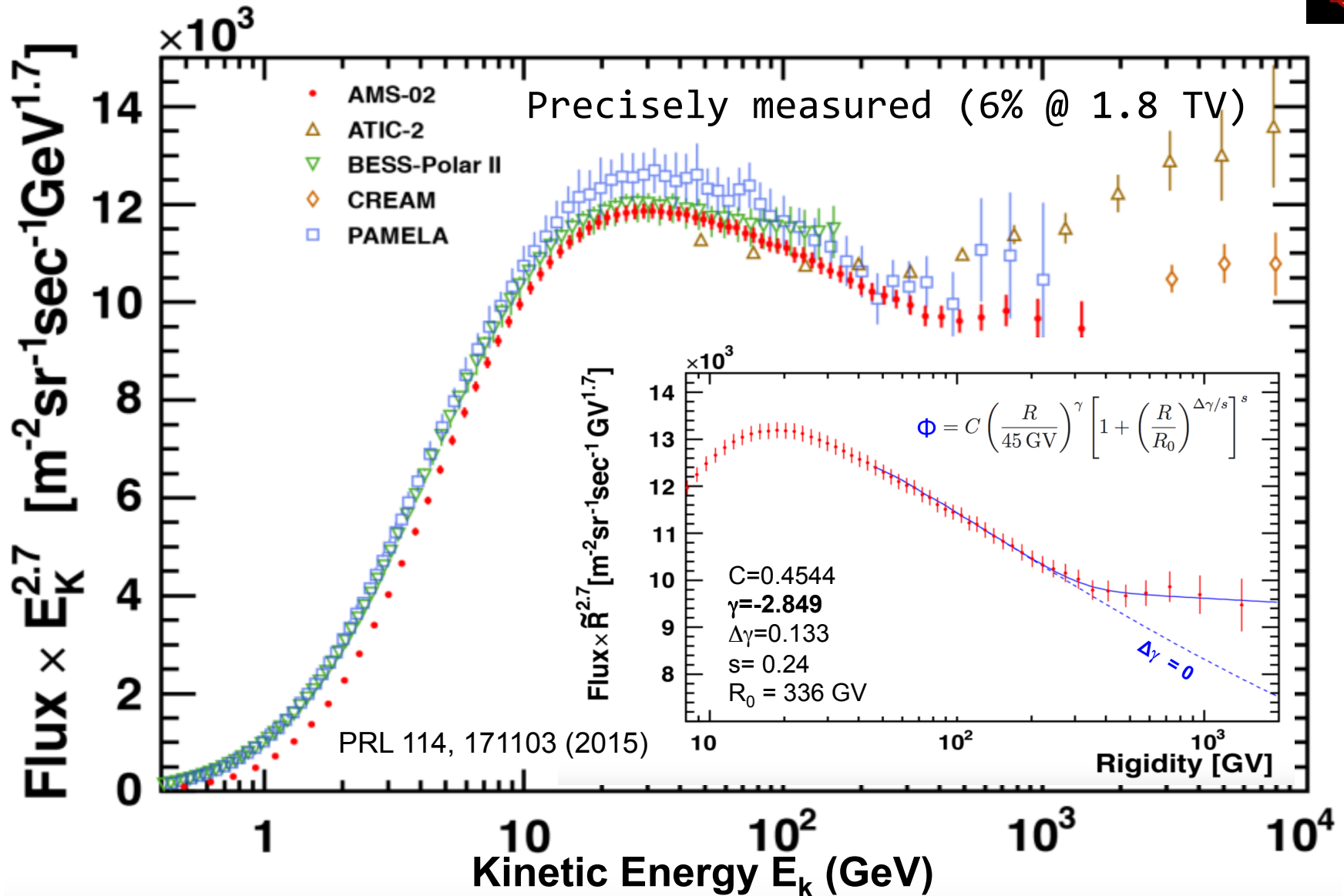


# Accelerazione dei raggi cosmici



# Proton flux



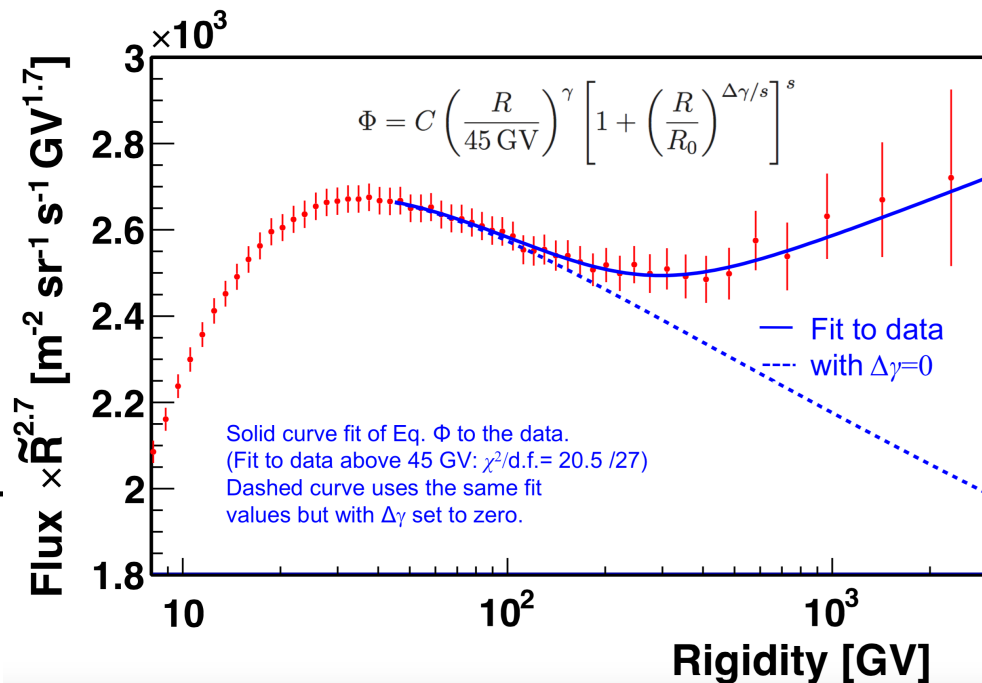
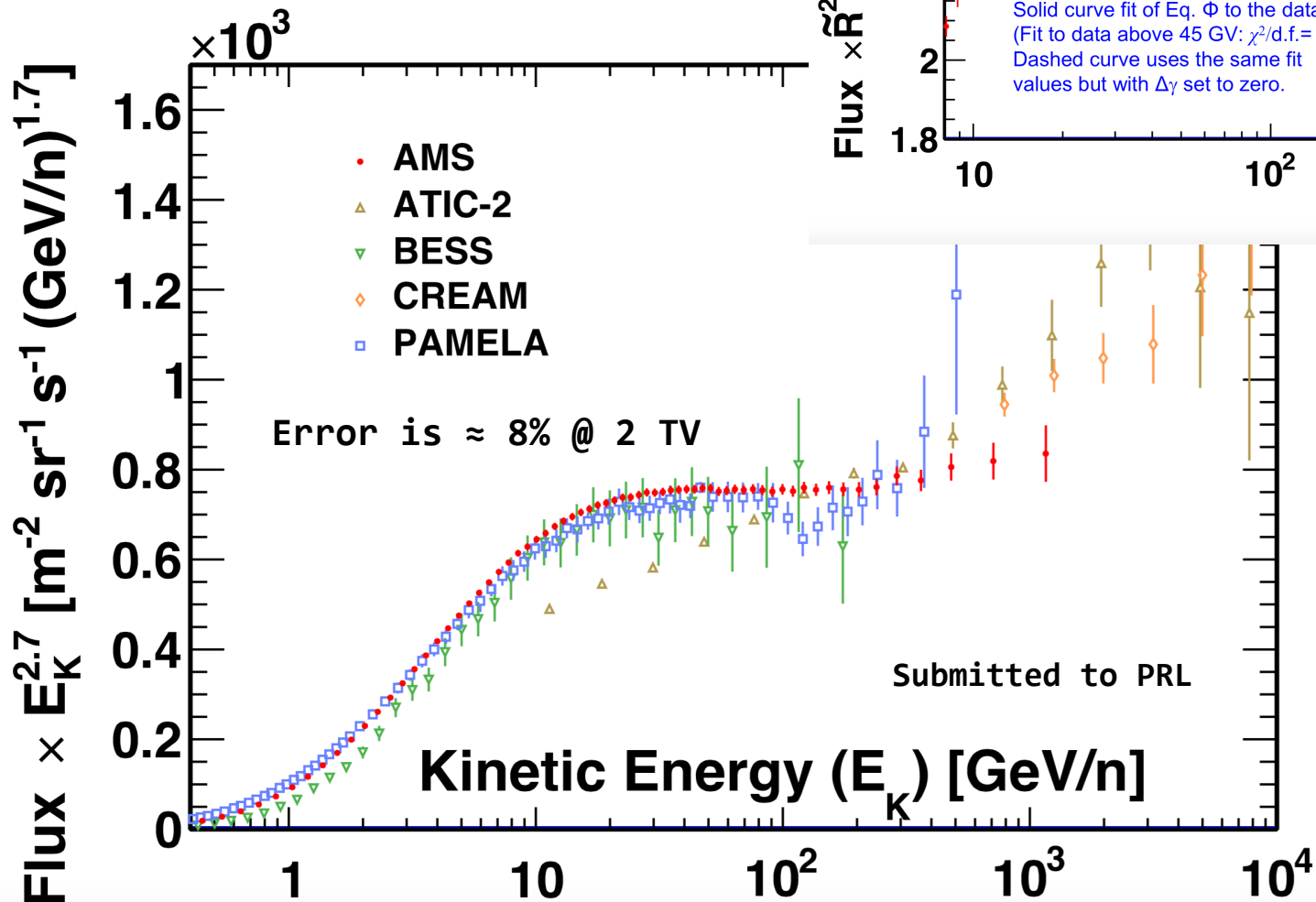
A change in the the spectral index is observed at 336 GV



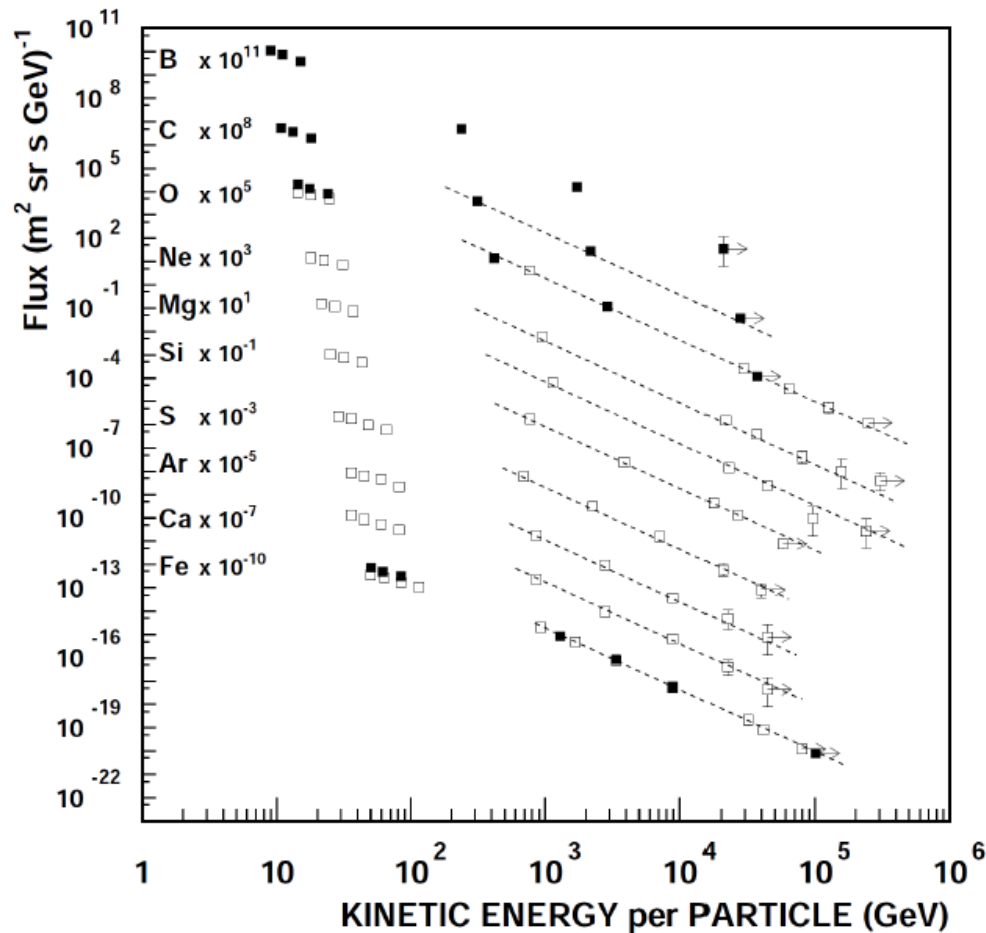


# He Flux

Transition energy  $R_0$  and smoothness parameter  $s$  are the same as for protons



# Nuclear species flux



TRACER 2 flights data:  
POWER\_LAW fit above 20 GeV/n:

$$\text{index } 2.65 \pm 0.05$$

# Energy spectra of particles

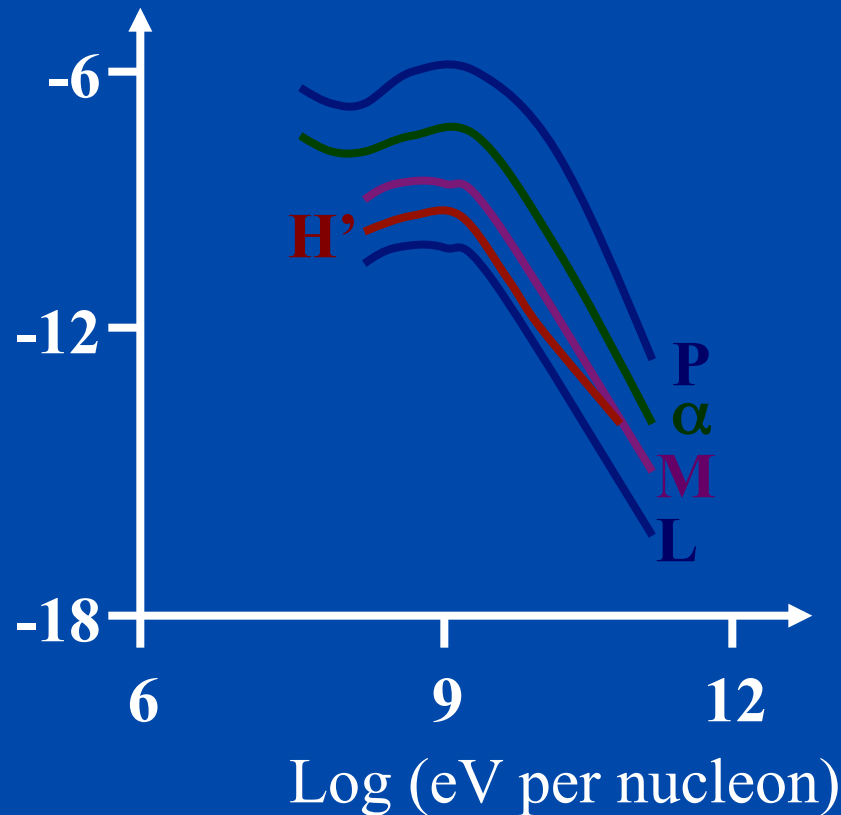
**L**  $3 \leq Z \leq 5$

**M**  $6 \leq Z \leq 9$

**H**  $Z \geq 10$

Log Particle flux

$\text{m}^{-2} \text{s}^{-1} \text{ster}^{-1} \text{eV}^{-1}$

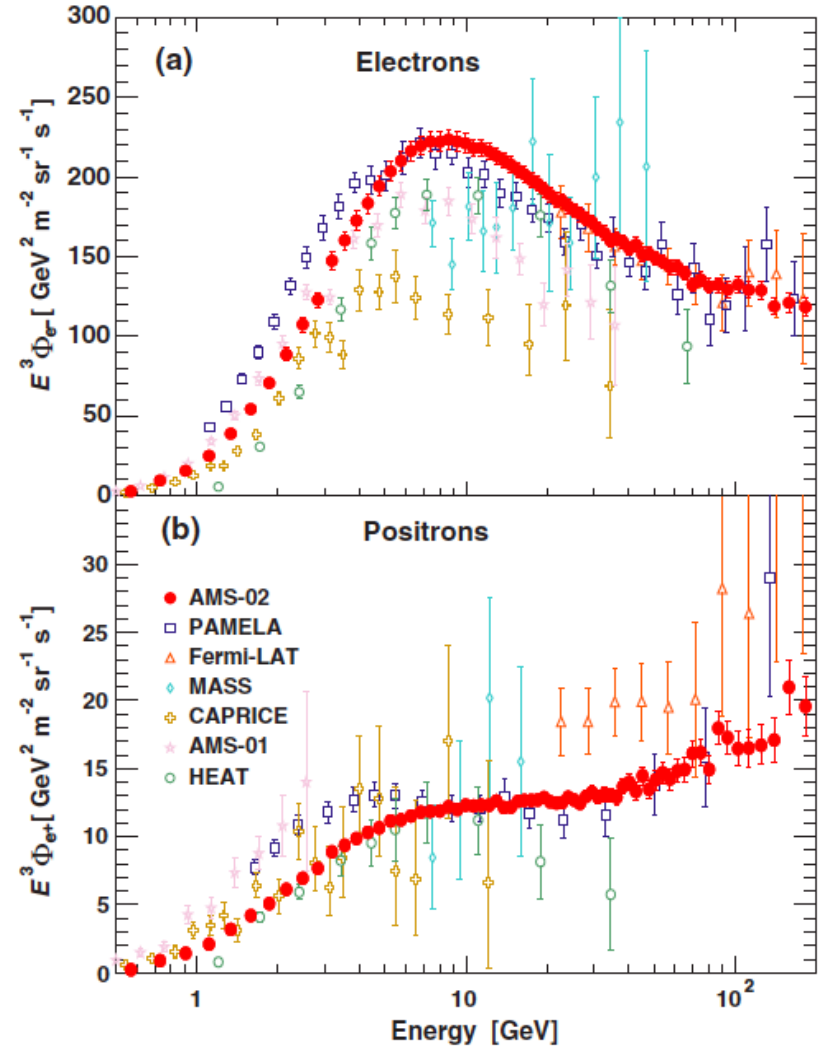
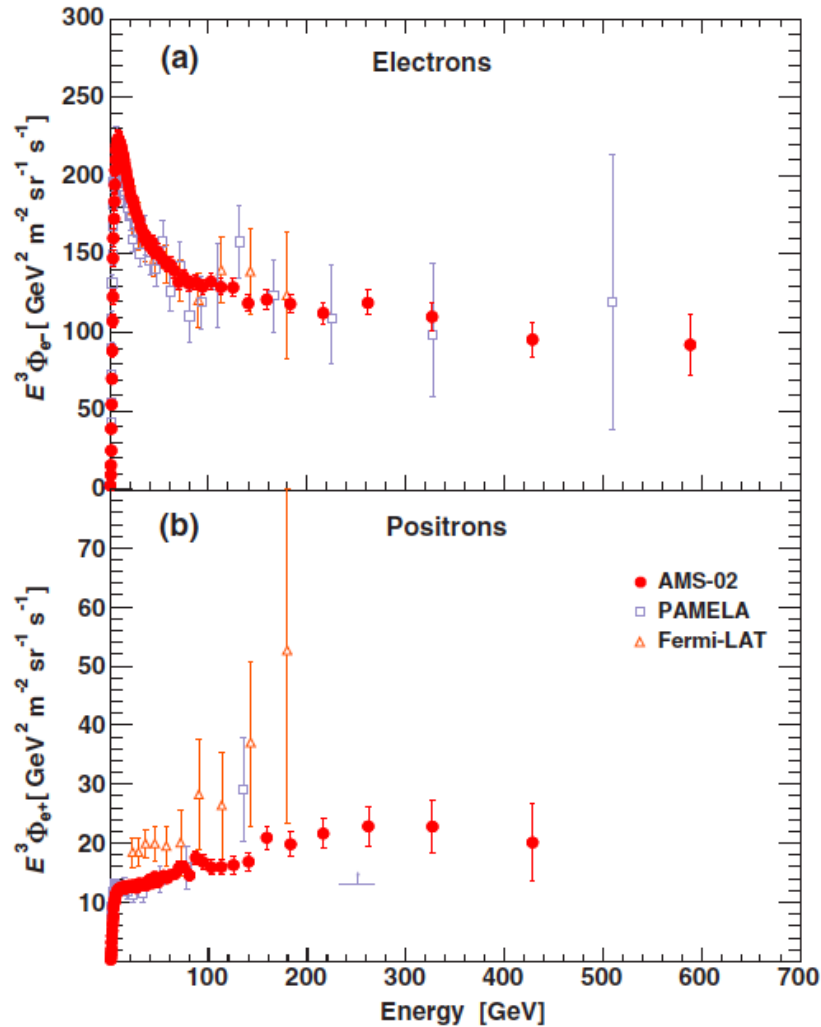


- Ci sono anche differenze negli spettri di energia
- Lo spettro di energia e' una legge di potenza  $N(E)dE = E^{-x}dE$  con
  - $x = 2.73 \pm 0.05$  per p
  - $x = 2.87 \pm 0.08$  per He
  - $X \sim 2.5$  per  $Z > 2$

- Esse riflettono differenze nei processi di accelerazione alla sorgente

## Electron/Positron fluxes:

No sharp structures



- Densità numerica dei RC, stimata dalla misura del flusso:

$$n_{CR} = \frac{4\pi}{c} \cdot \Phi(> E_o) = \frac{4\pi \cdot 1}{3 \cdot 10^{10}} \approx 4 \cdot 10^{-10} (cm^{-3})$$

- Stima della densità di energia dei RC:

$$w_{CR} = \int_0^{\infty} E \cdot n(E) \cdot dE = \int_0^{\infty} E \cdot \left( \frac{4\pi}{c} k E^{-\gamma} \right) \cdot dE = \text{Esercizio: analisi dimensionale}$$

$$= \frac{4\pi}{c} \frac{k}{\gamma - 2} \left[ E^{-\gamma+2} \right]_{3 \times 10^6 GeV}^{GeV} = \frac{4\pi}{c} \frac{1.8}{2.7 - 2} (1)^{-0.7} \cong 10^{-9} GeV / cm^3$$

$$w_{CR} \cong 1 \quad eV / cm^3$$

- Domanda: Come possiamo confrontare questo numero?
- E' “grande” o “piccolo” su scala dei fenomeni astrofisici?

- Densità di energia del campo magnetico galattico ( $B=3 \times 10^{-6}$  G)

$$w_B = \frac{1}{8\pi} B^2 = \frac{1}{8\pi} (3 \cdot 10^{-6})^2 = 4 \cdot 10^{-13} \frac{\text{erg}}{\text{cm}^3} = 0.2 \frac{\text{eV}}{\text{cm}^3}$$

- Densità di energia della radiazione cosmica di fondo a 3 K

$$w_{CMB} = \left( 500 \frac{\text{fotoni}}{\text{cm}^3} \right) \cdot 3kT = 500 \times 3 \times 3 \times 8 \cdot 10^{-5} (\text{eV} / K) = 0.4 \frac{\text{eV}}{\text{cm}^3}$$

- Luce delle stelle (da misure fotometriche)

$$w_o = 10^{-2} \frac{\text{eV}}{\text{cm}^3}$$

- La densità di energia che compete ai RC ( $1 \text{ eV}/\text{cm}^3$ ) è dunque importante su scala galattica. ■<sub>8</sub>

# Confinamento:

$$r_{Larmor} = \frac{1}{300} \frac{E}{ZB} (eV / Gauss) [cm]$$

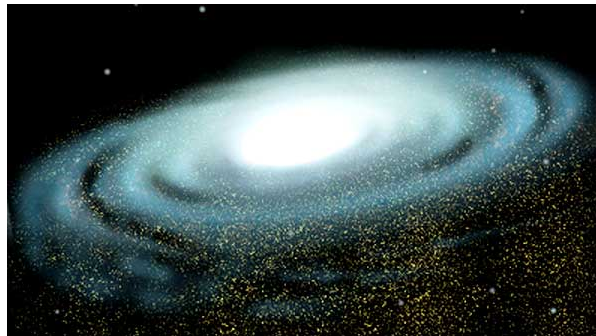
- Utilizziamo i valori tipici del campo B ( $3 \times 10^{-6}$  G) galattico per protoni:

$$r_L = \begin{cases} (E = 10^{12} eV) & = 10^{15} cm = 3 \cdot 10^{-4} pc \\ (E = 10^{15} eV) & = 10^{18} cm = 0.3 pc \\ (E = 10^{18} eV) & = 10^{21} cm = 300 pc \end{cases}$$

- I p hanno un raggio di Larmor sempre minore dello spessore del disco galattico (300 pc) se  $E < 10^{18}$  eV. Per questo motivo tutti i RC (meno quelli di energia estrema) sono *confinati* nel piano Galattico dal campo magnetico.

# Isotropia dei RC

- I RC primari al di sotto di  $10^{18}$  eV hanno una distribuzione di arrivo completamente isotropa sulla sommità della nostra atmosfera. **Qualè il motivo?**
  - Campi magnetici **galattici**:
    - $B \cong 3 \times 10^{-6}$  G
    - coerenti su scale di distanza 1-10 pc
    - NOTA:  $1 \text{ pc} = 3 \times 10^{18} \text{ cm}$
  - **Galassia**  $\equiv$  disco di raggio  $R = 15 \text{ kpc}$ , spessore  $h = 300\text{-}1000 \text{ pc}$



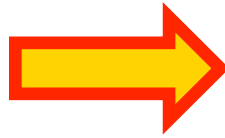


# Richiamo: moto di un RC nel campo magnetico Galattico

$$mv^2 / r = pv / r = Ze v B / c$$

$$r = pc / ZeB$$

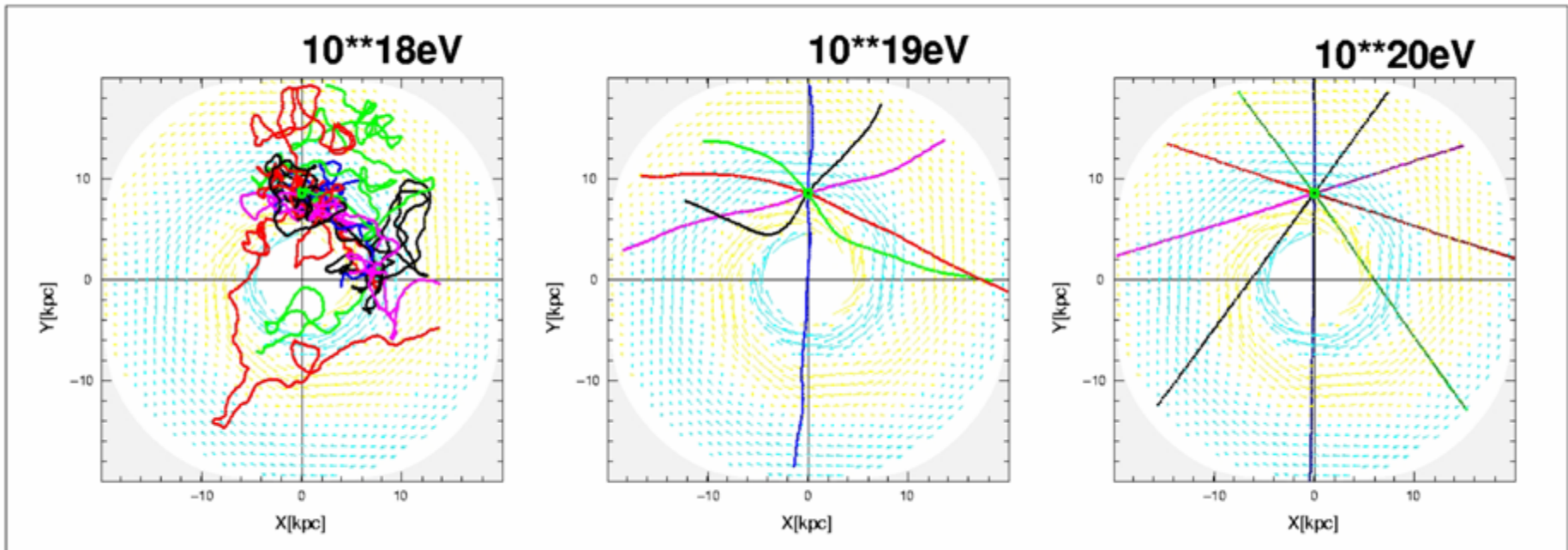
$$r(cm) = \frac{1}{300} \frac{E(eV)}{ZB(G)}$$



$$(10^{12} eV) = 10^{15} cm = 3 \times 10^{-4} pc$$

$$r = (10^{15} eV) = 10^{18} cm = 3 \times 10^{-1} pc$$

$$(10^{18} eV) = 10^{21} cm = 300 pc$$



# Equazione di propagazione (1)

$$\frac{\partial N_i}{\partial t} - \vec{\nabla} \cdot (\hat{D}_i \vec{\nabla} N_i) + \frac{\partial}{\partial E} (b_i N_i) + m v \sigma_i N_i + \frac{N_i}{\tau_i} = q_i + \sum_{j < i} m v \sigma_{ij} N_j + \sum_j \frac{N_j}{\tau_{ji}}$$

- I CR non sono accelerati nell'ISM, sono accelerati da sorgenti puntiformi
- La loro potenza e distribuzione spazio-temp. e' descritta dalle funzioni  $q_i(t, \vec{r}, E)$  (per la specie  $i$ )

- \*  $\hat{D}_i(\vec{r}, E)$  e' il tensore di diffusione
- \*  $b_i(\vec{r}, E)$  caratterizza le perdite continue di energia delle singole particelle cosi' che  $dE/dt = b_i$  ( $\equiv$  ionizz. + brems + sincrotr + Compton inverso)
- \*  $\sigma_i(E)$  e' la sez. d'urto inelastica del nucleo  $i$  con i nuclei dell'ISM
- \*  $n(\vec{r})$  e' la densita' dell'ISM
- \*  $\sigma_{ij}$  e' la sez. d'urto di prod. di nuclei di tipo  $i$  da nuclei piu' pesanti
- \*  $\tau_i$  e' la vita media rispetto a decad. radioatt.
- \*  $N_j/\tau_{ji}$  descrive l'apparizione di nuclei di tipo  $i$  a causa del decad. di altri nuclei

# 4.1 Accelerazione di particelle

- L'accelerazione di RC da parte di un qualche meccanismo Galattico (o extragalattico) deve tener conto dei seguenti fatti sperimentali:
  - Lo spettro di potenza (per tutti i tipi nucleari) del tipo  $dN/dE \sim E^{-\gamma}$  ( $\gamma=2.7$  per p sino a  $E \sim 4 \times 10^{15}$  eV)
  - lo spettro alle sorgenti ha un indice spettrale prossimo a 2
  - L'energia massima misurata ( $E \sim 10^{20}$  eV)
  - Le abbondanze relative tra gli elementi, tenendo conto degli effetti di propagazione

# Potenza delle sorgenti dei RC

- Il *confinamento* dei RC ci induce a sospettare che le sorgenti siano di origine Galattica (tranne che per i RC di energia estrema).
- Qual è *l'energetica* delle sorgenti? (necessaria per individuarle).
  - Il tempo di confinamento dei RC:  $\tau = 3 \times 10^7$  y (calc. Fra poco)
  - Volume della galassia (con o senza alone) :

$$V_G = (15 \text{ kpc})^2 \cdot \pi \cdot 300 \text{ pc} \cong 6 \times 10^{66} \text{ cm}^3$$

$$V_G^{\text{Alone}} = 4/3 \cdot \pi (10 \text{ kpc})^3 \cong 10^{68} \text{ cm}^3$$

- Potenza necessaria per mantenere uno *stato stazionario* di RC:

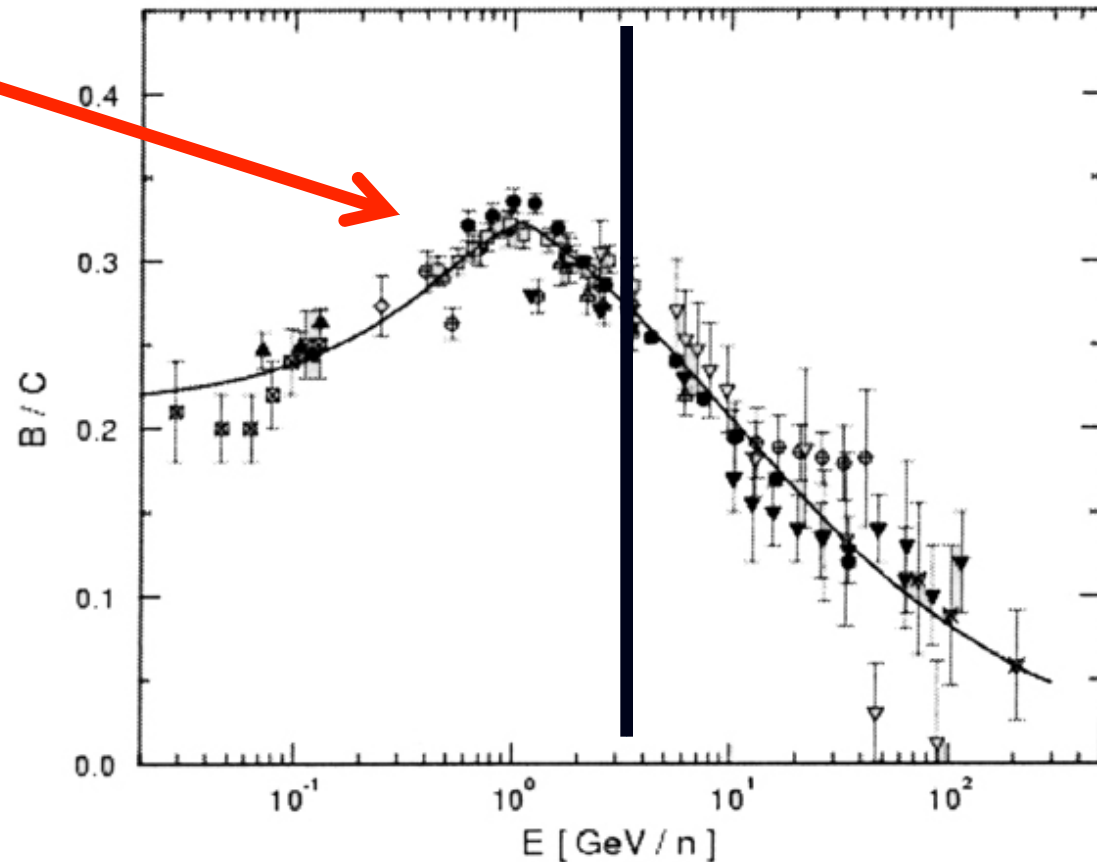
$$W_{CR} = \frac{w_{CR}(\text{erg} / \text{cm}^3) \cdot V_G^A(\text{cm}^3)}{\tau(s)} = \frac{10^{68} \times 1.6 \cdot 10^{-12}(\text{erg})}{3 \cdot 10^7 \times (3.15 \cdot 10^7 \text{ s})} \cong 10^{41} \frac{\text{erg}}{\text{s}}$$

A bassa energia, dove  $\xi_I \ll \xi_{esc}$ , in assenza di altri processi ci si aspetterebbe un rapporto costante  $N_L/N_M \sim P_{ML}(\xi_L/\xi_M)$ .

Cio' non avviene a causa di un ulteriore processo che puo' avvenire durante la propagazione: riaccelerazione di nuclei di bassa energia (qualche GeV/n) ad opera di nubi di plasma magnetizzato in movimento (casuale) presenti nel mezzo interstellare, grazie al meccanismo di Fermi del II ordine, che faremo fra un po'.

Nuclei di bassa E (centinaia di MeV/n) vengono accelerati fino a qualche GeV/n, impoverendo la popolazione a bassa E e incrementando quella intorno al 1 GeV/n

## Dipendenza del rapporto r vs. E



# Leaky Box Model

- A useful approximation is the following:
  - Assume there is no diffusion (eg  $D=0$ )
  - CR propagate freely in the galaxy volume, uniformly filled with ISM and regular B, until they reach the “border” and escape
  - A CR has a probability per unit of time to escape the galaxy  $p = 1/\tau_{\text{esc}}$ , where  $\tau$  is the measured residence time of CR in the galaxy

■ Then

$$D\nabla^2 \mathcal{N} \rightarrow -\frac{\mathcal{N}}{\tau_{\text{esc}}}$$

- As the diffusion coefficient  $D$  is energy dependent, also the characteristic escape time of CRs from the Galaxy  $\tau_{\text{esc}} = \tau_{\text{esc}}(E)$  is energy dependent



# Leaky Box Model

- The transport equation becomes

$$\frac{d\mathcal{N}_i}{dt} = -\frac{\mathcal{N}_i}{\tau_{\text{esc}}} + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i} \frac{P_{ji}}{\tau_j} \mathcal{N}_j.$$

- The leaky-box model provides the most common description of CR transport in the Galaxy at energies below  $\sim 10^{17}$  eV. The model is based on particles injected by sources  $Q$  distributed uniformly over the galactic volume (the *box*) filled with a uniform distribution of matter and radiation fields. The particles get-away from this volume with an escape time independent of their position in the box. The escape time  $\tau_{\text{esc}}(E)$  depends on the particle energy, charge, and mass number, but it does not depend on the spatial coordinates. Secondary nuclei are produced during the propagation as a function of the path length

# Spettro dei RC alle sorgenti

- Il modello Leaky Box permette di collegare lo spettro osservato a Terra con quello alle sorgenti
- In assenza di tutti i processi e in uno stato stazionario, tranne che la fuga dalla galassia, il modello ci da

$$0 = Q(E) - \frac{N(E)}{\tau(E)}$$

$$N(E) = Q(E)\tau(E)$$

- In questa approssimazione, lo spettro a Terra e' quello delle sorgenti  $Q(E)$  modificato dall'effetto della propagazione, descritta dal termine  $\tau(E)$



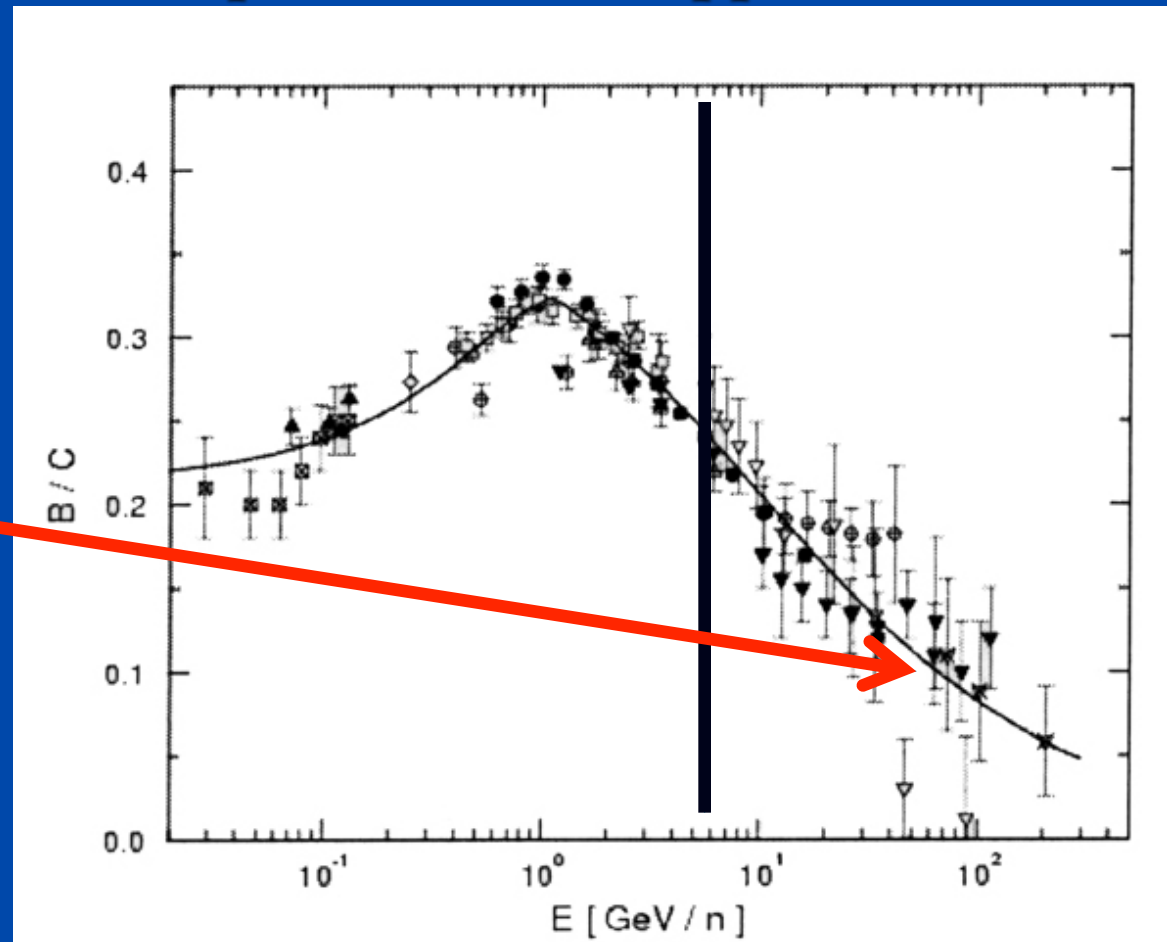
# Dipendenza del rapporto $r$ vs. $E$

- I dati sperimentali confermano questa ipotesi.
- In particolare, si ottiene che la probabilità di fuga dalla Galassia dipende dall'energia come:

$$\tau = \tau_o \left( \frac{R}{R_o} \right)^{-\delta}$$

- Ossia, poiché  $\tau \sim \xi$

$$\xi = \xi_o \left( \frac{R}{R_o} \right)^{-\delta}$$



Valori tipici  $\xi_o = 11.8 \text{ gr/cm}_2$ ,  $R_o = 5 \text{ GV}/c$ ,  $\delta = 0.6$

L'indice spettrale  $\delta$  ha un'incertezza piuttosto grande. Il suo valore potrebbe variare in un intervallo 0.15-0.8 a seconda del modello di diffusione. Per esempio un regime in cui  $D$  o  $\tau$  dipendono dalla posizione, come modelli a doppia regione (disco+alone), effetti non lineari,...

# Spettro dei RC alle sorgenti

- Il risultato appena ottenuto è estremamente importante, perché permette di avere informazioni sullo spettro energetico dei RC alle sorgenti.
- Poiché il flusso dei RC sulla Terra è stazionario, vi deve essere equilibrio tra:

■ Spettro energetico misurato:

$$\Phi(E) \propto E^{-2.7} (erg / cm^3 \cdot GeV)$$

■ Spettro energetico alle Sorgenti:

$$Q(E) \propto E^{-?} (erg / s \cdot GeV)$$

■ Probabilità di diffusione:

$$\tau(E) \propto E^{-0.6} (s)$$

$$\left[ \frac{erg}{cm^3} \right]$$

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

$$\frac{\left[ \frac{erg}{s \cdot GeV} \right] \cdot [s]}{[cm^3]}$$

# Spettro dei RC alle sorgenti

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

- Quindi, inserendo le dipendenze funzionali:

$$Q(E) = \frac{\Phi(E)}{\tau(E)} = \frac{E^{-2.7}}{E^{-0.6}} = E^{-2.1}$$

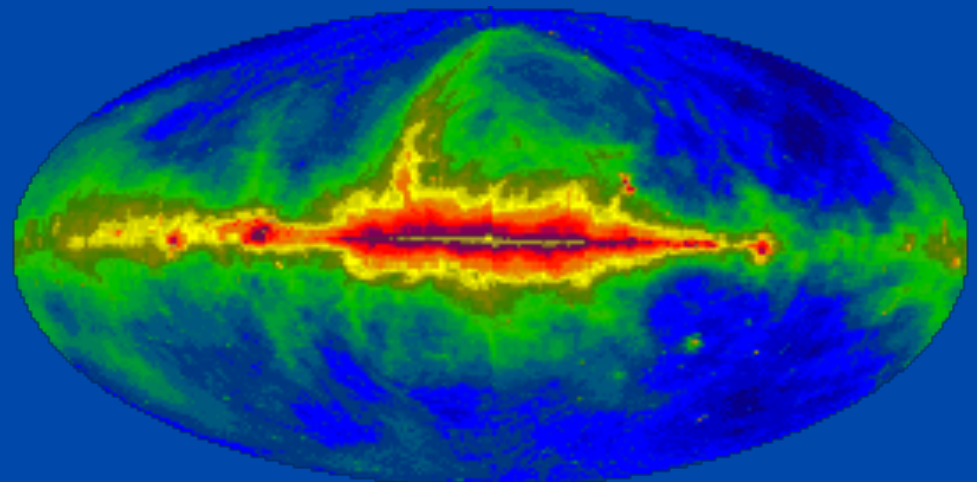
- Il modello che descrive le sorgenti di RC nella Galassia, dovrà prevedere una dipendenza con l'energia del tipo  $\sim E^{-2}$ .
- Occorre trovare un processo che produca uno spettro di questo tipo alla sorgente.
- Il modello di Fermi prevede proprio un andamento funzionale di questo tipo!

# Volume di confinamento

- Le particelle attraversano in media  $\xi = 50 \text{ kgm}^{-2}$  di ISM
- $\xi = \rho c \tau \rightarrow$  otteniamo  $\rho < 1 \text{ pcm}^{-3}$  tipico del disco galattico
- Questo implica che le part viaggino anche in regioni meno dense di quelle tipiche del disco (dove le abbondanze non cambiano) con un percorso casuale "tortuoso" per via delle irregolarita' del campo B galattico. Si puo' dire che esse non sono libere di sfuggire liberamente
- E' importante quindi stimare quanto e' grande il volume in cui i CR sono confinati

# Volume di confinamento (2)

- Il volume puo' essere:
  - il disco della galassia, cioe' approx un disco di raggio 10-15 kpc e spessore 300-500 pc
  - galassia + alone, una regione meno densa di raggio » 15 kpc che circonda il disco di forma sferico-ellissoidale → favorito dalle misure sperimentali



Other evidence of galactic halo: 408 MHz map of the sky :  
synchrotron emission  
of few GeV electrons in the galactic magnetic field<sub>23</sub>

# Elettroni e positroni

$$\frac{\partial N_i}{\partial t} - \vec{\nabla} \cdot (\hat{D}_i \vec{\nabla} N_i) + \frac{\partial}{\partial E} (b_i N_i) + m v r_i N_i + \frac{N_i}{\tau_i} = q_i + \sum_{j < i} m v b_{ij} N_j + \sum_j \frac{N_j}{\tau_{ji}}$$

$\sigma_i$  e' la sez d'urto per il processo  $p_{CR} p_{ISM} \rightarrow \pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$  ed  $n$  e' la densita' di  $p$  nell ISM,  $N_i$  e' la densita' di elettroni

Da dove vengono gli elettroni?

Da interazione dei RC con i protoni del mezzo interstellare

Da sorgenti primarie, cioe' siti di accelerazione

Da sorgenti esotiche(?)

Per gli elettroni, a differenza della componente nucleare, sono importanti i processi di perdita di energia durante la propagazione. Il termine che descrive le perdite di energia continue non puo' essere trascurato ma anzi diventa dominante



# Toy Model

o Caso semplice di: - soluz. stazionarie  $dN/dt = 0$

x C'è una distrib. infinita e uniforme di sorgenti che iniettano  $e^-$  con spettro di iniezione  $Q(E) = k E^{-p}$

$\Rightarrow$  La diffusione non è importante  $\Rightarrow$  L'equ. diventa

$$-\frac{d}{dE}[b(E)N(E)] = Q(E) \quad \Rightarrow \quad \int d[b(E)N(E)] = - \int Q(E) dE$$

x Assumiamo  $N(E) \rightarrow 0$  per  $E \rightarrow \infty \Rightarrow$

$$N(E) = \frac{kE^{-(p-1)}}{(p-1)b(E)} \quad \text{dove } b(E) = A_1 \left( \ln \frac{E}{mc^2} + 19.8 \right) + A_2 E + A_3 E^2$$

$\Rightarrow$  Se domina ionizz.  $N(E) \propto E^{-(p-1)}$  . spettro di  $E$  più piatto

Se " Brems  $N(E) \propto E^{-p}$  . lo spettro non cambia  
(o adiab. loss)

Se IC o sincr. dominano  $N(E) \propto E^{-(p+1)}$  spettro più ripido

o Nel caso di diffusione, le particelle si spostano in media di  $\lambda \approx (2D\tau)^{1/2}$

o In generale  $D = D(E) \Rightarrow \lambda(E, E_0) = \left( \int_0^{\tau(E)} D(E') d\tau' \right)^{1/2} = \left( \int_{E_0}^E \frac{D(E') dE'}{b(E')} \right)^{1/2}$

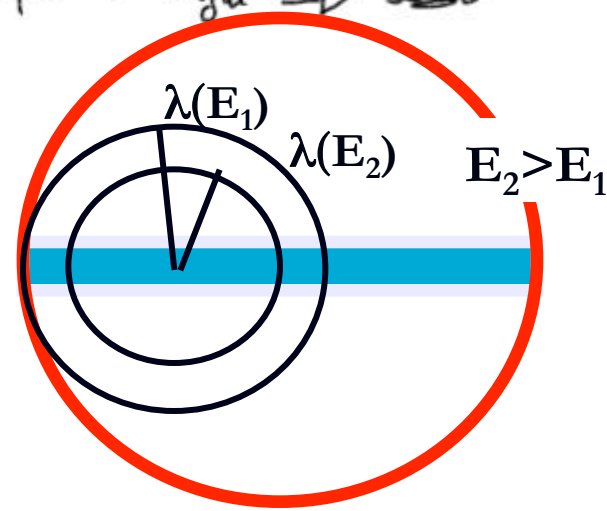
x  $D \approx 10^{29} \text{ cm}^2 \text{ s}^{-1}$  (val. medio galattico)  $\Rightarrow \lambda \approx 10 \text{ kpc}$  @  $1 \text{ GeV}$   
(solo 10 + Sincro i.e.  $\tau = \frac{1}{KE_0}$ )

x Trascurando la dipend. di  $D$  da  $E$   $\lambda \propto E^{-1/2} \Rightarrow$   
Lo spettro degli  $e^\pm$  nei differenti intervalli di energia dipende alle condizioni medie di propagazione nell'ISM in una regione di raggio  $R \sim \lambda(E)$

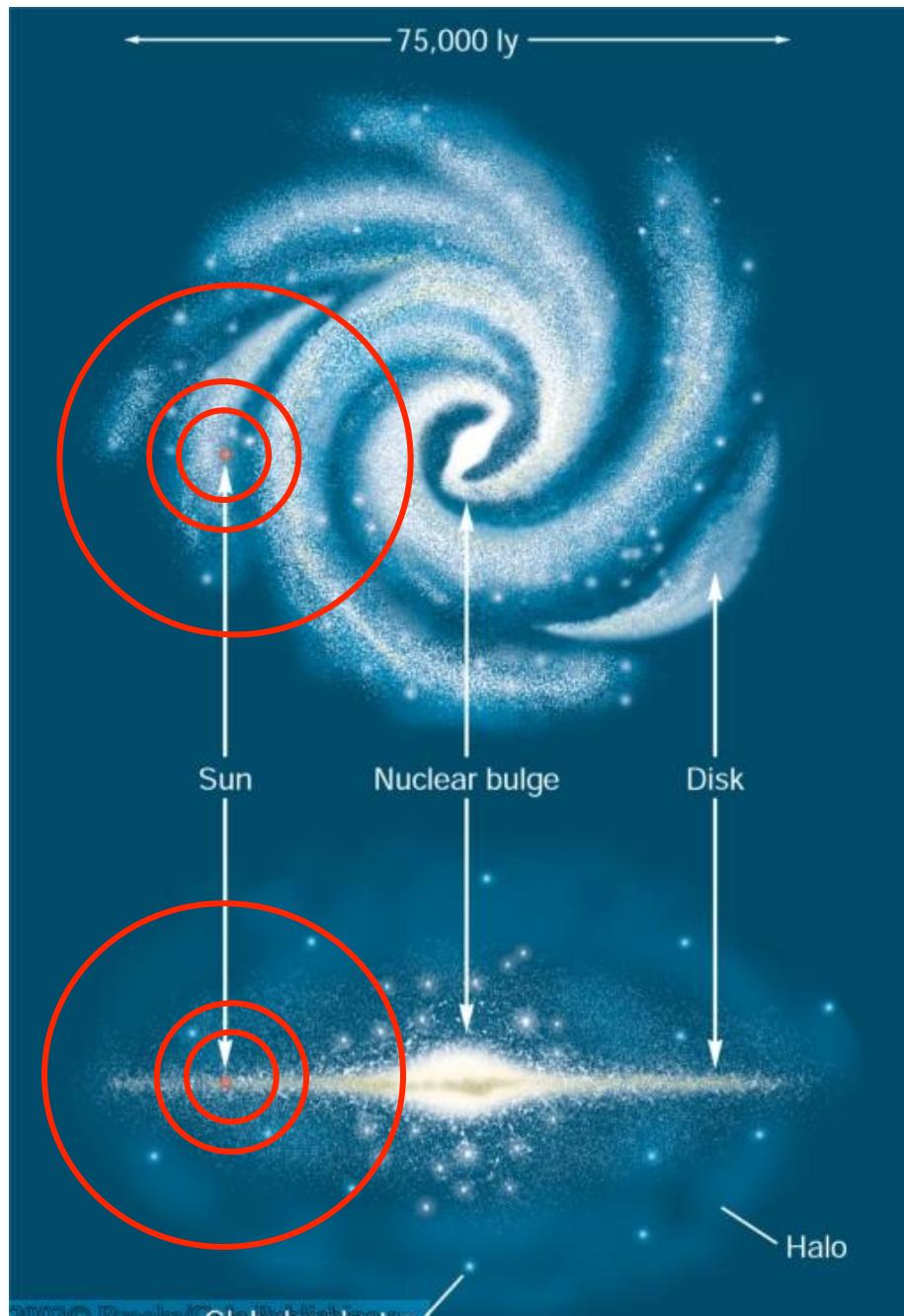
x Esempio: per  $e^-$  con  $E > 1 \text{ GeV}$ ,  $\lambda(E) \lesssim 10 \text{ kpc} = R_{\text{gal}} \Rightarrow$  ~~solo~~

$\Rightarrow$  Gli  $e^-$  sono galattici.

■ Quindi lo spettro osservato cambia parecchio durante la propagazione







$\approx 10^2$  ly @ 1 TeV

$\approx 3.16 \times 10^3$  ly @ 100 GeV

$\approx 10^4$  ly @ 10 GeV

$\approx 3 \times 10^4$  ly @ 1 GeV

Gli elettroni  
vengono da vicino!

# SNRs vicine

- Vicino al noi ci sono numerose potenziali sorgenti di elettroni

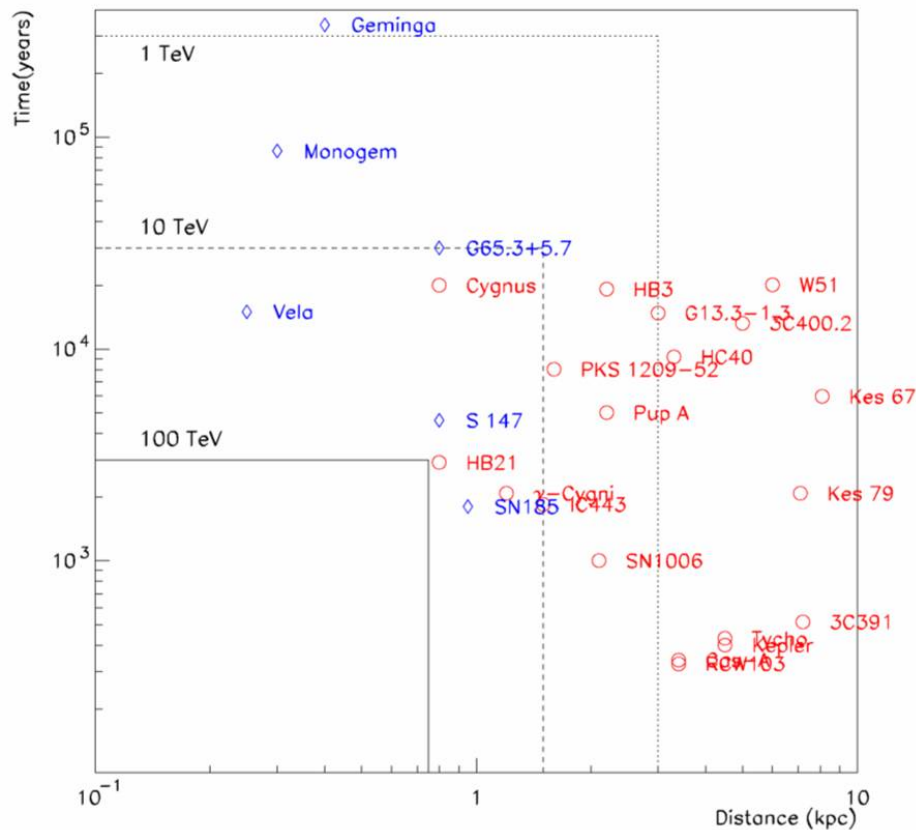


TABLE 1  
LIST OF NEARBY SNRs

| SNR              | Distance (kpc) | Age (yr)          | $E_{\max}^a$ (TeV) | Reference |
|------------------|----------------|-------------------|--------------------|-----------|
| SN 185.....      | 0.95           | $1.8 \times 10^3$ | $1.7 \times 10^2$  | 1         |
| S147.....        | 0.80           | $4.6 \times 10^3$ | 63                 | 2         |
| HB 21.....       | 0.80           | $1.9 \times 10^4$ | 14                 | 3, 4      |
| G65.3+5.7.....   | 0.80           | $2.0 \times 10^4$ | 13                 | 5         |
| Cygnus Loop..... | 0.44           | $2.0 \times 10^4$ | 13                 | 6, 7      |
| Vela.....        | 0.30           | $1.1 \times 10^4$ | 25                 | 8         |
| Monogem.....     | 0.30           | $8.6 \times 10^4$ | 2.8                | 9         |
| Loop I.....      | 0.17           | $2.0 \times 10^5$ | 1.2                | 10        |
| Geminga.....     | 0.4            | $3.4 \times 10^5$ | 0.67               | 11        |

<sup>a</sup> Maximum energy limited by the propagation of electrons in the case of the prompt release after the explosion. The delay of the release time gives the larger value.

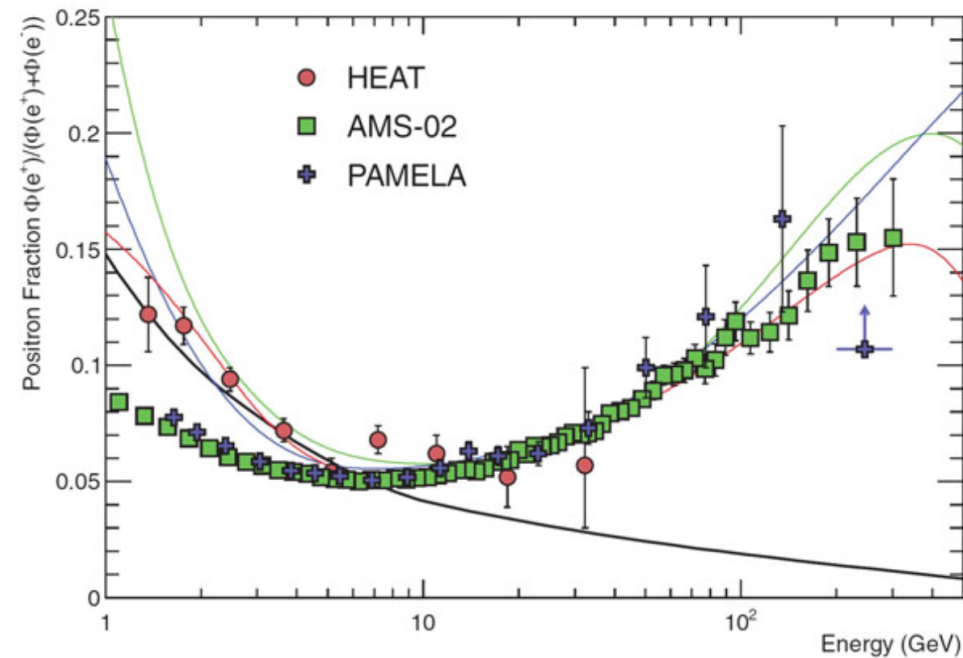
REFERENCES.—(1) Strom 1994; (2) Braun et al. 1989; (3) Tatematsu et al. 1990; (4) Leahy & Aschenbach 1996; (5) Green 1988; (6) Miyata et al. 1994; (7) Blair et al. 1999; (8) Caraveo et al. 2001; (9) Plucinsky et al. 1996; (10) Egger & Aschenbach 1995; (11) Caraveo et al. 1996.

## Eccesso rispetto a cosa?

Produzione secondaria nell'ISM di  $e^+$  ed  $e^-$

Sorgenti primarie di  $e^-$ :  $n_-(E) = K_{ep} N_{cr}(E) + \sigma_{pe^-} n_{ISM} c n_{cr}(E)$ ,  $K_{ep} \approx 10^{-3}$

No sorgenti primarie di  $e^+$ :  $n_+(E) = \sigma_{pe^+} n_{ISM} c n_{cr}(E)$



Leaky box equilibrium density  $n_{cr}(E) = N_{cr}(E) \tau_{esc}(E)$ ,  $\tau_{esc}(E) = \tau_0 E^{-\delta}$

$$\frac{n_-(E)}{n_+(E)} = \frac{(K_{ep} N_{cr}(E) + \sigma_{pe^-} n_{ISM} c N_{cr}(E) \tau_{esc}(E))}{\sigma_{pe^+} n_{ISM} c N_{cr}(E) \tau_{esc}(E)} = \frac{\sigma_{pe^-}}{\sigma_{pe^+}} + \frac{K_{ep}}{\tau_{esc}(E)}$$

$$\frac{n_+(E)}{(n_+(E) + n_-(E))} = \frac{1}{(1 + n_-(E)/n_+(E))} = \frac{1}{(1 + \sigma_{pe^-}/\sigma_{pe^+} + K_{ep}/\tau_{esc}(E))}$$

$$\sigma_{pe^-}/\sigma_{pe^+} \approx 0.5 - 0.3 \rightarrow \frac{n_+(E)}{(n_+(E) + n_-(E))} \approx \frac{1}{(1.5 + (K_{ep}/\tau_0) E^\delta)}$$

→ Senza produzione primaria di  $e^+$  alle sorgenti la frazione decresce con l'energia

## Comparing our data with a minimal model, as an example.

In this model the  $e^+$  and  $e^-$  fluxes,  $\Phi_{e^+}$  and  $\Phi_{e^-}$ , are parametrized as the sum of individual diffuse power law spectra and the contribution of a single common source of  $e^\pm$ :

$$\Phi_{e^+} = C_{e^+} E^{-\gamma_{e^+}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad \blacksquare \text{ pulsar source} \quad \text{Eq(1)}$$

$$\Phi_{e^-} = C_{e^-} E^{-\gamma_{e^-}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad (E \text{ in GeV}) \quad \text{Eq(2)}$$

Coefficients  $C_{e^+}$  and  $C_{e^-}$  correspond to relative weights of diffuse spectra for positrons and electrons.

$C_s$  is the weight of the source spectrum.

$\gamma_{e^+}$ ,  $\gamma_{e^-}$  and  $\gamma_s$  are the corresponding spectral indexes.

$E_s$  is a characteristic cutoff energy for the source spectrum.

With this parametrization the positron fraction depends on 5 parameters.

Possiamo dare un quadro generale degli spettri alle sorgenti:

Per la componente nucleare  $p \approx 2$

Per la componente  $e^+e^-$   $p \approx 2.5$

Fino ad almeno qualche centinaio di GeV

Gli spettri alla sorgente sono differenti per  $e^-$  e protoni (cum grano salis)

Per entrambe le componenti c'è evidenza sperimentale di hardening dello spettro a qualche  $O(100 \text{ GeV})$

Per  $e^+$  ed  $e^-$  l'evidenza è netta: c'è una sorgente che immette  $e^+$  ed  $e^-$  di alta energia nell'ISM

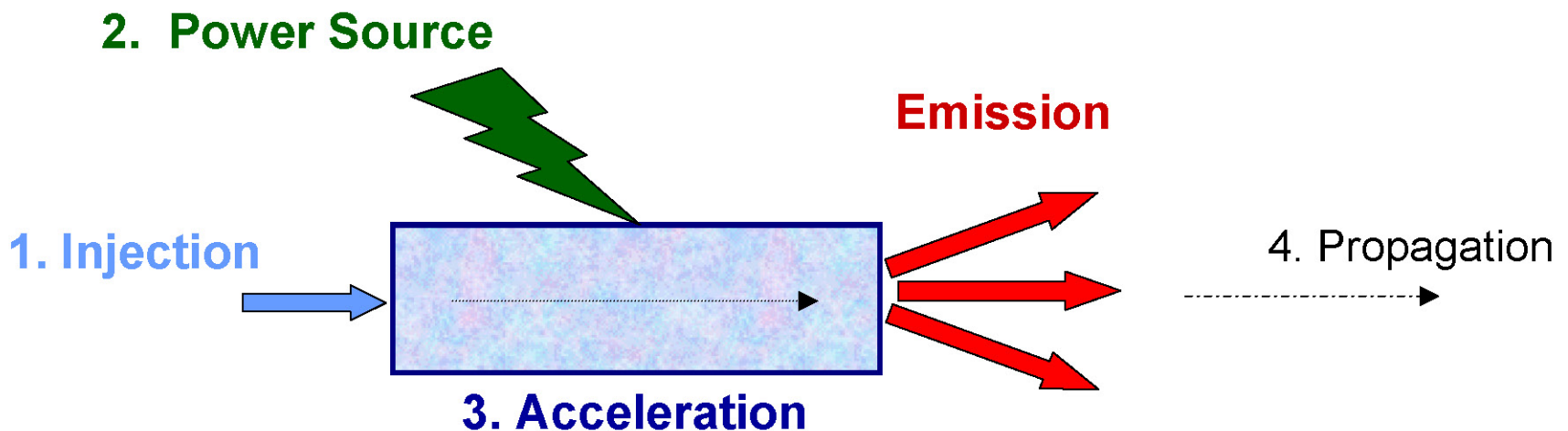
Per i nuclei la situazione è più incerta: nuova classe di sorgenti? Effetto di propagazione?

Servono più dati.

Le pulsar accelerano  $e^+$  ed  $e^-$  ma non protoni (per esempio)

# Origine dei raggi cosmici

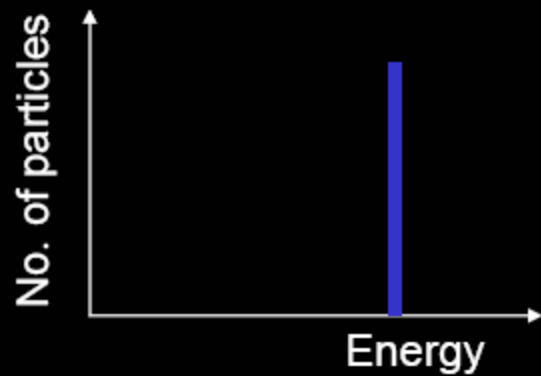
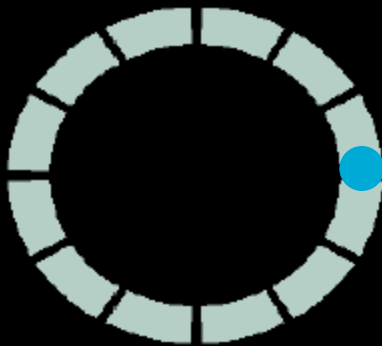
Per costruire un "acceleratore cosmico" abbiamo bisogno di:





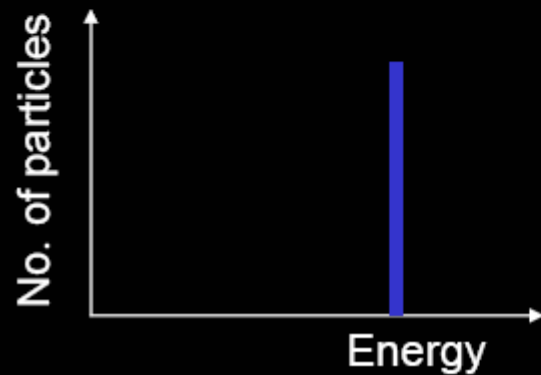
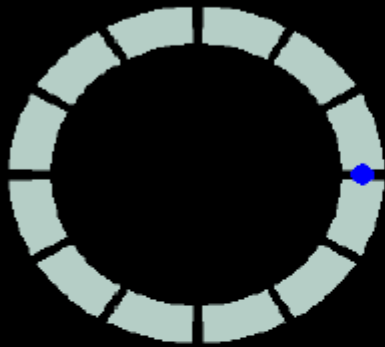
# How might such cosmic accelerators work?

Man-made accelerators

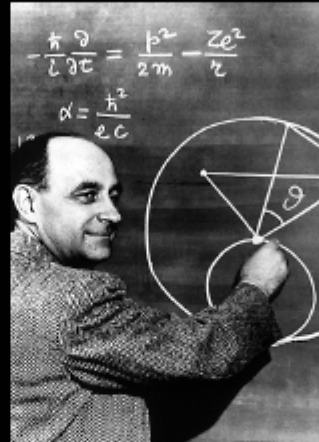


# How might such cosmic accelerators work?

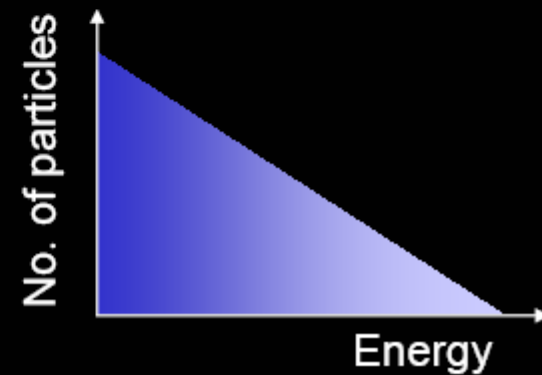
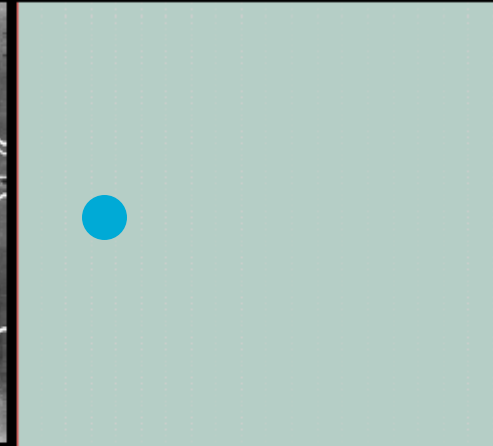
Man-made accelerators



Nature's accelerators



Enrico Fermi



- In ambiente astrofisico (in presenza di particelle ionizzate, plasmi) *campi elettrostatici non possono essere mantenuti* a causa dell'alta conducibilità dei plasmi stessi
- Sono possibili meccanismi in cui f.e.m. sono prodotte tramite  $\nabla \times \mathbf{E} = \delta \mathbf{B} / \delta t$
- Il meccanismo “idrodinamico” descrive accelerazione stocastica di RC da parte di ripetuti urti delle particelle con un'onda di shock, ad esempio emessa dall'esplosione di una SN.
- Questo meccanismo venne utilizzato per i RC per la prima volta da parte di E. Fermi (1949), e prende per questo il suo nome.
- Le particelle cariche sono riflesse da “specchi” magnetici dovute alla presenza dell'irregolare campo magnetico galattico.
- Ad ogni riflessione, le particelle guadagnano (in media) energia
- Il meccanismo predice il corretto andamento del flusso *vs.* E

# Electromotive Acceleration

Fundamentally, Lorentz Force law  $\Rightarrow$

$$\frac{d}{dt}(\gamma m \vec{v}) = e(\vec{E} + \vec{v} \wedge \vec{B})$$

But, on macroscopic scales, no electrostatic fields, i.e.,  $\langle \vec{E} \rangle = 0$ .

Also, for a stationary  $\vec{B}$ ,  $\frac{d}{dt}|\vec{v}| = 0$ .

Need “moving” magnetic fields, i.e., acceleration by electromotive force.

Expect, rate of energy gain by relativistic particles = work done by the electromotive force =  $\frac{d\mathcal{E}}{dt} = \xi Z e c B . c$ , with  $\xi < 1$ .

Fermi's original suggestion: Encounters of charged particles with moving, magnetized, interstellar clouds.

# ■ Fermi Acceleration



- Stochastic Mechanism
- Charged particles collide with clouds in ISM and
  - are reflected from irregularities
- in galactic magnetic field
  - 2<sup>nd</sup> order

- Charged particles can be accelerated to high energies in astrophysical shock fronts
- 1<sup>st</sup> order acceleration

- Tra i siti possibili di accelerazione dei raggi cosmici possiamo includere (ad energia crescente):
  - i venti stellari
  - le esplosioni di Supernovae
  - le “remnants” di tali esplosioni: stelle di neutroni ruotanti, pulsar con nebulose, ...
  - altri oggetti esotici, quali i “mini-black holes”, se esistono.
  - I raggi cosmici osservati con energie  $E > 10^{19}$  eV, potrebbero essere stati accelerati da meccanismi extragalattici, quali jets di nuclei Galattici attivi o GRB
- Il meccanismo di Fermi può essere attivo in molte di queste situazioni astrofisiche, e lo analizzeremo in qualche dettaglio per le esplosioni di Supernovae
- Il fattore comune a queste classi di sorgenti e' la presenza di onde di shock e/o zone di campo magnetico turbolento in moto ( $\langle \Delta B \rangle = 0$ , ma  $\langle \Delta B^2 \rangle \neq 0$ )



## On the Origin of the Cosmic Radiation

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(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

### I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller<sup>1</sup> has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.<sup>2</sup> The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

I propose in the present note to discuss a hypothesis on the origin of cosmic rays which attempts to meet in part this objection, and according to which cosmic rays originate and are accelerated primarily in the interstellar space, although they are assumed to be prevented by magnetic fields from leaving the boundaries of the galaxy. The main process of acceleration is due to the interaction of cosmic particles with wandering magnetic fields which, according to Alfvén, occupy the interstellar spaces.

Such fields have a remarkably great stability because of their large dimensions (of the order of magnitude of light years), and of the relatively high electrical conductivity of the interstellar space. Indeed, the conductivity is so high that one might describe the magnetic lines of force as attached to the matter and partaking in its streaming motions. On the other hand, the magnetic field itself reacts on the hydrodynamics<sup>3</sup> of the interstellar matter giving it properties which, according to Alfvén, can pictorially be described by saying that to each line of force one should attach a material density due to the mass of the matter to which the line of force is linked. Developing this point of view, Alfvén is able to calculate a simple formula for the velocity  $V$  of propagation of magneto-elastic waves:

$$V = H / (4\pi\rho)^{1/2}, \quad (1)$$

<sup>1</sup> Nuclear Physics Conference, Birmingham, 1948.

<sup>2</sup> Alfvén, Richtmyer, and Teller, *Phys. Rev.*, to be published.

<sup>3</sup> H. Alfvén, *Arkiv Mat. f. Astr., o. Fys.* 29B, 2 (1943).

where  $H$  is the intensity of the magnetic field and  $\rho$  is the density of the interstellar matter.

One finds according to the present theory that a particle that is projected into the interstellar medium with energy above a certain injection threshold gains energy by collisions against the moving irregularities of the interstellar magnetic field. The rate of gain is very slow but appears capable of building up the energy to the maximum values observed. Indeed one finds quite naturally an inverse power law for the energy spectrum of the protons. The experimentally observed exponent of this law appears to be well within the range of the possibilities.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

### II. THE MOTIONS OF THE INTERSTELLAR MEDIUM

It is currently assumed that the interstellar space of the galaxy is occupied by matter at extremely low density, corresponding to about one atom of hydrogen per cc, or to a density of about  $10^{-24}$  g/cc. The evidence indicates, however, that this matter is not uniformly spread, but that there are condensations where the density may be as much as ten or a hundred times as large and which extend to average dimensions of the order of 10 parsec. (1 parsec. =  $3.1 \times 10^{18}$  cm = 3.3 light years.) From the measurements of Adams<sup>4</sup> on the Doppler effect of the interstellar absorption lines one knows the radial velocity with respect to the sun of a sample of such clouds located at not too great distance from us. The root mean square of the radial velocity, corrected for the proper motion of the sun with respect to the neighboring stars, is about 15 km/sec. We may assume that the root-mean-square velocity

<sup>4</sup> W. S. Adams, *A.p.J.* 97, 105 (1943).

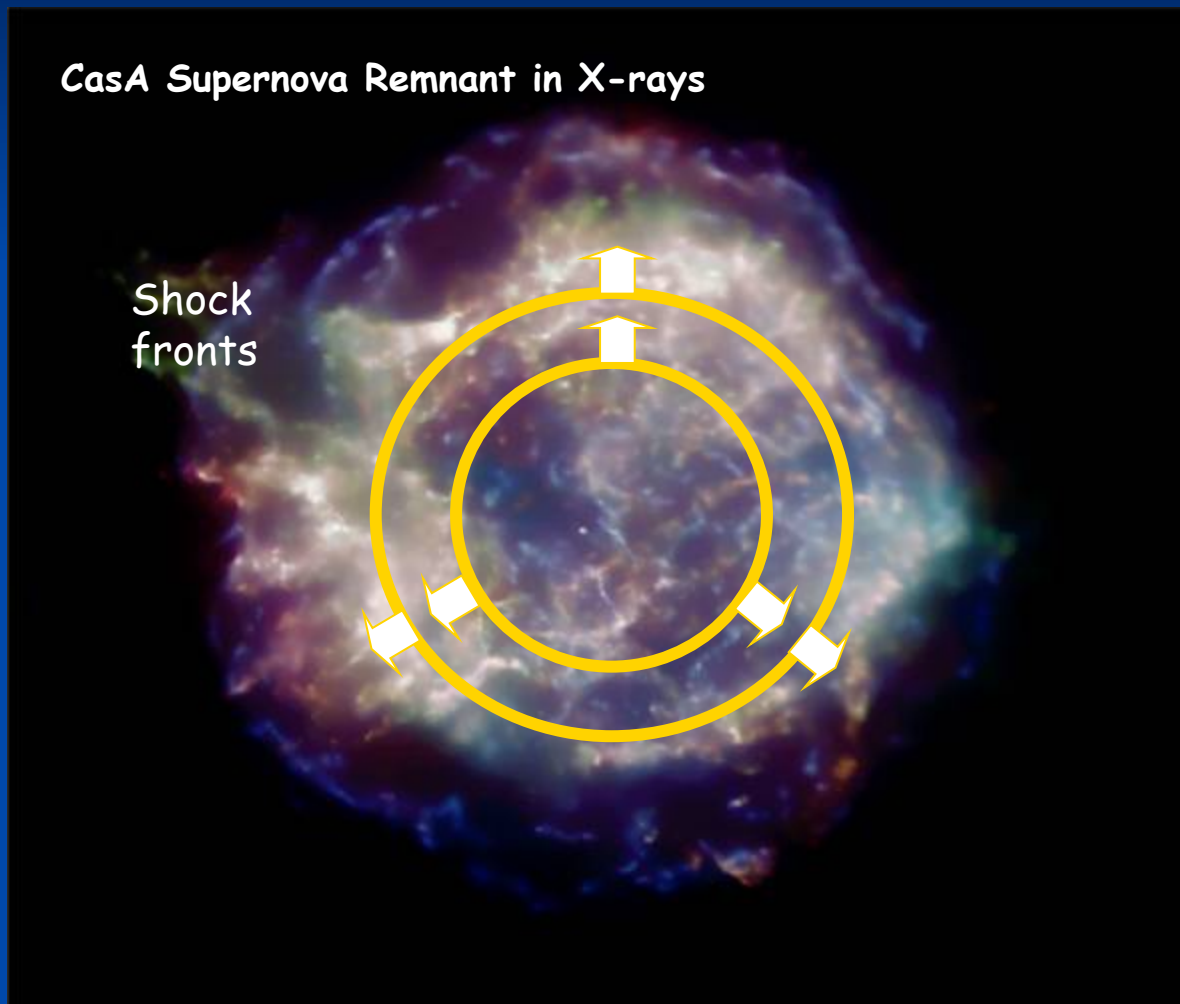
# 4.2 Il meccanismo di Fermi

Una “collisione” con una nube magnetica può causare un aumento dell’energia della particella. Un gran numero di collisioni possono far crescere l’energia fino a valori molto elevati. Guadagno di energia per collisione:

$$\Delta E/E = \varepsilon$$



# Shocks interlude



# Summary: Equations describing ideal (self-)gravitating fluid

■ Equation of Motion:

$$\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla P - \rho \nabla \Phi$$

■ Continuity Equation:  
■ behavior of mass-density

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

■ Ideal gas law  
■ &  
■ Adiabatic law:  
■ Behavior of pressure  
■ and temperature

$$P(\rho, T) = nk_{\text{b}}T = \frac{\rho \mathcal{R}T}{\mu}$$

$$P \rho^{-5/3} = \text{constant}$$

■ Poisson's equation: self-gravity

$$\nabla^2 \Phi_{\text{self}}(\mathbf{x}, t) = 4\pi G \rho(\mathbf{x}, t)$$

# Shocks: non-linear fluid structures

■ One of the peculiarities of the hydrodynamics is that it admits discontinuous solutions, that is such that on some special surfaces, called discontinuity surfaces, all the physical observables that characterize the state of the fluid ( $\rho$ ,  $p$ ,  $V$ ,  $T$ ,...) are discontinuous



■ From a mathematical point of view, these solutions have true steps, while from a physical point of view the discontinuity is not sharp but has a finite thickness, very small with respect to all the other dimensions of the system

# Shock properties

1. Shocks are sudden transitions in flow properties such as density, velocity and pressure;
2. In shocks the kinetic energy of the flow is converted into heat, (pressure);
3. Shocks are inevitable if sound waves propagate over long distances;
4. Shocks always occur when a flow hits an obstacle supersonically
5. In shocks, the flow speed along the shock normal changes from supersonic to subsonic

# Shocks waves

- In the limit of small disturbances, where the non-linear term  $(\mathbf{V} \cdot \nabla)\mathbf{V}$  can be neglected, we got the wave propagation for them
- But when the approximation breaks down, the behavior of the fluid changes rapidly because intrinsic non linearities play an essential role
- These lead to shock formation in a natural way: in practice, they are unavoidable if the perturbations to the fluid are not infinitesimal
- Shock waves have an enormous importance in astrophysics because they are present everywhere and because the matter immediately after the transit of the shock wave emits much more than before, making it easily detectable by us

# Shock waves

- Roughly, two different types of discontinuity exist:
  - i) tangential discontinuity, when two adjacent fluids are in relative motion without mass flow through the discontinuity. This discontinuity can occur at any speed (as in the case of slender jets)
  - ii) shock wave in which there is a discontinuity between two fluids with distinct properties, but with a flow of mass, momentum and energy across the surface

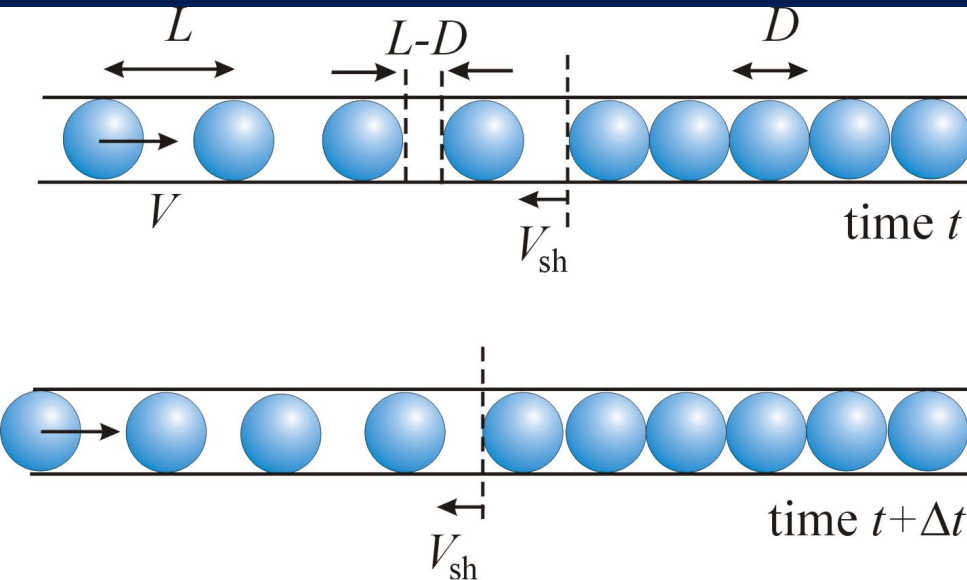




- Shocks occur whenever a flow hits an obstacle
- at a speed larger than the (adiabatic) sound speed

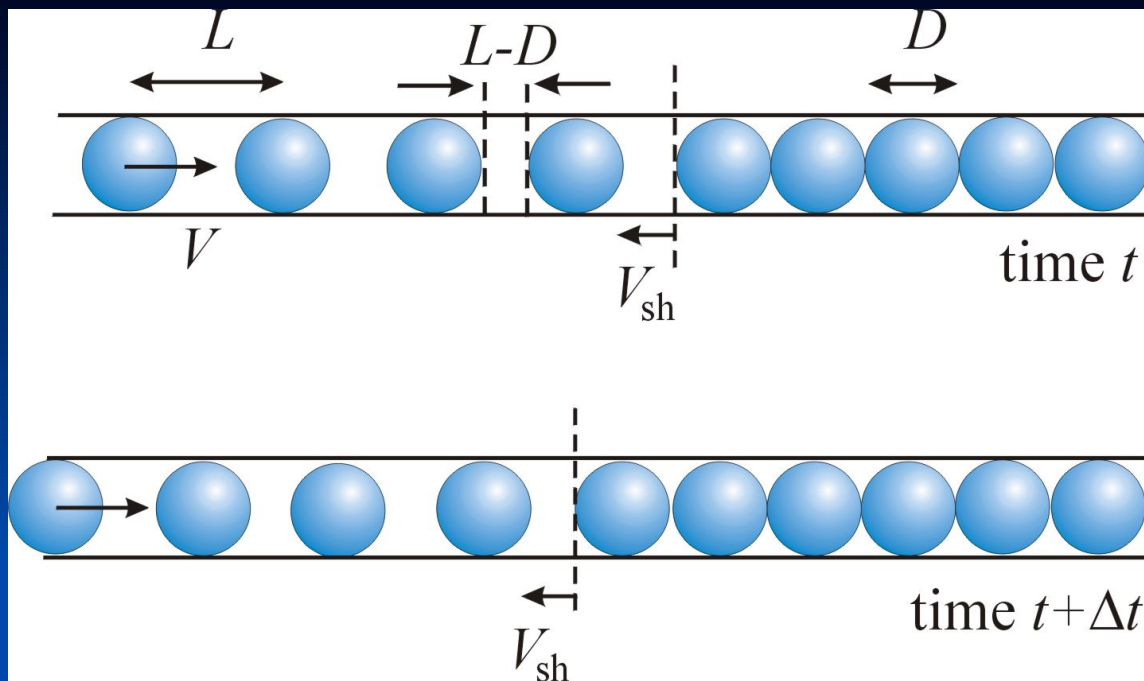


# The marble-tube analogy for shocks



- As a simple mechanical model for shock formation, consider in a hollow (semi-infinite) tube spherical marbles with diameter  $D$ , separated by a distance  $L > D$  which roll with speed  $V$
- The end of the tube is plugged, forming an obstacle that prevents the marbles from continuing onward

- As a result, the marbles collide, lose their speed and accumulate in a stack at the plugged end of the tube. The transition between freely moving and stationary marbles is the analogue of a shock surface
- Far ahead of the obstacle, where the marbles still move freely, the line density is  $n_1 = 1/L$
- In the stack, the density is instead  $n_2 = 1/D > n_1 \rightarrow$  the density increases when the marbles are added to the stack



■ The growth of the stack is easily calculated: in order to collide, two adjacent marbles have to close the separation distance  $\Delta D = L - D$  between the surfaces  $\rightarrow$  the time between two collisions is

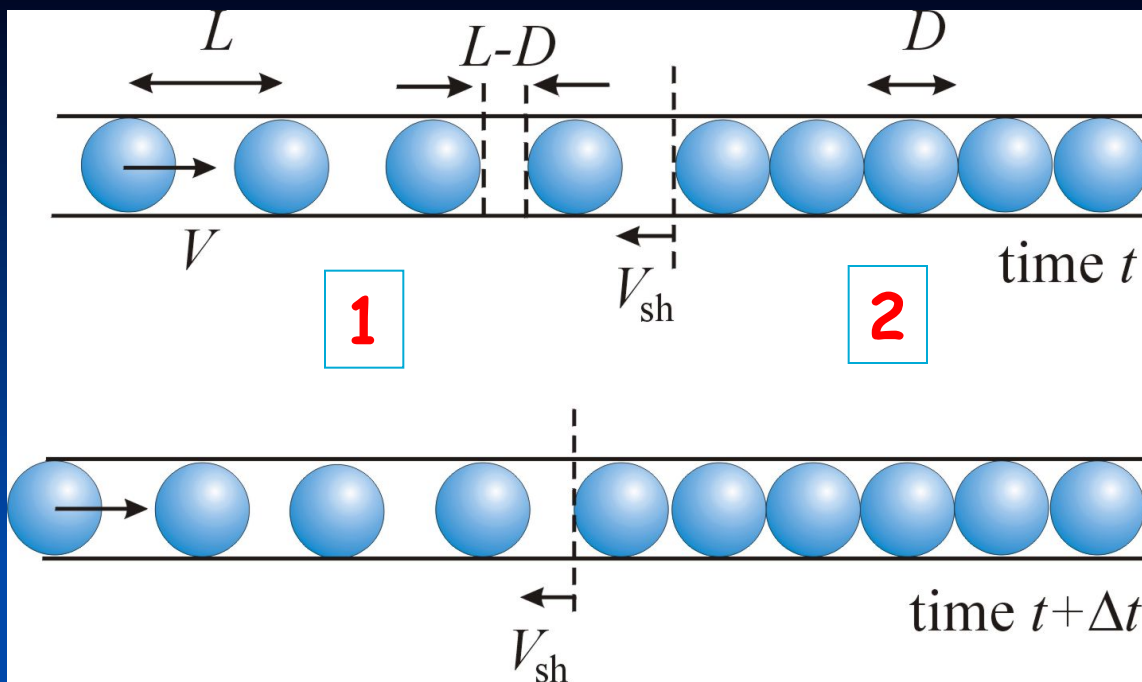
$$\Delta T = (L - D) / V$$

■ At every collision, one marble is added to the stack  $\rightarrow$  the length of the stack increases by  $D \rightarrow$  the average speed with which the length of stack increases is

$$V_{sh} = -\frac{D}{\Delta T} = -\frac{D V}{L - D}$$

■ 'Shock speed' = growth velocity of the stack.

■ The imaginary surface at the front-end of the stack, that separates a region of "low" density  $n_1$  from a region of "high" density  $n_2$  of marbles, is the analogue of a hydrodynamical shock wave



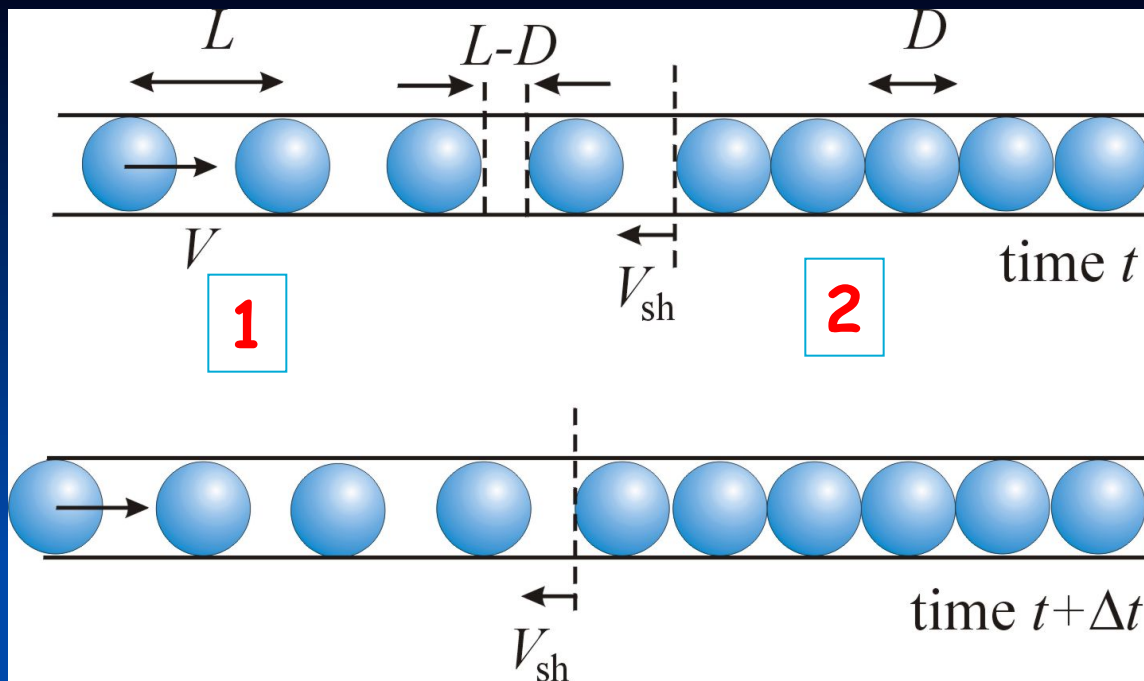
- Let transform to a reference where the shock is stationary and neglect the fact that the stack grows impulsively each time a marble is added

■ In this reference frame the incoming marbles have a speed:

$$V_1 = V - V_{sh} = V \left( \frac{L}{L - D} \right)$$

■ Marbles in stack, stationary in lab move away from the shock with speed:

$$V_2 = -V_{sh} = V \left( \frac{D}{L - D} \right)$$



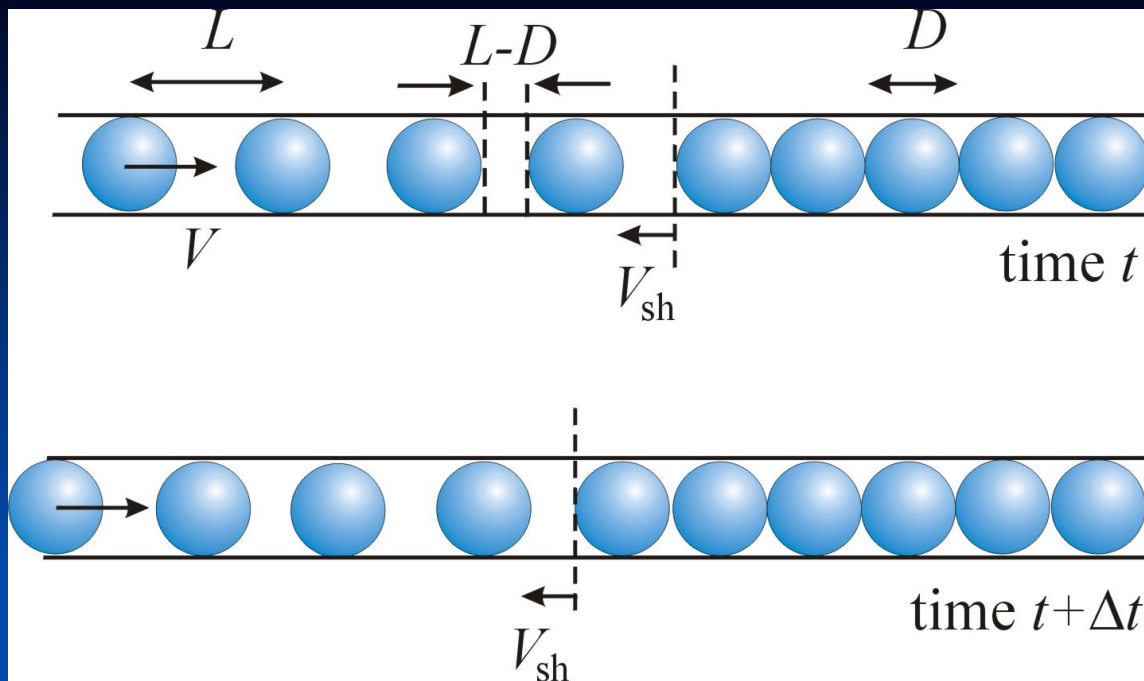
■ In any frame the flux is  $\text{Flux} = \text{density} \times \text{velocity}$

■ Incoming flux:

$$F_1 = n_1 \times V_1 = \left( \frac{1}{L} \right) \times V \left( \frac{L}{L - D} \right) = \frac{V}{L - D}$$

■ Outgoing flux:

$$F_2 = n_2 \times V_2 = \left( \frac{1}{D} \right) \times V \left( \frac{D}{L - D} \right) = \frac{V}{L - D}$$



## ■ Conclusions:

1. The density increases across the shock
2. The flux of incoming marbles equals the flux of outgoing marbles in the shock rest frame:

$$F_1 = F_2$$

■ This equality has a simple interpretation: the nbr of marbles crossing the shock surface in  $\Delta T$  is  $\Delta N = F \Delta t$

■ Since an infinitely thin surface has no volume, the nbr of marbles entering the surface at the front must exactly equal the nbr that leaves in the back...nothing can be "stored" in the shock!  $\rightarrow \Delta N_{in} = \Delta N_{out}$

■ Many of the concepts introduced here can be immediately translated to the physics of a shock in a gas, in particular the flux conservation

# Wave Breaking

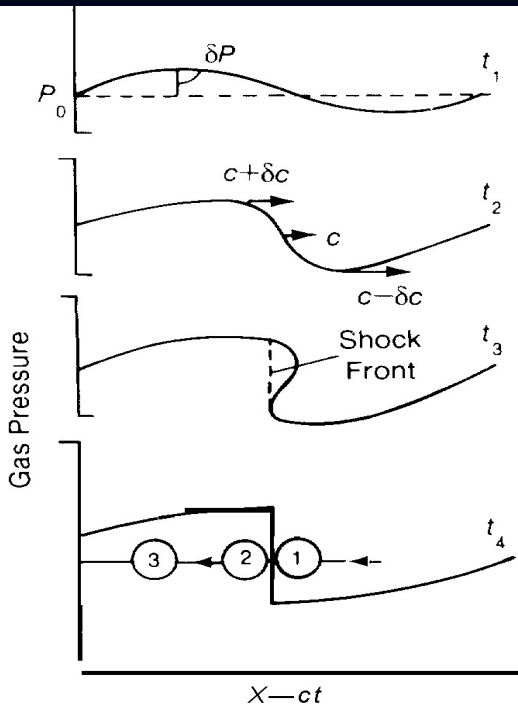


Fig. A. Self-steepening of a finite-amplitude sound wave. In the region where the state variables of the wave (here, pressure) would become multi-valued, irreversible processes dominate to create a steep, single-valued shock front (vertical dashed line).

- The formation of a shock wave depends on the non-linearity of the motion equations
- Let consider a perturbation with finite amplitude in a fluid otherwise homogeneous

Shock must form

- It is possible to show that the propagation speed is higher where there is an overdensity and lower where there is an underdensity: so the wave crests will move faster than the ventral part

$$u = \frac{2c_{s0}}{\gamma - 1} \left[ \left( \frac{\rho}{\rho_0} \right)^{(\gamma-1)/2} - 1 \right]$$

High-pressure/density regions move faster

$$\approx c_{s0} \left( \frac{\Delta \rho}{\rho} \right)$$

- Therefore the crest will reach the the ventral in a finite time, forming a discontinuity surface

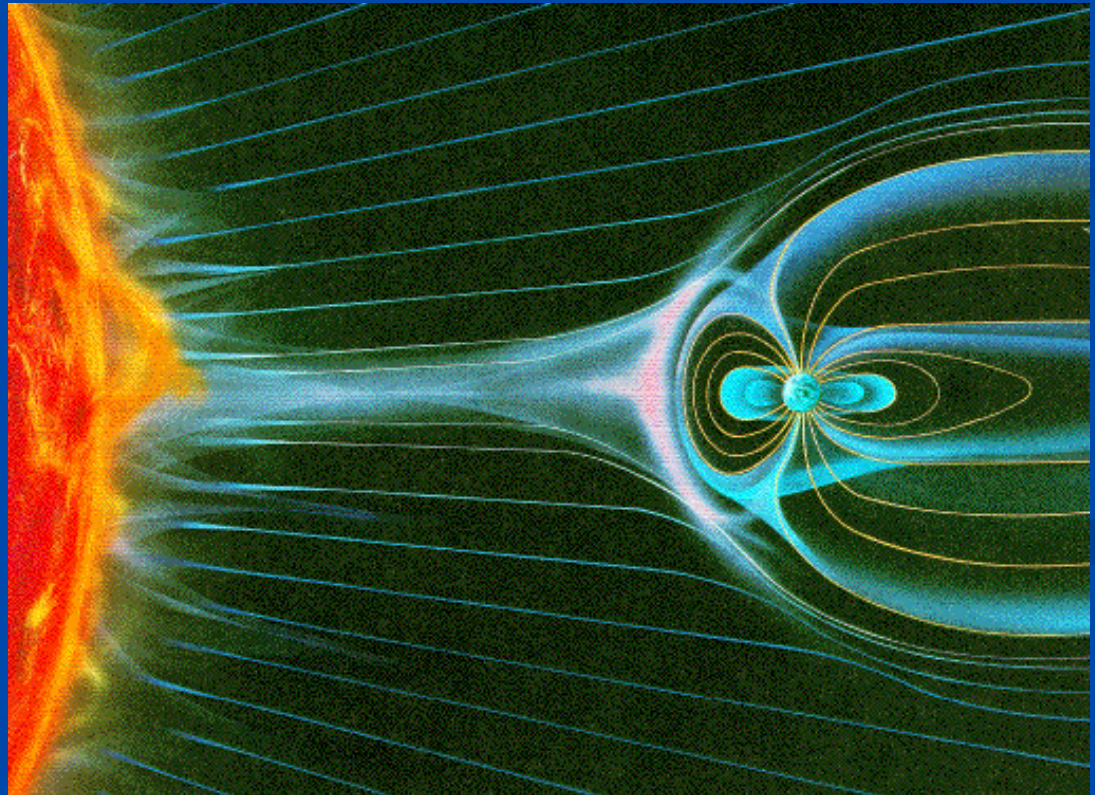
So, unless  $\rho$  is always constant, every perturbation of finite amplitude evolves toward a discontinuity (nb: a discontinuity DOES NOT imply a shock wave) if it can travel enough.

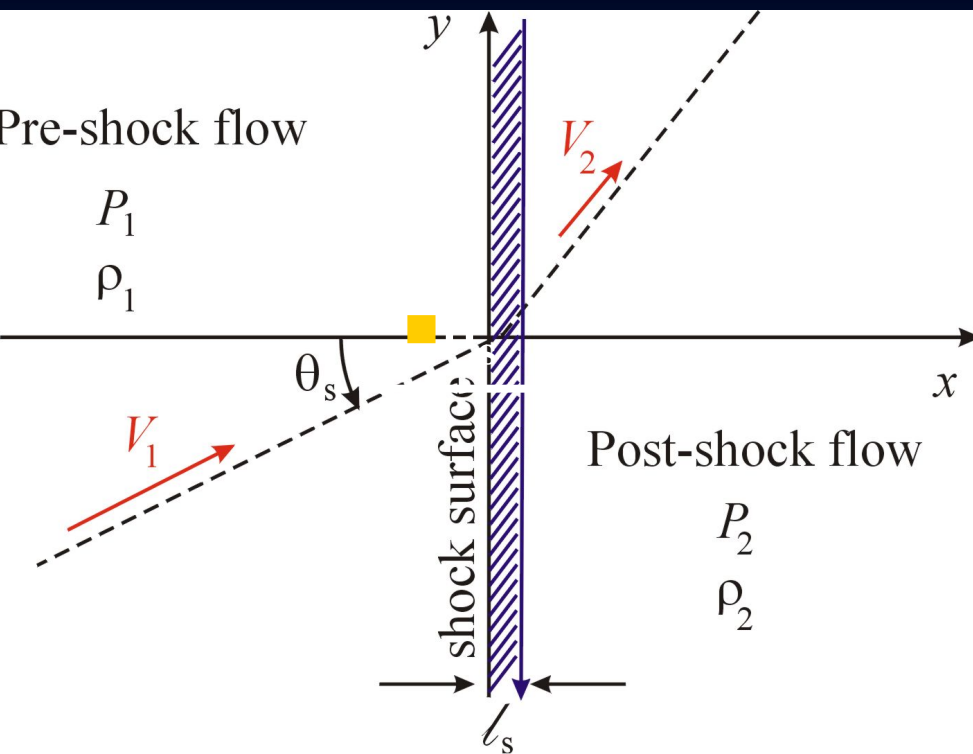
In practice, this does not occur always because of the damping due to viscous dissipation and<sup>58</sup> heat conduction



# Shock formation

- Shock waves are a feature of supersonic flows with a Mach number exceeding the unity  $M_s = |V|/c_s > 1$
- They occur when a supersonic flow encounters an obstacle which forces it to change its speed
- For instance a bow shock forms around the Earth in the tenuous supersonic solar wind when the ionized wind material "hits" the strongly magnetized earth's magnetosphere





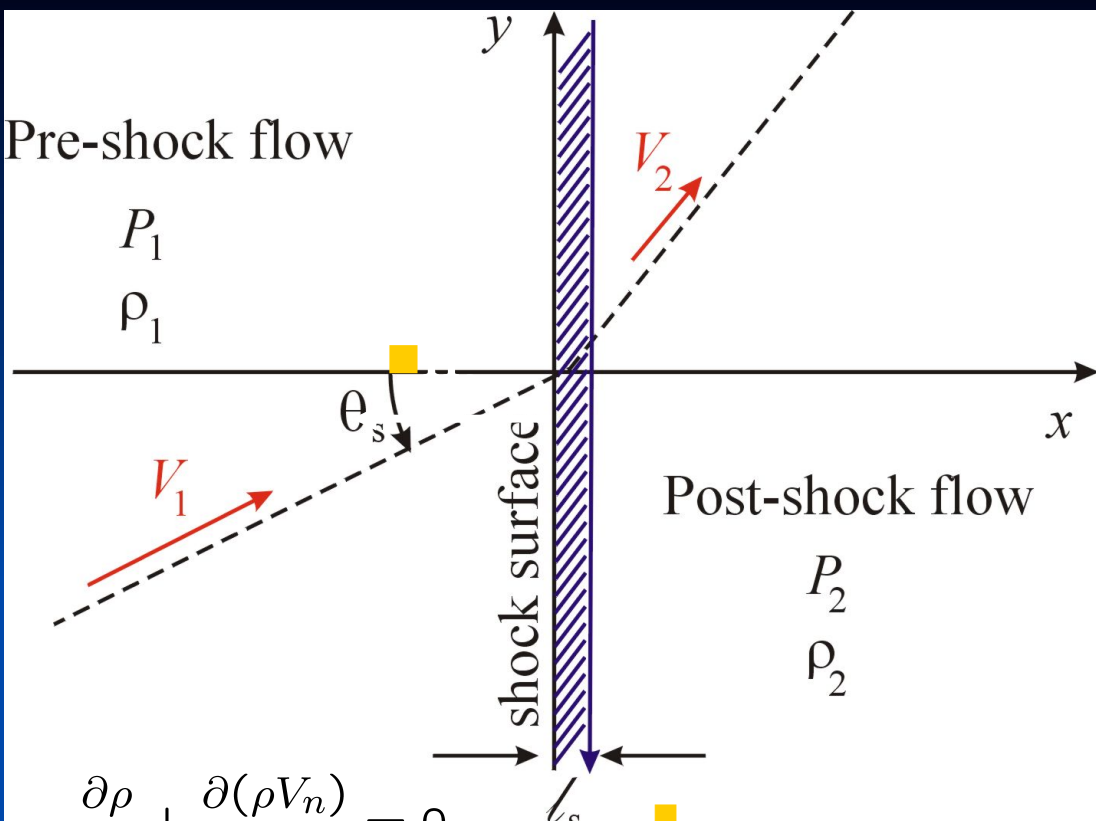
- Let consider a simple fluid described by a polytropic relation  $P=a\rho^\gamma$  on either side of the shock, but not with the same constant  $a$  as a result of dissipation in the shock (as we will see)
- Assume the shock is planar, located in a fixed position in the  $y$ - $z$  plane
- The flow is from left-to-right, so that pre-shock flow occurs at  $x<0$  and post shock at  $x>0$

- The normal  $\mathbf{n}_s$  to the shock front coincides with  $x$ -axis toward negative  $x$ ,

$$\mathbf{n}_s = -\mathbf{e}_x$$

- Let assume that flow properties (speed, density,...) depend only on the  $x$  coord:  $\partial/\partial y = \partial/\partial z = 0$  and that the speed lies in the  $x$ - $z$  plane (always possible with suitable choice of reference frame)

$$\vec{V} = V_n \vec{e}_x + V_t \vec{e}_z$$



$$\vec{V} = V_n \vec{e}_x + V_t \vec{e}_z$$

■ Neglecting the effect of gravity and dissipation in the flow the equations describing the flux are the mass, momentum and energy conservation, which reduce to

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_n)}{\partial x} = 0$$

$$\frac{\partial(\rho V_n)}{\partial t} + \frac{\partial(\rho V_n^2 + p)}{\partial x} = 0$$

■ Momentum conservation ? shock front

$$\frac{\partial(\rho V_t)}{\partial t} + \frac{\partial(\rho V_n V_t)}{\partial x} = 0$$

■ Momentum conservation k shock front

$$\frac{\partial}{\partial t} [\rho (\frac{V^2}{2} + e)] + \frac{\partial}{\partial x} [\rho V_n (\frac{V^2}{2} + h)] = 0$$

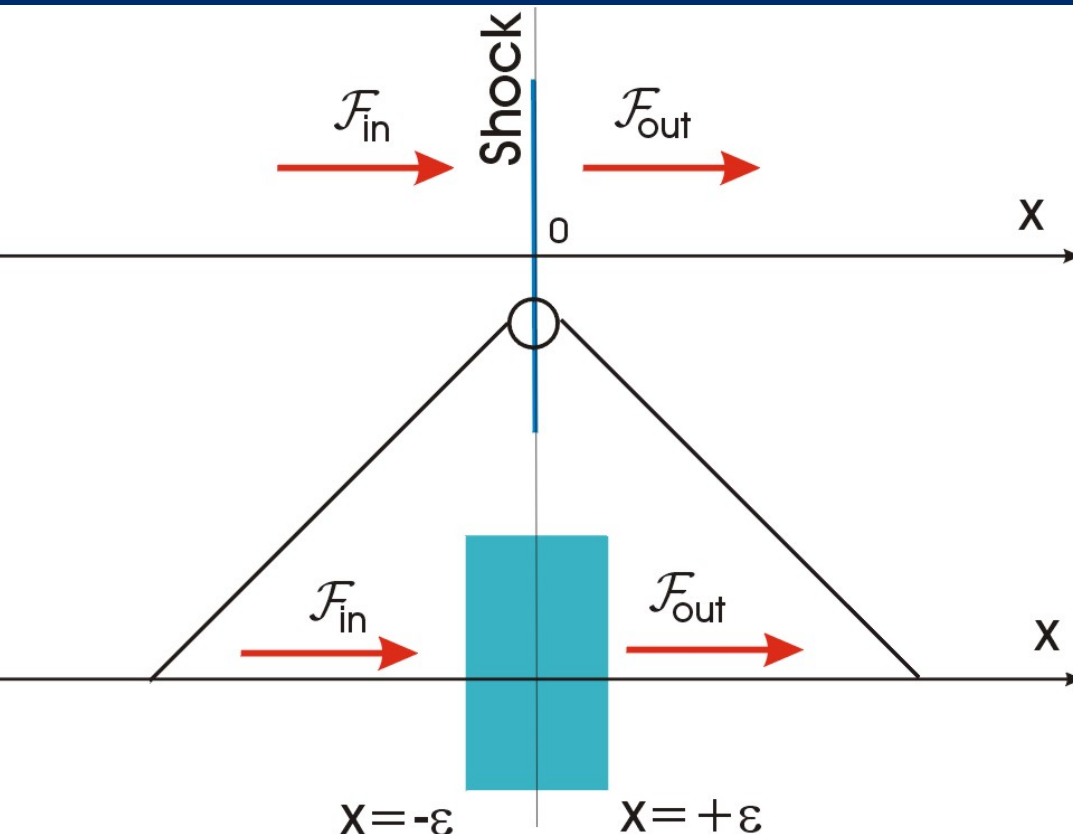
■ Energy conservation

With  $e = \frac{p}{(\gamma - 1)\rho}$   $h = \frac{\gamma p}{(\gamma - 1)\rho}$  ■ Specific internal energy and enthalpy

# Effect of a sudden transition on the conservation law

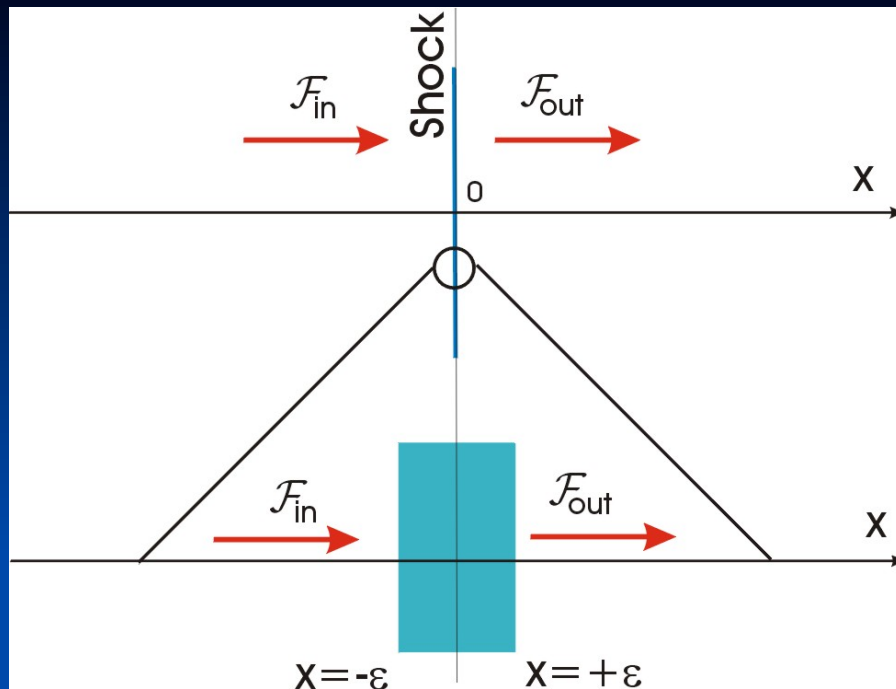
- All the equations have the form

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = 0$$



- Where is some quantity, like mass density, momentum density or energy density and  $F$  is the corresponding flux in the  $x$  direction

- Let assume that the shock has a thickness  $\varepsilon$  around  $x=0$  so that it extends in the range from  $-\varepsilon/2$  to  $+\varepsilon/2$



- The integrated version of conservation law reads

$$\int_{-\varepsilon}^{+\varepsilon} dx \left( \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} \right) = 0 \Leftrightarrow$$

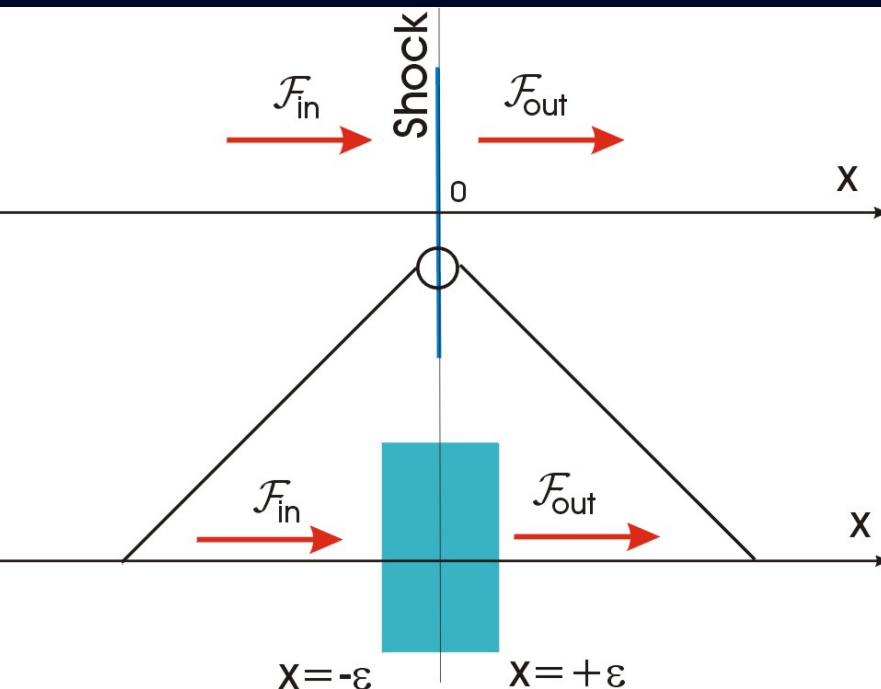
$$\begin{aligned} \frac{\partial}{\partial t} \left( \int_{-\varepsilon}^{+\varepsilon} dx Q \right) &= F(x = -\varepsilon) - F(x = \varepsilon) \\ &= F_{\text{in}} - F_{\text{out}} \end{aligned}$$

flux in - flux out

Change of  
amount in  
layer

- If the shock thickness is small, one can estimate the integral using the mean value of  $\partial Q / \partial t$

$$-\Delta F = \int_{\varepsilon/2}^{\varepsilon/2} dx \frac{\partial Q}{\partial t} \approx \frac{\varepsilon}{2} \left( \frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t} \right)$$



Infinitely thin layer:

■ Formal proof: limiting process

$$\lim_{\epsilon \rightarrow 0} \left[ \frac{\partial}{\partial t} \left( \int_{-\epsilon}^{+\epsilon} dx Q \right) \right] = F(x = -\epsilon) - F(x = \epsilon)$$

$\Leftrightarrow$

$$0 = F_{\text{in}} - F_{\text{out}}$$

■ Flux in = Flux out

$$-\Delta F = \int_{\epsilon/2}^{\epsilon/2} dx \frac{\partial Q}{\partial t} \approx \frac{\epsilon}{2} \left( \frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t} \right) \rightarrow 0 \text{ when } \epsilon \rightarrow 0$$

■ What goes in must come out!

■ NB: in the case of steady flux ( $\partial/\partial t = 0$ ), the integral is identically zero



Shock conditions: **what goes in must come out!**

(1 = in front of shock, 2= behind shock)

■ Three conservation laws means three conserved fluxes!

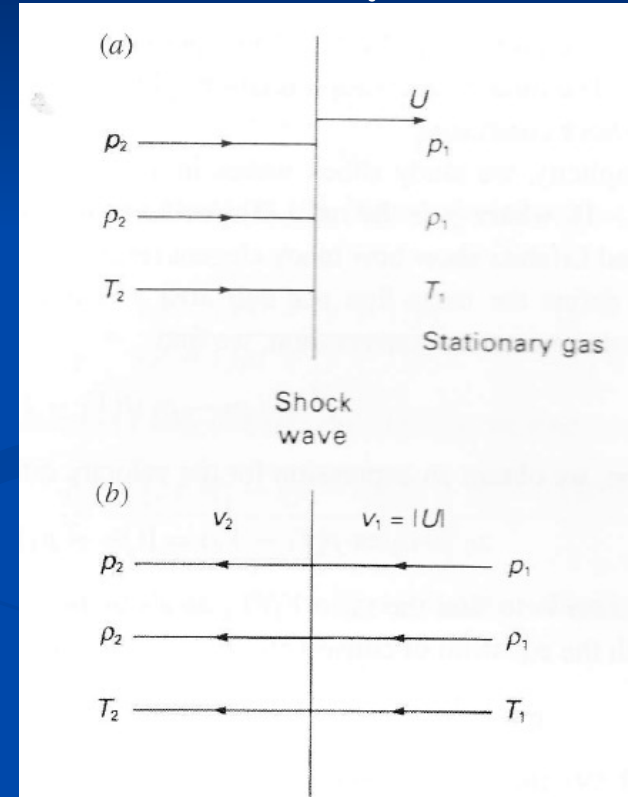
$\rho_1 V_1 = \rho_2 V_2$  ■ Mass flux

$\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2$  ■ Momentum flux

$$\rho_1 V_1 \left( \frac{V_1^2}{2} + h_1 \right) = \rho_2 V_2 \left( \frac{V_2^2}{2} + h_2 \right)$$

$$h = \frac{\gamma p}{(\gamma - 1)\rho}$$

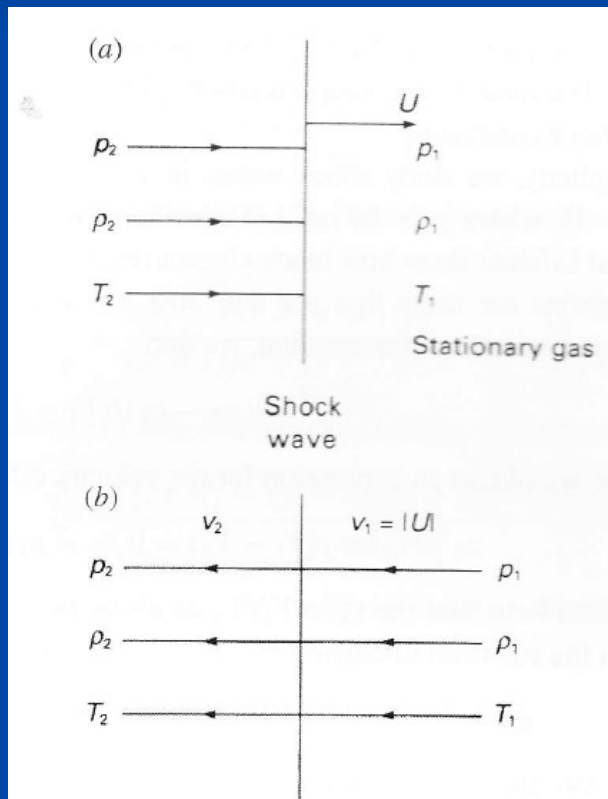
■ Energy flux



■ Three ordinary equations for three unknowns: post-shock

■ state (2) is uniquely determined by pre-shock (1) state!

- A shock is a transition layer where the fluid flow velocity changes discontinuously.
- An object moving with supersonic speed through a medium will drive a shock through that medium.
- Shocks are ubiquitous in astrophysical media because sound speed is low (because of low density), so it's easy to exceed sound speed in such media.



## Shock wave propagating through a Stationary gas at supersonic velocity $U$

Flow of gas in a reference frame in which the shock front is stationary

$$\rho_1 v_1 = \rho_2 v_2 .$$

$$\rho_1 v_1 \left( \frac{1}{2} v_1^2 + w_1 \right) = \rho_2 v_2 \left( \frac{1}{2} v_2^2 + w_2 \right) .$$

- Energy flux through
- a surface normal to  $v$

$w = \varepsilon_m + pV$ ,  $\varepsilon_m$  is the internal energy per unit mass and  $V$  is the specific volume

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 .$$

Momentum flux through shock wave

# Shock compression ratio

$$r \equiv \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

■ Definition compression ratio

Substituting  $v_1 = r v_2$   $\frac{\gamma}{\gamma - 1}(p_2 v_2 - p_1 v_1) = \frac{1}{2}(v_2 - v_1)(p_2 - p_1)$  ■ (cfr. pag 233)

express the compression ratio as a function of pressures pre- and

$$r = \frac{\rho_2}{\rho_1} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

■ Shock jump condition

# Summary

$$r \equiv \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

$$r = \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_s^2}{2 + (\gamma - 1)M_s^2} = \frac{(\gamma + 1)}{2/M_s^2 + (\gamma - 1)}$$

$$M_s = \frac{v_1}{(\gamma p_1 / \rho_1)^{1/2}} = \frac{v_1}{c_{s1}} > 1$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$

$$r = \frac{\rho_2}{\rho_1} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{p_2}{p_1}\right) = \frac{[2 + (\gamma + 1)^2 M_s^2][2\gamma M_s^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_s^2}$$

i)  $r > 1 \rightarrow \rho_2 > \rho_1$  but limited at  $(\gamma+1)/(\gamma-1)$   
in the limit  $M_s \rightarrow \infty$  so shock waves  
compress moderately the fluid

ii)  $r > 1 \rightarrow V_2 < V_1 \rightarrow (\gamma-1)/(\gamma+1)$  when  
 $M_s \rightarrow \infty$

iii)  $p_2 > p_1$  and  $p_2/p_1 \propto M_s^2 \rightarrow$  large  
compression of the fluid

iv)  $T_2 > T_1$  and  $T_2/T_1 \propto M_s^2 \rightarrow$  large  
heating of the fluid

# Physical interpretation

■ Shocks decelerate the bulk flow speed (ii) and at same time heat and compress (i,iii) the fluid immediately after the shock (iv).

i)  $\rho_2 > \rho_1$

ii)  $V_2 < V_1$

iii)  $p_2 > p_1$

iv)  $T_2 > T_1$

■ It is clear that the energy to make the heating must come from kinetic energy of the fluid: infact the speed is decreased by the same factor  $r$  by which density is increased

■ → shocks transform kinetic energy in internal (thermal) energy

# Physical interpretation

- But this is not all the history...entropy change is also involved at the shock front

- First, note that, when  $M_s=1$ , all the ratios are =1:  
in this case the shock does not make any transformation on the fluid

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_s^2}{2 + (\gamma - 1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$

- What does happen if  $M_s < 1$ ?

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$

- The Rankine-Hugoniot eqns tell us that  $V_1 < V_2$  and  $T_1 < T_2$ :
- the fluid is accelerated and cooled!

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma + 1)M_s^2][2\gamma M_s^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_s^2}$$

- This means that a fraction of the internal energy is transformed in kinetic energy, without having done anything else

■ BUT...



# Physical interpretation

■ We have seen that in an ideal polytropic gas the specific entropy is  $s = c_v \ln(p\rho^{-\gamma})$

■ Since we have neglected dissipation in deriving the equations (ie adiabatic transformations), the entropy is constant on either sides of the shock:

■  $s(x < 0) = \text{const.} = s_1, s(x > 0) = \text{const.} = s_2$

■ However we can calculate the entropy jump at the shock

$$\Delta s = s_2 - s_1 = c_v \ln \left[ \left( \frac{p_2}{p_1} \right) \left( \frac{\rho_1}{\rho_2} \right)^\gamma \right]$$

■ It is clear that:

■ i)  $\Delta s > 0$ , provided  $\rho_2 > \rho_1$ , that is  $M_s > 1$

■ ii)  $\Delta s = 0$  when  $M_s = 1 \rightarrow$  again the fluid properties do not change across the shock

■ i) clearly indicates that some form of dissipation occurs at the shock: kinetic energy of upstream flow is irreversibly dissipated into thermal (internal) energy of the shock-heated gas downstream. Nevertheless, the details of the dissipation mechanism do not enter into the final equations

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_s^2}{2 + (\gamma - 1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$

# Physical interpretation

$$\Delta s = s_2 - s_1 = c_v \ln \left[ \left( \frac{p_2}{p_1} \right) \left( \frac{\rho_1}{\rho_2} \right)^\gamma \right]$$

■ If  $M_s < 1$ , then  $\rho_2 < \rho_1 \rightarrow$  we have  $\Delta s < 0$ !

■ This is impossible: 2nd law of thermodynamics states that the only possible transformations are those for which  $\Delta s \geq 0$

■ Therefore, we CANNOT have shock waves with  $M_s < 1$

■ Shocks occurs ONLY for supersonic speeds

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_s^2}{2 + (\gamma - 1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$

# Strong shocks

■ In many astrophysical applications, the normal Mach number is large,  $M_s \gg 1$

■ In this limit the Rankine-Hugoniot jump conditions simplify considerably

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)}{2/M_s^2 + (\gamma - 1)}$$



$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma + 1)}{(\gamma - 1)}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$



$$\frac{p_2}{p_1} \approx 2\gamma M_s^2$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$



$$\frac{V_2}{V_1} \approx \frac{\gamma - 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma - 1)M_s^2][2\gamma M_s^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_s^2}$$



$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2$$

$$M_{s2} = \frac{2 + (\gamma - 1)M_s^2}{2\gamma M_s^2 - \gamma + 1}$$



$$M_{s2} \approx \frac{\gamma - 1}{2\gamma}$$

# Strong shocks

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma + 1)}{(\gamma - 1)}$$

■ Fluid density changes moderately

$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma + 1)}$$

■ Fluid is strongly compressed

$$\frac{V_2}{V_1} \approx \frac{\gamma - 1}{\gamma + 1}$$

■ Speed flow is slowed

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2$$

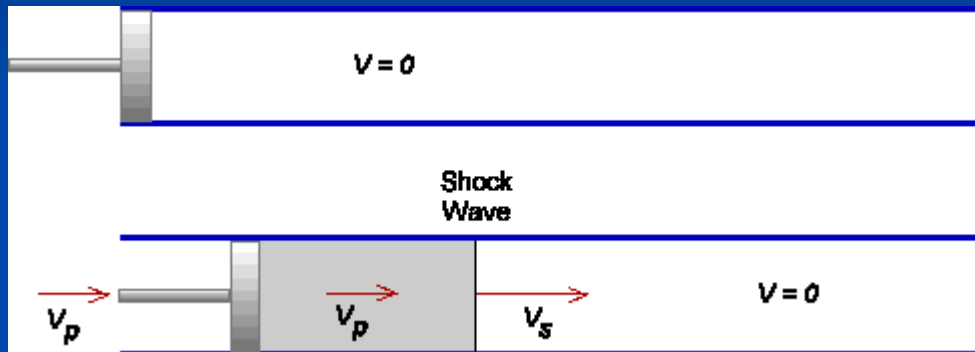
■ Fluid temperature is highly increased

$$M_{s2} \approx \frac{\gamma - 1}{2\gamma}$$

■ Mach number is  $<1 \rightarrow$  fluid is subsonic

# The supersonic piston

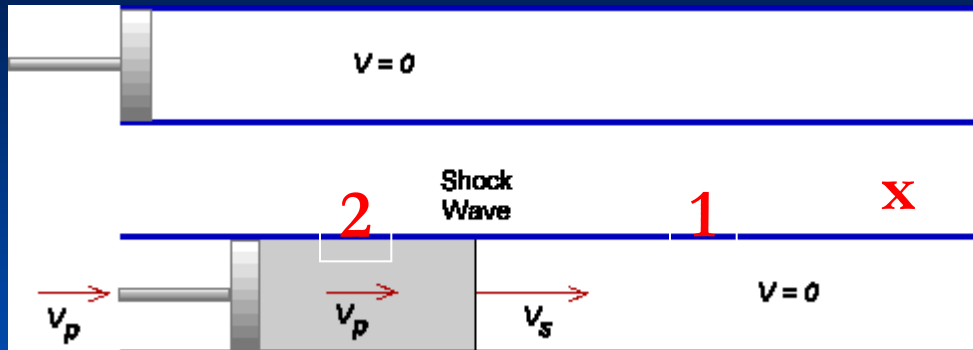
- A common situation in high energy astrophysics is one in which an object is driven supersonically into a gas, or equivalently, a supersonic flow past a stationary object (ie what is important is the relative speed between fluid and object), as for instance in the case of supernovae explosions



- A good illustrative example is a piston driven supersonically into a tube filled with stationary gas

- Basically, a hollow tube is filled with a uniform gas at rest and fitted with a piston at one end. At time  $t = 0$ , the piston is suddenly put into motion with a constant speed,  $V_p (> c_s)$ .
- The motion of the piston creates a shock wave, ahead of the piston, that moves in the same direction as the piston, but at faster, constant speed,  $V_s$  (faster than the speed of sound).
- The problem is to calculate the shock wave speed

# The supersonic piston



■ Let work out the problem in the shock wave reference frame, which is moving in the observer frame at speed  $+V_s$

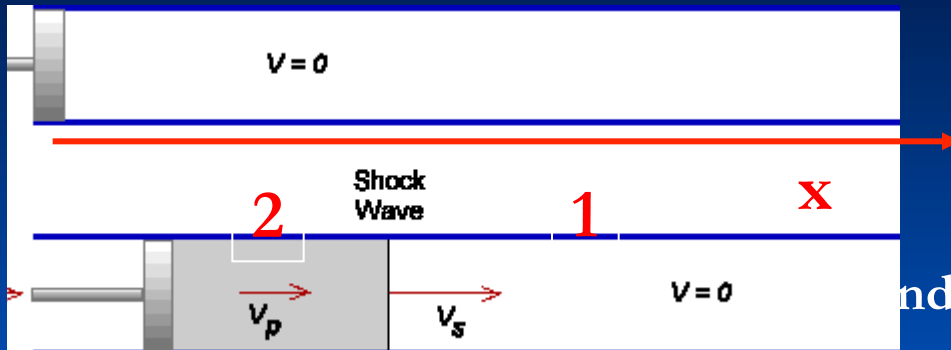
■ In the obs frame, the pre-shock fluid is at rest  $V_1=0$ , while for the post-shock fluid is reasonable to assume that is moving at the same speed of the piston  $V_2=V_p > c_s$  because the particles of the fluid are swept up when the piston reach them, very much like snow is swept up by a snowplow

■ The galileo's tranformation is  $V_{sh}=V_{obs} - V_s$



■ In the shock frame, the pre-shock fluid moves with speed  $V_1'=-V_s$  and the post shock fluid moves at (unknown) speed  $V_2'$

# The supersonic piston



■ From mass flux conservation

$$V_1'^2 = J^2 v_1^2$$

and

$$J^2 = \frac{p_2 - p_1}{v_1 - v_2} = \frac{p_2 - p_1}{v_1(1 - v_2/v_1)} \quad \text{■ (cfr. pag 233)}$$

but 
$$\frac{v_1}{v_2} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

■ (cfr. pag 234)



$$J^2 = \frac{(\gamma - 1)p_1 + (\gamma + 1)p_2}{2v_1}$$



$$V_1'^2 = \frac{v_1}{2}(\gamma - 1)p_1 + (\gamma + 1)p_2 = \frac{p_1 v_1}{2}[(\gamma - 1) + (\gamma + 1)(p_2/p_1)]$$

■ But

$$c_1^2 = \gamma p_1 v_1$$

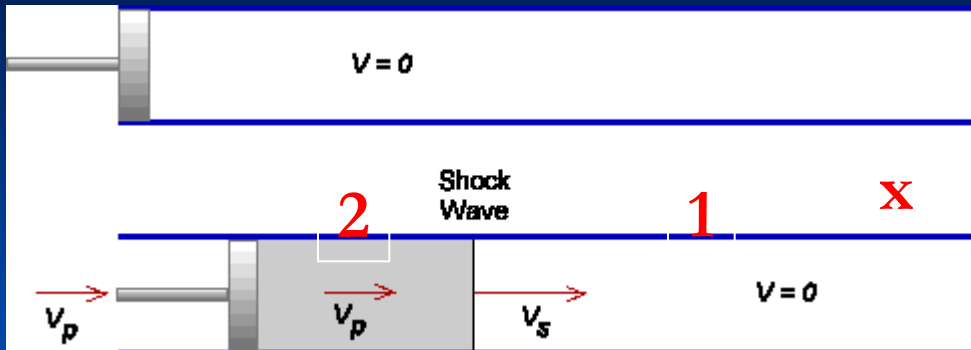


$$V_1'^2 = \frac{c_1^2}{2\gamma}[(\gamma - 1) + (\gamma + 1)(p_2/p_1)]$$

■ Then we have to find the pressure ratio



# The supersonic piston



We don't know  $V_2'$ , but we know the speed difference

$$V_1' - V_2' = V_p$$

From Galileo's transformation

From mass flux conservation we have  $\rho_1 V_1' = \rho_2 V_2' \equiv J \quad \Rightarrow \quad (\rho_1/\rho_2) V_1' = V_2'$

$$V_1' - V_2' = V_p = V_1' - (\rho_1/\rho_2) V_1'$$

$$V_1' - V_2' = V_p = V_1' [1 - (\rho_1/\rho_2)] = \rho_1 V_1' [1/\rho_1 - 1/\rho_2] = \rho_1 V_1' [v_1 - v_2] = J [v_1 - v_2]$$

$$\Rightarrow \frac{V_1' - V_2'}{v_1 - v_2} = J$$

but

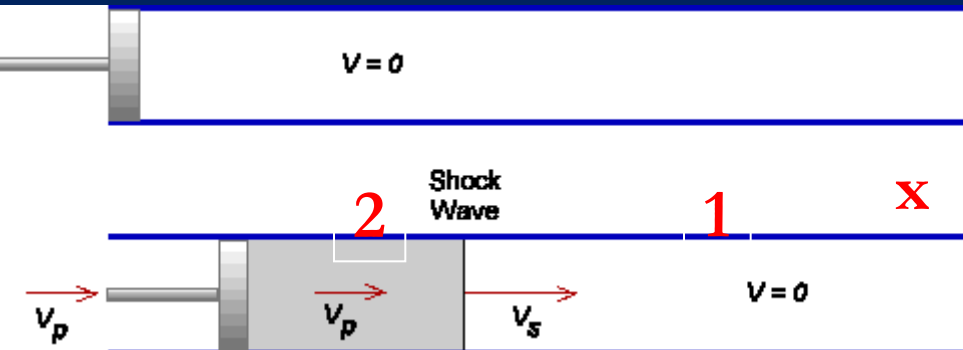
$$J^2 = \frac{p_2 - p_1}{v_1 - v_2} \quad \blacksquare \text{ (See pag 234)}$$

$$\frac{(V_1' - V_2')^2}{(v_1 - v_2)^2} = \frac{p_2 - p_1}{v_1 - v_2}$$



$$V_1' - V_2' = (p_2 - p_1)(v_1 - v_2)^{1/2}$$

# The supersonic piston



$$V_1' - V_2' = V_p$$

$$V_1' - V_2' = (p_2 - p_1)(v_1 - v_2)^{1/2} = V_p \quad (a)$$

Now eliminate specific volume  $v_2$  in (a) by using

$$\frac{v_1}{v_2} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

■(cfr. pag 234)

Then square (a) and solve for  $p_2/p_1$

$$(p_2/p_1)^2 - (p_2/p_1)[2 + (\gamma+1)\frac{V_p^2}{2p_1 v_1'}] + [1 - \frac{(\gamma-1)V_p^2}{2p_1 v_1'}] = 0$$

The sound speed in the pre-shock region is  $\gamma v_1' p_1 = c_1^2 \Rightarrow$  we can solve for  $p_2/p_1$

$$(p_2/p_1) = 1 + \frac{\gamma(\gamma+1)V_p^2}{4c_1^2} + \frac{\gamma V_p}{c_1} [1 + \frac{(\gamma+1)^2 V_p^2}{16c_1^2}]^{1/2}$$

# The supersonic piston

$$(p_2/p_1) = 1 + \frac{\gamma(\gamma+1)V_p^2}{4c_1^2} + \frac{\gamma V_p}{c_1} \left[ 1 + \frac{(\gamma+1)^2 V_p^2}{16c_1^2} \right]^{1/2} \quad \blacksquare (a)$$

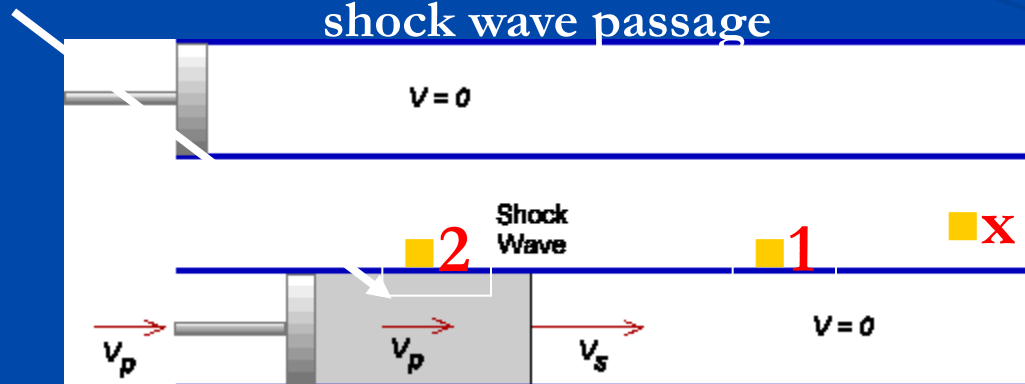
$$V_1^2 = \frac{c_1^2}{2\gamma} [(\gamma-1) + (\gamma+1)(p_2/p_1)] \quad \blacksquare (b)$$

■ Inserting (a) into (b) after some simple algebra we have

$$V_1' \equiv |V_s| = \frac{(\gamma+1)}{4} V_p + \left[ c_1^2 + \frac{(\gamma+1)^2 V_p^2}{16} \right]^{1/2}$$

■ The shock wave travels at higher speed with respect to the piston

■ → there is a layer of shocked material in between the shock wave and the piston, traveling at the piston speed, compressed and heated by the shock wave passage



# The supersonic piston

$$V_1' \equiv |V_s| = \frac{(\gamma + 1)}{4} V_p + [c_1^2 + \frac{(\gamma + 1)^2 V_p^2}{16}]^{1/2}$$

In the limit of strong shocks  $V_p \gg c_s$ , the expression reduces to

$$|V_s| \approx \frac{(\gamma + 1)}{2} V_p$$

→ the ratio between the shock position and the piston position  
is

$$|V_s|/V_p \approx \frac{(\gamma + 1)}{2}$$

e.g. for a monoatomic gas  $\gamma=5/3 \rightarrow V_s/V_p=4/3$

- All the gas originally in the tube between  $x=0$  and the shock position is squeezed into a smaller distance  $(V_s - V_p)t$
- So behind the shock wave and ahead of the obstacle (the piston in this example) there is a layer of material compressed and heated
- It is also seen that there is a stand-off distance of a shock front from a blunt object placed in the flow, as for instance in the case of solar wind past the Earth's magnetic dipole
- This is what is expected to occur when a supernova ejects a sphere of hot gas into the ISM

# Blast shock waves

- A blast shock wave is a shock wave formed by a hot gas bubble expanding supersonically in the ambient medium

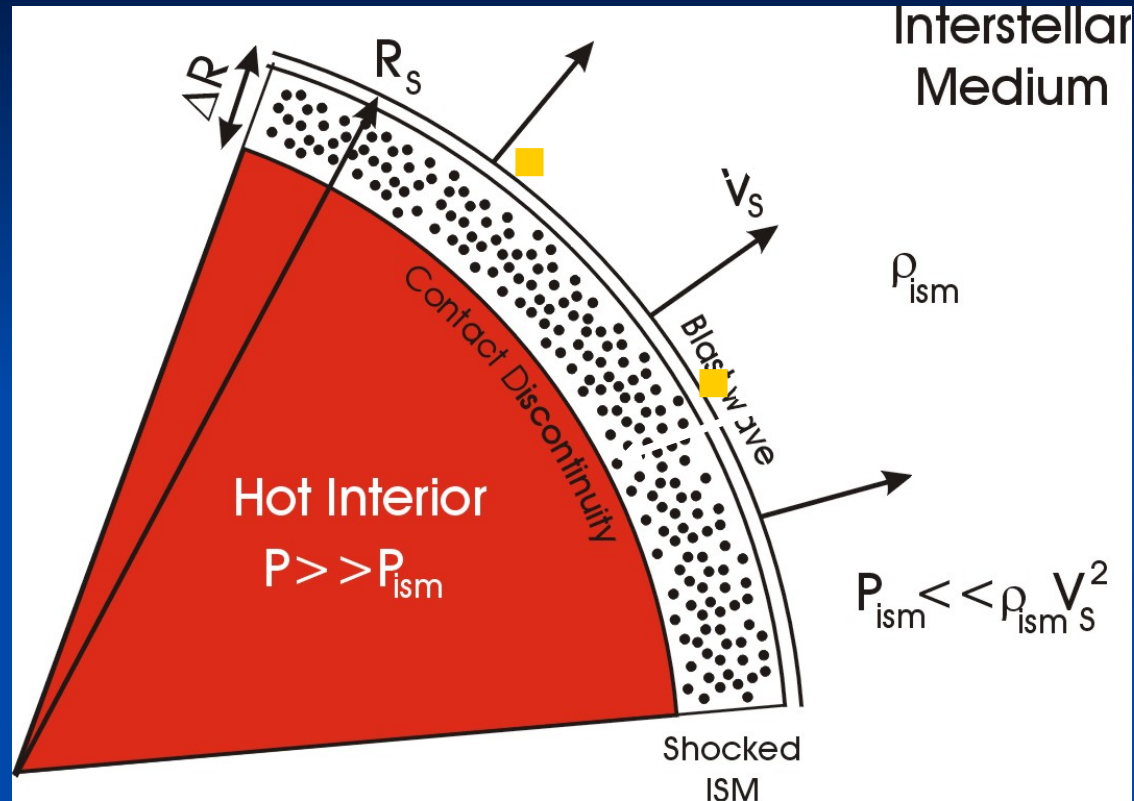
- Let us assume that the expansion occurs in a uniform stationary polytropic medium with density  $\rho_0$  and pressure  $p_0$

- how does the shock wave evolve in time?

- First, since the surrounding medium is uniform, the expansion will have spherical symmetry

# Blast shock waves

- Consider a spherical bubble containing a low density, very hot gas with internal pressure  $p_i$  and density  $\rho_i$  embedded in a cold, dense stationary medium with low pressure  $p_o \ll p_i$  and a high density  $\rho_o \gg \rho_i$  (...we can have low pressure with high density at low  $T$  because  $p=nkT$ )

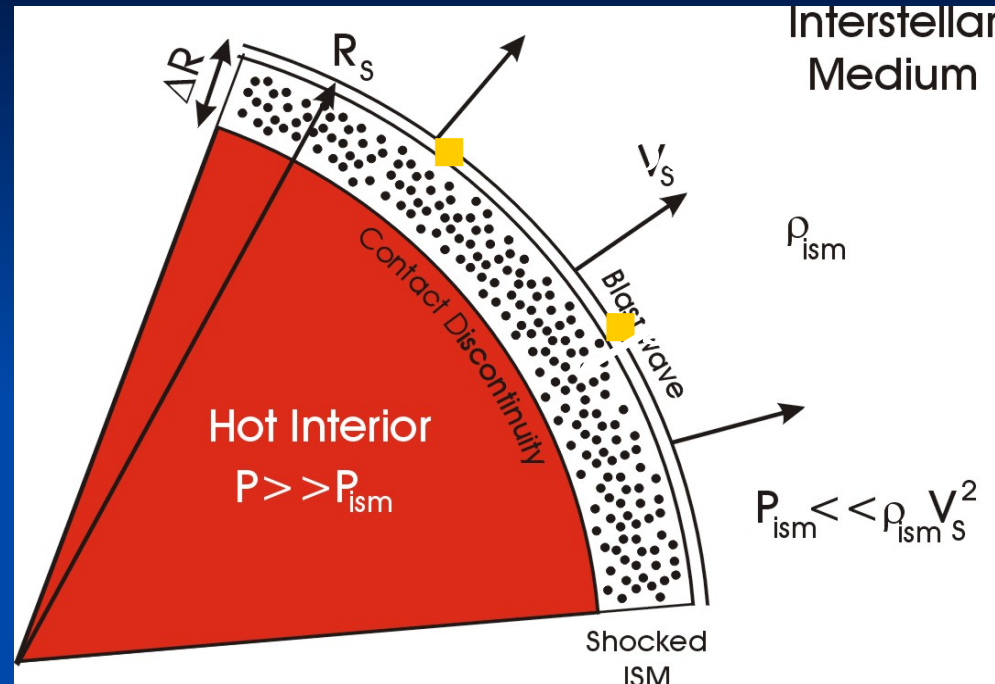


- Because of the high pressure difference, the bubble will start to expand rapidly

- A 3-D supersonic piston

# Blast shock waves

- If the difference between internal and external pressure is sufficiently large, the expansion speed will be supersonic with respect to sound speed of the surrounding medium,
- For instance, the typical observed expansion speed of a supernova remnant is  $\sim 10000$  km/s, while the sound speed in the ISM ranges 10-100 km/s
- Because of supersonic speed, a shock will form at the outer edge of the bubble (which acts as a supersonic piston).
  - This shock is usually called blast wave

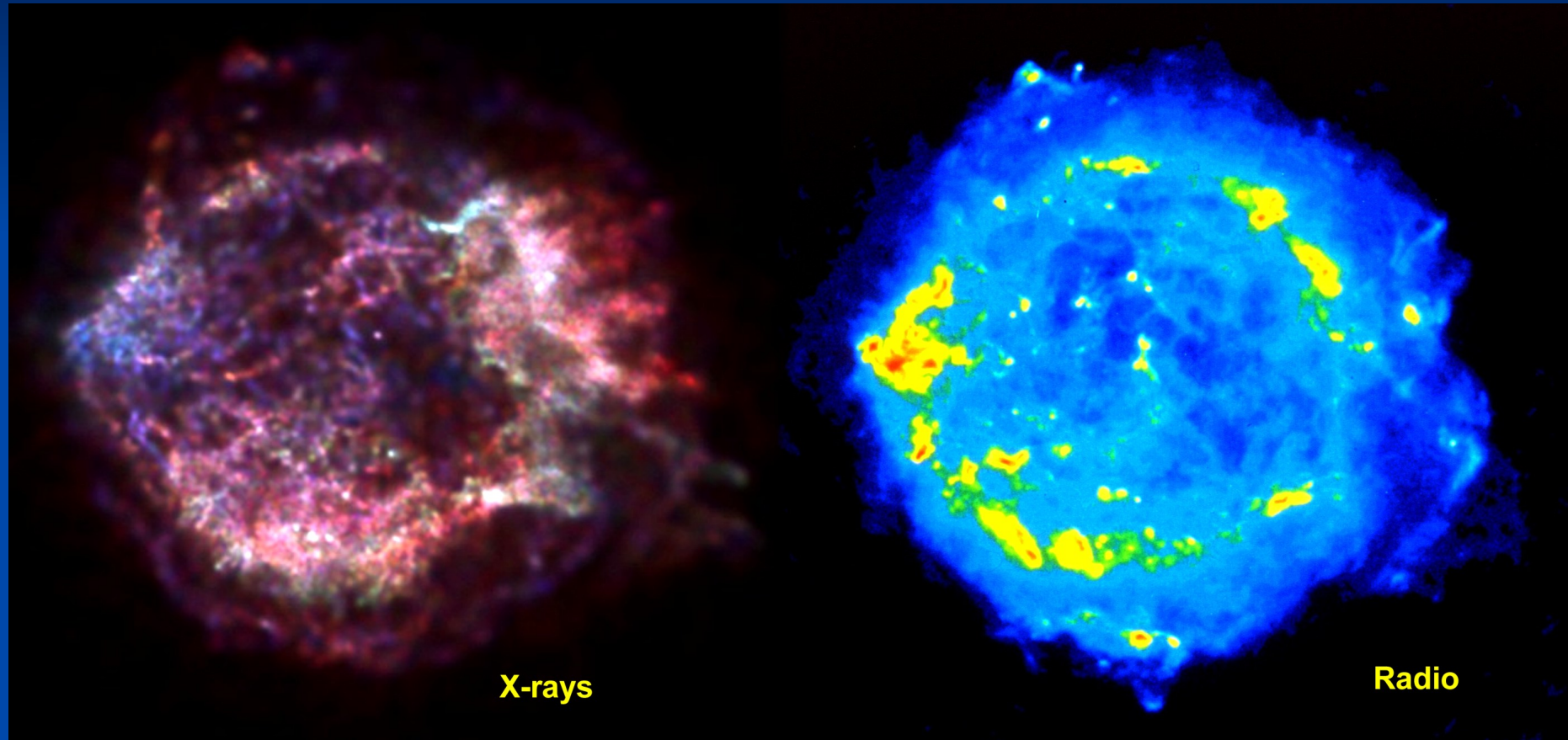




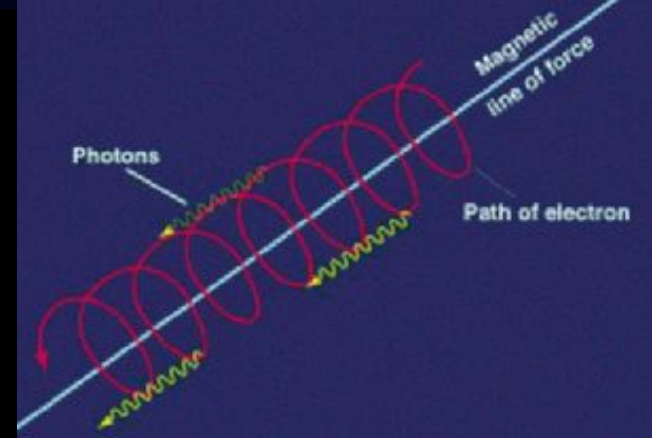
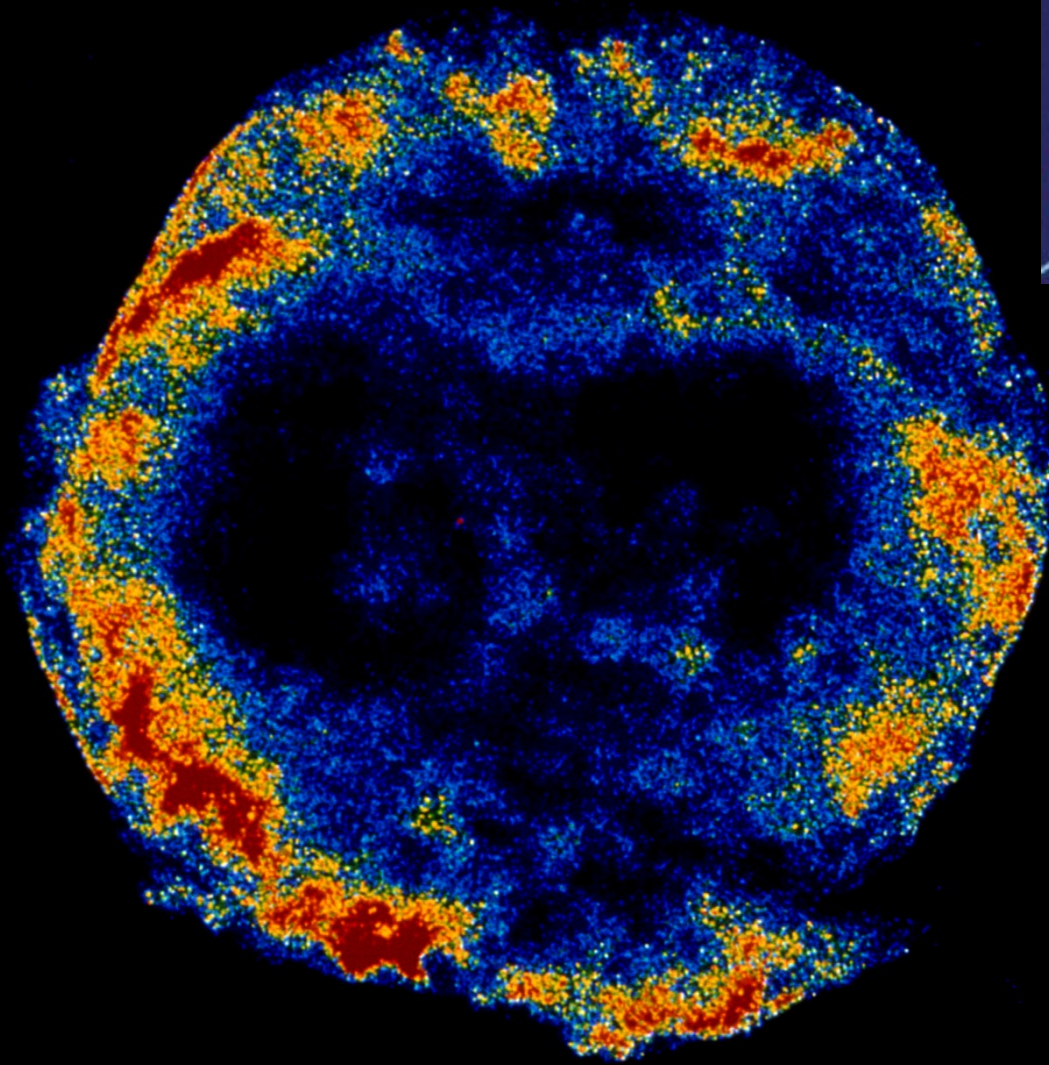
# SuperNova Remnants (SNRs)

- A supernova remnant (SNR) consists essentially of the stellar ejecta of the SN explosion embedded in a hot expanding bubble, preceded by swept-up interstellar material and an outer blast wave (strong shock) propagating into the interstellar medium

# Supernova Remnant Cassiopeia A



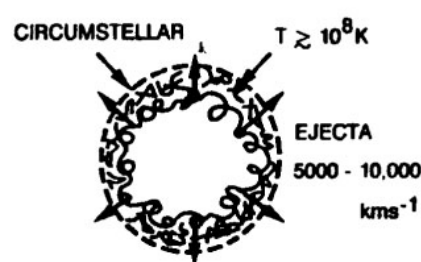
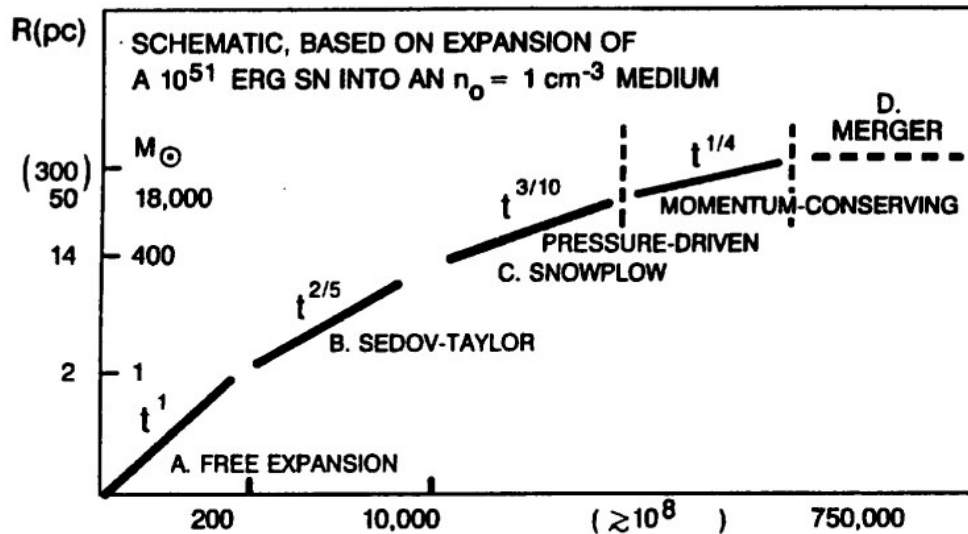
2.9 pc, exploded 350 years ago



■ Remnant of Tycho's supernova of 1572 AD



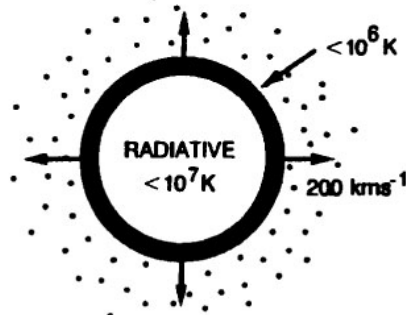
# STANDARD SNR EVOLUTION



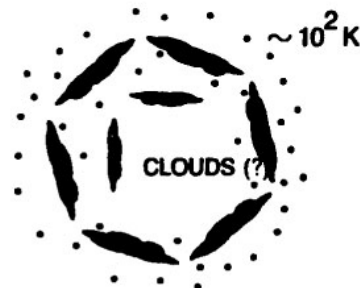
A. FREE EXPANSION



B. BLAST WAVE



C. SNOWPLOW



D. DEATH → COALESCENCE

## Main properties:

### Different expansion stages:

- Free expansion stage ( $t < 1000 \text{ yr}$ )

$$R \propto t$$

- Sedov-Taylor stage ( $1000 \text{ yr} < t < 10,000 \text{ yr}$ )

$$R \propto t^{2/5}$$

- Pressure-driven snowplow ( $10,000 \text{ yr} < t < 250,000 \text{ yr}$ )

$$R \propto t^{3/10}$$

- Momentum-conserving ( $250,000 < t < 750,000 \text{ yr}$ )

$$R \propto t^{1/4}$$

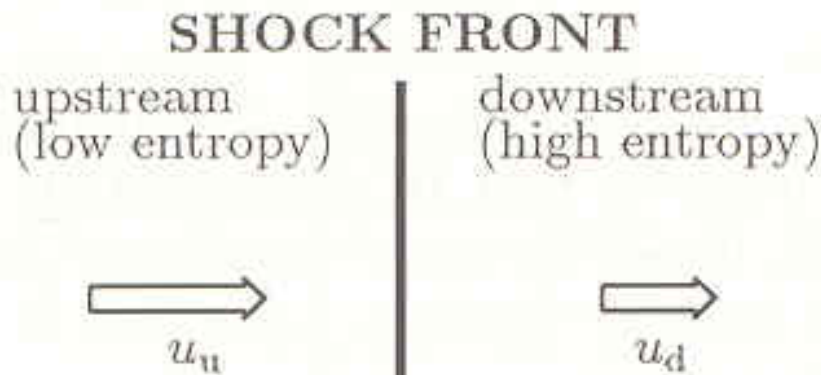
# Acceleration at shock waves

At shock waves, acceleration of particles occurs. The ingredients are:

- a physical mechanism to change energy at each collisionless interaction with the shock wave and
- a statistical process which gives an average gain of energy with an enough high number of encounters with the shock wave front, i.e. suitable interaction time

The main mechanisms are:

- shock drift acceleration (SDA) or scatter-free accel. in the  $E$  induction field in the shock front
- diffusive shock acceleration (DSA) due to repeated reflections in the plasma converging at the shock front
- stochastic acceleration (SA) in the turbulence behind the shock front



The relative contributions of these mechanisms depends on the properties of the shock, e.g.

SDA is maximal for normal shocks, SA requires strong turbulence in the downstream, DSA requires scattering centers also upstream

The accel. proc. is stochastic and the problem of feeding the particles back into the acceleration mech. is a recurrent problem in the study of shock accel. independent of the actual accel. mechanism

# Shock Drift Acceleration

In the shock rest frame, a particle feels an induction field  $\mathbf{E} = -\mathbf{u}_u \times \mathbf{B}_u = -\mathbf{u}_d \times \mathbf{B}_d \rightarrow$  normal to  $\mathbf{u}$  and  $\mathbf{B}$  in the shock plane

→ Maximal for normal shocks, vanishes for parallel shocks.

→ The energy gain is largest if the particle can interact with the shock front for a long time, depending on the normal  $\mathbf{u}$ : if small, particles stick to the shock, if large it escapes before gaining large amount of energy. The gain can be enhanced if turbulence is present.

The average gain factor is between 1.5-5, even if for repeated crossing may lead to higher values

→ The details of the particle trajectory, in particular whether particle is transmitted or reflected depend on its initial energy and pitch angle, leading to characteristic angular distribution of accel. particles: an initially isotropic distr. is converted to a very strong field-parallel beam in the upstream and to a smaller beam normal to field in the downstream

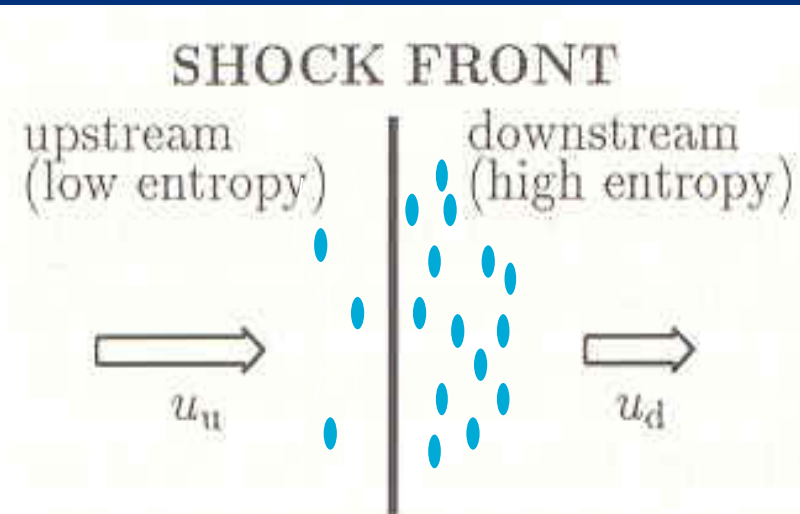
→ Shock drift accel. out of the solar wind up to  $E$  of a few KeV to some tens of KeV can be observed at the Earth's bow shock and in interplanetary traveling shocks.

This process does not lead to large energy gains, compared to DSA and SA

# Diffusive Shock and Stochastic Acceleration

These processes are dominated by scattering in the irregularities downstream and upstream flux.

The shock front does not play any direct role.



The description given so far is a geometrical one, i.e. the shock front is just a plane in which a discontinuity occurs, in the real life turbulences and fluctuations in the B parameters (i.e.  $\langle \Delta B^2 \rangle \neq 0$ ) downstream and irregularities in the upstream flux are present.

They act as “scattering centers” for collisionless interactions, frozen into the plasma flux. Because of this they may be approximated as “walls” on which particles are “reflected”

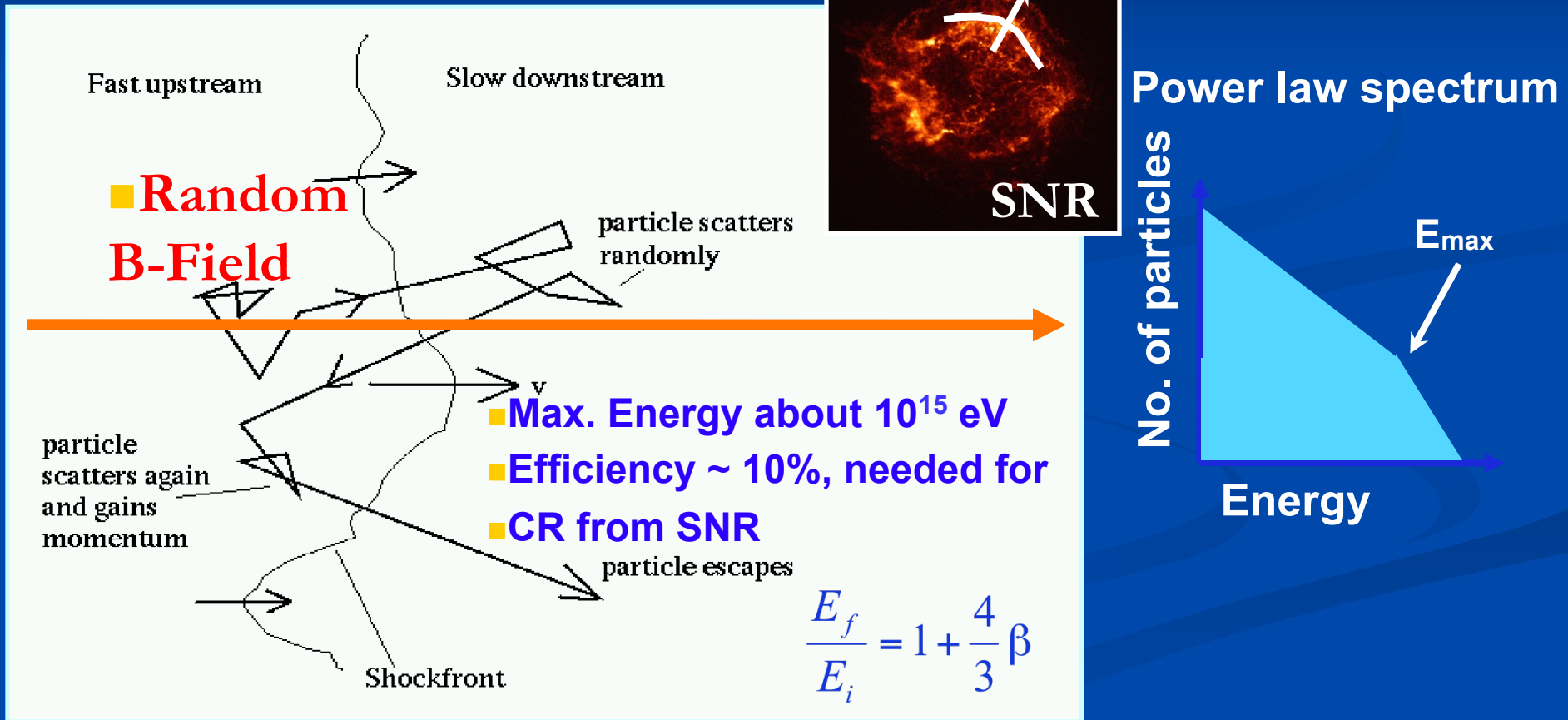
The basic mechanism is the stochastic Fermi mechanism in which scattering occurs at these collisionless centers.



# Shock acceleration mechanism

(by Enrico Fermi)

Particles (electrons and hadrons) get scattered many times in shock front and gain energy in each cycle  
(TeV energies  $\rightarrow$  several 100 years)



# Onda di shock

$v \cos \theta$

*Campi magnetici*

$v_{cl}$

*Scattering elastico*

These processes are dominated by scattering in the irregularities downstream and upstream flux.

$v_{cl}$

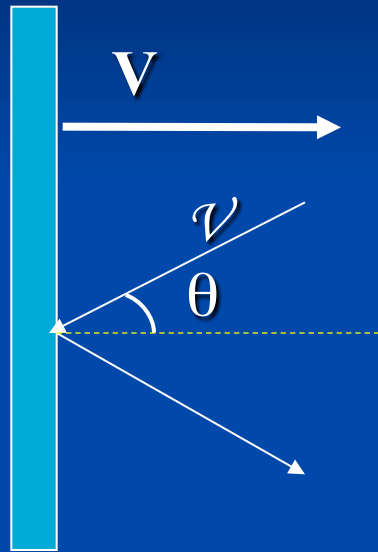
They act as “scattering centers” for collisionless interactions, frozen into the plasma flux. Because of this they may be approximated as “walls” on which particles are “reflected”

At shock waves, acceleration of particles occurs. The ingredients are:

- a physical mechanism to change energy at each collisionless interaction with the shock wave and
  - a statistical process which gives an average gain of energy with an enough high number of encounters with the shock wave front, i.e. suitable interaction time

The shock front does not play any direct role.

## 4.3 Un esercizio: incremento di energia in urto con onda di shock



- *Onda di shock* = perturbazione che si propaga con velocità  $V >$  velocità del suono nel mezzo.
- Assumeremo l'approssimazione di onda piana e con massa  $M \gg$  massa particella
- L'urto è elastico nel SR di quiete di un osservatore sull'onda di shock ( $S'$ ).

- Considereremo il processo nei due SR:

- $S$  = Sistema di riferimento dell'osservatore
- $S'$  = Sistema di riferimento dell'onda di shock

# SR osservatore

# SR onda shock

## ■ Quadrimpulso particella

$$(E, p_x)$$

$$(E', p'_x)$$

$$\begin{aligned} E' &= \gamma(E + Vp_x) \\ p'_x &= \gamma\left(p_x + \frac{V}{c^2}E\right) \end{aligned}$$

## ■ Urto elastico:

$$\begin{aligned} E' &\xrightarrow{\text{urto}} E' \\ p'_x &\xrightarrow{\text{urto}} -p'_x \end{aligned}$$

## ■ Conseguenze dell'urto:

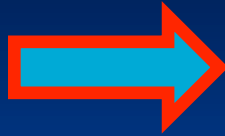
$$E = \gamma(E' - Vp'_x) \xrightarrow{\text{urto}} \gamma(E' - V(-p'_x)) \equiv E^*$$

## ■ dove $E^*$ = energia della particella dopo l'urto:

$$E^* \equiv \gamma(E' - V(-p'_x)) \rightarrow E^* = \gamma \left[ \gamma(E + Vp_x) + V\gamma\left(p_x + \frac{V}{c^2}E\right) \right]$$

■ Ricordando che:

$$\begin{aligned} p_x &= mv\gamma \cos \theta \\ E &= mc^2 \gamma \end{aligned}$$



$$\frac{p_x}{E} = \frac{mv\gamma \cos \theta}{mc^2 \gamma} = \frac{v}{c^2} \cos \theta$$

$$E^* = \gamma \left[ \gamma(E + Vp_x) + V\gamma(p_x + \frac{V}{c^2} E) \right] = \gamma^2 \left[ (E + 2Vp_x + \frac{V^2}{c^2} E) \right] =$$

$$= \gamma^2 E \left[ 1 + 2V \frac{p_x}{E} + \frac{V^2}{c^2} \right] = \gamma^2 E \left[ 1 + 2V \frac{v \cos \theta}{c^2} + \frac{V^2}{c^2} \right] \stackrel{Taylor}{\cong}$$

$$= \left( 1 + \frac{V^2}{c^2} \right) E \left[ 1 + 2V \frac{v \cos \theta}{c^2} + \frac{V^2}{c^2} \right] \stackrel{Taylor}{\cong}$$

$$\stackrel{2^{ordine}}{=} E \left[ 1 + 2 \frac{Vv \cos \theta}{c^2} + 2 \frac{V^2}{c^2} \right]$$

- L' energia guadagnata dalla particella nell' urto con l' onda di shock nel sistema S (Galassia):

$$\Delta E = E^* - E = \left[ 2 \frac{Vv \cos \theta}{c^2} + 2 \frac{V^2}{c^2} \right] \cdot E$$

$$\underset{v \approx c}{\cong} \left( 2 \frac{V \cos \theta}{c} \right) \cdot E$$

$$\frac{\Delta E}{E} = \frac{E^* - E}{E} = \left( 2 \frac{V \cos \theta}{c} \right)$$

- In altri termini, il rapporto tra energia finale e iniziale è  $>1$  nel caso in cui la particella si diriga contro l' onda ( $\cos \theta > 0$ ) :

$$\frac{E^*}{E} = \left( 1 + 2 \frac{V}{c} \cos \theta \right)$$

- Mediando (ossi, integrando) su tutti gli angoli per cui  $\cos \theta > 0$  :

$$\langle \cos \theta \rangle = \frac{\int_0^1 \cos \theta \cdot \cos \theta d\theta}{\int_0^1 \cos \theta d\theta} = \frac{2}{3}$$

$$\left\langle \frac{E^*}{E} \right\rangle = \left( 1 + \frac{4}{3} \frac{V}{c} \right) = \mathbf{B}$$

$$\langle E^* \rangle = \mathbf{B} \langle E \rangle$$

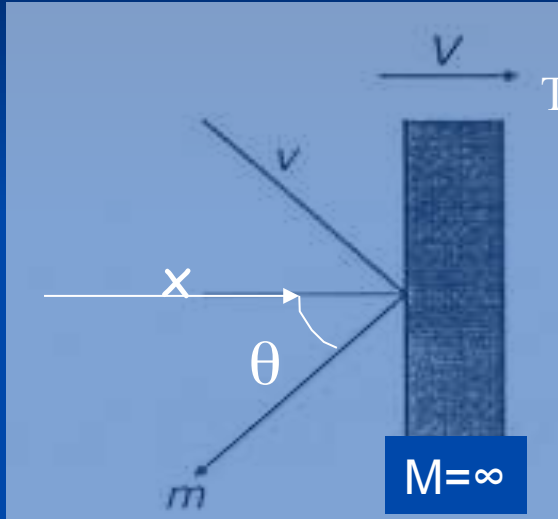
eq. 4.1

# Shock waves: Energy gain

The mechanism for energy gain is always the same

The gain is given by “elastic” collision in the wall rest frame and a relativistic boost to the lab.

- In the ref. frame of the walls,  $E'_i = \gamma(E_i + Vp_{ix})$ ,  $P'_{ix} = \gamma(p_{ix} + VE_i)$  ( $c = 1$ )



The collision is elastic (i.e. collisionless):  $E'_f = E'_i$  and  $p'_f = -p'_i$

Going back to the lab frame

$$E_f = \gamma(E'_f - Vp'_{fx}) = \gamma(E'_i + Vp'_{ix})$$

$$p_{fx} = \gamma(p'_{fx} - VE'_f) = \gamma(-p'_{ix} - VE'_i)$$

$$\text{Therefore } E_f = \gamma(E'_i + Vp'_{ix}) = \gamma^2[(E_i + Vp_{ix}) + V(p_{ix} + VE_i)] = \gamma^2[E_i + 2Vp_{ix} + V^2E_i]$$

$$\text{But } p_{ix} = v_{ix}E_i \rightarrow E_f = \gamma^2[E_i + 2Vv_{ix}E_i + V^2E_i] = \gamma^2E_i[1 + 2Vv_{ix} + V^2]$$

$\rightarrow$  the relative gain per each scattering is (put back  $c$ )

$$\Delta E/E_i = (E_f - E_i)/E_i = \gamma^2[1 + 2Vv_{ix} + V^2] - 1 = \gamma^2[1 + 2Vv \cos \theta / c^2 + V^2/c^2] - 1$$

$$\gamma^2 = [1 - (V/c)^2]^{-1} \approx 1 + (V/c)^2 \text{ if } V \ll c$$

$$\Delta E/E_i \approx [1 + (V/c)^2][1 + 2Vv \cos \theta / c^2 + (V/c)^2] - 1$$

$$= 2Vv \cos \theta / c^2 + (V/c)^2 + (V/c)^2 + (V/c)^2[2Vv \cos \theta / c^2 + (V/c)^2] \approx 2Vv \cos \theta / c^2 + 2(V/c)^2$$

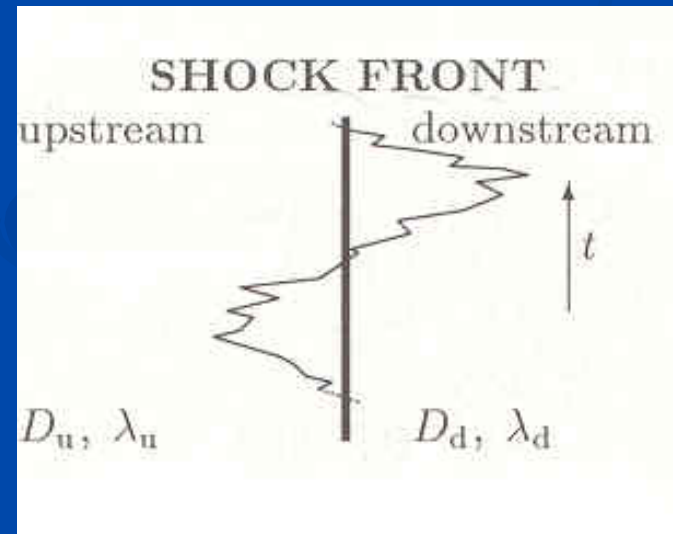
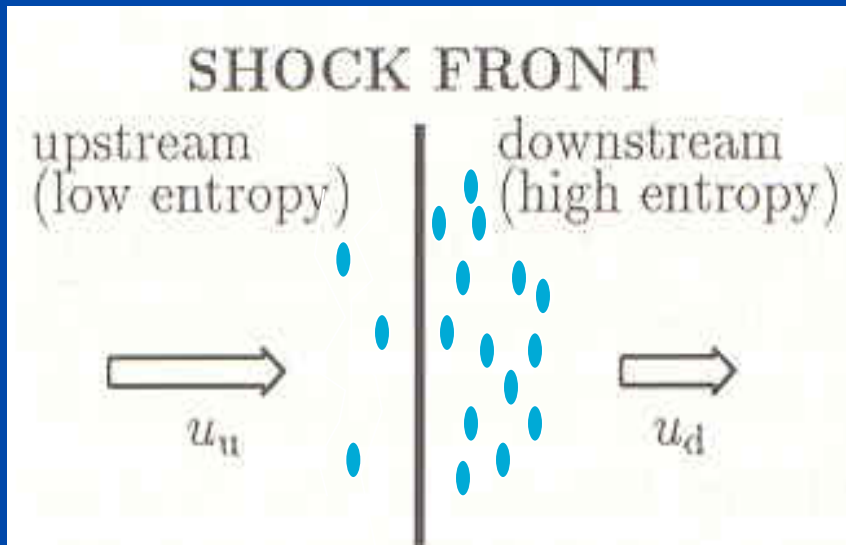
It is the gain for non-relativistic shock waves, i.e.  $V \ll c$



# Shock waves: Energy gain

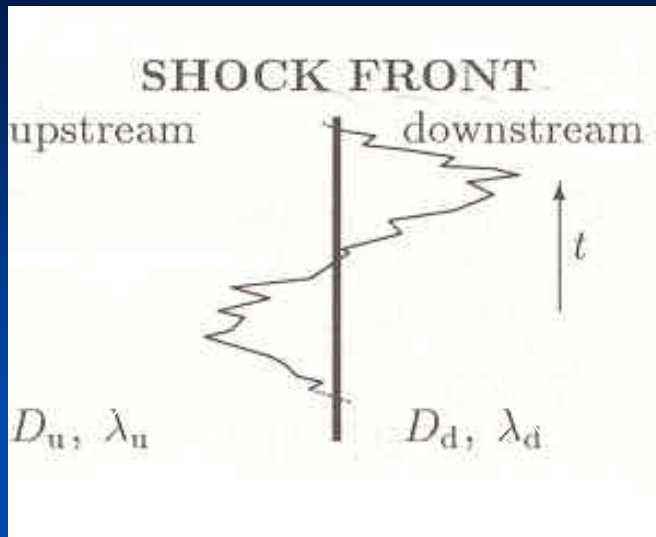
Because of the stochastic nature of the process we cannot follow a single particle, but we can work out the average energy gain.

The basic property is that when a particle enters in the scattering region, because of the random distribution of the scattering centers, also its direction gets very rapidly randomized –key point– so that the average speed of the test particle is the same as for the flow and an isotropic distribution of directions can be assumed in a suitable ref. frame (the shock rest frame).



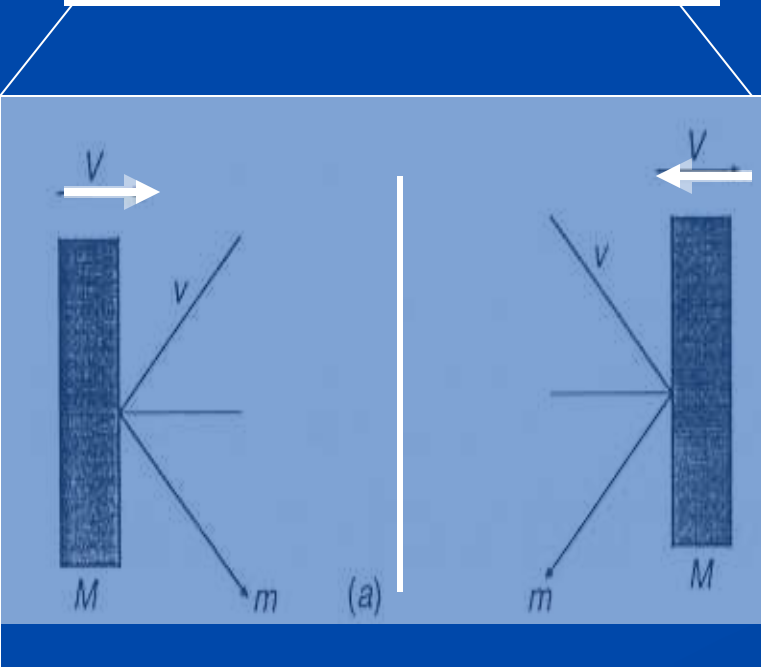
$$\langle \Delta E/E \rangle = \int_{\theta_1}^{\theta_2} P(\Omega) \frac{\Delta E}{E} (\cos \theta) d\Omega$$

# Diffusive Shock Acceleration



DSA is the dominant mech. for quasi-parallel shocks. In DSA particle scattering up- and down-stream is crucial  $\rightarrow$  high level of turbulence and irregularities are requested on both sides of the shock.

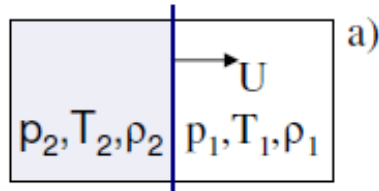
The magnetic fields on both sides are turbulent, so that the resulting scattering is quantified by the diffusion coefficient  $D$  or by the mean free paths  $\lambda$



The crucial point is that in the ref frame in which one of the fluxes is at rest, a particle always sees the other side moving toward it (in a symmetric way for the 2 sides), so that the “collision” is always head-on.

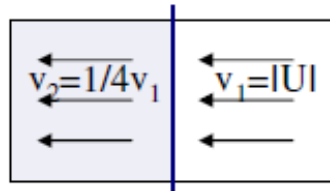
Since the scattering centers are frozen into the plasma, particle scatt. back and forth through the shock front can be understood as repeated reflections between converging scattering centers (i.e. “walls” with infinite mass).

# Infinite plane shock with a magnetic field perpendicular to it



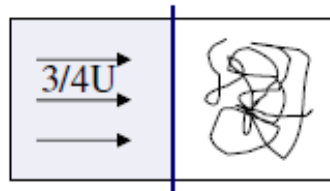
a)

$U \gg c_s$  through stationary interstellar gas  
mass conservation:  $\rho_1 v_1 = \rho_2 v_2$



b)

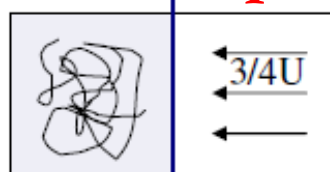
reference frame of the shock front at rest:  
for monoatomic or fully ionised gas  $\gamma=5/3$ ,  $\rho_2/\rho_1=(\gamma+1)/(\gamma-1)$



c)

upstream gas stationary:  
isotropic distribution of velocities  
the gas behind the shock moves with velocity  $V=-v_1+v_2$

**Upstream**

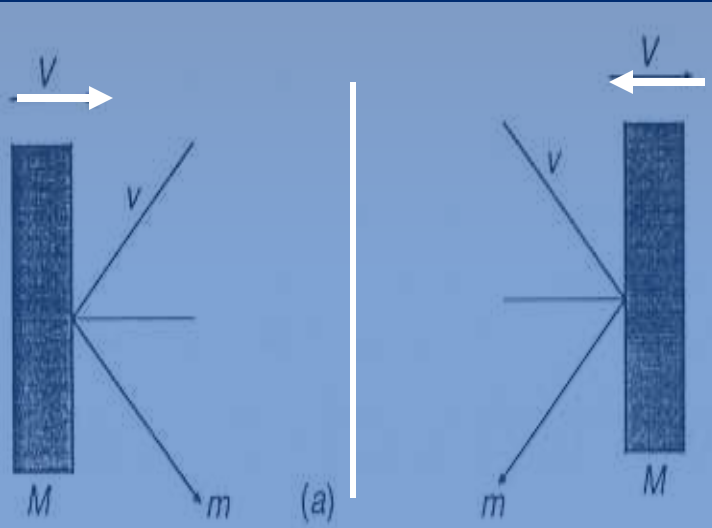


d)

downstream gas stationary:  
isotropic distribution of velocities

**Up -> Down and vice versa : increase in energy,  
increment of same order**

# DSA: Energy gain (1st order Fermi mechanism)



In such a case, in the center of mass of the shock, i.e. the shock rest frame (SRF), the particle distribution is isotropic.

The same happens in the Lab. Ref. Frame because the speed of center of mass vanishes.

Then,  $P(\theta)$  is prop. to the normal comp. of the particle velocity along the normal to the shock,  $v \cos \theta$ :

$$P(\Omega) d\Omega = (\cos \theta / \pi) (2\pi \sin \theta d\theta) = 2 \cos \theta d(\cos \theta) \text{ with } 0 < \theta < \pi/2$$

$$\langle \Delta E / E \rangle \approx \int p(\Omega) d\Omega [2Vv \cos \theta / c^2 + 2(V/c)^2]$$

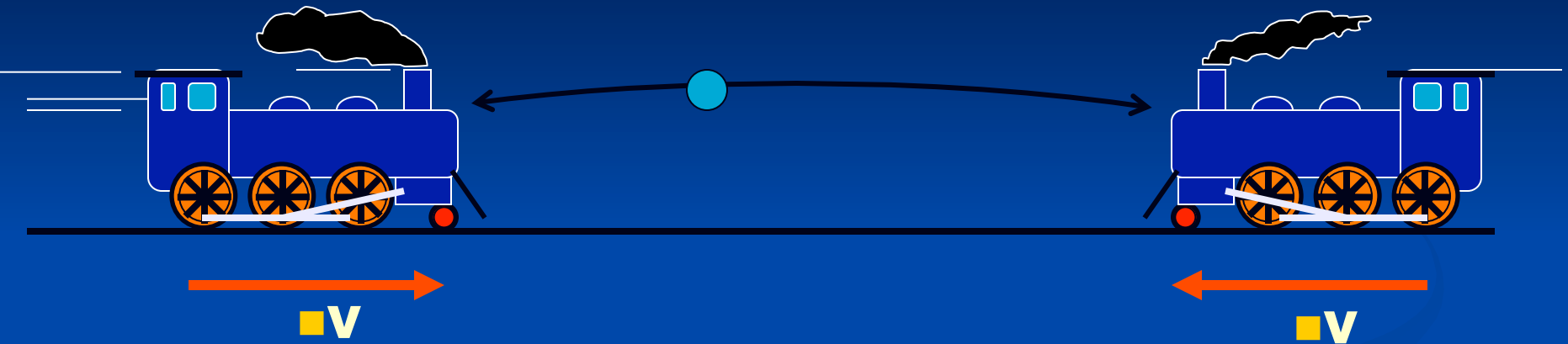
$$\langle \Delta E / E \rangle = \frac{4}{3} \left( \frac{V}{c} \right) \left( \frac{v}{c} \right) \quad V \ll c$$

For  $V \ll c$ ,  $\langle \Delta E / E \rangle$  is of 1st order in  $V/c$  and is maximal for relativistic particles  $v \approx c$

$V$  is NOT the shock speed, but relative speed  $|u_u - u_d|$  of the flow speed upstream and downstream in the SRF (i.e. Standing shocks). In the lab,  $u_u = U$ , the speed of shock front and downstream con  $u_d = (\rho_u / \rho_d) U \rightarrow V = (1 - \rho_u / \rho_d) U$

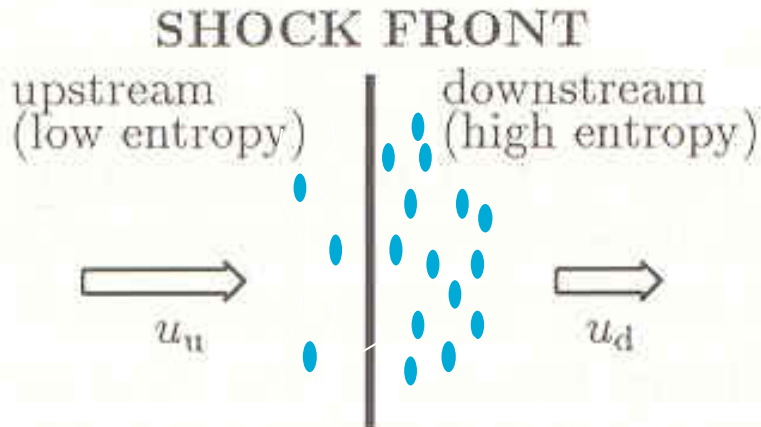
$$\langle \Delta E / E \rangle = \frac{4}{3} \left( \frac{V}{c} \right)$$

- Un modello giocattolo per il meccanismo
- di Fermi



| Urto n. | Velocità<br>nel lab. |
|---------|----------------------|
| 0       | 0                    |
| 1       | $+2v$                |
| 2       | $-6v$                |
| 3       | $+14v$               |
| ....    |                      |

# SA: Energy gain



In such a case, the turbulence downstream is the essential ingredient and the shock crossing is not important.

The scattering process takes place only the turbulence region in that region, where the scattering centers are moving with the flux.

This can be schematized as two reflecting walls moving with same speed  $V$  in lab. Frame, so that both head-on and follow-on collisions occur.

The angular particle distribution is not isotropic in the lab frame but it is in the rest frame of the walls.

In the lab frame, we get

$$p(\theta)d\theta = (v + V\cos\theta)d(\cos\theta)$$

(because of the Doppler effect due to the relativistic motion of the particles), so that

$$\langle \Delta E/E \rangle = 2 \left( \frac{V}{c} \right)^2 \left( 1 + \frac{1}{3} \frac{v}{c} \right)$$

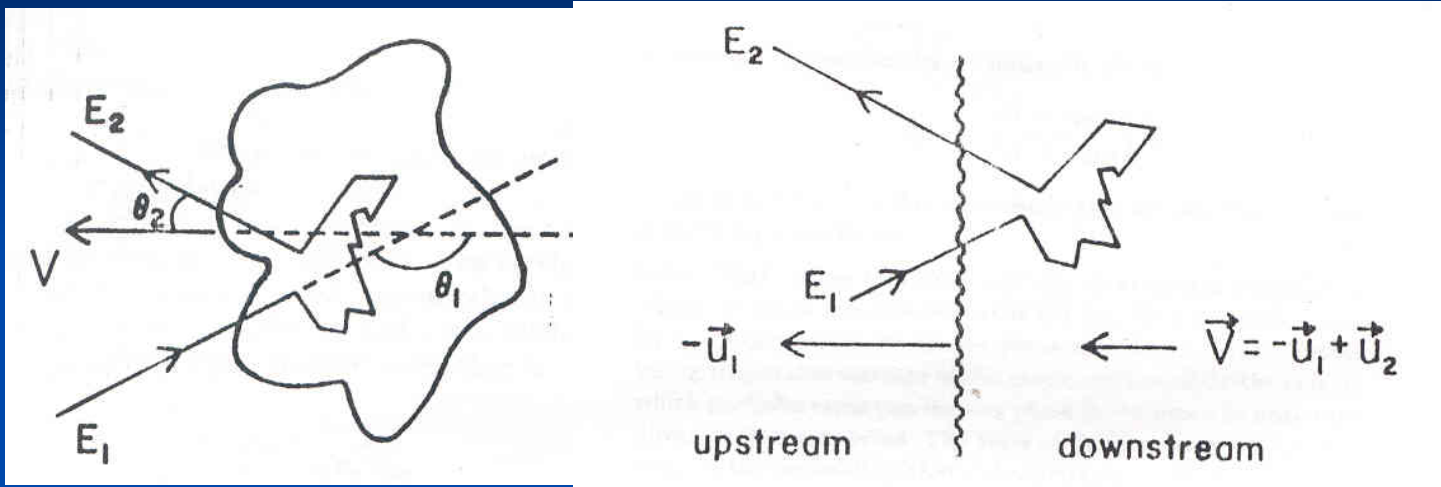
$$\langle \Delta E/E \rangle = \frac{8}{3} \left( \frac{V}{c} \right)^2$$

In such a case  $V = u_d$  in the SFR, so that  $V = (\rho_u/\rho_d)U$

for  $v \approx c$

# SW energy gain: another approach

Let consider another approach: the same physical mechanism is working both at a shock front both in a cloud of magnetized plasma with strong magnetic irregularities acting as collisionless scattering centers (e.g. this can be thought as the turbulent region ahead a shock front or even the whole galactic disk)



Downstream the shocked gas flows to the left with speed  $u_2$  relative to the shock front  $\rightarrow$  in the lab it moves to the left with  $V = -u_1 + u_2$

In the rest frame of the moving cloud or in the shock rest frame, we have

$$E'_1/c = \gamma(E_1/c - \beta_V p_{x1}) = \gamma(E_1/c)(1 - \beta_V \cos \theta_1) \quad \text{Because for ultra-relat. particles } E/c=p$$

The collisions are elastic (because collision-less in the rest frame of the cloud/SW)

$E'_1 = E'_2$  and going back to the labo

$$E_2/c = \gamma(E'_2/c + \beta_V p'_{x2}) = \gamma(E'_2/c)(1 + \beta_V \cos \theta'_2) = \gamma(E'_1/c)(1 + \beta_V \cos \theta'_2)$$

$$= \gamma^2(E_1/c)(1 - \beta_V \cos \theta_1)(1 + \beta_V \cos \theta'_2) = \gamma^2[(E_1/c)(1 + \beta_V \cos \theta'_2 - \beta_V \cos \theta_1 - \beta_V^2 \cos \theta_1 \cos \theta'_2)]$$

therefore 
$$\frac{\Delta E}{E} = \frac{E_2 - E_1}{E_1} = \gamma^2(1 + \beta_V \cos \theta'_2 - \beta_V \cos \theta_1 - \beta_V^2 \cos \theta_1 \cos \theta'_2) - 1$$



# SW energy gain: another approach

$$\frac{\Delta E}{E} = \frac{E_2 - E_1}{E_1} = \gamma^2(1 + \beta_V \cos \theta'_2 - \beta_V \cos \theta_1 - \beta_V^2 \cos \theta_1 \cos \theta'_2) - 1$$

The crucial difference between the two cases comes when we take the angular averages to obtain the average fractional energy gain per encounter.

First, the average over  $\theta'_2$  has to be done

Cloud

In  $S'$  (where the particle is at rest in average), the particle distr. is isotropic because of the random motion inside the cloud, therefore  $dn/d(\cos \theta'_2) = \text{const.}$ ,  $-1 < \cos \theta'_2 < 1$

Then  $\langle \cos \theta'_2 \rangle = 0$  and

$$\Delta E/E_1 = \gamma^2 (1 - \beta_V \cos \theta_1) - 1$$

Shock front

In  $S'$  we have again a isotropic distr. but we have to project on a plane (the shock front), therefore  $dn/d(\cos \theta'_2) = 2 \cos \theta'_2$ ,  $0 < \cos \theta'_2 < 1$

Then  $\langle \cos \theta'_2 \rangle = 2/3$  and

$$\Delta E/E_1 = \gamma^2 [1 + (2/3)\beta_V - \beta_V \cos \theta_1 - (2/3)\beta_V^2 \cos \theta_1] - 1$$

# SW energy gain: another approach

Shock front

In  $S'$  we have again a isotropic distr. but we have to project on a plane (the shock front), therefore

$$dn/d(\cos\theta_2') = 2\cos\theta_2', \quad 0 < \cos\theta_2' < 1$$

Then  $\langle \cos\theta_2' \rangle = 2/3$  and

$$\Delta E/E_1 = \gamma^2 [1 + (2/3)\beta_v - \beta_v \cos\theta_1 - (2/3)\beta_v^2 \cos\theta_1] - 1$$

The prob of the particles which cross the shock arrive in a solid angle  $d\Omega$  around  $\theta$  dir per unit of time is:

$$dn = v \cos\theta d\Omega dt \rightarrow dp(\theta) \propto \cos\theta d(\cos\theta)$$

Normalizing so that prob is 1 for all the particles approaching the shock (ie those with  $0 < \theta < \pi/2$ ) one gets

$$dp(\theta) = 2\cos\theta d(\cos\theta)$$

# SW energy gain: another approach

Next we need to average over  $\cos\theta_1$

## Cloud

The probability of a collision is / to the relative speed between the cloud and the particle

$$dn/d(\cos\theta_1) = (c - V\cos\theta_1)/2c, \quad -1 < \cos\theta_1 < 1$$

Then  $\langle \cos\theta_1 \rangle = -\beta_V/3$  and

$$\begin{aligned} \Delta E/E_1 &= \gamma^2 (1 - \beta_V \cos\theta_1) - 1 = (1 + \beta_V^2/3)/(1 - \beta_V^2) - 1 = \\ &= (4/3)\gamma^2 \beta_V^2 \approx (4/3)\beta_V^2 \text{ if } \beta_V \ll 1 \end{aligned}$$



2nd order

## Shock front

We have again a isotropic distr. but we have to project on a plane (the shock front), therefore

$$dn/d(\cos\theta_1) = 2 \cos\theta_1, \quad -1 < \cos\theta_1 < 0$$

Then  $\langle \cos\theta_1 \rangle = -2/3$  and

$$\begin{aligned} \Delta E/E_1 &= \gamma^2 [1 + (4/3)V - (4/9)V^2] - 1 = \\ &= \gamma^2 [(4/3)\beta + (5/9)\beta^2] \sim (4/3)\beta \text{ if } \beta \ll 1 \end{aligned}$$

1st order



$$v_{\text{rel}} = |\vec{v}_{\text{cloud}} - \vec{v}_{\text{particle}}|$$

$$= \sqrt{(c - v \cos \theta_i)^2 + v^2 \sin^2 \theta_i}$$

$$= c \sqrt{(1 - \beta \cos \theta_i)^2 + \beta^2 \sin^2 \theta_i}$$

$$= c \sqrt{1 + \beta^2 - 2\beta \cos \theta_i}$$

$$\simeq c \sqrt{1 - 2\beta \cos \theta_i}$$

$$\simeq c (1 - \beta \cos \theta_i)$$

$$\beta \ll 1$$

# Basic Phenomenology of Acceleration

- Let us consider strong shock : SNR exploding into a medium.
- HE particles front and behind the shock
- Shock velocity  $\ll$  velocity of particles
- Particles get scattered : vel. Distribution become isotropic on either side of shock

## 4.4 Accelerazione ricorsiva

- Dalla eq. 4.1 abbiamo ottenuto che in ogni urto frontale, la particella guadagna energia:

$$E^f = \mathbf{B} \cdot E_o$$

$\mathbf{B} = \Delta E / E = \text{gain for single crossing}$

- La particella inoltre rimane nella zona di accelerazione con una certa probabilità  $P$

$$P$$

- Dopo  $k$  collisioni:

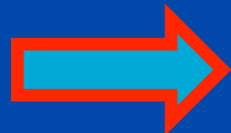
- Energia in possesso della particella

$$E = E_o \mathbf{B}^k$$

- Numero di particelle con energia  $E$

$$N = N_o P^k$$

$$\begin{aligned} \ln(E / E_o) &= k \ln B \\ \ln(N / N_o) &= k \ln P \end{aligned}$$



$$\frac{\ln(N / N_o)}{\ln(E / E_o)} = \frac{\ln P}{\ln B} \equiv \alpha$$

eq. 4.2

$$\frac{N}{N_o} = \left( \frac{E}{E_o} \right)^\alpha$$

eq. 4.3

- La formula trovata si riferisce al numero  $N$  di particelle con energia  $>E$ , ossia  $N=N(>E)$  è la funzione integrale di:

$$\frac{dN(E)}{dE} \propto E^{\alpha-1}$$

eq. 4.4

- La 4.4 rappresenta la distribuzione differenziale del numero di particelle in un certo intervallo di energia.
- La 4.4 ha la forma di uno spettro di potenza, con  $\gamma=\alpha-1$ .
- Questo è quanto cercavamo per lo spettro (osservato) dei RC. Il problema è ora determinare il valore di  $\gamma$ . Dalla 4.2:

$$\gamma = \alpha - 1 = \frac{\ln P}{\ln B} - 1$$

eq. 4.5

- Quindi, occorre determinare il valore del rapporto tra  $\ln P/\ln B$

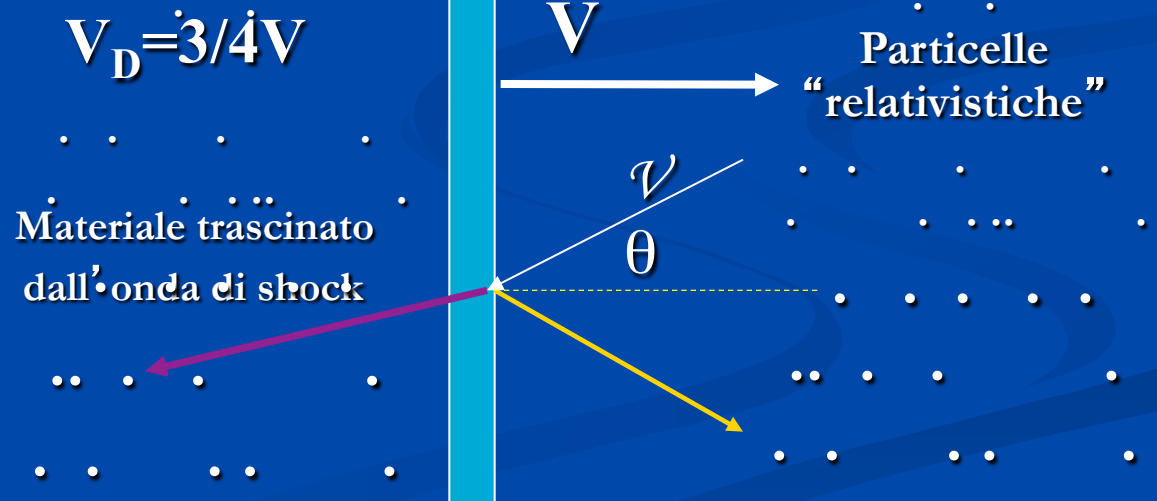


## 4.5 Stima del coefficiente $\alpha = \ln P / \ln B$

- *NOTA: la determinazione di  $\alpha$  come presentata in questa sezione è over-semplificata e in qualche modo non del tutto soddisfacente. Per una trattazione completa: vedere Longair*

### Onda di shock= Pistone

Il materiale (plasma ionizzato) si comporta come un gas monoatomico, con  $\gamma = c_p / c_v = 5/3$ . Dalla teoria degli shocks, la velocità con cui il materiale viene trascinato è  $v_D = 3/4V$



*Particella che passa downstream. Non sarà soggetta di nuovo ad accelerazione*

*Particella che rimbalza di nuovo upstream. Sarà soggetta di nuovo ad accelerazione*

- Per determinare  $\alpha \rightarrow$  conservazione del flusso di particelle.
- Flusso di particelle relativistiche VERSO il fronte d'onda:

$$F[s^{-1}] = \rho[cm^{-3}] \cdot c[cm/s] \cdot A[cm^2]$$

- Le particelle nella regione *downstream* non vengono di nuovo accelerate. Il flusso di queste particelle verso sinistra è:

$$F' = \rho \cdot v_D \cdot A = \rho \cdot \frac{3}{4} V \cdot A$$

- La probabilità che il RC oltrepassi il fronte d'onda e venga persa (ossia NON venga riaccelerato):

$$\bar{P} = \frac{F'}{F} = \frac{\rho \cdot (3/4) \cdot V \cdot A}{\rho \cdot c \cdot A} = \frac{3}{4} \frac{V}{c}$$

- Quindi, la probabilità che il RC rimanga nella regione di accelerazione:

$$P = 1 - \bar{P} = 1 - \frac{3}{4} \frac{V}{c}$$

eq. 4.6

# Il valore stimato di $\alpha$

■  $\gamma(\alpha)$  definito dalla eq. 4.5:

$$\gamma = \alpha - 1 = \frac{\ln P}{\ln B} - 1$$

■ L'equazione 4.6

$$P = 1 - \bar{P} = 1 - \frac{3}{4} \frac{V}{c}$$

■ B dalla eq. 4.1

$$\left\langle \frac{E^*}{E} \right\rangle = \left( 1 + \frac{4}{3} \frac{V}{c} \right) = B$$

■ Quindi, se  $(V/c) \ll 1$ :

$$\alpha = \frac{\ln P}{\ln B} = \frac{\ln \left( 1 - \frac{4}{3} \frac{V}{c} \right)}{\ln \left( 1 + \frac{4}{3} \frac{V}{c} \right)} \stackrel{Taylor}{\cong} \frac{-\frac{4}{3} \frac{V}{c}}{\frac{4}{3} \frac{V}{c}} = -1$$

$$\gamma = \alpha - 1 = -2$$

eq. 4.7

# Spettro energetico alle sorgenti

- Il modello di Fermi predice quindi uno spettro energetico delle particelle in prossimità delle sorgenti (eq. 4.4) del tipo:

$$\frac{dN(E)}{dE} \propto E^{\alpha-1} = E^{-2}$$

- Si tratta di una predizione che si accorda coi dati sperimentali. La pagina seguente riporta una slide già vista: 
- Occorre ora mostrare che:
  - La velocità dell'onda di shock NON è relativistica
  - Come le particelle vengono fatte “rimbalzare” verso l'onda di shock
  - La massima energia cui si può giungere con questo modello

## 3.9 Spettro dei RC alle sorgenti

- Il risultato appena ottenuto è estremamente importante, perché permette di fare previsioni sullo spettro energetico dei RC alle sorgenti.
- Poiché il flusso di CR alla Terra è stazionario, vi deve essere equilibrio tra produzione e perdita.

**?=2 dal  
modello di  
Fermi!**

- Spettro energetico misurato:
- Spettro energetico alle Sorgenti:
- Probabilità di diffusione:

$$\Phi(E) \propto E^{-2.7} (\text{erg} / \text{cm}^3 \cdot \text{GeV})$$

$$Q(E) \propto E^{-?} (\text{erg} / \text{s} \cdot \text{GeV})$$

$$\tau(E) \propto E^{-0.6} (\text{s})$$

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{\text{Volume}} dE$$

## 4.6 Parametri caratteristici di un'onda di shock da Supernova

- Osservazioni di Supernovae (da altre Galassie):  
 $\tau = 1 \text{ SN} / 30 \text{ anni}$
- Energia emessa sotto forma di energia cinetica:  
 $K = 10^{51} \text{ erg}$
- Massa caratteristica delle Supernovae:  
 $M = 10 M_s (= 10 \times 2 \times 10^{33} \text{ g})$
- “Potenza” alimentata dalle esplosioni di SN:  
 $W = K / \tau = 10^{51} / 30 (3 \times 10^7 \text{ s}) = 10^{42} \text{ erg/s}$
- Velocità di propagazione dell'onda di shock:

$$V \cong \sqrt{\frac{2K}{M}} = \sqrt{\frac{2 \cdot 10^{51} \text{ erg}}{10 \cdot (2 \cdot 10^{33} \text{ g})}} \cong 3 \cdot 10^8 \text{ cm/s}$$



$$\frac{V}{c} \cong 10^{-2}$$

eq. 4.8



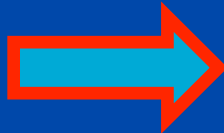
# Dimensioni lineari e durata caratteristica dell'onda di shock

- Per la stima seguente, assumeremo che l'onda di shock perde la sua “spinta propulsiva” quando, espandendosi sino ad un raggio  $R_{OS}$  la sua densità  $\rho_{SN}$  uguaglia quella del mezzo interstellare  $\rho_{IG} \sim 1 \text{p/cm}^3$ .

$$\rho_{SN} = \frac{10M_s}{Vol} = \frac{10M_s}{4/3\pi R_{OS}^3}$$

$$\rho_{IG} = \frac{1p}{cm^3} = 1.6 \times 10^{-24} \text{ g/cm}^3$$

$$\rho_{SN} = \rho_{IG}$$



$$R_{OS} = \left( \frac{3 \cdot 10M_s}{4\pi\rho_{ig}} \right)^{1/3} = \left( \frac{3 \cdot 10 \cdot 2 \times 10^{33}}{4\pi \cdot 1.6 \times 10^{-24}} \right)^{1/3} = 1.4 \times 10^{19} \text{ cm}$$

■NB: questa e' una sottostima!

$$R_{OS} = 5 \text{ pc}$$

eq. 4.9

- Questo valore corrisponde alle dimensioni lineari (raggio) in cui l'onda di shock riesce ad accelerare particelle.



- La durata caratteristica del processo di accelerazione:

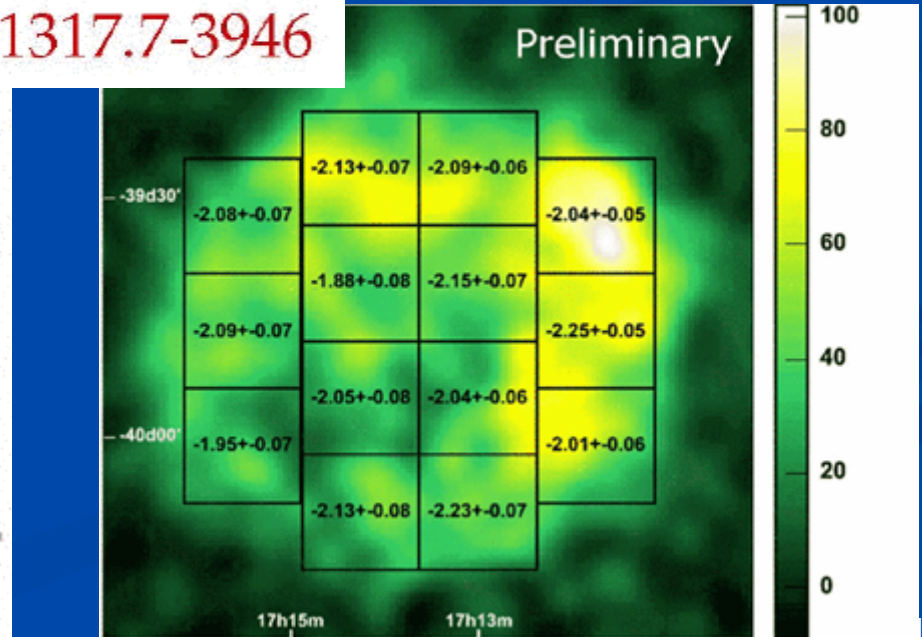
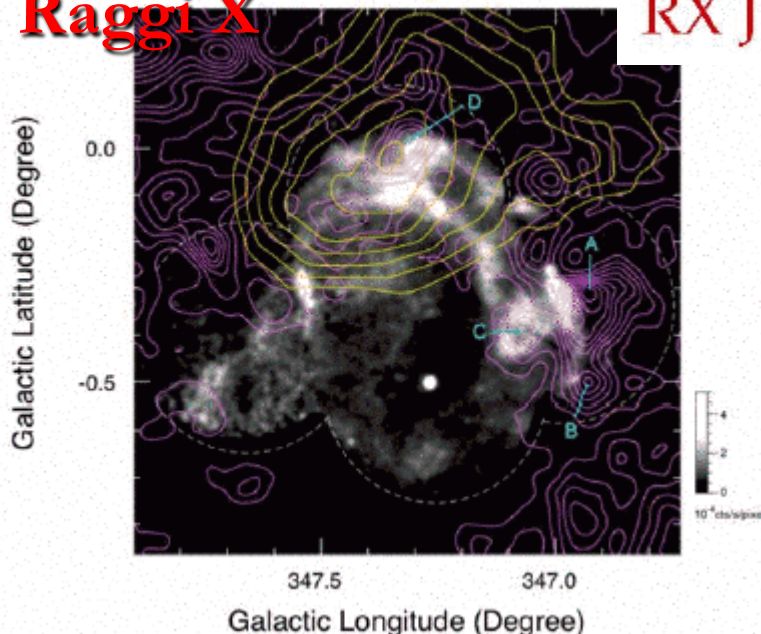
$$T_{OS} = \frac{R_{OS}}{V} = \frac{1.4 \times 10^{19} \text{ cm}}{5 \times 10^8 \text{ cm/s}} = 3 \times 10^{10} \text{ s} = 1000 \text{ y}$$

eq. 4.10

- Utilizzando  $R_{OS}$  e  $T_{OS}$ , è possibile stimare la *energia massima* a cui le particelle (RC) possono essere iniettate nella Galassia
- **Esercizio:** *RXJ1317.7-3946 è probabilmente il “remnant” della SN esplosa nel 393 d.c. Sapreste verificare  $R_{OS}$  e  $T_{OS}$ ?*

**Raggi X**

**RX J1317.7-3946**



**Raggi γ**

# Massima energia per i RC da SN

- Incremento di energia in un singolo urto (eq.4.1):

$$\langle E \rangle = \mathbf{B} \langle E_o \rangle = \left( 1 + \frac{4}{3} \frac{V}{c} \right) \langle E_o \rangle$$

$$\Delta E = \langle E \rangle - \langle E_o \rangle = \frac{4}{3} \frac{V}{c} \langle E_o \rangle = \eta \langle E_o \rangle; \quad \eta \approx 10^{-2}$$

- Tempo che intercorre tra due urti successivi:  $T_{\text{ciclo}}$ ;
- Numero massimo di urti possibili:  $N_{\text{cicli}} = T_{\text{OS}} / T_{\text{ciclo}}$ ;
- La massima energia raggiungibile è dunque:

$$E_{\text{max}} = N_{\text{cicli}} \Delta E = \frac{\eta E_o \cdot T_{\text{OS}}}{T_{\text{ciclo}}}$$

eq. 4.11

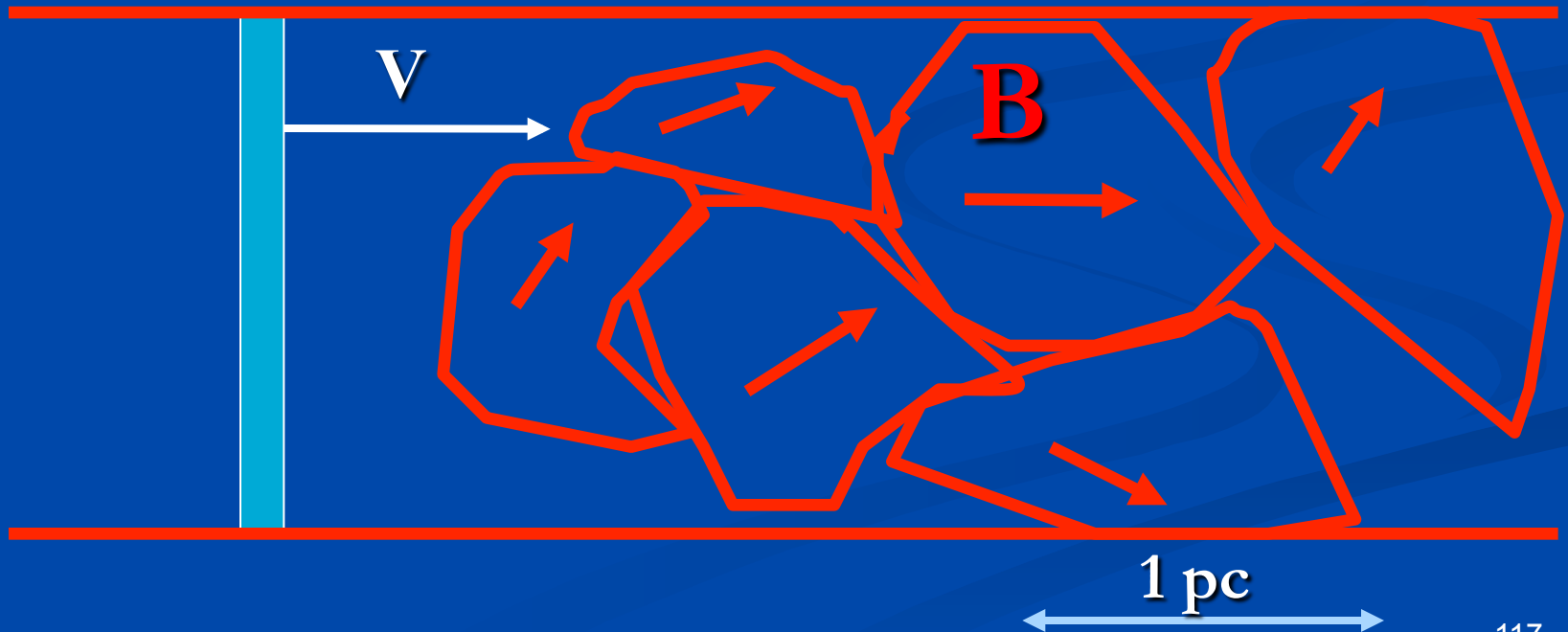
- Occorre dunque stimare il parametro  $T_{\text{ciclo}}$ ;

# Stima di $T_{\text{ciclo}}$

$$T_{\text{ciclo}} = \frac{\lambda_c}{V}$$

$\lambda_c$  = Lunghezza caratteristica della particella  
confinata = raggio di Larmor nel campo  
magnetico Galattico

$$\lambda_c \approx r_L = \frac{E}{ZeB}$$



■ Se assumiamo:

$$\lambda_c \approx r_L = \frac{E}{ZeB}$$

■ Allora:

$$T_{ciclo} = \frac{\lambda_c}{V} = \frac{E}{ZeBV}$$

■ Possiamo determinare la massima energia (eq. 4.11):

$$E_{\max} = N_{cicli} \Delta E = \frac{\eta E \cdot T_{OS}}{T_{ciclo}}$$

$$E_{\max} = \frac{\eta E \cdot T_{OS}}{T_{ciclo}} = \cancel{\eta E} \left( \frac{ZeBV}{\cancel{E}} \right) \cdot T_{OS}$$

$$\eta = \frac{4}{3} \frac{V}{c}$$

$$E_{\max} = \frac{4}{3} \frac{ZeB}{c} V^2 \cdot T_{OS}$$

$$B = 3 \times 10^{-6} G$$

$$V = 5 \times 10^8 cm / s$$

$$T_{OS} = 10^3 y = 3 \times 10^{10} s$$

$$E_{\max} = \frac{4}{3} \frac{ZeB}{c} V^2 \cdot T_{OS} = 480 \cdot Z \quad erg = 3 \times 10^{14} Z \quad eV$$

$$E_{\max} = 300 \times Z \quad TeV$$

eq. 4.12

# Hillas plot

Irrespective of the precise acceleration mechanism, there is a simple dimensional argument, given by Hillas, which allows one to restrict attention to only a few classes of astrophysical objects as possible sources capable of accelerating particles to a given energy.



# Derivation of Hillas relation

The maximum energy supplied to a particle with charge  $Ze$  is

$$E_{\max} = \int Ze \mathcal{E} dx$$

If the energy is given by induction

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$



$$\frac{\mathcal{E}}{L} \cong \frac{B \omega_0}{c}$$

$\mathcal{E}$ =electrical field induced in a region of length  $L$

We obtain

$$E_{\max} \cong Ze \frac{BL \omega_0}{c} L = Ze \beta_0 BL$$

$$E_{\max} (EeV) \cong Z \beta_0 B(\mu G) L(kpc)$$

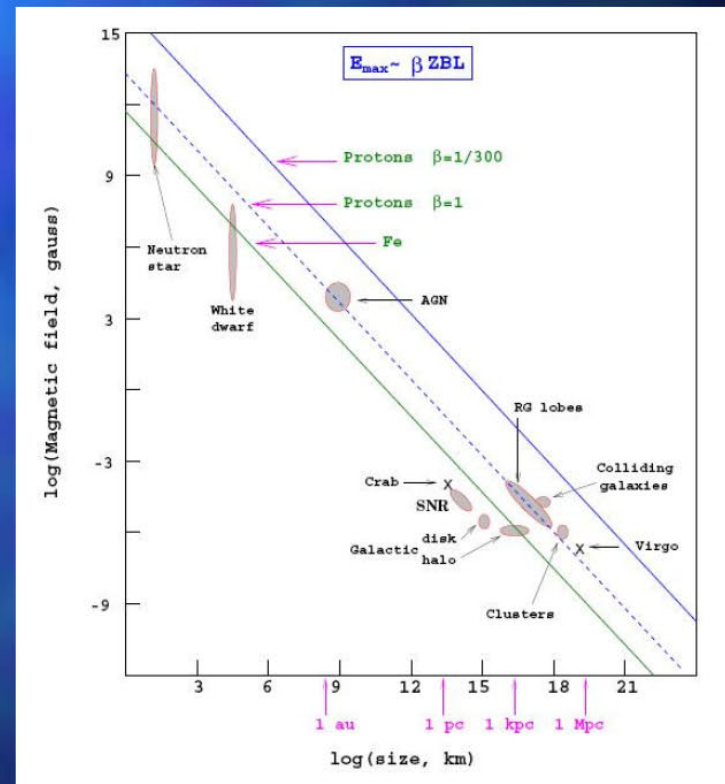
# Hillas plot

Irrespective of the precise acceleration mechanism, there is a simple dimensional argument, given by Hillas, which allows one to restrict attention to only a few classes of astrophysical objects as possible sources capable of accelerating particles to a given energy.

In any statistical acceleration mechanism, there must be a magnetic field,  $B$ , to keep the particles confined within the acceleration site. The size  $R$  of the acceleration region must be larger than the diameter of the orbit of the particle. One gets the general condition

$$E(EeV) < Z \beta B(\mu G) R(kpc)$$

The above condition also applies to direct acceleration scenarios in which the electric field arises due to a moving magnetic field.

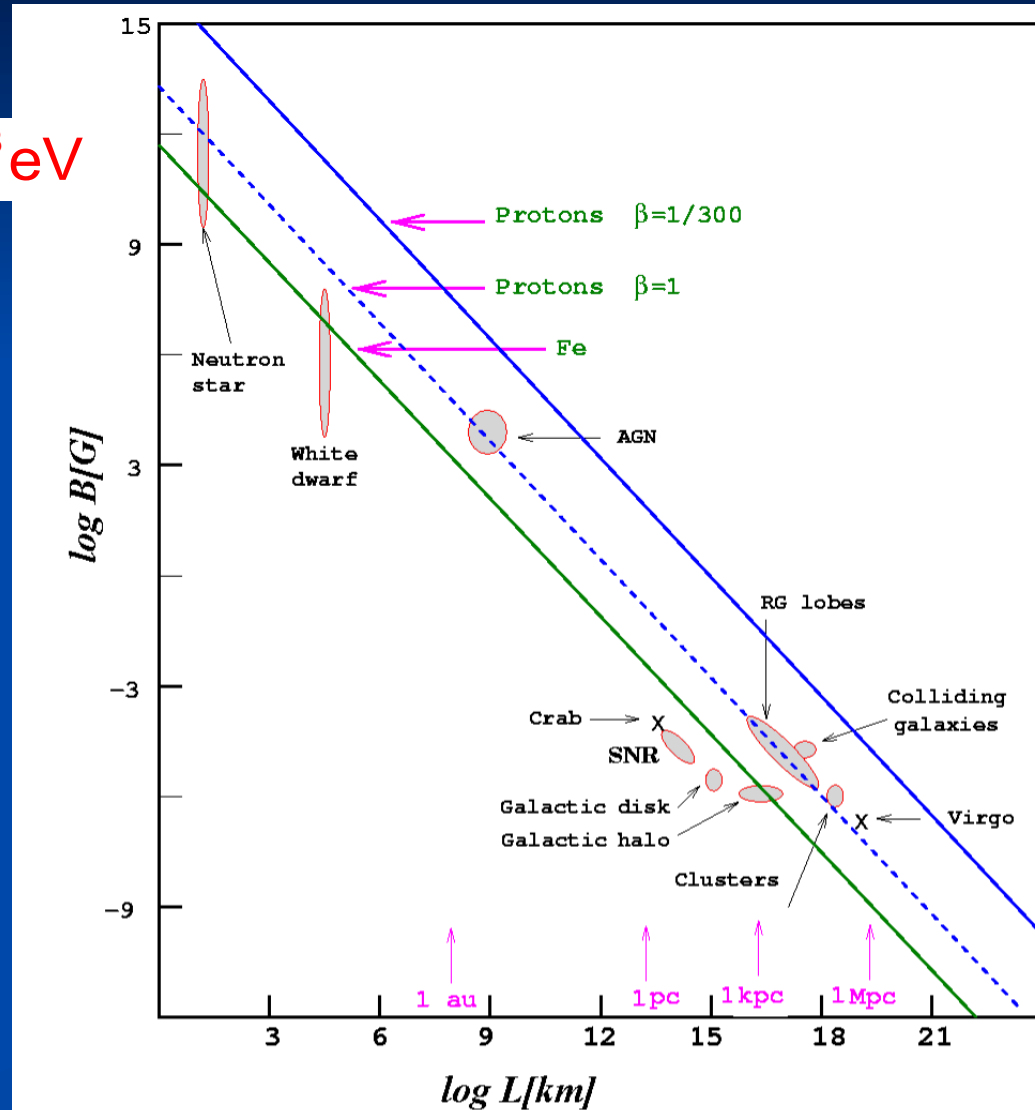




# “Hillas Plot”

$$E_{\text{max}} \approx \beta_{\text{shock}} Z \cdot B[\mu\text{G}] \cdot L[\text{kpc}] \cdot 10^{18} \text{ eV}$$

- Fissata la massima energia (in fig.  $10^{20}$  eV), i meccanismi astrofisici candidati devono avere campi magnetici intensi o grandi regioni di accelerazione.



# Modelli top-down

- Durante e dopo l'inflazione,  $\exists$  particelle con  $m \sim m_{\text{inflatone}}, 10^{13}-10^{14} \text{ GeV}$
- Possono essere rimaste intrappolate in difetti topologici, o impossibilitate a decadere per l'esistenza di qualche simmetria discreta

- Simmetrie discrete  $\rightarrow X$  accumulate nell'alone galattico (materia oscura?)  $\rightarrow$  no GZK, anisotropia galattica (più flusso da centro galattico)

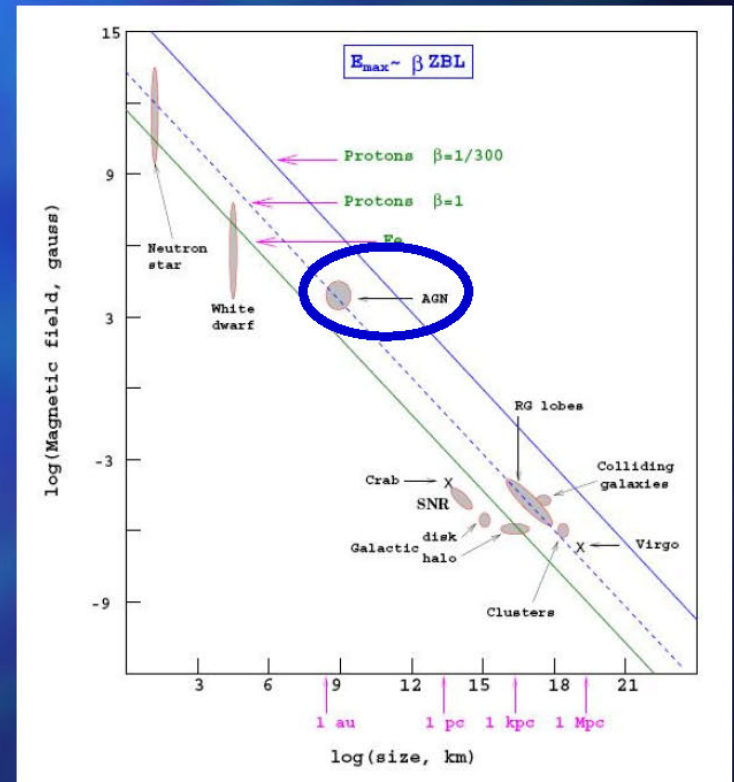
- Intrappolamento  $\rightarrow$  distanze cosmologiche  $\rightarrow$  problema GZK e...
  - Decadimento  $X \rightarrow qq \rightarrow \text{jets}$

Il processo di adronizzazione produce una piccola percentuale di nucleoni, il resto pioni ( $\rightarrow \nu, \gamma, e$ )

Considerando la propagazione, fotoni dovrebbero dominare a energie superiori a  $10^{20} \text{ eV}$

# AGNs

An active galactic nucleus (AGN) is a compact region at the centre of a galaxy which has a much higher than normal luminosity over some or all of the electromagnetic spectrum (radio, infrared, optical, ultra-violet, X-ray and/or  $\gamma$ -ray). A galaxy hosting an AGN is called an active galaxy.





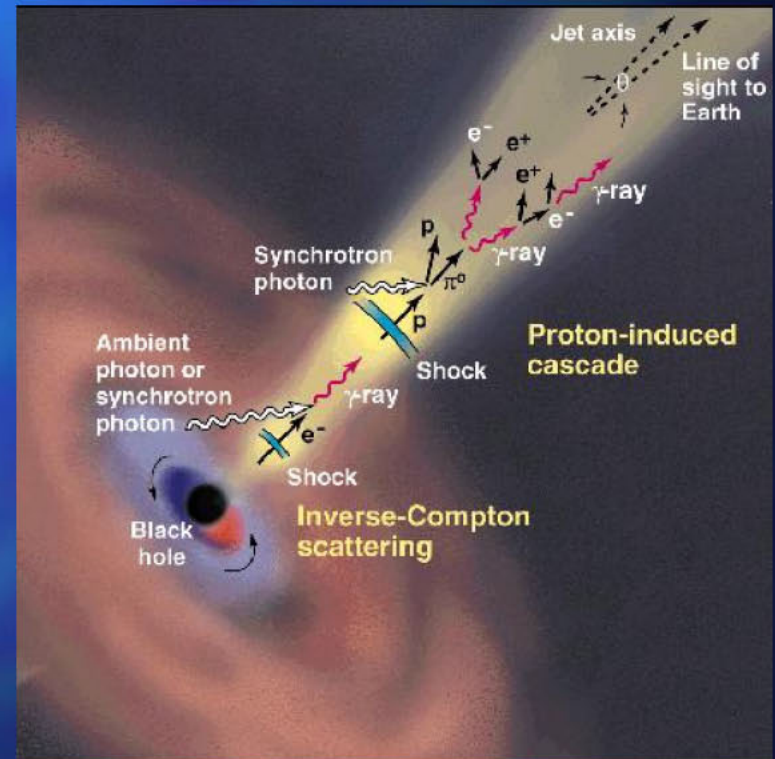
# AGNs

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The radiation from AGN is believed to be a result of accretion onto the supermassive black hole at the centre of the host galaxy.

$\gamma$ -rays above 10 TeV have been observed from a certain class of AGN, BL Lacertae. These can be produced by the decay of pions from the interaction of accelerated protons.

On the other end, the fast variability of flares observed in AGN may favor the acceleration of electrons. Moreover, accelerated protons are severely degraded due to the interactions with the central engine.

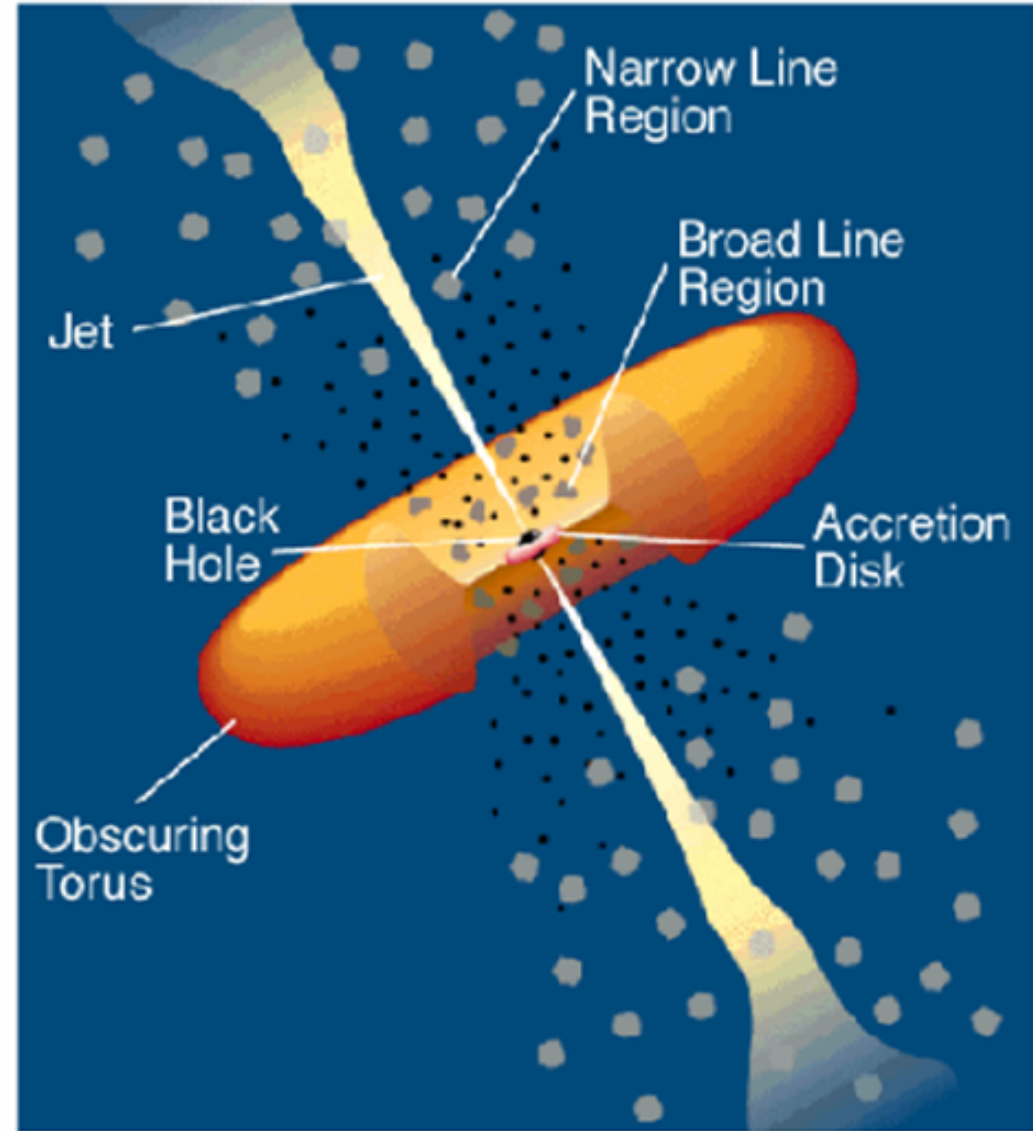


# AGNs

- Galassie attive:
  - Buco nero al centro (?)
  - Disco di accrescimento del buco nero
  - Getti ultrarelativistici

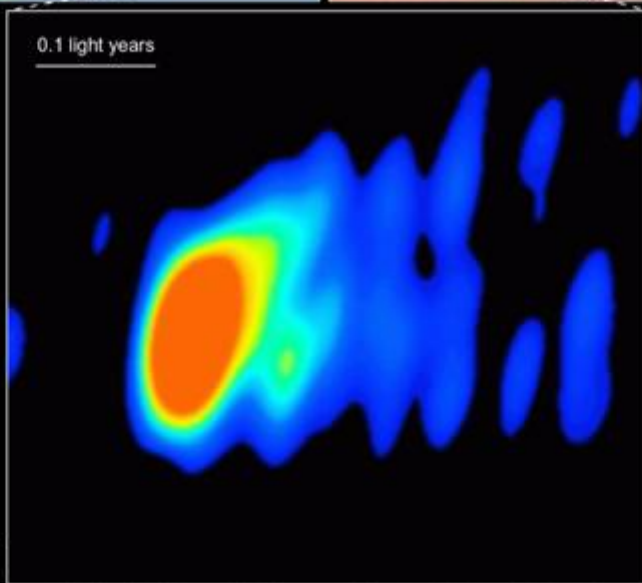
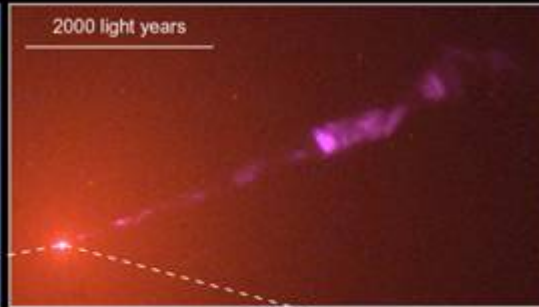
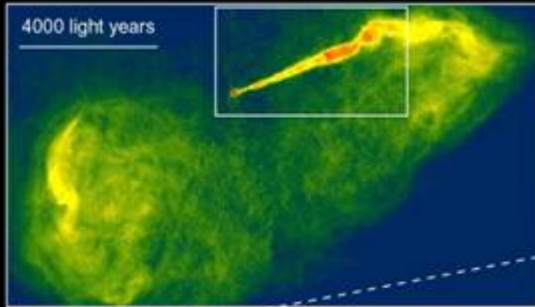


Accelerazione da shock  
ultrarelativistico

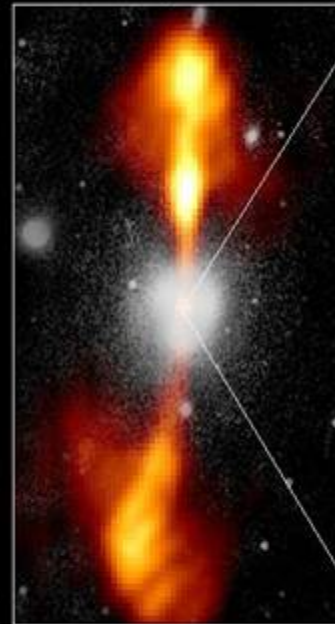


# AGNs

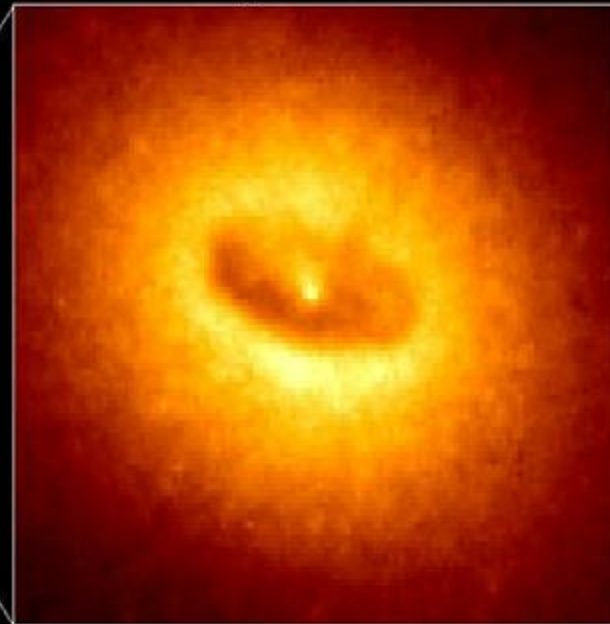
Galaxy M87



Ground-Based Optical/Radio Image



HST Image of a Gas and Dust Disk

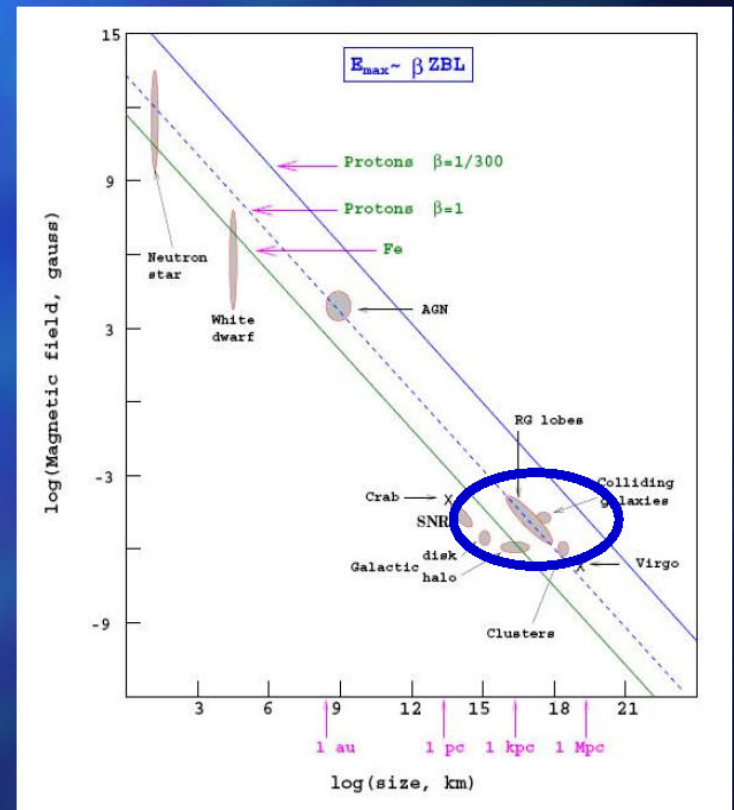


NASA, NRAO and J. Biretta (STScI) • STScI-PRC99-43



# Radio-galaxies (hot-spots)

Radio galaxies and their relatives, radio-loud quasars and blazars, are types of active galaxy that are very luminous at radio wavelengths. The radio emission is due to the synchrotron process.





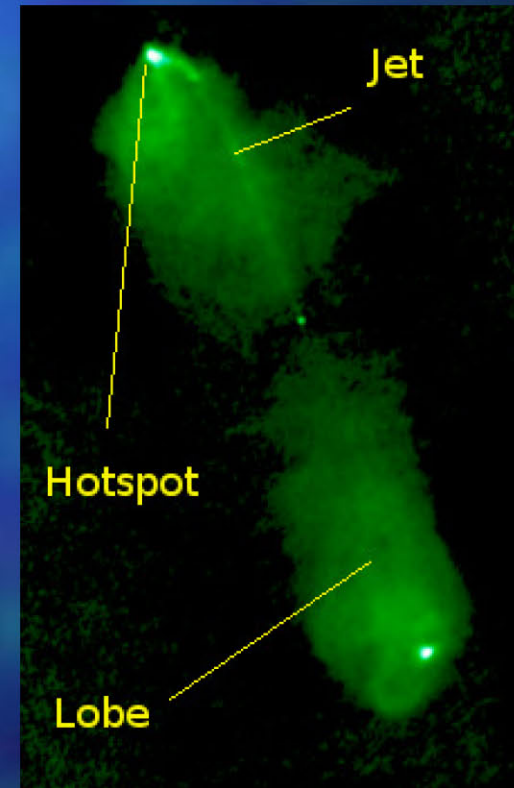
# Radio-galaxies (hot-spots)

Radio galaxies and their relatives, radio-loud quasars and blazars, are types of active galaxy that are very luminous at radio wavelengths. The radio emission is due to the synchrotron process.

The observed structure in radio emission is determined by the interaction between twin jets and the external medium, modified by the effects of relativistic beaming.

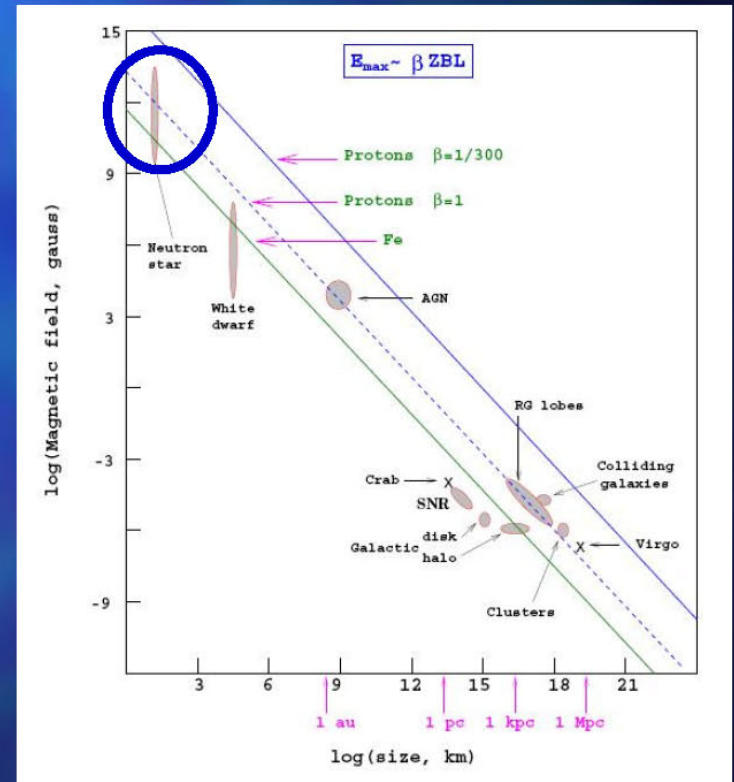
The maximum energy of even up to  $\sim 10^{21}$  eV seems to be possible.

The main problem with radio-galaxies is their location: large cosmological distances ( $> 100$  Mpc) from the earth.



# Pulsars

Pulsars are highly magnetized rotating neutron stars which emit a beam of detectable electromagnetic radiation in the form of radio waves. Their observed periods range from 1.5 ms to 8.5 s. The radiation can only be observed when the beam of emission is pointing towards the Earth.





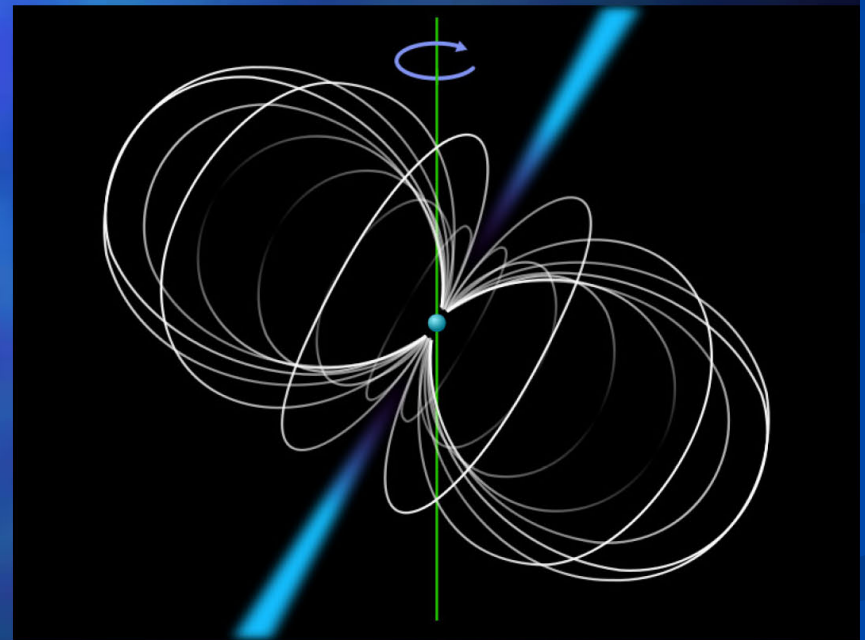
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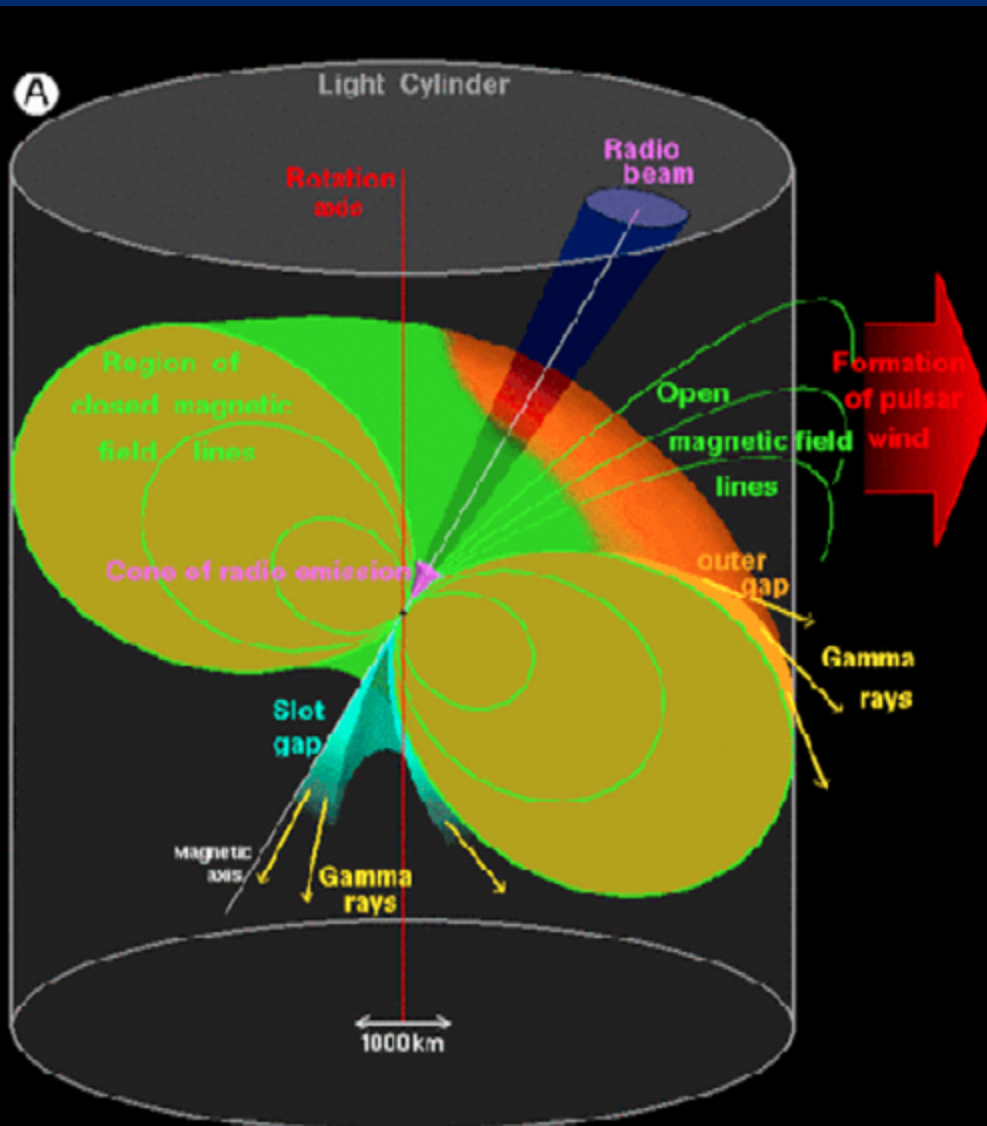
Most of the acceleration scenarios involving pulsars rely upon direct acceleration of particles in the strong electrostatic potential drop at the surface of the neutron star.

The maximum potential drop for typical pulsars can in principle be as high as  $\sim 10^{21}$  eV.

However, in realistic models, this potential drop would be significantly short-circuited by electrons and positrons moving in the opposite directions along the field lines.



# Stelle di neutroni



- Stelle di neutroni con intensi campi magnetici superficiali rotanti

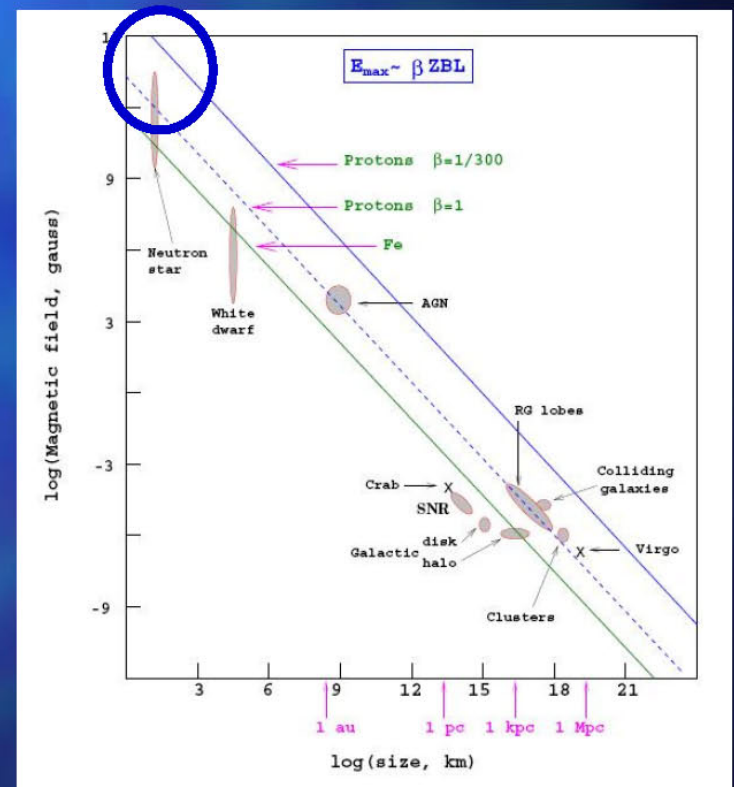
- Ad una certa distanza  $B$  non è più chiuso perchè non può essere in rotazione con  $v > c$



- Formazione di vento relativistico ( $\Gamma \gg 10^7 - 10^{10}$ ), e accelerazione forse fino a  $10^{20}$  eV

# Magnetars

A magnetar is a neutron star with an extremely powerful magnetic field (about 100 to 1000 times stronger than the Crab Nebula pulsar), the decay of which powers the emission of copious amounts of high-energy electromagnetic radiation, particularly X-rays and gamma-rays.



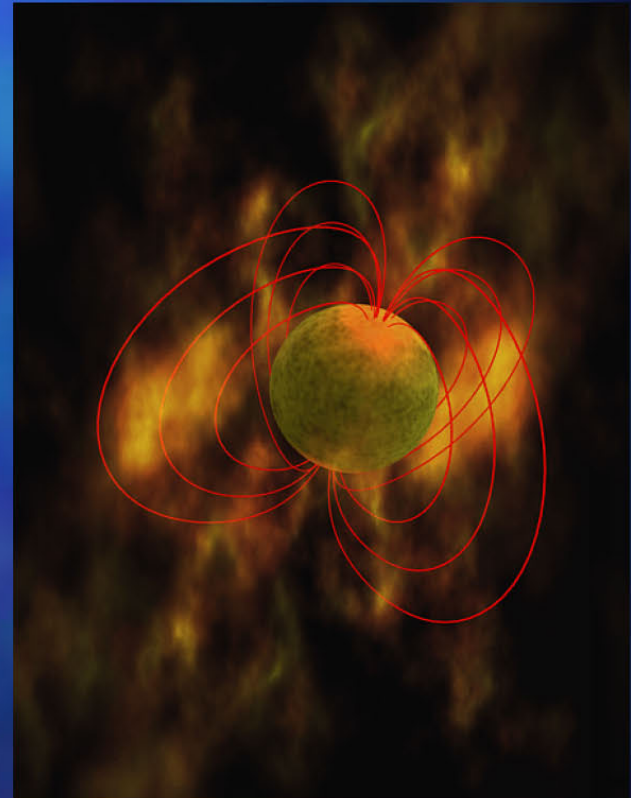


# Magnetars

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There are indications of the existence of pulsars with dipole magnetic fields approaching  $\sim 10^{15}$  G.

It is not clear if the energy loss processes, a generic problem with acceleration around compact object, can be gotten around.



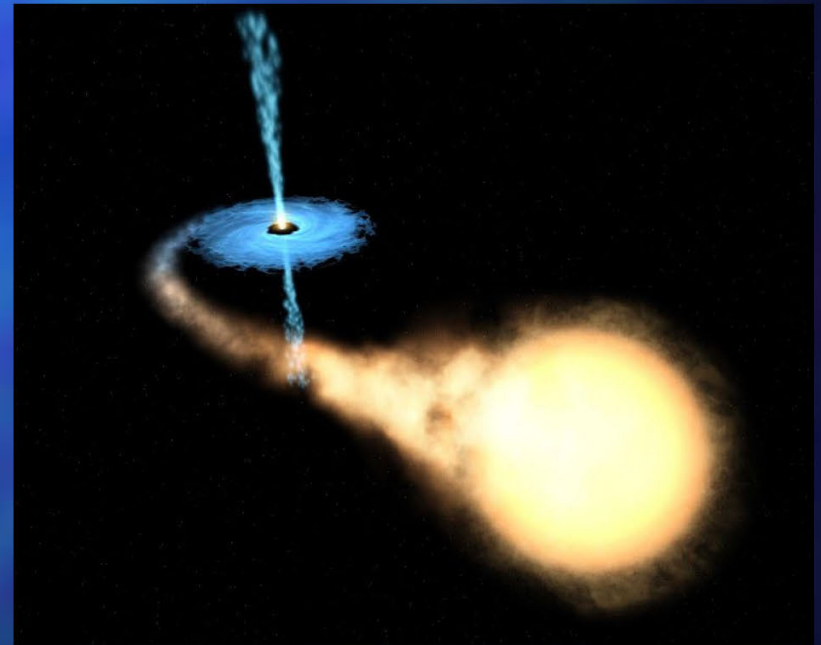


# Accretion disks

An accretion disk is a structure formed by diffuse material in orbital motion around a central body which, in our case, can be a neutron star or a black hole. Material in the disk spirals inward towards the central body. Gravitational energy released in that process is transformed into heat and emitted at the disk surface in form of electromagnetic radiation.

The acceleration is produced by large electric fields produced by induction in the disk.

However, energy loss through interaction of the accelerated particles with the ambient photon field prevents the maximum achievable energy from existing  $\sim 10^{15}$  eV.

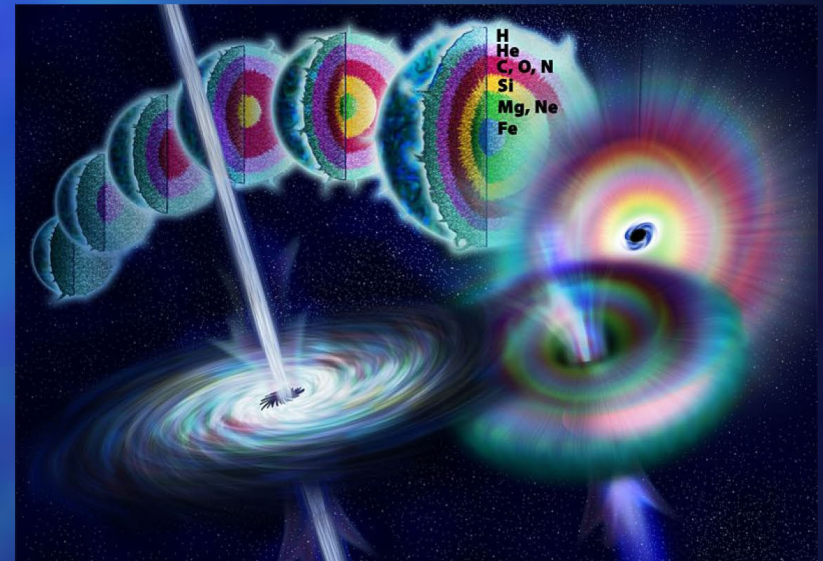


# GRBs

Gamma-ray bursts (GRBs) are the most luminous events occurring in the universe since the Big Bang. They are flashes of gamma rays emanating from seemingly random places in deep space at random times. The duration of GRBs usually ranges between a few milliseconds to minutes, often followed by an afterglow emission at longer wavelengths (X-ray, ultra violet, optical, infrared, and radio).

The majority of observed GRBs appear to be collimated emissions from the collapsing cores of rapidly rotating, high-mass stars into black holes. A subclass of GRBs (the "short" bursts) appear to originate from a different process, the leading candidate being the collision of neutron stars orbiting in a binary system.

GRBs most likely contribute a negligible fraction to the low energy CR flux.





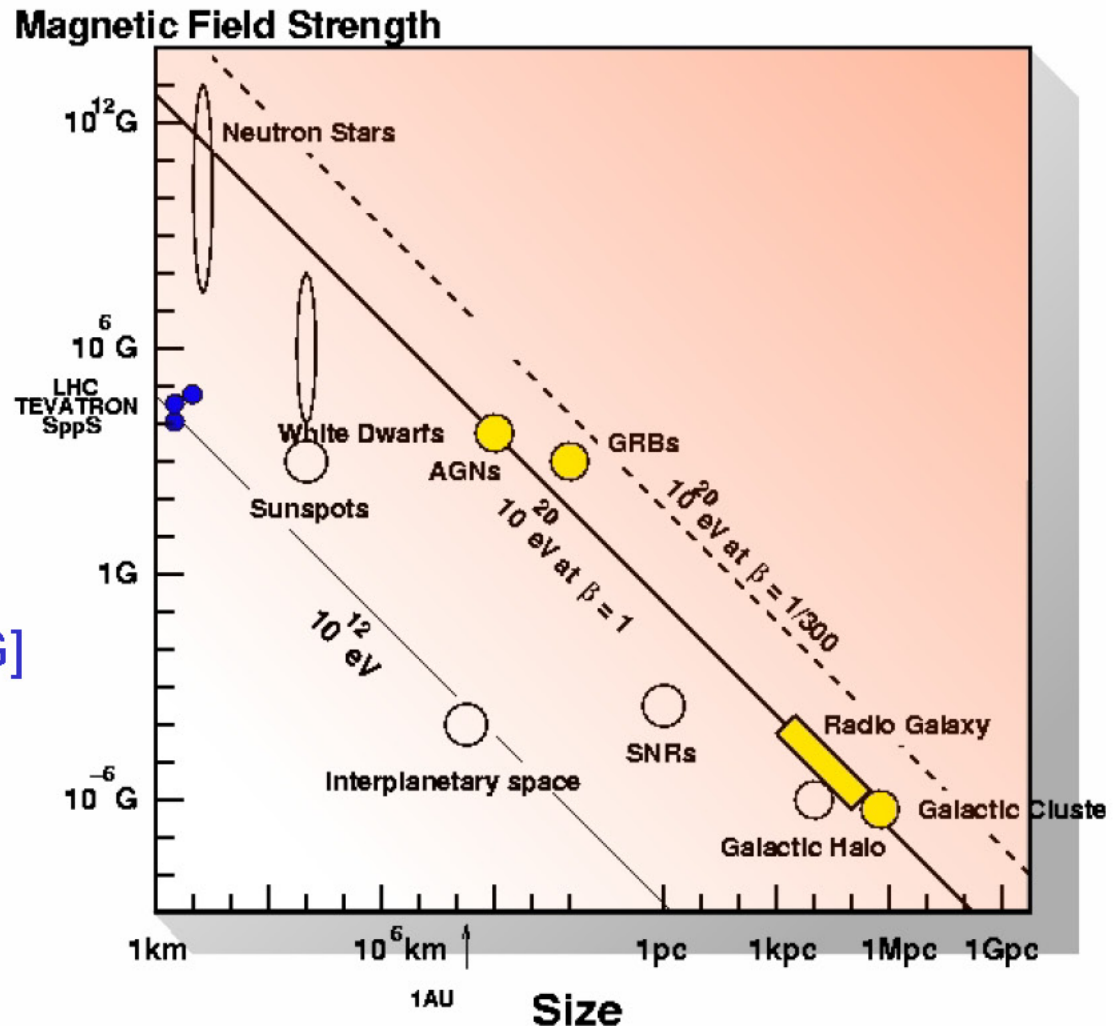
# Magnetic Confinement

## “Hillas Plot”

Minimum size of B field to contain particles being accelerated.

Achievable energy:

$$E [\text{EeV}] \sim Z R [\text{kpc}] B [\mu\text{G}]$$



## 4.7 Conclusioni circa il modello

- Il modello di accelerazione dei RC da parte di SN fonda la sua giustificazione sulla concordanza tra energia cinetica emessa ( $10^{42}$  erg/s) e la “potenza” sotto forma di RC nella Galassia:  
 $W_{CR} = 5 \times 10^{40}$  erg/s
- Un meccanismo che trasferisca il  $\sim 5\%$  di energia verso particelle relativistiche (RC) è sufficiente per spiegare i RC galattici sino ad energie  $\sim 10^{15}$  eV.
- Il meccanismo di Fermi ha proprio una efficienza
- Nella regione di accelerazione, lo spettro energetico dei RC è descritto da una legge di potenza:

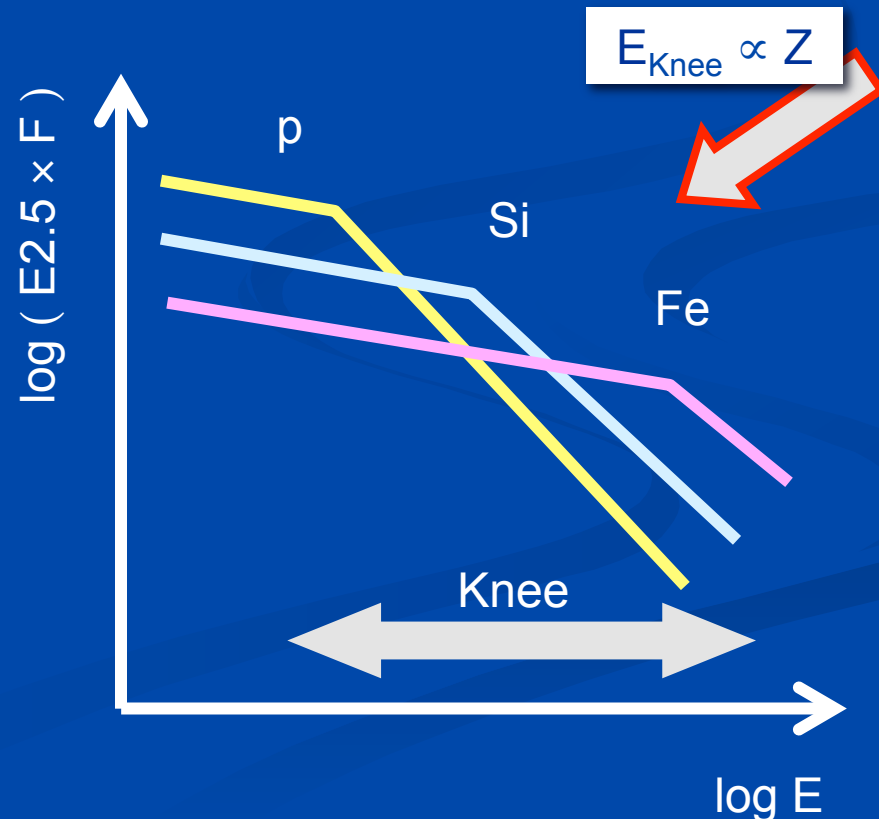
$$\eta = \frac{V}{c} \cong 5 \times 10^{-2}$$

$$\frac{dN(E)}{dE} \propto E^{\alpha-1} = E^{-2}$$

- La legge di potenza alla sorgente del tipo  $E^{-2}$  si confronta con l'osservazione sperimentale di uno spettro del tipo  $E^{-2.7}$  sulla Terra, tenendo conto della probabilità di fuga dalla Galassia vs.  $E$
- L'energia *massima* che i RC possono acquisire in queste regione di accelerazione è

$$E_{\text{max}} = 300 \times Z \quad \text{TeV}$$

- In corrispondenza di questa energia, si trova una struttura nello spettro osservato (ginocchio). La previsione del modello è che il ginocchio dipende dalla rigidità (ossia, da  $Z$ ) della particella



# Efficienza

- Il meccanismo di Fermi  
funziona in assenza di  
collisioni
- Occorre tenere conto delle  
perdite di energie dovute  
ad altri processi  
contemporaneamente in  
atto: le particelle cariche se  
accelerate emettono  
radiazione  
elettromagnetica, perdono  
energia per urto, ....

## Particle Acceleration

Need for Efficient and Dynamic Acceleration:  
Overcome Ionization Losses and Radiation reaction

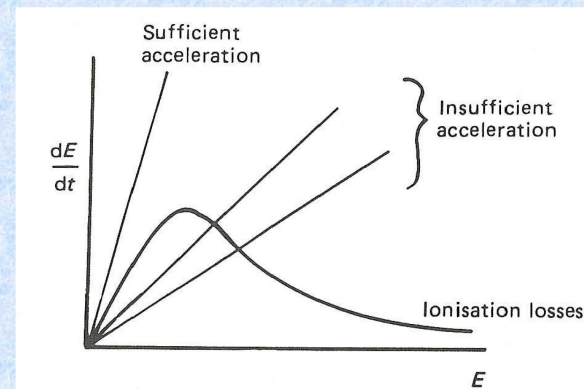


Figure 21.2. Comparison of the acceleration rate and energy loss rate due to ionisation losses for a high energy particle.





# 6.5 Osservazioni sperimentali di acceleratori astrofisici nella Galassia: astronomia $\gamma$ (TeV) con telescopi Imaging Cherenkov

- *Cos'è un telescopio Cherenkov?* Un riflettore ottico con un fotomoltiplicatore (PMT) nel fuoco + elettronica veloce (ns)
- Perché “imaging”? Perché permette di ricostruire i parametri dello sciame di particelle:
  - Informazione sulla direzione di arrivo
  - Possibile separazione di sciame di adroni da quelli di fotoni
  - Largo Field of view (FoV), ossia area di raccolta
- Energia di soglia: 1 TeV

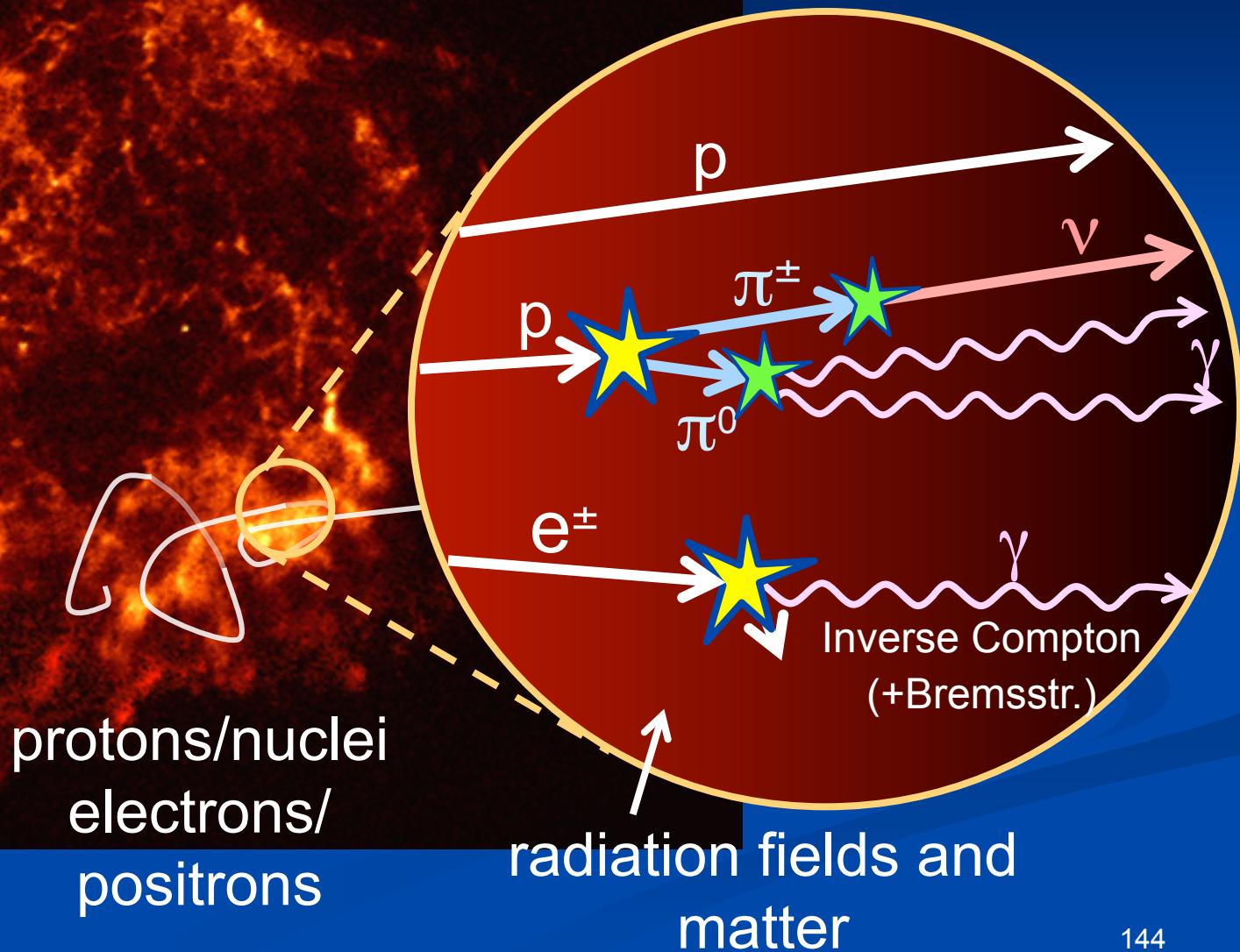
# Cherenkov Telescopes

above some  
10...100 GeV

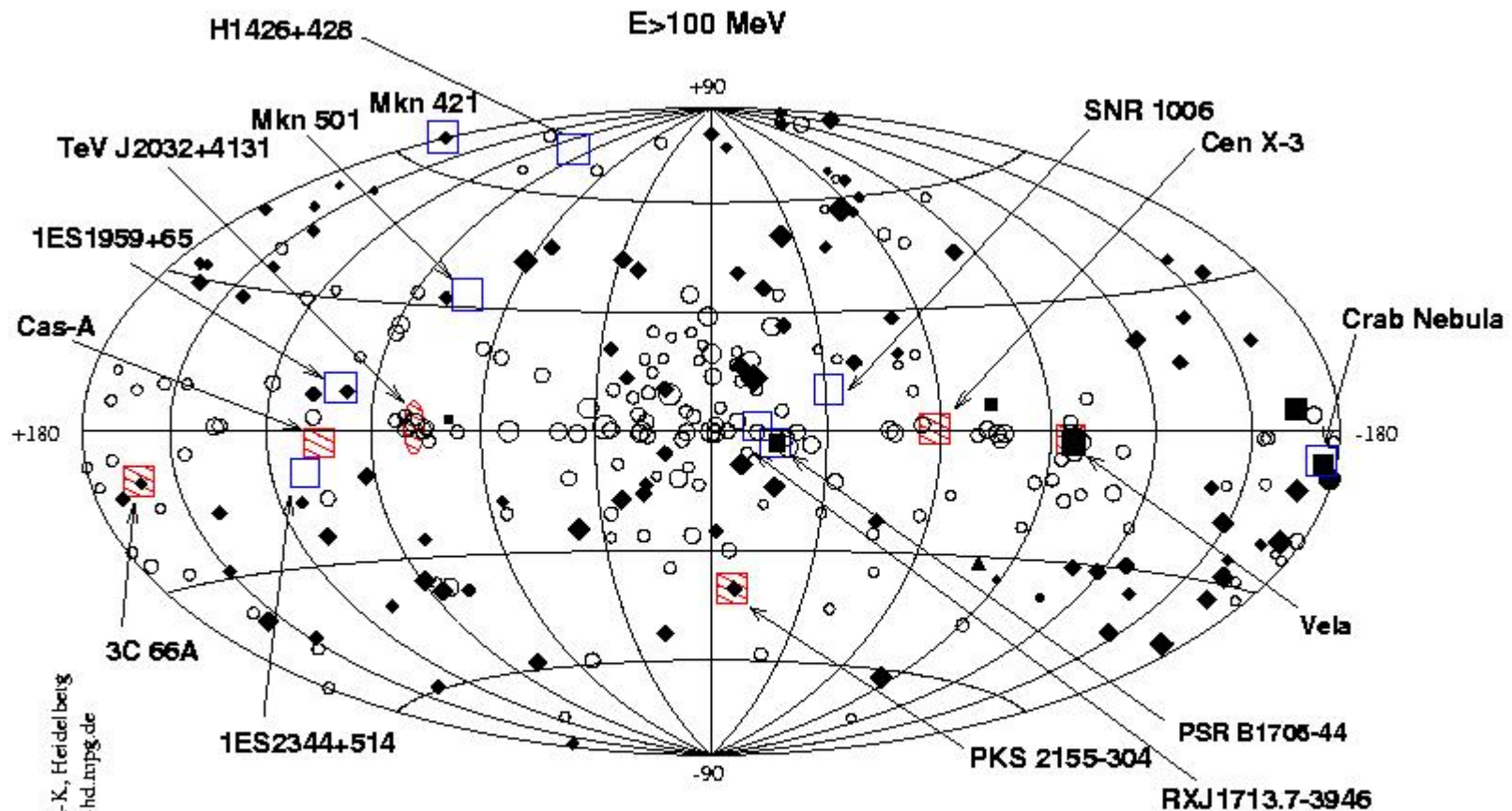
Large detection area:  $\sim 10^5 \text{ m}^2$   
High sensitivity:  $\sim 1\%$  Crab  
Angular resolution:  $\sim 5'$   
Small fov: few msr  
Modest duty cycle:  $\sim 10\%$



# Produzione di $\gamma$ in acceleratori cosmici



# The emerging VHE gamma-ray sky



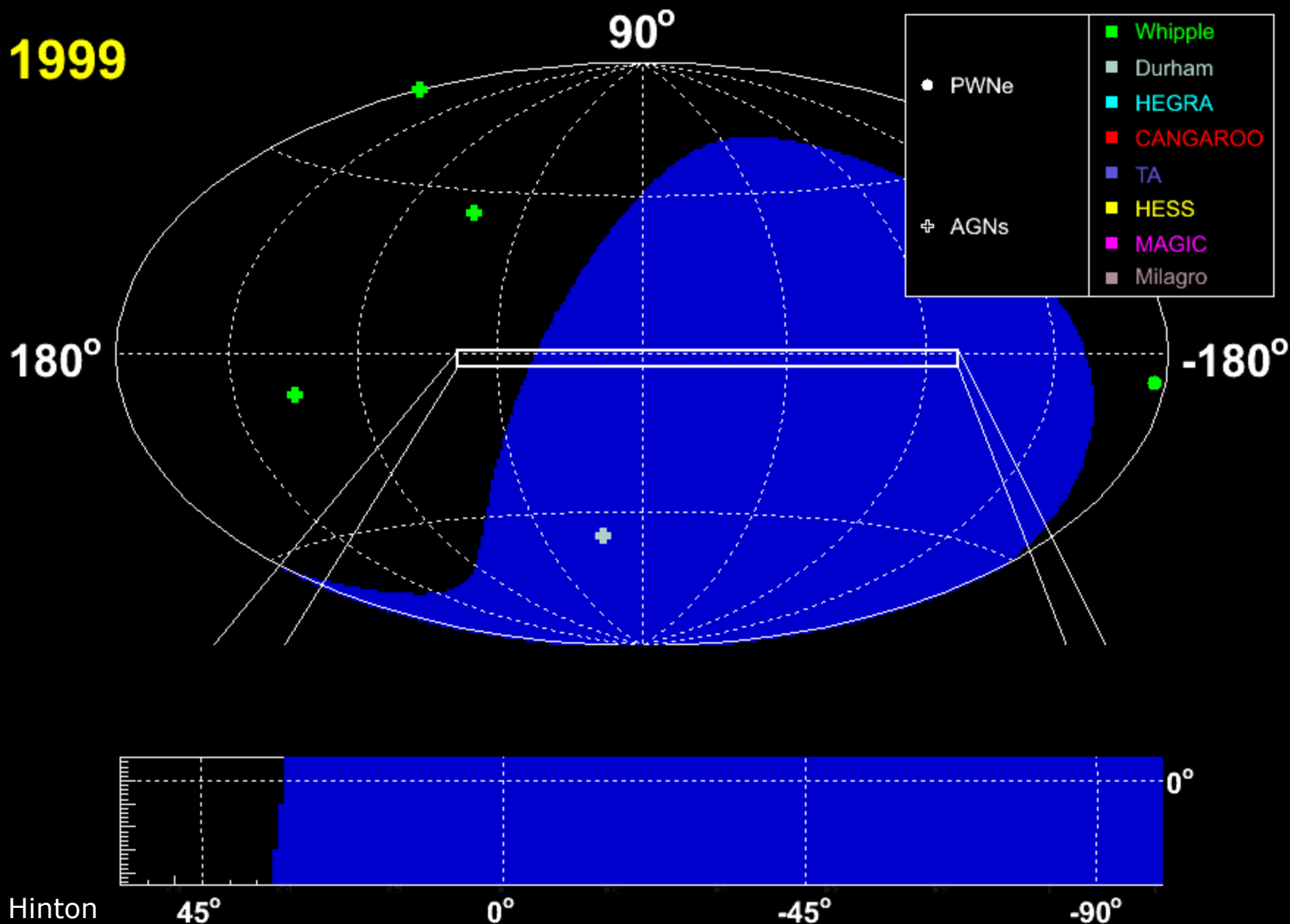
3rd EGRET catalogue:

- ◆ AGN
- Unidentified EGRET sources
- solar flare
- Pulsars
- ▲ LMC

Ground based detections:

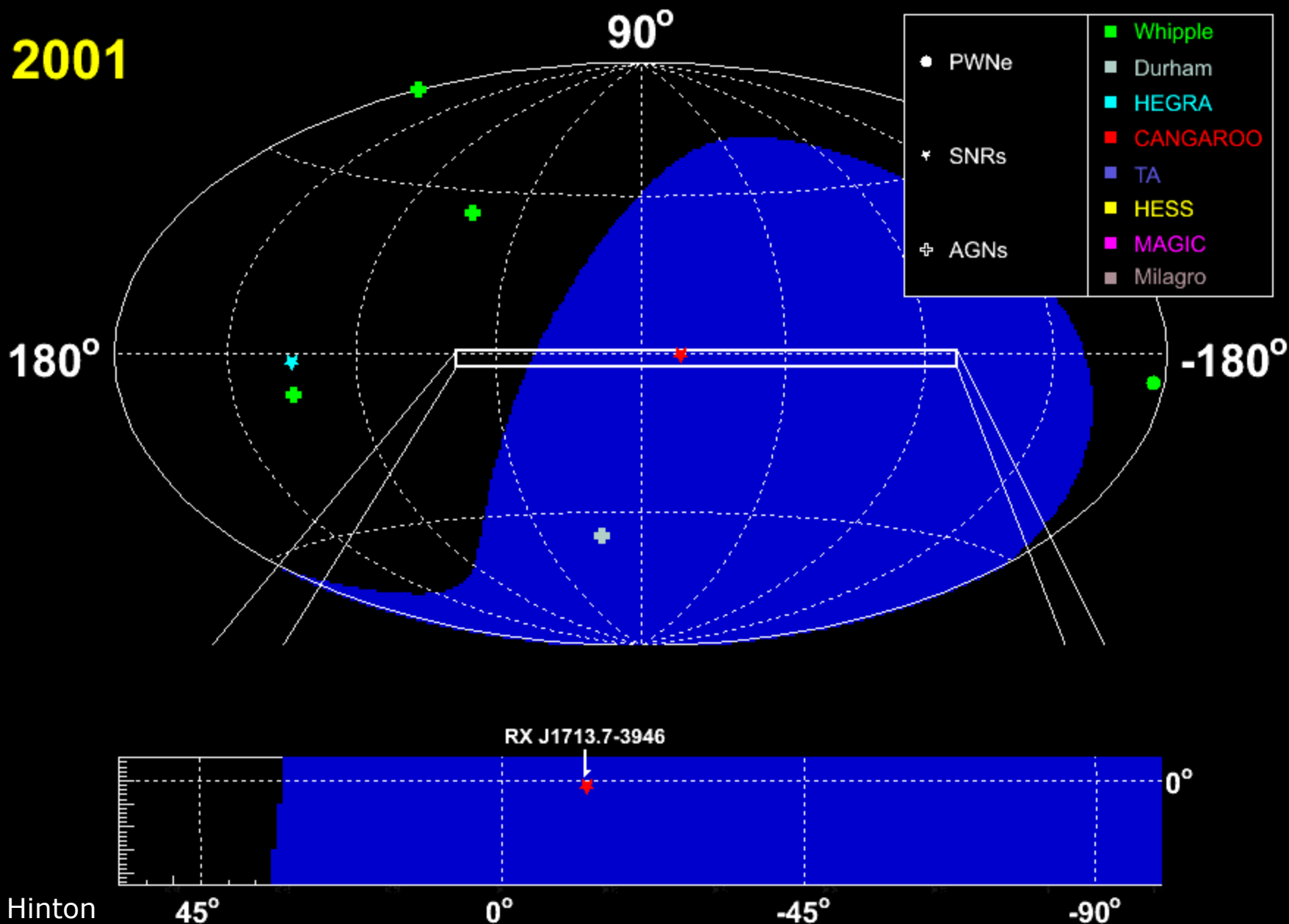
- TeV sources (confirmed)
- ▨ TeV sources (not confirmed)
- ▨ Unidentified TeV source (not conf.)

1999

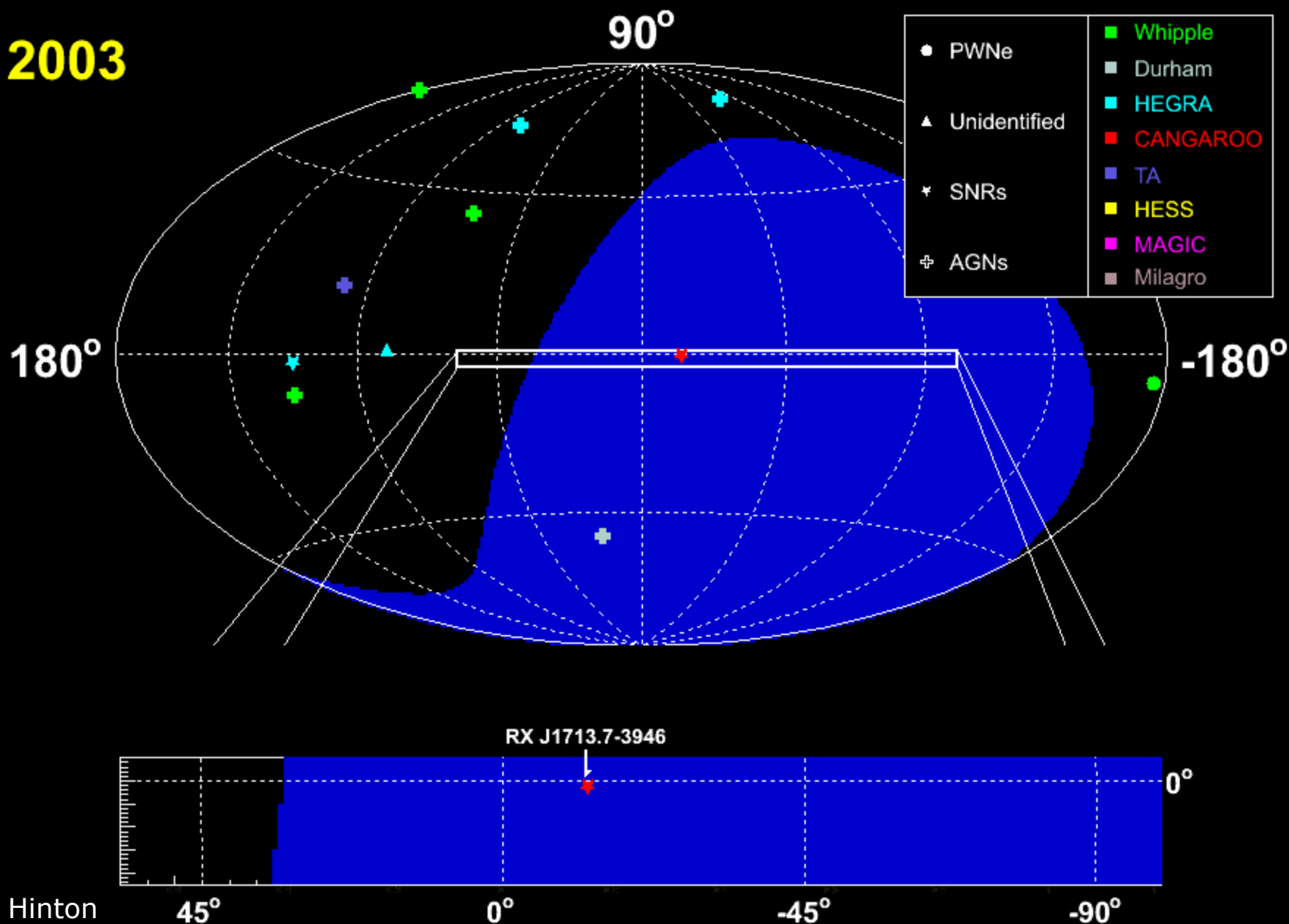




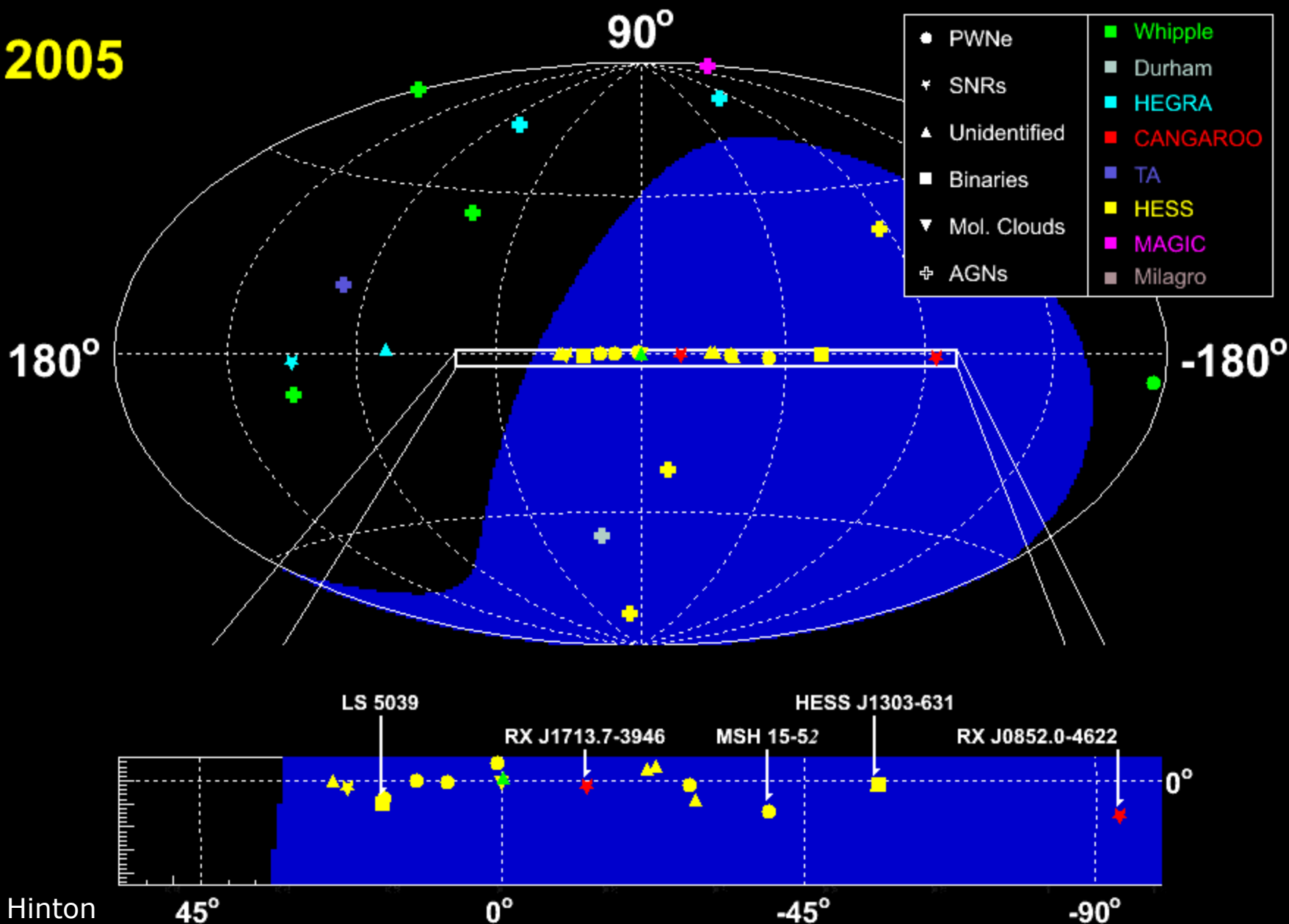
2001



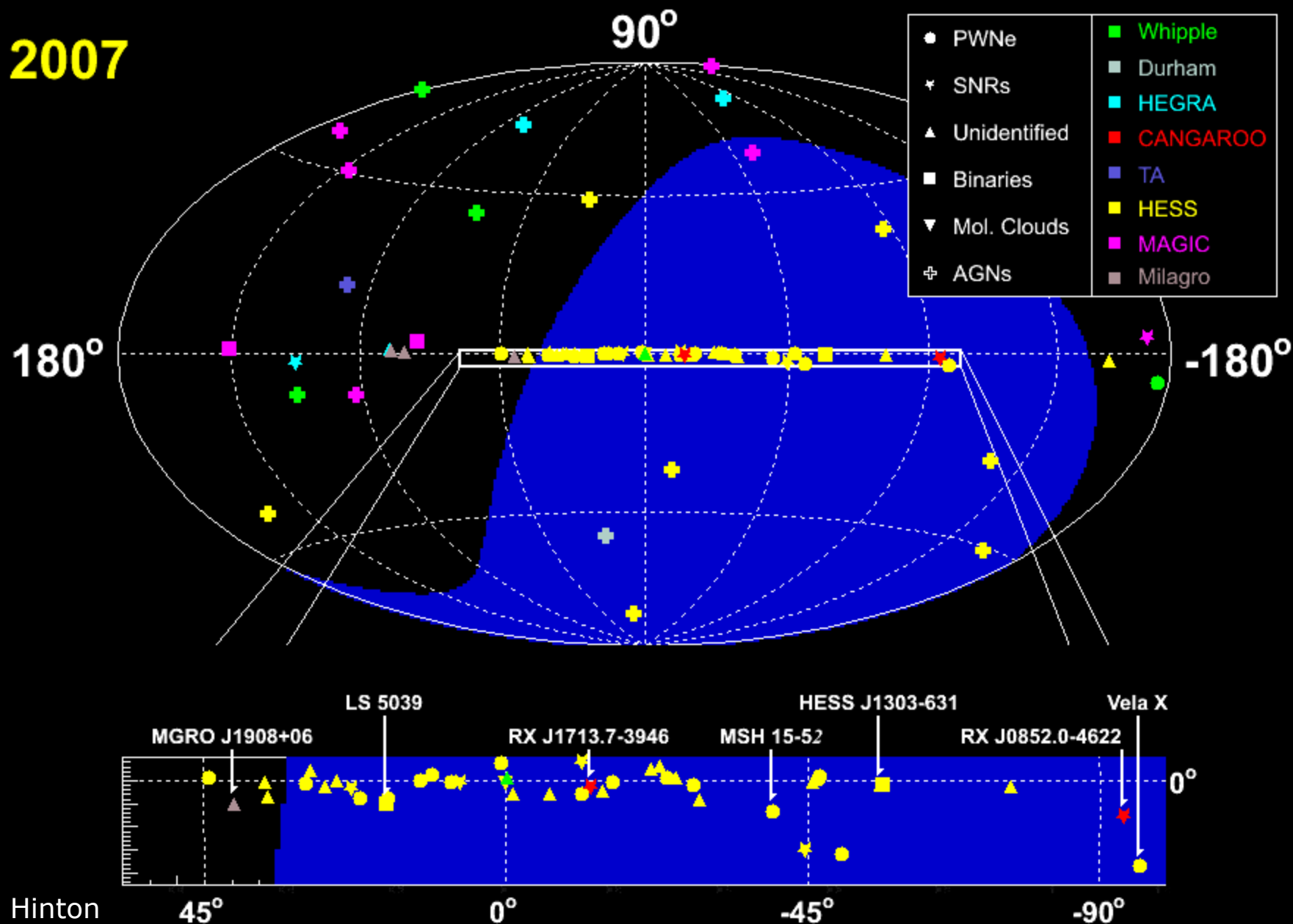
2003



2005



2007



2007

**71 VHE sources -  
each a cosmic particle accelerator,  
where gamma rays trace  
primary electrons or nuclei**

**Supernova Remnants (SNR) 7**

**Pulsar Wind Nebulae (PWN) 18**

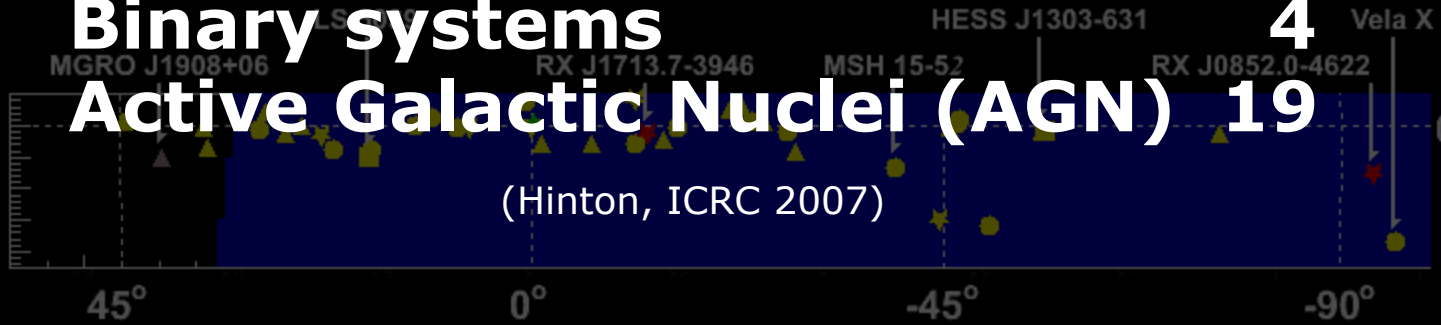
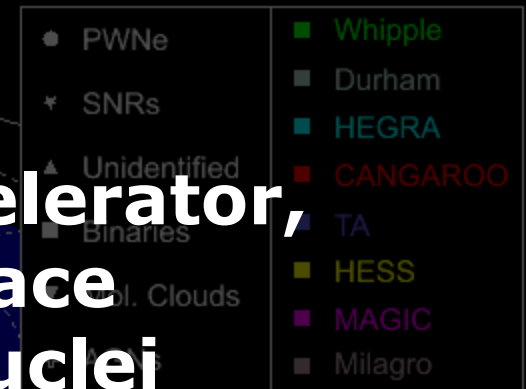
**Unidentified Galactic Sources 21**

**Diffuse Sources 2**

**Binary systems 4**

**Active Galactic Nuclei (AGN) 19**

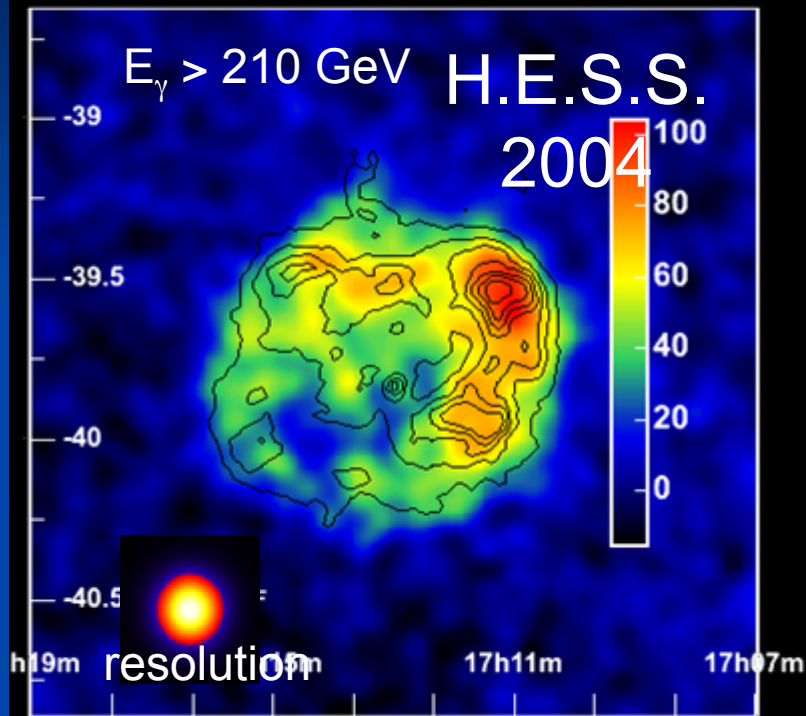
(Hinton, ICRC 2007)



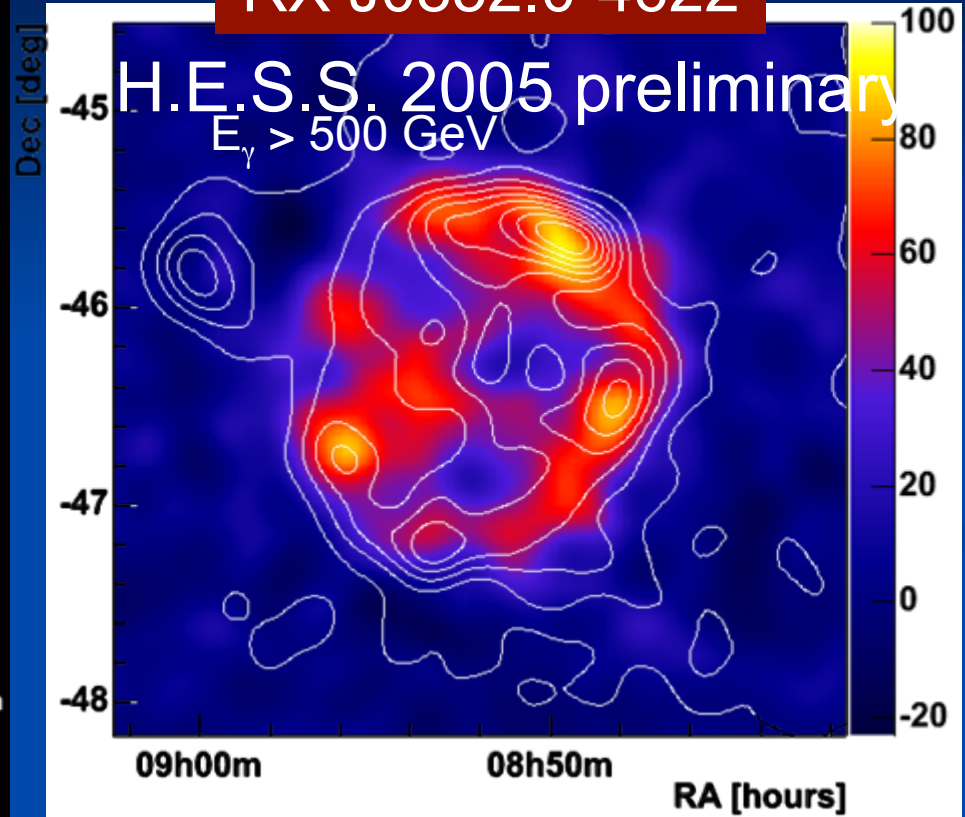
**Want to know:**

# Supernova Shells: Acceleratori di RC?

RX J1713.7-3946



RX J0852.0-4622

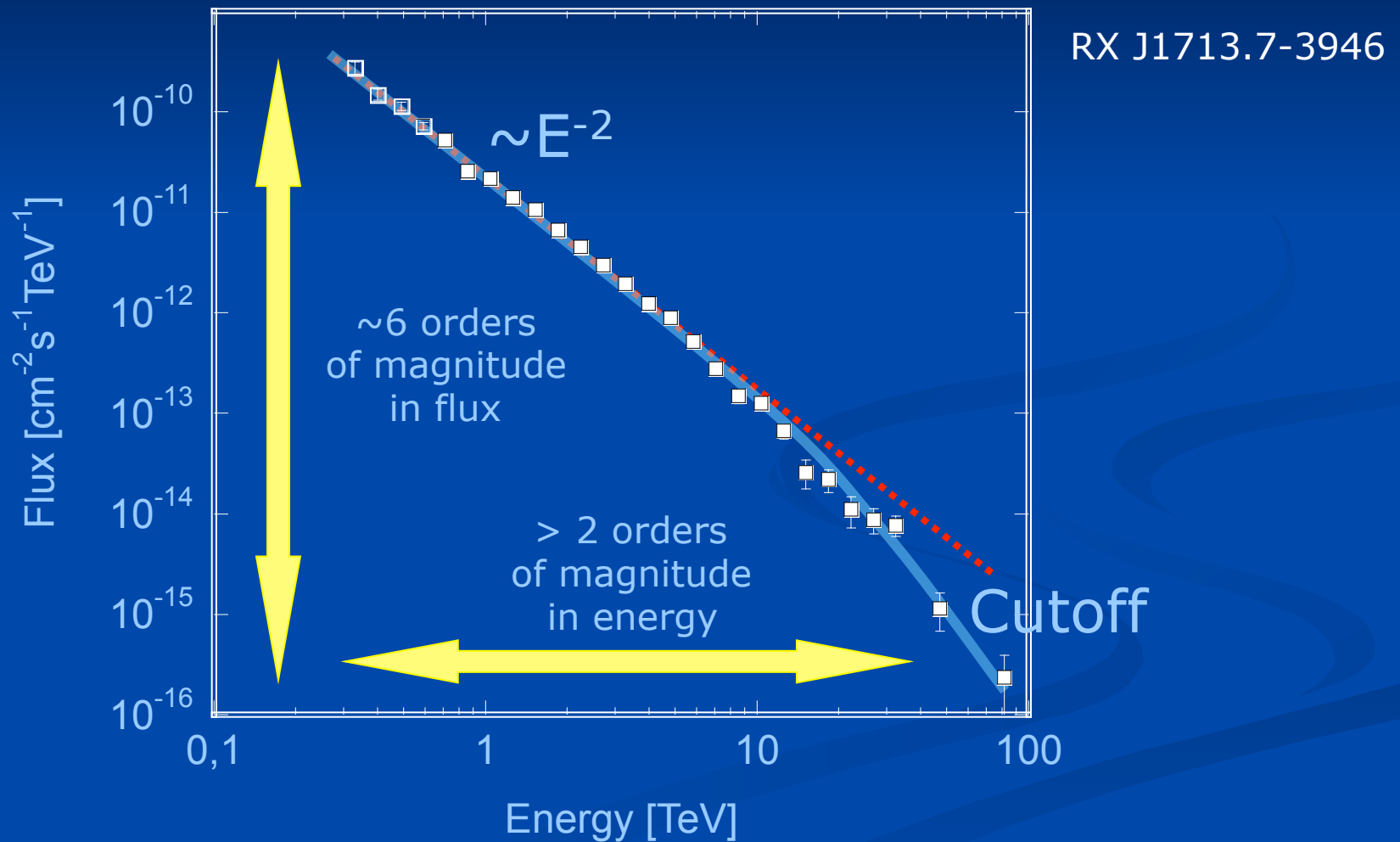


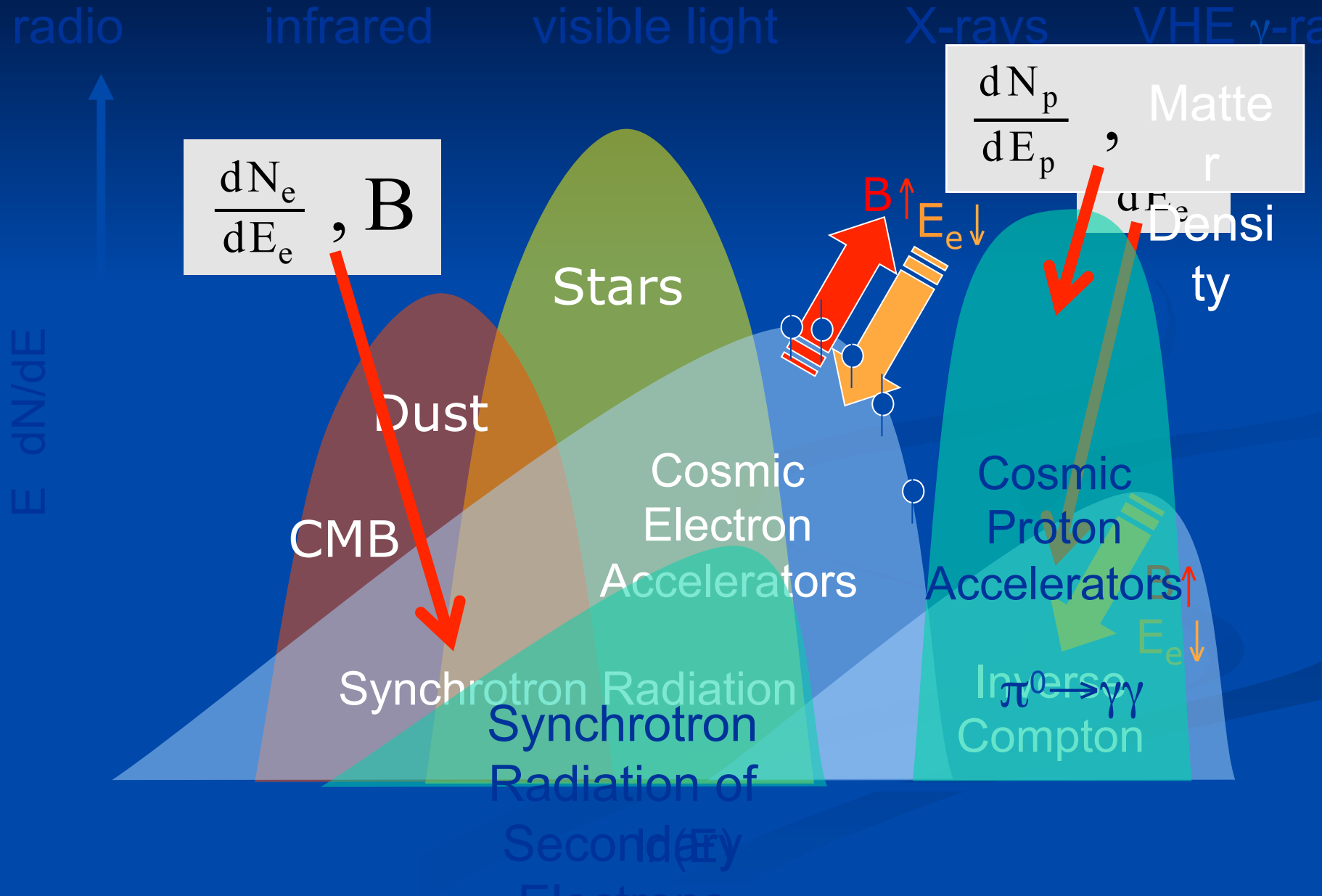
Strong Correlation with X-ray Intensities

- SN-Shells sono acceleratori di particelle sino almeno 100 TeV
- Ma le particelle sono protoni/nuclei o elettroni?



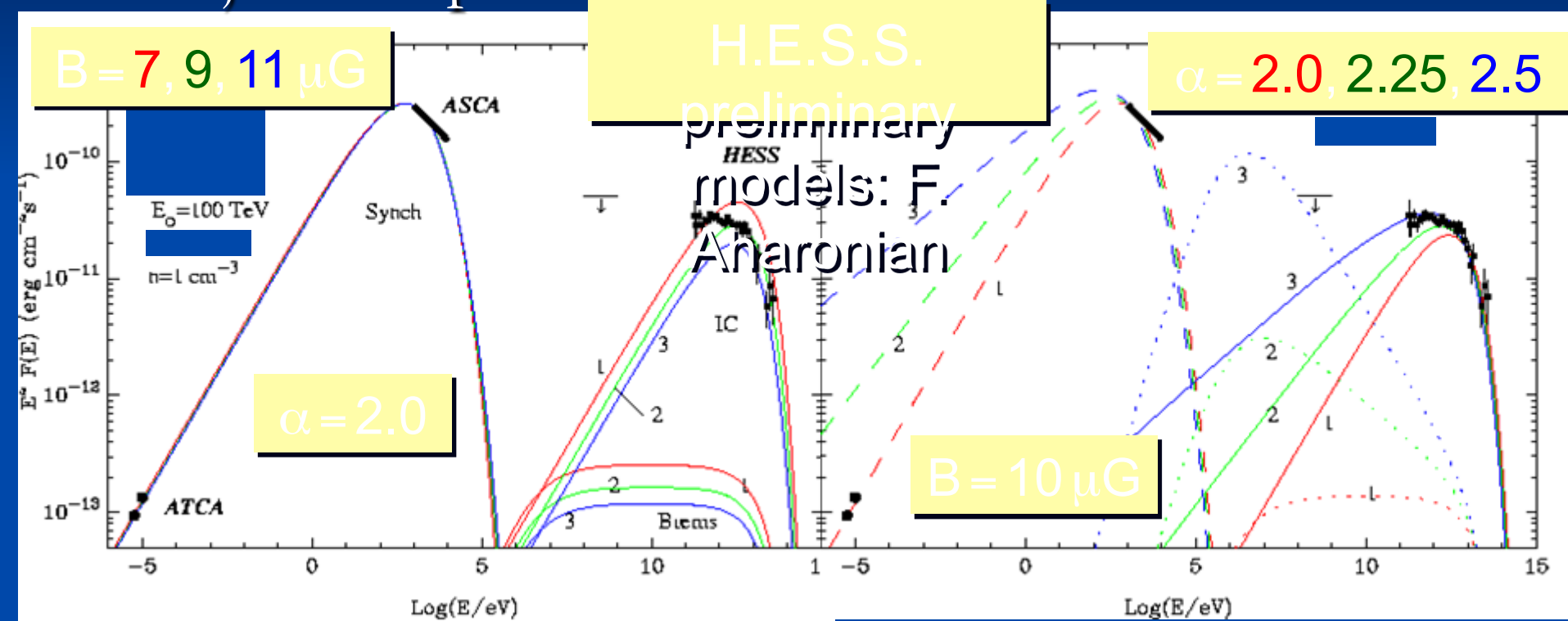
# Spettro energetico RX J1713.7-3946





# Electron accelerator fits for **RX J1713.7-3946**:

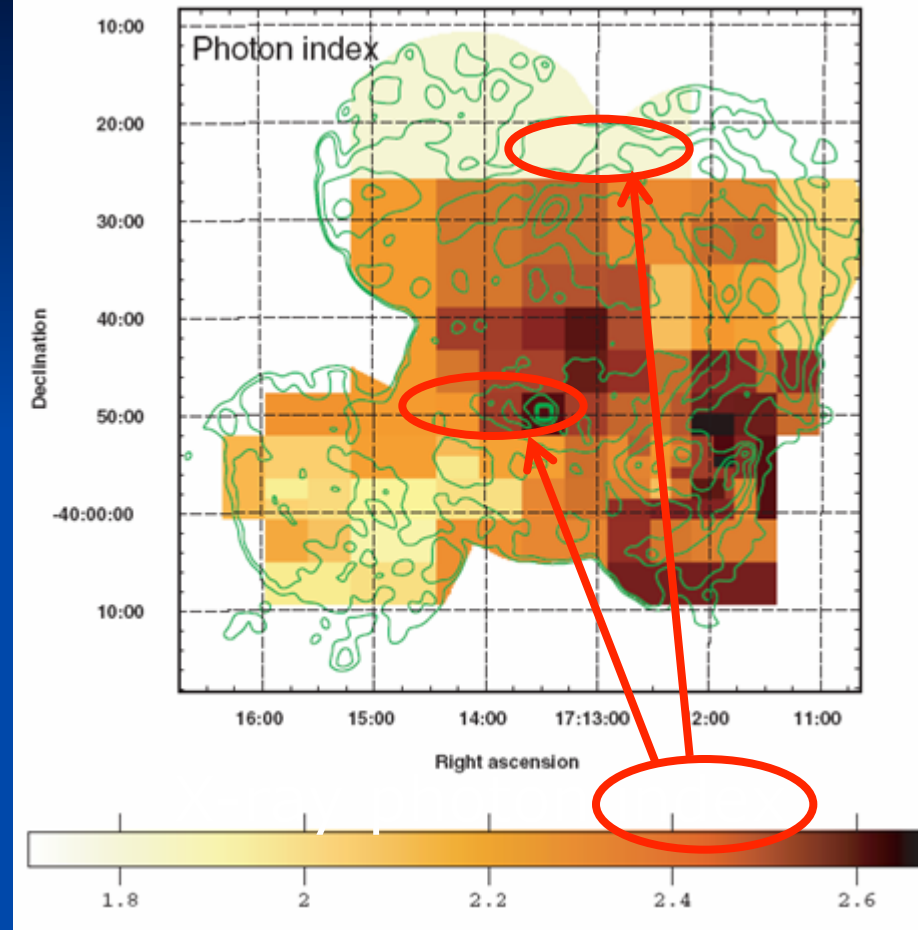
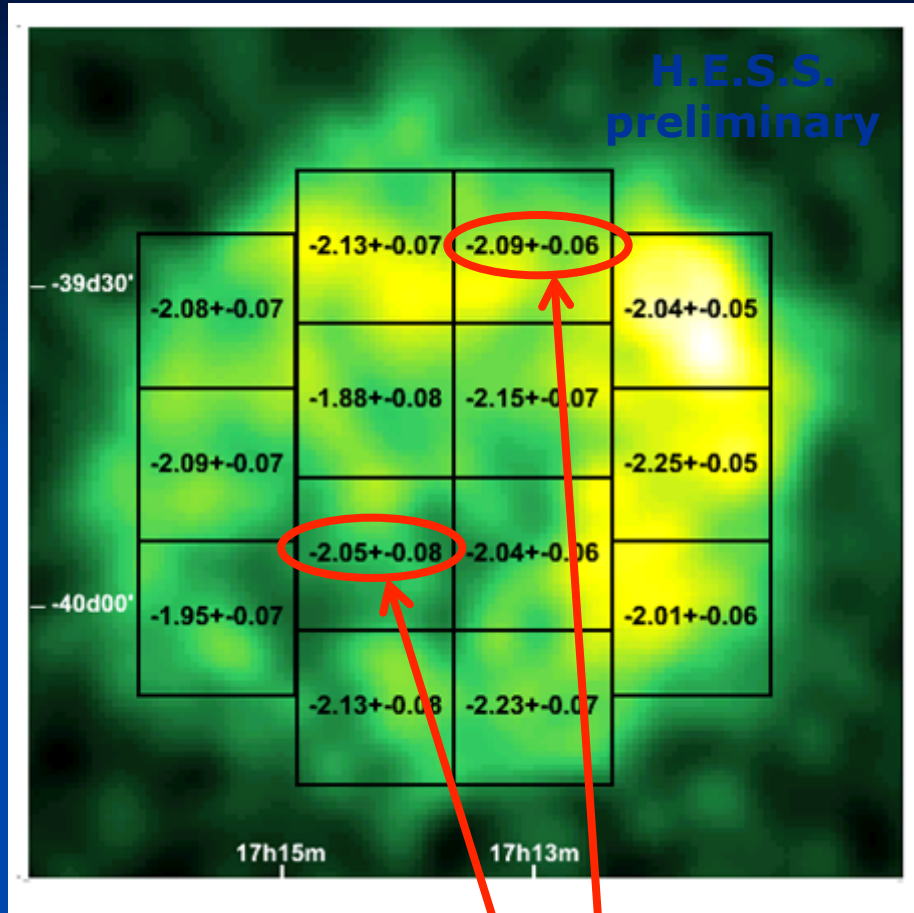
- Continuous electron injection over 1000 years
- Injection spectrum: power law  $E_e^{-\alpha}$  with cutoff



- IC peak not well described
- B-field low for SNR shell

- large & injection rate
- bremsstrahlung important
- needs tuning at low E

# Spatially resolved spectra of RX J1713.7-3946

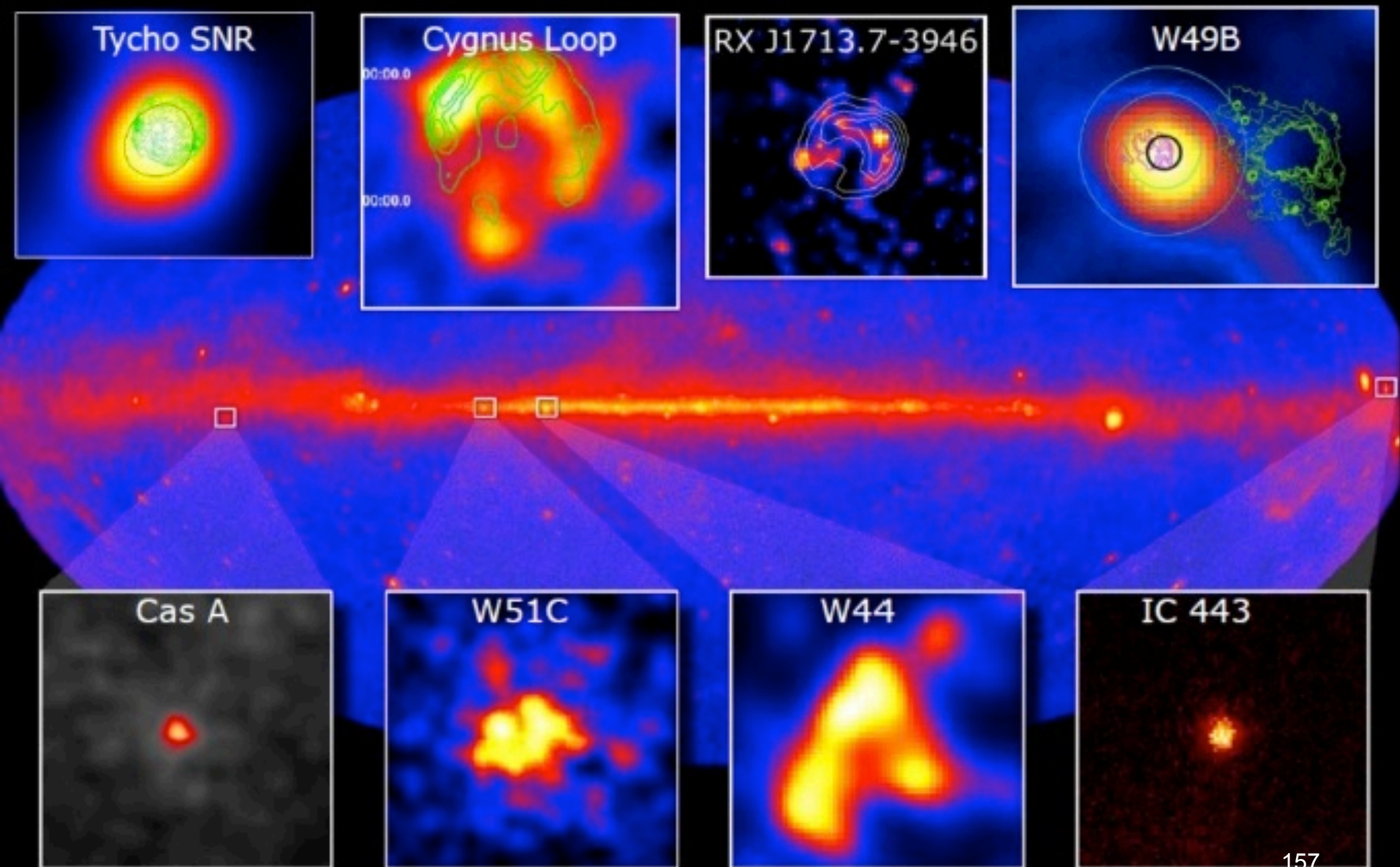


G. Cassam-Chenaï A&A 427, 199 (2004)

TeV photon index const

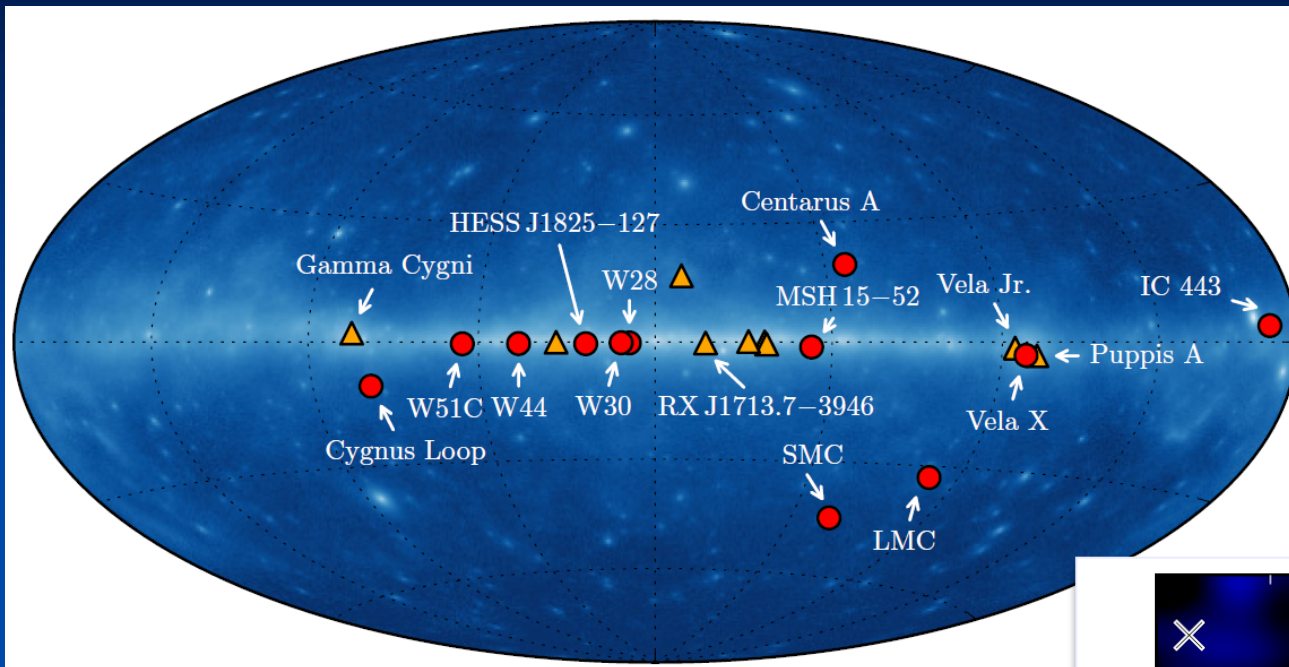
TeV / X-ray intensities correlate, but NOT the spectral shapes  
 $\Rightarrow$  very hard to understand for pure electron accelerator !

## ■ Supernova Remnants - Spatially Resolved





# Spatially Extended Sources



■ Identify 2FGL sources extended beyond the PSF

■ e.g., Gamma Cygni SNR

■ 12 extended sources reported in 2FGL

■ 6 SNRs (W51C, IC 443, W28, W44, Cygnus Loop, W30), 3 PWNe (Vela-X, MSH 15-52,

■ HESS J1825-137), LMC, SMC, Cen A

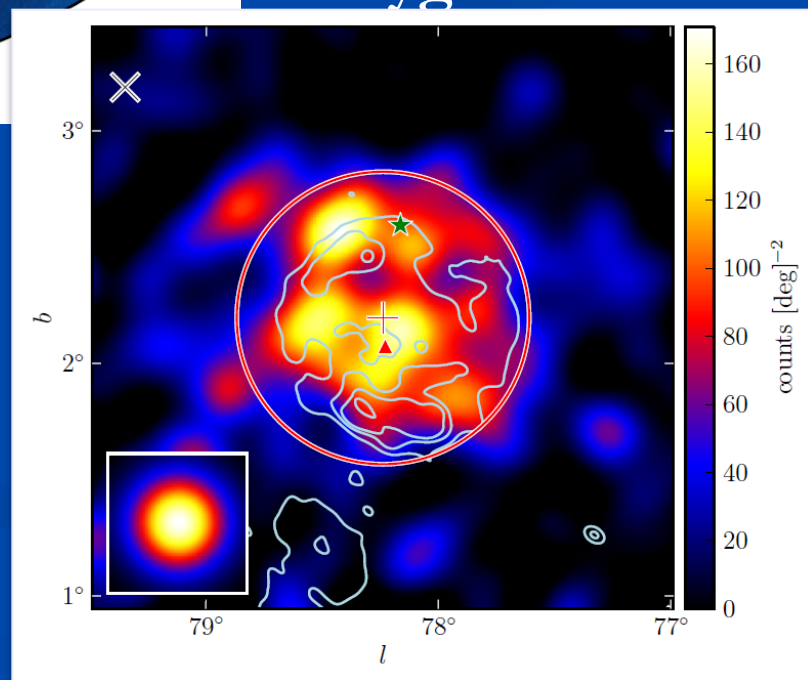
■ + Vela Jr. and RXJ 1713.7-3946 SNRs

■ + 7 new spatially extended GeV sources including the

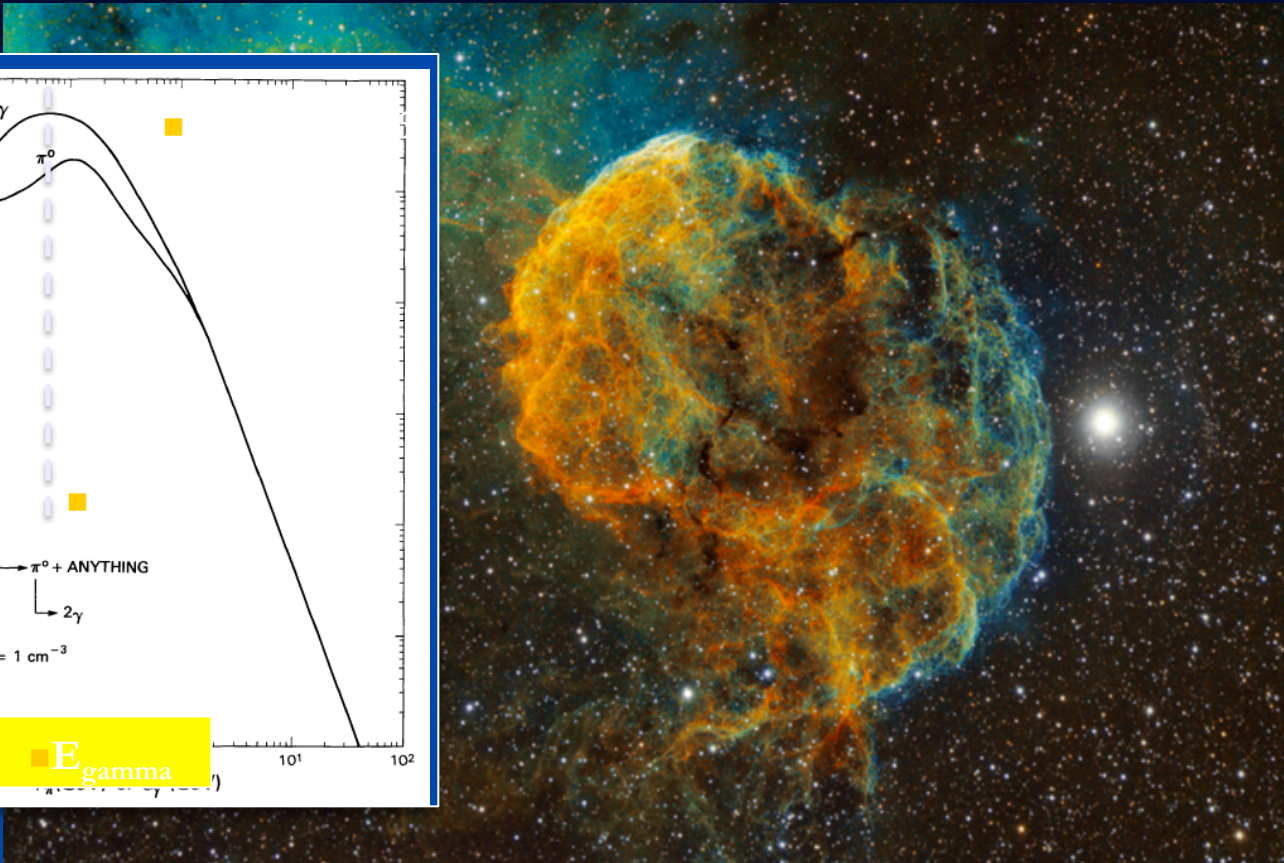
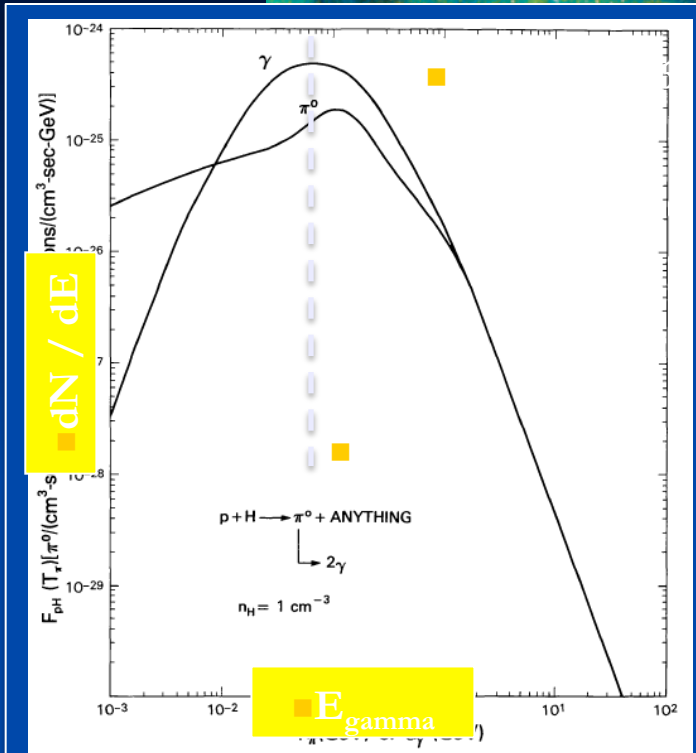
Puppis A and Gamma Cygni SNRs

■ Lande et al. 2012, ApJ, 756, 5

158



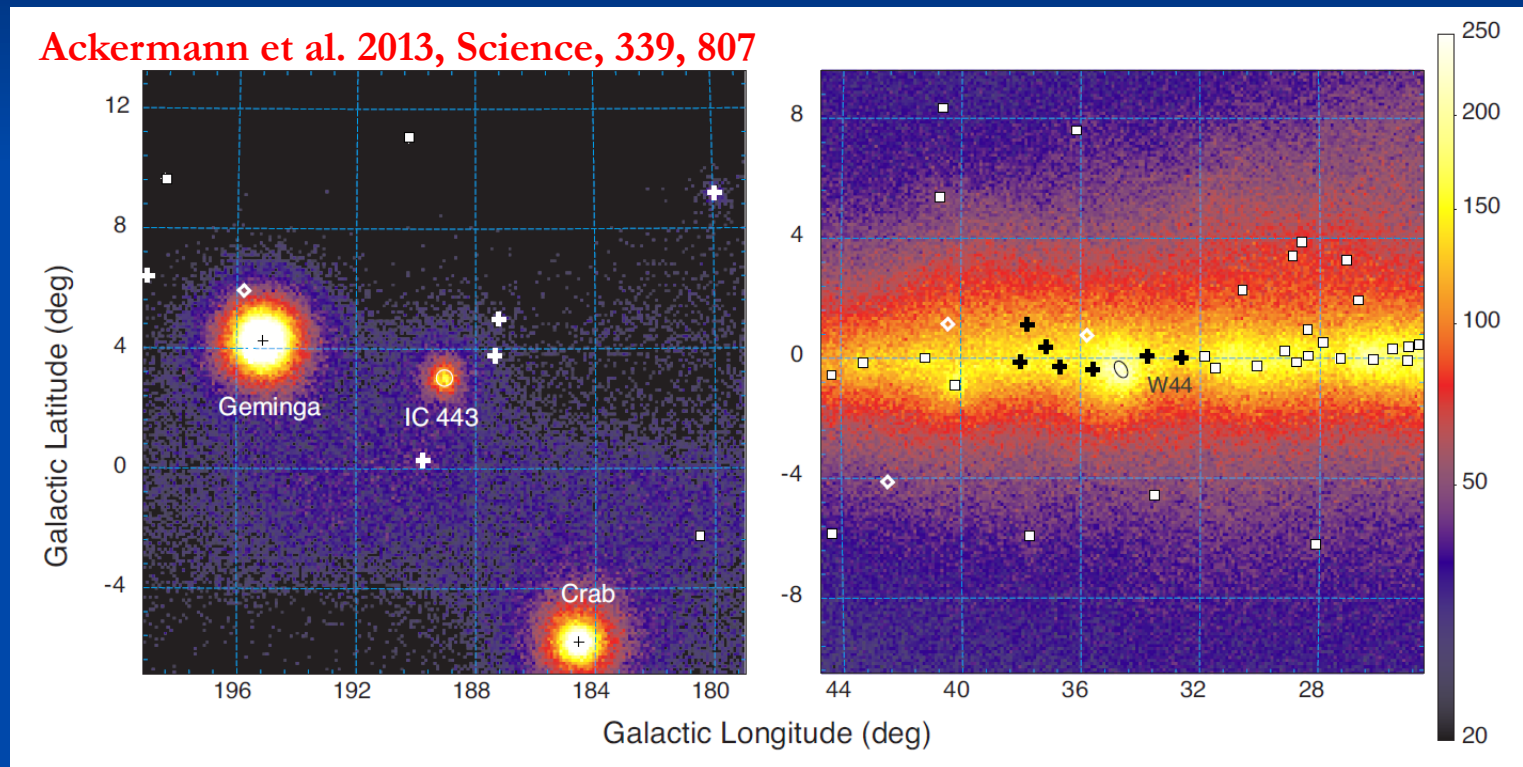




# DETECTION OF THE PION- DECAY CUTOFF IN SUPERNOVA REMNANTS

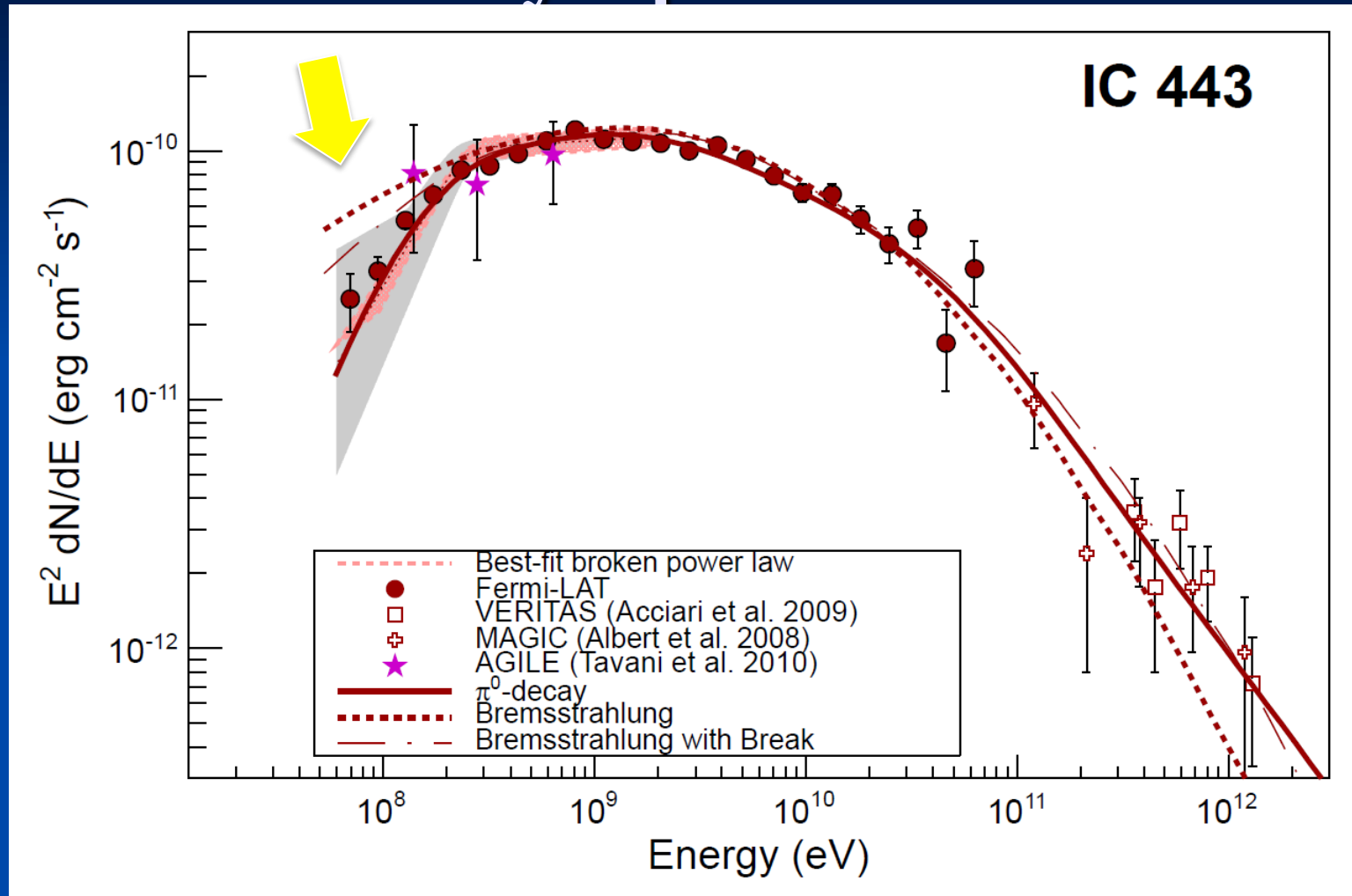
# Top SNR Candidates: IC 443 &

- Both SNRs interacting with clouds, ages of  $\sim 10^4$  years, and highest significance SNRs in 2FGL



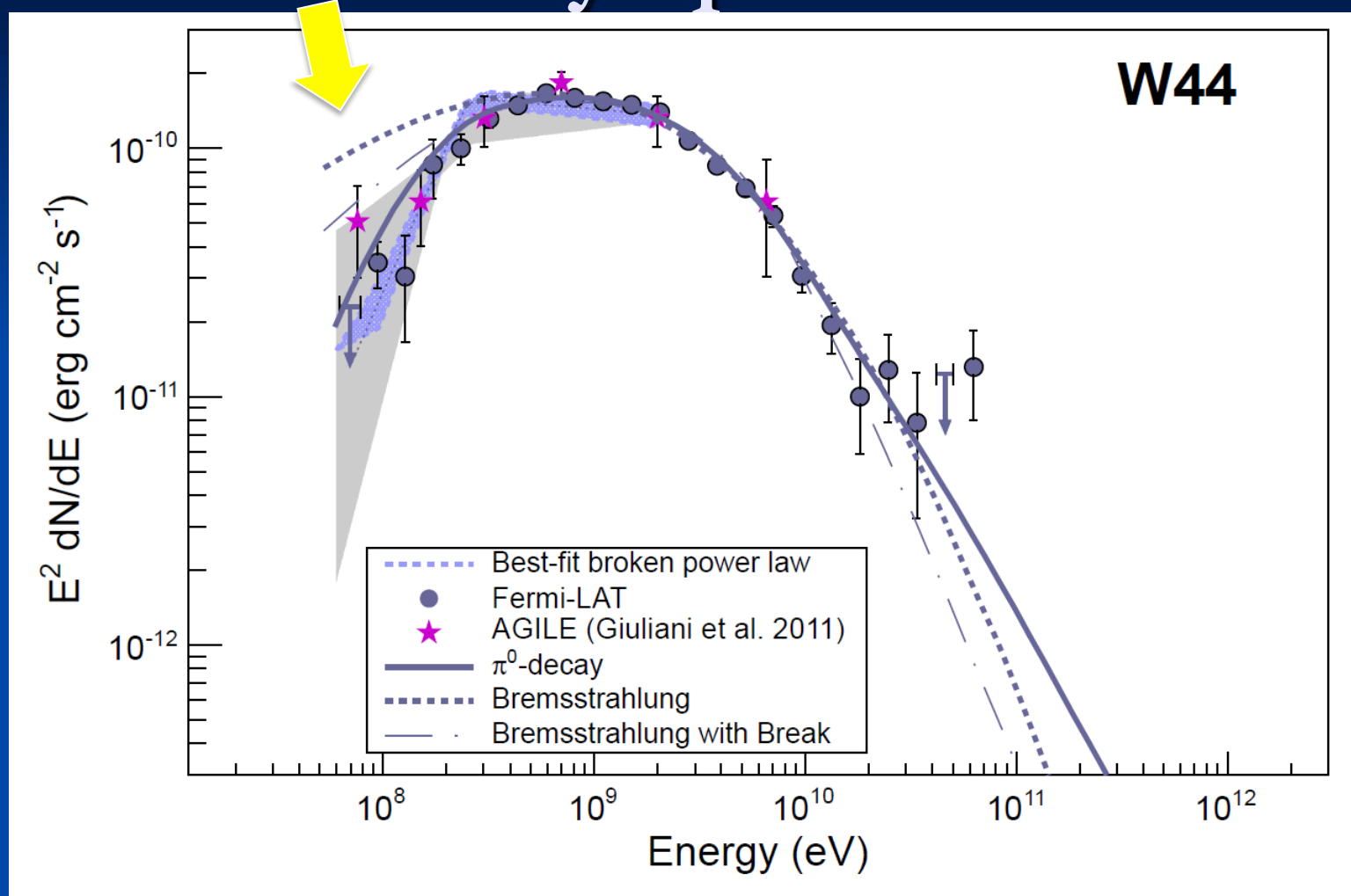
- Counts map in  $0.1 \text{ deg} \times 0.1 \text{ deg}$  pixels, 60 MeV – 2 GeV
  - Diamonds indicate previously undetected sources
  - Crosses and diamonds indicate sources with normalization free in the fit

# Gamma-ray Spectrum IC 443



- Color (Gray) shaded region indicates statistical (systematic) uncertainty < 2 GeV.
- Systematic uncertainty associated with Galactic diffuse modeling was estimated by using several alternative diffuse models based on GALPROP.
- Ackermann et al. 2013, Science, 339, 807

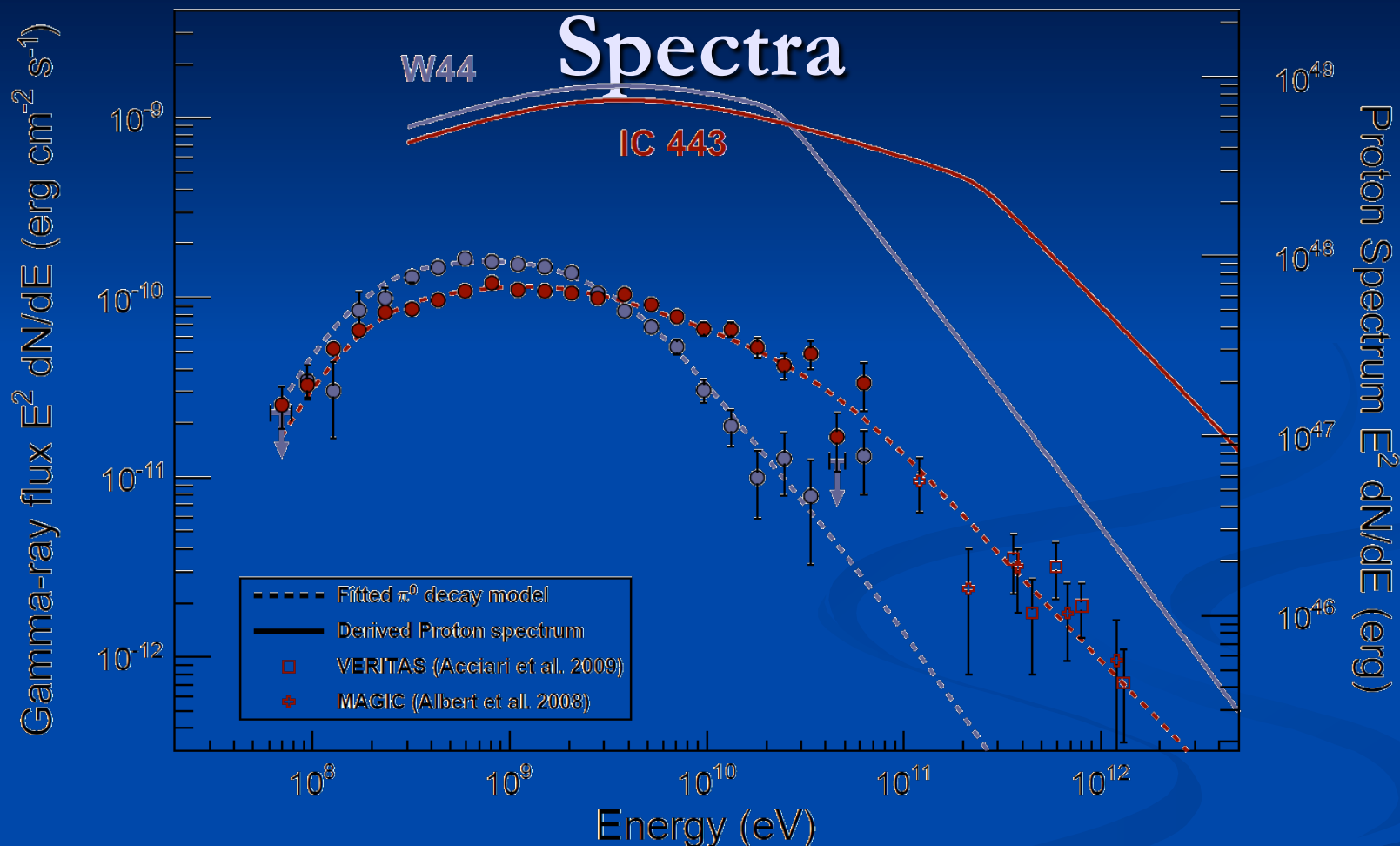
# Gamma-ray Spectrum W44



- Color (Gray) shaded region indicates statistical (systematic) uncertainty  $< 2$  GeV.
- Systematic uncertainty associated with Galactic diffuse modeling was estimated by using several alternative diffuse models based on GALPROP.
- Ackermann et al. 2013, Science, 339, 807



# Gamma-ray and Inferred Proton Spectra



- Assume average gas densities of  $20 \text{ cm}^{-3}$  (IC 443) and  $n = 100 \text{ cm}^{-3}$  (W44) and distances of 1.5 kpc (IC 443) and 2.9 kpc (W44), factor 1.85 enhancement from heavier nuclei. Inferred CR acceleration efficiencies of 1-10% (protons with  $p > 0.8 \text{ GeV c}^{-1}$ ).

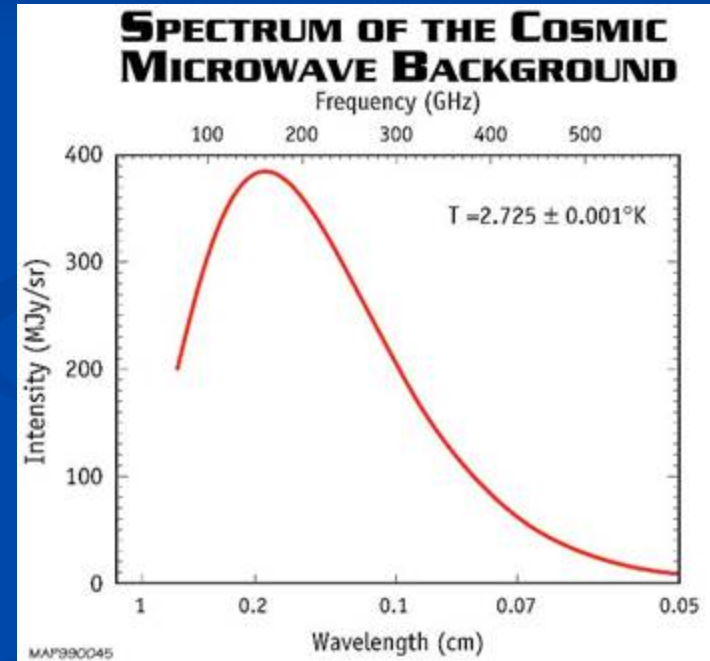
# 5.9 Volume di confinamento dei RC di origine extragalattico: il Cut-off di Greisen (GZK)

- L' universo è permeato dalla Radiazione Cosmica di Fondo a 3° K (CMBR) → 160.2 GHz

- CMBR: fotoni di energia

$$E_{\gamma}^{\text{CMBR}} = h\nu = 2\pi \times 6 \cdot 10^{-22} \text{ MeV} \cdot \text{s} \times 160.2 \cdot 10^9 \text{ Hz} = 6 \cdot 10^{-4} \text{ eV}$$

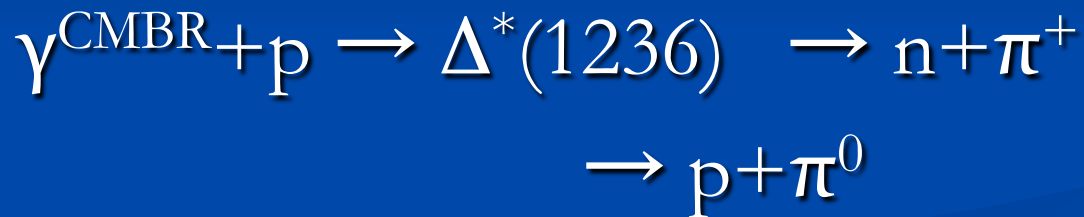
- La densità dei fotoni di fondo è  $\sim 400/\text{cm}^3$
- Il fondo di radiazione pone un limite sulla distanza massima da cui i RC possono provenire.





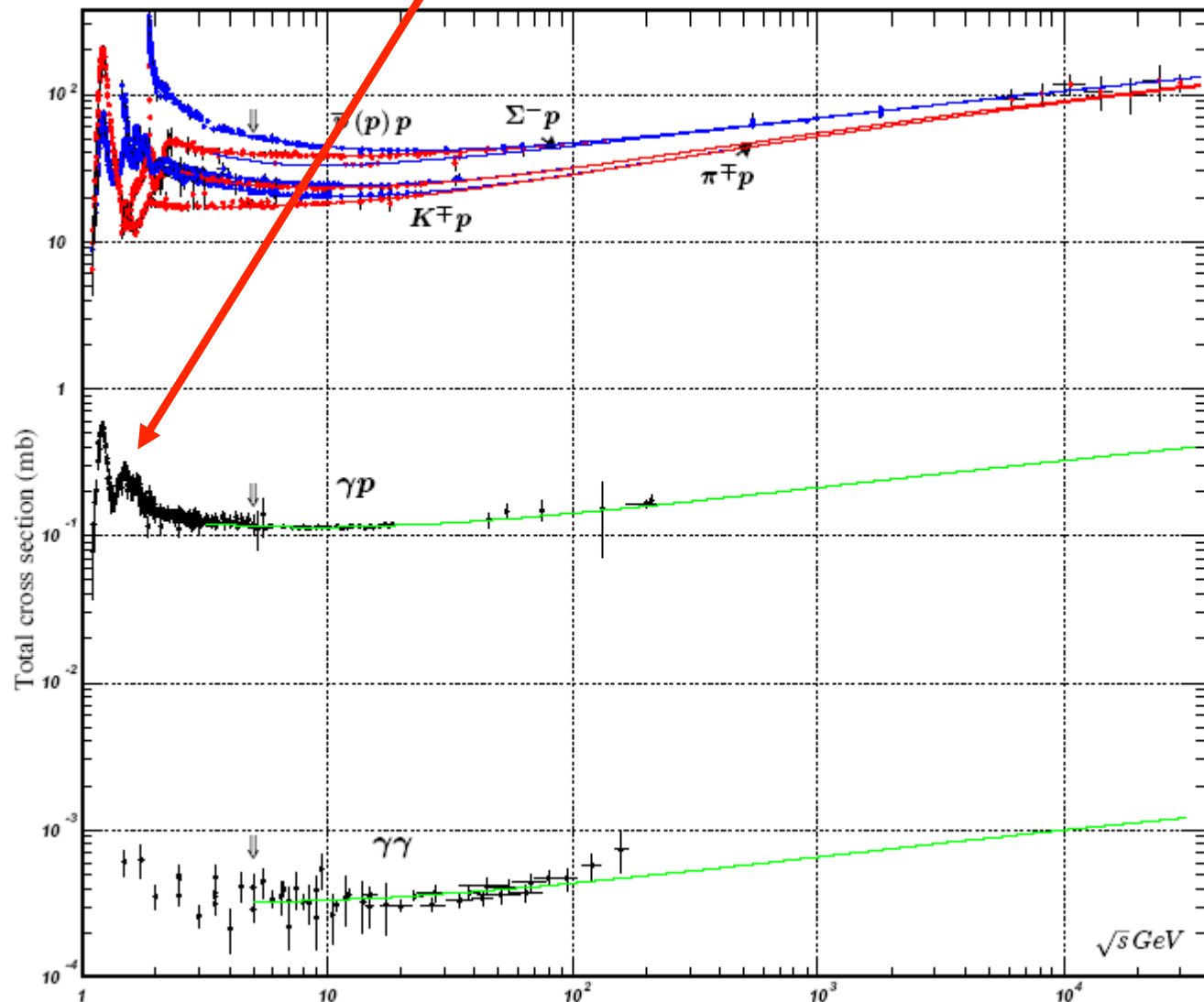
# $G_{\text{reisen}} Z_{\text{atsepin}} K_{\text{uzmin}}$ cutoff Soglia per reazioni di fotoproduzione

- **Fotoproduzione:** Protoni di alta energia possono interagire con fotoni, producendo un pione:



- È necessario essere sopra la soglia di fotoproduzione nel sistema del CM:  $E_0^{\text{FP}} \approx 300 \text{ MeV}$
- Il processo ha una sezione d'urto in risonanza  $\sigma_0^{\text{P}} \approx 250 \mu\text{b}$

# Sezione d'urto per la $\Delta$



# Soglia per la fotoproduzione (FP)

| SdR Laboratorio                                                  | SdR protone in quiete                                |         |
|------------------------------------------------------------------|------------------------------------------------------|---------|
| $p^\mu = (\gamma m_p \vec{v}_p c, \gamma m_p c^2)$               | $\vec{p}'^\mu = (0, m_p c^2)$                        | Protone |
| $k^\mu = (\frac{E_\gamma^{CMBR}}{c} c \hat{k}, E_\gamma^{CMBR})$ | $\vec{k}'^\mu = (E_0^{FP} c \hat{k}' / c, E_0^{FP})$ | Fotone  |

■ Vogliamo determinare il valore di  $\gamma$  tale che si abbia un valore  $E_0^{FP} > 300 \text{ MeV}$

■ La trasformazione di Lorentz tra i due SdR:  $\left[ x_0 = \gamma(x_0' + \frac{V}{c} x_1') \right]$

$$E_0^{FP} = \gamma(E_\gamma^{CMBR} + \frac{v_p}{c} E_\gamma^{CMBR}) =$$

$$= \gamma E_\gamma^{CMBR} (1 + v_p / c) \approx 2\gamma E_\gamma^{CMBR}$$

$E_0^{FP}$  = Soglia fotoproduzione  
(~300 MeV)

$E_\gamma^{CMBR}$  = Energia fotone =  $h\nu$

$\gamma$  = boost Lorentz del protone

- Dalla relazione, si ricava il valore di  $\gamma$  necessario per la fotoproduzione nel sistema di riferimento del laboratorio

$$\gamma = \frac{E_0^{FP}}{2E_\gamma^{CMBR}} \approx \frac{300MeV}{2 \cdot 4.7 \cdot 10^{-4} eV} \approx 3 \cdot 10^{11}$$

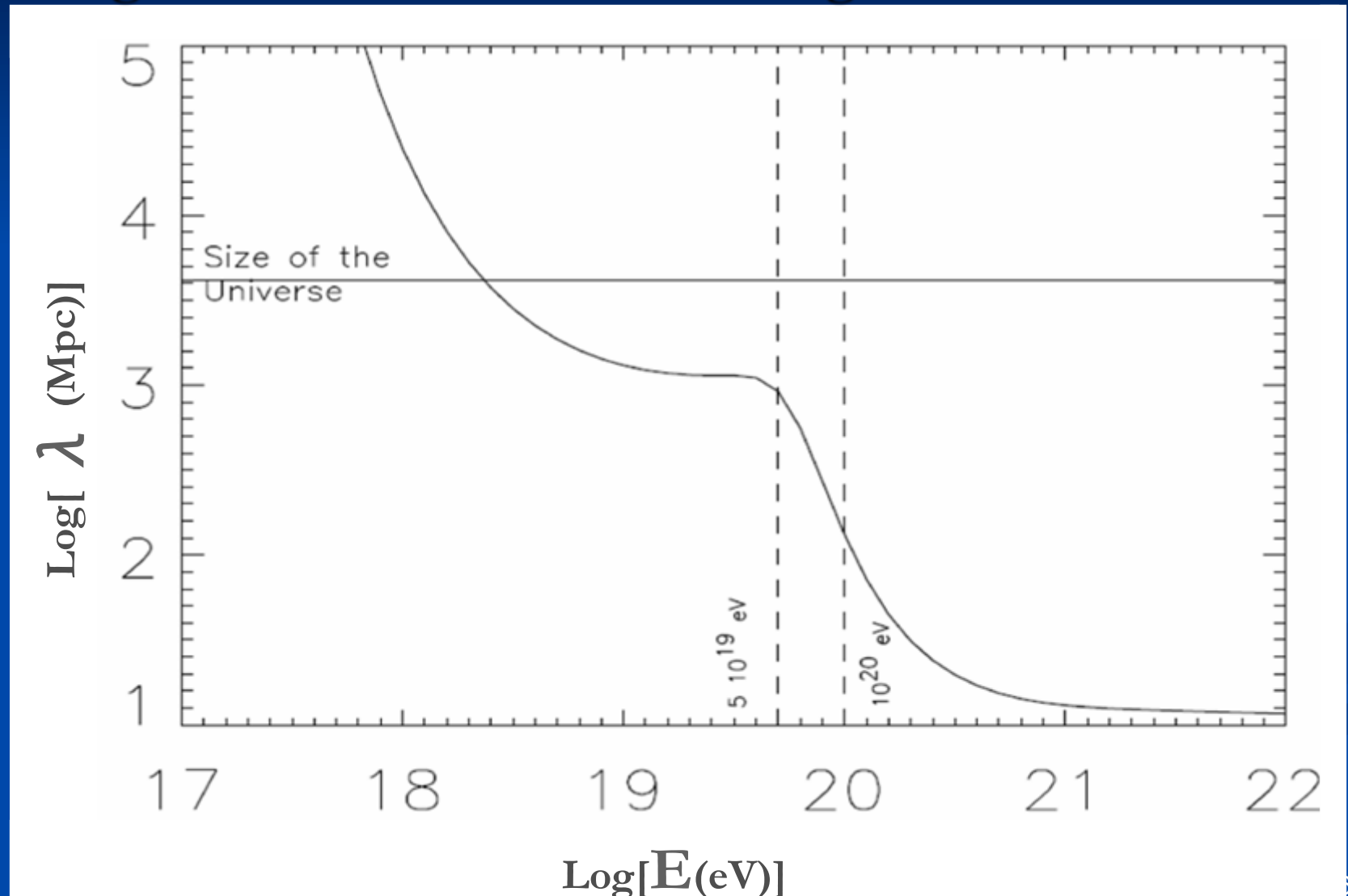
- L'energia di soglia per i protoni per produrre  $\pi$  è quindi

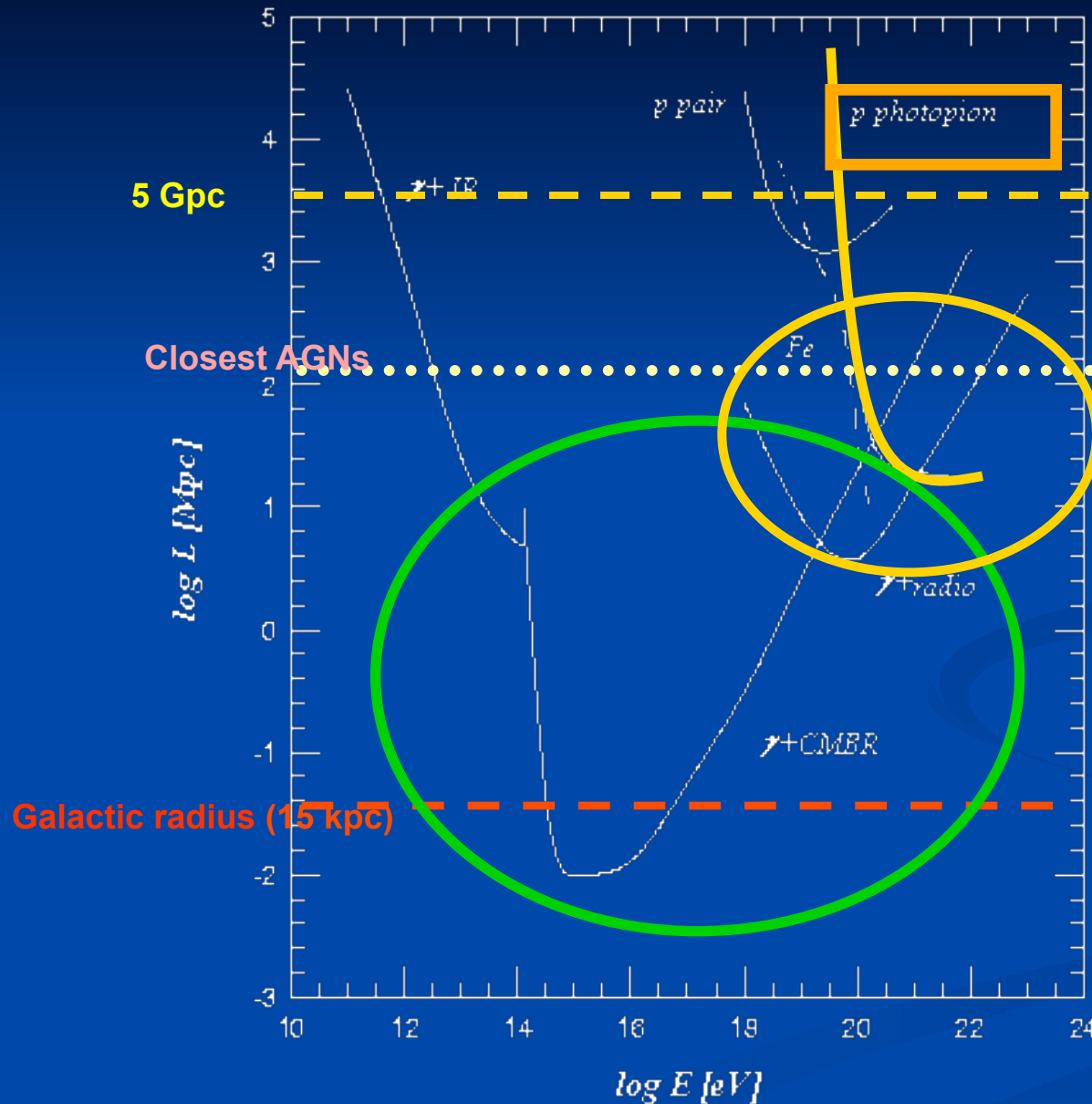
$$E_0^{GZK} = \gamma(m_p c^2) \approx 3 \cdot 10^{20} eV$$

- Se l'energia del protone supera  $E_0^{GZK}$ , si innesca la FP.
- In ogni processo, il p perde circa 1/10 della sua energia
- Nota la densità numerica della CMBR ( $n_\gamma = 400 \text{ cm}^{-3}$ ), si stima il cammino libero medio del p:

$$\lambda = (\sigma_{\gamma p} n_\gamma)^{-1} = 10^{25} \text{ cm} = 3 \text{ Mpc}$$

- Si può dunque stimare che i p **NON** possano giungere da distanze superiori a  $10 \times 3 \text{ Mpc} = 30 \text{ Mp}$
- Figura: Risultato di calcoli dettagliati.





**L'Universo NON è  
trasparente ai  
protoni di  
altissima energia**

**L'Universo NON è  
trasparente ai  
fotoni di alta  
energia**



# Ricerca delle sorgenti

- Nella reazione di fotoproduzione (responsabile del cutoff di GZK) sono prodotti  $\pi^\pm$ ,  $\pi^0$  che decadono:

$$\pi^+ \rightarrow \nu_\mu \mu^+ \rightarrow \nu_\mu \nu_\mu \nu_e e^+$$
$$\pi_0 \rightarrow \gamma\gamma$$

- Neutrini e fotoni di altissima energia possono quindi essere studiati per confermare il meccanismo GZK, e per localizzare le sorgenti di RC a  $E > 10^{19}$  eV

- Il RC di più alta energia osservato:  $\gtrsim 3.2 \cdot 10^{20}$  eV  
(?)

- Se le sorgenti non possono essere troppo lontane (<30 Mpc), possiamo cercare di localizzarle tramite:

- Studi di anisotropia con esperimenti di RC
- Confronto con altre misure astronomiche

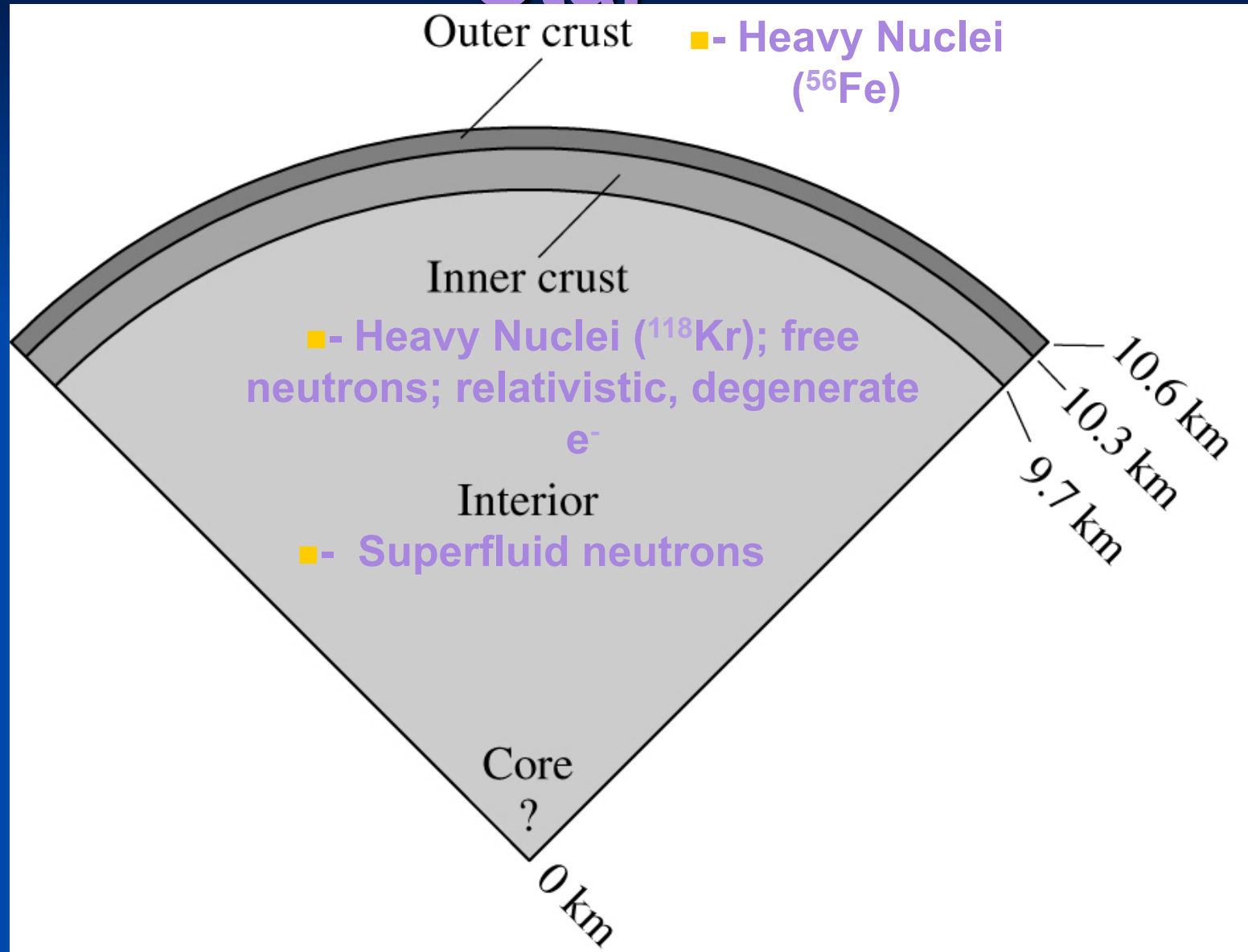
# Neutron Stars



## Gradual compression of a stellar iron core

[illegible]

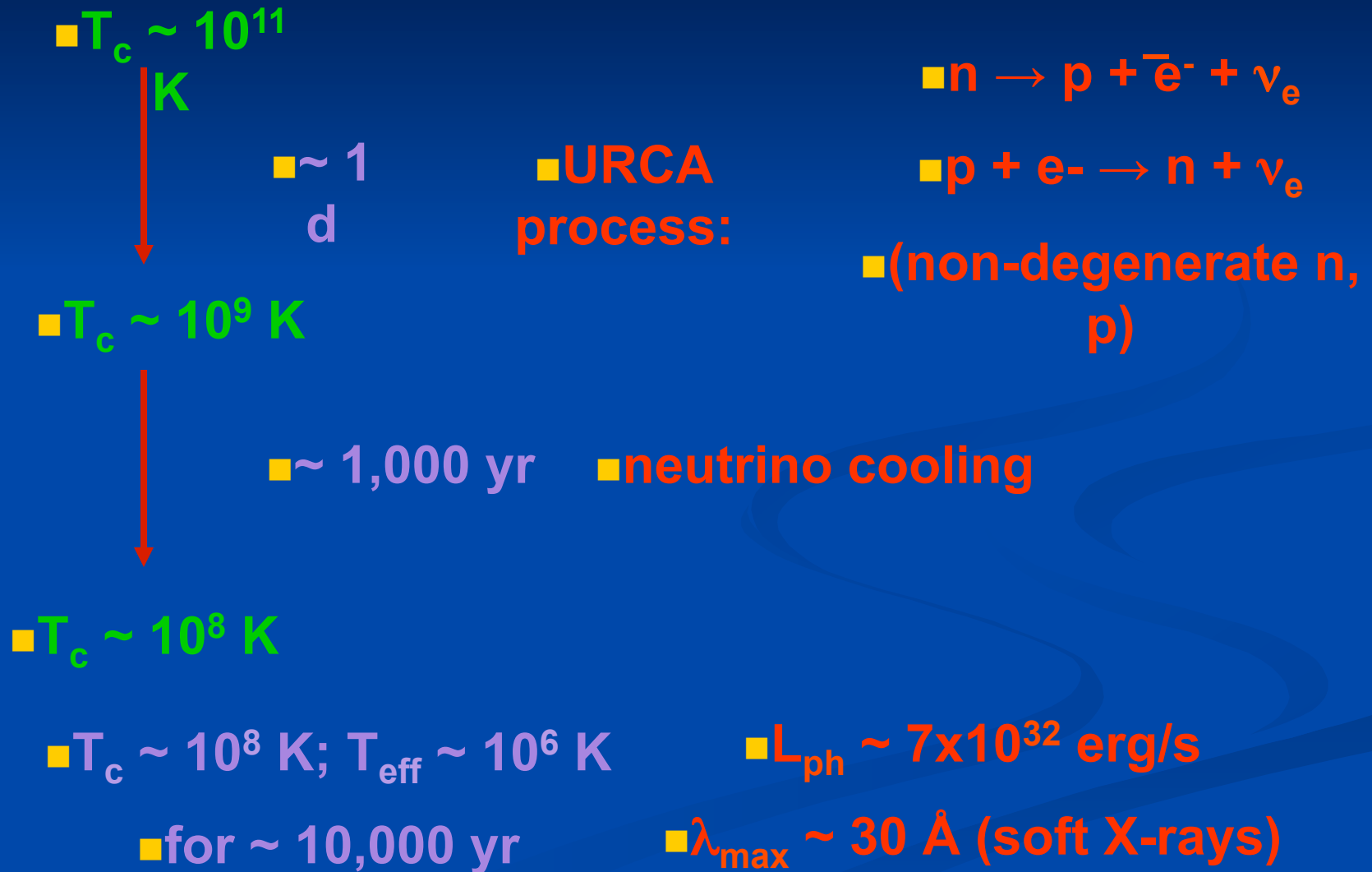
# Radial Structure of a Neutron Star



# Properties of Neutron Stars

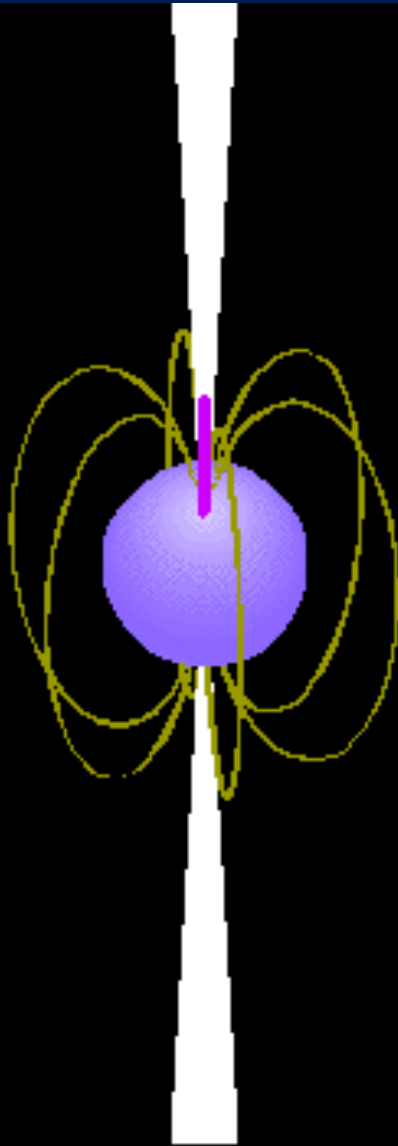
- Typical size:  $R \sim 10 \text{ km}$
- Mass:  $M \sim 1.4 - 3 M_{\text{sun}}$
- Density:  $\rho \sim 4 \times 10^{14} \text{ g/cm}^3$
- → **1 teaspoon** full of NS matter has a **mass of ~ 2 billion tons!!!**
- Rotation periods: ~ a few ms – a few s
- Magnetic fields:  $B \sim 10^8 - 10^{15} \text{ G}$ 
  - (Atoll sources; ms pulsars)
  - (magnetars)

# Neutron Star Cooling





# The Lighthouse Model of Pulsars



- A Pulsar's magnetic field has a dipole structure, just like Earth.

- Radiation is emitted mostly along the magnetic poles.

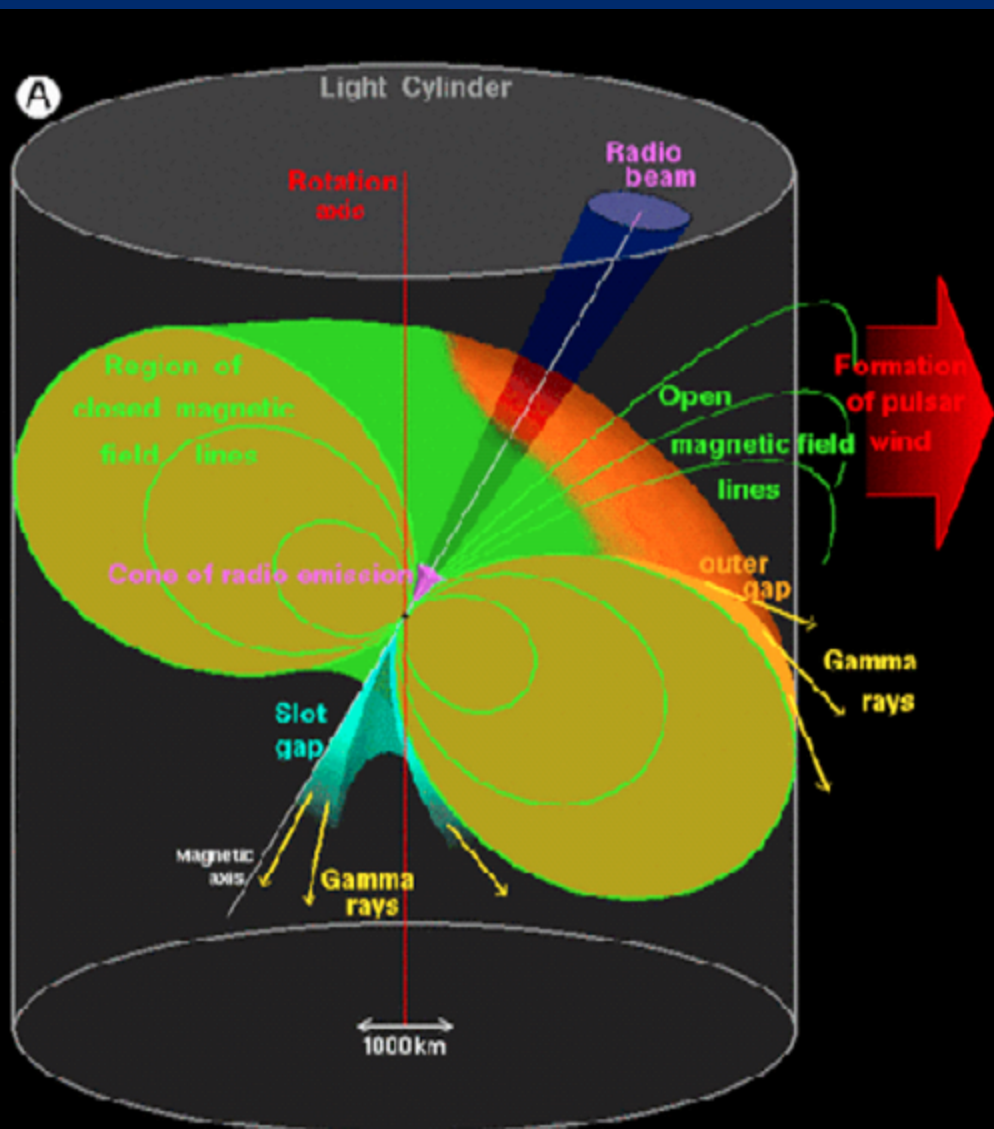
- Rapid rotation along axis not aligned with magnetic field axis

- → Light house model of pulsars

- Pulses are not perfectly regular

- → gradual build-up of average pulse profiles

# Stelle di neutroni



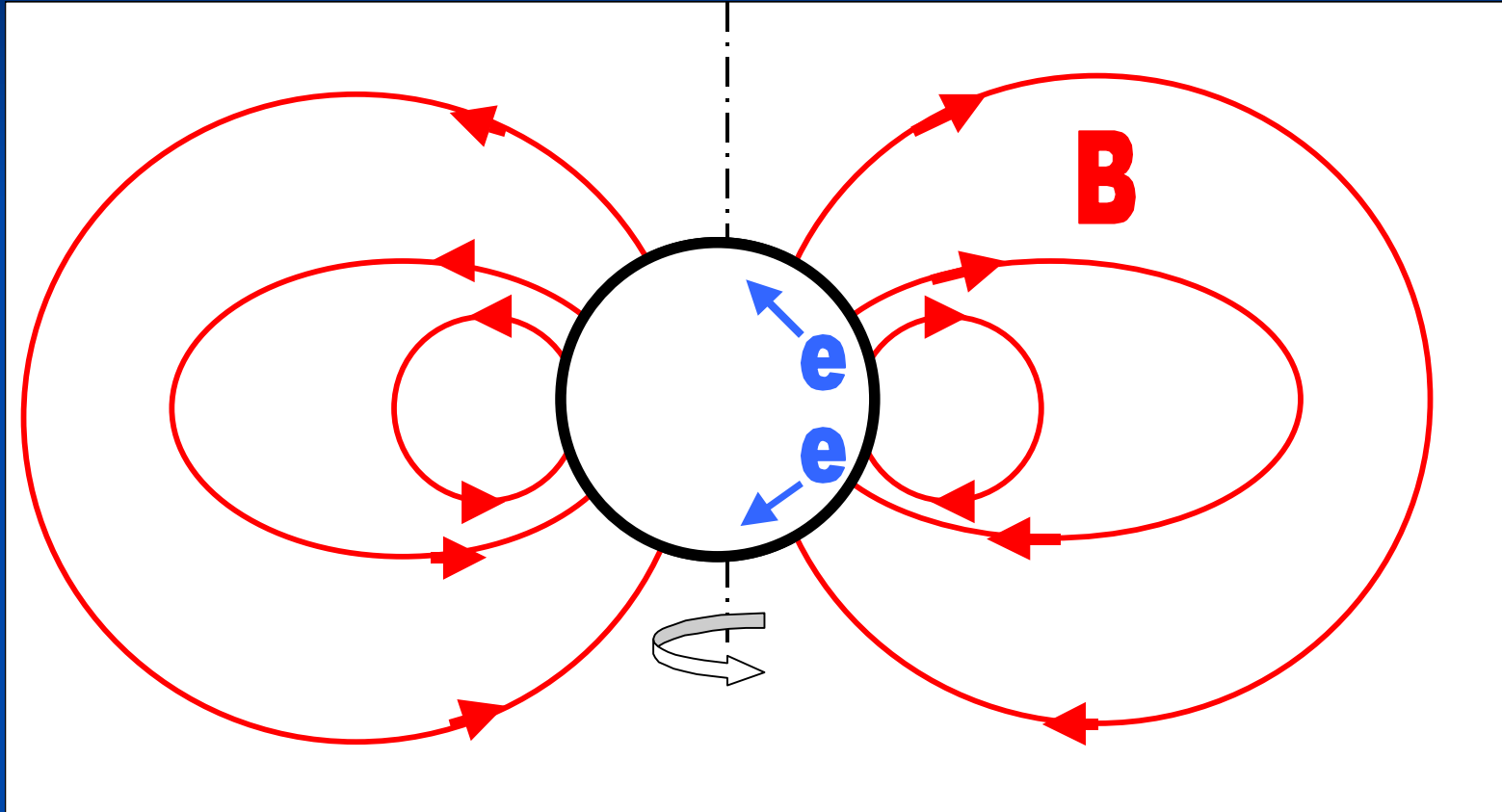
- Stelle di neutroni con intensi campi magnetici superficiali rotanti

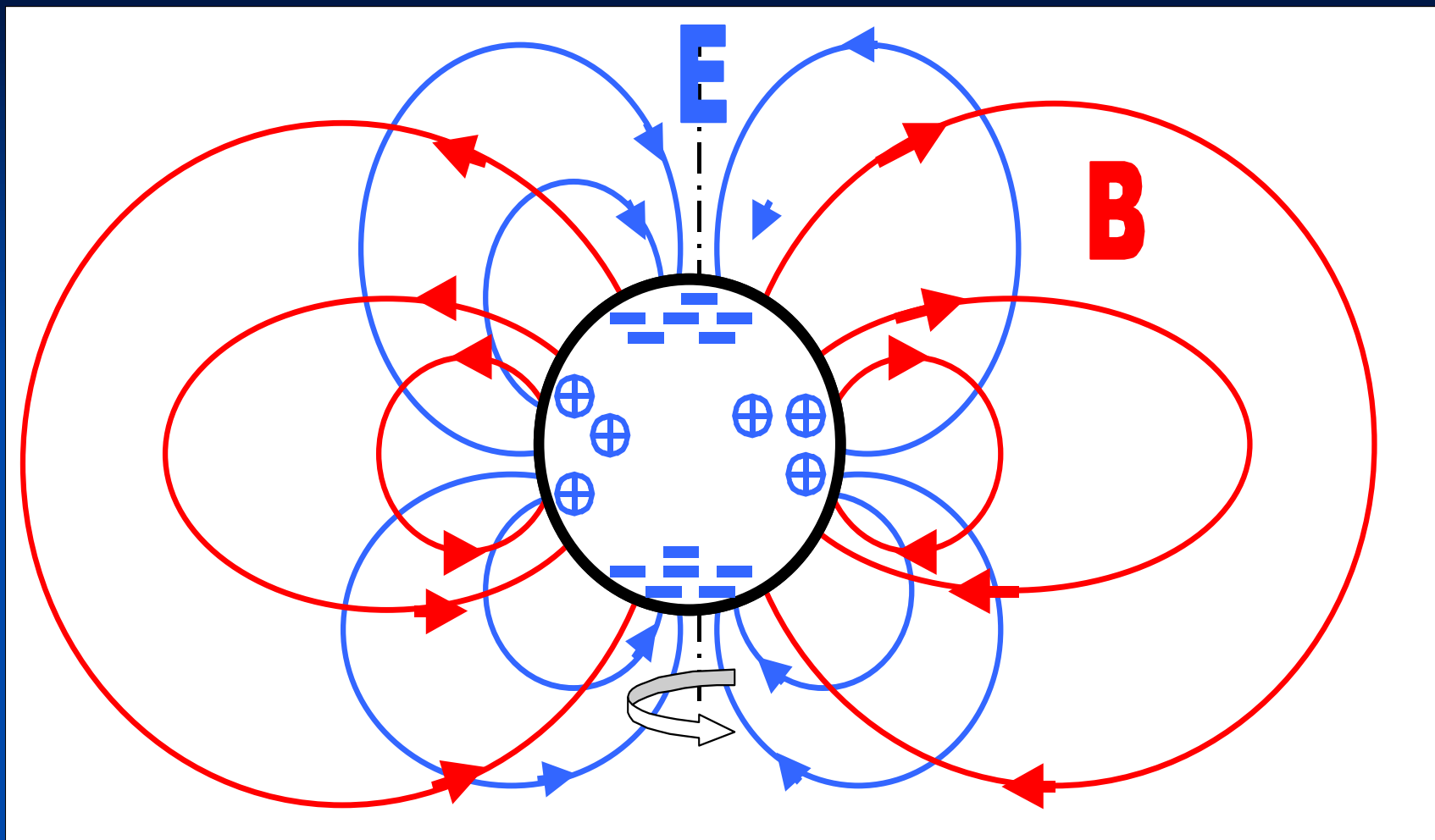
- Ad una certa distanza  $B$  non è più chiuso perchè non può essere in rotazione con  $v > c$



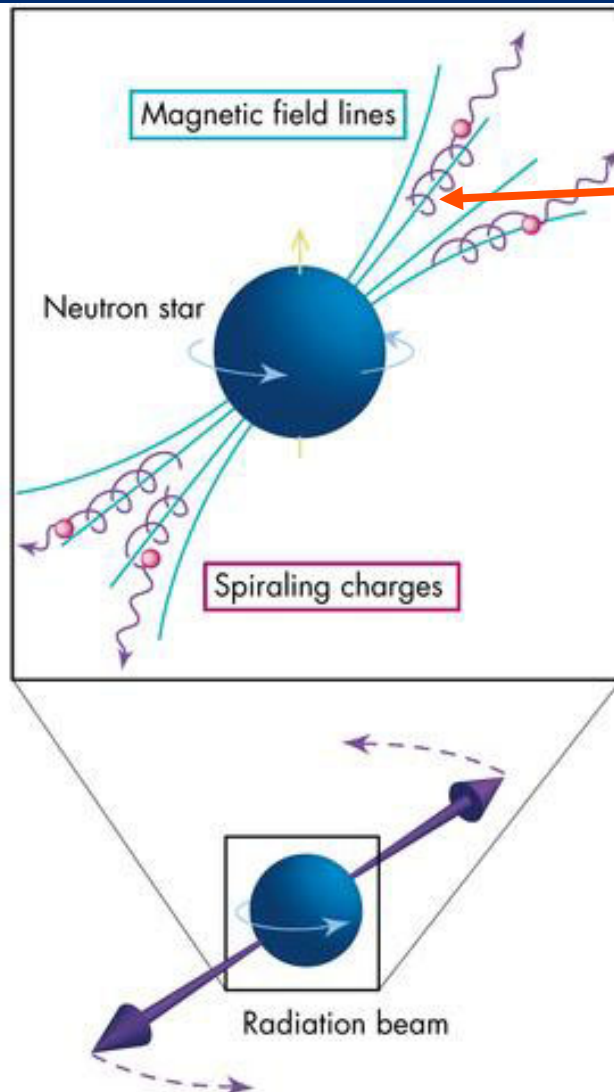
- Formazione di vento relativistico ( $\Gamma \gg 10^7 - 10^{10}$ ), e accelerazione forse fino a  $10^{20}$  eV

# Rotating Magnetic Field





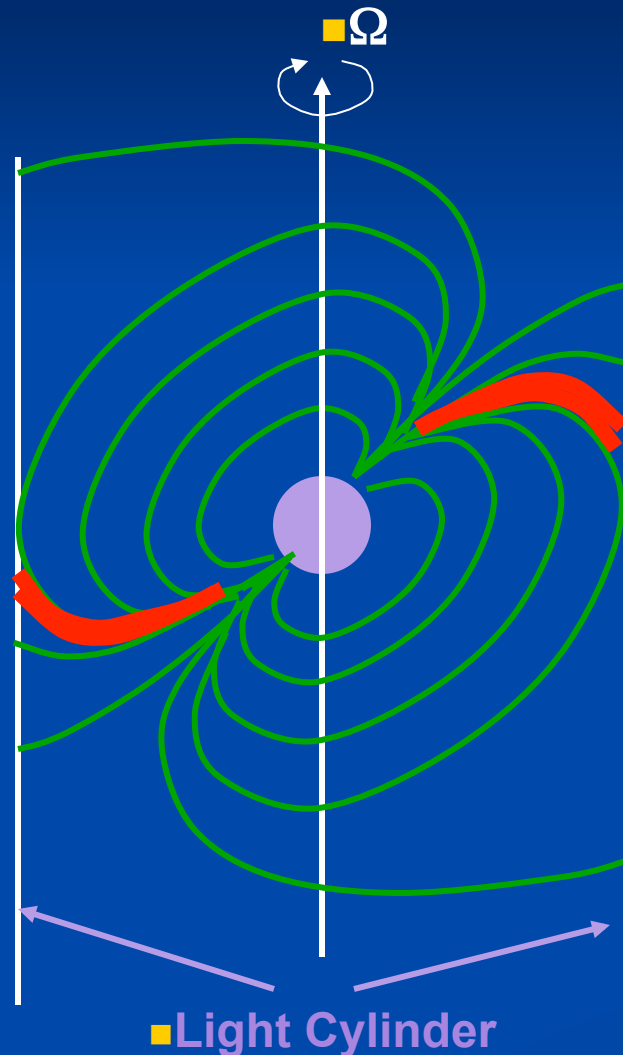
# Pulsar Emission Models: Polar Cap model



- Particle acceleration along magnetic field lines
- Synchrotron emission
- Curvature radiation
- Pair production
- Electromagnetic cascades

# ■ Pulsar Emission Models:

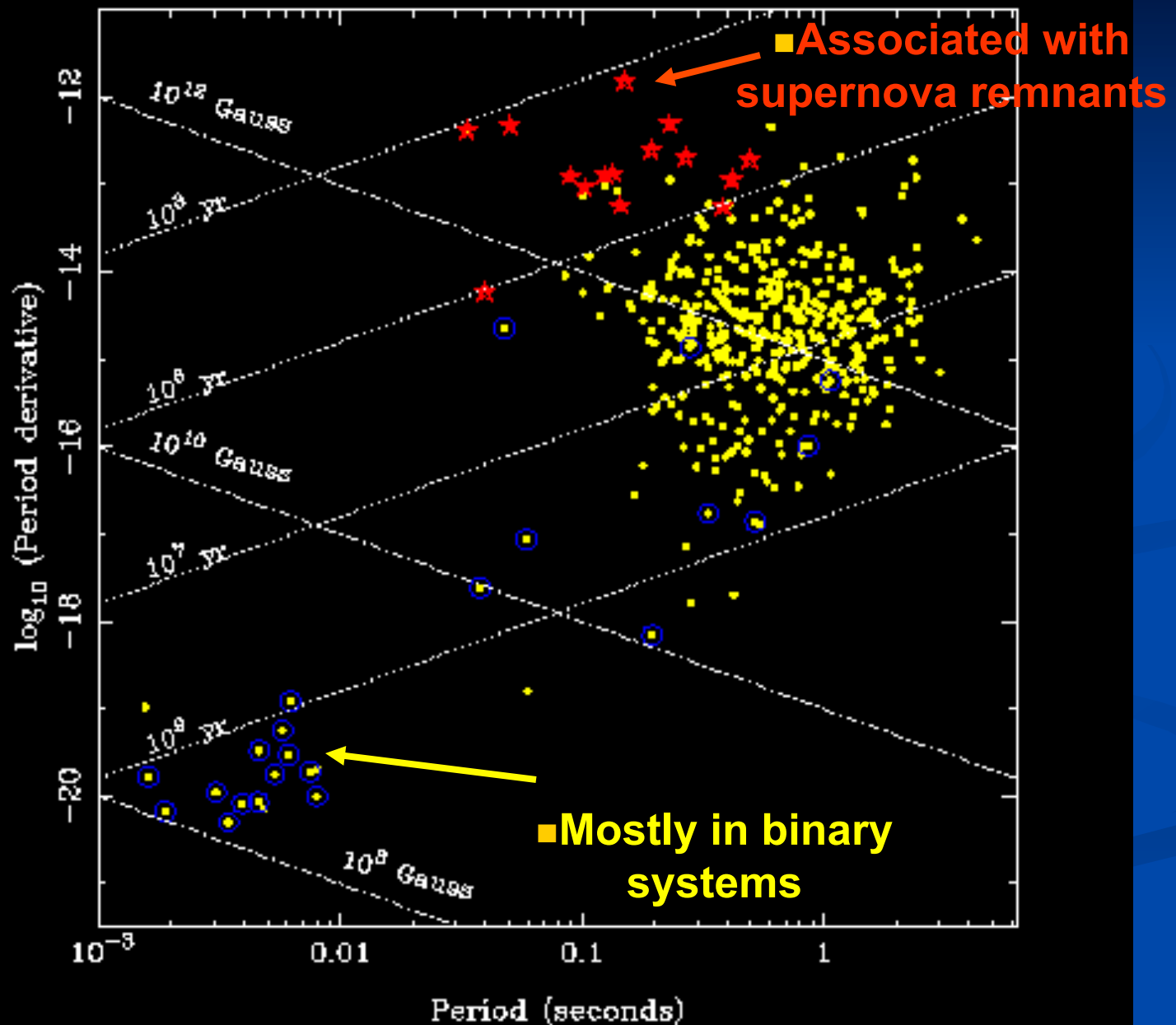
## Outer Gap model



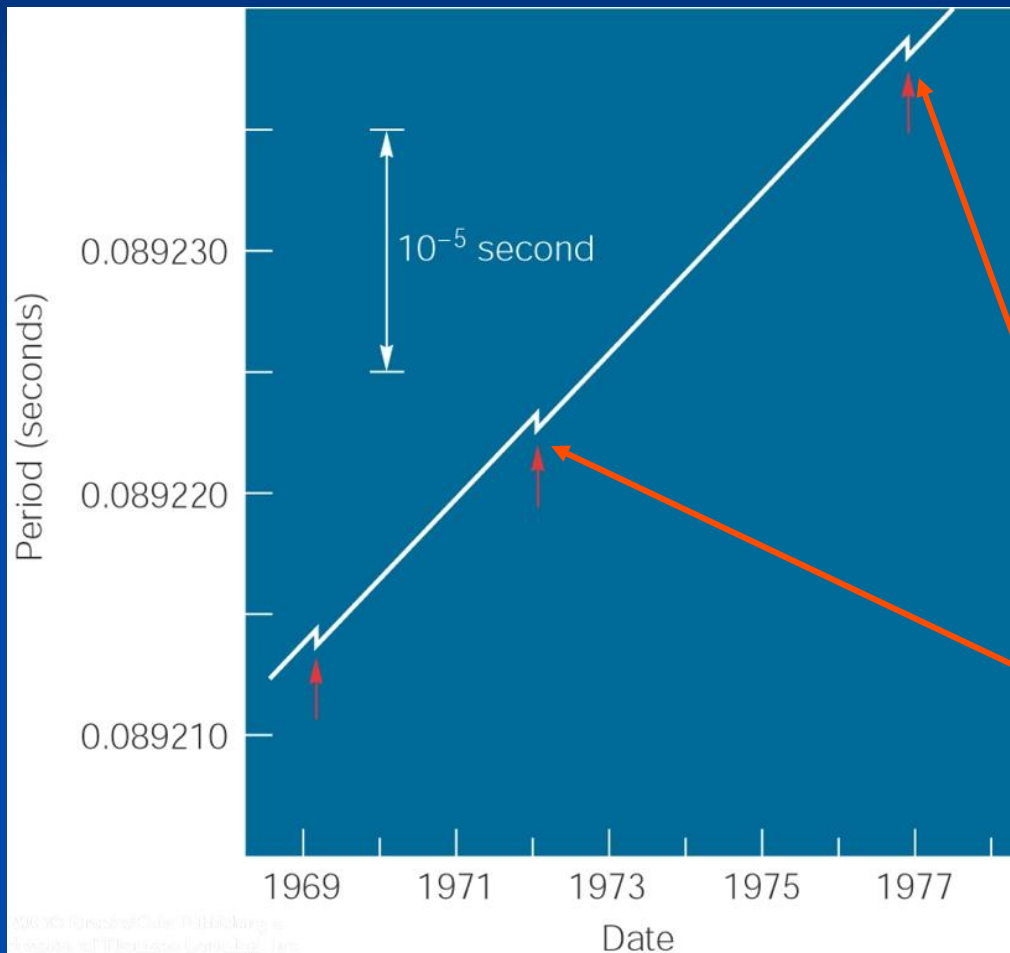
- Electrons are bound to magnetic fields co-rotating with the pulsar
- At a radial distance  $r = c/\Omega$
- co-rotation at the speed of light
  - → “light cylinder”
- → Particles ripped off magnetic fields
- **Synchrotron emission**
- **Curvature radiation**



# Pulsar periods and derivatives



# Pulsar periods



■ Over time, pulsars lose energy and angular momentum

■ => Pulsar rotation is gradually slowing down.

■  $dP/dt \sim 10^{-15}$

■ Pulsar

Glitches:  $\Delta P/P \sim 10^{-8}$

# Energy Loss of Pulsars

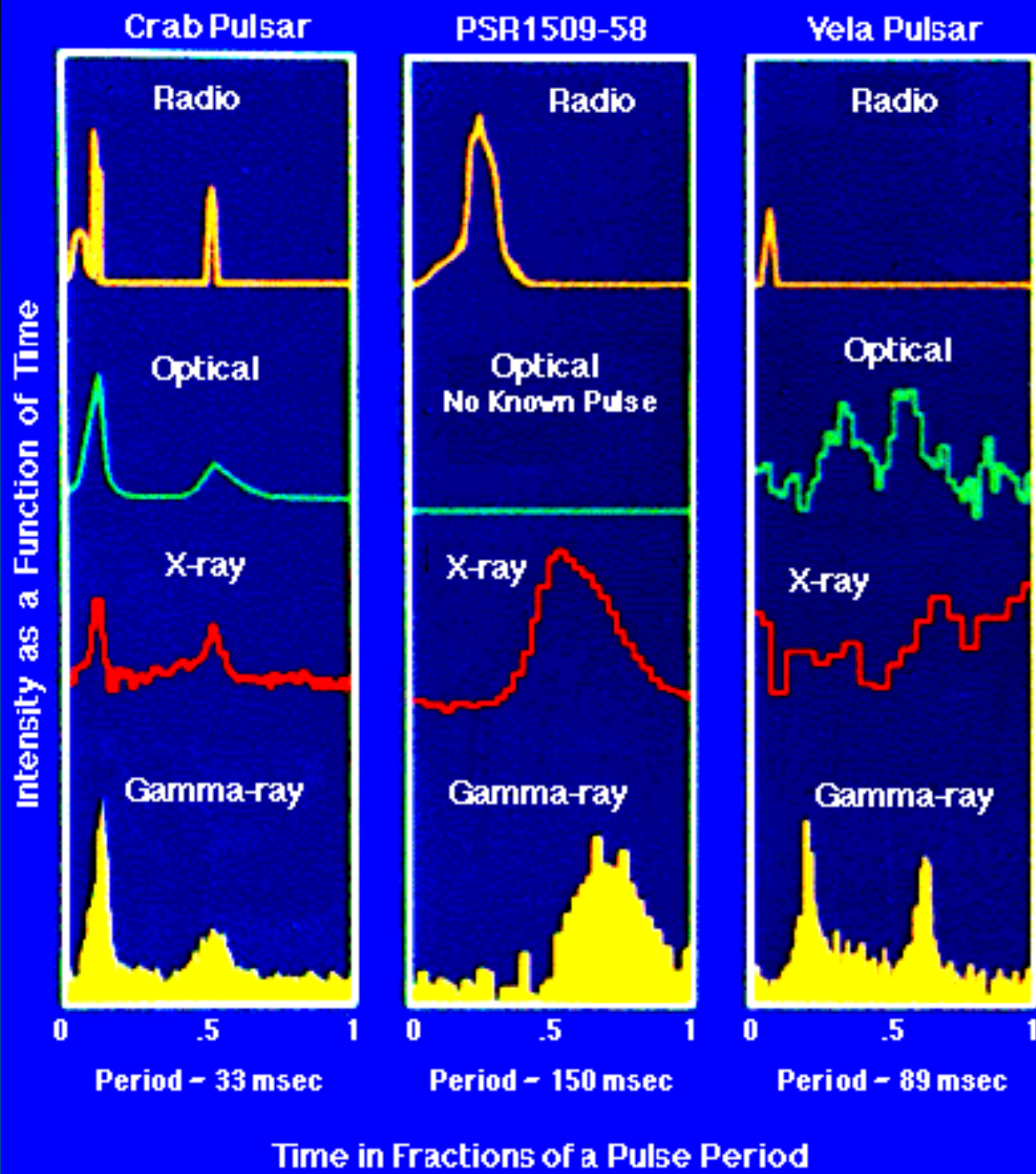
■ From the gradual spin-down of pulsars:

$$\frac{dE}{dt} = \frac{d}{dt} \left( \frac{1}{2} I \omega^2 \right) = I \dot{\omega} \omega = - \frac{1}{6} \mu_{\perp}^2 \omega^4 r^4 c^{-3}$$

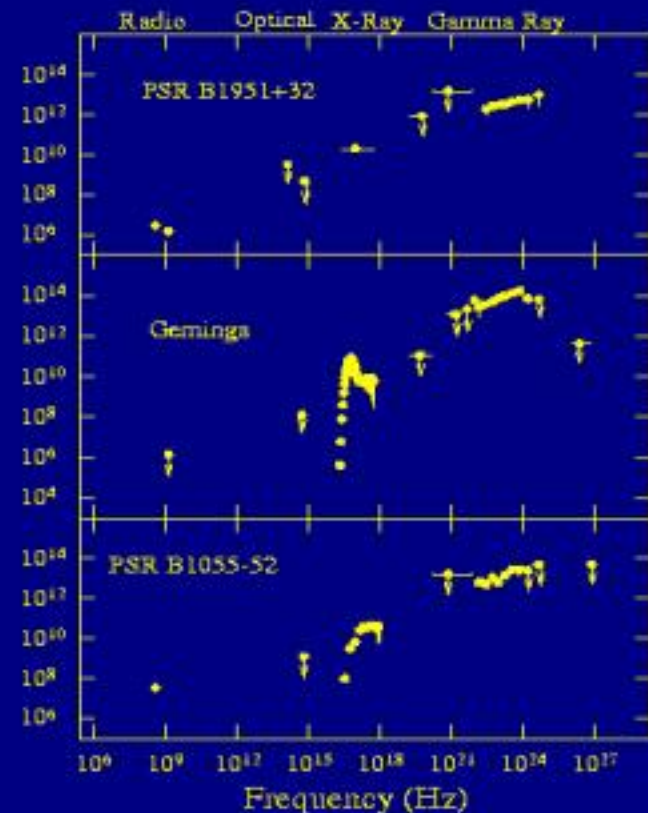
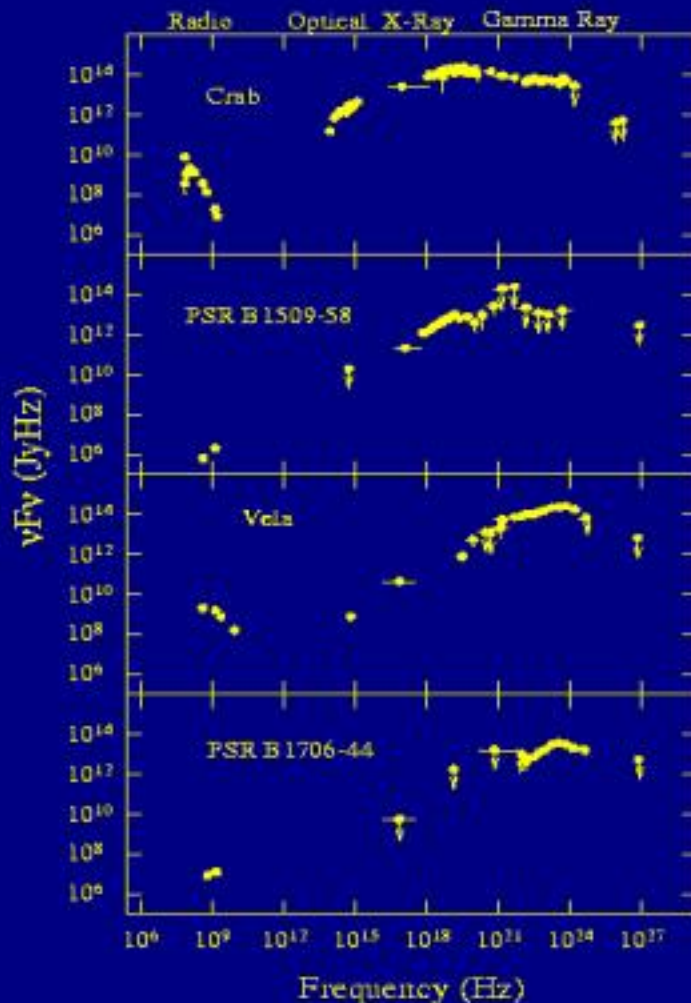
$$\mu_{\perp} \sim B_0 r \sin \alpha$$

■ One can estimate the magnetic field of a pulsar as

$$B_0 \approx 3 \times 10^{19} \sqrt{\dot{P} P} \text{ G}$$



# $\gamma$ -Ray Pulsars



Multiwavelength Energy Spectra  
of Pulsars

(Summary from Thompson, 1996)

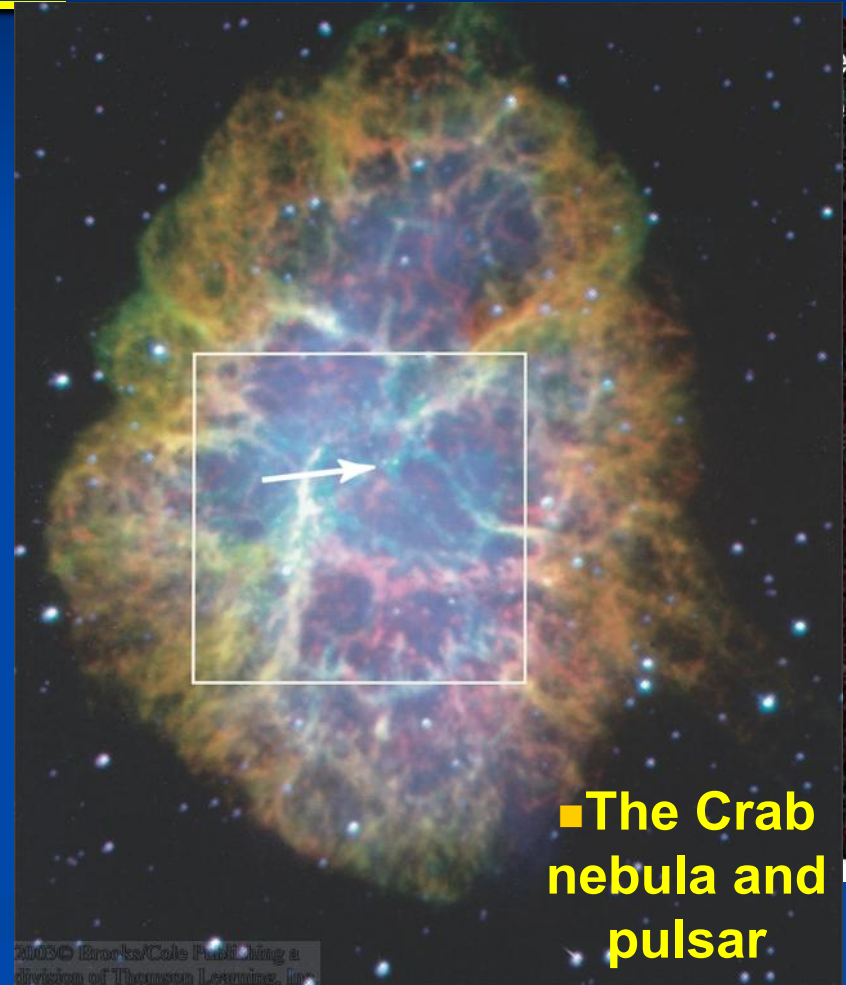
- Detected pulsars have highest  $\dot{E} = I \cdot \Omega \cdot \dot{\Omega}$  !



# Images of Pulsars and other Neutron Stars



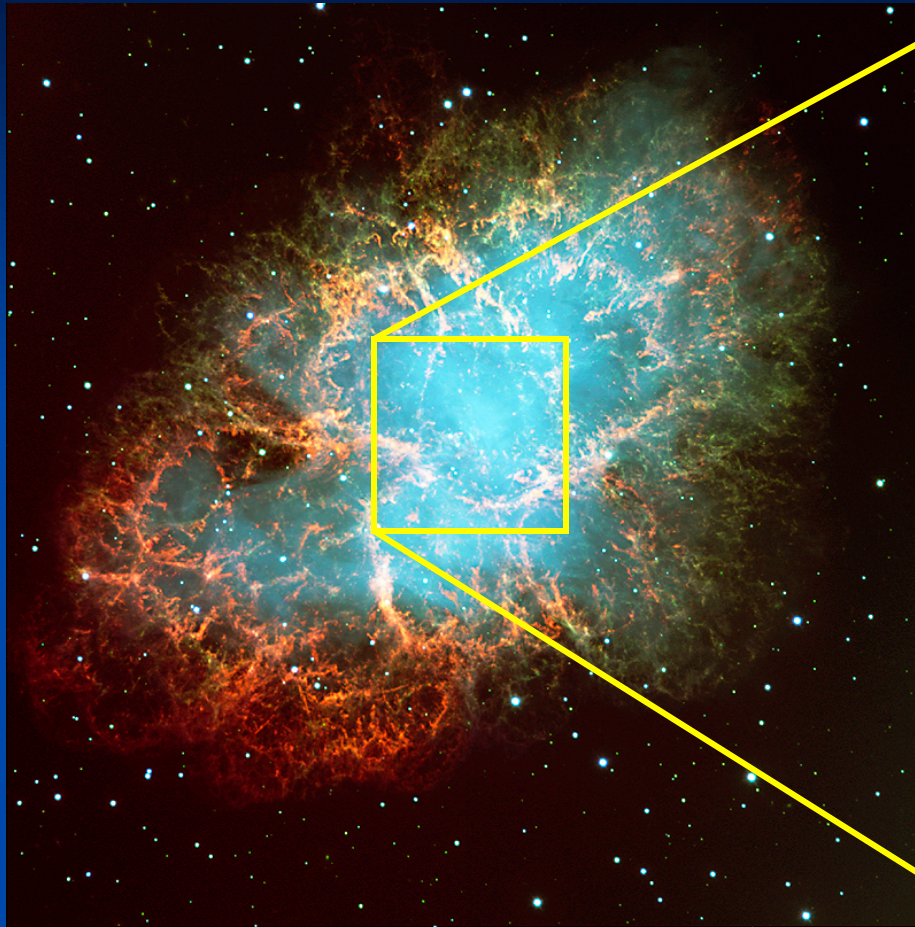
■ The Vela Pulsar moving  
through interstellar  
space



■ The Crab  
nebula and  
pulsar



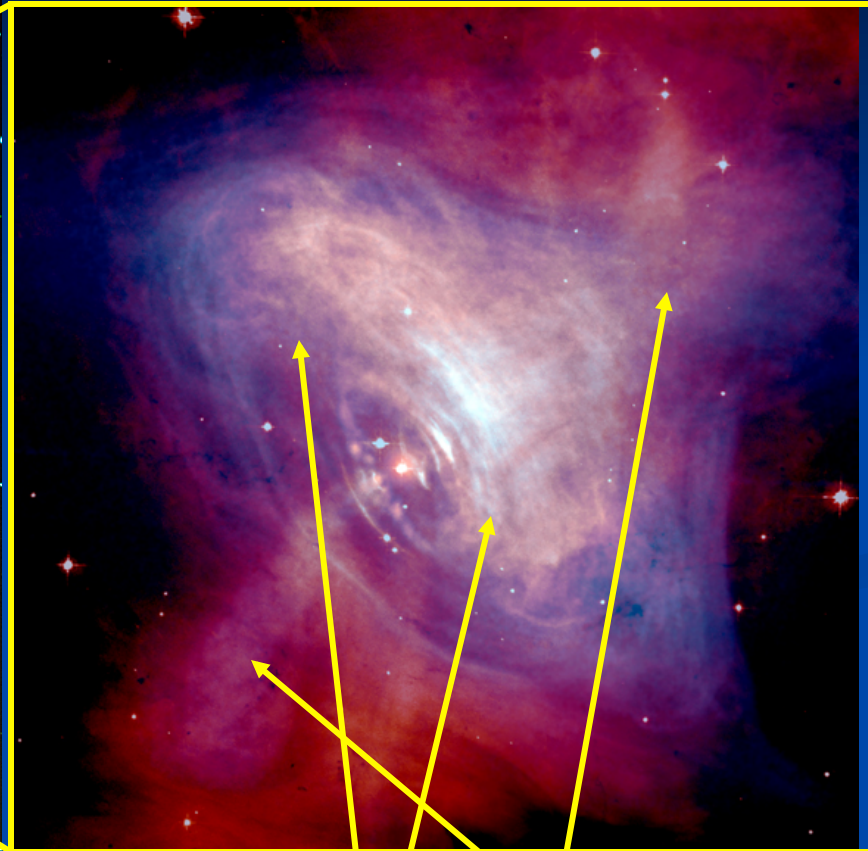
# The Crab Pulsar



The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

ESO PR Photo 40f/99 ( 17 November 1999 )

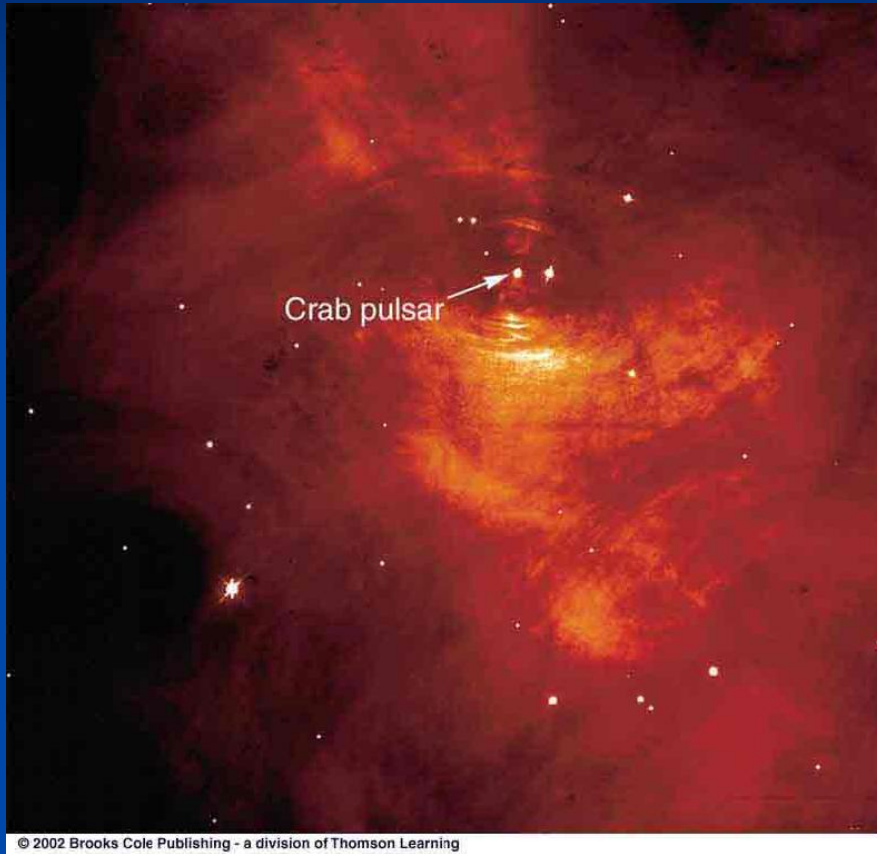
© European Southern Observatory



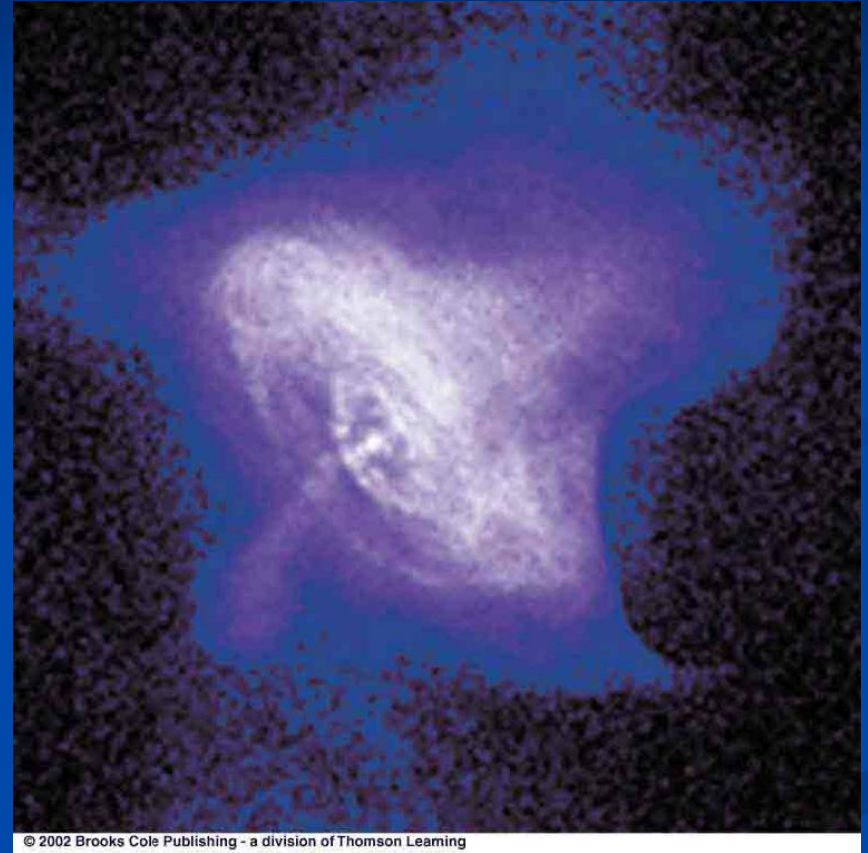
■ Pulsar wind + jets

■ Remnant of a supernova observed in A.D.  
1054

# The Crab Pulsar



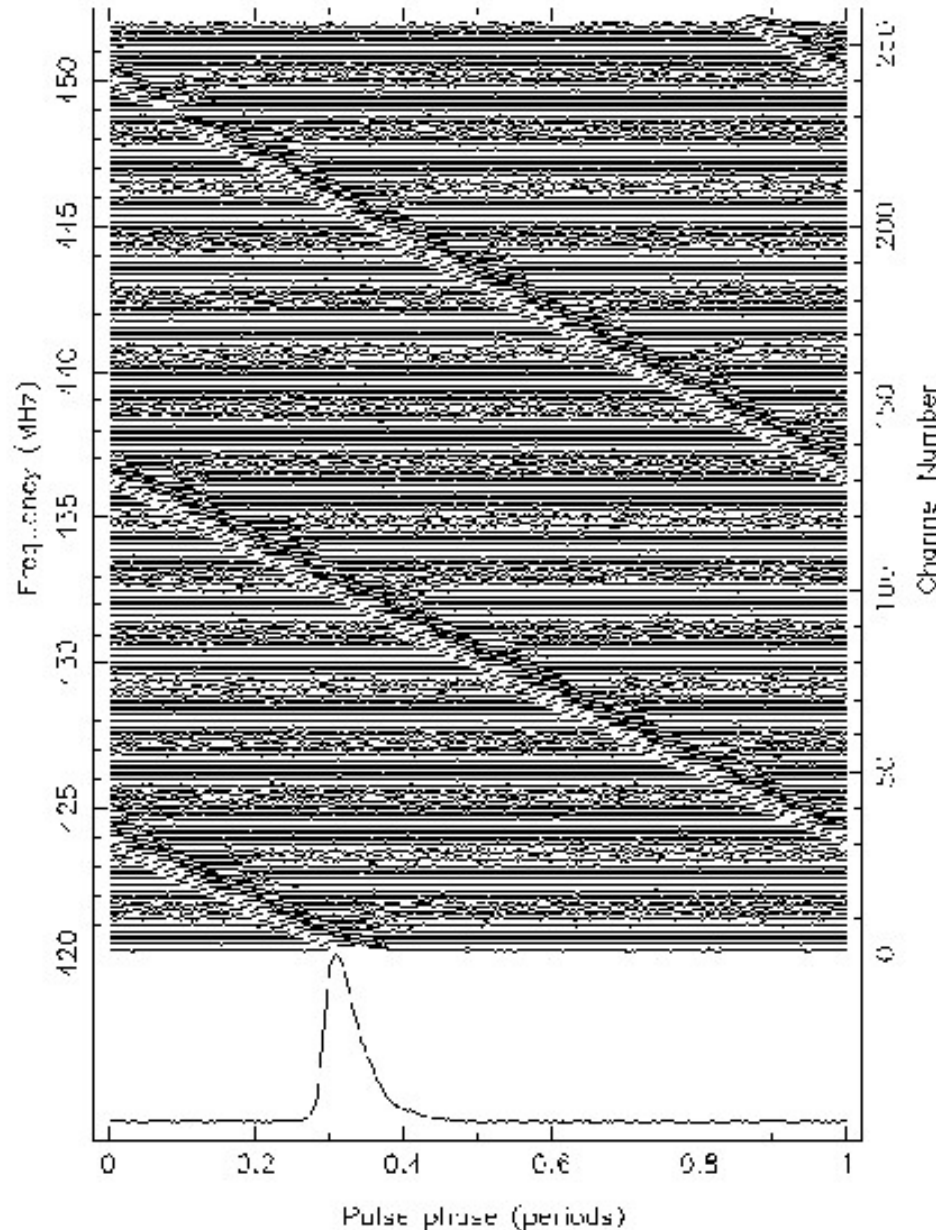
■ Visual image



■ X-ray image



# Dispersion of Pulsar Signals



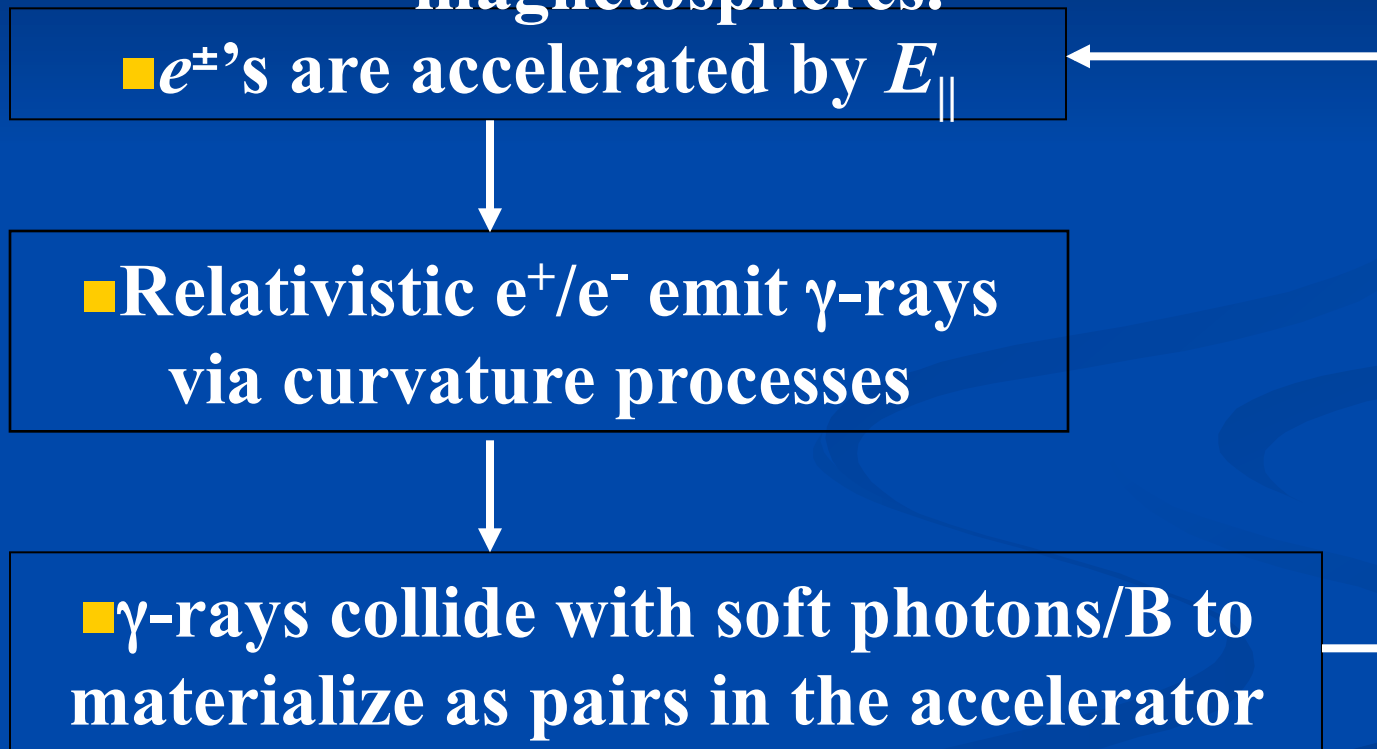
$$\delta t = \left( \frac{4\pi e^2}{m_e c \omega_1^3} \right) \delta \omega \text{ DM}$$

$$\text{DM}^d = \int n_e(s) ds$$

DM = Dispersion Measure

### §3 New gap theory: *Self-consistent* approach

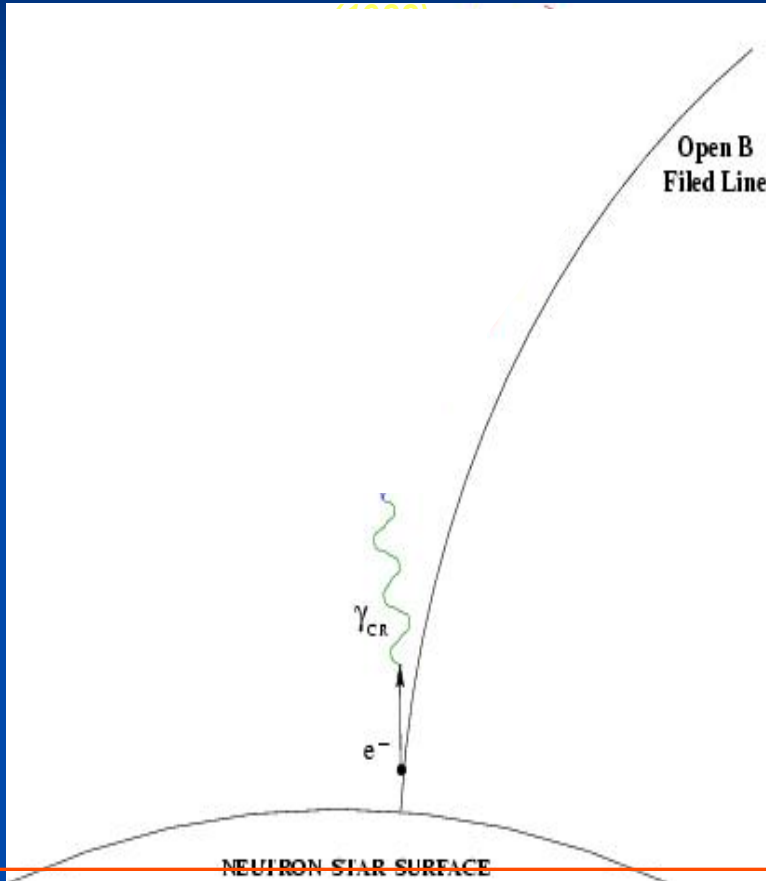
- How to construct a **self-consistent** particle accelerator theory in pulsar magnetospheres.



# ■ Models of $\gamma$ -ray emission in Pulsars

## Polar Cap Model

Sturrock (1971); Ruderman & Sutherland (1975); Harding (1981); Daugherty & Harding (1982)



- Acceleration of electrons
- Cooling mechanisms
  - a) Curvature radiation
  - b) Synchrotron, I.C. of X-rays
- $\gamma$ -rays interact with magnetic field, via *Magnetic pair production*

$$\gamma + \vec{B} \rightarrow e^+ + e^-$$

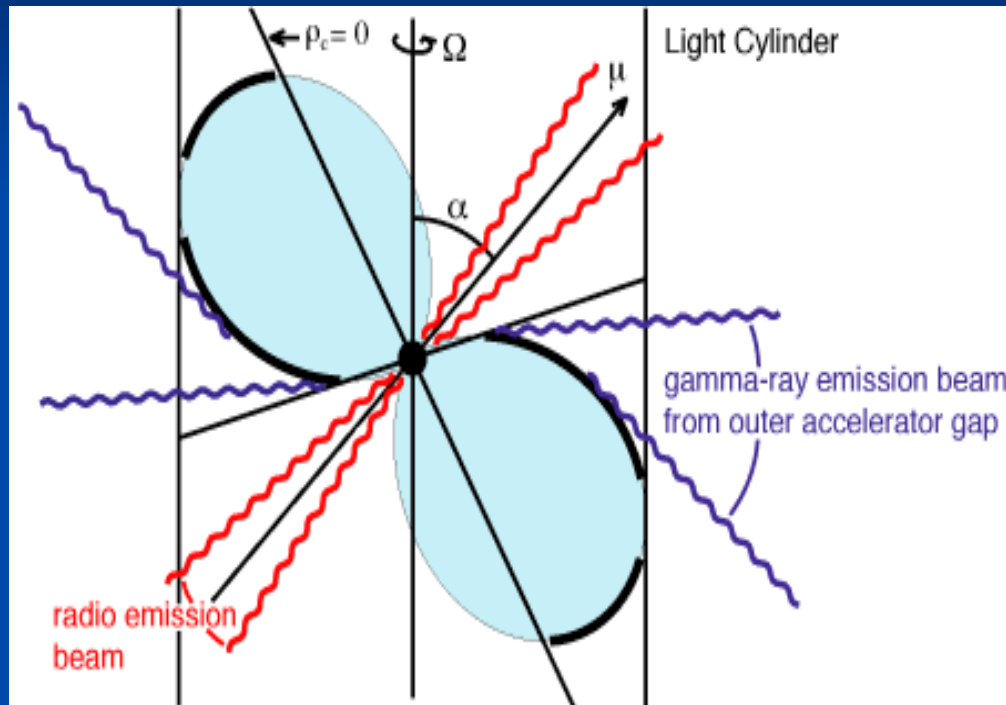
$$\kappa \propto B_p \cdot \exp(-1 / E_\gamma B_p)$$

Polar Cap model predicts **super-exponential cutoff** in high energy  $\gamma$ -ray

# ■ Models of $\gamma$ -ray emission in Pulsars

## Outer Gap model

Cheng, Ho & Ruderman (1986); Romani (1996)



■  $\gamma$ -ray emission occurs near LC

■ Charges accelerated in vacuum gap

→  $\gamma$ -rays via *Curv. rad.*

■ B not strong enough for pair-production. But, curvature photons interact with non-thermal X-

$$\gamma\gamma \rightarrow e^+e^-$$

**Softer *exponential cutoff* in the high energy  $\gamma$ -ray spectra !**

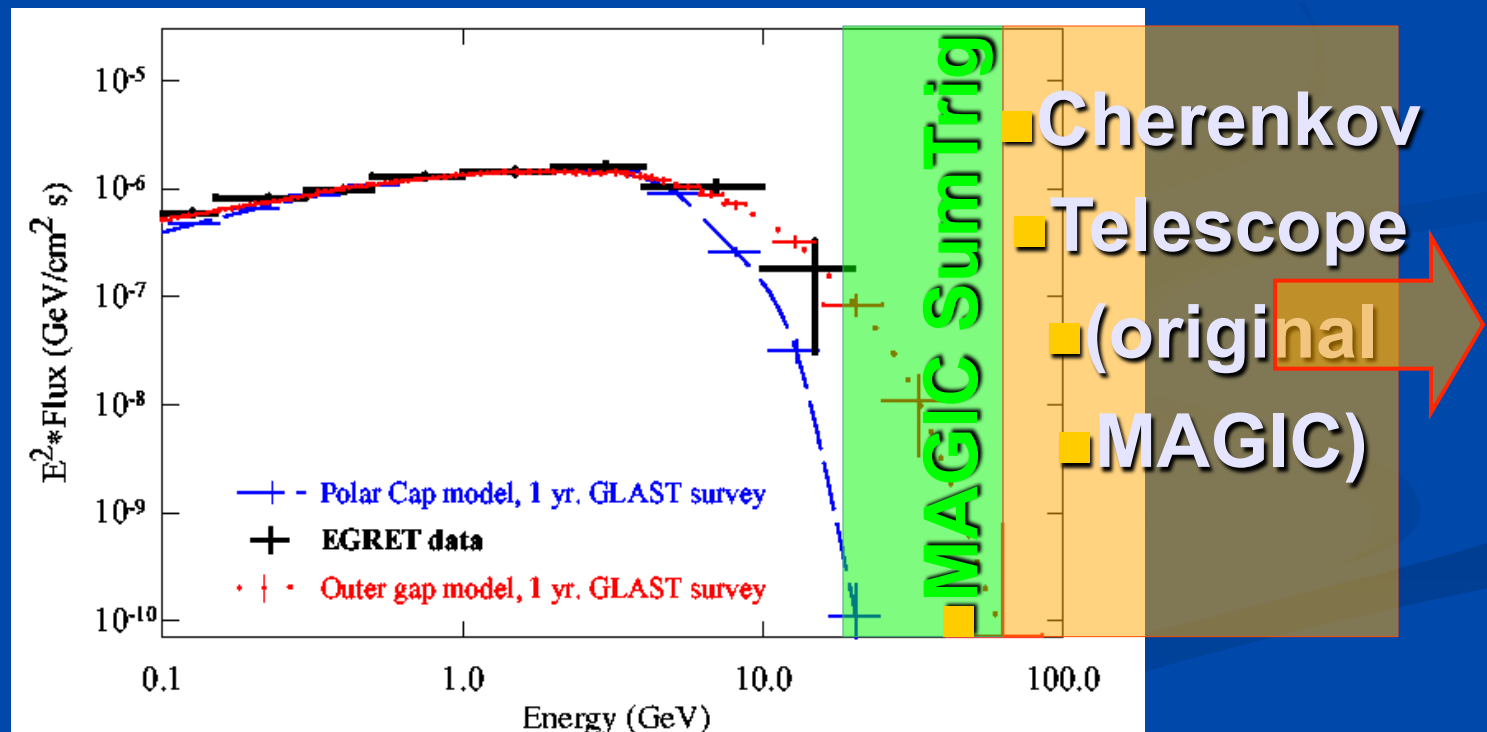
Electrons may up scatter IR photons to TeV Gamma-rays !



# ■ Where do $\gamma$ -rays come from? Outer gap or polar cap?

## Discrimination between models

- Different models predict different spectral cutoff.
- Measuring the spectral tail is possible to distinguish between models.





# ■ Astroparticle Physics : Fermi's Theories of Shock Acceleration - II

Pratik Majumdar  
SINP, Kolkata

## Outline:

- Shock Acceleration
- Fermi's theory of 2<sup>nd</sup> and 1<sup>st</sup> order acceleration
- Application to simple cases

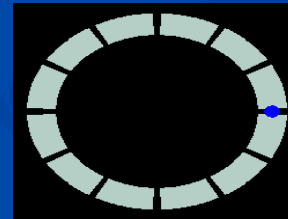
# Reading Materials

- Longair : High Energy Astrophysics
- T. Stanev : High Energy Cosmic Rays
- T. Gaisser : Particle Physics and Cosmic rays
- Many review articles on the subject

# ■ Acceleration of ■ Cosmic Rays



## ■ Man-made accelerators



■ No. of particles

■ Energy  
y

# Electromotive Acceleration

Fundamentally, Lorentz Force law  $\Rightarrow$

$$\frac{d}{dt}(\gamma m v) = e(\vec{E} + \vec{v} \wedge \vec{B})$$

But, on macroscopic scales, no electrostatic fields, i.e.,  $\langle \vec{E} \rangle = 0$ .

Also, for a stationary  $\vec{B}$ ,  $\frac{d}{dt}|\vec{v}| = 0$ .

Need “moving” magnetic fields, i.e., acceleration by electromotive force.

Expect, rate of energy gain by relativistic particles = work done by the electromotive force =  $\frac{d\mathcal{E}}{dt} = \xi Z e c B . c$ , with  $\xi < 1$ .

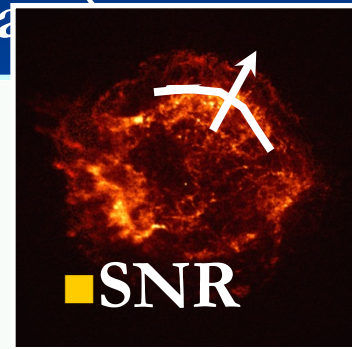
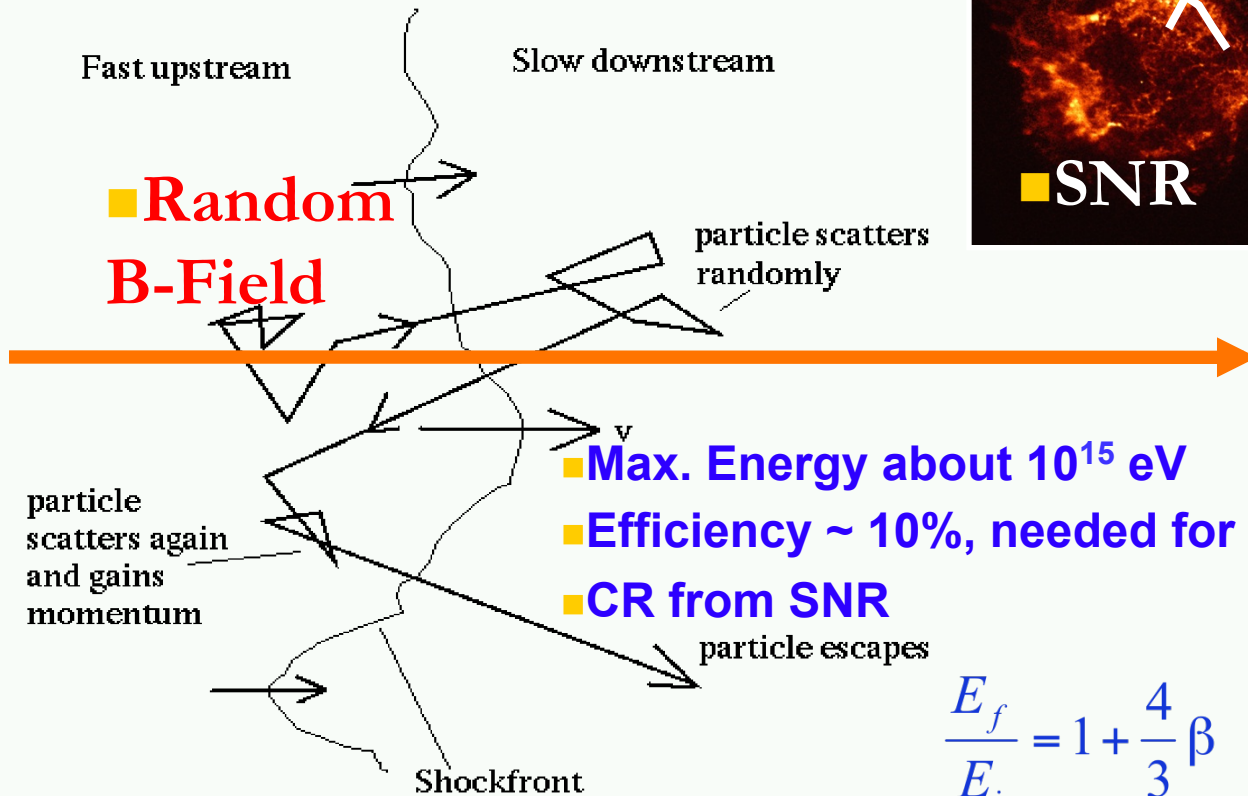
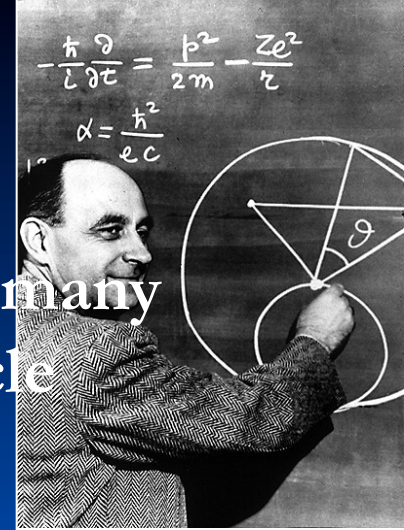
Fermi's original suggestion: Encounters of charged particles with moving, magnetized, interstellar clouds.



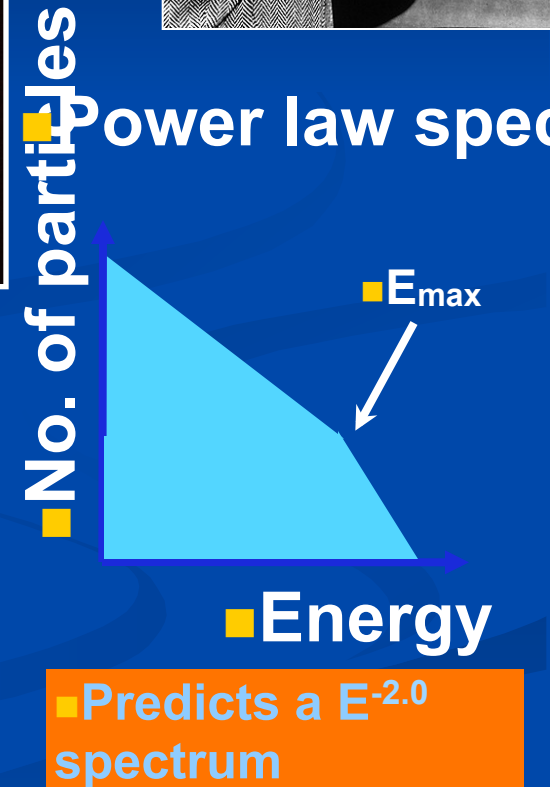
# Shock acceleration mechanism

(by Enrico Fermi)

- Particles (electrons and hadrons) get scattered many times in shock front and gain energy in each cycle (TeV energies → several 100 years)



$$\frac{E_f}{E_i} = 1 + \frac{4}{3} \beta$$



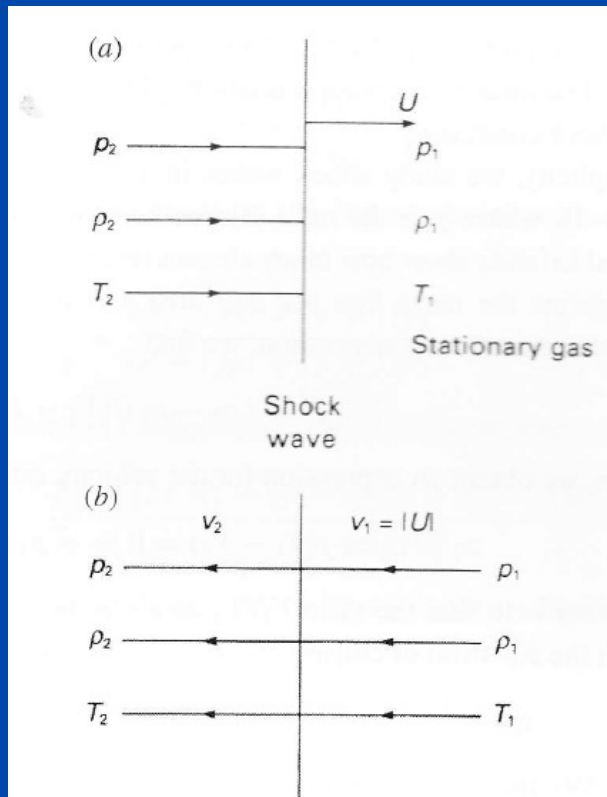
# ■ Fermi Acceleration



- Stochastic Mechanism
- Charged particles collide with clouds in ISM and
  - are reflected from irregularities
- in galactic magnetic field
  - 2<sup>nd</sup> order

- Charged particles can be accelerated to high energies in astrophysical shock fronts
- 1<sup>st</sup> order acceleration

- A shock is a transition layer where the fluid flow velocity changes discontinuously.
- An object moving with supersonic speed through a medium will drive a shock through that medium.
- Shocks are ubiquitous in astrophysical media because sound speed is low (because of low density), so it's easy to exceed sound speed in such media.



- Shock wave propagating through a
- Stationary gas at supersonic velocity  $U$
- Flow of gas in a reference frame
- In which the shock front is stationary

$$\rho_1 v_1 = \rho_2 v_2 .$$

$$\rho_1 v_1 \left( \frac{1}{2} v_1^2 + w_1 \right) = \rho_2 v_2 \left( \frac{1}{2} v_2^2 + w_2 \right) .$$

- Energy flux through
- a surface normal to  $v$

$w = \varepsilon_m + pV$ ,  $\varepsilon_m$  is the internal energy per unit mass and  $V$  is the specific volume

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 .$$

- Momentum flux through shock wave

# Shock Conditions Solutions

- Solve for shock in perfect gas.

$$\gamma p V / (\gamma - 1)$$

- Enthalpy =

the mass flux per unit area  $j = \rho_1 v_1 = \rho_2 v_2$ .

$$\frac{V_2}{V_1} = \frac{p_1(\gamma + 1) + p_2(\gamma - 1)}{p_1(\gamma - 1) + p_2(\gamma + 1)},$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{V_2}{V_1} = \frac{p_2}{p_1} \frac{p_1(\gamma + 1) + p_2(\gamma - 1)}{p_1(\gamma - 1) + p_2(\gamma + 1)}.$$

$$v_1^2 = j^2 V_1^2 = \frac{V_1}{2} [(\gamma - 1)p_1 + (\gamma + 1)p_2],$$

$$v_2^2 = j^2 V_2^2 = \frac{V_2}{2} \frac{[p_1(\gamma + 1) + p_2(\gamma - 1)]^2}{p_1(\gamma - 1) + p_2(\gamma + 1)}.$$

$$M_1 = U/c_1 = v_1/c_1$$

$$c_1 = (\gamma p_1 / \rho_1)^{1/2}.$$

$$\frac{v_2}{v_1} = \frac{\gamma + M_1^{-2} \pm (1 - M_1^{-2})}{\gamma + 1}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1 + 2M_1^{-2}}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)][\gamma - 1 + 2M_1^{-2}]}{(\gamma + 1)^2}$$

In the limit of very strong shocks,  $M_1 \gg 1$ , we find the following results

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{(\gamma + 1)},$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)}{(\gamma - 1)},$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)M_1^2}{(\gamma + 1)^2}.$$

■ For monatomic gas : show that  $\frac{\rho_2}{\rho_1} = 4$

■ Heating of gas takes place

## Problem:

We have a number  $N$  of particles all with Energy  $E_0$  inside an “accelerator”. At regular intervals the particles acquire an energy  $\Delta E = \xi E$ , but with probability  $P$  the particle will exit the accelerators, and the acceleration process stops.

What is the energy Spectrum of the particles that exit from the accelerator:

$$\Delta E = \xi E$$

$$E \rightarrow E (1 + \xi) \rightarrow E (1 + \xi) (1 + \xi) \rightarrow \dots$$



$$E_0$$

$$E_1 = E_0 (1 + \xi)$$

$$E_2 = E_0 (1 + \xi)^2$$

...

$$E_k = E_0 (1 + \xi)^k$$

The Probability to have energy  $E_k$  is the probability of having received the acceleration *exactly*  $k$  times:

$$P_k = (1 - P) (1 - P) \dots (1 - P) P$$

$$= (1 - P)^k P$$

# Acceleration theory, Contd...

- Probability of escape :  $P_{\text{esc}}$  after  $k$  encounters, so prob of remaining in the source :  $(1 - P_{\text{esc}})^k$
- So, no. of encounters needed to reach  $E$

$$k = \log_{1+\xi} \left[ \frac{E_k}{E_0} \right]$$
$$= \frac{\ln(E_k/E_0)}{\ln(1 + \xi)}$$

# Acceleration Theory Contd...

- No. of particles accelerated to energies  $> E$  after  $k$  interactions

$$n_k \equiv n(E_k) = N_0 P (1 - P)^k$$

$$= N_0 P (1 - P)^{\frac{\ln(E_k/E_0)}{\ln(1+\xi)}}$$

$$= N_0 P \exp \left[ \frac{\ln(1 - P) \ln(E_k/E_0)}{\ln(1 + \xi)} \right]$$

$$= N_0 P \left( \frac{E_k}{E_0} \right)^{\frac{\ln(1-P)}{\ln(1+\xi)}}$$

$$n(E_k) = N_0 P \left( \frac{E_k}{E_0} \right)^{\frac{\ln(1-P)}{\ln(1+\xi)}}$$

Discrete Spectrum  
of the toy model

$$\frac{dn}{dE} \simeq \frac{n(E_k)}{\Delta E_k} \propto E^{\frac{\ln(1-P)}{\ln(1+\xi)} - 1}$$

Differential Spectrum  
slope  $\alpha = \gamma - 1$

$$n(> E) \propto E^{-\gamma}$$

Integral Spectrum  
slope  $\gamma$

$$\frac{dn(E)}{dE} \propto E^{-(\gamma+1)}$$

$$\gamma = -\frac{\ln(1-P)}{\ln(1+\xi)} \simeq \frac{P}{\xi}$$

$$\ln(1+x) \simeq x + \frac{x^2}{2} - \frac{x^3}{3} + \dots$$

$$\ln(1-P) \simeq -P + \dots$$

$$\ln(1+\xi) \simeq \xi + \dots$$

# What did we learn ???

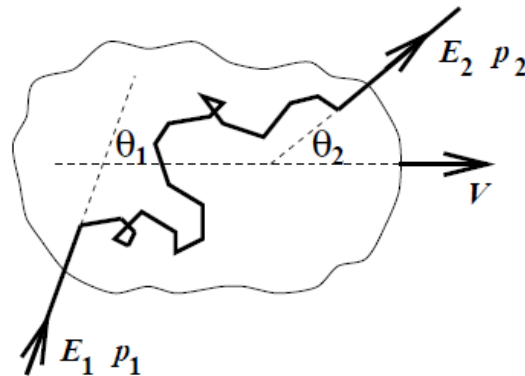
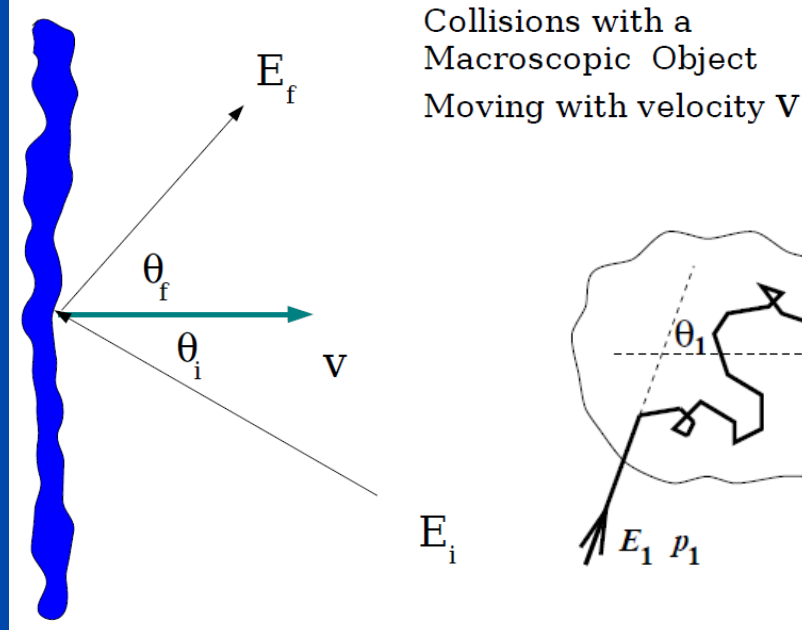
- Now  $P_{\text{esc}} = T_{\text{cycle}} / T_{\text{esc}}$  acceleration / escape :  
prob per encounter of escape from acceleration region  
After acceleration for time  $t$ ,

$$n_{\text{max}} = t / T_{\text{cycle}}, \text{ So, } E \leq E_0 (1 + \epsilon) t / T_{\text{cycle}}$$

- Higher energy particles take longer time to accelerate

Let's apply these to some specific astrophysical cases.....

**Problem :** A Particle scatters on a moving wall  
 (a moving MACROSCOPIC OBJECT  $M \gg m$ )  
 The particle has initial Energy  $E_i$  and initial direction  
 (with respect to the wall velocity)  $\theta_i$ .  
 Compute the final state energy  $E_f$  as a function of  $\theta_f$   
 (the scattering angle).



- Consider a magnetized cloud, velocity  $V$

In the cloud rest frame,

- (a) magnetic scattering  $\Rightarrow$  no energy change of the particle.
- (b) particle's direction is randomized  $\Rightarrow$  equal prob of emerging in any direction.



In the system where the wall is at rest  
(indicated by \*) the problem is of course trivial:  
 $E_f^* = E_i^*$ .

$$E_i^* = \gamma (E_i - \beta p_{z,i})$$

$$\simeq \gamma E_i (1 - \beta \cos \theta_i)$$

$$E_f = \gamma E_i^* (1 + \beta \cos \theta_f^*)$$

$$= \gamma^2 (1 - \beta \cos \theta_i)(1 + \beta \cos \theta_f^*) E_i$$

$$\langle E_f \rangle = E_i \gamma^2 (1 - \beta \langle \cos \theta_i \rangle) (1 + \beta \langle \cos \theta_f^* \rangle)$$

$$\langle \cos \theta_f^* \rangle = 0$$

The computation of  $\langle \cos \theta_i \rangle$  is a little more difficult, but it is obvious that “front” encounters are more likely than “back” encounters and therefore  $\langle \cos \theta_i \rangle < 0$ . In fact the probability of  $\theta_i$  is proportional to the *relative velocity* between the particle and the cloud.

$$v_{\text{rel}} = |\vec{v}_{\text{cloud}} - \vec{v}_{\text{particle}}|$$

$$= \sqrt{(c - v \cos \theta_i)^2 + v^2 \sin^2 \theta_i}$$

$$= c \sqrt{(1 - \beta \cos \theta_i)^2 + \beta^2 \sin^2 \theta_i}$$

$$= c \sqrt{1 + \beta^2 - 2\beta \cos \theta_i}$$

$$\simeq c \sqrt{1 - 2\beta \cos \theta_i}$$

$$\simeq c (1 - \beta \cos \theta_i)$$

$$\beta \ll 1$$

$$\frac{dN}{d \cos \theta_i} \propto v_{\text{rel}} \propto (1 - \beta \cos \theta_i)$$

$$\langle \cos \theta_i \rangle = \frac{\int_{-1}^{+1} d \cos \theta_i \cos \theta_i (1 - \beta \cos \theta_i)}{\int_{-1}^{+1} d \cos \theta_i (1 - \beta \cos \theta_i)} = -\frac{\beta}{3}$$

$$\frac{\langle E_f \rangle}{E_i} = \gamma^2 (1 - \beta \langle \cos \theta_i \rangle) (1 + \beta \langle \cos \theta_f^* \rangle)$$

$$\simeq \gamma^2 \left( 1 - \beta \left( -\frac{\beta}{3} \right) \right) (1 + \beta \times 0)$$

$$\simeq \frac{1}{1 - \beta^2} \left( 1 + \frac{\beta^2}{3} \right)$$

$$\simeq (1 + \beta^2 + \dots) \left( 1 + \frac{\beta^2}{3} \right)$$

$$\simeq 1 + \frac{4}{3} \beta^2 + \dots$$

$$\boxed{\frac{\langle E_f \rangle}{E_i} \simeq 1 + \frac{4}{3} \beta^2 + \dots}$$

Fermi 2<sup>nd</sup> order method is very inefficient

Moreover :

We have not considered energy losses due to ionisation !!!

No satisfactory injection mechanism of particles proposed !!!!

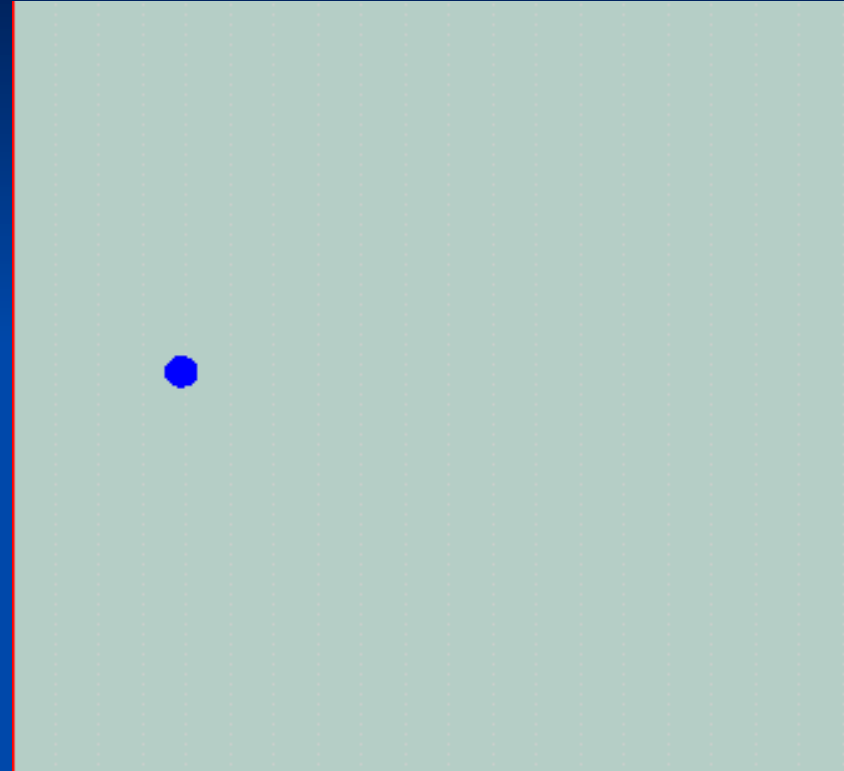
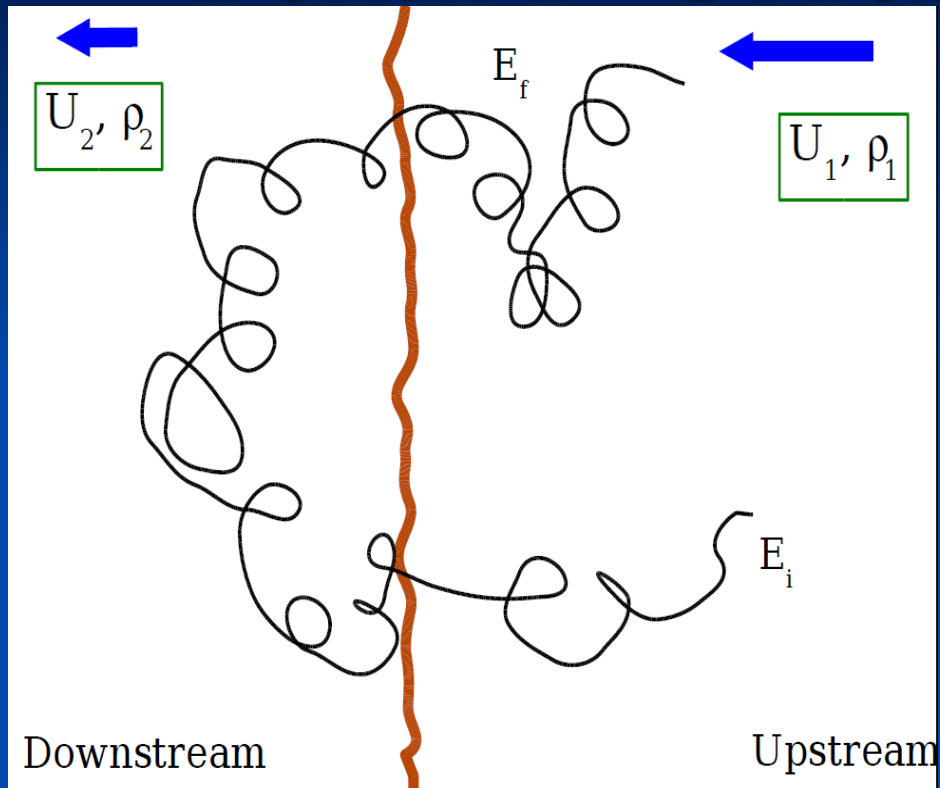
# Fermi 1<sup>st</sup> Order Acceleration

Strong shock caused by a supernova explosion, propagating through the interstellar medium:

- $v_{\text{particles}} \gg V_{\text{front}}$
- gyroradius  $\gg$  thickness of the front (plane approximation)
- flux of particles both in front and behind the shock front:
  - downstream: turbulence caused by the shock front passing through the medium
  - upstream: turbulence in the form of Alfvén waves excited by energetic particles which pass through the shock front and attempt to escape upstream
- each time a particle goes through one cycle its energy increases



# Fermi 1<sup>st</sup> Order Acceleration



# Fermi 1<sup>st</sup> Order Acceleration

$$E_f = \gamma^2 [1 - \beta \cos \theta_i] [1 + \beta \cos \theta_f^*]$$

$$-1 \leq \cos \theta_i \leq 0$$

$$0 \leq \cos \theta_f \leq 1$$

$$\frac{dN}{d \cos \theta_i} \propto \cos \theta_i$$

$$\frac{dN}{d \cos \theta_f^*} \propto \cos \theta_f^*$$

$$\langle \cos \theta_i \rangle = \frac{\int_{-1}^0 dz \, z}{\int_{-1}^0 dz} = \frac{-1/3}{1/2} = -\frac{2}{3}$$

$$\langle \cos \theta_f^* \rangle = \frac{\int_0^1 dz \, z}{\int_0^1 dz} = \frac{+1/3}{1/2} = +\frac{2}{3}$$

$$\frac{E_f}{E_i} = \gamma^2 [1 - \beta \langle \cos \theta_i \rangle] [1 + \beta \langle \cos \theta_f^* \rangle]$$

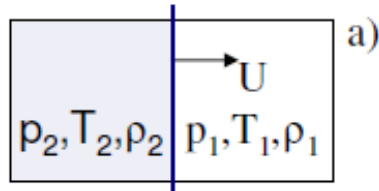
$$= \frac{1}{1 - \beta^2} \left( 1 + \beta \frac{2}{3} \right) \left( 1 + \beta \frac{2}{3} \right)$$

$$\simeq 1 + \frac{4}{3} \beta + O(\beta^2)$$

# Basic Phenomenology of Acceleration

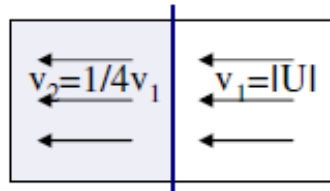
- Let us consider strong shock : SNR exploding into a medium.
- HE particles front and behind the shock
- Shock velocity  $\ll$  velocity of particles
- Particles get scattered : vel. Distribution become isotropic on either side of shock

# Infinite plane shock with a magnetic field perpendicular to it



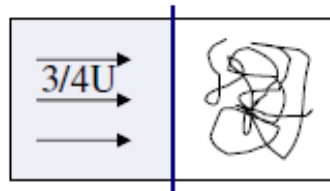
a)

$U \gg c_s$  through stationary interstellar gas  
mass conservation:  $\rho_1 v_1 = \rho_2 v_2$



b)

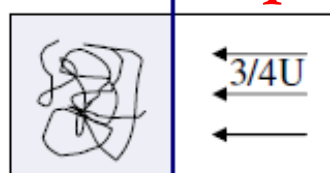
reference frame of the shock front at rest:  
for monoatomic or fully ionised gas  $\gamma=5/3$ ,  $\rho_2/\rho_1=(\gamma+1)/(\gamma-1)$



c)

upstream gas stationary:  
isotropic distribution of velocities  
the gas behind the shock moves with velocity  $V=-v_1+v_2$

■ **Upstream**



d)

downstream gas stationary:  
isotropic distribution of velocities

■ **Up -> Down and vice versa : increase in energy,  
increment of same order**

# An example : SNR explosion

- Frequency of SuperNovae explosions:  
 $f = 1 \text{ SN} / (30 - 70) \text{ yr}$
- *Typical* Kinetic energy of the ejected mass:  
 $K \sim 10^{51} \text{ erg}$
- *Typical* mass:  
 $M = 10 \text{ solar masses } (20 \cdot 10^{33} \text{ g})$
- Power emitted in the form of kinetic energy:  
 $W = K \cdot f = 10^{51} \text{ erg} / (30 \cdot 3 \cdot 10^7 \text{ s}) \sim 10^{42} \text{ erg/s}$
- Speed of shock front:  
 $V = (2K/M)^{1/2} \sim 3 \cdot 10^8 \text{ cm/s}$
- One can roughly assume that the shock front gets “extinguished” when the mass of the ejecta reach a density equal to the average interstellar density ( $\rho_{\text{IG}} \sim 1 \text{ p/cm}^3 = 1.6 \cdot 10^{-24} \text{ g/cm}^3$ ):  
 $\rho_{\text{SN}} = M/\text{Volume} = M/(4/3\pi R^3) = \rho_{\text{IG}} \Rightarrow \text{Volume} \Rightarrow R \sim 1.4 \cdot 10^{19} \text{ cm} \sim 5 \text{ pc}$
- How long does it take to extinguish the accelerator (time of free expansion of the ejecta)?  
 $T_{\text{acc}} = R/V \sim 1000 \text{ years}$  (NB these are just “typical” order of magnitude numbers...)



# Maximum energy from shock acceleration

- The energy gain per collision (cfr. Exercise):
  - $\Delta E = 4/3 V/c E = \eta \cdot E$  with  $\eta \sim 10^{-2}$
- If we know the typical time in between two consecutive collisions  $T_{\text{cycle}}$ , the maximum number of possible collision is on average:
  - $N_{\text{collisions}} = T_{\text{acc}}/T_{\text{cycle}}$  with  $T_{\text{acc}}$  as calculated before
- And the maximum reachable energy is:
  - $E_{\text{max}} = \Delta E \cdot N_{\text{collisions}} = \eta \cdot E \cdot T_{\text{acc}}/T_{\text{cycle}}$
- $T_{\text{cycle}}$  (or also the shock front crossing-time) depends on the shock velocity  $V$  and the mean free path for magnetic scattering of the particles  $\lambda_s$ 
  - $T_{\text{cycle}} = \lambda_s / V$
- On the other hand, acceleration can only continue if the particle is confined, that means that  $\lambda_s < \text{gyroradius}$  ( $E/ZeB$ ,  $Ze$  charge of the particle,  $B$  magnetic field  $\sim 3 \mu\text{G}$ ), which leads to:
  - $E_{\text{max}} \sim 4/3 ZeB/c V^2 \cdot T_{\text{acc}} \sim 300 \cdot Z [\text{TeV}]$

# Pulsars

Gamma-ray pulsars are predicted to produce energetic electron-positron pairs with a harder spectrum than that from secondary cosmic-ray induced origin, leading to the possibility that such sources may dominate the cosmic ray positron spectrum at high energies.

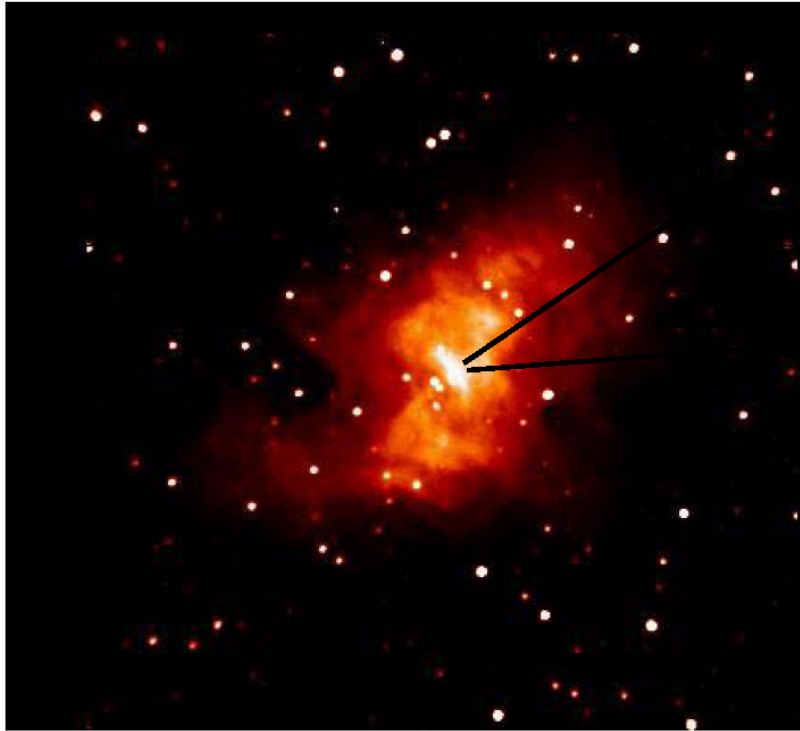
Una Pulsar è una giovane *stella di neutroni* (NS) rapidamente ruotante rispetto ad un asse.

Una NS ha un'altissima densità (quella dei nuclei), massa pari a  $\sim 1.4 M_{\odot}$  (massa sole), e raggio  $R_{NS} \sim 10$  km

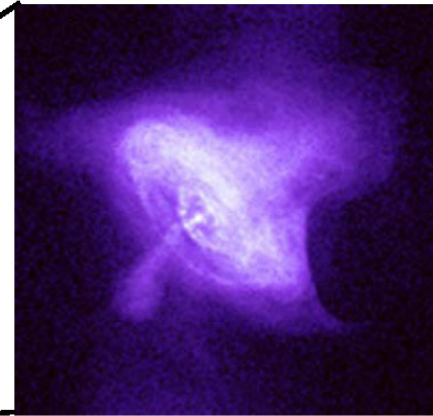
Supponendo un campo magnetico della stella prima del collasso pari a  $B \sim B_{\odot} \sim 10^{-2}$  T, per la legge di Gauss:

$$B_{NS} = B \left( \frac{R}{R_{NS}} \right)^2 \cong 10^{-2} T \cdot 10^{10} \cong 10^8 T$$

# Power Source: Pulsar

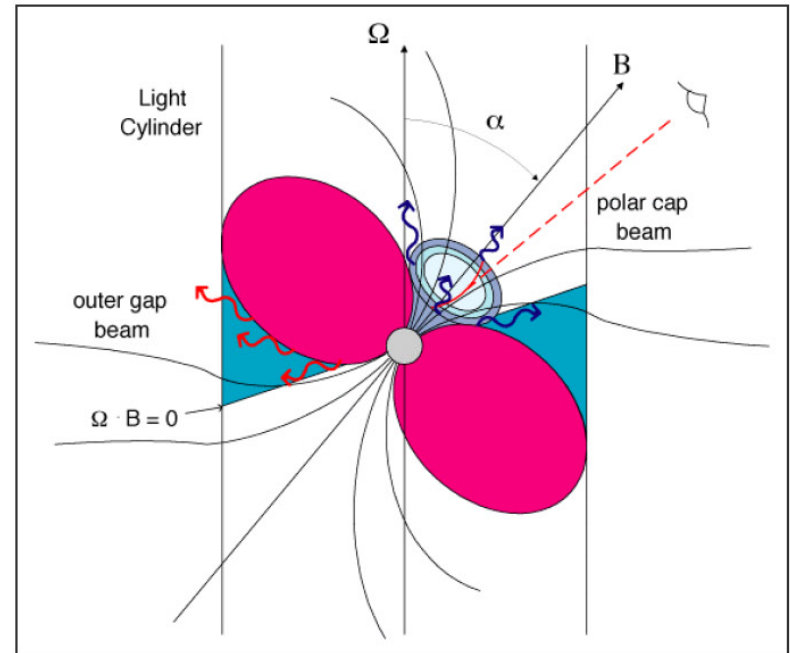


Crab Nebula

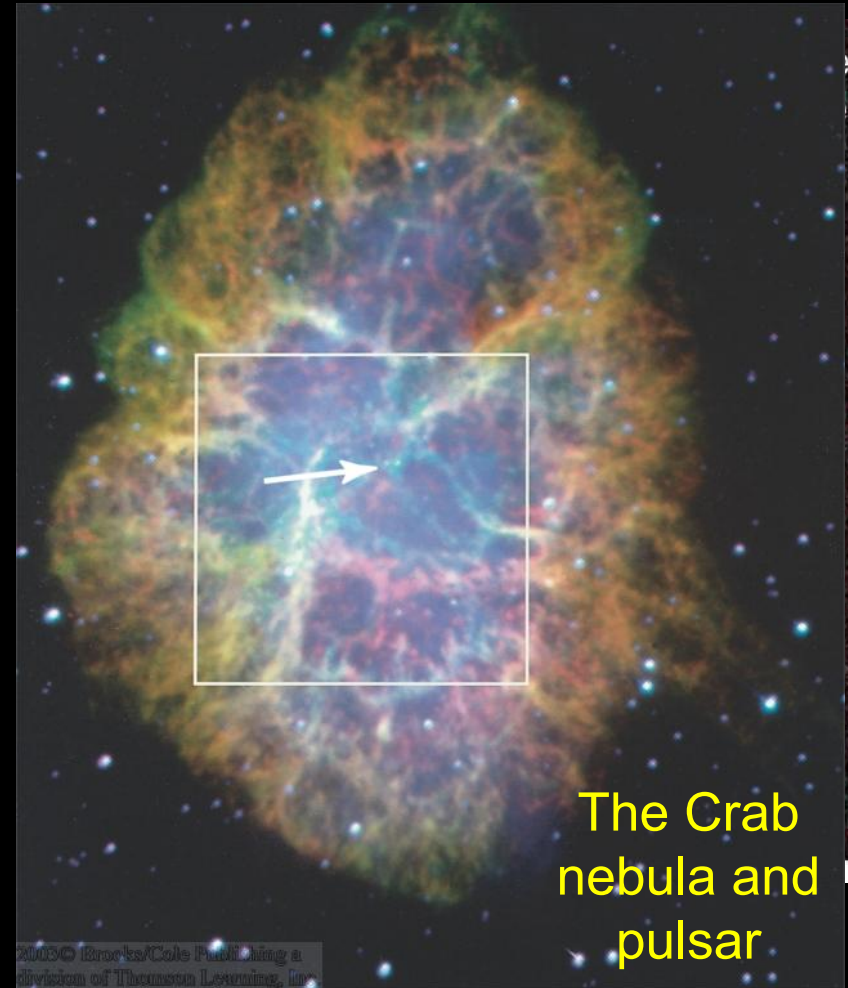
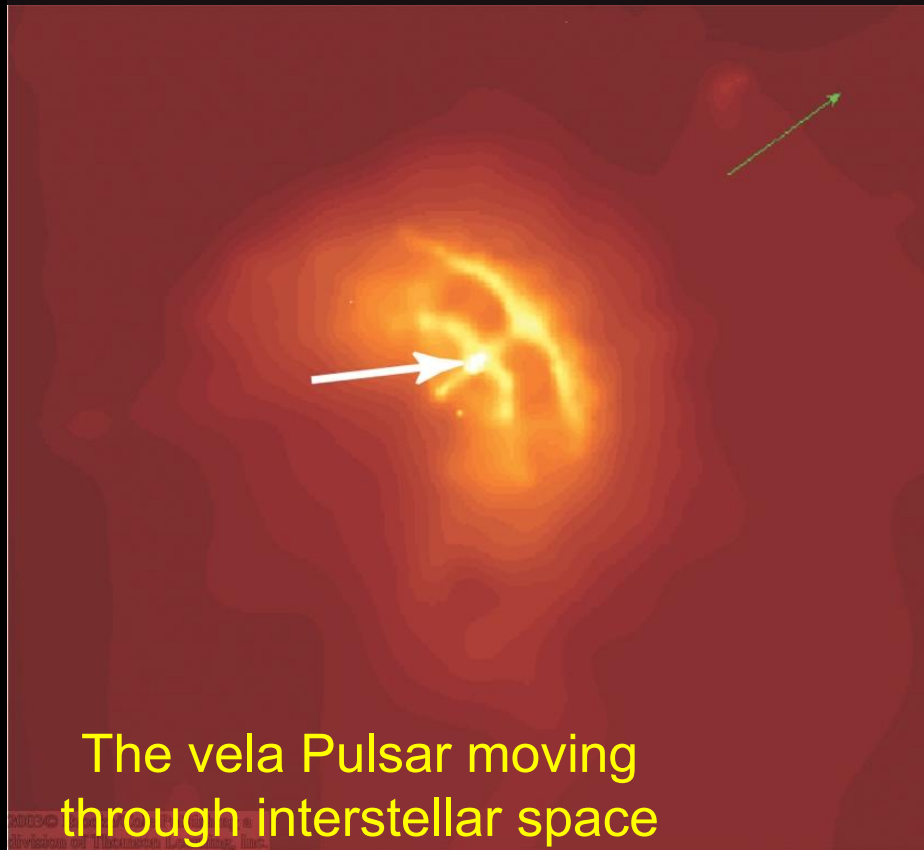


Pulsar

Emitted beams

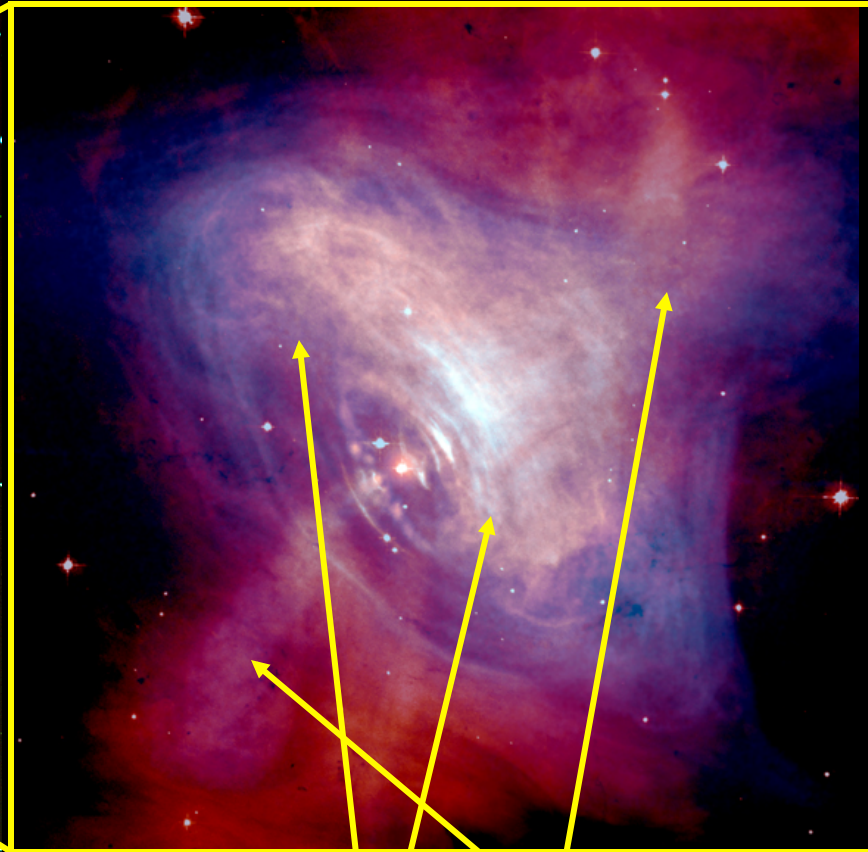
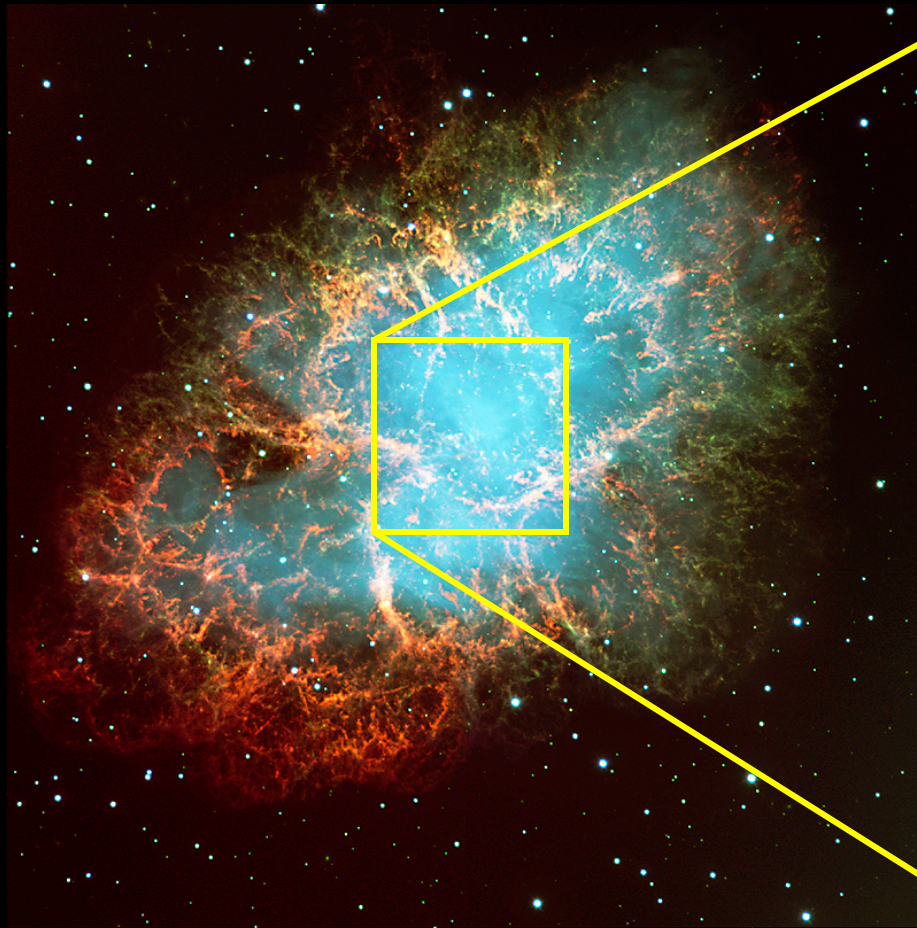


# Images of Pulsars and other Neutron Stars





# The Crab Pulsar



Pulsar wind + jets

The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

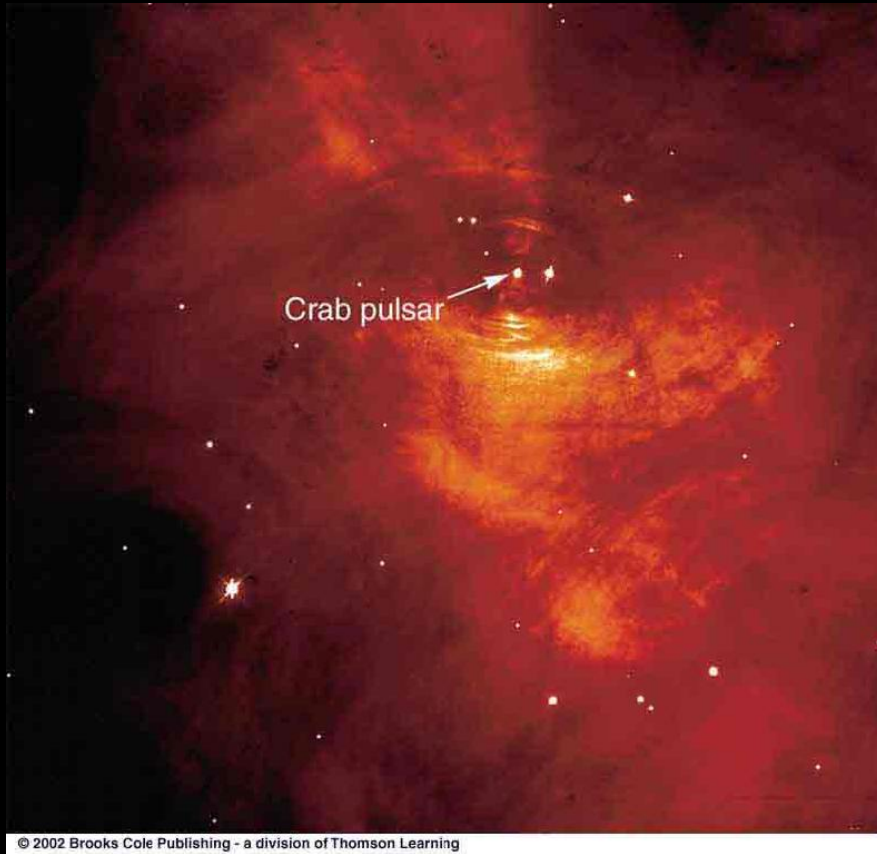
ESO PR Photo 40f/99 ( 17 November 1999 )

© European Southern Observatory

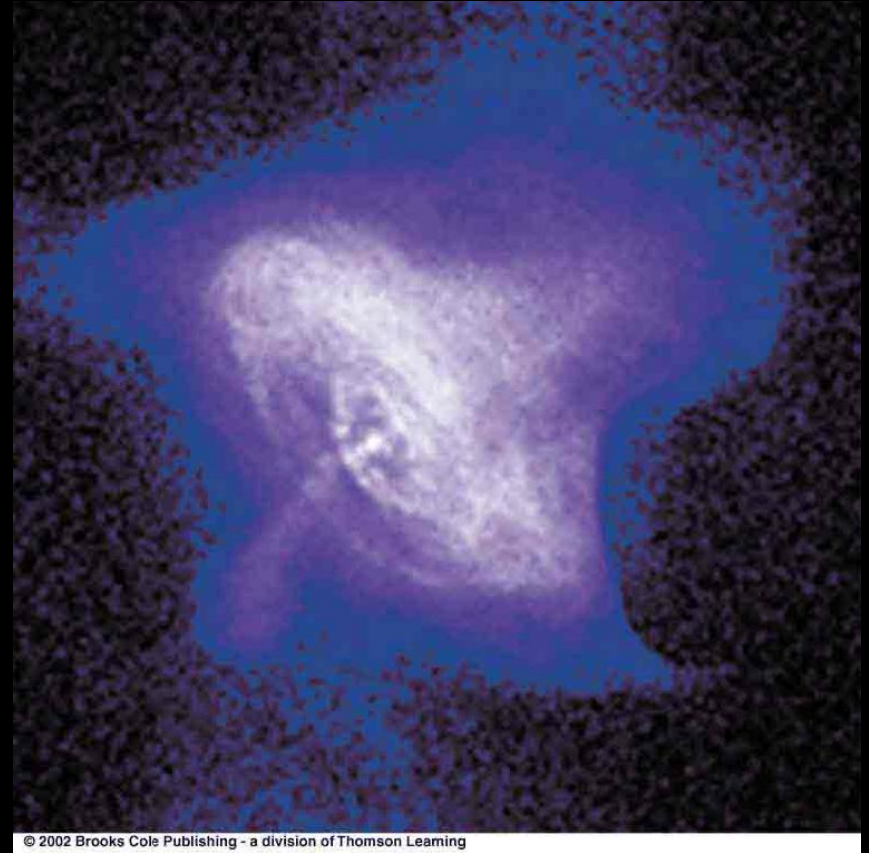


Remnant of a supernova observed in A.D. 1054

# The Crab Pulsar



Visual image



X-ray image



# Properties of Neutron Stars

Typical size:  $R \sim 10 \text{ km}$

Mass:  $M \sim 1.4 - 3 M_{\text{sun}}$

Density:  $\rho \sim 4 \times 10^{14} \text{ g/cm}^3$

→ 1 teaspoon full of NS matter has a mass of ~ 2 billion tons!!!

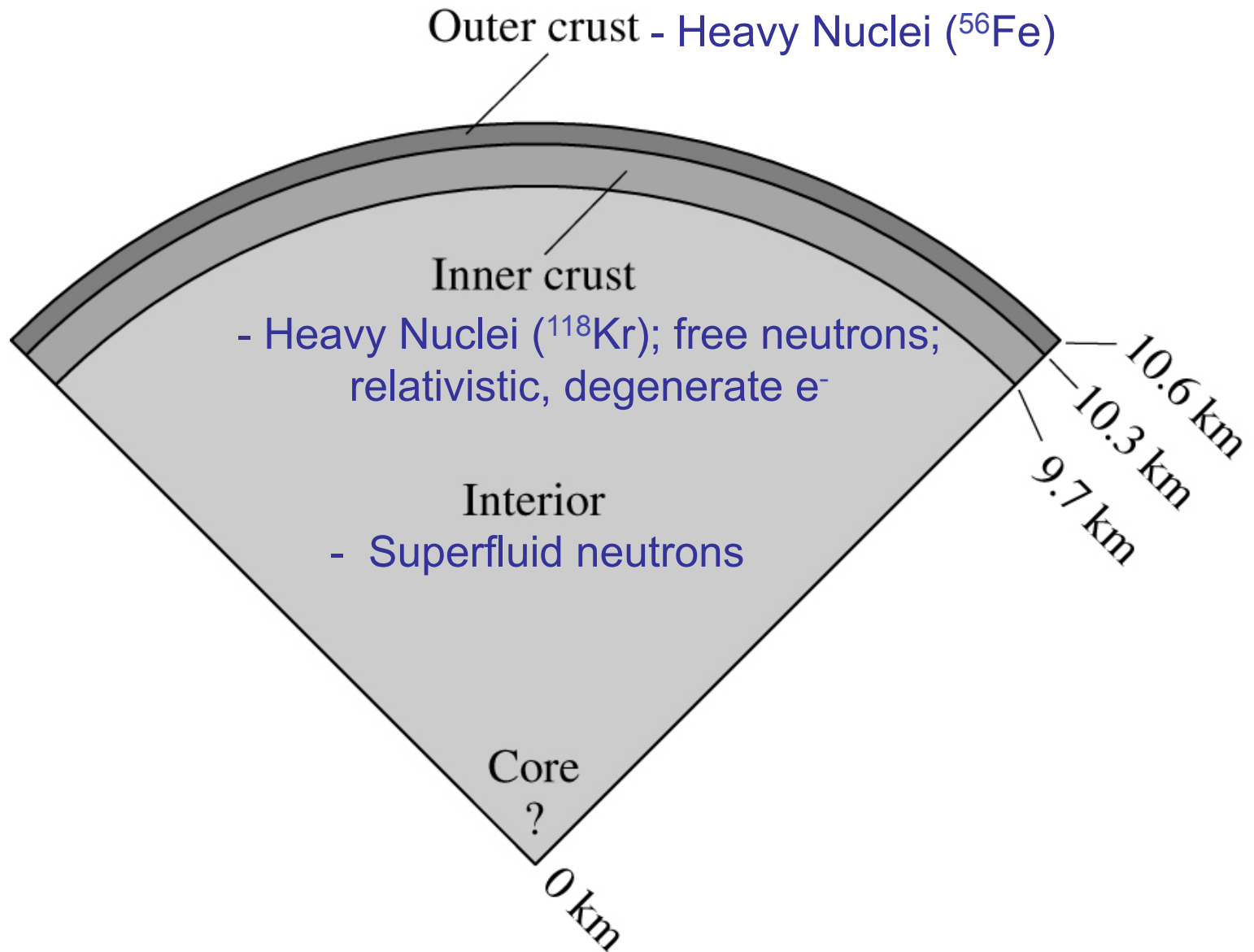
Rotation periods: ~ a few ms – a few s

Magnetic fields:  $B \sim 10^8 - 10^{15} \text{ G}$

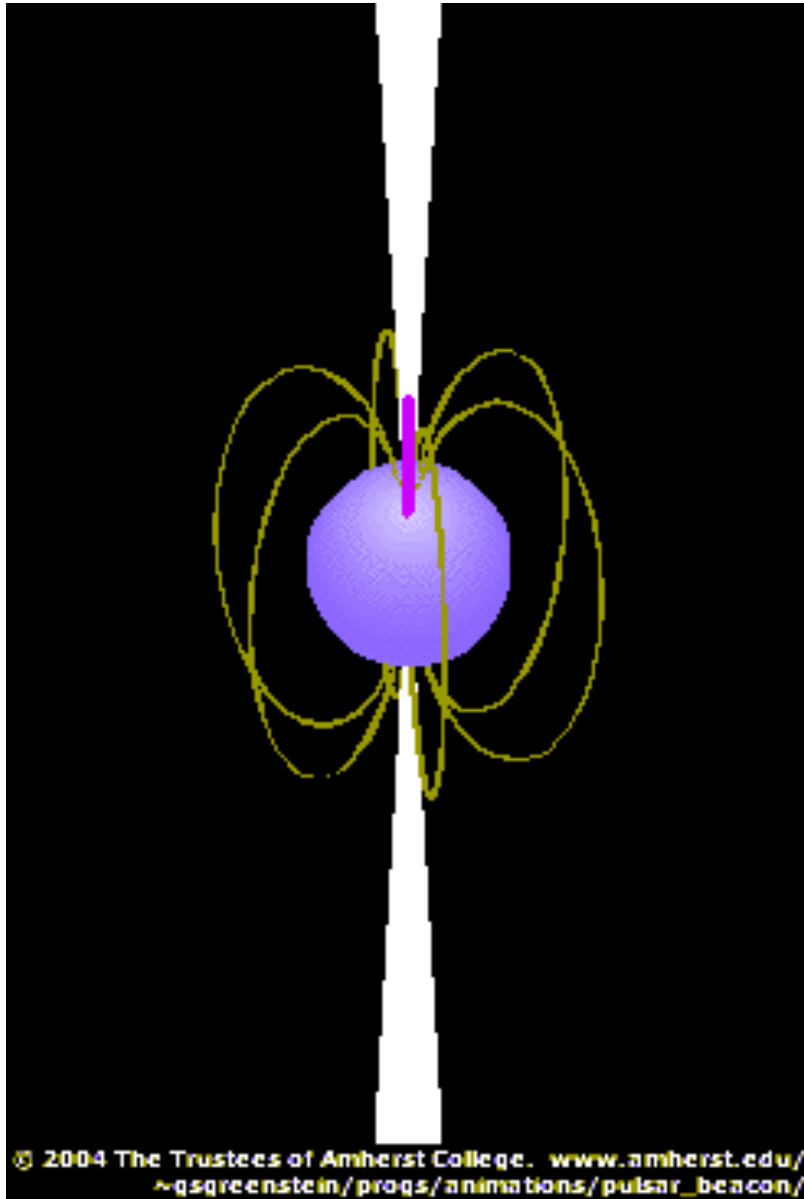
(Atoll sources;  
ms pulsars)

(magnetars)

# Radial Structure of a Neutron Star



# The Lighthouse Model of Pulsars



A Pulsar's magnetic field has a dipole structure, just like Earth.

Radiation is emitted mostly along the magnetic poles.

Rapid rotation along axis not aligned with magnetic field axis

→ Light house model of pulsars

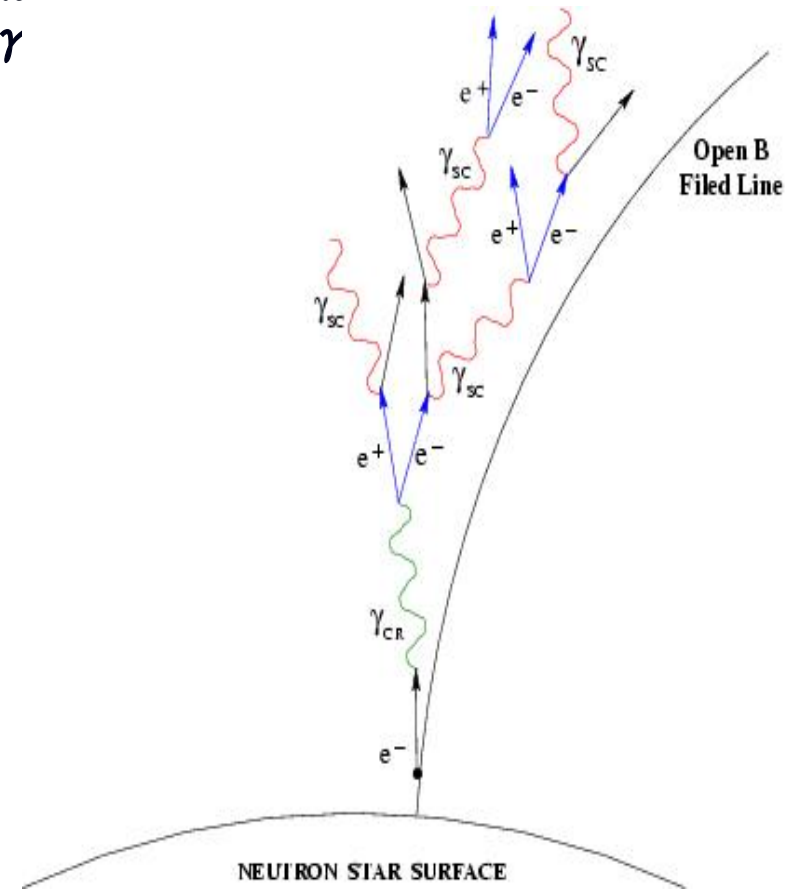
Pulses are not perfectly regular

→ gradual build-up of average pulse profiles

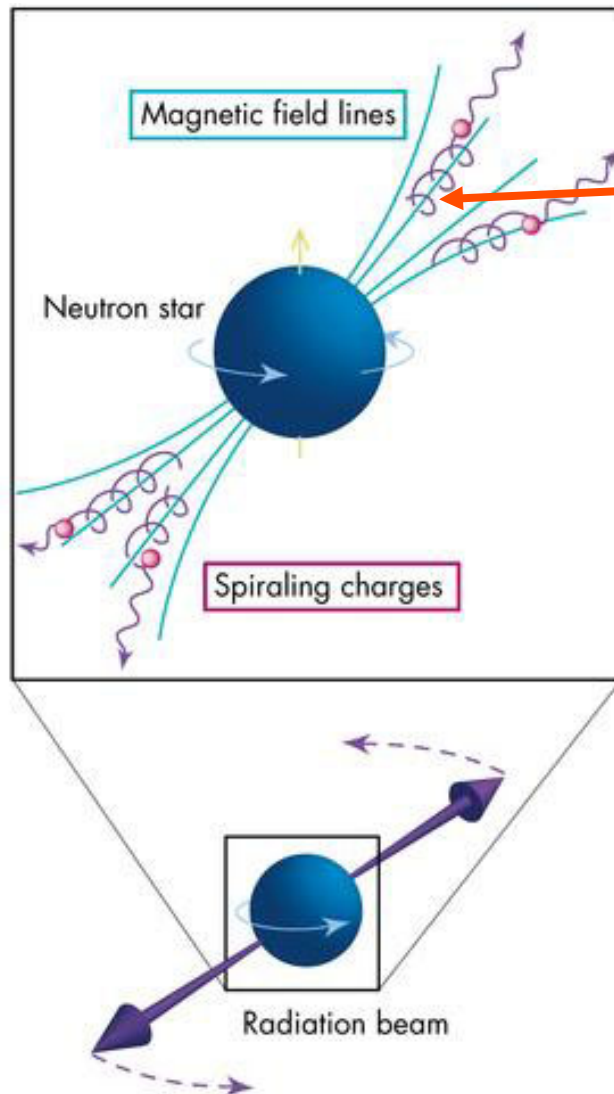
# $e^+e^-$ production in neutron stars

**Mechanism:** the spinning  $\mathbf{B}$  of the pulsar strips  $e^-$  that accelerated at the polar cap or at the outer gap emit  $\gamma$  that make production of  $e^\pm$  that are trapped in the cloud, further accelerated and later released at  $\tau \sim 10^5$  years.

In both models of polar gap and outer gap, electrons can be accelerated in different regions of the pulsar magnetosphere due to induced electric fields and produce an electromagnetic cascade through the emission of curvature radiation, which in turn results in production of photons which are above threshold for pair production in the strong pulsar magnetic field. This process results in lower energy electrons and positrons that can escape the magnetosphere either through the open field lines [25] or after joining the pulsar wind. In this second case, the electrons and positrons lose part of their energy adiabatically because of the expansion of the wind.



# Pulsar Emission Models: Polar Cap model



Particle acceleration along  
magnetic field lines

Synchrotron emission

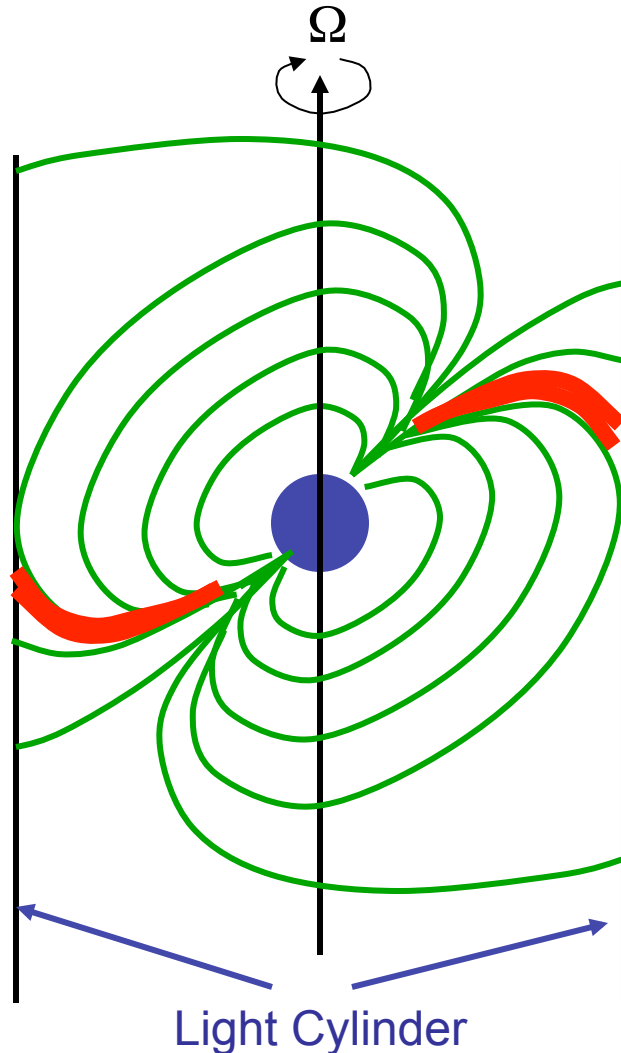
Curvature radiation

Pair production

Electromagnetic cascades

# Pulsar Emission Models:

## Outer Gap model



Electrons are bound to magnetic fields co-rotating with the pulsar

At a radial distance  $r = c/\Omega$   
co-rotation at the speed of light  
→ “light cylinder”

→ Particles ripped off magnetic fields

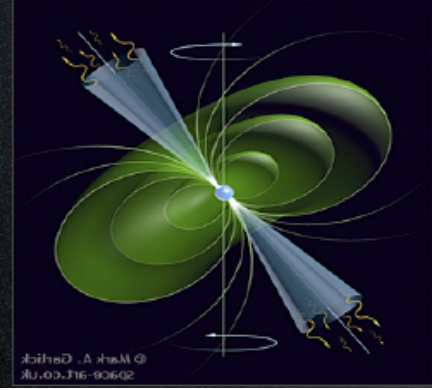
Synchrotron emission

Curvature radiation



# Astrophysical Explanation Pulsars

*S. Profumo Astro-ph 0812-4457*



Mechanism: the spinning  $\mathbf{B}$  of the pulsar strips  $e^-$  that accelerated at the polar cap or at the outer gap emit  $\gamma$  that make production of  $e^\pm$  that are trapped in the cloud, further accelerated and later released at  $\tau \sim 10^5$  years.

Young ( $T \sim 10^5$  years) and nearby ( $< 1\text{kpc}$ )

If not: too much diffusion, low energy, too low flux.

Geminga: 157 parsecs from Earth and 370,000 years old

B0656+14: 290 parsecs from Earth and 110,000 years old

Many others after Fermi/GLAST

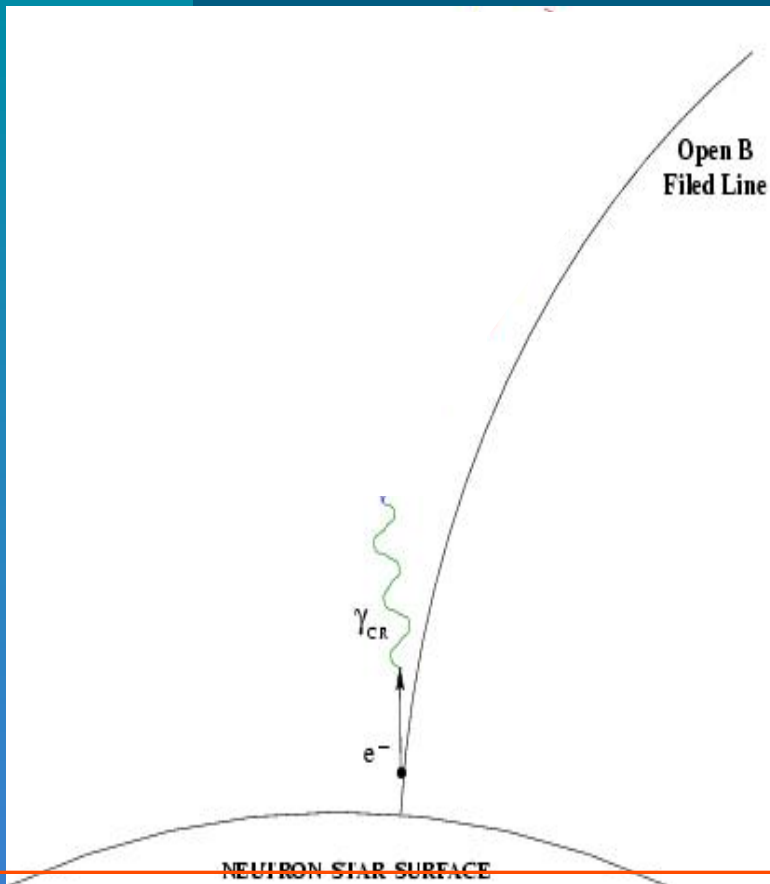
Diffuse mature pulsars



# Models of $\gamma$ -ray emission in Pulsars

## Polar Cap Model

Sturrock (1971); Ruderman & Sutherland (1975);  
Harding (1981); Daugherty & Harding (1982)



- Acceleration of electrons
- Cooling mechanisms
  - a) Curvature radiation
  - b) Synchrotron, I.C. of X-rays
- $\gamma$ -rays interact with magnetic field, via *Magnetic pair production*

$$\gamma + \vec{B} \rightarrow e^+ + e^-$$

$$\kappa \propto B_p \cdot \exp(-1 / E_\gamma B_p)$$

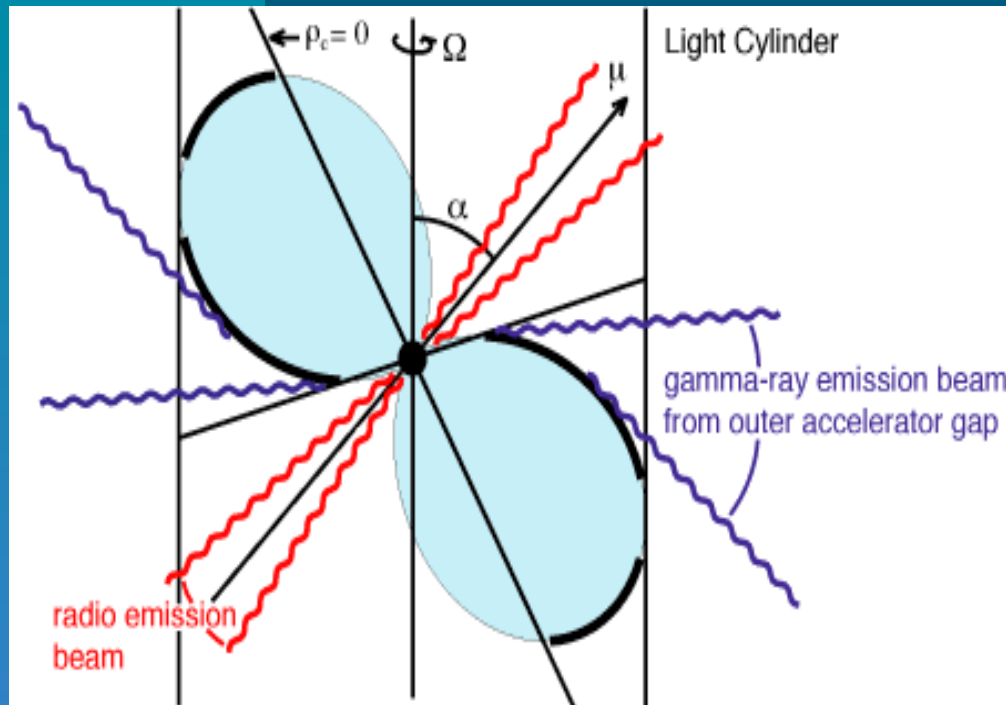
Polar Cap model predicts **super-exponential cutoff** in high energy  $\gamma$ -ray spectra !



# Models of $\gamma$ -ray emission in Pulsars

## Outer Gap model

Cheng, Ho & Ruderman (1986); Romani (1996)



- $\gamma$ -ray emission occurs near LC
- Charges accelerated in vacuum gap  
→  $\gamma$ -rays via *Curv. rad.*
- **B** not strong enough for pair-production. But, curvature photons interact with non-thermal X-rays

$$\gamma\gamma \rightarrow e^+e^-$$

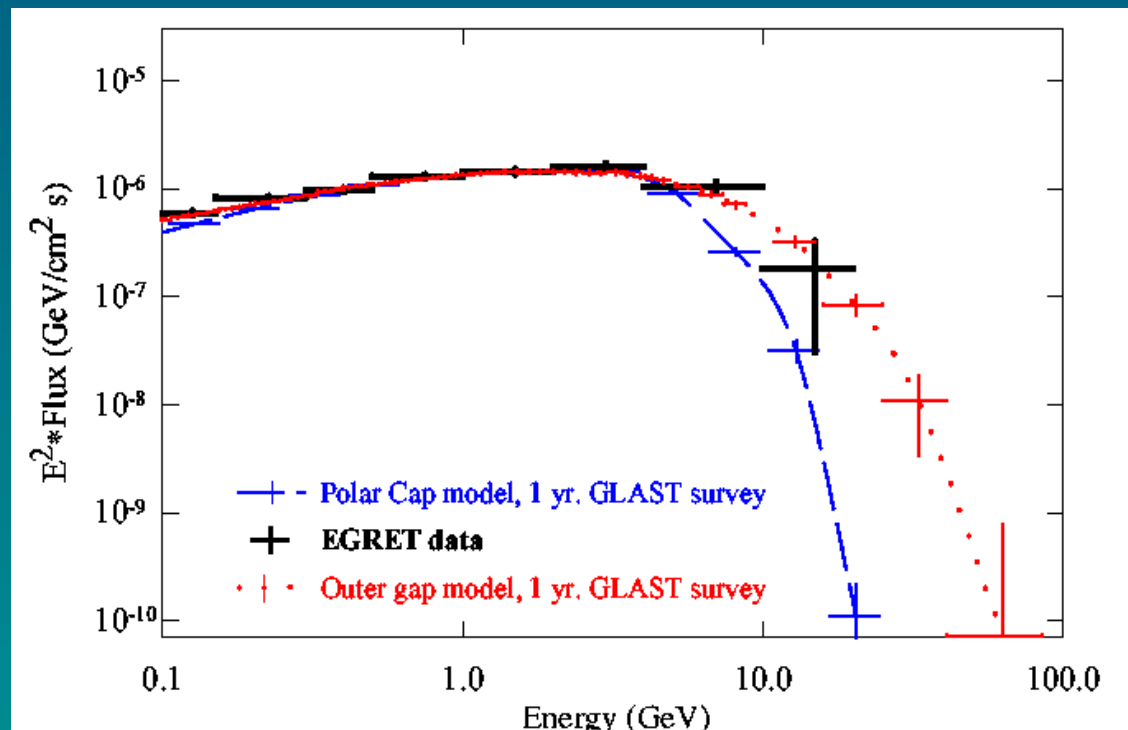
Softer **exponential cutoff** in the high energy  $\gamma$ -ray spectra !

Electrons may up scatter IR photons to TeV Gamma-rays !

# Where do $\gamma$ -rays come from? Outer gap or polar cap?

## Discrimination between models

- Different models predict different spectral cutoff.
- Measuring the spectral tail is possible to distinguish between models.



- La velocità di rotazione delle pulsar è nota sperimentalmente, e può essere facilmente stimata:

$$\frac{GM^2}{R^2} = M\omega_{NS}^2 R \longrightarrow \omega_{NS} \approx 10^3 - 10^4 \text{ s}^{-1}$$

La Pulsar fornisce energia tramite induzione EM.  
 L'energia massima fornita ad una particella di carica Ze:

$$\nabla \times \varepsilon = -\frac{1}{c} \frac{\partial B}{\partial t} \longrightarrow \frac{\varepsilon}{L} = \frac{B\omega_o}{c}$$

(ε = campo elettrico indotto su una regione lineare L)

$$E_{Max} = \int Ze \cdot \varepsilon \cdot dx = Ze \cdot B\omega_o L \cdot L / c$$

PULSAR: Massima energia acquisita dalla particella accelerata nel caso di p (Z=1),  
 B=10<sup>8</sup> T, ω=10<sup>3</sup> s<sup>-1</sup>, R=10 km

$$\begin{aligned} E_{Max} &= Ze \cdot B\omega_o L^2 / c = \\ &= 1.6 \times 10^{-19} (C) \cdot 10^8 (T) \cdot 10^3 (s^{-1}) \cdot [10^4 (m)]^2 \\ &= 1.6 (J) = 10^{19} eV \end{aligned}$$

The energy spectrum injected by a single pulsar depends on the environmental parameters of the pulsar, but some attempts to calculate the average spectrum injected by a population of mature pulsars suggest that the spectrum may be relatively hard, having a slope of  $\sim 1.5$ -  $1.6$ .

This spectrum, however, results from a complex interplay of individual pulsar spectra, of the spatial and age distributions of pulsars in the Galaxy, and on the assumption that the chief channel for pulsar spin down is magnetic dipole radiation. Due to the related uncertainties, variations from this injection spectra cannot be ruled out.

$$\frac{dN_e}{dE_e} \approx 8.6 \times 10^{38} \dot{N}_{100} (E_e/\text{GeV})^{-1.6} \exp(-E_e/E_c) \text{ GeV}^{-1} \text{ s}^{-1},$$

With  $E_c \sim 10$ - $1000$  GeV and  $\dot{N}_{100}$  is rate of pulsar generation per century

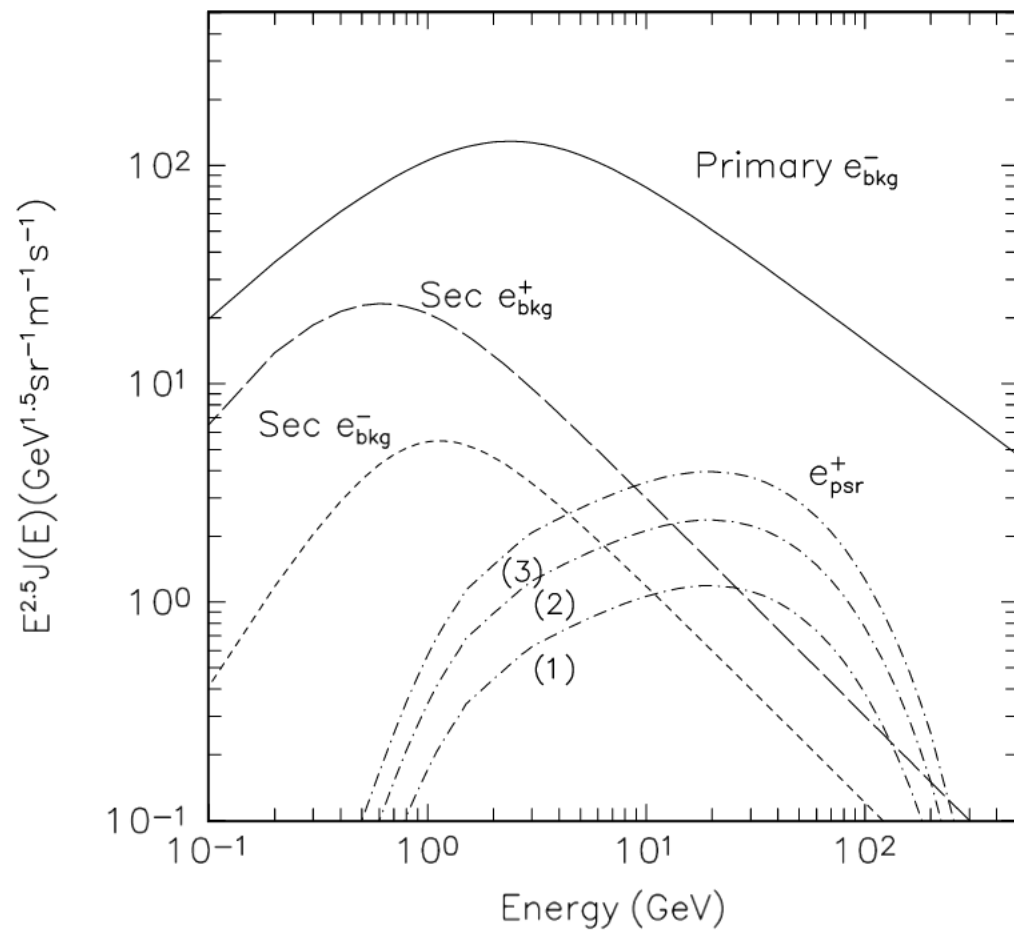
This expression corresponds to an average energy output in electron-positron pairs of approximately  $6 \times 10^{46}$  erg per pulsar, to be compared with the energy injected by the pulsar of the order of  $10^{49}$  erg

■ For details of derivation: Cosmic-ray positrons from mature gamma-ray pulsars, L. Zhang  
And K.S. Cheng, *Astronomy & Astrophysics*, 368, 1063-1070 (2001),

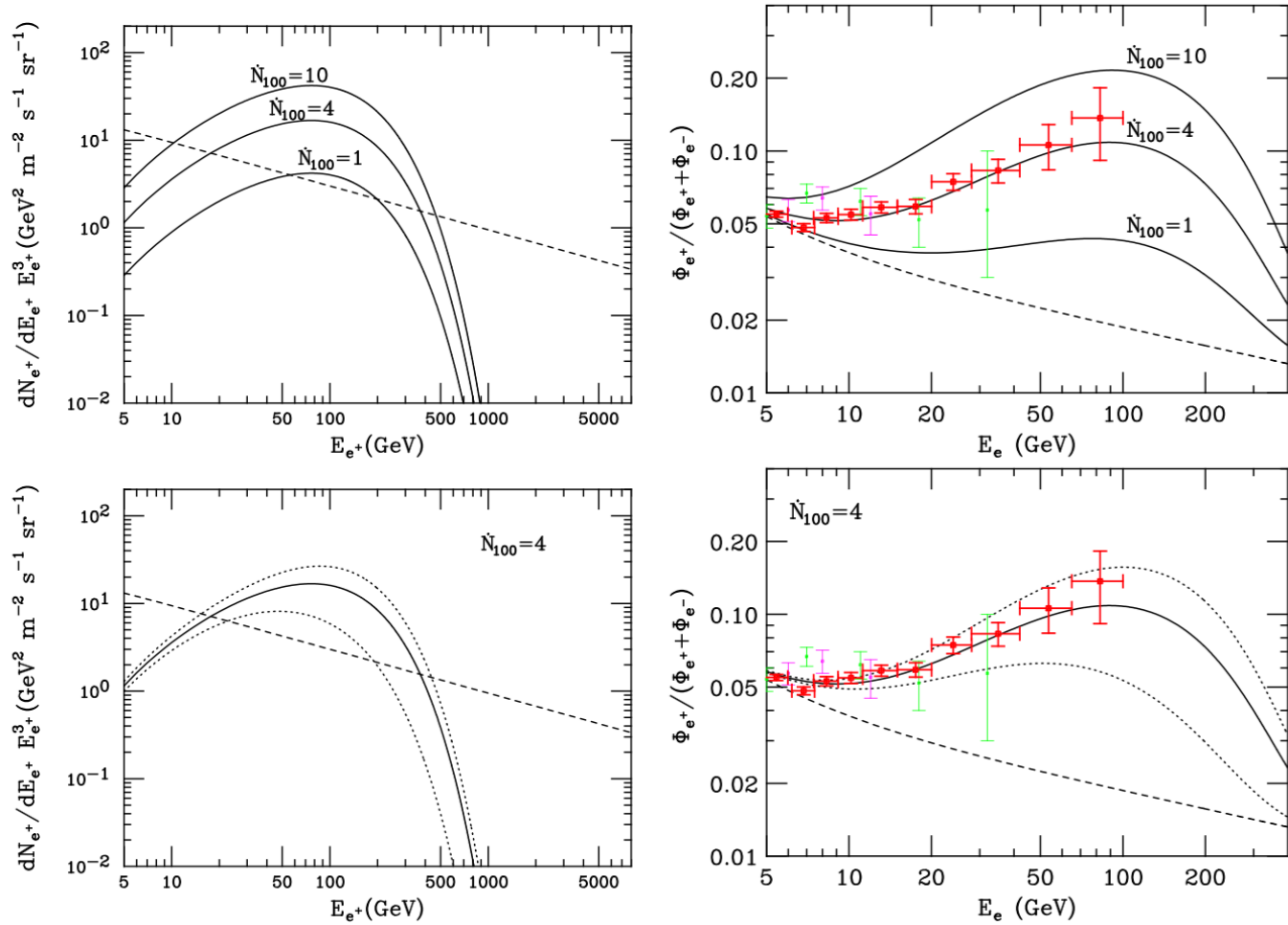
Pulsars as the sources of high energy cosmic ray positrons, Dan Hooper,<sup>a,b</sup> Pasquale Blasia,<sup>c,d</sup> and Pasquale Dario Serpico, *Journal of cosmology and astrophysics*, 2009



- This cutoff is determined by the details of the development of the electromagnetic cascade in the pulsar magnetosphere and, even more importantly, by the distribution of periods, magnetic fields, and radii of mature pulsars. The exact value of the cutoff energy should, therefore, not be considered to be a robust prediction of the theory, although it represents a good estimate of the order of magnitude of the cutoff energy. For instance, in some models it is argued that the typical energy of electrons and positrons in the cascade associated with the polar gap is  $E_e \approx 8B_{12}^{5/7} P_o^{-17/7}$  GeV which, for a period of 200 ms, would yield a maximum energy of the resulting pairs of ~ 400 GeV. On the other hand, other models lead to different maximum energies (~80 GeV) and even different scalings of the cutoff with the parameters of the pulsar.



**Fig. 2.** The spectra of cosmic-ray electrons and positrons. The background of cosmic-ray electrons and positrons are approximated by those given by Moskalenko & Strong (1998): primary  $e_{\text{bkg}}^{-}$  (solid curve), secondary  $e_{\text{bkg}}^{-}$  (short-dashed curve) and secondary  $e_{\text{bkg}}^{+}$  (long-dashed curve). The positron spectra from the mature  $\gamma$ -ray pulsars for various birth rate of the neutron stars in the Galaxy are labeled as (1) ( $\dot{N}_{100} = 1$ ), (2) ( $\dot{N}_{100} = 2$ ) and (3) ( $\dot{N}_{100} = 10/3$ )



**Figure 1:** The spectrum of cosmic ray positrons (left) and the positron fraction (right) resulting from the sum of all pulsars throughout the Milky Way. Also shown as a dashed line is the prediction for secondary positrons (and primary and secondary electrons in the right frames) as calculated in ref. [27]. In the right frames, the measurements of HEAT [3] (light green and magenta) and measurements of PAMELA [2] (dark red) are also shown. We have used the injected spectrum reported in eq. (2.7). In the lower frames, the upper (lower) dotted line represents the case in which the injection rate within 500 parsecs of the Solar System is doubled (neglected), providing an estimate the variance resulting from the small number of nearby pulsars contributing to the spectrum.

# Case study: Crab nebula

■ La luminosita' totale della Crab e'  $L=5 \times 10^{38} \text{ ergs}^{-1}$

■ Da dove viene l'energia irradiata?

■ Se si considera la pulsar come una sfera rigida di raggio  $R \gg 10 \text{ km}$  e massa  $M=1$  massa solare  $=1.988 \times 10^{33} \text{ gr}$ , l'energia rotazionale e'

$$I = \frac{3}{5}MR^2 = 1.2 \times 10^{45} \text{ gr cm}^2$$

$$E = \frac{1}{2}I\omega^2 = 4.8 \times 10^{49} \text{ erg}$$

■ La frequenza di rotazione misurata e'  $\omega = 190 \text{ s}^{-1}$

■ Il tasso di diminuzione di velocita' angolare  $\frac{d\omega}{dt} = -2.4 \times 10^{-9} \text{ s}^{-2}$

■ L'energia dissipata (che viene trasferita al mezzo circostante) e' quindi  $\frac{dE}{dt} = I\omega \frac{d\omega}{dt}$

■ tramite processi che coinvolgono campi magnetici molto intensi e rapidamente variabili in prossimita' della superficie della pulsar

$$= (1.2 \times 10^{45}) \times (190) \times (2.4 \times 10^{-9}) \text{ ergs}^{-1} \approx 5.5 \times 10^{38} \text{ ergs}^{-1}$$

■ Valore molto vicino alla potenza totale irradiata dalla pulsar  $\rightarrow$  la sorgente di energia e proprio la rotazione della pulsar che rallenta il suo moto

# Backup Slides

## ■ Energy Spectrum : Simplified Case

- $E = \beta^k E_0$ , average energy of particles after  $k$  collisions
- $N = P^k N_0$ ,  $P^k$  is prob. that the particle remains within the acceleration region after  $k$  collisions

$$\rightarrow N/N_0 = (E/E_0) \ln P / \ln \beta$$

$$N(E) dE = \text{const. } E^{-1 + \ln P / \ln \beta} dE$$

$$\beta = 1 + \Delta E / E = 1 + 4V / 3c$$

$$P = 1 - U/c \dots\dots \rightarrow N(E) dE \sim E^{-2} dE$$



# Rate of acceleration

- For a large plane shock, rate of encounters is given by projection of an isotropic cosmic ray flux on plane shock front. Can be shown :

$$c\rho_{\text{CR}}/4$$

- Acceleration rate is  $dE/dt = xE/T_{\text{cycle}}$

- It can be shown :

$$T_{\text{cycle}} = 4/c (k1/u1 + k2/u2)$$

$u1$  = fluid velocity (-ve) relative to shock front, for downstream region :  
average the residence time of those particles which do not escape.

# Rate of Acceleration

Define a cycle  $\equiv U \rightarrow \text{shock} \rightarrow D \rightarrow \text{shock} \rightarrow U$ , so  $\tau_{\text{acc}} = E/(dE/dt) = \tau_{\text{cycle}}/\xi$ . But,  $\tau_{\text{cycle}} = ?$

Let  $t_2$  be the average time spent in D before returning to the shock, i.e., for  $(S \rightarrow D \rightarrow S)$ .

Average distance in D through which particle diffuses in time  $t$  is  $\approx \sqrt{k_2 t}$ , with  $k_2$  = diffusion coefficient in D.

Average distance through which the particle is advected in D in time  $t$  is  $= u_2 t$ .

The particle is lost downstream (i.e., low probability of returning to shock) if  $u_2 t > \sqrt{k_2 t}$ . Thus, 'returning zone boundary' is given by the condition  $u_2 t \approx \sqrt{k_2 t}$ . Thus only the particles within a distance  $d_2 \approx \frac{k_2}{u_2}$  from the shock return to the shock.

Number of particles per unit area within the 'returning zone' is  $n_{\text{CR}} d_2 = n_{\text{CR}} \frac{k_2}{u_2}$ . Thus,

$$t_2 \approx n_{\text{CR}} d_2 / r_{\text{cross}}, \text{ i.e., } t_2 = \frac{4 k_2}{c u_2}.$$

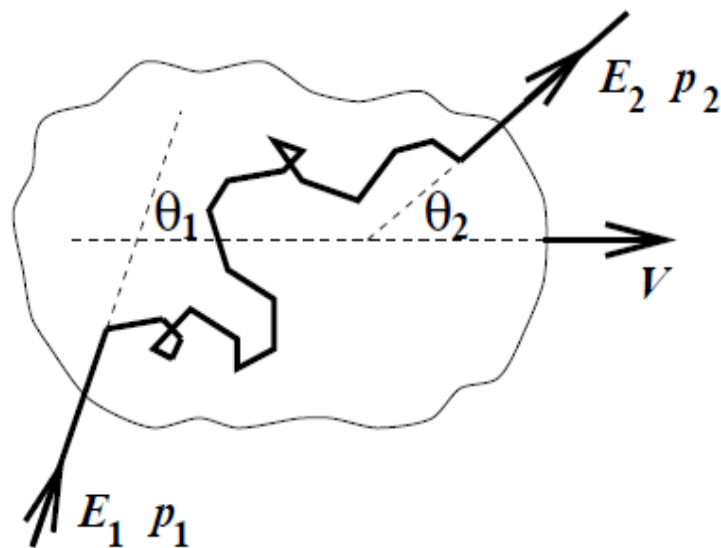
Similarly, for  $S \rightarrow U \rightarrow S$ , we have  $t_1 = \frac{4 k_1}{c u_1}$ . Thus,  $\tau_{\text{cycle}} = \frac{4}{c} \left( \frac{k_1}{u_1} + \frac{k_2}{u_2} \right)$ .

$$\text{Thus, } r_{\text{acc}} = \frac{R-1}{3R} u_1 \left( \frac{k_1}{u_1} + \frac{k_2}{u_2} \right)^{-1}.$$

# SNR Parameters

- Mean ejecta speed :  $v = (2E_{\text{SN}}/M_{\text{ej}})^{1/2}$
- Radius swept away :  $R = (3M_{\text{ej}}/4\Pi\rho)^{1/3}$
- Sweep time :  $t_0 = R/v$
- ISM density :  $\rho = 1.4m_p n_h$

(Gaisser, Protheroe, ...)



- Consider a magnetized cloud, velocity  $V$

In the cloud rest frame,

- (a) magnetic scattering  $\Rightarrow$  no energy change of the particle.
- (b) particle's direction is randomized  $\Rightarrow$  equal prob of emerging in any direction.

In cloud rest frame (assume already relativistic particles,  $pc \approx E$ )

$$E'_2 = E'_1 = \gamma E_1 (1 - \beta \cos \theta_1), \quad \text{where } \beta = V/c \text{ and } \gamma = 1/\sqrt{1 - \beta^2}.$$

Transform back to Lab frame:

$$E_2 = \gamma E'_2 (1 + \beta \cos \theta'_2).$$

Fractional change in LAB-frame energy  $(E_2 - E_1)/E_1$

$$\frac{\Delta E}{E} \equiv \frac{E_2 - E_1}{E_1} = \frac{1 - \beta \cos \theta_1 + \beta \cos \theta'_2 - \beta^2 \cos \theta_1 \cos \theta'_2}{1 - \beta^2} - 1.$$

In any individual encounter, the particle may either gain or lose energy depending on the values of  $\cos \theta_1$  and  $\cos \theta'_2$ .

But, need to average over many “encounters”, i.e., over  $\cos \theta_1$  and  $\cos \theta'_2$ .

Clearly,  $\langle \cos \theta'_2 \rangle = 0$ , since  $\frac{dn}{d \cos \theta'_2} = \text{constant}$ , and  $-1 \leq \cos \theta'_2 \leq 1$ .

But  $\langle \cos \theta_1 \rangle = ?$

Note,  $\frac{dn}{d \cos \theta_1} \propto (c - V \cos \theta_1)$  = relative velocity between the cloud and the particle, with  $-1 \leq \cos \theta_1 \leq 1$ .

Normalized distribution:  $\frac{dn}{d \cos \theta_1} = (c - V \cos \theta_1)/2c$ ,

$$\langle \cos \theta_1 \rangle = \int_{-1}^1 \cos \theta_1 \frac{dn}{d \cos \theta_1} d \cos \theta_1 = -\beta/3.$$

$$\text{Thus, } \xi \equiv \frac{\langle \Delta E \rangle}{E} = \frac{1 + \beta^2/3}{1 - \beta^2} - 1 \simeq +\frac{4}{3}\beta^2, \text{ since } \beta \ll 1.$$

# How does the spectrum look like ?

Recall,  $N(E) \propto E^{-\Gamma}$ , with  $\Gamma \approx 1 + P_{\text{esc}}/\xi$ . So,  $P_{\text{esc}} = ?$

$P_{\text{esc}} = (\text{rate of particle loss downstream}) / (\text{rate of particles crossing from U to D})$

$$r_{\text{loss}} = n_{\text{CR}} u_2 = n_{\text{CR}} \frac{V_s}{R} \quad (\text{in units of area} \cdot \text{time})^{-1}.$$

$$r_{\text{cross}}(U \rightarrow D) = n_{\text{CR}} \frac{1}{4\pi} \int_{V_s/c}^1 2\pi(V_s + c \cos \theta) d(\cos \theta) = \frac{1}{4} n_{\text{CR}} c$$

$$P_{\text{esc}} = \frac{r_{\text{loss}}}{r_{\text{cross}}(U \rightarrow D)} = 4 \frac{V_s}{Rc} = \frac{V_s}{c}, \quad \text{for } R = 4.$$

$$\text{Thus, } \Gamma = 1 + \frac{P_{\text{esc}}}{\xi} = 2, \quad \text{since } \xi = V_s/c.$$

In general  $\Gamma \geq 2$  for  $R \leq 4$ .



