\delta/2 = (dE/dX)_{pol} - (dE/dX)_{nopol}$ cosi' che nel limite di alte energie

$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln \beta\gamma - 1/2$$

E La perdita di energia diventa

$$-\frac{dE}{dx} = \rho K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{2mc^2 \beta^2 \gamma^2}{I} - \beta^2 - \frac{\delta(\gamma)}{2} \right]$$

Con queste correzioni si vede subito che ad alti $\beta\gamma$ dE/dX cresce come $\ln(\beta\gamma/h\omega_p)$ piuttosto che come $\ln((\beta\gamma)^2/I)$ (ie risalita relativitica) → Dato che ω_p dipende da $N_e^{1/2}$, materiali con la stessa densita' di e- hanno la stessa perdita indipendentemente dal potenziale di ionizzazione.
Dato che $Ne = \rho N_A(Z/A)$, la risalita e' piu' rapida per materiali meno densi, p. es. gas rispetto a liquidi e solidi e da cui il nome "effetto densita'"

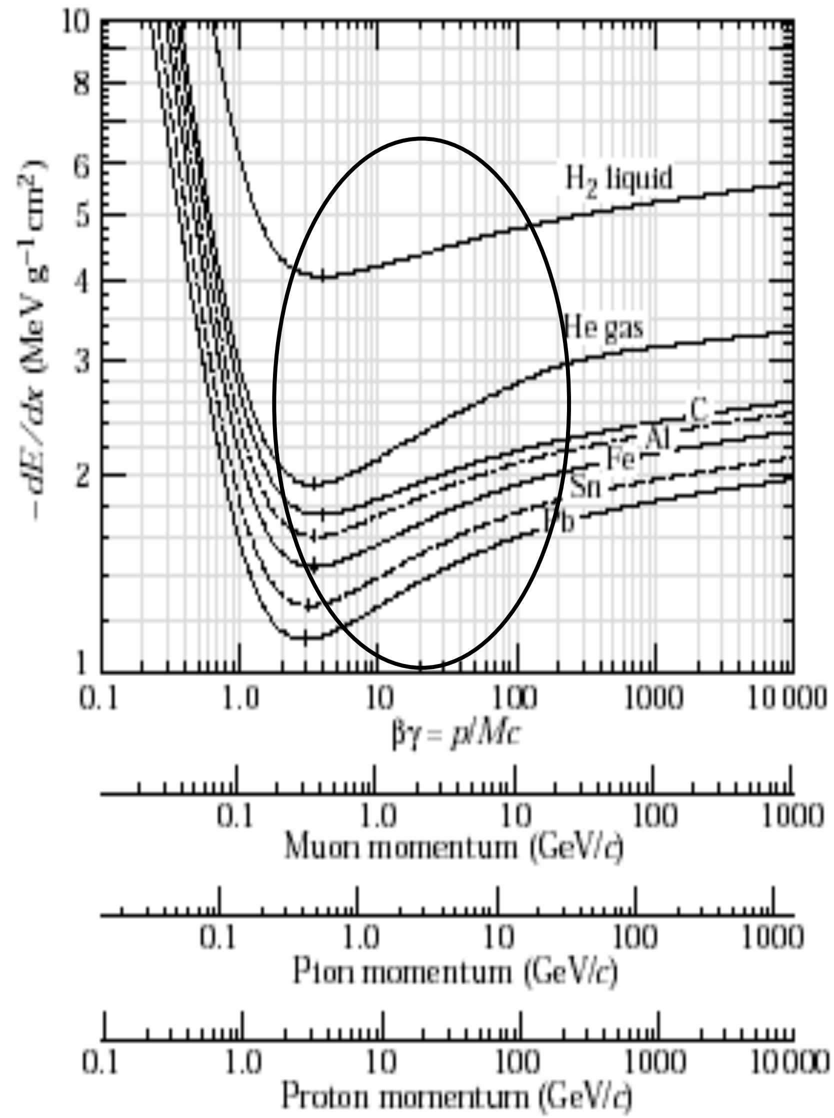
Bethe Bloch: effetto densita'

$$-\frac{dE}{dx} = \rho K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{2mc^2\beta^2\gamma^2}{I} - \beta^2 - \frac{\delta(\gamma}{2}\right.$$

La salita relativistica satura crescendo γ → **plateau di Fermi.**

Più denso è il mezzo tanto prima si raggiunge il plateau di Fermi → la salita relativistica è più importante nei gas che nei liquidi e nei solidi.

Funziona fino al % per particelle fino al nucleo di α per β 0.1 - 1.0. Per basse velocità (β~0.05) non è più valida in quanto non sono più valide molte delle assunzioni di Bethe Block (ie non posso piu' trascurare l'energia di legame degli e-).



Bethe Bloch: effetto densita'–formule pratiche

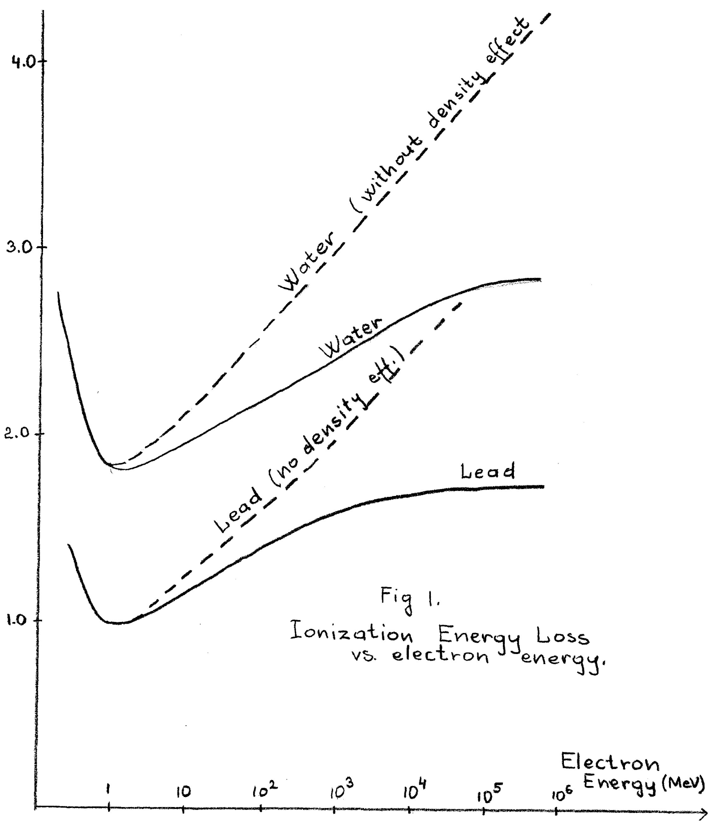
Nella pratica δ viene tabulato in funzione di $\beta\gamma$ per differenti materiali

$$-\frac{dE}{dx} = \rho K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{2mc^2\beta^2\gamma^2}{I} - \beta^2 - \frac{\delta(\gamma)}{2}\right]$$

Al plateau di Fermi,

- $(dE/dX)_{plateau} \approx$ 1.05-1.1 $(dE/dX)_{MIP}$ per i solidi
- $(dE/dX)_{plateau} \approx$ 1.5-1.1 $(dE/dX)_{MIP}$ per i gas

$$\delta = \begin{cases} 0 & X < X_o \\ 4.0652X + C + a(X_1 - X)^m) & X_o < X < X_1 \\ 4.0652X + C & X > X_1 \end{cases}$$



X = ln($\beta\gamma$)

Table 2.1. Constants for the density effect correction

Material	I [eV]	$-C$	a	m	X_1	X_0
Graphite density = 2	78	2.99	0.2024	3.00	2.486	−0.0351
Mg	156	4.53	0.0816	3.62	3.07	0.1499
Cu	322	4.42	0.1434	2.90	3.28	−0.0254
Al	166	4.24	0.0802	3.63	3.01	0.1708
Fe	286	4.29	0.1468	2.96	3.15	−0.0012
Au	790	5.57	0.0976	3.11	3.70	0.2021
Pb	823	6.20	0.0936	3.16	3.81	0.3776
Si	173	4.44	0.1492	3.25	2.87	0.2014
NaI	452	6.06	0.1252	3.04	3.59	0.1203
N_2	82	10.5	0.1534	3.21	4.13	1.738
O_2	95	10.7	0.1178	3.29	4.32	1.754
H_2O	75	3.50	0.0911	3.48	2.80	0.2400
Iucite	74	3.30	0.1143	3.38	2.67	0.1824
Air	85.7	10.6	0.1091	3.40	4.28	1.742
BGO	534	5.74	0.0957	3.08	3.78	0.0456
Plastic Scint.	64.7	3.20	0.1610	3.24	2.49	0.1464

Bethe Bloch Formula

- Bethe-Bloch law for "stopping power":

NB: it has a slightly different but equivalent form

$$-\frac{dE}{dX} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\gamma^2\beta^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$ Units: MeV g^{-1} cm^2

constant $4\pi N_A r_e^2 m_e c^2$ = 0.31 MeV g^{-1} cm^2

- Where:
  - $X = \rho x$, where ρ is the density of the absorber material
  - N_A = Avagadro no.
  - $r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2}$ = classical electron radius = 2.82×10^{-13} cm
  - z = charge of incident particle in units of e
  - Z, A = atomic no. and atomic weight of the absorber
  - T_{max} = maximum kinetic energy which can be imparted to a free electron in a single collision. For heavy particles ($m \gg m_e$), $T_{max} = 2m_e c^2\beta^2\gamma^2$

Energy loss of electrons and positrons

Calculation of electron and positron energy loss due to ionization and excitation complicated due to:

- spin ½ in initial and final state
- small mass of electron/positron
- identical particles in initial and final state for electrons

Form of equation for energy loss similar to "heavy particles"

$$-\frac{dE}{dX}\bigg|_c = 2\pi N_a r_e^2 m_e c^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I^2/m_e c^2)}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

τ is kinetic energy of incident electron in units of $m_e c^2$. $\tau = E_{ke}/m_e c^2 = \gamma - 1$

$$F(\tau)_{e^-} = 1 - \beta^2 + \frac{\tau^2/8 - (2\tau+1)\ln 2}{(\tau+1)^2} = 1 - \beta^2 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 + \frac{(2\gamma-1)\ln 2}{\gamma^2}$$

Note:
Typo on P38 of Leo
2r®2t

$$F(\tau)_{e^+} = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{(\tau+2)} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{(\gamma+1)} + \frac{10}{(\gamma+1)^2} + \frac{4}{(\gamma+1)^3}\right)$$

For both electrons and positrons *F*(t) becomes a constant at very high incident energies.

Comparison of electrons and heavy particles (assume β=1):

$$-\frac{dE}{dx}\bigg|_c \propto \left[2\ln\left(\frac{2m_e c^2}{I}\right) + A\ln\gamma - B\right]$$

	A	B
electrons:	3	1.95
heavy	4	2

Bethe Bloch: elettroni

E_{max}, nel caso degli e-, si ottiene dalla formula ottenuta nel caso di particelle pesanti

$$E_c = 2m_e c^2 \gamma_{cm}^2 (v_{cm}/c)^2 \qquad v_{cm} = \frac{M\gamma v}{(M\gamma + m_e)}$$

Sostituendo si ha

$$E_c = \frac{2m_e M^2\gamma^2 v^2}{(m_e+\gamma M)^2} \cdot \frac{1}{1 - \frac{M^2\gamma^2 v^2}{c^2(m_e+\gamma M)^2}} \Rightarrow E_c = \frac{2m_e M^2\gamma^2 v^2}{(m_e+\gamma M)^2 - M^2\gamma^2\beta^2}$$

$$E_c = \frac{2m_e M^2\gamma^2 v^2}{[m_e^2 + 2\gamma M m_e + M^2\gamma^2(1-\beta^2)]} \qquad E_c = \frac{2m_e M^2\gamma^2 v^2}{[m_e^2 + 2\gamma M m_e + M^2]}$$

Se $M=m_e$ → $$E_c = \frac{m_e\gamma^2 v^2}{[1+\gamma]}$$

Bethe Block

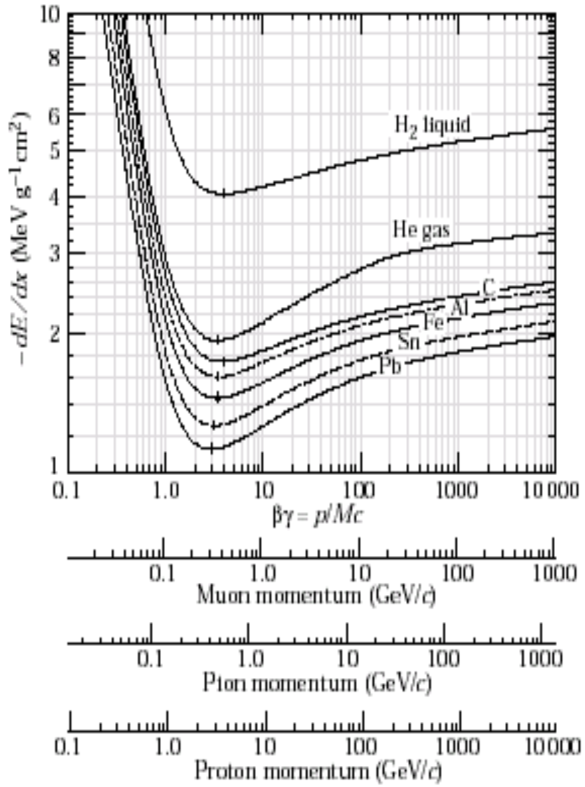


Figure 26.3: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 26.20.

Particelle della stessa velocità hanno praticamente la stessa dE/dx in materiali diversi, se escludiamo l'idrogeno. È presente una piccola diminuzione della perdita di energia all'aumentare di Z.

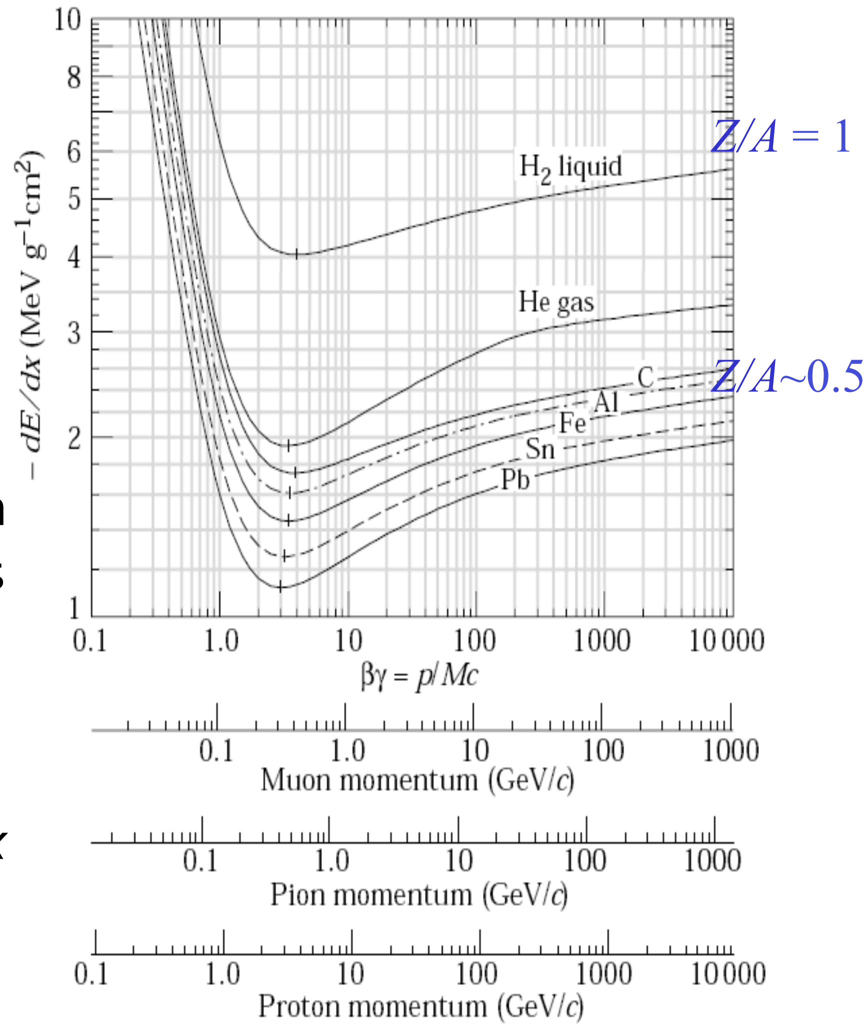
In pratica, la maggioranza delle particelle relativistiche hanno una perdita di energia simile a quella del minimo MIP (minimum ionizing particle).

La perdita di energia è normalmente espressa in termini della densità di area $dS=\rho dx$ e le particelle ionizzanti al minimo perdono in media 1.94 MeV/(gr/cm^2) in He, 1.08 in Uranio e ~4 MeV/(gr/cm^2) in H_2.

dE/dx dependence on medium

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \frac{z^2}{\beta^2}\frac{Z}{A}\left\{\frac{1}{2}\ln\left[\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2}\right] - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right\}$$

- Curves scale reasonably well
- Scaling with medium ~Z/A, but *I* rises with Z
- Note larger relativistic rise in Helium gas; less rise in liquids and solids (screening due to density)
- Recall that linear distance *y* related to *x* via mass density: $x = \rho y$



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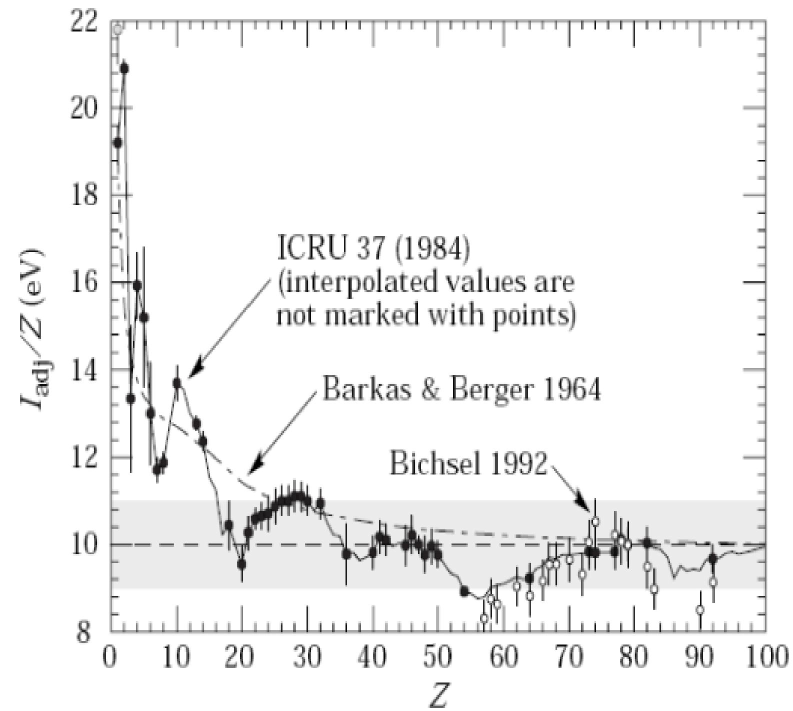
Ionization potential

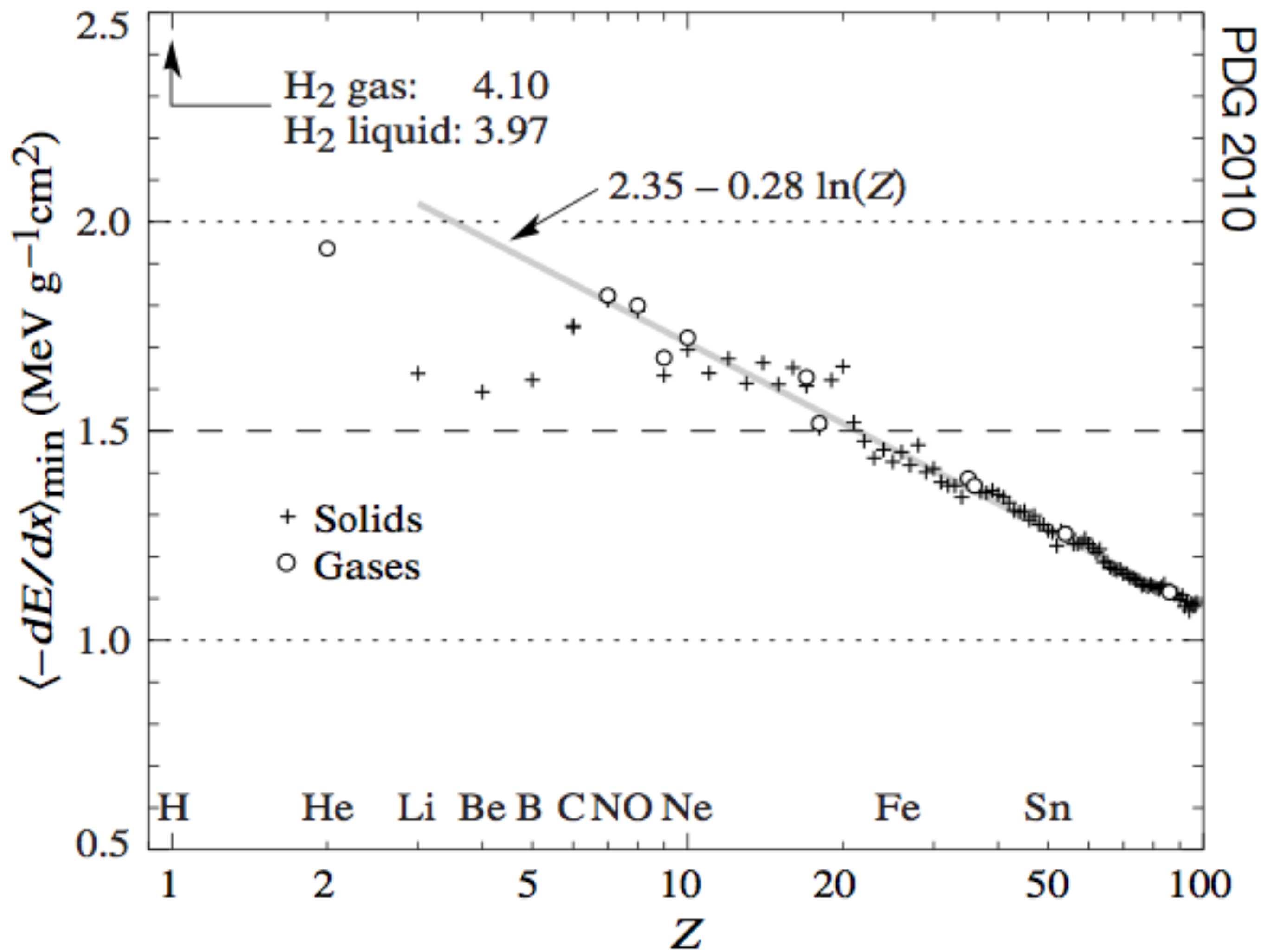
- Measured values indicate non-trivial structure vs Z (note suppressed zero)
- Crude approximation at large Z is I = 10Z, accurate to about 10%
- An approximate empirical relation is:

$$I \approx I_0 Z^{-0.9},$$

with I_0=16 eV



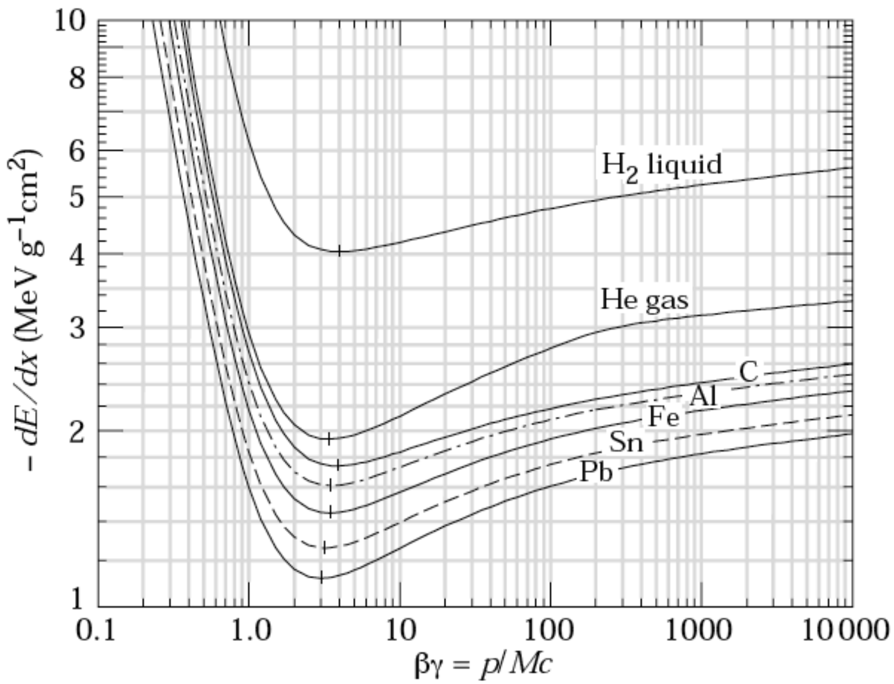
MIP dependence on chemical species



Stopping power at minimum ionization for the chemical elements. The straight line is fitted for Z > 6.
A simple functional dependence on Z is not to be expected, since <–dE/dx> also depends on other variables.

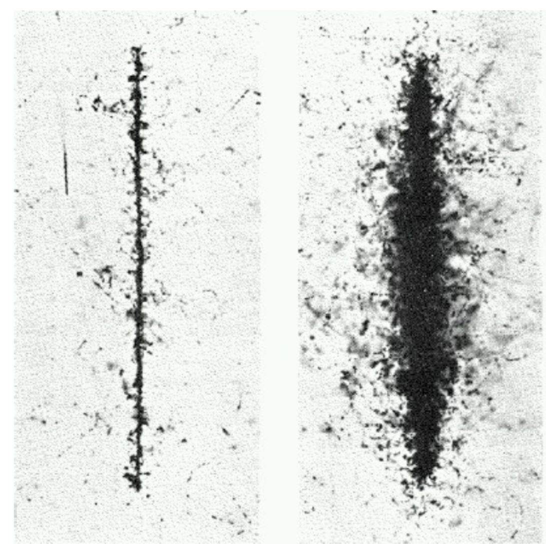
dE/dx dependence on medium

$$-\frac{dE}{dX} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\gamma^2\beta^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$



Depends quadratically on z of incident particle: the higher is the charge, the more the mean energy deposit is high

Tracks of ions in emulsion



Iron Z = 26 Thorium Z = 90

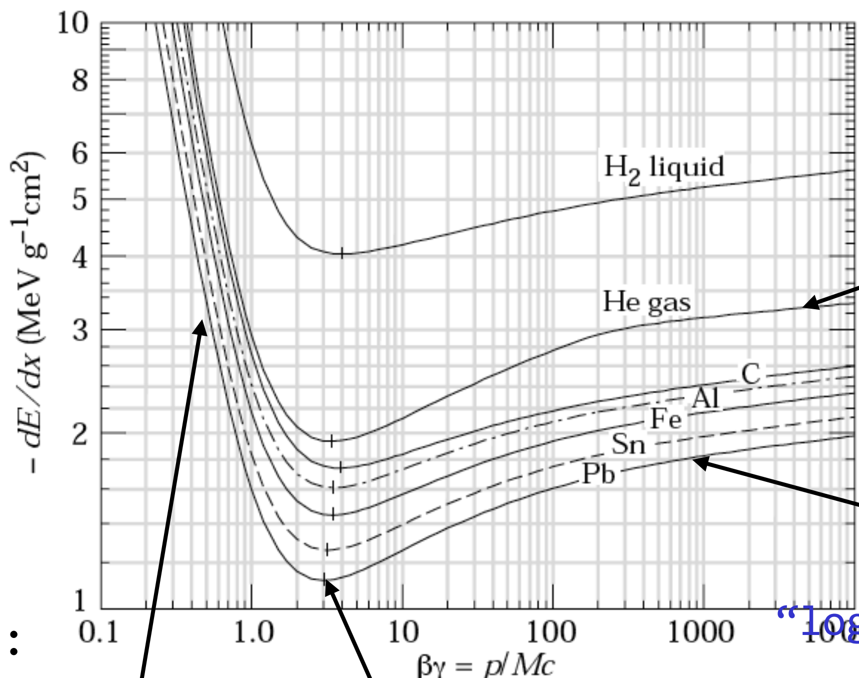
This means that, in principle, particle charge can be measured provided speed is known

Bethe Bloch

0.31 MeV g^{-1} cm^2

$$-\frac{dE}{dX} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\gamma^2\beta^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$

- Only valid for "heavy" particles ($m \geq m_\mu$) i.e. not electrons
- For a given material, dE/dX depends only on β – independent of mass of particle
- First approximation: medium characterized by electron density ~Z/A



$Z/A = 1$

"Fermi plateau" at large γ

$Z/A \sim 0.5$

$$\frac{dE}{dX} \propto \ln \gamma^2$$

"log relativistic rise"

$$\frac{dE}{dX} \propto \frac{1}{\beta^2}$$

"kinematical term"

$\beta\gamma \approx$ 3-4 ($v \approx 0.96c$)

Minimum ionizing particles, MIPs

$$\left.\frac{dE}{dX}\right|_{\min} \sim 1\text{-}2 \text{ MeV g}^{-1}\text{ cm}^2$$

True for **all** particles with same charge

dE/dx phenomena

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \frac{z^2}{\beta^2}\frac{Z}{A}\left\{\frac{1}{2}\ln\left[\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2}\right] - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right\}$$

- Energy loss is small (~few MeV cm^2/gm) except for $\beta\gamma<1$
- At very low βγ binding effects become important (keep curves finite); dE/dx magnitude peaks at a few hundred MeV cm^2/gm near **β ~ 0.01**
- Relativistic charged particles traverse large amounts of material; 1 GeV muon penetrates ~500 gm/cm^2; for water this corresponds to a length of 500 cm.

Ionizzazione: correzioni di shell

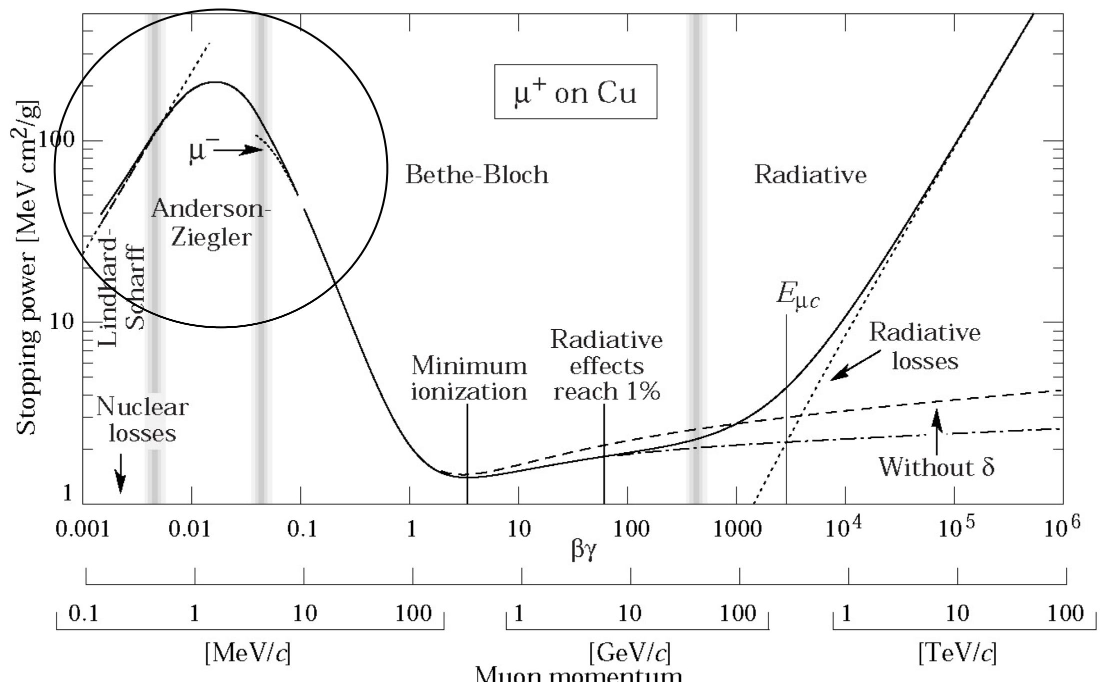
A basse energie, quando la velocita' della particella incidente e' paragonabile a quella orbitale dell'e-, c'e' un'altra correzione alla BB, dovuta al fatto che l'assunzione che l'e- sia in quiete durante l'interazione non e' piu' valida→ correzioni di shell

$$\frac{dE}{dX} = \frac{z^2 e^4 N_e}{4\pi\epsilon_0^2 m_e v^2}[ln(\frac{2\gamma^2 m_e v^2}{I}) - \frac{v^2}{c^2} - \delta - C/Z]$$

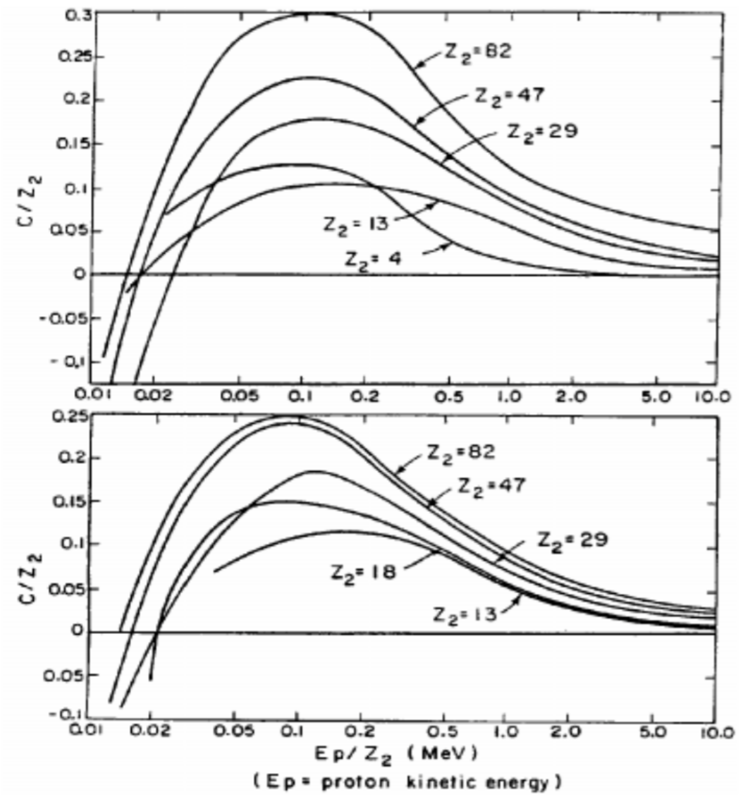
$$C(I,\eta) = (0.4223\eta^{-2} + 0.0304\eta^{-4} - 0.00038\eta^{-6}) \times 10^{-6} I^2 + (3.8501\eta^{-2} + 0.1668\eta^{-4} - 0.00158\eta^{-6}) \times 10^{-9} I^3$$

Valida per $\eta = \beta\gamma > 0.1$

In ambito astrofisico, le correzioni di densita' e di shell sono in genere trascurabili, ma sono importanti nelle applicazioni di rivelatori di particelle



Shell correction $\frac{-dE}{dX} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} [\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z}]$



Shell corrections (C/Z) constitute a correction to slow particles (e.g for protons in the energy range of 1-100 MeV)

Correction for the assumption that particle velocity >> bound electron velocity

Assumption that electron is at rest not valid

Maximal about 6%

Calculation with various approximations

J. F. Ziegler, Applied Physics Reviews / J. Applied Physics, 85, 1249-1272 (1999).

http://www.srim.org/SRIM/SRIMPICS/IONIZ.htm

Copper: about 1% at βγ=0.3 (6 MeV Pion), decreases rapidly with velocity

Charged particle energy loss in matter

- At very low $\beta\gamma$, large energy loss due to atomic effects
- For large (and relevant) range of relativistic $\beta\gamma$, energy loss is small (minimium ionizing particle – "mip")
- Ultra-relativistic particles lose energy mostly via gamma radiation

Taken from RPP2008; pdg.lbl.gov

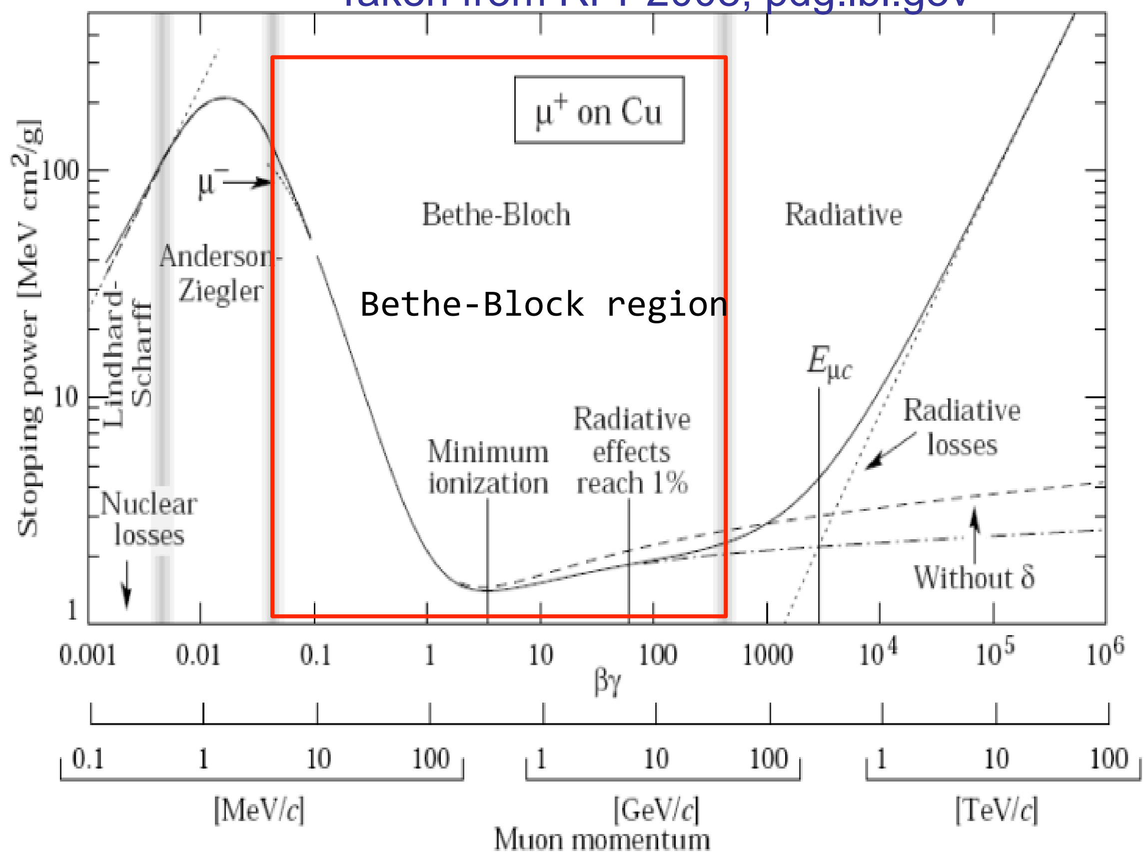
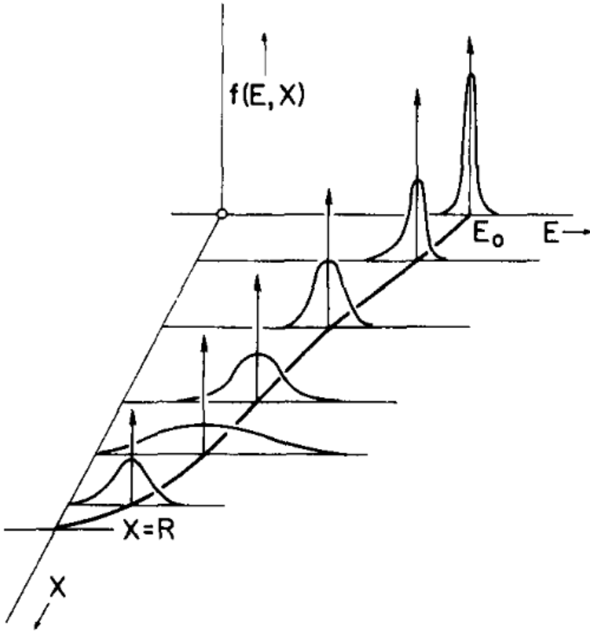


Fig. 27.1: Stopping power ($= \langle -dE/dx \rangle$) for positive muons in copper as a function of $\beta\gamma = p/Mc$ over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy). Solid curves indicate the total stopping power. Data below the break at $\beta\gamma \approx 0.1$ are taken from ICRU 49 [2], and data at higher energies are from Ref. 1.

Energy loss fluctuations

We learned that as a beam of particles travels in a slab of material loses energy.
Because of statistical nature of the process, not all the particles have the same loss (eg different nbr of collisions and different energy loss in each collision) at the same depth: BB describes the **average energy loss**, not the actual loss in a single collision, that is $\langle\Delta E\rangle = \langle(dE/dX)\rangle\Delta X$ if medium is homogeneous.

Therefore, a spread in energies always results after a beam of monoenergetic charged particles has passed through a given thickness of absorber. The width of this energy distribution is a measure of energy straggling, which varies with the distance along the particle track.

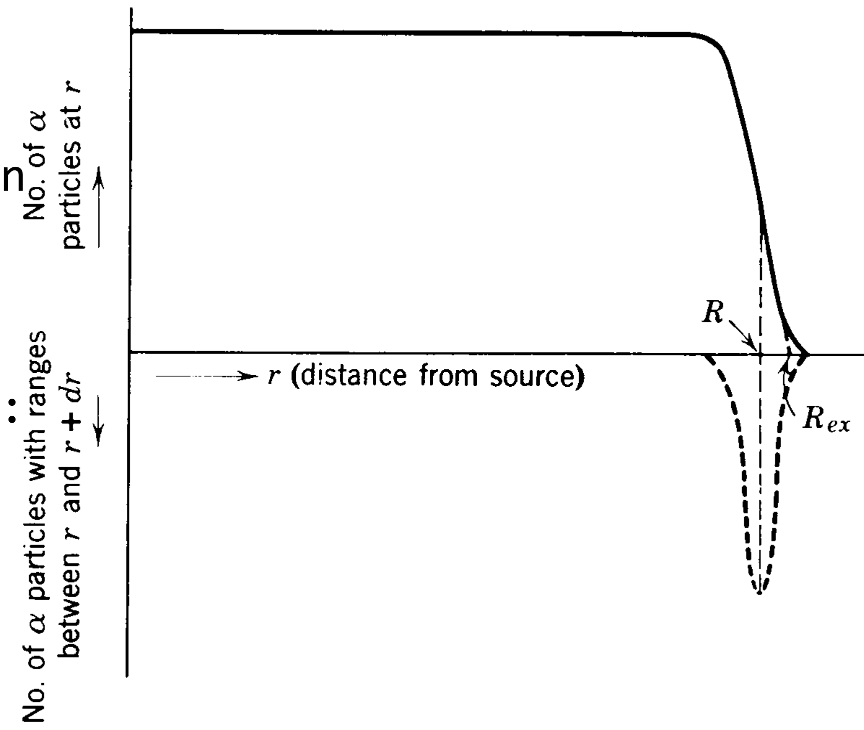


If the slab is thick enough, the particles will lose all their energy and will be stopped in the absorber. For the same reason as above, not all the particles with the same initial energy will travel the same distance in the absorber before stopping

Range

- Since particles loose energy in matter, a particle will travel a finite distance, called "range", before it loses all its energy.
- Experimentally, the range can be determined by passing a beam of particles through different thicknesses L and measuring the ratio **r** of transmitted to incident particles

- For small L, **r**=1: all (or practically all) the particles pass through.
- At the range R, r drops to 0
- The drop is not sharp, but slopes down over a certain spread of L, since the energy loss process is statistical. Particles with the same energy will travel different distances because of:
  - ✓ different nbr of collisions and
  - ✓ different energy loss in each collision.
- This is known as "**range straggling**"

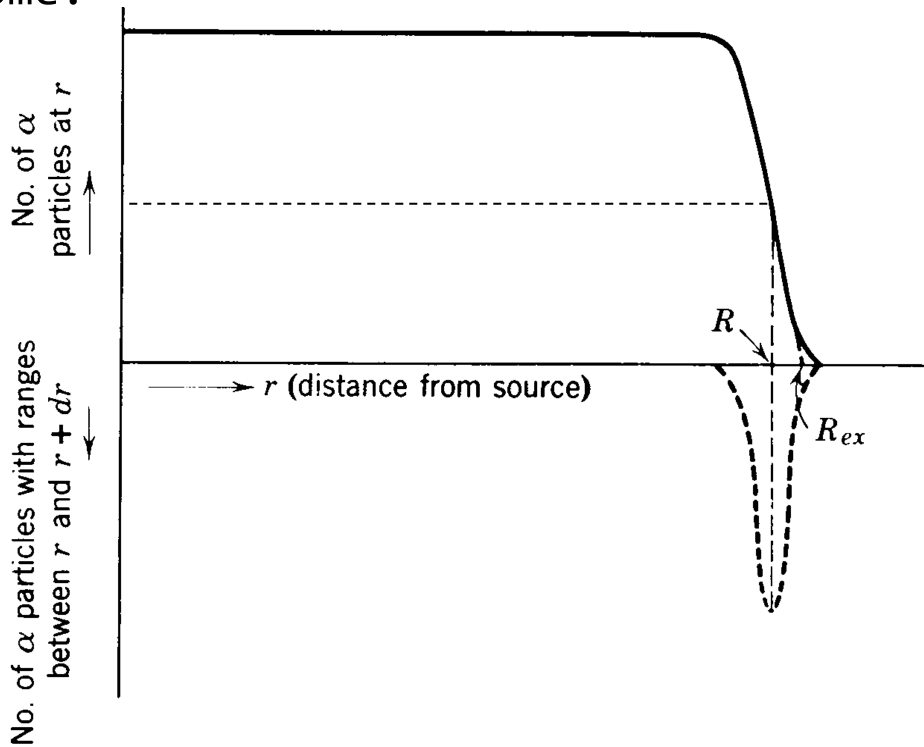


Range

Dato un fascio monoenergetico, **la profondita' alla quale le particelle sono ridotte alla meta' si chiama range medio.**

Formalmente il range e' definito come:

$$R(E) = \int_E^0 \frac{1}{dE/dx} dE$$



Il range rappresenta la distanza percorsa ed e' diversa dallo spessore a causa delle deviazioni che subisce negli urti. Si misura in gr/cm^2.
La dispersione intorno al range medio e' gaussiana. Il calcolo e' complicato. (Evans, Atomic Nucleus, pp. 600-667, available on the web, 1002 pages)

Distance a 10 MeV particle can cross

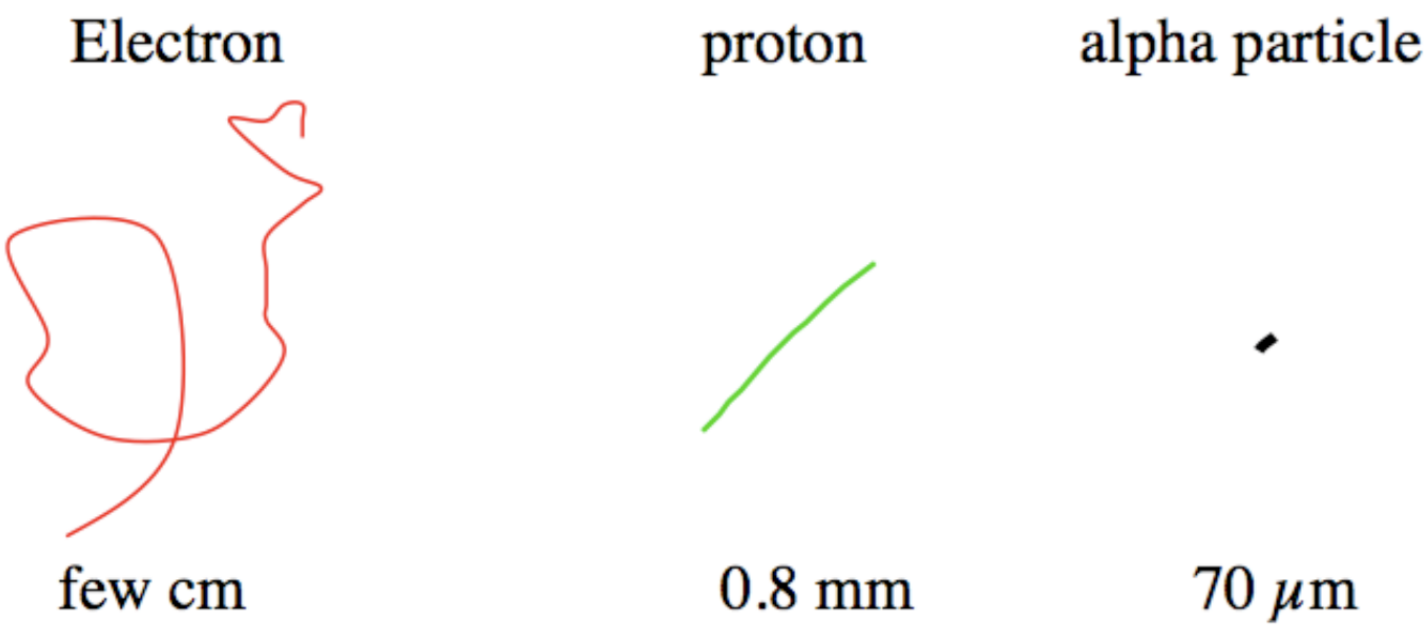


Figure 2.2.2: *A typical trajectory for an electron, a proton and an alpha particle of 10 MeV in silicon. The electron trajectory is drawn on a scale 10 times smaller than the trajectory of the proton and the alpha particle.*

Note that the trajectory is not a straight line because of the collisions against nuclei, i.e. multiple scattering (later).

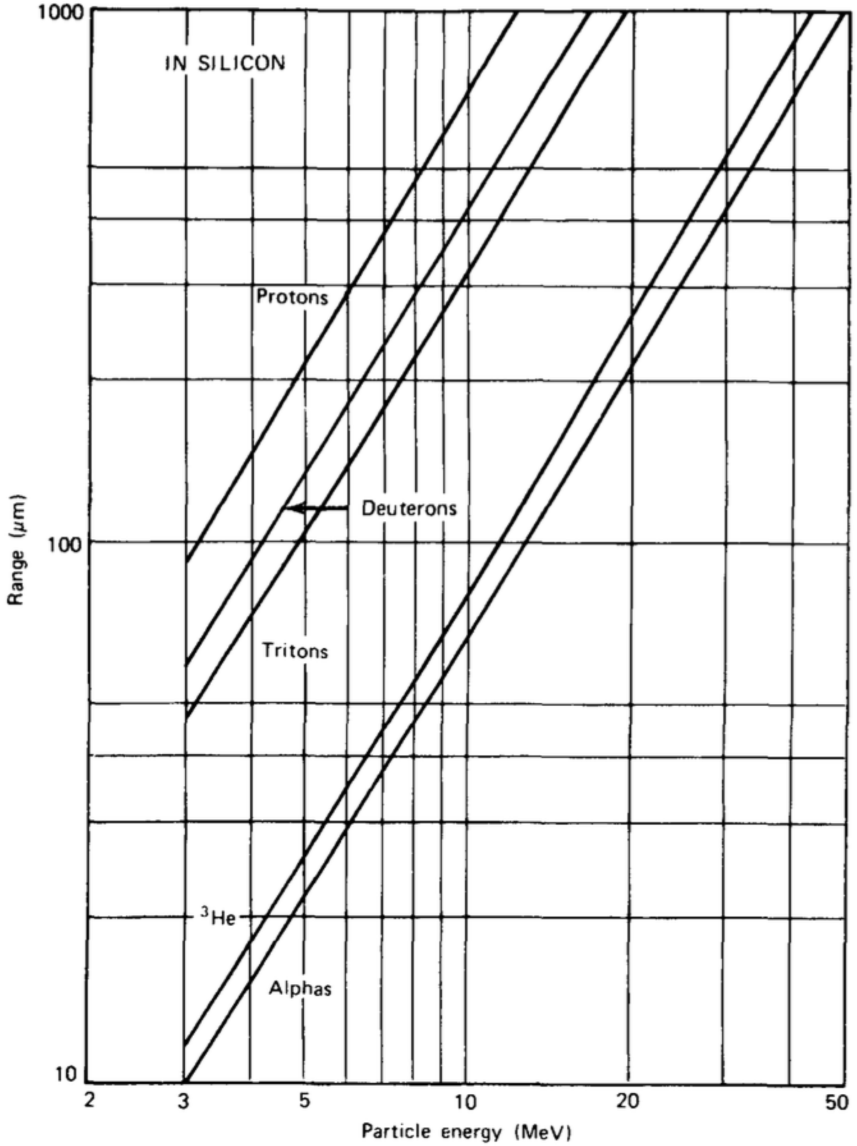


Figure 2.7 Range–energy curves calculated for different charged particles in silicon. The near-linear behavior of the log–log plot over the energy range shown suggests an empirical relation to the form $R = aE^b$, where the slope-related parameter b is not greatly different for the various particles. (From Skyrme.[4])

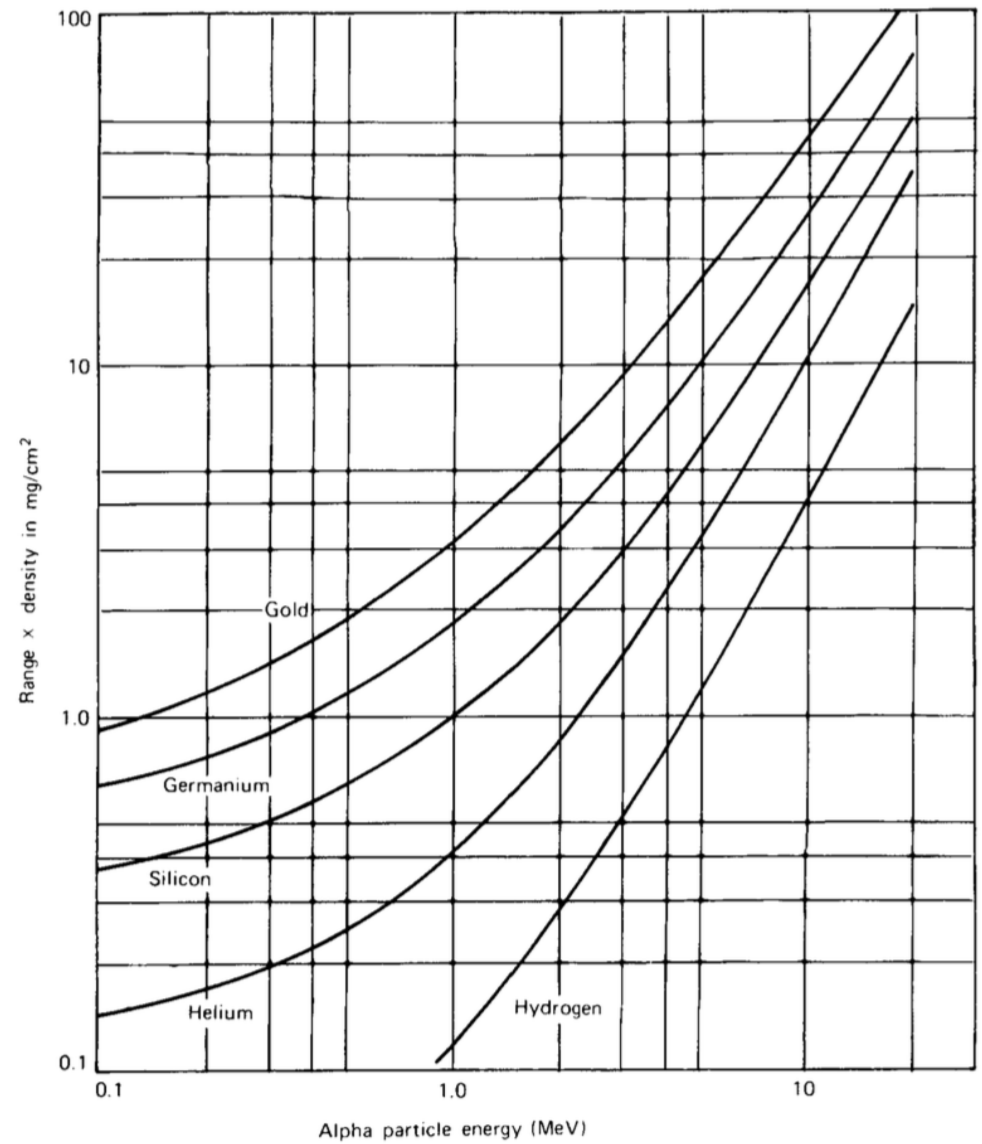


Figure 2.8 Range–energy curves calculated for alpha particles in different materials. Units of the range are given in mass thickness (see p. 54) to minimize the differences in these curves. (Data from Williamson et al.[5])

Range

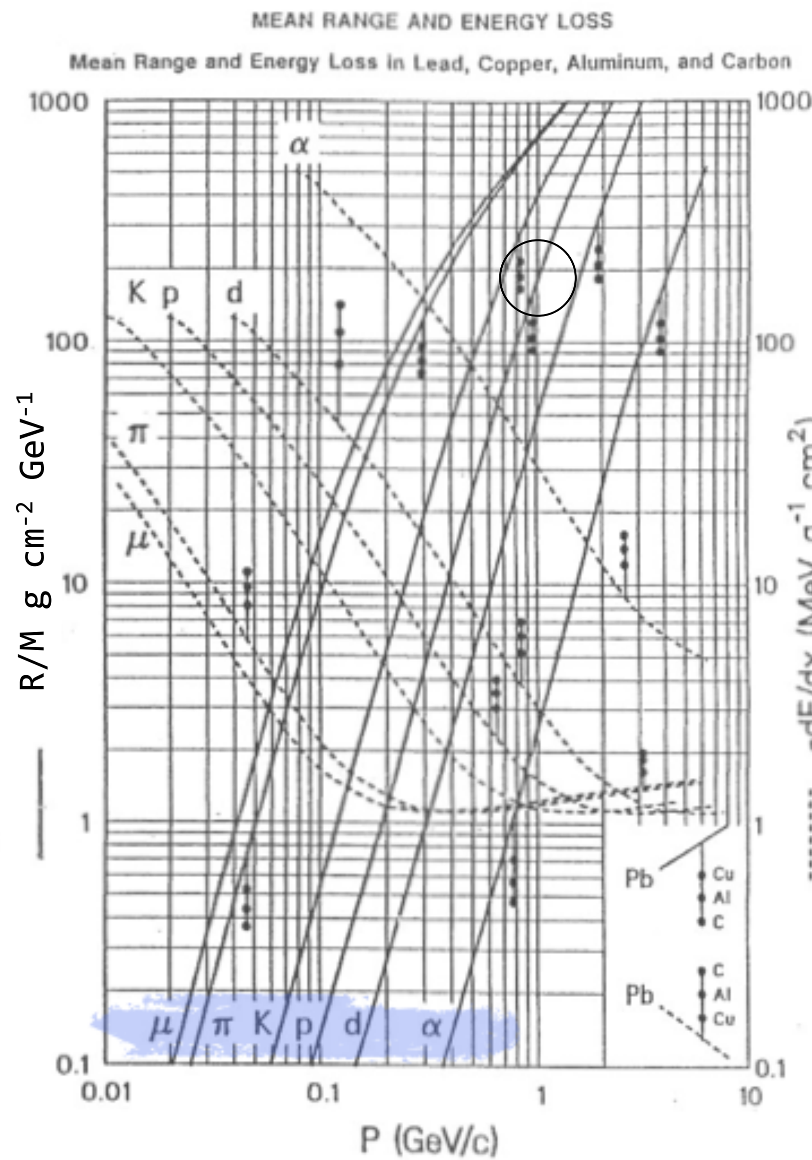
Integrate over energy loss
from E down to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

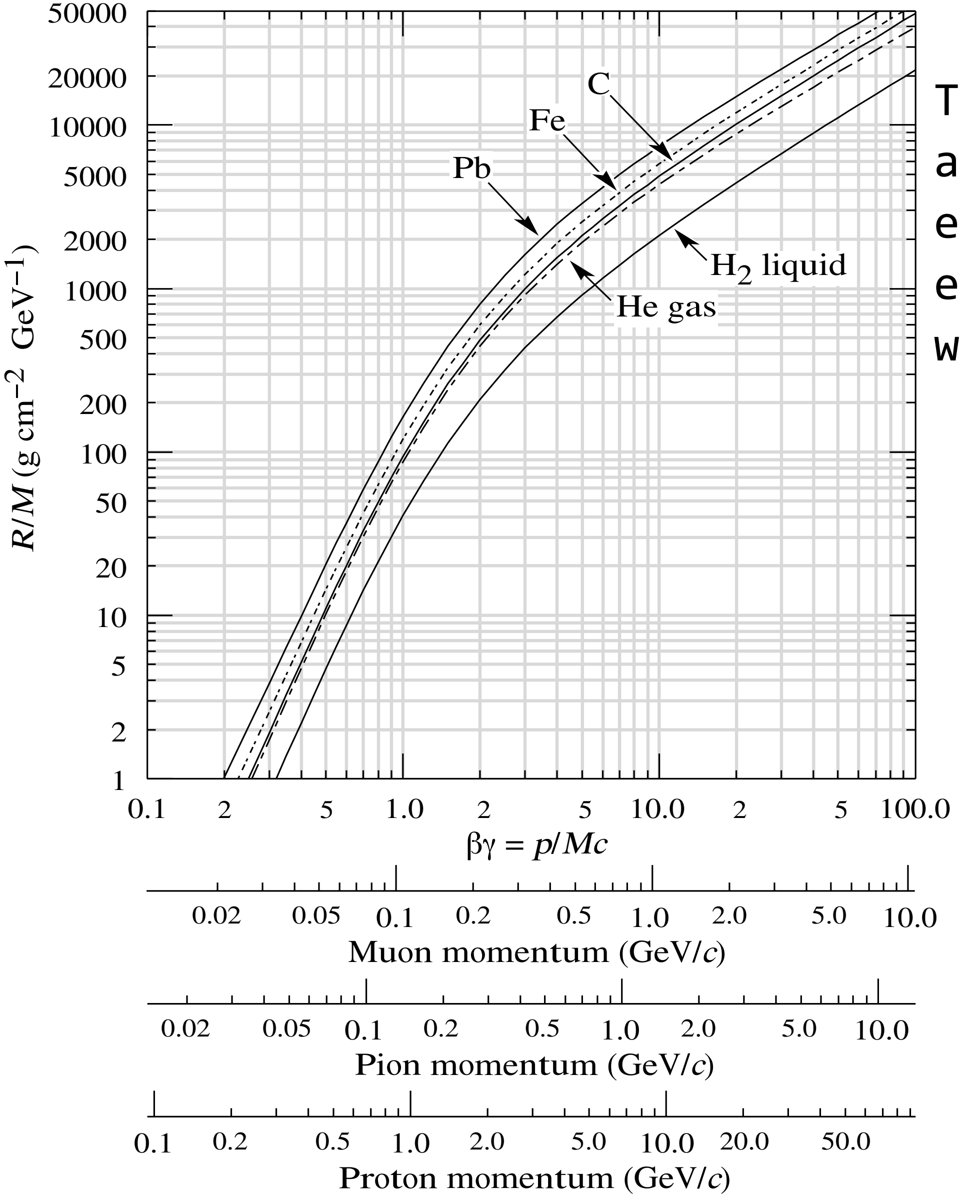
Example:

Proton with p = 1 GeV
Target: lead with $\rho = 11.34\ \mathrm{g/cm^3}$

R/M = 200 g $\mathrm{cm^{-2}}$ $\mathrm{GeV^{-1}}$
→ R = 200/11.34/1 cm ~ 20 cm



Percorso delle particelle (Range)



The range-energy relationships are often expressed as $R(E)=(E/E_o)^n$ e.g. the range in meters of low energy protons can be approximated with $n=1.8$ and $E_o=9.3$ MeV.

$$R(E) \cong \left(\frac{E}{9.3}\right)^{1.8}$$

E in MeV, R in meters of air

Scaling laws

- Sometimes data are not available on the range or energy loss characteristics of precisely the same particle-absorber combination needed in a given experiment. Recourse must then be made to various approximations, most of which are derived based on the Bethe formula and on the assumption that the dE/dX per atom of compounds or mixtures is additive. This latter assumption, known as the Bragg-Kleeman rule, may be written

$$\frac{1}{N_c}\left(\frac{dE}{dx}\right)_c = \sum_i W_i \frac{1}{N_i}\left(\frac{dE}{dx}\right)_i$$

- In this expression, N is the atomic density, and *Wi* represents the atom fraction of the ith component in the compound C.
- As an example of the application of BB, the linear stopping power of alpha particles in a metallic oxide could be obtained from separate data on dE/dX in both the pure metal and in oxygen.
- Some caution should be used in applying such results, however, since some measurements for compounds have indicated a stopping power differing by as much as 10-20% from that calculated from BB.

dE/dx per composti e miscugli.

Una buona approssimazione della perdita di energia per composti e miscugli è data dalla regola di Bragg: una media pesata delle perdite di energia degli elementi i del composto M, pesate con la frazione di elettroni dell'elemento

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{w_1}{\rho_1}\left(\frac{dE}{dx}\right)_1 + \frac{w_2}{\rho_2}\left(\frac{dE}{dx}\right)_2 + \cdots \qquad w_i = a_i \frac{A_i}{A_M} \qquad A_M = \sum_i a_i A_i$$

Dove a_1, a_2, … e' il nr. di elettroni nello i-esimo elemento del composto M e A_i e' il nr atomico dell'elemento

Possiamo definire dei valori efficaci come segue:

$$Z_{eff} = \sum a_i Z_i \qquad A_{eff} = \sum a_i A_i$$

$$\ln I_{eff} = \sum \frac{a_i Z_i \ln I_i}{Z_{eff}} \qquad \delta_{eff} = \sum \frac{a_i Z_i \delta_i}{Z_{eff}}$$

E riscrivere la dE/dx in termini dei valori efficaci.

dE/dX vs. depth

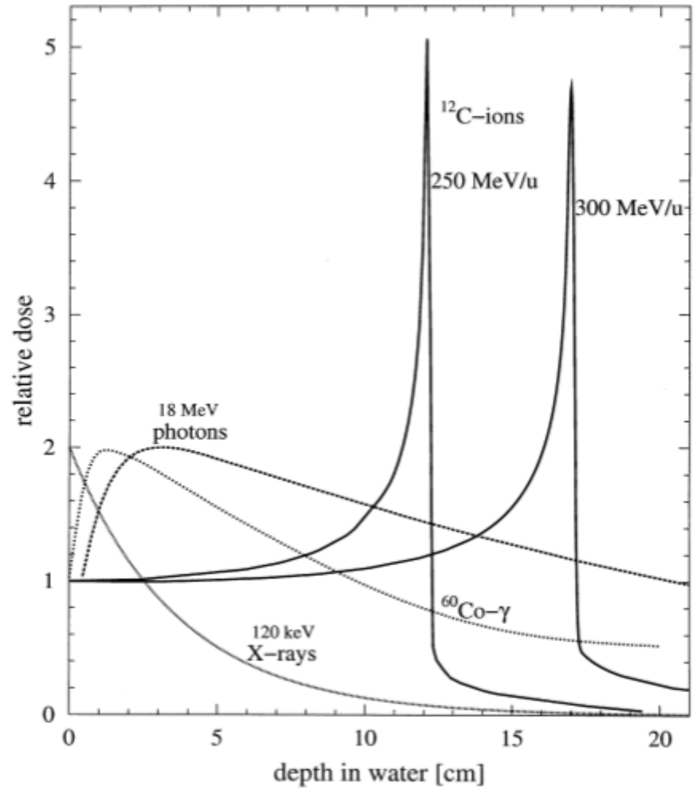
A plot of the specific energy loss along the track of a charged particle is known as a *Bragg* curve.

Until the particle is a MIP, its energy loss stays constant (more exactly it varies slowly –logarithmically-with $\beta\gamma$. Remember: $dE/dX \propto 1/\beta^2$ and $\beta \approx 1$ for MIPs)

As $\beta\gamma$ goes down below MIP, the E loss increases very rapidly because of $1/\beta^2$ dependence or as $1/T$ (kinetic E) as the particle becomes non relativistic and there is a peak in energy deposit : Bragg peak

$\beta\gamma > 3.5$: $\langle dE/dX \rangle \approx (dE/dX)_{min}$
$\beta\gamma < 3.5$: $\langle dE/dX \rangle \gg (dE/dX)_{min}$

Near the end of the track, the charge is reduced through electron pickup and the curve falls off: particle becomes very slow and easily captures electrons from medium and becomes neutral before stopping.



dE/dX vs. depth

Particles with high charge begin to pick up electrons early in their slowing-down process.
Note that in an Al absorber, singly charged H ions (protons) show strong effects of charge pickup below about 100 keV, but doubly charged ^{3}He ions show equivalent effects at about 400 keV.

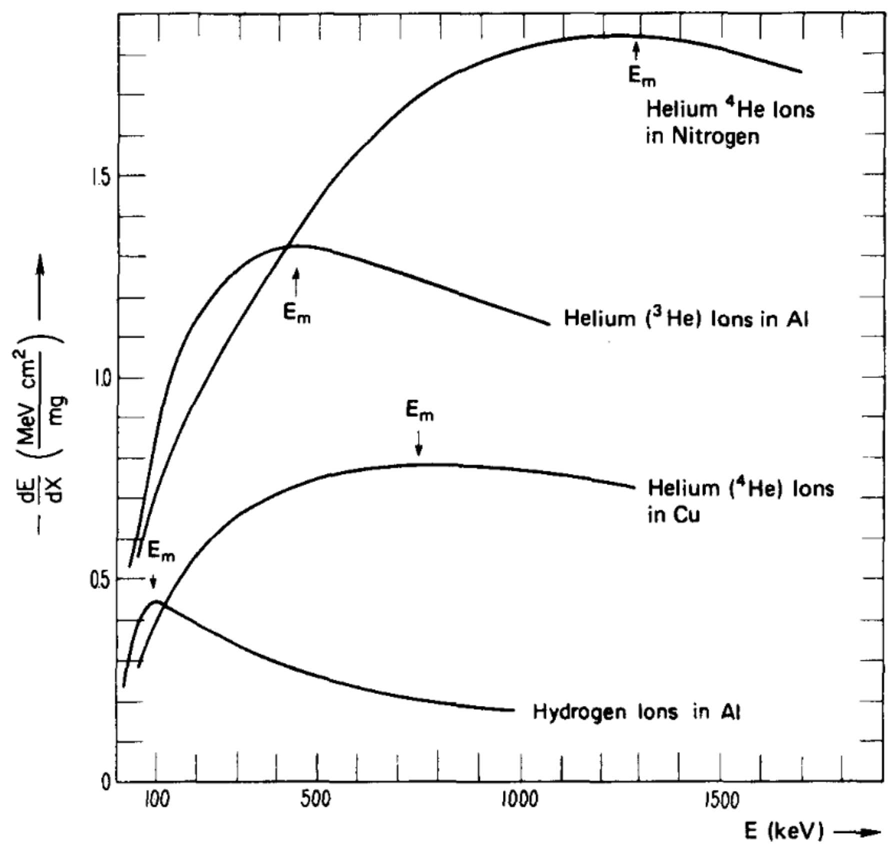


Figure 2.3 Specific energy loss as a function of energy for hydrogen and helium ions. E_m indicates the energy at which dE/dx is maximized. (From Wilken and Fritz.[3])

dE/dx phenomena

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \frac{z^2}{\beta^2}\frac{Z}{A}\left\{\frac{1}{2}\ln\left[\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2}\right] - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right\}$$

- Slow particles lose most of their energy in a short distance, since kinetic energy T ~ β^2

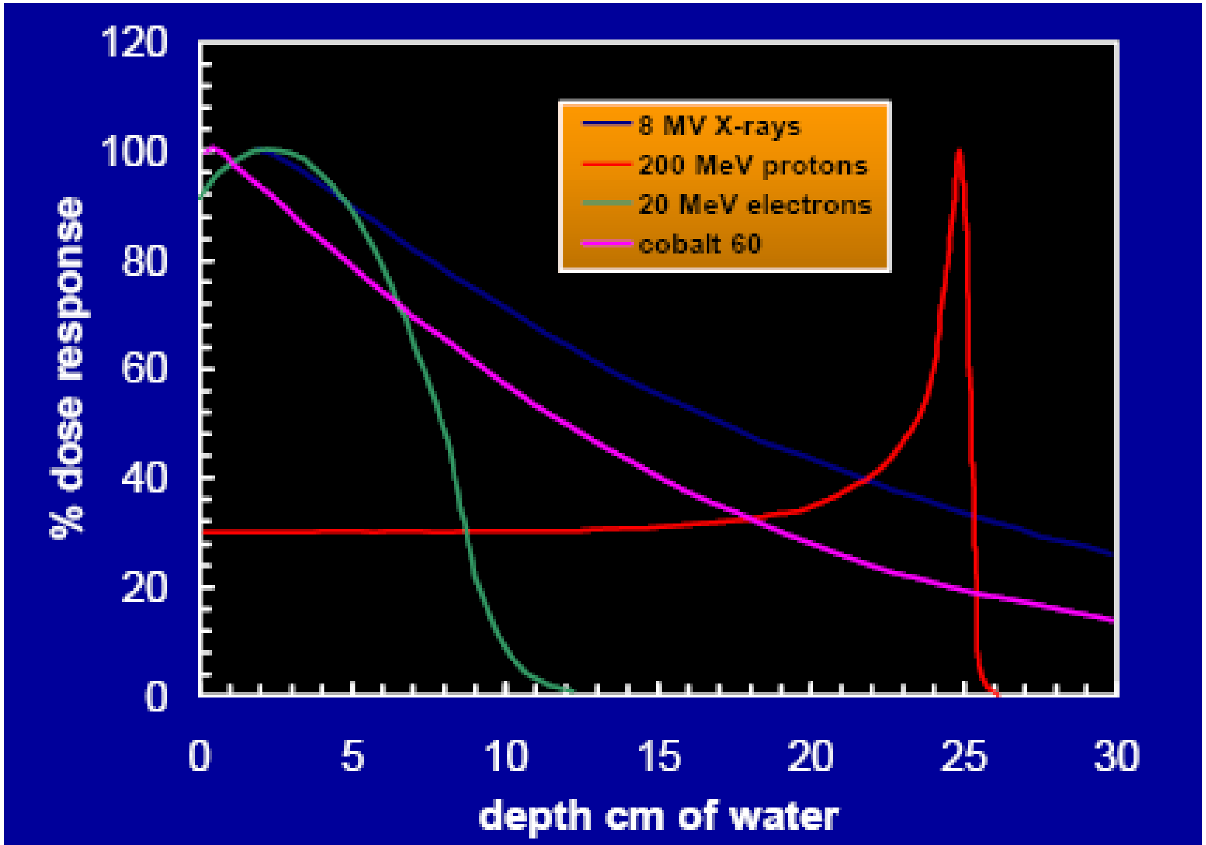
$$-\frac{dE}{dx} = -\frac{dT}{dx} = \left\langle \frac{dE}{dx} \right\rangle_{T_0} \frac{T_0}{T} \qquad \Delta X = -\int_{T_0}^{0} \frac{dT}{dT/dx} \qquad \Delta X = -\int_{T_0}^{0} \frac{TdT}{(dE/dx)_o T_0}$$

$$\Delta X = -\frac{1}{(dE/dx)_o T_0}\int_{T_0}^{0} T\,dT = -\frac{T_0}{2(dE/dx)_o} = \rho\Delta y$$

- For 30 MeV protons in water, <dE/dx> ~ 50 MeV cm^2/gm, so Δy ~ 0.3 cm

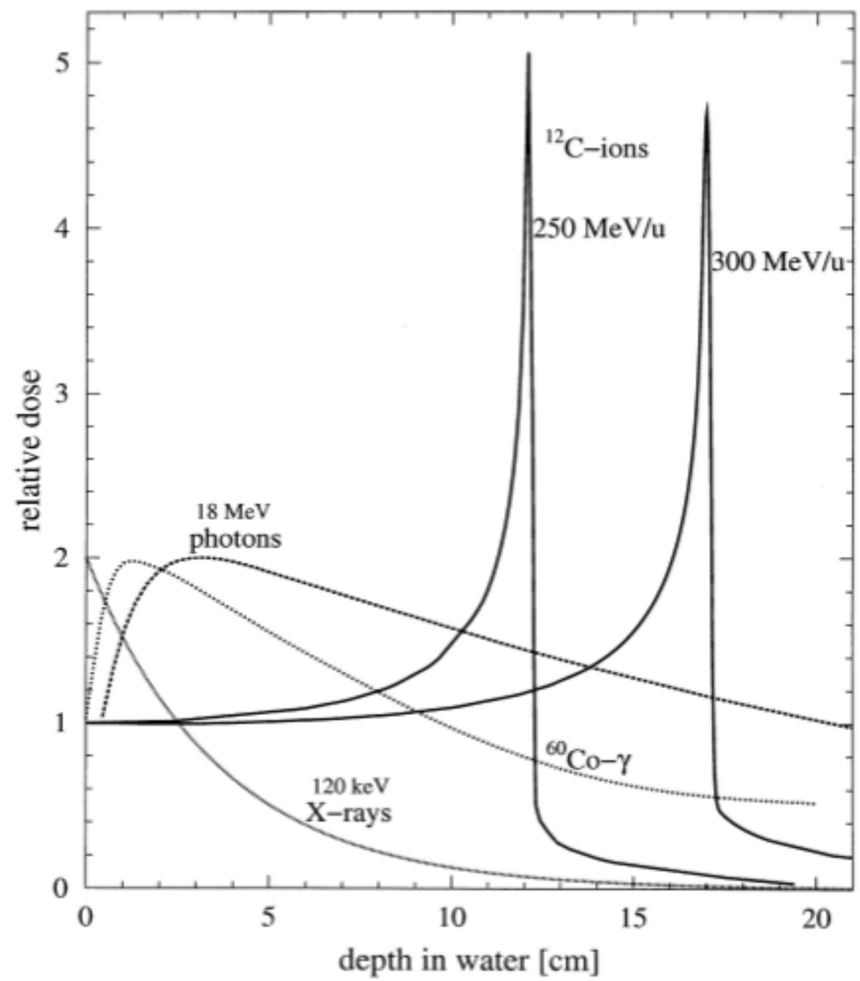
Application of Range

- The localized energy deposition of heavy charged particles can be useful therapeutically = proton radiation therapy



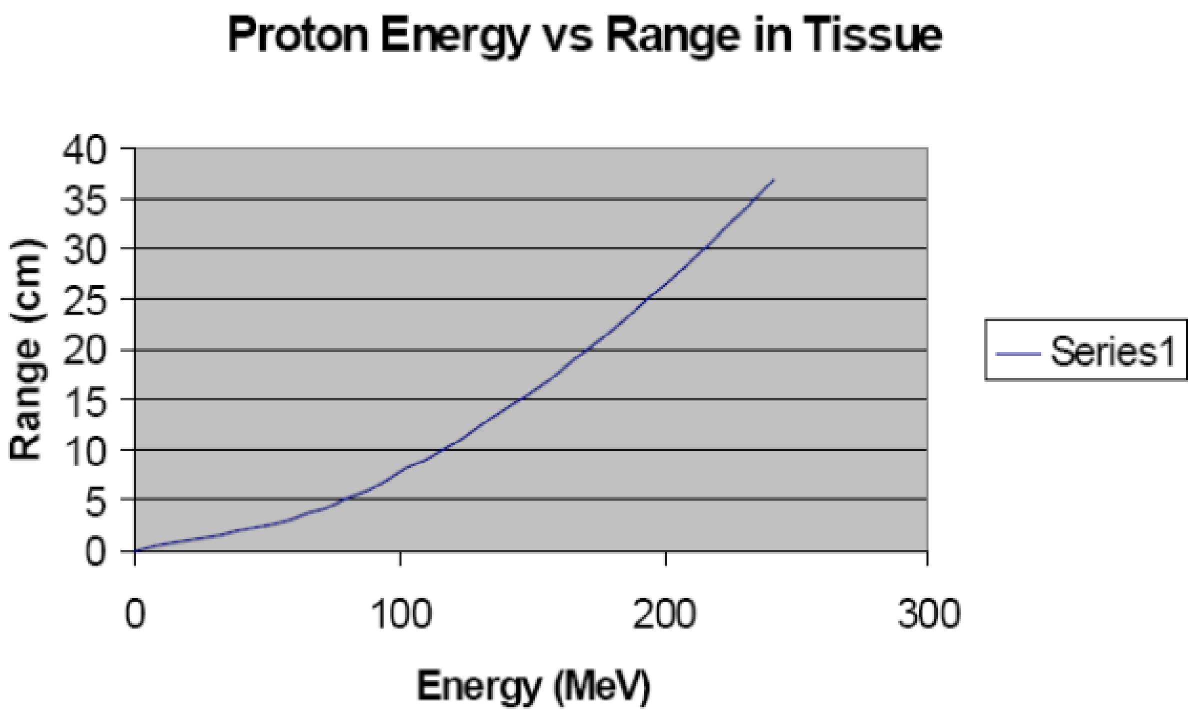
Application of Range

- Monoenergetic proton beam loses energy more rapidly as it slows down; gives sharp Bragg peak in ionization versus depth
- Using a range of proton energies allows a varied profile versus depth
- Photon beam (x-rays) deposits most energy near entrance into tissue
- Tumor therapy with hadrons (adro-therapy)



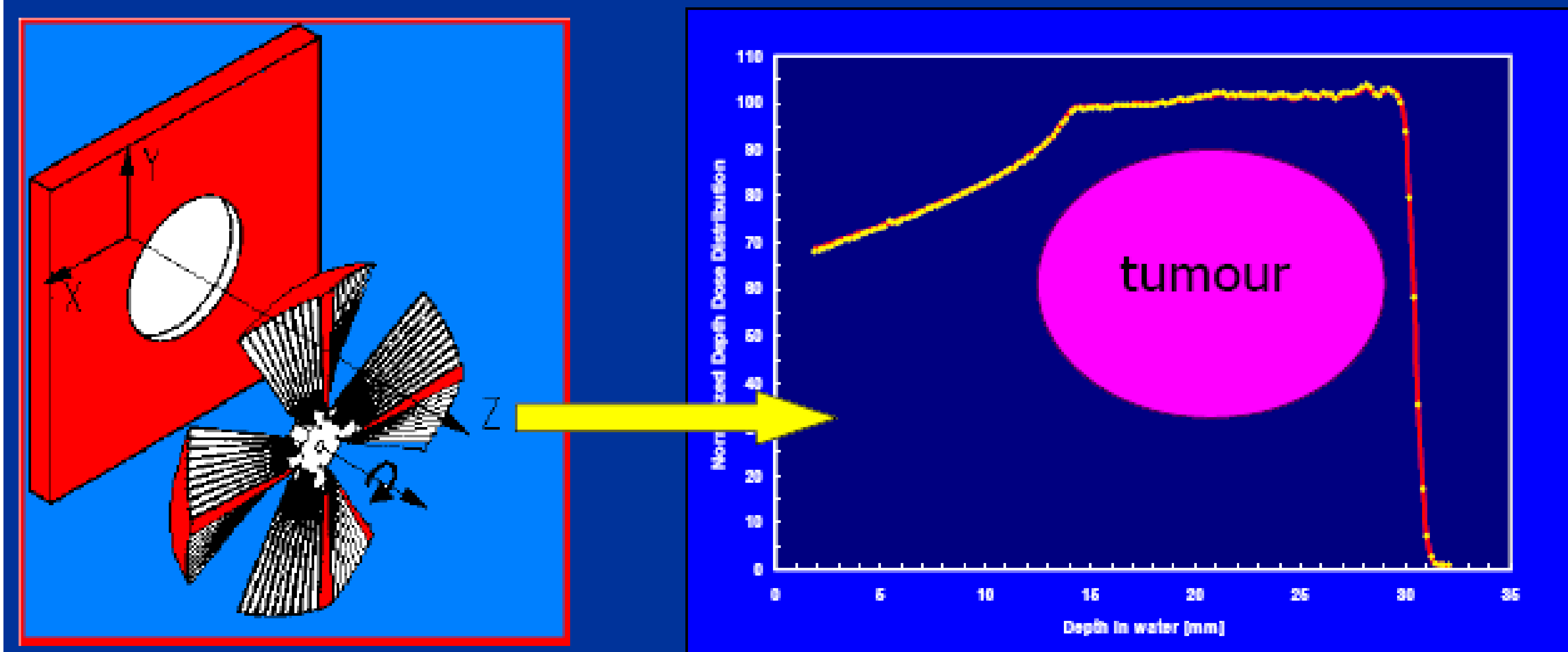
Proton Therapy

- Energy range of interest from 50 (eye) – 250 (prostate) MeV

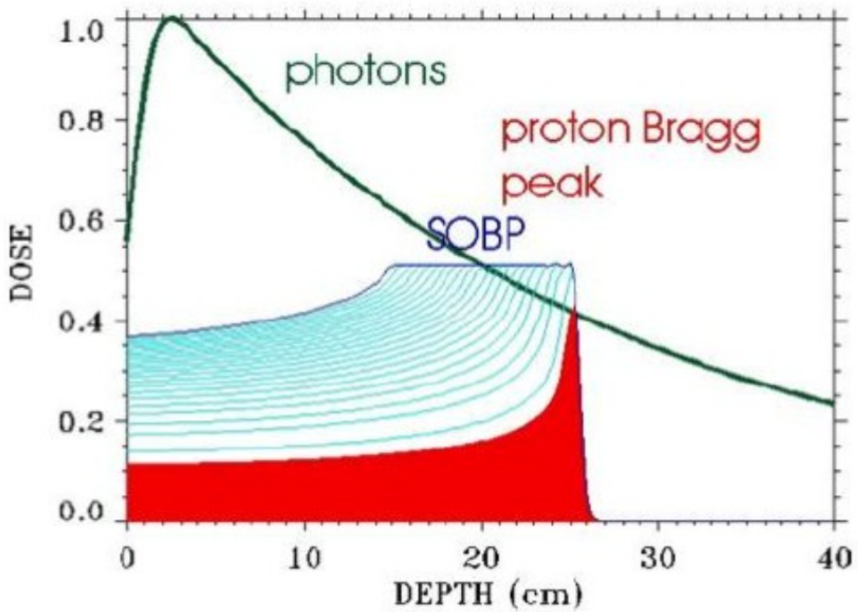


Proton Therapy

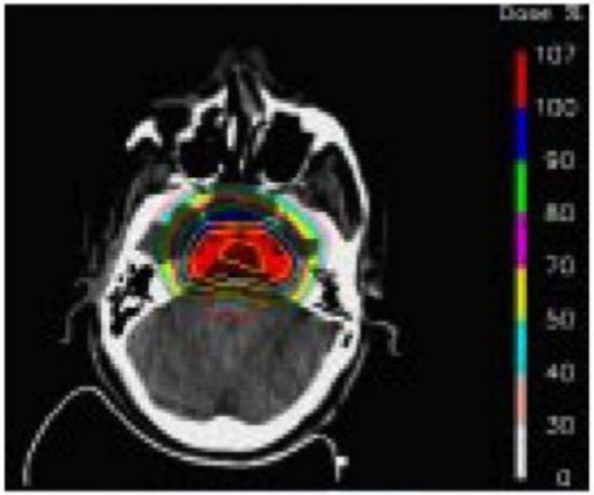
- Simulation of the Modulator Wheel to Obtain the Therapeutical Spread Out Bragg Peak

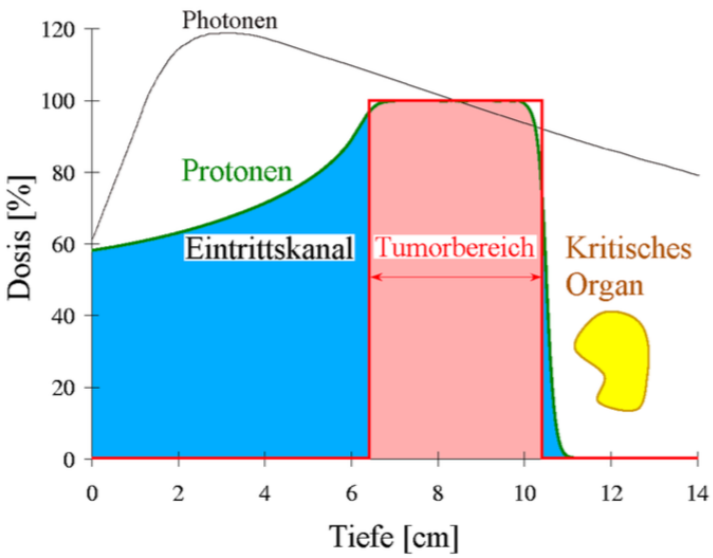


Proton therapy at PSI



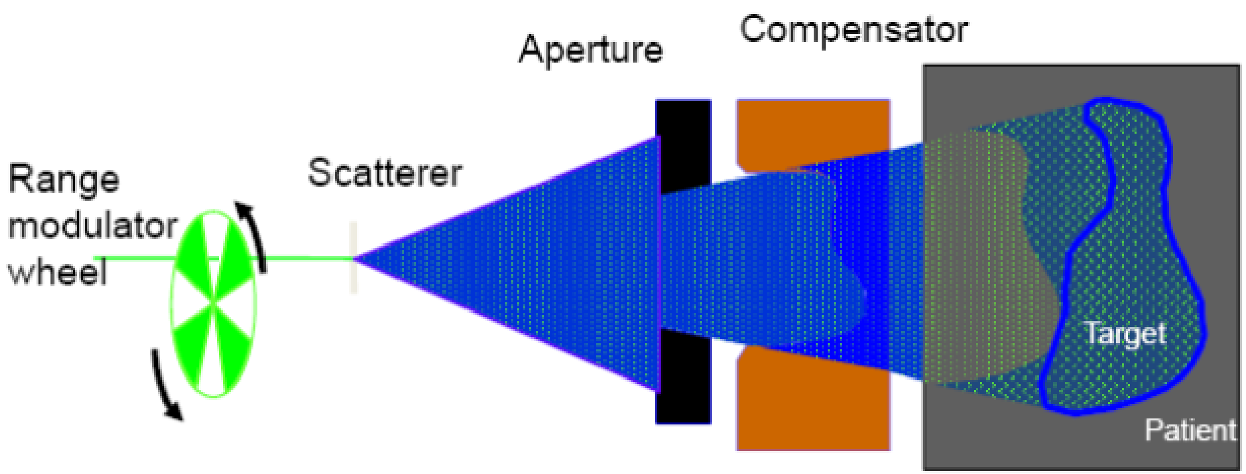
Spread out Bragg peaks (SOBP)
Different absorbers → almost constant dose in region of tumour





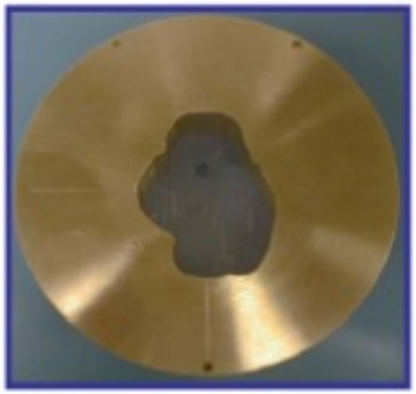
Proton Therapy

- Schematic apparatus for hadron-therapy

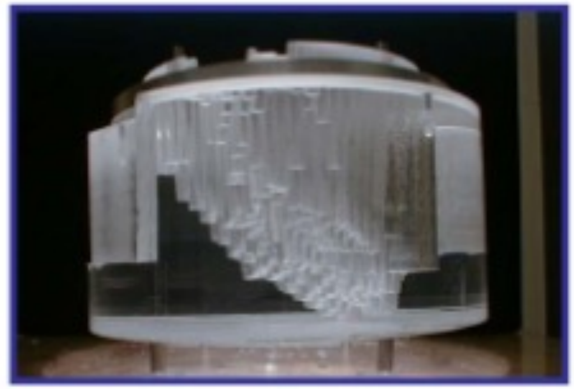


Proton Therapy

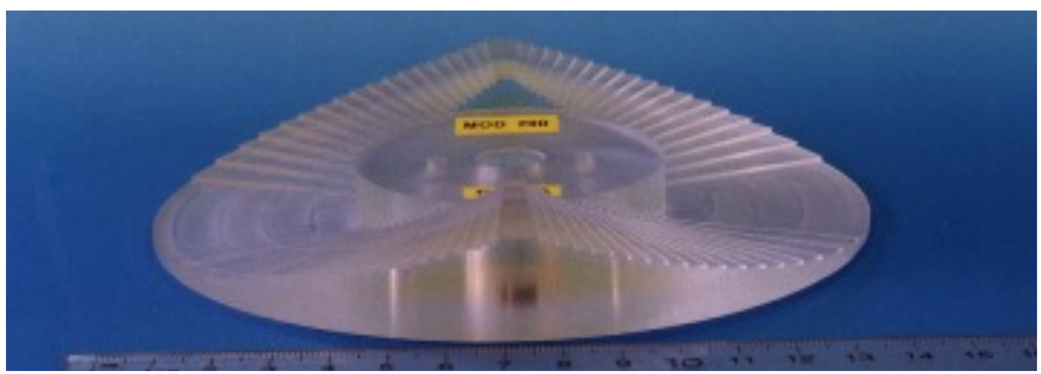
- Modulator, aperture, and compensator



Brass aperture

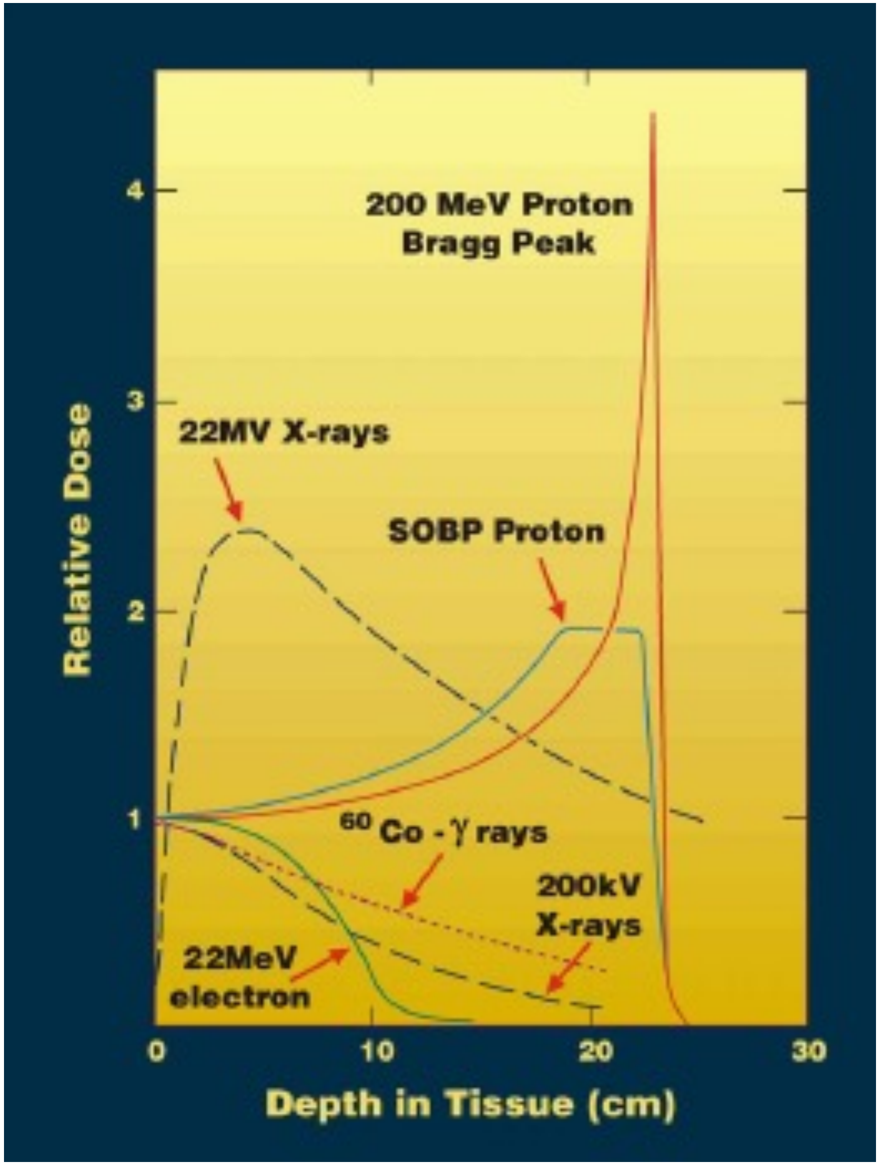


Lucite range compensator



Modulator

Proton Therapy

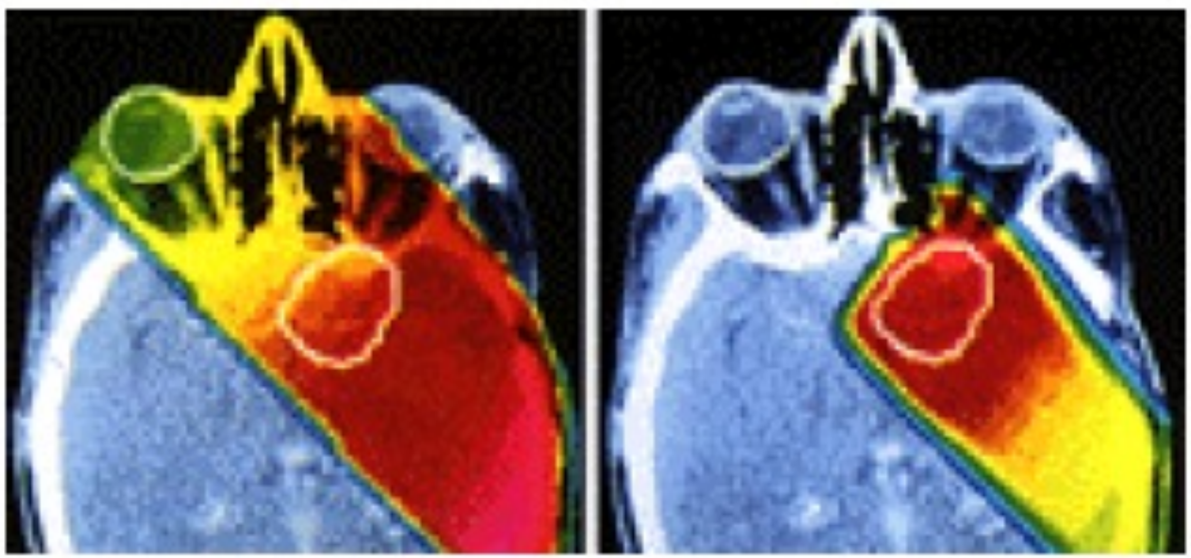


Proton Therapy

Comparison of Treatment Planning using Protons vs. X-rays

Highest dose
- red

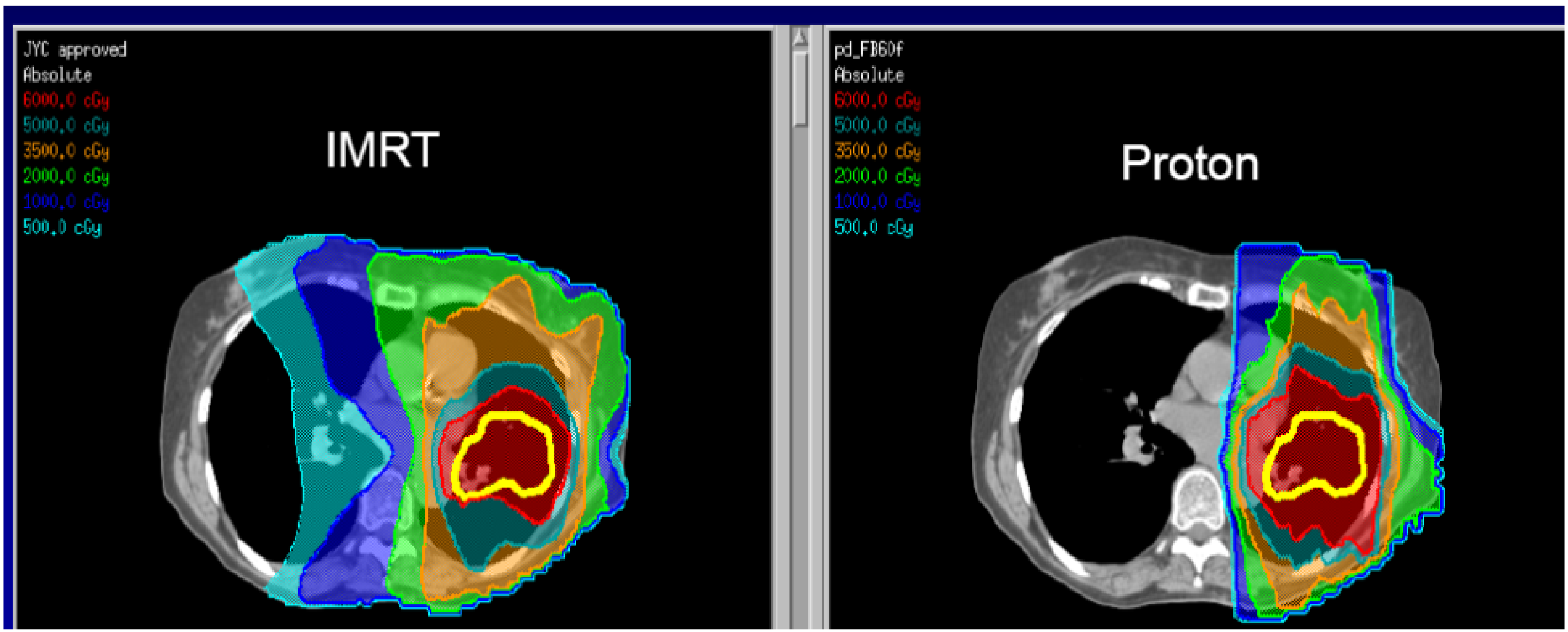
Lowest dose
- yellow



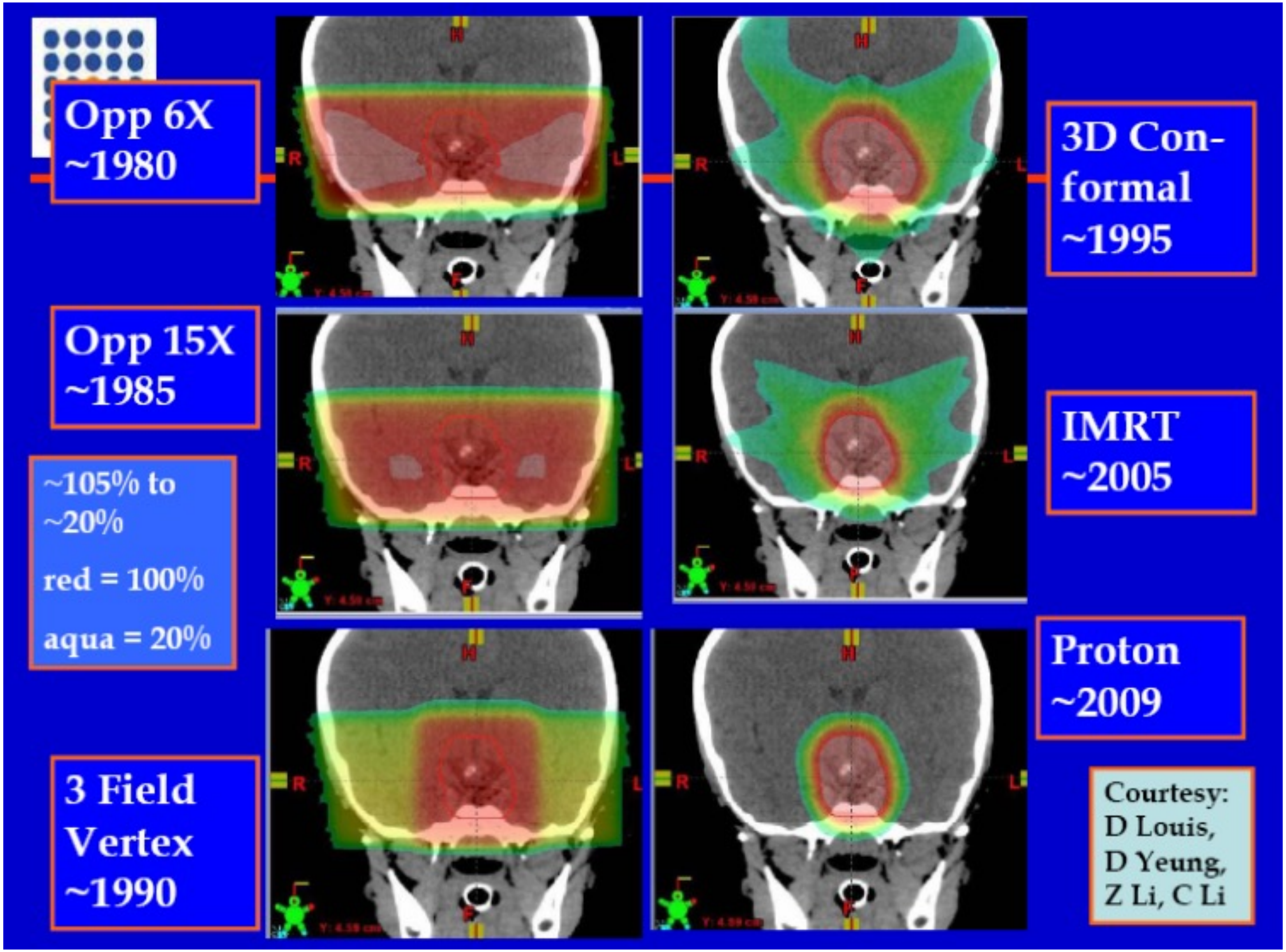
X-rays Protons

Proton Therapy

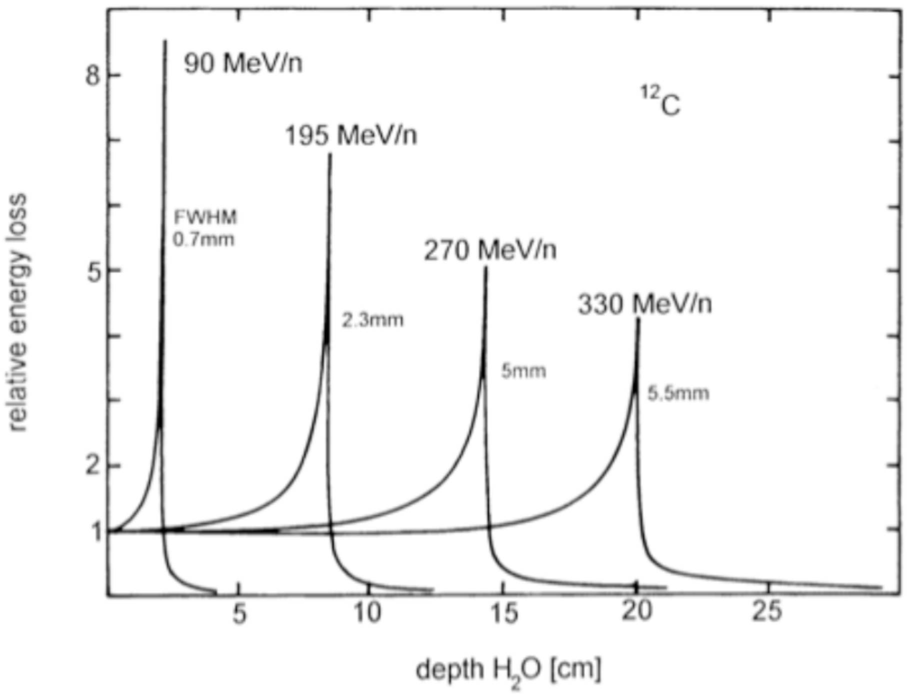
- Lung cancer treatment
  - Intensity modulated radiation therapy vs proton therapy



Proton Therapy



Hadron-therapy for cancer

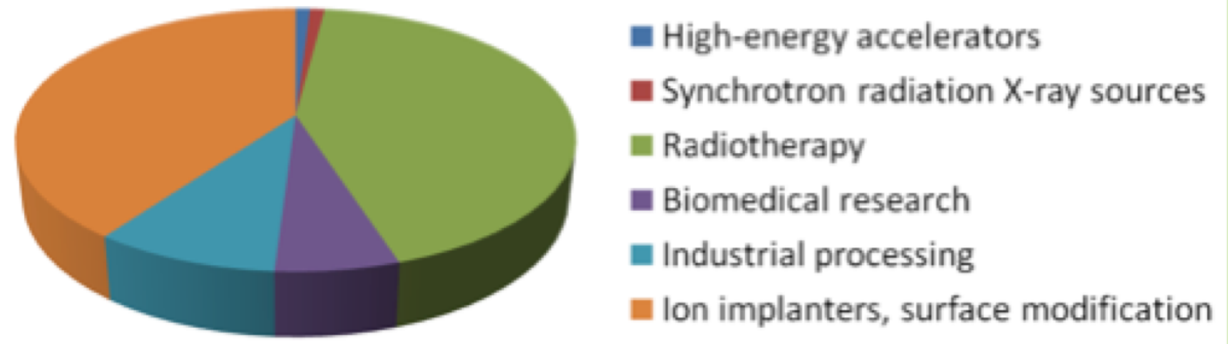


^{12}C ions in water. Treatment of deep-seated tumors.

Proton Cancer Therapy

Accelerators Worldwide

The number of accelerators worldwide exceed 20000



- Market for **medical and industrial** accelerators exceeds **$3.5 billion.** All products that are processed, treated, or inspected by particle beams have a collective annual value of more than $500 billion [1]

[1] http://www.acceleratorsamerica.org/

Accelerators are not only for high-energy physics

Accelerators for Medicine

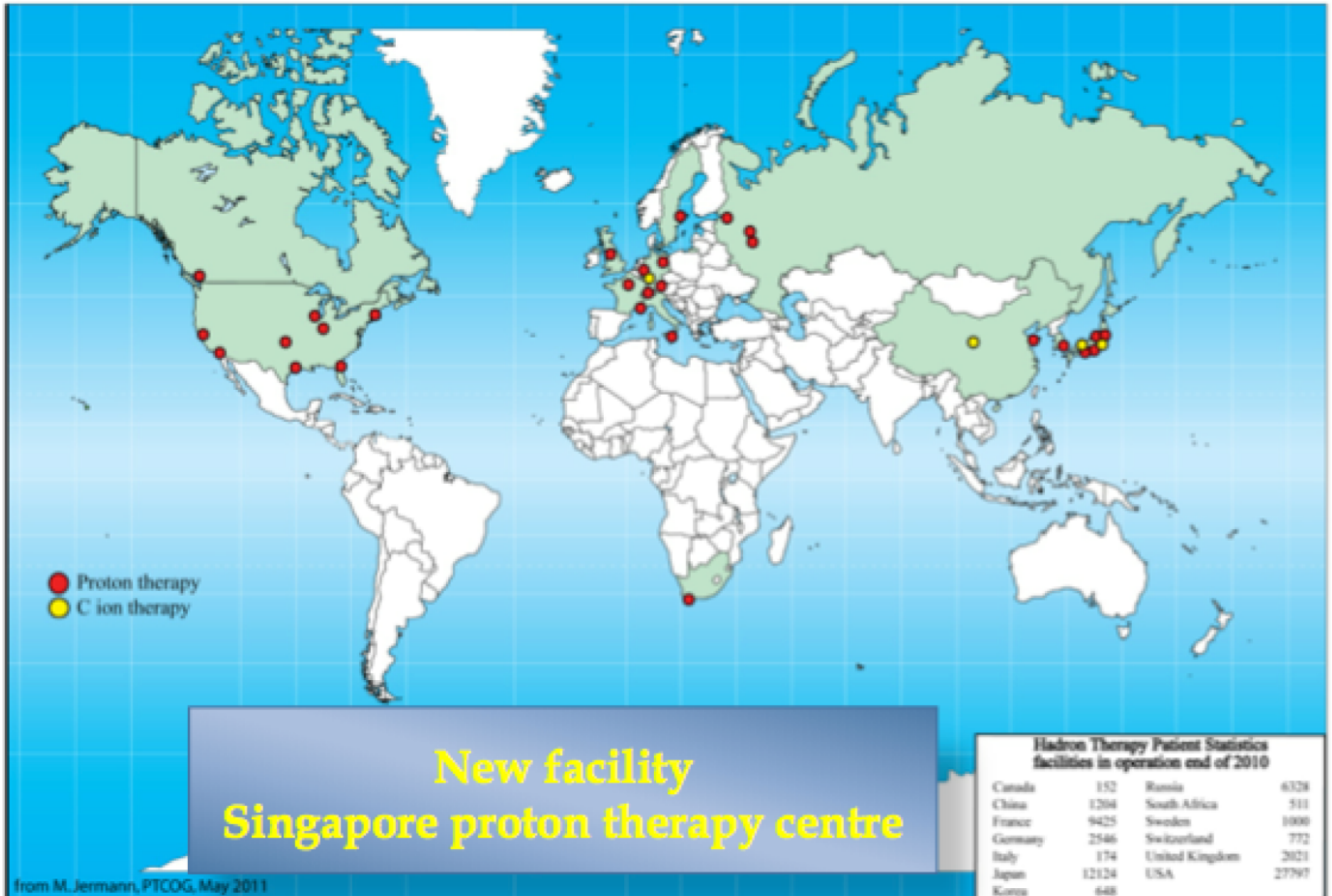
- **Medical Therapy**
  - X-rays have been used for decades to destroy tumours.
  - For deep-seated tumours and/or minimizing dose in surrounding healthy tissue use hadrons (protons, light ions).
  - Accelerator-based hadrontherapy facilities.

Accelerator cancer therapy

Loma Linda Proton Treatment Centre
Constructed at FNAL

Centers for HADRON Therapy

in operation end of 2010



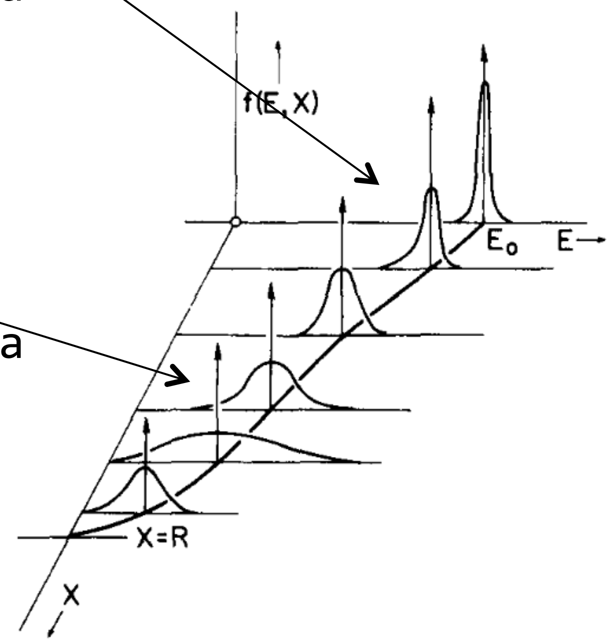
Hadron Therapy Patient Statistics
facilities in operation end of 2010

Canada	152	Russia	6328
China	1204	South Africa	511
France	9425	Sweden	1000
Germany	2546	Switzerland	772
Italy	174	United Kingdom	2021
Japan	12124	USA	27797
Korea	648		

Worldwide: 30 centres (4 have C-ions): ~ 65'000 patients
Europe: 9 centres (with C-ions at GSI and Heidelberg): ~ 16'000 patients

Energy loss fluctuations

- Quando una particella attraversa un materiale di spessore L si hanno due casi estremi:
  - lo spessore del materiale e' molto minore del range medio, L<<R: la particella attraversa il mezzo perdendo solo una frazione piccola della sua energia $\Delta E \ll E_{inc}$. In tal caso, fissato lo spessore attraversato, si osservano fluttuazioni nell'energia ΔE depositata nel mezzo.
  - il materiale e' cosi' spesso che la particella vi deposita tutta la sua energia ($\Delta E = E_{inc}$) e si ferma. In tal caso si osserva, come abbiamo appena visto, una distribuzione nel range delle particelle a causa delle fluttuazioni statistiche del numero di urti e dell'energia persa in ciascun urto. Questo accade quando il range medio della particella e' molto minore dello spessore del materiale: R << L



Energy loss fluctuations

La perdita di energia dE/dx (Bethe Block) descrive la perdita media di energia per una particella nell'attraversare uno spessore dX di materiale. Cio' significa che in un fascio, una particella in media perde <dE/dX>

$$\left\langle -\frac{dE}{dx} \right\rangle = \rho K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{2mc^2\beta^2\gamma^2}{I} - \beta^2 - \frac{\delta(\gamma)}{2} \right]$$

In un mezzo omogeneo, dalla BB si ha <dE> ∝ X $\quad \langle dE \rangle \approx \rho K z^2 \frac{Z}{A} \frac{1}{\beta^2} X$

Se fissiamo l'attenzione su una singola particella, in generale

$-\frac{dE}{dx} \neq \left\langle -\frac{dE}{dx} \right\rangle$ ovvero la perdita totale in uno spessore ΔX, ΔE ≠ <ΔE>

a causa delle fluttuazioni statistiche che avvengono

i. nel numero di collisioni con gli elettroni del mezzo

ii. nella perdita di energia nelle singole collisioni

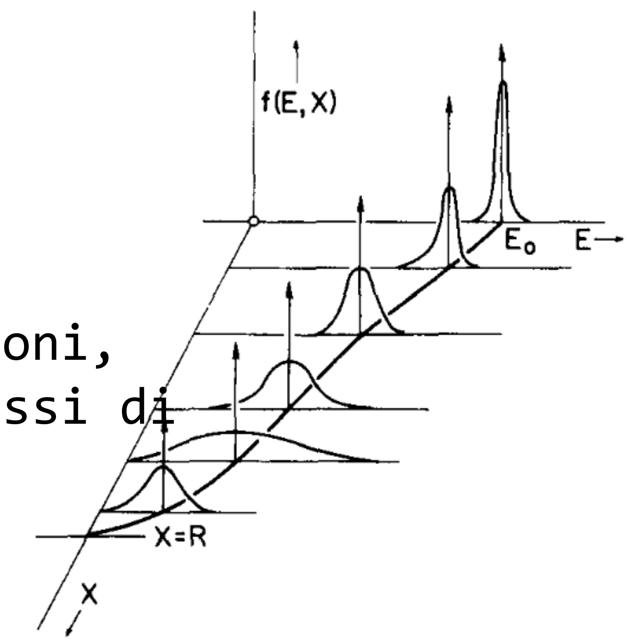
$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N : number of collisions
δE : energy loss in a single collision

δE distribuite statisticamente

Energy loss fluctuations for R<<L

Un fascio di particelle monoenergetiche con $E = E_0$ mostrera' uno sparpagliamento σ intorno a $E = E_0 - \langle dE \rangle$, dopo aver attraversato un spessore (costante) di materiale e non sara' piu' monoenergetico (una delta di Dirac) con $E = E_0 - \langle dE \rangle$, ma mostrera' una distribuzione di perdite intorno a quella media.



La forma della distribuzione dipende dallo spessore attraversato

Nel trattare il problema delle fluttuazioni, tradizionalmente si hanno due grandi classi di assorbitori:

1. assorbitori spessi
2. assorbitori sottili

(nel limite di cui sopra)

Energy loss fluctuations for R<<L

- La distinzione tra assorbitori spessi e sottili non e' netta.
- L'assorbitore diventa spesso quando il # di collisioni N $\rightarrow \infty$ ma nella pratica e' sempre finito e non e' facile distinguere sempre bene i due casi (anche perche' siamo nell'ipotesi che la particella perda una frazione piccola dell'energia, cioe' R<<L)

Energy loss fluctuations for R<<L

There is another way to define thick and thin absorbers:
Roughly, for a MIP the energy loss in a thickness X g/cm^2 of absorber is $\Delta E = \langle dE/dX \rangle X$ and also $\Delta E = \sum_0^N \delta E_i$
The δE_i are distributed statistically according to their probability between δE_{min} and δE_{max}

If $N \rightarrow \infty$, $\Delta E \gg \delta E_i$, for any i and thus also for the max cinematically allowed δE_{max}: $\Delta E \gg \delta E_{max}$

$$\langle dE/dX \rangle X \gg \delta E_{max}$$

Therefore a detector can be said "thick" if

$$X \gg \delta E_{max} / \langle dE/dX \rangle$$

In other words, the detector is thick if single collisions with large energy transfers are not important for the total energy loss

- Is a 1 cm scintillator thick or thin for 1 GeV muon?

1 GeV μ

$$W_{max} = 2 m_e c^2 \beta^2 \gamma^2$$

$$\beta = p/E \qquad \gamma = E/m \qquad E^2 = p^2 + m^2$$

$$E^2 = 1^2 + (.105)^2 \Rightarrow E = 1.0055$$

$$\beta = 1/1.0055 = .9945$$

$$\gamma = 1.0055/.105 = 9.576$$

$$W_{max} = \frac{2(.511)}{\cancel{c^2}} \cancel{c^2} (.9945)^2 (9.576)^2$$

$$= 92.69 \text{ MeV}$$

$\Delta E = (dE/dX)(\rho x)$

$$= 1.956 \cdot 1.032 \cdot 1 = 2 \text{ MeV}$$

$W_{max} >> \Delta E$
So 1 cm plastic scintillator is NOT thick

Actually, even 1 cm of Pb is not thick since $\Delta E = (dE/dX)(\rho x) = 1.1*11.35*1 = 12.5$ MeV!!

Energy loss probability

- Trascurando lo spin dell'e- (ie scattering di Coulomb), la probabilita' di espellere un e- di energia fra E ed E + dE e' :

$$P(E)dE = k\frac{X}{E^2}dE \quad \text{where} \quad k = 2\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2} \qquad (1)$$

Si ricordi la distribuzione del # di e- espulsi con E fra E ed E+dE in una collisione $n(E) = -\frac{dN}{dEdx} = \pi N_e(\frac{e^4 Z^2}{8\pi^2\epsilon_o^2 m_e v^2})\frac{1}{E^2}$

Infatti il # di interazioni e' dN = ndV.
dV e' il volume di interazione disponibile: per definizione di sezione d'urto, tutte gli atomi in dV = Ldσ interagiscono con la particella incidente , quindi nLdσ e' il # di interazioni.

Poiche' osserviamo la distribuzione di energia, dσ = (dσ/dE)dE ≡P(E)dE e quindi la (1)

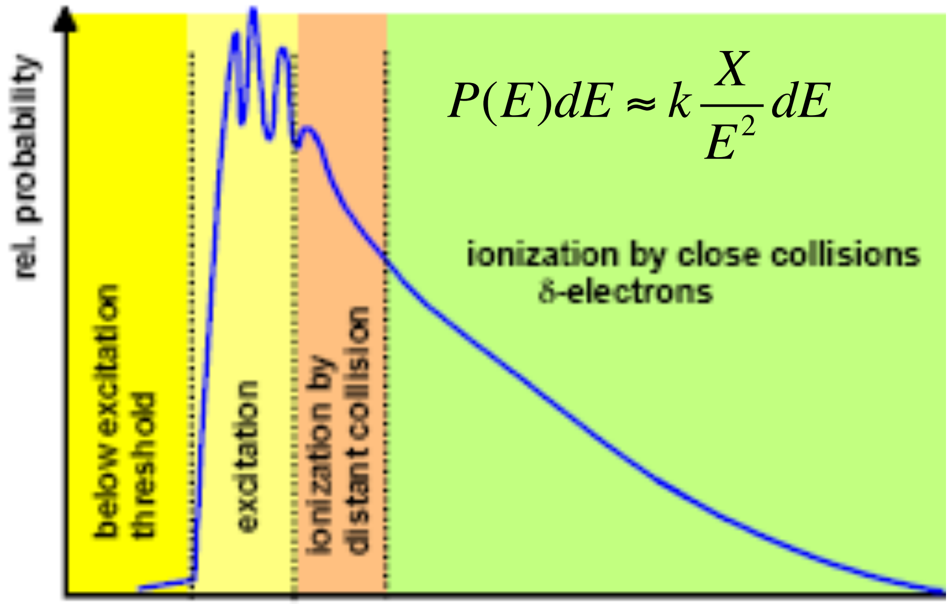
La distribuzioni esatta per elettroni e' $P(E)dE = k\frac{X}{E^2}(1-\beta^2\frac{E}{E_{\max}})dE$

La distribuzioni esatta per particelle pesanti di spin ½ e'

$$P(E)dE = k\frac{X}{E^2}(1-\beta^2\frac{E}{E_{\max}}+\frac{1}{2}\frac{E}{E+mc^2})dE$$

La probabilita' di emissione (cosi' come lo spettro) e' limitata dalla cinematica fra $E_{min} < E < E_{max}$

Energy loss probability



Andamento schematico della probabilita' di perdere un'energia W in una collisione.

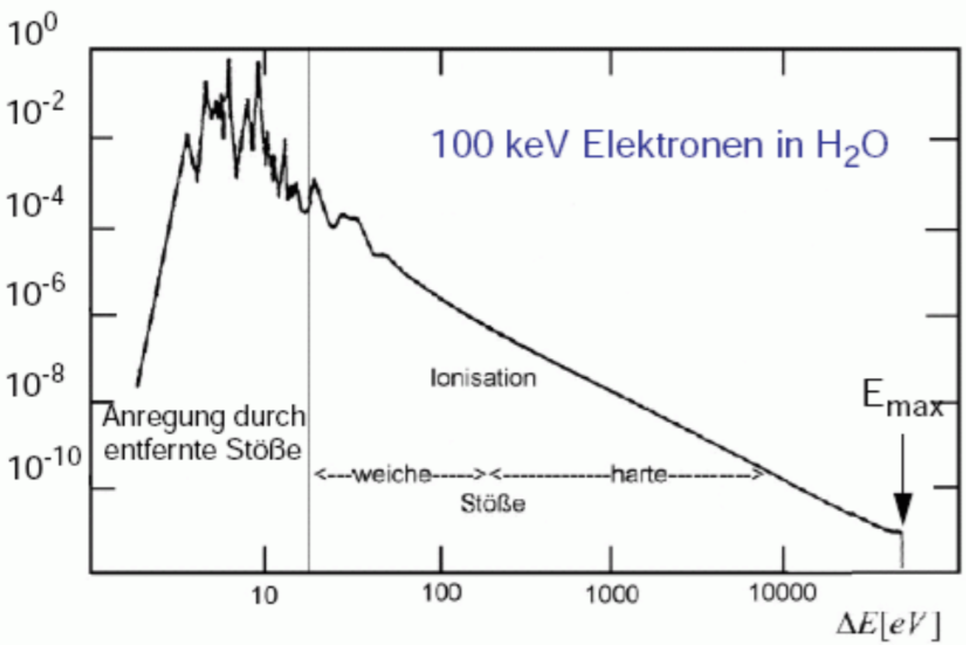
La distribuzione descrive bene la probabilita' finche' il trasferimento di energia e' grande rispetto all'energia di legame atomico (regione verde).

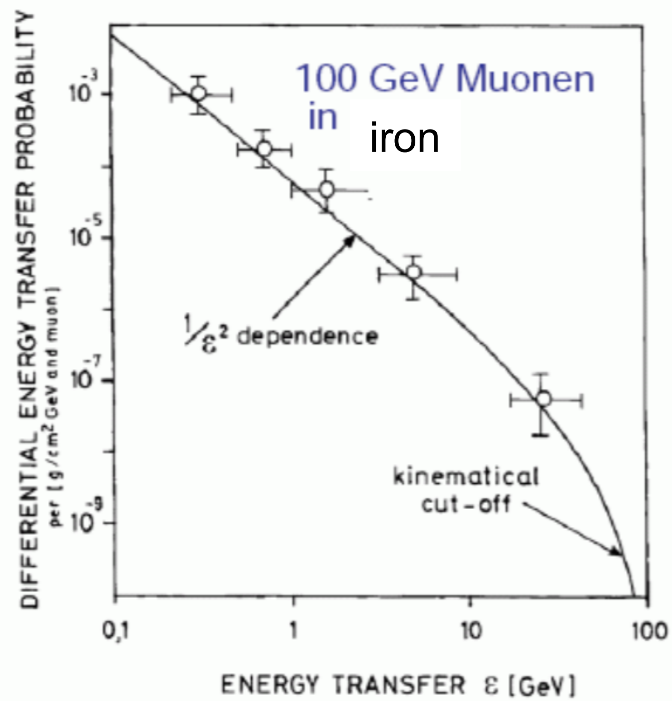
A basse energie (es. per collisioni distanti) entrano in gioco altri effetti (screening/ polarizzazione del mezzo, eccitazione di elettroni atomici, oscillazioni atomiche).

δ Electrons

Energy distribution of secondary electrons: $$\frac{d N^2}{dT\, dx} = \frac{1}{2} k\, z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2}$$

$$F(T) = 1 - \beta^2 T / T_{max}$$ (Spin 0)





Example: 500 MeV pion in 300 μm Si: 5% produce an electron with T>166 keV
important background in Cherenkov counter
Number of δ-electrons proportional z^2/β^2

Energy loss fluctuations for R<<L

Il numero di collisioni nel mezzo determina la distribuzione delle perdite di energia intorno al valore medio <dE>

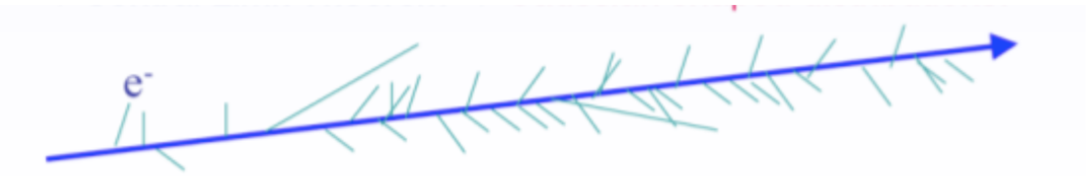
Ciò deriva direttamente dal teorema del limite centrale: la somma di N variabili casuali, che seguono tutte la stessa distribuzione statistica, diventa una gaussiana nel limite di $N \rightarrow \infty$.

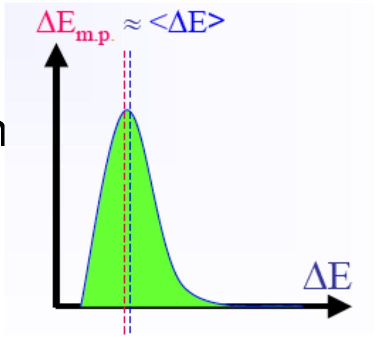
Se consideriamo come variabile casuale la dE, cioè l'energia persa in una collisione singola, che le collisioni sono statisticamente indipendenti e che in ogni collisione la velocità β del proiettile non è cambiata in maniera apprezzabile in modo che $d\sigma(E)/dE$ è costante ➔ l'energia totale persa è la somma di tutte le dE, tutte con la stessa distribuzione ed e' quindi gaussiana.

Per assorbitori relativamente spessi la distribuzione della perdita di energia è gaussiana.

Energy loss probability (1)

- Real detector measures the energy ΔE deposited in a layer of finite thickness δx.
- For thick layers and high density materials:
  - Many collisions
  - Central limit theorem: distribution → Gaussian





$$P(E)dE = k\frac{X}{E^2}dE \qquad \text{con} \qquad k = 2\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}$$

La probabilita' e' piccata a piccoli valori di E, cioe' per valori ~ Emin: la maggior parte delle collisioni comporta piccole perdite di E, ma c'e' una probabilita' finita che in una singola collisione avvenga una perdita "grande", vicina a Emax
Se N e' sufficientemente elevato, in media le perdite di energia saranno simmetriche intorno al valore medio: singole collisioni con alta E sono compensate da tante collisioni a bassa E, in modo che la distribuzione tenda a una gaussiana, in cui il valore medio <dE> e la perdita di energia piu' probabile dE_{mp} coincidono

Assorbitori spessi: limite gaussiano.

Se il materiale è spesso (ma non troppo) o denso ➔ N è grande quindi vale il teorema del limite centrale e la perdita totale di energia E è distribuita secondo una gaussiana

$$f(X,E) \propto \exp\left(-\frac{\left(E-\bar{E}(X)\right)^2}{2\sigma^2(X)}\right)$$

Con X spessore del materiale, E perdita di energia nell'assorbitore, $\bar{E}$ perdita di energia media = (dE/dX)X, e σ la sua deviazione standard.

Assorbitori spessi: limite gaussiano.

Bohr ha calcolato la deviazione standard σ_0 per particelle non relativistiche:

$$\sigma_0^2 = 4\pi Z_{inc}^2 Z_{target} n_a e^4 x = 4\pi r_e^2 N_A (m_e c^2)^2 Z_{inc}^2 \frac{Z_t}{A_t} \rho x = 0.157 Z_{inc}^2 \frac{Z}{A} \rho x \left[MeV^2 \right]$$

Dove N_A è il numero di Avogadro, r_e il raggio classico dell'elettrone, ρ la densità, A_t il peso atomico (in kg/mol) e Z_t il numero atomico dell'assorbitore.

Estesa a particelle relativistiche diventa:

$$\sigma^2 = \sigma_0^2 \frac{\left(1 - \tfrac{1}{2}\beta^2\right)}{1 - \beta^2}$$

- For a Gaussian distribution resulting from N random events the ratio of the width/mean $\propto 1/\sqrt{N}$
- Increasing the thickness of the detector decreases the relative width of the Gaussian peak