Particle Detectors

Lecture 2
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a.a. 2015-2016
Emanuele Fiandrini

Particle glossary

- Most important particle properties from the detector point of view are:
  - Mass
  - Charge (electric, "strong", "weak")
  - Interactions (EM, strong)
  - Lifetime
  - But also: **x**, **p**, E, β, radiation emission

Stable particles, life time $\tau = \infty$

- Can be used as beam particles
- Decay prohibited by *conservation laws*
  - Photon (γ)
  - Neutrinos (ν)
  - Electron/positron
- Proton/antiproton

Conservation Laws

Noether's Theorem: Every symmetry of nature has a conservation law associated with it, and vice-versa.

Emmy Noether
1882-1935

- **Energy, Momentum and Angular Momentum**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: translations in time and space; rotations in space
- **Charge conservation e.g. electric charge Q, colour charge**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: gauge transformation - underlying symmetry in QM description of electromagnetism / strong force
- **Lepton Flavour L_e, L_μ, L_τ and total quark number N_q**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: mystery!
- **Quark Flavour $N_u, N_d, N_s, N_c, N_b, N_t$, Parity, π**
  - ➡ Conserved in strong and electromagnetic interactions
  - ➡ Violated in weak interactions
  - ➡ Symmetry: unknown!

Quark and Lepton Flavour Quantum Numbers

- **Lepton Number**, L: Total number of leptons – total number of anti-leptons
  - ➡ Electron number, L_e
  - ➡ Muon number, L_μ
  - ➡ Tau number, L_τ
  - $L = L_e + L_\mu + L_\tau$

$$L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$
$$L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$$
$$L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$$

- **Quark Number**, N_q: Total number of quarks – total number of anti-quarks
  - ➡ Up quark number, N_u: *e.g.* $N_u = N(u) - N(\overline{u})$
  - ➡ Down quark number, N_d
  - ➡ Strange quark number, N_s
  - ➡ Charm quark number, N_c
  - ➡ Bottom quark number, N_b
  - ➡ Top quark number, N_t
  - $N_q = N_u + N_d + N_s + N_c + N_b + N_t$

- The lepton flavour quantum numbers (L, L_e, L_μ, L_τ) are conserved in **all** Standard Model interactions: strong, electromagnetic and weak.
- Quark number (N_q) is also conserved in all interactions.
- [Individual quark flavours (N_u, N_d, N_s, N_c, N_b, N_t) are conserved in strong and electro-magnetic interactions. They are not (necessarily) conserved in weak interactions.]

Average Life Time $\tau < \infty$

An unstable particle which is moving travels a distance before it decays. The average path length is:

$$\lambda_d = \gamma\beta c\tau = \left(\frac{p}{mc}\right)c\tau$$

The nbr of particles decaying in the length dx at the position x is proportional to the nbr of particles at x and to the probability of having an interaction in dx

$$dN(x) = -N(x)\cdot\frac{dx}{\lambda_d} \Rightarrow N(x) = N_0 e^{-x/\lambda_d}$$

→ This means that the number of particles surviving after a path legth x is an expo with slope λ_d (decay length)

Weakly decaying particles

- Decay "parameter" $\gamma c\tau = E c\tau / mc^2 = pc\tau / mc$
  - So $c\tau/m$ gives the mean decay distance for E = 1 GeV energy or $c\tau$ gives the mean distance for $E = mc^2$
- Neutron and muon ***n: 3×10^{11}m $\mu^{\pm}$: 6km***
- Light quark mesons: **$\pi^{\pm}, K^{\pm}, K^{0}_{L}$: 5-50m**

At high energy, 90% of detected particles from an hadronic interaction are charged pions!

- Strange baryons or "Hyperons" ***1-10cm***
- Heavy quark hadrons, τ lepton ***50-200μm***

Very short-lived particles $\tau < 10^{-12}$ s, $\lambda_d < \sim O(100\ \mu m)$

- Detectable *only* by their decay products, ie never reach the detectors
- Electromagnetic decays to photons or lepton pairs
  - Includes π^0 giving high-energy photons

Ex: π^0: $c\tau = 180 nm$

- Strongly decaying “resonances”

Very massive fundamental particles (τ~10^{-25} s)

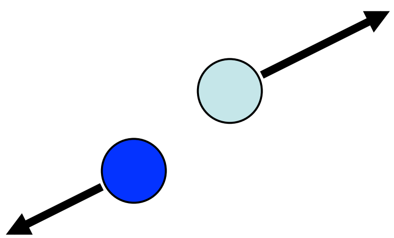
- $W^{\pm}, Z^{0}$
- top quark
- Higgs boson
- Super-symmetric particles, …

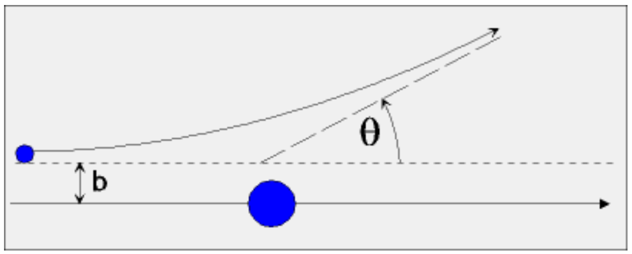
- Decay indiscriminately to lighter known (and possibly unknown) objects – leptons, quark "jets" (pions plus photons) etc.

What to measure, why?

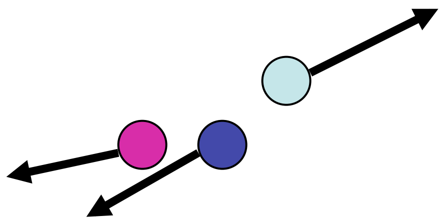
- Tipically in particle experiments we study interactions by:
  - particle-particle collisions
  - the decay products of unstable particles (eg radioactive decays)

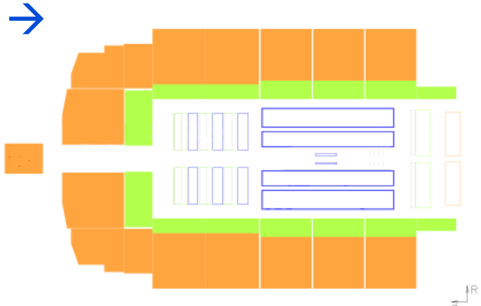
- The effects are:
  - *Change of the flight directions/ of the energy / of momentum of the original particles(eg e+e-→e+e- Bhabha)*





  - Production of new particles (e+e- → qq-bar → hadronization)

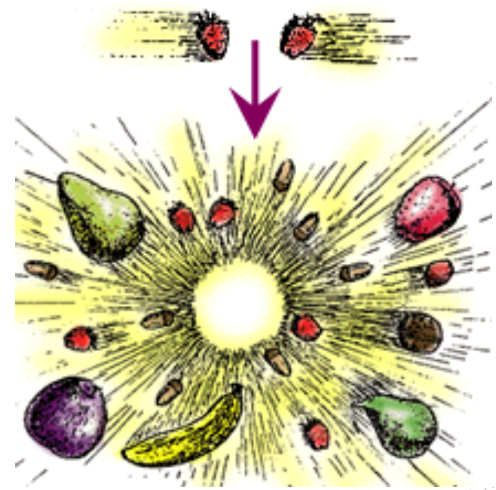




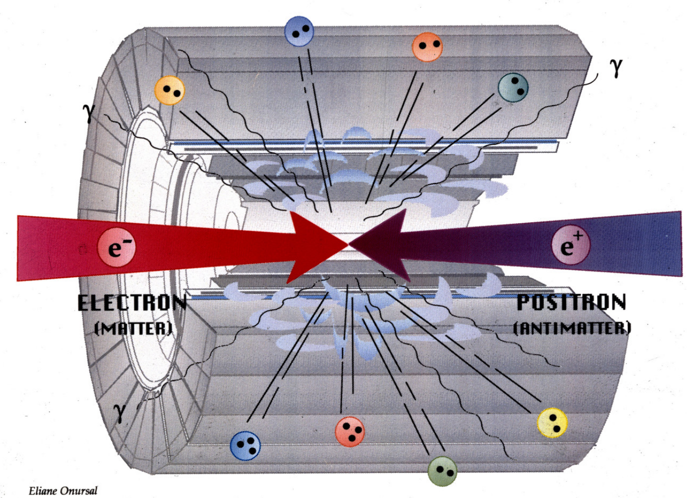
What can we detect?

- Directly observable particles must:
  - Be long lived ($\gamma c\tau$ sufficient to pass through sensitive elements of the detector)
  - Undergo strong or e.m. interactions

Collide



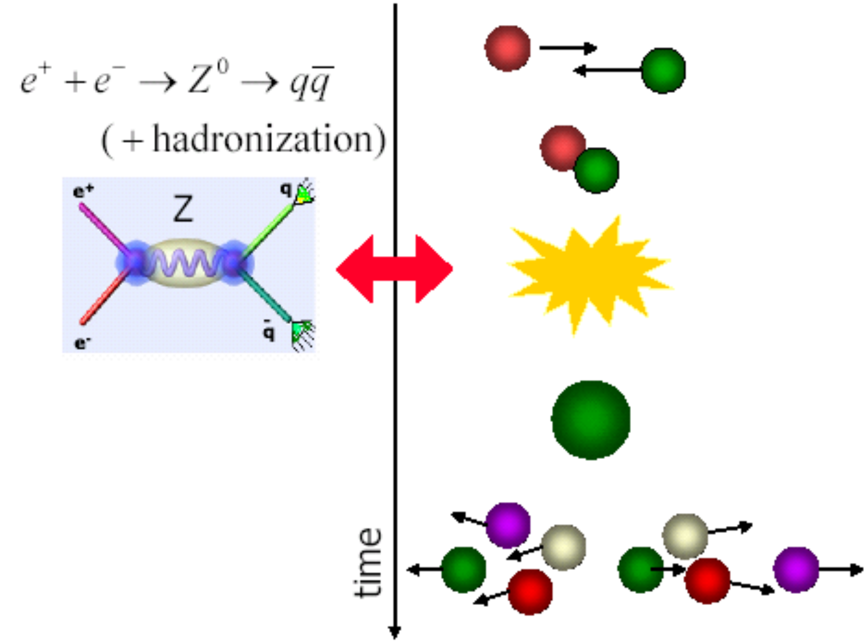
- We can directly observe:
  - electrons
  - muons
  - photons
  - neutral and charged hadrons/jets
    - $\pi^{\pm}$, K^0, $K^{\pm}$, p, n,…
    - Many physics analyses treat **jets** from quark hadronization collectively as single objects



- We can indirectly observe long lived weakly interacting particles (e.g. neutrinos) through **missing transverse energy**

What to measure, why?

Usually we can only 'see' the end products of the reaction, but not the reaction itself.



In order to reconstruct the reaction and the properties of the involved particles, we want the maximum information about the end products

- → **need to identify the particle type and measure the energy, direction, charge and momentum of all final state products as precisely as possible to reconstruct the decay chains and "to see" the parent particles.**

What to measure, why?

- Particle detectors need to provide:
  - Detection and identification of different particle types (mass, charge)
  - Measurement of particle momentum (track) and/or energy (calorimeter)
  - Coverage of full solid angle without cracks ("hermiticity") in order to measure missing E_T (neutrinos, supersymmetry) (if at accelerators) or large counting power (if not at accelerators)
  - Fast response (minimum dead time, eg LHC bunch crossing interval 25ns!)

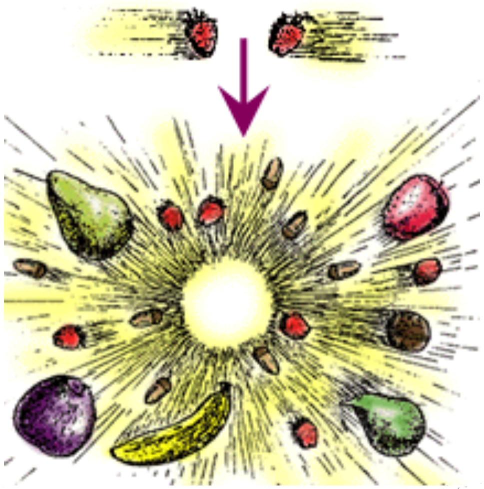
What to measure, why?

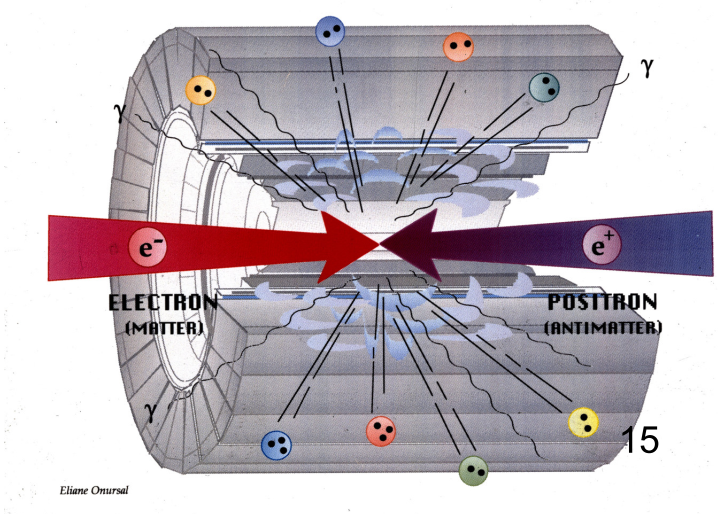
- A perfect detector should reconstruct any interaction of any type with 100% efficiency and unlimited resolution (get “4-momenta”of basic physics interaction)
- BUT not all particles are detected, some leave the detector without any trace (neutrinos), some escape through not sensitive detector areas (holes, cracks for e.g. water cooling and gas pipes, electronics, mechanics), some do not give a signal, limited resolution.
- Efficiency is never 100%, errors are never negligible

Detected Particles: only stable or long lived

- Different particle types interact differently with matter (detector)
- (eg. photons do not feel a magnetic field)
  - need different types of detectors to measure different types of particles

Collide





Detection of Charged Particles

- Ultimately all detectors end up detecting charged particles:
  - Photons are detected via electrons produced through:
    - Photoelectric effect
    - Compton effect
    - e^+e^- pair production (dominates for E>5GeV)
  - Neutrons are detected through transfer of energy to charged particles in the detector medium (shower of secondary hadrons at high E, recoiling nuclei at low E)
- Charged particles are detected via e.m. interaction with electrons or nuclei in the detector material:
  - Inelastic collisions with atomic electrons → energy loss
  - Elastic scattering from nuclei → change of direction

Detection of Charged Particles

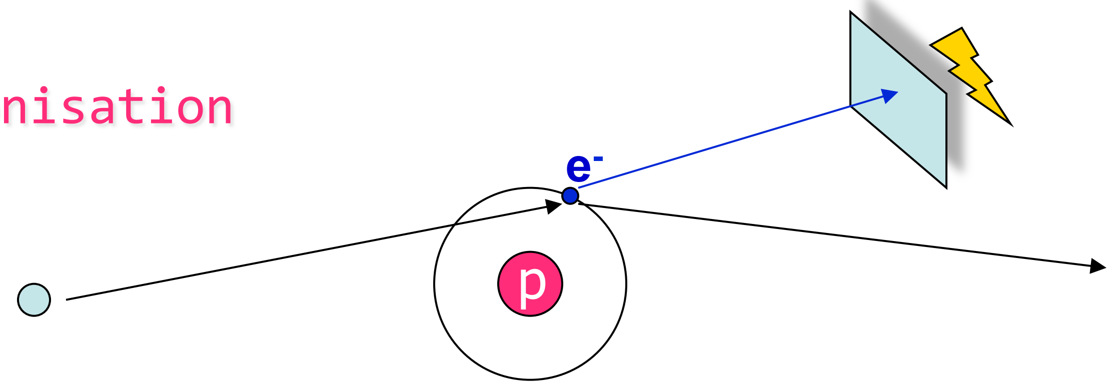
- Which kind of particle do we have to detect?
- What is the required dimension of the detector?
- Which “property” of the particle do we have to know?
  - Position, trajectory
  - Time
  - Number
  - Energy
  - Momentum

  } **What is the required resolution?**
- What is the maximum count rate?
- What is the time distribution of the events?
- And last, but not least, how much does it cost?

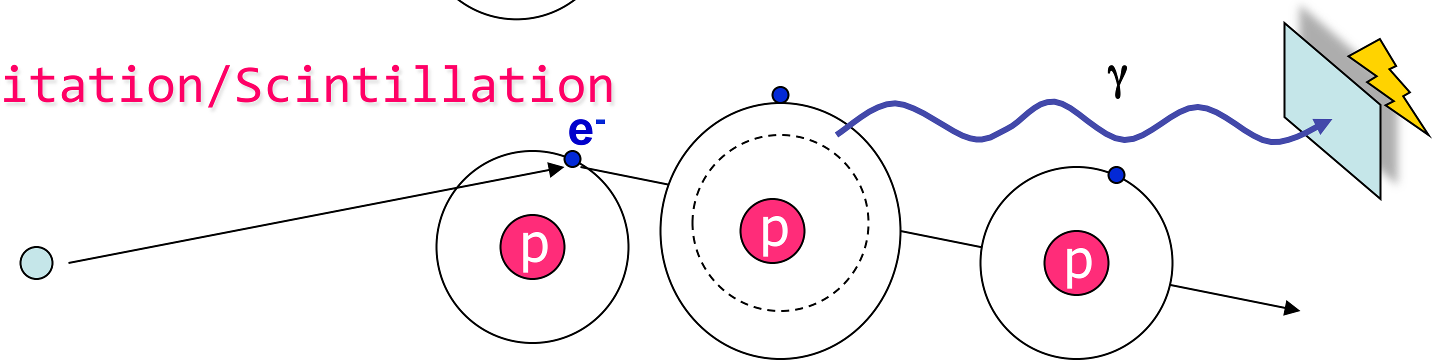
Principles of a measurement

- Measurement occurs via the interaction (again…) of a particle with the detector(material)
  - creation of a measureable signal

- Ionisation



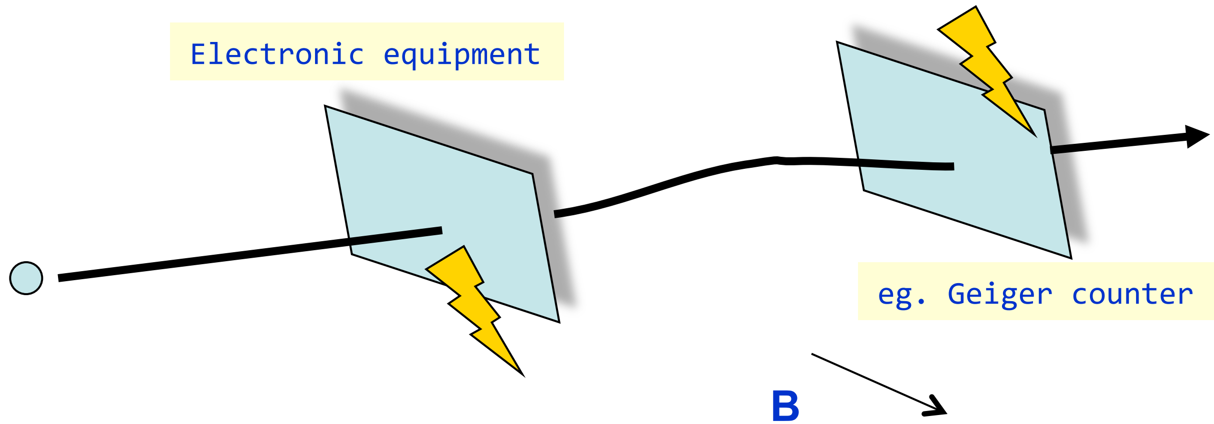
- Excitation/Scintillation



- Change of the particle trajectory
  - curving in a magnetic field,energy loss
  - scattering, change of direction, absorption

Measured quantities

- The creation/passage of a particle (--> type)



- Its four-momentum

$$\begin{pmatrix} \text{Energy} \\ \text{momentum in x-dir} \\ \text{momentum in y-dir} \\ \text{momentum in z-dir} \end{pmatrix} = \begin{pmatrix} E \\ \vec{p} \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

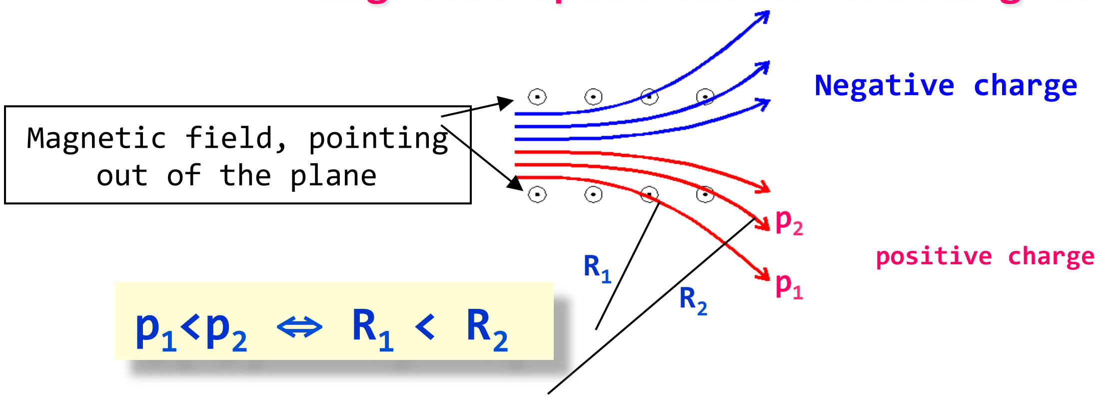
- Its velocity $\beta = v/c$

How measure the four-momentum?

- Energy : from "**calorimeter**"→ E is absorbed in the active medium (ie showers)

Momentum :

- from "**magnetic spectrometer+tracking detector**"



$p_1 < p_2 \Leftrightarrow R_1 < R_2$

Lorentz-force

$q\,v\,B = m\,v^2/R$

⇓

$q\,B\,R = m\,v = p$

velocity :

- **time of flight** or **Cherenkov radiation**

$v\,t = L$

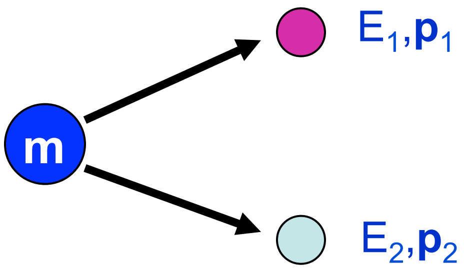
Derived properties

- Mass
  - in principle, if E and **p** measured: $E^2 = m^2c^4 + p^2c^2$

▌ if v and **p** measured: $p = mv/\sqrt{(1 - \beta^2)}$

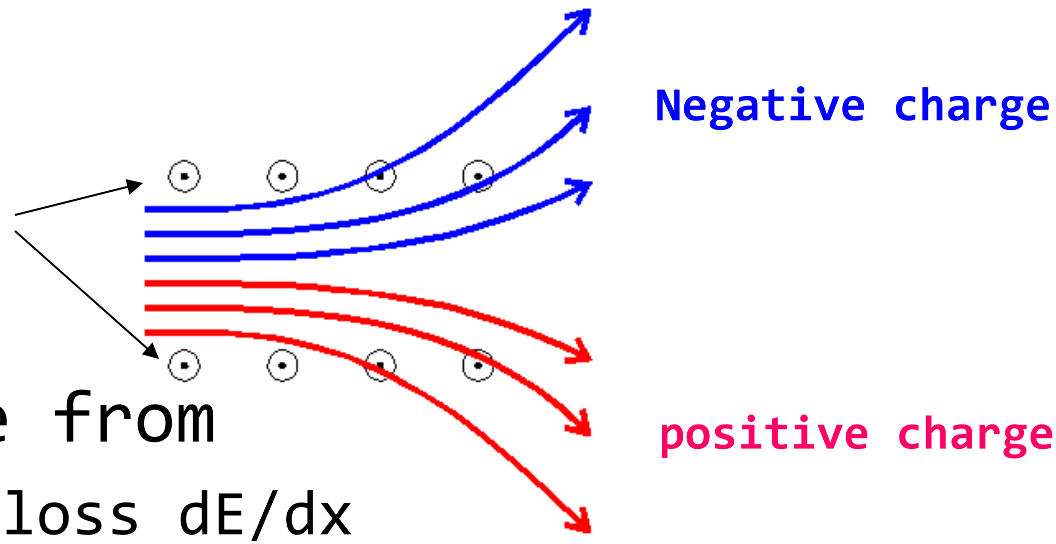
▌ from E and **p** of decay products:

 | $m^2c^4 = (E_1+E_2)^2 - (cp_1 + cp_2)^2$



Further properties...

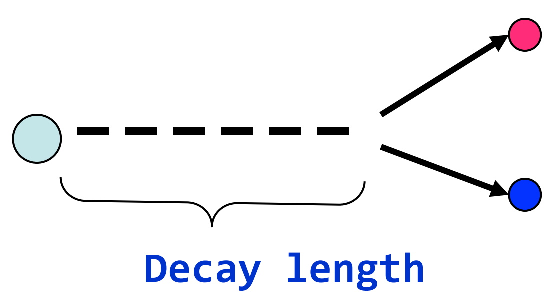
- The charge (at least the sign…)
  - from curvature in a magnetic field



- The charge value from
  - Specific energy loss dE/dx
  - Cherenkov radiation

The lifetime τ

- from flight path before decay

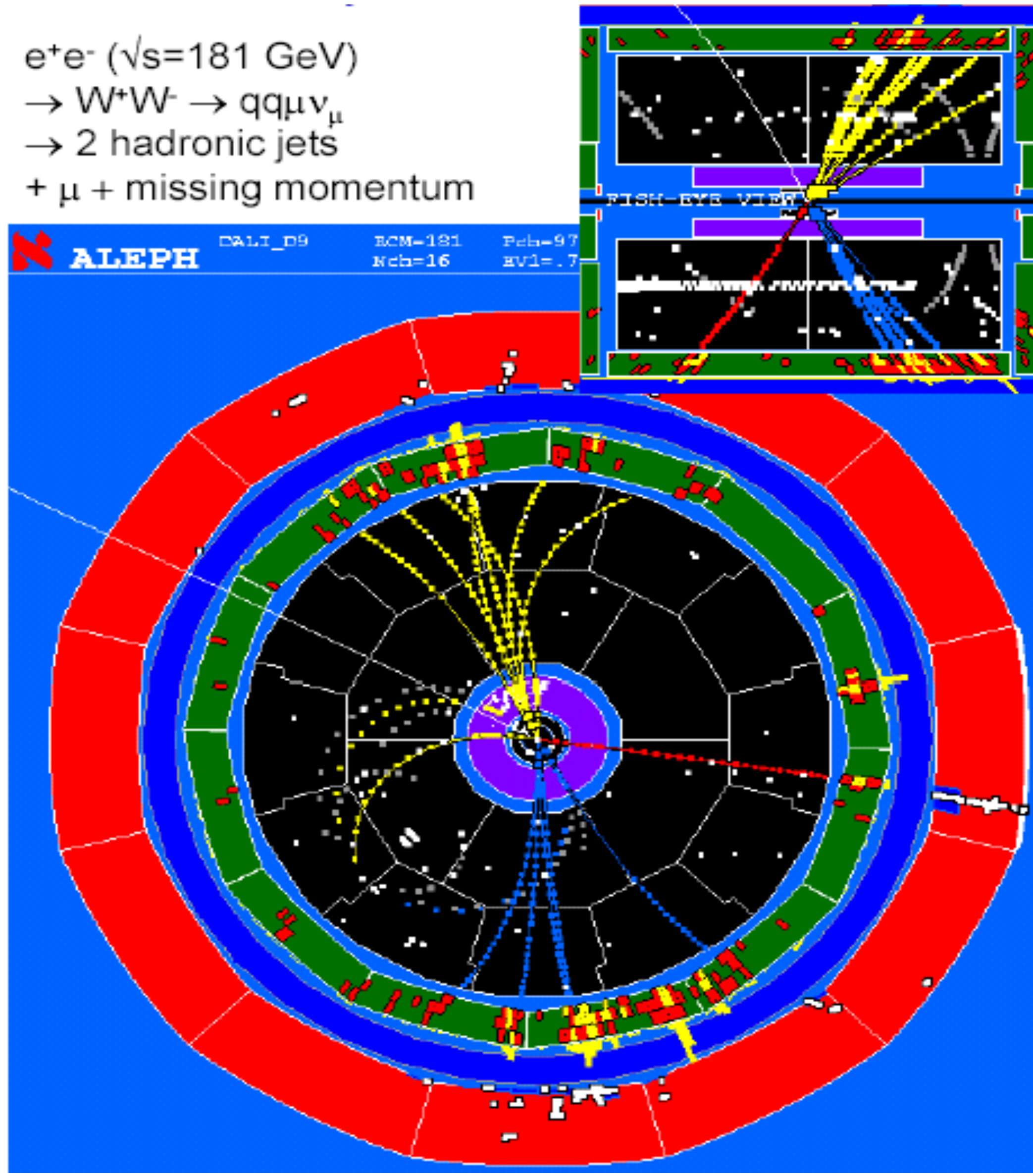


What to measure, why?

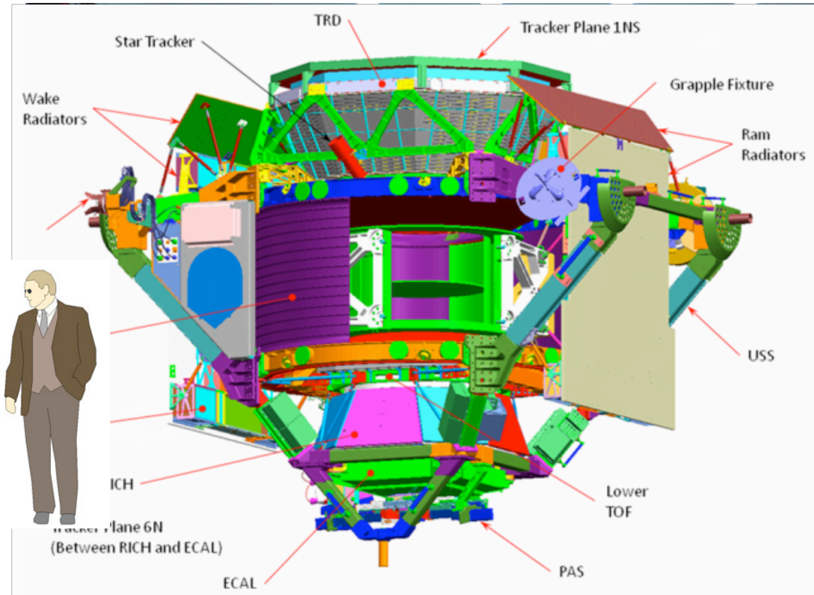
The key element for an experimental apparatus is the combination of different detectors to obtain a detector system: many (ie >2) detectors that works sinchronously/in parallel, providing a set of signals correlated spatially and temporally.

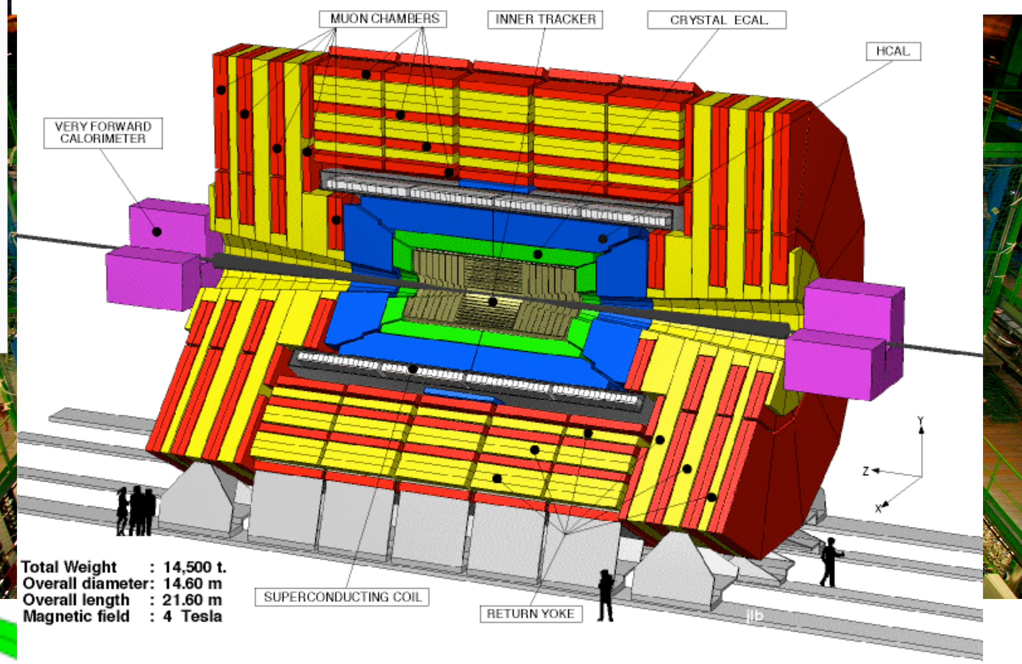
By the correlation of signals it is possible to measure some cinematic observables (as speed, energy, momentum, charge,...)

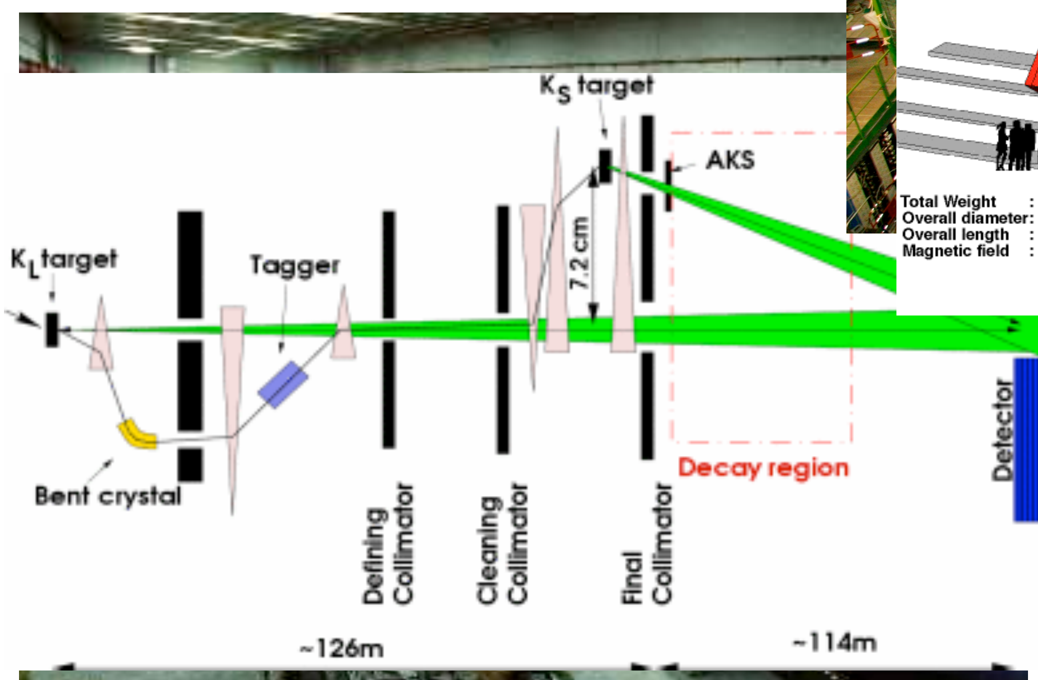
Example: a magnetic spectrometer is the combination of a tracking detector in a B field and a Time Of Flight (TOF) detector* in a suitable geometric setup



*which in turn is the combination of at least 2 scintillators with a time coincidence within a time window


TRD
Tracker Plane 1NS
Star Tracker
Grapple Fixture
Wake
Radiators
Ram
Radiators
USS
ICH
Lower
TOF
lane 6N
(Between RICH and ECAL)
ECAL
PAS


MUON CHAMBERS
INNER TRACKER
CRYSTAL ECAL
HCAL
VERY FORWARD
CALORIMETER
Total Weight : 14,500 t.
Overall diameter: 14.60 m
Overall length : 21.60 m
Magnetic field : 4 Tesla
SUPERCONDUCTING COIL
RETURN YOKE


K_S target
AKS
K_L target
Tagger
7.2 cm
Bent crystal
Defining Collimator
Cleaning Collimator
Final Collimator
Decay region
Detector
~126m
~114m

Useful Reference Frames

- **CM frame** is Centre-of-Mass or Centre-of-Momentum
  - "Rest frame" for a system of particles
  - I.e. $\sum \underline{\mathbf{p}}_i = 0$ (where $\underline{\mathbf{p}}$ is the usual 3-vector)
- **LAB frame** – may be:
  - Rest frame of some initial particle

Invariant Quantities – Invariant Mass

- **Lorentz *invariant*** quantities exist for individual particles **and** systems.
- **Invariant mass** of a system:

$$s = p^2 = (\sum_{i=1..N} p_i^{\mu})(\sum_{i=1..N} p_{i\mu})$$

$$\sum_{i=1..N} p_i^{\mu} = (\sum_{i=1..N} E_i, \sum_{i=1..N} \underline{p})$$

$$s = \left(\sum_{i=1..N} E_i\right)^2 - \left(\sum_{i=1..N} \underline{p}_i\right)^2$$

Invariant Mass

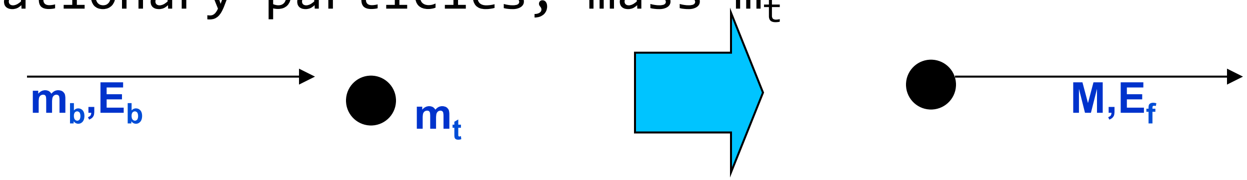
- **Invariant mass** is equivalent to the CM frame energy for a particle system
  - If $(\Sigma \underline{\mathbf{p}}_i)=0$ then

$$s = \left(\sum_{i=1..N} E_i \right)^2 - \left(\sum_{i=1..N} \underline{p} \right)^2 = \left(\sum_{i=1..N} E_i \right)^2$$

- NB within a frame Σp^{μ}_{i} =constant
  - (conservation of momentum)

Total CM Energy in Fixed Target

- "Fixed target" experiment with a beam of particles, energy E_b, mass m_b incident on a target of stationary particles, mass m_t



$$p_\mu = (E_b + m_t, \vec{p}_b) \qquad s = (E_b + m_t)^2 - p_b^2$$

$$s = m_t^2 + m_b^2 + 2E_b m_t \qquad E_b \gg m_i \qquad \sqrt{s} \approx \sqrt{2m_t E_b}$$

Only a fraction of the energy is available for particle production. Where does the rest of E go? It appears as motion of the particle system as a whole, that is as energy of the CM in the lab

- If we want a fixed target experiment to have a CM energy, √s, higher than M then the beam energy E_b :

$$E_b \geq \frac{M^2 - m_t^2 - m_b^2}{2m_t}$$

Center of Mass Collisions

The cross sections and the energy available for new particle production depend on the total energy in the center of mass (CM) frame.
By definition then, in the CM frame we have for two 4-vectors (p_1, p_2):

$$(p_1+p_2)= (E_1+E_2, \mathbf{p}_1+\mathbf{p}_2) = (E_1+E_2, 0) \text{ since } \mathbf{p}_1 = -\mathbf{p}_2$$

If the masses of the two particles are equal as in the case of proton anti-proton collisions then the above reduces to:
$(p_1+p_2)= (E_1+ E_2, p_1+ p_2) = (E_1+ E_2, 0)= (2E, 0)$ (twice energy of either particle)

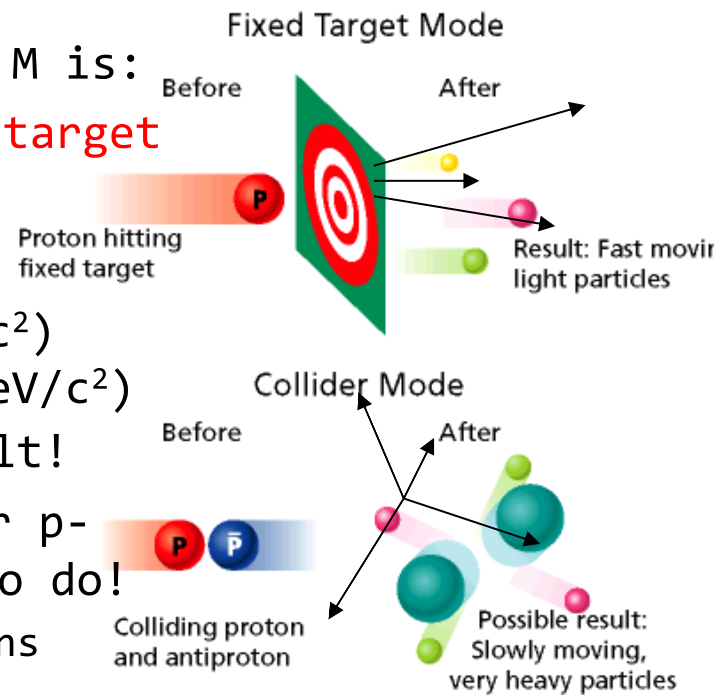
How much energy is available in the CM from a 10 GeV/c anti-proton colliding with a proton at rest?
Since (p_1+p_2) is a Lorentz invariant we evaluate in any frame we please!
We are given values in the lab frame: $(p_1+ p_2) =(E_1+m_p, p_1+0)$
The magnitude of this 4-vector is: $s = (E_1+m_p)^2- p_1^2= (10.044+0.938)^2-10^2= 20.6\text{GeV}$
Thus the total energy in the CM is 4.54GeV
We could have gotten the same CM energy with two beams = 2.27 GeV !
In general the energy available for new particle production increases as:

$(2m_{target}E_{beam})^{1/2}$ for *fixed target* experiments

$2E_{beam}$ for *colliding beam* experiments

Types of collisions

- The available energy for the reactions is the energy of the projectile-target system on in Center of Mass reference frame.
- The minimum E to create a particle of mass M is:
- Fixed target: $E > M^2c^2/2m_t$, where m_t is the target mass
- "Head on": $E > Mc^2/2$
- For example to create a W boson ($M \approx 80$ GeV/c^2) with p collisions on a fixed target ($m \approx 1$ GeV/c^2) $E > M^2c^2/2m = 80^2/(2*1) = 3200$ GeV: difficult!
- While in a head-on collision of protons (or p-antip), $E > Mc^2/2 = 40$ GeV (each) → easy to do!
  - All the beam energy is available for reactions



Even More Relativistic Kinematics

In the early 1950's many labs were trying to find evidence of the anti-proton. At Berkeley a new proton accelerator (BEVATRON) was being designed for this purpose. Assuming fixed target proton-proton collisions would be used to create the antiproton what energy proton beam (E_b) is necessary ?
The simplest reaction that conserves all the necessary quantities (energy, momentum, electric charge, baryon number) is:

$$p\,p \rightarrow p\,\bar{p}\,p\,p \quad \text{with } \bar{p} = \text{anti-proton}$$

The total energy in the CM is given by: $(p_b+p_t)^2$ the sum of the 4-vectors of the beam and target proton (assumed to be at rest):

$$(p_b+p_t)^2 = (E_b+m_p,\ p_b)^2 = m_b^2+m_t^2+2m_tE_b = 2m_p^2+2m_pE_b$$

The trick now is to remember that $(p_b+p_t)^2$ is Lorentz invariant and can be evaluated in any frame we choose. The most convenient frame is the one where all the final state particles are produced at rest. Here we have:

$$(p_b+p_t)^2_{initial} = (E_b+m_p,\ p_b)^2 = (p_b+p_t)^2_{final} = (4m_p)^2$$

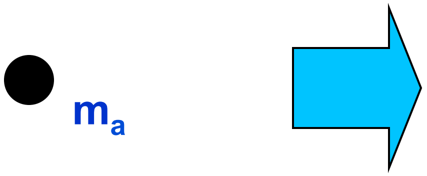
$$2m_p^2+2m_pE_b = (4m_p)^2$$

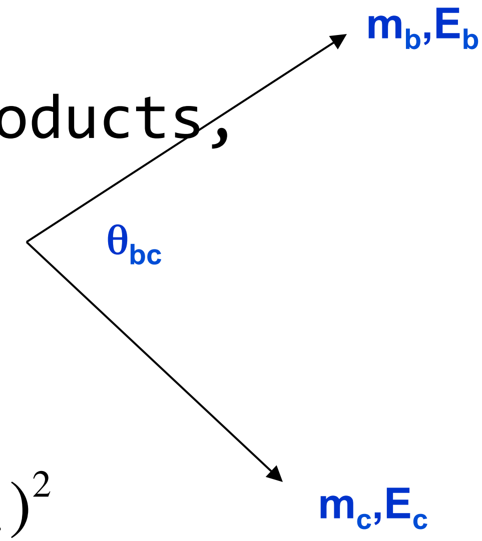
$$E_b = 7m_p = 6.6 \text{ GeV}$$

The anti-proton was discovered at Berkeley in 1955 (Nobel Prize 1959)

Mass of Short-lived Particle

- We can’t see short-lived particles (that is they don’t reach the detectors) but rather their decay products
- From invariant mass of its decay products, e.g. 2-body:
- How to measure m_a?

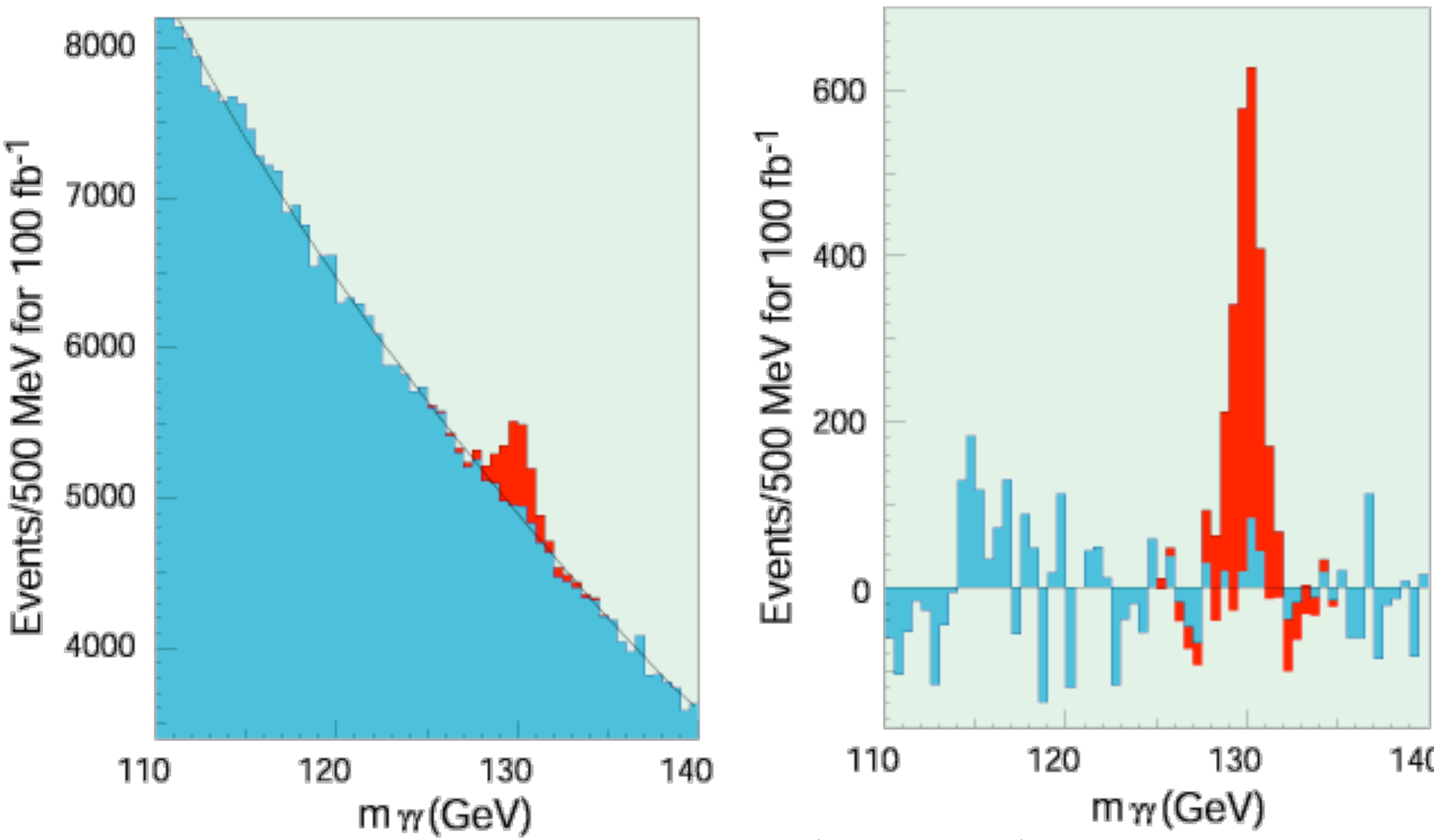




- Initial invariant mass s = m_a^2
- Final invariant mass = $(E_b + E_c)^2 - (\vec{p}_b + \vec{p}_c)^2$
- If E_b, E_c >> m_c , m_c then E_b, E_c~ p_b, p_c
- So, $$m_a^2 = 2E_bE_c(1 - Cos\theta_{ab})$$

- If we measure E_b,E_c and the angle, we can determine the mass of the parent particle

Higgs boson discovery in γγ decay channel



$$m_a{}^2 = 2E_bE_c(1 - Cos\,\theta_{ab})$$

Interaction Rates and Cross-sections

- No matter what experiment, at the end, one ends up counting particles
- Experiments measure rates of reactions – these depend on both
  - “kinematics” e.g. energy available to final state particles, and
  - “dynamics”, e.g. strength of interaction, propagator factors etc.

Cross section σ

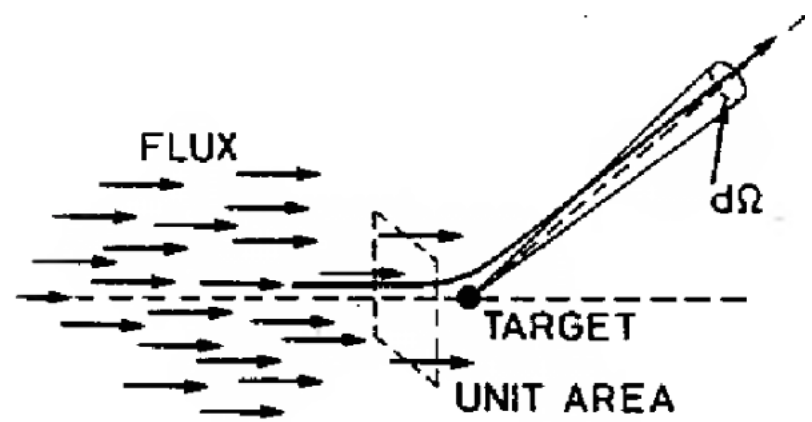
- Cross section gives the probability for a given process to occur
- Cross section incorporates:
  - Strength of underlying interaction (vertices)
  - Propagators for virtual exchange factors
  - Phase space factors (available energy)
  - Does **not** depend on rate of incoming particles.
- Called the "cross-section" because it has units of area.
  - Normally quoted in units of **barns** (10^{-28}m^2)

  … or multiples eg. nanobarns (nb), picobarns (pb)

Cross section σ

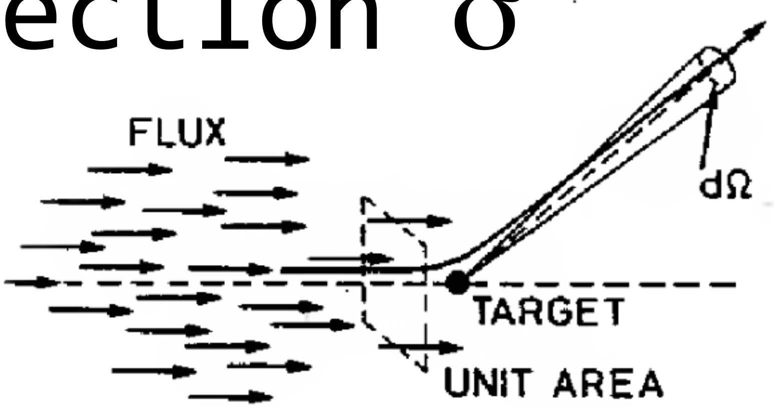
- Formally, the cross-section is defined in the following manner: Consider a beam of particles 1 incident upon a target particle 2. Assume that the beam is much broader than the target and that the particles in the beam are uniformly distributed in space and time.
- We can speak of a flux F of incident particles ($cm^{-2}s^{-1}$).
- Now look at the number N_s of particles scattered into the solid angle dΩ per unit time, $dN_s/d\Omega$.
- Because of the randomness of the impact parameters, this number will fluctuate over different finite periods of measuring time. However, if we average many finite measuring periods, this number will tend towards a fixed $dN_s/d\Omega$ where Ns is the average number scattered per unit time. The differential cross section is then defined as the ratio

$$\frac{d\sigma}{d\Omega}(E, \Omega) = \frac{1}{F}\frac{dN_s}{d\Omega},$$



Cross section σ

$$\frac{d\sigma}{d\Omega}(E, \Omega) = \frac{1}{F}\frac{dN_s}{d\Omega},$$



- dσ/dΩ is the average fraction of the particles scattered into dΩ per unit time per unit flux F.

- Note that because of the dimensions of F, dσ has dimensions of area, which leads to the heuristic interpretation of dσ as the geometric cross sectional area of the target intercepting the beam. That fraction of the flux incident on this area will then obviously interact while all those missing da will not. This is only a visual aid, however, and should in no way be taken as a real measure of the physical dimensions of the target.

Cross-Section – "physical" interpretation

σ is used to measure the probability of interactions between elementary particles.

- If we play dartboard, the important parameter is the target dimension, that is the surface area that incident dart beam sees.
- Similarly, if we shoot an electron beam on a H tank, the important parameter is the proton dimension, that is the surface area seen by the incident beam. But the proton has not a well defined section; the more the e- gets close, the higher is the interaction probability. The cross section depends on projectile and target species, on energy, spin,… .
  - elastic cross section (if the energy is low we will have only $e+p \rightarrow e+p$)
  - anelastic cross section (if the energy is enough we can have $e+p \rightarrow e+p+\gamma$ or $e+p \rightarrow e+p+\pi$ etc)

  - **1 barn (b) =10^{-24} cm^2**

For a linear momentum in the lab of 10 GeV/c we have:

- **σ_t ($\pi^+ p$) ~ 25 mb (forte)**
- **σ_t (γp) ~ 100 μb (e.m.)**
- **σ_t (νp) ~ 0.1 pb (debole)**

Cross-Section – "physical" interpretation

- Can be thought of as an effective area centred on the target – if the incident particle passes through this area an interaction occurs.
  - Physical picture only realistic for short range interactions. (target behaves like a featureless extended ball)
  - For long range interactions, like EM, integrated cross-section is infinite.

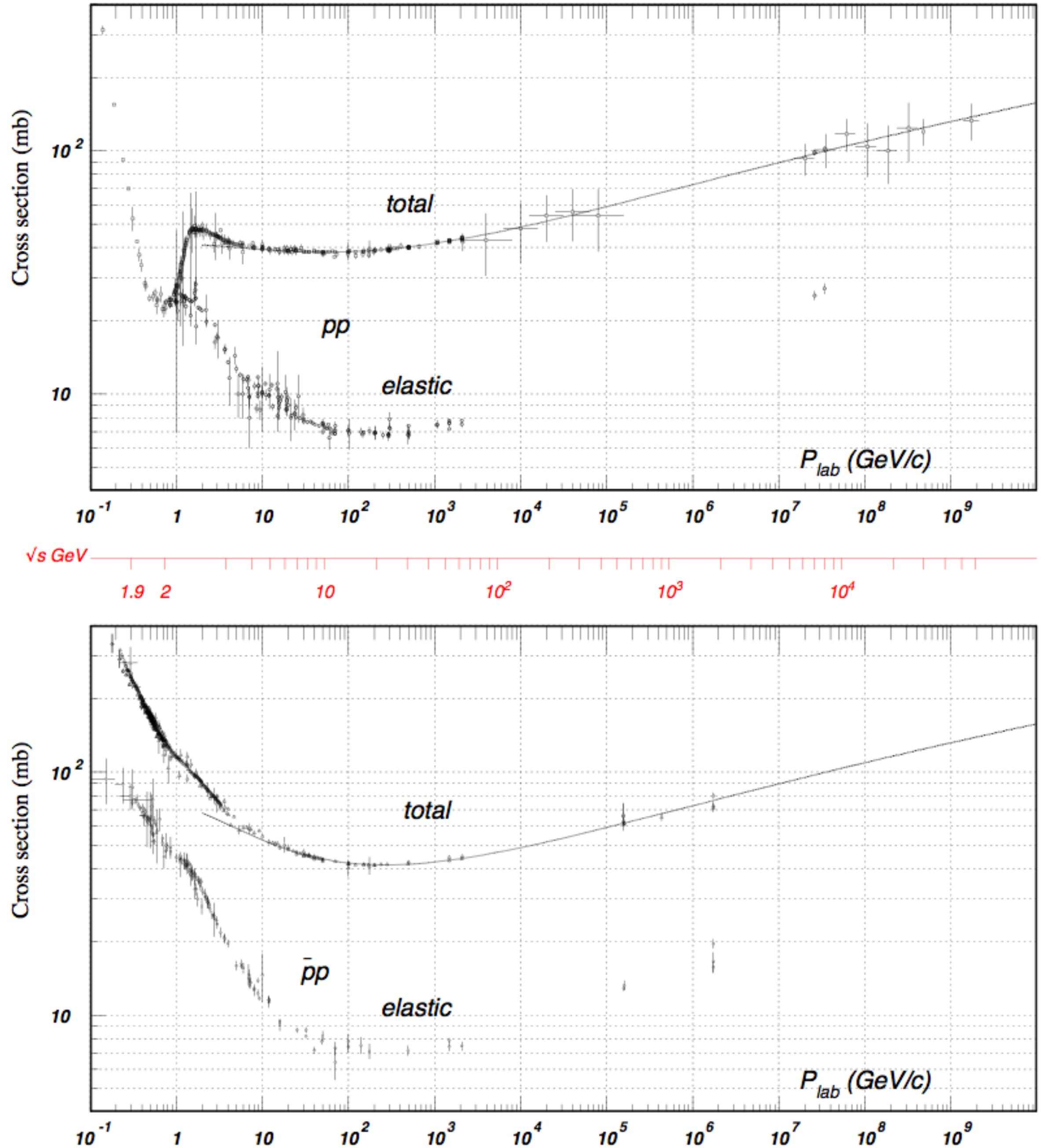


Figure 49.9D11: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding data files may be found at

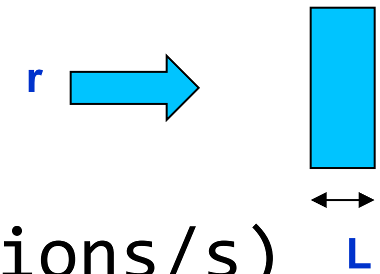
- In real situations, the target is usually a slab of material containing many scattering centers and it is desired to know how many interactions occur on the average.
- Assuming that the target centers are uniformly distributed and the slab is not too thick so that the likelihood of one center sitting in front of another is low, the number of centers per unit perpendicular area which will be seen by the beam is then Ndx where N is the density of centers and dx is the thickness of the material along the direction of the beam. If the beam is broader than the target and A is the total perpendicular area of the target, the number of incident particles which are eligible for an interaction is then FA. The average number scattered into dΩ per unit time is then

$$N_s(\Omega) = F A N \delta x \frac{d\sigma}{d\Omega} .$$

If the beam is smaller than the target, then we need only set A equal to the area covered by the beam. Then FA = n_{inc}, the total number of incident particles per unit time. In both cases, now, if we divide by FA, we have the probability for the scattering of a single particle in a thickness dx, Prob. of interaction in dx = $N\sigma\, dx$

Cross-section and Interaction rate.

- For fixed target, with a target larger than the beam



- $W = r n_t L \sigma$
  - W =interaction rate (# interactions/s)
  - r =rate of incoming particles (# part/s)
  - n_t =number density of target particles (# part m^{-3})
  - L = thickness of target (m)
  - σ = cross-section for interaction (m^2)

Cross-section and Interaction rate quick calc

- Given a single target, by definition of x-section, all the particles in the volume $\sigma dx = \sigma v dt$ will interact with target.
- If the projectile particles have density n_p, then the # interactions is $dN = n_p \sigma v dt$.
- If there is a nbr density of targets n_t, the interacion rate per unit of volume is

$$dN/dVdt = n_t n_p \sigma v$$

- If the interaction volume is V = S L, with S section of projectile beam and L target thickness, then interaction rate is $W = n_t \, n_p \, \sigma \, v \; S \; L$, but the rate of incident part is $r = n_t \, v \; S$ ➔ $W = r \; n_t \; L \; \sigma$

Cross-section and Interaction rate.

- $W = n_t n_p \sigma c S L$
- For fixed target, in terms of particle flux, $J = n_p c$
- $W = J n \sigma$
  - W = interaction rate
  - J = Flux: particles per unit area per unit time.
  - n = total number density of particles in target.
  - σ = cross-section for interaction

Sezione d'urto

- **Esempio numerico: $p^- p \rightarrow \pi^0 n$**
  - $N_p = 10^7$ particelle incidenti a burst (impulso dell'acceleratore)
  - 1 burst ogni 10 s
  - 8 giorni di presa dati (N_{days})
  - Bersaglio di Be (ρ =1.8 gr/cm^3) L=10 cm
  - Dati raccolti 7.49×10^{10}

La relazione da usare e': $W = r\, n_t\, L\, \sigma$

Se prendiamo dati per un tempo dT, il numero di interazioni e'

$N_{rac} = WdT = (rdT)\, n_t\, L\, \sigma$, $rdT = N_{fascio} = (N_p * 86400/10) * N_{days}$

Ci serve in nr. di protoni del bersaglio:

La densita' di numero di atomi di Be e' $n_{Be} = \rho/m_{Be}$

La massa m_{Be} si ottiene dalla relazione $m_{Be} = M_{mol}/N_A = A/N_A$, M_{mol} = massa molare (Kg/mol) = A kg/mol $\rightarrow n_{Be} = \rho N_A/A$,

Il numero atomico e' Z quindi ci sono $n_p = Zn_{Be} = \rho N_A(Z/A)$ protoni, con Z/A = 4/9 per unita' di volume

$\sigma_T = (N_{rac}/N_{fascio}) \times (1/n_A)$ ($N_{rac} = 7.49 \times 10^{10}$, $N_{fascio} = 6.9120 \times 10^{11}$)

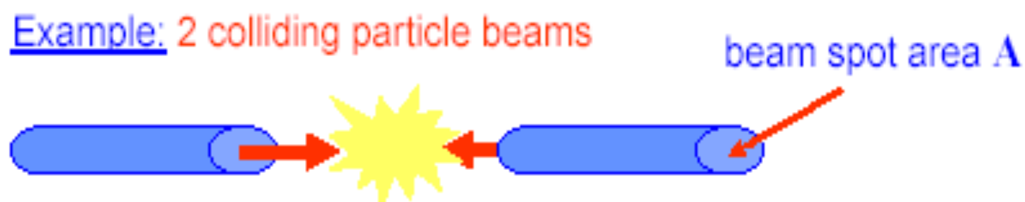
$n_A = Ln_p$ (numero di protoni/cm^2 visti dal fascio)

$\sigma_T = (7.49 \times 10^{10})/(69120 \times 10^7 \times 48.18 \times 10^{23}) \sim 2.25 \times 10^{-26}$ cm^2=22.5 mb

Colliding Beam Interaction Rate

- In a colliding beam accelerator, particles in each beam stored in bunches (see accelerators, later).
  - Bunches pass through each other at interaction point, with a frequency f
  - Have an effective overlap area, A → interaction rate is

$$W = \frac{n_1 n_2 f}{A}\sigma$$



  - Can express in terms of beam currents I=nf

$$W = \frac{I_1 I_2}{Af}\sigma$$

- Factors n_1n_2f/A normally called the Luminosity, *L*

$$W = L\sigma$$

Colliding Beam Interaction Rate quick calc

- In one revolution, a particle of the bunch 1 "sees" a bunch of N_2 particles, distributed over an effective surface A → sees a particle density of N_2/A.
- Since there are N_1 particles, circulating with freq f, the # of encounters per unit of time and surface is $(N_2/A)fN_1$
- In each encounter, the probability for a given process to occur is σ ➔

$$W = \frac{n_1 n_2 f}{A}\sigma$$