Lecture 16 281119

- Il pdf delle lezioni puo’ essere scaricato da
- http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1920/

The slides are taken from http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1920/bibliography/hydrodynamics_achterberg.pdf

Shock waves

- So far, we have considered the properties of shocks in the case where there is no magnetic field in the fluid. We now explore what happens when we relax this assumption. To begin with, we assume that ideal MHD applies everywhere but within the shock-front itself, and that the velocities of the ions and the neutrals are the same (i.e. we can treat them as a single fluid).

- We can use the MHD version of the fluid equations to derive jump conditions relating conditions in the pre-shock gas to those in the post-shock gas. In the absence of a magnetic field, the conditions that we arrive at simply require that the flux of mass, momentum and energy is conserved across the shock. In the MHD case, however, we also need to consider what happens to the magnetic field, and so it is useful to look at the full derivation.

Shock waves

The description of discontinuities in MHD is more complex of pure hydrodynamics, due to the presence of magnetic fields.
The MHD shock waves form

However, in astrophysics exists the following circumstance that simplifies considerably the problem

Except that around the pulsars, astrophysical fluids are never dominated by the magnetic field, in the sense that the plasma-β parameter is such that

$$\frac{B^2}{8\pi p} = \frac{\gamma v_A^2}{2c_s^2} < 0.1 - 0.3$$

(take the defs of v_A and c_s)

This implies that the dynamical importance of B field can be approximated as small

Thanks to this we can also neglect the fact that the shock waves must be superalfvenic, that is $V_s > v_A$

In effect, since $v_A = (B^2/4\pi\rho)^{1/2}$ and from previous relation we have $B^2/8\pi < \rho c_s^2$, the condition $V_s > v_A$ is equivalent to $V_s > c_s$, which is absolutely necessary for shock waves

Equation of Motion

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g}$$

(1) (2) (3) (4)

(i) $\dfrac{(2)}{(3)} = \beta = \dfrac{p}{B^2/(2\mu)}$ * **Plasma beta** *

When $\beta << 1$, $\mathbf{j} \times \mathbf{B}$ dominates

(ii) $(1) \approx (3) \rightarrow v \approx v_A = \dfrac{B}{\sqrt{\mu\rho}}$ * **Alfvén speed** *

- We start by noting that the continuity equation has the same form regardless of whether or not a magnetic field is present. Therefore, the associated shock jump condition in the MHD case is the same as in the hydrodynamical case:

$$\rho_1 v_{1,\perp} = \rho_2 v_{2,\perp}. \tag{422}$$

- The momentum equation, in the form that we derived it in lecture 2, is given by

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla p + \frac{1}{4\pi}(\vec{B}\cdot\nabla)\vec{B} - \frac{1}{8\pi}\nabla\left(|\vec{B}|^2\right). \tag{423}$$

In component form, this can be written as

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\partial_j v_i\right) = -\partial_i p + \frac{1}{4\pi}(B_j\partial_j B_i) - \frac{1}{8\pi}(\partial_i|\vec{B}|^2). \tag{424}$$

- The left-hand side of this equation can be rewritten as

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\partial_j v_i\right) = \frac{\partial}{\partial t}(\rho v_i) - v_i\frac{\partial \rho}{\partial t} + \partial_j\rho v_j v_i - v_i\partial_j\rho v_j, \quad (425)$$

$$= \frac{\partial}{\partial t}(\rho v_i) + \partial_j\rho v_j v_i - v_i\left(\frac{\partial \rho}{\partial t} + \partial_j\rho v_j\right) \quad (426)$$

The final term on the right-hand side of this expression is simply v_i times the continuity equation, and hence is zero. Therefore, the momentum equation becomes

$$\frac{\partial}{\partial t}(\rho v_i) + \partial_j\rho v_j v_i = -\partial_i p + \frac{1}{4\pi}(B_j\partial_j B_i) - \frac{1}{8\pi}(\partial_i|\vec{B}|^2). \quad (427)$$

- The magnetic pressure term on the right-hand side can be written as

$$\frac{1}{4\pi}(B_j\partial_j B_i) = \frac{1}{4\pi}(\partial_j B_j B_i - B_i\partial_j B_j) \tag{428}$$

Since the magnetic field satisfies $\nabla \cdot \vec{B} = 0$, we have $\partial_j B_j = 0$, and hence can write the momentum equation as

$$\frac{\partial}{\partial t}(\rho v_i) + \partial_j \rho v_j v_i = -\partial_i p + \frac{1}{4\pi}(\partial_j B_j B_i) - \frac{1}{8\pi}(\partial_i |\vec{B}|^2). \tag{429}$$

Finally, collecting terms together and using the identity $\partial_j \equiv \partial_i \delta_{ij}$, where δ_{ij} is the Kronecker delta, we arrive at the following form for the MHD momentum equation:

$$\frac{\partial}{\partial t}(\rho v_i) + \partial_j (\rho v_j v_i + p\delta_{ij} - T_{ij}) = 0, \tag{430}$$

where T_{ik} is the **Maxwell stress tensor**

$$T_{ij} = \frac{1}{4\pi}\left(B_i B_j - \frac{1}{2}|\vec{B}|^2 \delta_{ij}\right). \tag{431}$$

- Now consider a small cylindrical volume with cross-sectional area A oriented perpendicular to the shock, with one end in the pre-shock gas and the other in the post-shock gas. From the above form of the momentum equation, we see that

$$\int_V \frac{\partial}{\partial t}(\rho v_i)\,\mathrm{d}V = -\int_V \partial_j\left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right)\mathrm{d}V, \tag{432}$$

where V is the volume of the cylinder. If we allow the volume of the cylinder to tend to zero, then the integral of the time derivative term vanishes, and we have simply

$$\int_V \partial_j\left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right)\mathrm{d}V = 0. \tag{433}$$

Applying Gauss' theorem then yields

$$\int_S \left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right) n_j \mathrm{d}A = 0, \tag{434}$$

where $\vec{n}$ is the vector normal to the area element $\mathrm{d}A$, and S is the closed surface of the cylinder. Since we can make the length of the cylinder as short as we like, the only surfaces we need consider are the two ends. We therefore find that

$$A\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_1 - A\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_2 = 0, \tag{435}$$

where the subscripts denote that the contents of the brackets are evaluated in the pre-shock and post-shock gas, respectively. From this, our desired jump condition follows trivially:

$$\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_1 = \left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_2. \tag{436}$$

- To translate this from component notation back into something more useful, note that we can locally decompose the fluid velocity into two components, $v_\perp$ and $v_\|$, where $v_\perp$ is oriented perpendicular to the normal direction* and $v_\|$ is ordered parallel to it. Similarly, $\vec{B}$ can also be decomposed into perpendicular and parallel components, $B_\perp$ and $B_\|$.

- If we let both i and j represent the perpendicular component, then $i = j$ and the jump condition tells us that

$$\rho_1 v_{1,\perp}^2 + p_1 - \frac{1}{8\pi}\left(B_{1,\perp}^2 - B_{1,\|}^2\right) = \rho_2 v_{2,\perp}^2 + p_2 - \frac{1}{8\pi}\left(B_{2,\perp}^2 - B_{2,\|}^2\right). \tag{437}$$

Alternatively, if we let i represent the perpendicular component and j represent the parallel component, then $i \neq j$ and the momentum jump condition yields

$$\rho_1 v_{1,\perp} v_{1,\|} - \frac{1}{4\pi} B_{1,\perp} B_{1,\|} = \rho_2 v_{2,\perp} v_{2,\|} - \frac{1}{4\pi} B_{2,\perp} B_{2,\|}. \tag{438}$$

* Normal direction with respect to the shock surface

- A similar analysis applied to the constraint that $\nabla \cdot \vec{B} = 0$ gives us another jump condition for the magnetic field

$$B_{1,\perp} = B_{2,\perp}, \tag{439}$$

and allows us to simplify the first of the momentum jump conditions to

$$\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi} B_{1,\parallel}^2 = \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2. \tag{440}$$

We therefore see that if the flow of the gas is perfectly parallel to the field lines, so that the shock is oriented perpendicularly to them and $B_{\parallel} = 0$, then the momentum jump condition that we obtain is the same as in the hydrodynamical case. This makes sense on physical grounds – in this scenario, the field exerts no net force on the gas, so it is not surprising that the jump conditions remain unaltered. We also see that when the parallel component of the field is non-zero, then our jump condition for the momentum in the perpendicular direction *does* depend on the magnetic field, which provides an additional source of pressure.

- A further jump condition on the velocity and the magnetic field comes from the induction equation
$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{B} \times \vec{v}\right) = 0. \tag{441}$$
If we consider the same small cylindrical volume as before and require the time derivative of the magnetic field to vanish within it, then it follows that
$$\int_V \nabla \times \left(\vec{B} \times \vec{v}\right) \mathrm{d}V = 0. \tag{442}$$
This can be converted to the following surface integral
$$\int_S \vec{n} \times \left(\vec{B} \times \vec{v}\right) \mathrm{d}S = 0, \tag{443}$$

where $\vec{n}$ is the normal to the surface, and the vector identity

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \tag{444}$$

then allows us to write this as

$$\int_S \left[(\vec{n} \cdot \vec{v})\vec{B} - (\vec{n} \cdot \vec{B})\vec{v}\right] \mathrm{d}S = 0. \tag{445}$$

- As before, we can choose our volume V and surface S so that the only parts of the surface integral that we need to worry about are the ends of the cylinder, and hence can simply take $\vec{n}$ to be perpendicular to the shock front. We therefore have

$$\int_S \left[v_\perp \vec{B} - B_\perp \vec{v} \right] \mathrm{d}S = 0. \tag{446}$$

From this, we obtain the jump condition

$$v_{1,\perp} B_{1,\|} - B_{1,\perp} v_{1,\|} = v_{2,\perp} B_{2,\|} - B_{2,\perp} v_{2,\|}. \tag{447}$$

(Taking the other component of the vector simply yields the trivial result that $v_\perp B_\perp - B_\perp v_\perp = 0$, and hence tells us nothing new).

- Finally, it is possible to write the energy equation for the flow in conservative form as[10]

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \epsilon + \frac{B^2}{8\pi}\right) + \nabla \cdot \left[\rho\vec{v}\left(h + \frac{1}{2}v^2\right) + \frac{1}{4\pi}(\vec{B}\times\vec{v})\times\vec{B}\right] = 0 \qquad (448)$$

from which a final jump condition follows:

$$\rho_1 v_{1,\perp}\left(h_1 + \frac{1}{2}v_1^2\right) - \frac{1}{4\pi}CB_{1,\parallel} = \rho_2 v_{2,\perp}\left(h_2 + \frac{1}{2}v_2^2\right) - \frac{1}{4\pi}CB_{2,\parallel}, \qquad (449)$$

where $C = B_{1,\perp}v_{1,\parallel} - B_{1,\parallel}v_{1,\perp} = B_{2,\perp}v_{2,\parallel} - B_{2,\parallel}v_{2,\perp}$ is conserved through the shock.

- The full set of jump conditions for an MHD shock is therefore

$$
\begin{aligned}
\rho_1 v_{1,\perp} &= \rho_2 v_{2,\perp}, \\
B_{1,\perp} &= B_{2,\perp}, \\
\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi} B_{1,\parallel}^2 &= \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2, \\
\rho_1 v_{1,\perp} v_{1,\parallel} - \frac{1}{4\pi} B_{1,\perp} B_{1,\parallel} &= \rho_2 v_{2,\perp} v_{2,\parallel} - \frac{1}{4\pi} B_{2,\perp} B_{2,\parallel}, \\
v_{1,\perp} B_{1,\parallel} - B_{1,\perp} v_{1,\parallel} &= v_{2,\perp} B_{2,\parallel} - B_{2,\perp} v_{2,\parallel}, \\
\rho_1 v_{1,\perp} \left(h_1 + \frac{1}{2} v_1^2 \right) - \frac{1}{4\pi} C B_{1,\parallel} &= \rho_2 v_{2,\perp} \left(h_2 + \frac{1}{2} v_2^2 \right) - \frac{1}{4\pi} C B_{2,\parallel}.
\end{aligned}
$$

- It is clear from the form of these jump conditions that MHD shocks are considerably more complicated than their hydrodynamical equivalents. On reflection, however, this should not be surprising. In a hydrodynamical flow, we have only a single characteristic velocity, the sound speed c_s, at which signals can propagate. In an MHD flow, however, there are *three* characteristic velocities: the sound speed, and also v_F and v_S, the speeds of the fast and slow MHD waves. Shocks are associated with the jump of the flow velocity from above the phase velocity of a given wave to below, and hence there are *six* possible types of MHD shock.

- The first type of MHD shock is known as a **fast shock**. In this case, we have $v_{1,\perp} > v_{\mathrm{F}}$ and $v_{\mathrm{F}} > v_{2,\perp} > v_{\mathrm{A},\perp}$, where $v_{\mathrm{A},\perp}$ is the Alfven velocity in the perpendicular direction. The flow therefore jumps from above to below the velocity of the fast MHD wave, but remains faster than the Alfven velocity.

- Another type of MHD shock is a **slow shock**, where $v_{\mathrm{A},\perp} > v_{1,\perp} > v_{\mathrm{S}}$ and $v_{2,\perp} < v_{\mathrm{S}}$. In this case, both pre-shock and post-shock velocities are slower than the Alfven velocity.

- Finally, there are four different types of **intermediate shock**:

$$
\begin{aligned}
v_{1,\perp} > v_{\mathrm{F}}, &\qquad v_{\mathrm{A},\perp} > v_{2,\perp} > v_{\mathrm{S}}, \\
v_{1,\perp} > v_{\mathrm{F}}, &\qquad v_{2,\perp} < v_{\mathrm{S}}, \\
v_{\mathrm{F}} > v_{1,\perp} > v_{\mathrm{A},\perp}, &\qquad v_{\mathrm{A},\perp} > v_{2,\perp} > v_{\mathrm{S}}, \\
v_{\mathrm{F}} > v_{1,\perp} > v_{\mathrm{A},\perp}, &\qquad v_{2,\perp} < v_{\mathrm{S}}.
\end{aligned}
$$

  In all four of these shocks, the pre-shock flow is super-Alfvenic (i.e. faster than the Alfven velocity) and the post-shock flow is sub-Alfvenic.

- If $B_\perp = 0$, then the shock jump conditions reduce to

$$\rho_1 v_{1,\perp} = \rho_2 v_{2,\perp}, \tag{456}$$

$$\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi} B_{1,\parallel}^2 = \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2, \tag{457}$$

$$v_{1,\parallel} = v_{2,\parallel} \tag{458}$$

$$v_{1,\perp} B_{1,\parallel} = v_{2,\perp} B_{2,\parallel}, \tag{459}$$

$$\rho_1 v_{1,\perp} \left(h_1 + \frac{1}{2} v_1^2 \right) + \frac{1}{4\pi} B_{1,\parallel}^2 v_{1,\perp} = \rho_2 v_{2,\perp} \left(h_2 + \frac{1}{2} v_2^2 \right) + \frac{1}{4\pi} B_{2,\parallel}^2 v_{2,\perp}. \tag{460}$$

Using these, it is possible to show that in this case, the only type of shock that is physically possible is a fast shock, with $v_{1,\perp} > \left(v_{A,\perp,1}^2 + c_{s,1}^2 \right)^{1/2}$. The compression ratio produced by this shock can be written as

$$\frac{\rho_2}{\rho_1} = \frac{2(\gamma + 1)}{D + \left[D^2 + 4(\gamma + 1)(2 - \gamma) \mathcal{M}_{A,1}^{-2} \right]^{1/2}}, \tag{461}$$

where
$$D = (\gamma - 1) + \left(2\mathcal{M}_1^{-2} + \gamma \mathcal{M}_{A,1}^{-2} \right), \tag{462}$$

and $\mathcal{M}_{A,1} \equiv v_{1,\perp}/v_{A,1}$ is the Alfven Mach number of the shock. In the limit where $\mathcal{M} \gg 1$ and $\mathcal{M}_A \gg 1$, $D \to (\gamma - 1)$ and the compression ratio becomes

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}, \tag{463}$$

just as for a purely hydrodynamical shock. For weaker shocks, however, it is clear that we get less compression when the magnetic field is strong and $\mathcal{M}_{A,1}$ is small than when the magnetic field is weak or absent and $\mathcal{M}_{A,1}$ is very large. This is a consequence of the additional resistance to compression provided by the magnetic pressure in this scenario.

- The other important special case is when $B_{\parallel,1} = 0$. In this case, the shock jump conditions become

$$
\begin{aligned}
\rho_1 v_{1,\perp} &= \rho_2 v_{2,\perp}, & (464)\\
B_{1,\perp} &= B_{2,\perp}, & (465)\\
\rho_1 v_{1,\perp}^2 + p_1 &= \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2, & (466)\\
\rho_1 v_{1,\perp} v_{1,\parallel} &= \rho_2 v_{2,\perp} v_{2,\parallel} - \frac{1}{4\pi} B_{2,\perp} B_{2,\parallel}, & (467)\\
-B_{1,\perp} v_{1,\parallel} &= v_{2,\perp} B_{2,\parallel} - B_{2,\perp} v_{2,\parallel}, & (468)\\
\rho_1 v_{1,\perp}\left(h_1 + \frac{1}{2} v_1^2\right) &= \rho_2 v_{2,\perp}\left(h_2 + \frac{1}{2} v_2^2\right) - \frac{1}{4\pi} B_{1,\perp} B_{2,\parallel} v_{1,\parallel}. & (469)
\end{aligned}
$$

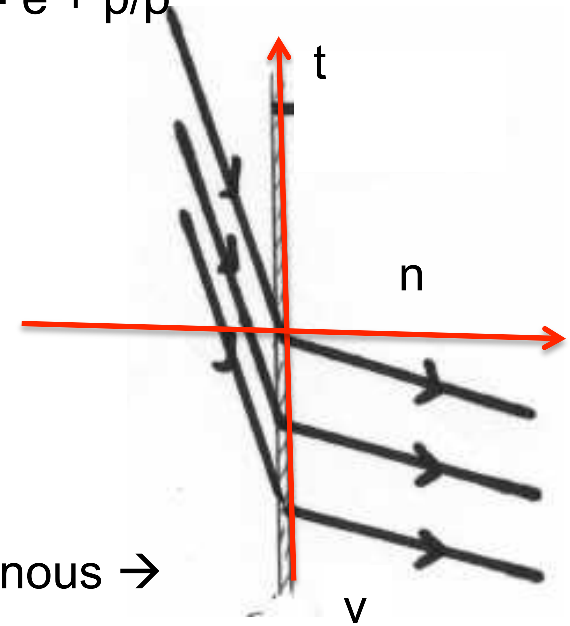
  Note that in this case, we cannot automatically assume that $B_{2,\parallel} = 0$, as $B_{\parallel}$ is not necessarily conserved through the shock.

- One possible solution consistent with these jump conditions has $B_{2,\parallel} = 0$. In this case, the jump conditions simplify further, becoming the same as for a purely hydrodynamical shock, and the magnetic field plays no role in determining the shock properties. This "hydrodynamical" solution exists whenever $v_1 > c_{s,1}$, and produces a compression ratio, temperature ratio, etc. that are the same as in the absence of a magnetic field. This hydrodynamical solution is a fast shock if $c_{s,1} > v_{A,1}$, and can be either a fast, slow or intermediate shock if $c_{s,1} < v_{A,1}$.

Shock waves

The jump conditions at the discontinuity surface are the same as for hydrodynamics:
Conservation of:
i) mass flux
ii) momentum flux (parallel and normal flux)
iii) energy flux

w = specific hentalpy (previously called "*h*")= e + p/ρ



Putting $[X] = X_2 - X_1$ We get

$$[\rho v_n] = 0$$

$$[p + \rho v_n^2 + \frac{1}{8\pi}(B_t^2 - B_n^2)] = 0 \qquad [\rho v_n v_t + \frac{1}{4\pi} B_t B_n] = 0$$

$$[\rho v_n (v^2/2 + w) + \frac{1}{4\pi}(v_n B^2 - B_n(\vec{v} \cdot \vec{B}))] = 0$$

There additional conditions due to electromagnetic fields:
From Maxwell eqns, at the discontinuity, B_n and E_t are continous →

$$[B_n] = 0 \qquad [E_t] = 0$$

Since we are in the ideal limit $E = -v \times B/c$ (a) we get $[B_n v_t - B_t v_n] = 0$

To get E_t, multiply vectorially of (a) with n

Shock waves

$$[\rho v_n] = 0$$

$$[p + \rho v_n^2 + \frac{1}{8\pi}(B_t^2 - B_n^2)] = 0 \quad [\rho v_n v_t + \frac{1}{4\pi} B_t B_n] = 0 \quad [B_n] = 0$$

$$[\rho v_n(v^2/2 + w) + \frac{1}{4\pi}(v_n B^2 - B_n(\vec{v} \cdot \vec{B}))] = 0 \quad [B_n v_t - B_t v_n] = 0$$

The analysis is simple in two limiting cases

The first is one in which the B field before the shock is normal to the shock surface, $B_{t1}=0$

This case is called "parallel" shock since the B field is parallel to shock normal direction

In such a case it is easy to show that $B_{t2} = 0$ behind the shock

Since both B_n and B_t are continous with $B_{n1} = B_{n2}$ and $B_{t1}=B_{t2}=0$, it follows that the parallel shocks reduce to the pure hydrodynamic case, as if the B field is not present

Shock waves

$$[\rho v_n] = 0$$

$$[p + \rho v_n^2 + \frac{1}{8\pi}(B_t^2 - B_n^2)] = 0 \quad [\rho v_n v_t + \frac{1}{4\pi} B_t B_n] = 0$$

$$[B_n] = 0$$

$$[\rho v_n (v^2/2 + w) + \frac{1}{4\pi}(v_n B^2 - B_n(\vec{v} \cdot \vec{B}))] = 0$$

$$[B_n v_t - B_t v_n] = 0$$

The analysis is simple in two limiting cases

The 2nd case is when the B field is parallel to the shock surface, ie perpendicular to the normal, B_{n1} =0

In such a case, from 3rd equation we see that v_t is continous → therefore we can choose a reference frame in which v_t=0 and the shock is a normal shock

From $[B_n v_t - B_t v_n] = 0$ And $[\rho v_n] = 0$ We get

$B_{t1}/B_{t2} = \rho_1/\rho_2$ While the others reduce to

$$[\rho v] = 0 \qquad [p + \rho v^2 + \frac{B_t^2}{8\pi}] = 0 \qquad [v^2/2 + w + \frac{B_t^2}{4\pi\rho}] = 0$$

Which are the hydrodynamics eqns with the additional magnetic terms is momentum and energy flux

If the plasma is made of electrons and protons (or ions), there is the question about the particle temperature after the shock, that is if we have $T_e=T_i$

We have seen that what transforms the the ordered kinetic energy in internal kinetic energy are not the collisions, but are time-varying induced electromagnetic fields

The electrons are subjected to same forces as ions, but, due to their much lower mass, the accelerations are much higher

In there conditions, it is perfectly possible that electron irradiate so that they do not retain, ie dissipate, the internal kinetic energy transferred by the shock wave → it is possible that the electrons come out form shock front with $T_e<T_i$ or even $T_e \ll T_i$

This means that the shock dissipates a fraction of its energy, but since before the shock, where the kinetic energy is mainly ordered, the electrons have only a fraction $m_e/m_p \ll 1$ of the total energy, even if almost all this energy would be dissipated, we have not important dynamic consequences on the jump conditions

We can say that, without radiative losses within the shock thickness and with no heat transfer from protons to electrons, the electron temperature behind the shock is a factor m_e/m_p lower than for protons, which is the temperature we get from RH conditions

Consider a subsonic disturbance moving through a conventional neutral fluid. As is well-known, *sound waves* propagating ahead of the disturbance give advance warning of its arrival, and, thereby, allow the response of the fluid to be both smooth and adiabatic. Now, consider a supersonic disturbance. In this case, sound waves are unable to propagate ahead of the disturbance, and so there is no advance warning of its arrival, and, consequently, the fluid response is sharp and non-adiabatic. This type of response is generally known as a *shock*. Let us investigate shocks in MHD fluids.

Since information in such fluids is carried via three different waves--namely, *fast* or compressional-Alfvén waves, *intermediate* or shear-Alfvén waves, and *slow* or magnetosonic waves we might expect MHD fluids to support three different types of shock, corresponding to disturbances traveling faster than each of the aforementioned waves. This is indeed the case.

(iv) Shock Waves

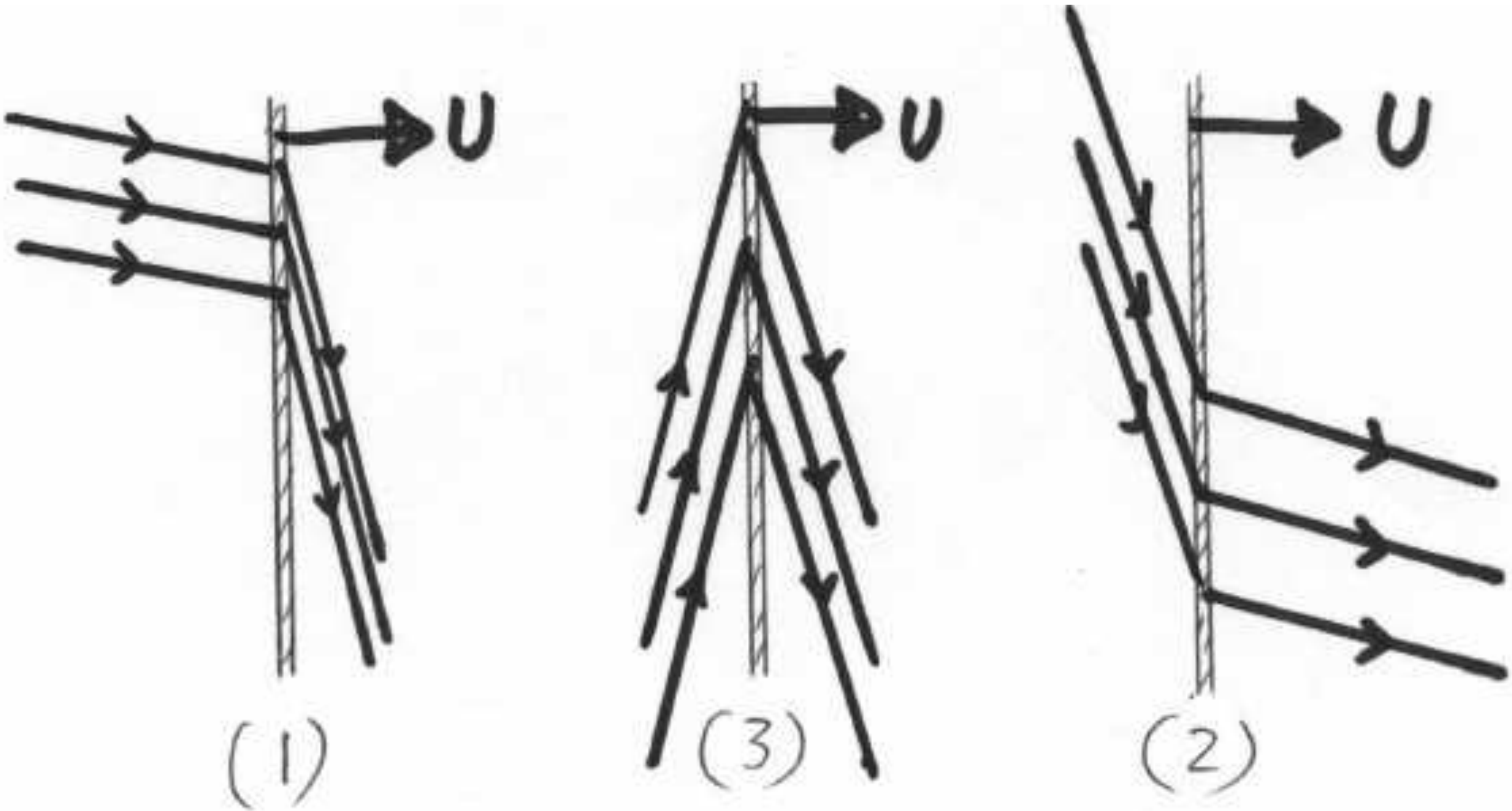
- Nonlinear sound wave can steepen to a shock wave
  -- propagates at speed > c_s

- In MHD 3 modes:

(1) **Slow-mode shock**
- propagates faster than slow-mode speed
- turns B towards normal

(2) **Fast-mode shock**
- propagates faster than fast-mode speed
- turns B away from normal

(3) **Finite-amplitude Alfvén Wave**
- no change in p - reverses tangential magnetic field



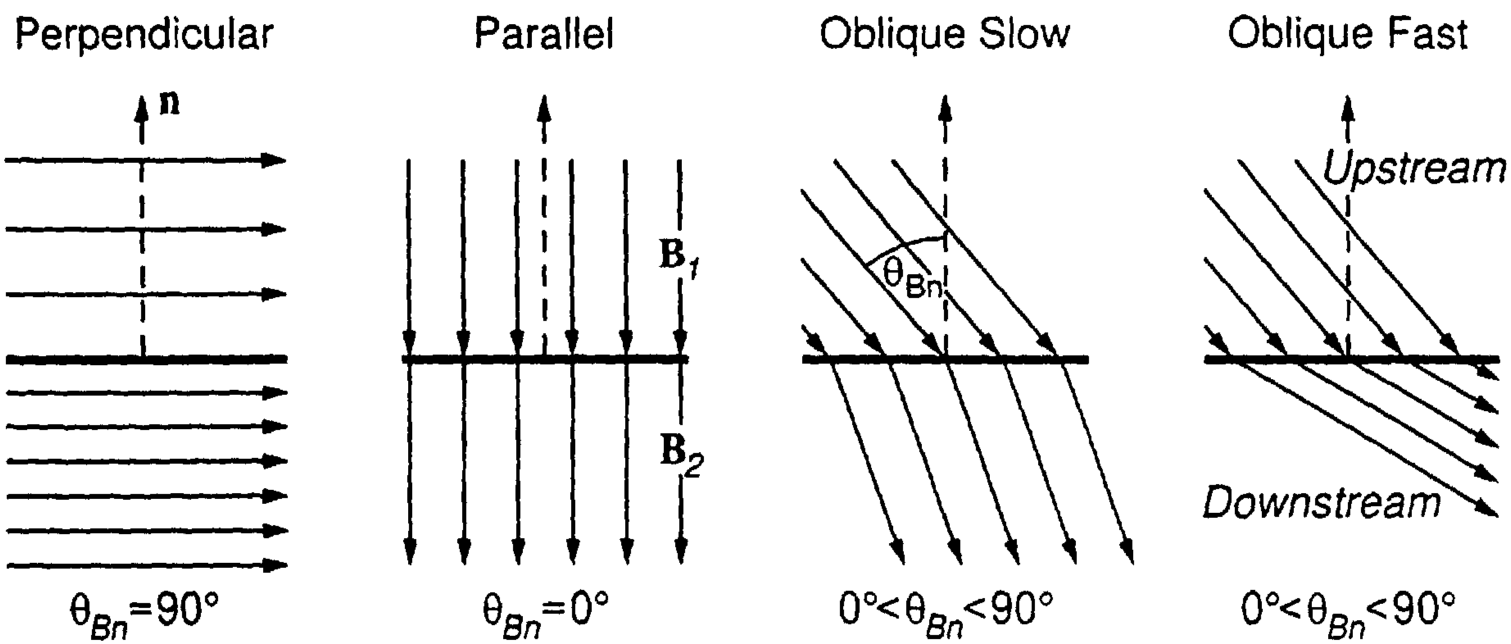
Slow-mode **Alfvén** **Fast-mode**

- The jump conditions are a set of 6 equations. If we want to find the downstream quantities given the upstream quantities then there are 6 unknowns ($\rho, v_{n,}, v_t, p, B_n, B_t$).
- The solutions to these equations are not necessarily shocks. These are conservation laws and a multitude of other discontinuities can also be described by these equations.

Types of Discontinuities in Ideal MHD		
Contact Discontinuity	$v_n = 0 \; , B_n \neq 0$	Density jumps arbitrary, all others continuous. No plasma flow. Both sides flow together at v_t.
Tangential Discontinuity	$v_n = 0, \; B_n = 0$	Complete separation. Plasma pressure and field change arbitrarily, but pressure balance
Rotational Discontinuity	$v_n \neq 0 \; , B_n \neq 0$ $v_n = B_n / (\mu_0 \rho)^{\frac{1}{2}}$	Large amplitude intermediate wave, field and flow change direction but not magnitude.

Types of Shocks in Ideal MHD		
Shock Waves	$v_n \neq 0$	Flow crosses surface of discontinuity accompanied by compression.
Parallel Shock ($\vec{B}$ along shock normal)	$B_t = 0$	B unchanged by shock.
Perpendicular Shock	$B_n = 0$	P and B increase at shock
Oblique Shocks	$B_t \neq 0, B_n \neq 0$	
Fast Shock		P, and B increase, B bends away from normal
Slow Shock		P increases, B decreases, B bends toward normal.
Intermediate Shock		B rotates 180^0 in shock plane. [p]=0 non-compressive, propagates at $u_{A,}$ density jump in anisotropic case.

•Configuration of magnetic field lines for fast and slow shocks. The lines are closer together for a fast shock, indicating that the field strength increases.



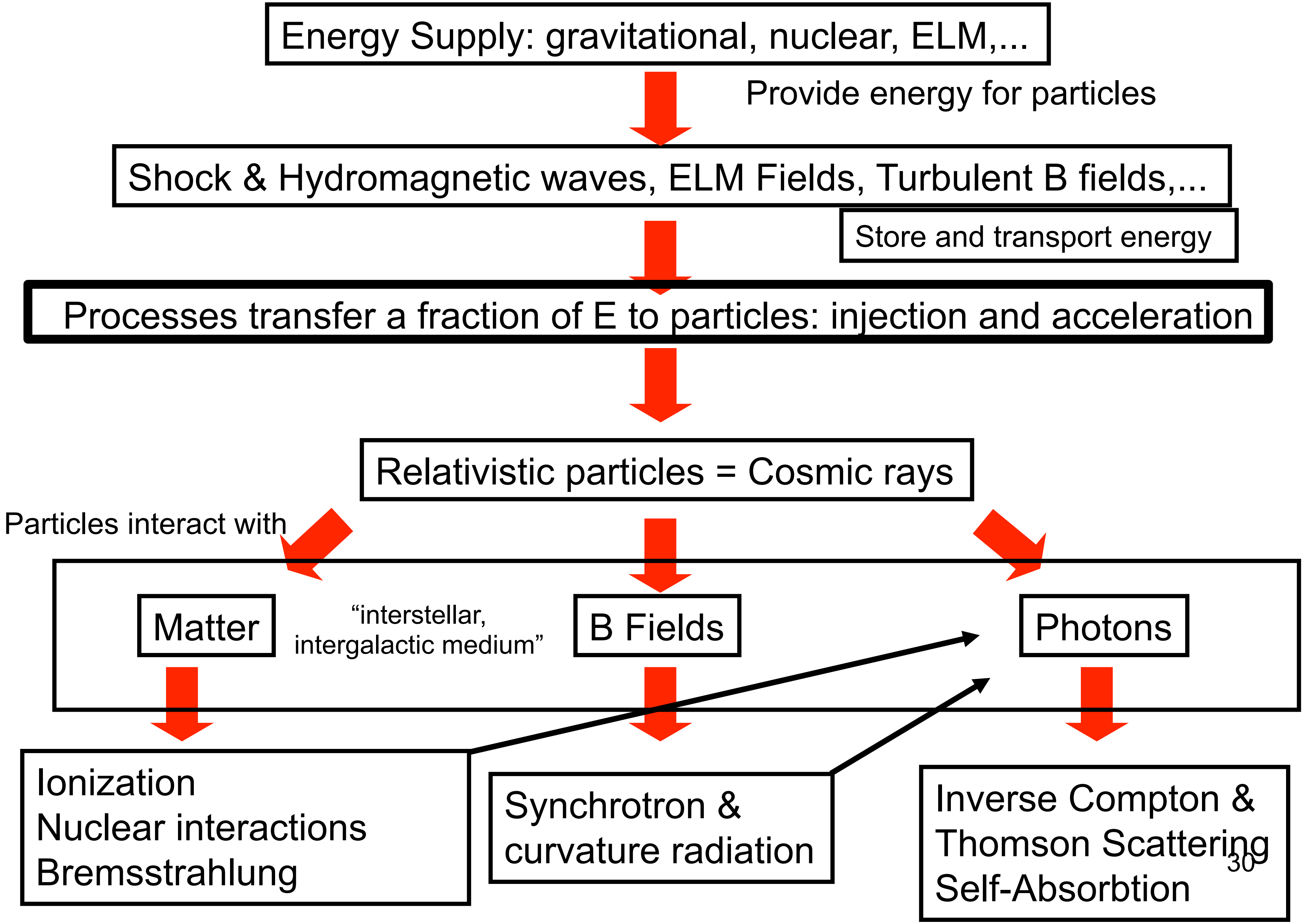
Equazione di propagazione (1)

$$\frac{\partial N_i}{\partial t} - \vec{\nabla}\cdot(\hat{D}_i \vec{\nabla} N_i) + \frac{\partial}{\partial E}(b_i N_i) + n v \sigma_i N_i + \frac{N_i}{\tau_i} = q_i + \sum_{j<i} n v \sigma_{ij} N_j + \sum_j \frac{N_j}{\tau_{ij}}$$

- I CR non sono accelerati nell'ISM, sono accelerati da sorgenti puntiformi
- La loro potenza e distribuzione spazio-temp. e' descritta dalle funzioni $q_i(t, \vec{r}, E)$ (per la specie i)

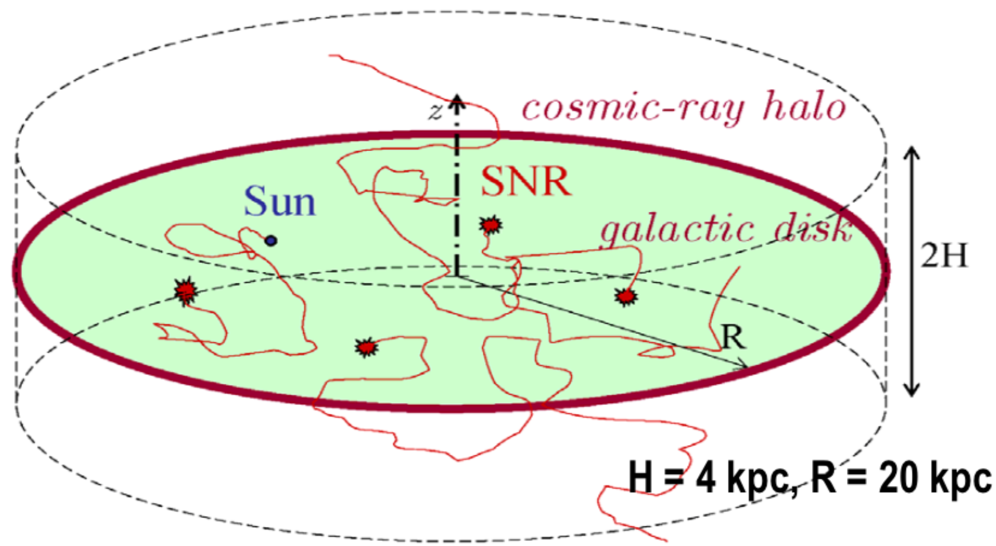
* $\hat{D}_i(\vec{r}, E)$ e' il tensore di diffusione
* $b_i(\vec{r}, E)$ caratterizza le perdite contine di energia delle <u>singole</u> particelle così che $dE/dt = b_i$ (≡ ionizz. + brems + sincrotr + Compton inverso)
* $\sigma_i(E)$ e' la sez. d'urto inelastica del nucleo i con i nuclei dell'ISM
* $n(\vec{r})$ e' la densita' dell'ISM
* σ_{ij} e' la sez. d'urto di prod. di nuclei di tipo i da nuclei più pesanti
* τ_i e' la vita media rispetto a decad. radioatt.
* N_i/τ_{ij} descrive l'apparizione di nuclei di tipo i a causa del decad. di altri nuclei

Formation and Interactions of CR's

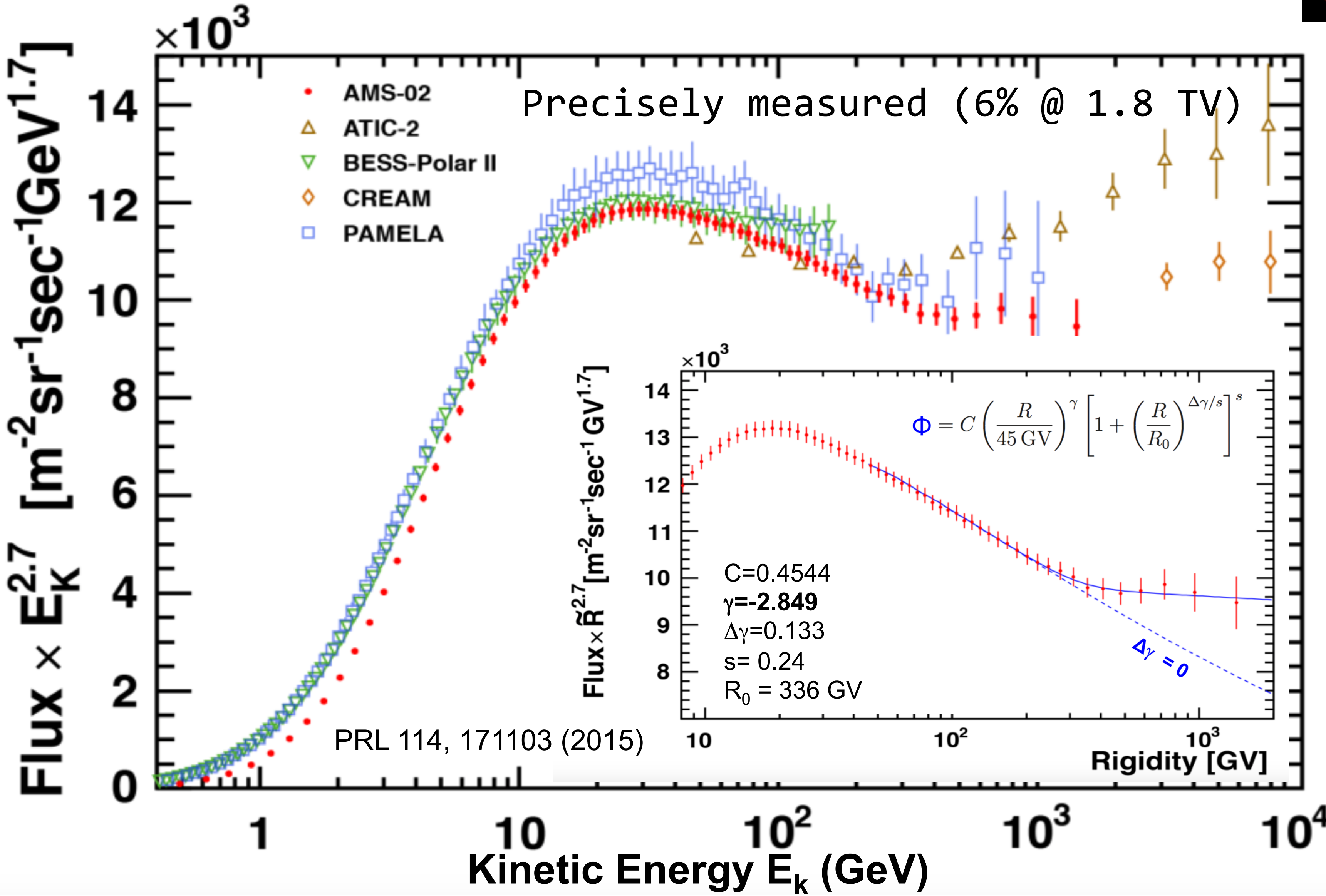


Formation and Interactions of CR's

- **Particles get accelerated at astrophysical sources**
- They leave sources and propagate in the ISM
- They interact with the ISM particles and/or decay producing new particles
- They loose energy in the ISM by elm processes (brems, IC, sync)
- They are bent randomly by ISM magnetic field
- Until they reach the galaxy border and escape the galaxy
- Therefore: Their spectum and composition at the Earth location are not representative of the source spectrum



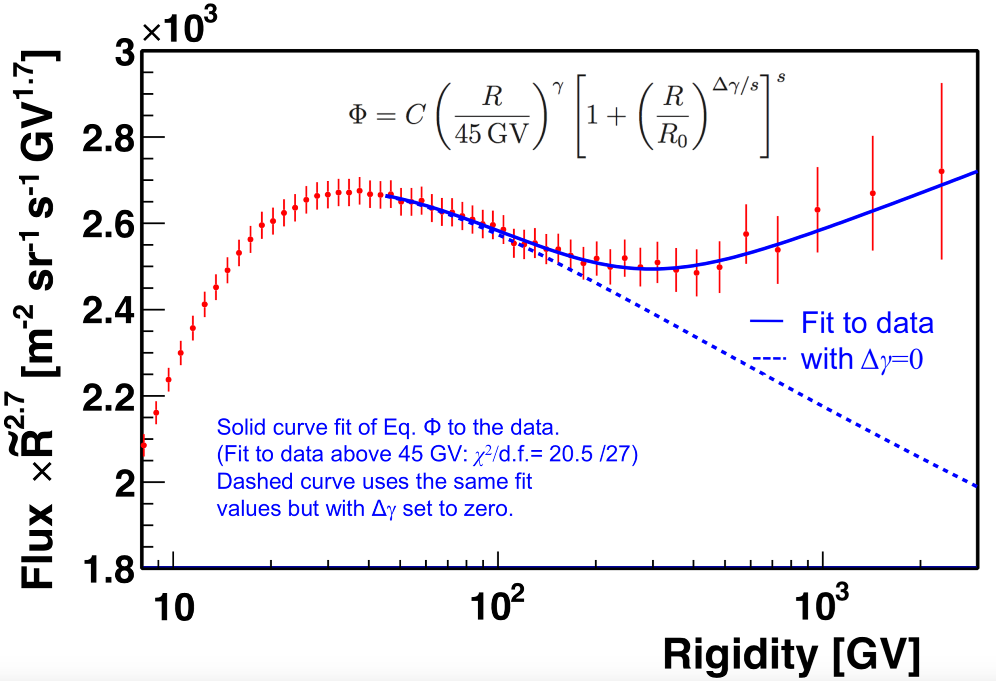
Proton flux

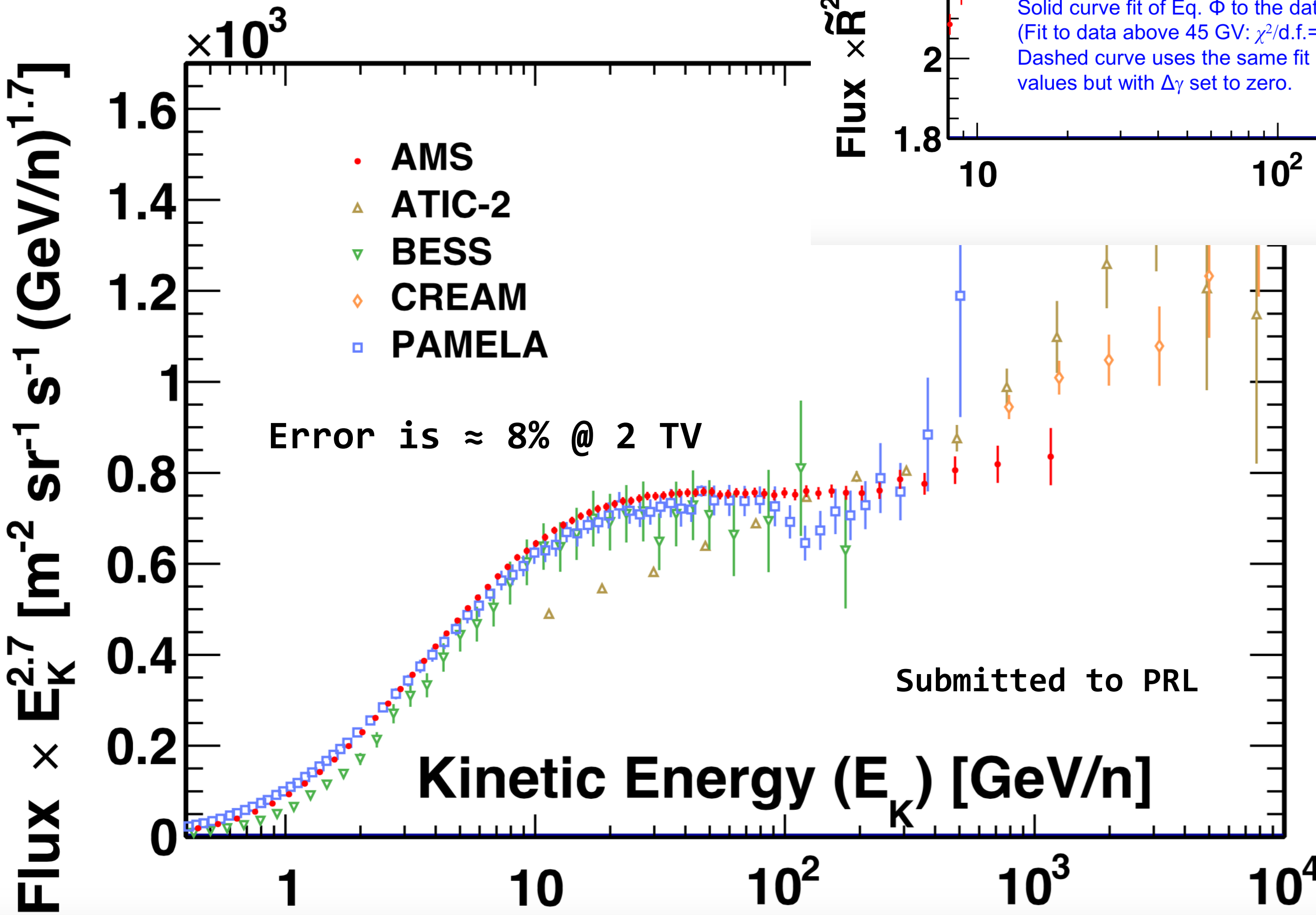


A change in the the spectral index is observed at 336 GV

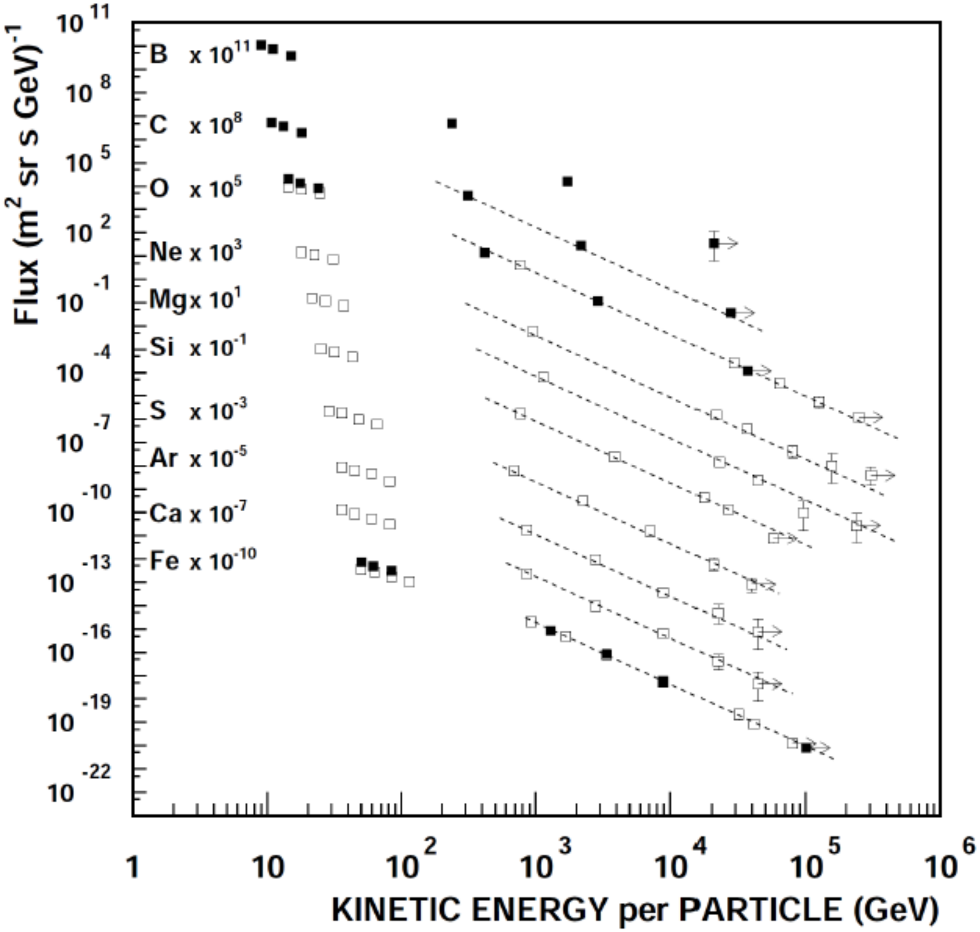
He Flux

Transition energy R_0 and smoothness parameter s are the same as for protons





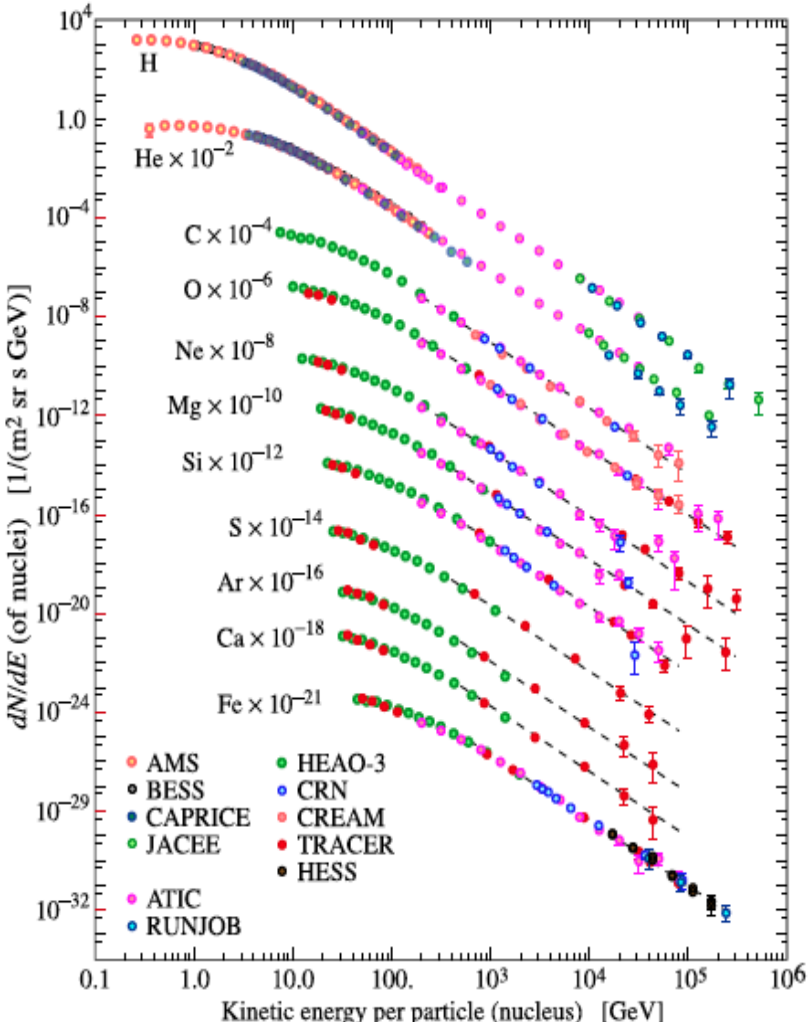
Nuclear species flux



TRACER 2 flights data:
POWER_LAW fit above 20 GeV/n:

$$\text{index } 2.65 \pm 0.05$$

Comparison between experimental results



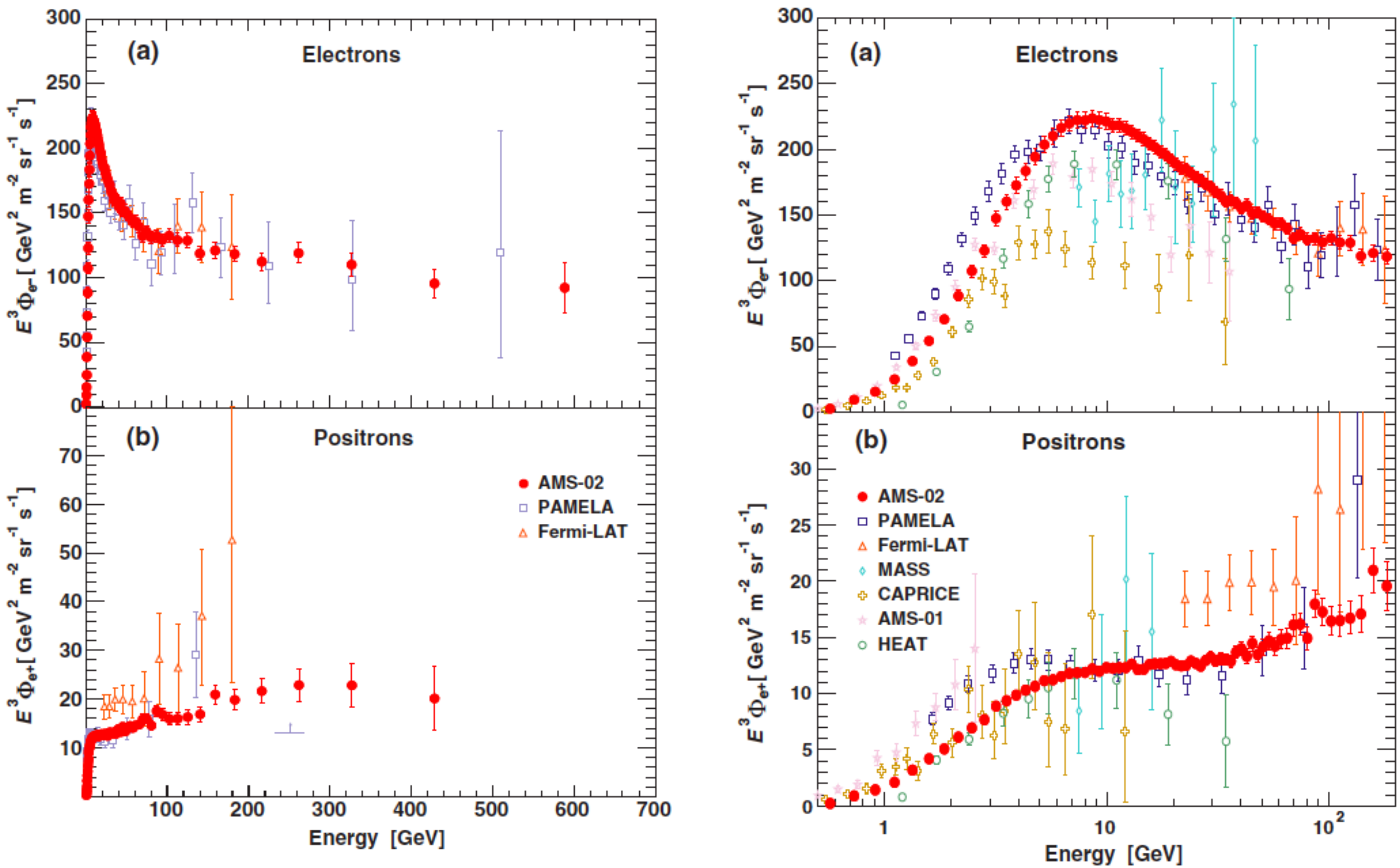
QUESTION:

Are all the hi-Z spectra simple power laws or is there beginning to be evidence for evolution (hardening), as now seems to be the case for H and He?

Figure courtesy of P.J. Boyle and D. Mueller

Electron/Positron fluxes:

No sharp structures



Accelerazione di particelle

- L'accelerazione di RC da parte di un qualche meccanismo Galattico (o extragalattico) deve tener conto dei seguenti fatti sperimentali:
  - Lo spettro di potenza (per tutti i RC) del tipo $dN/dE \sim E^{-\gamma}$ con $\gamma=2.7$ per p sino a $E \sim 4\times10^{15}$ eV e $\gamma \approx 3.1$ per e+ ed e-)
  - lo spettro alle sorgenti ha un indice spettrale prossimo a 2
  - L'energia massima misurata ($E \sim 10^{20}$ eV)
  - Le abbondanze relative tra gli elementi, tenendo conto degli effetti di propagazione

Confinamento: $r_{Larmor} = \frac{\mathbf{1}}{\mathbf{300}} \frac{E}{ZB} (eV / Gauss)$ [cm]

- Utilizziamo i valori tipici del campo B (3×10^{-6} G) galattico per protoni:

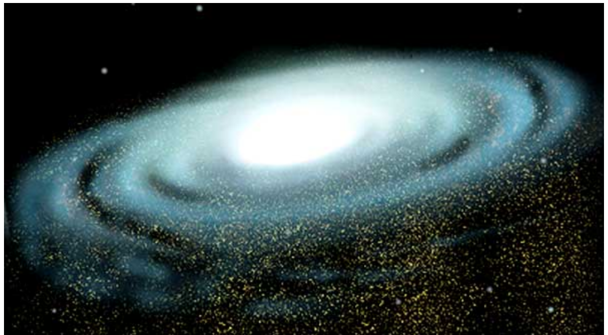
$$r_L = \begin{cases} (E = 10^{12} eV) & = 10^{15} cm = 3 \cdot 10^{-4}\ pc \\ (E = 10^{15} eV) & = 10^{18} cm = 0.3\ \ pc \\ (E = 10^{18} eV) & = 10^{21} cm = 300\ \ pc \end{cases}$$

- I p hanno un raggio di Larmor sempre minore dello spessore del disco galattico (300 pc) se E<10^{18} eV. Per questo motivo tutti i RC (meno quelli di energia estrema) sono *confinati* nel piano Galattico dal campo magnetico.

Isotropia dei RC

- I RC primari al di sotto di 10^{18} eV hanno una distribuzione di arrivo completamente isotropa sulla sommità della nostra atmosfera. **Qualè il motivo?**

- Campi magnetici **galattici**:
  - $B \cong 3\times10^{-6}$ G
  - coerenti su scale di distanza 1-10 pc
  - NOTA: 1 pc=3×10^{18} cm
- **Galassia≡** disco di raggio R=15 kpc, spessore h=300-1000 pc



Richiamo: moto di un RC nel campo magnetico Galattico

$$mv^2/r = pv/r = ZevB/c$$

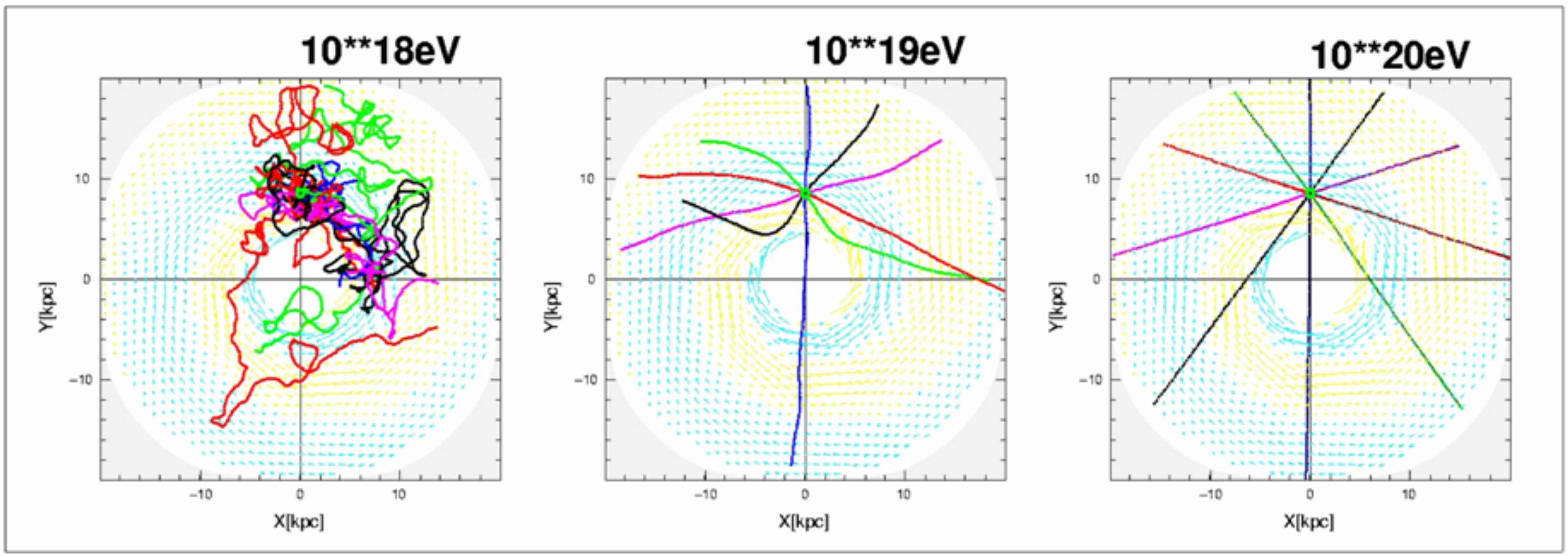
$$r = pc/ZeB$$

$$r(cm) = \frac{1}{300}\frac{E(eV)}{ZB(G)}$$

$$(10^{12}eV) = 10^{15}cm = 3\times10^{-4}\,pc$$

$$r = (10^{15}eV) = 10^{18}cm = 3\times10^{-1}\,pc$$

$$(10^{18}eV) = 10^{21}cm = 300\,pc$$



Le particelle diffondono e vengono isotropizzate dalle irregolarita' del campo magnetico grazie a risonanze (es. di ciclotrone) quando $\rho_g \sim \lambda$ delle onde di alfven con una lunghezza di diffusione $\lambda_{sc} = \rho_g(B_o/\delta B(k))^2$ con $\delta B(k)=\int w(k)dk$, $w(k) \sim k^{-\alpha}$ densita' spettrale di potenza

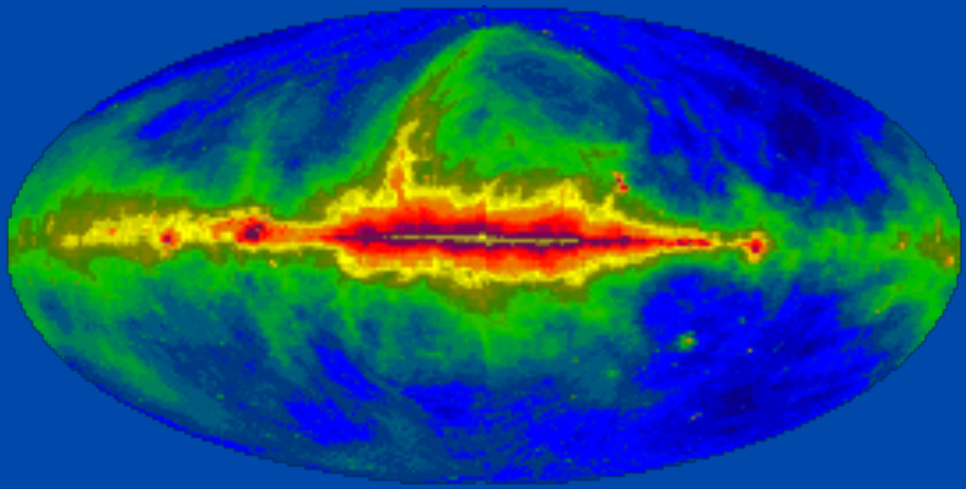
Il processo e' quindi diffusivo con $D= c\lambda_{sc}/3 \sim D_o(R/R_o)^{\alpha-2}$

Volume di confinamento

- Le particelle attraversano in media $\xi = 50$ kgm^{-2} di ISM
- $\xi = \rho c \tau \rightarrow$ otteniamo $\rho < 1$ pcm^{-3} tipico del disco galattico
- Questo implica che le part viaggino anche in regioni meno dense di quelle tipiche del disco (dove le abbondanze non cambiano) con un percorso casuale "tortuoso" per via delle irregolarita' del campo B galattico. Si puo' dire che esse non sono libere di sfuggire liberamente
- E' importante quindi stimare quanto e' grande il volume in cui i CR sono confinati

Volume di confinamento (2)

- Il volume puo' essere:
  - il disco della galassia, cioe' approx un disco di raggio 10-15 kpc e spessore 300-500 pc
  - galassia + alone, una regione meno densa di raggio ~15 kpc che circonda il disco di forma sferico-ellissoidale → favorito dalle misure sperimentali



Other evidence of galactic halo: 408 MHZ map of the sky : synchrotron emission of few GeV electrons in the galactic magnetic field

Potenza delle sorgenti dei RC

- Il *confinamento* dei RC ci induce a sospettare che le sorgenti siano di origine Galattica (tranne che per i RC di energia estrema).
- Qual è *l'energetica* delle sorgenti? (necessaria per individuarle).
  - Il tempo di confinamento dei RC: $\tau = 3\times10^7$ y (calc. Fra poco)
  - Volume della galassia (con o senza alone) :

$$V_G = (15kpc)^2 \cdot \pi \cdot 300\,pc \cong 6\times10^{66}\,cm^3$$

$$V_G^{Alone} = 4/3 \cdot \pi (10kpc)^3 \cong 10^{68}\,cm^3$$

- Potenza necessaria per mantenere uno *stato stazionario* di RC:

$$W_{CR} = \frac{w_{CR}(erg/cm^3)\cdot V^A{}_G(cm^3)}{\tau(s)} = \frac{10^{68}\times1.6\cdot10^{-12}(erg)}{3\cdot10^7\times(3.15\cdot10^7 s)} \cong 10^{41}\,\frac{erg}{s}$$

Esiste un meccanismo con una potenza tale da sostenere il flusso dei RC nella Galassia?

- Una esplosione di Supernova libera:
  - 10^{51} erg/esplosione
- La stima della frequenza di SN nella nostra Galassia è
  - $f_{SN} = 1/\tau_{SN} = 1/30\ y^{-1}$
- Si noti che $\tau_{SN} < \tau \cong 10^7$y. Le SN sono un fenomeno quasi continuo su scala dei tempi del confinamento dei RC.
- Potenza energetica liberata dalle SN:

$$W_{SN} = \frac{10^{51} erg}{30 \times 3.15 \cdot 10^{7} s} \cong 10^{42} erg/s$$

- Perché il quadro sia coerente, occorre trovare un meccanismo che trasferisca al più il 10% di energia dalle supernovae in energia cinetica di particelle (i RC)→ **Meccanismo di Fermi**

Leaky Box Model

A useful approximation is the following:

- Assume there is no diffusion (eg D=0)
- CR propagate freely in the galaxy volume, uniformly filled with ISM and regular B, until they reach the "border" and escape
- A CR has a probability per unit of time to escape the galaxy $p = 1/\tau_{esc}$, where τ is the measured residence time of CR in the galaxy

Then

$$D\nabla^2 \mathcal{N} \rightarrow -\frac{\mathcal{N}}{\tau_{esc}}.$$

As the diffusion coefficient D is energy dependent, also the characteristic escape time of CRs from the Galaxy $\tau_{esc} = \tau_{esc}(E)$ is energy dependent

Leaky Box Model

- The transport equation becomes

$$\frac{\mathrm{d}\mathcal{N}_i}{\mathrm{d}t} = -\frac{\mathcal{N}_i}{\tau_{\mathrm{esc}}} + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i} \frac{P_{ji}}{\tau_j}\mathcal{N}_j.$$

- The leaky-box model provides the most common description of CR transport in the Galaxy at energies below ~ 10^{17} eV. The model is based on particles injected by sources Q distributed uniformly over the galactic volume (the *box*) filled with a uniform distribution of matter and radiation fields. The particles get-away from this volume with an escape time independent of their position in the box. The escape time $\tau_{\mathrm{esc}}(E)$ depends on the particle energy, charge, and mass number, but it does not depend on the spatial coordinates. Secondary nuclei are produced during the propagation as a function of the path length

Spettro dei RC alle sorgenti

- Il modello Leaky Box permette di collegare lo spettro osservato a Terra con quello alle sorgenti
- In assenza di tutti i processi e in uno stato stazionario, tranne che la fuga dalla galassia, il modello ci da

$$0 = Q(E) - \frac{N(E)}{\tau(E)}$$

$$N(E) = Q(E)\tau(E)$$

- In questa approssimazione, lo spettro a Terra e' quello delle sorgenti Q(E) modificato dall'effetto della propagazione, descritta dal termine τ(E)

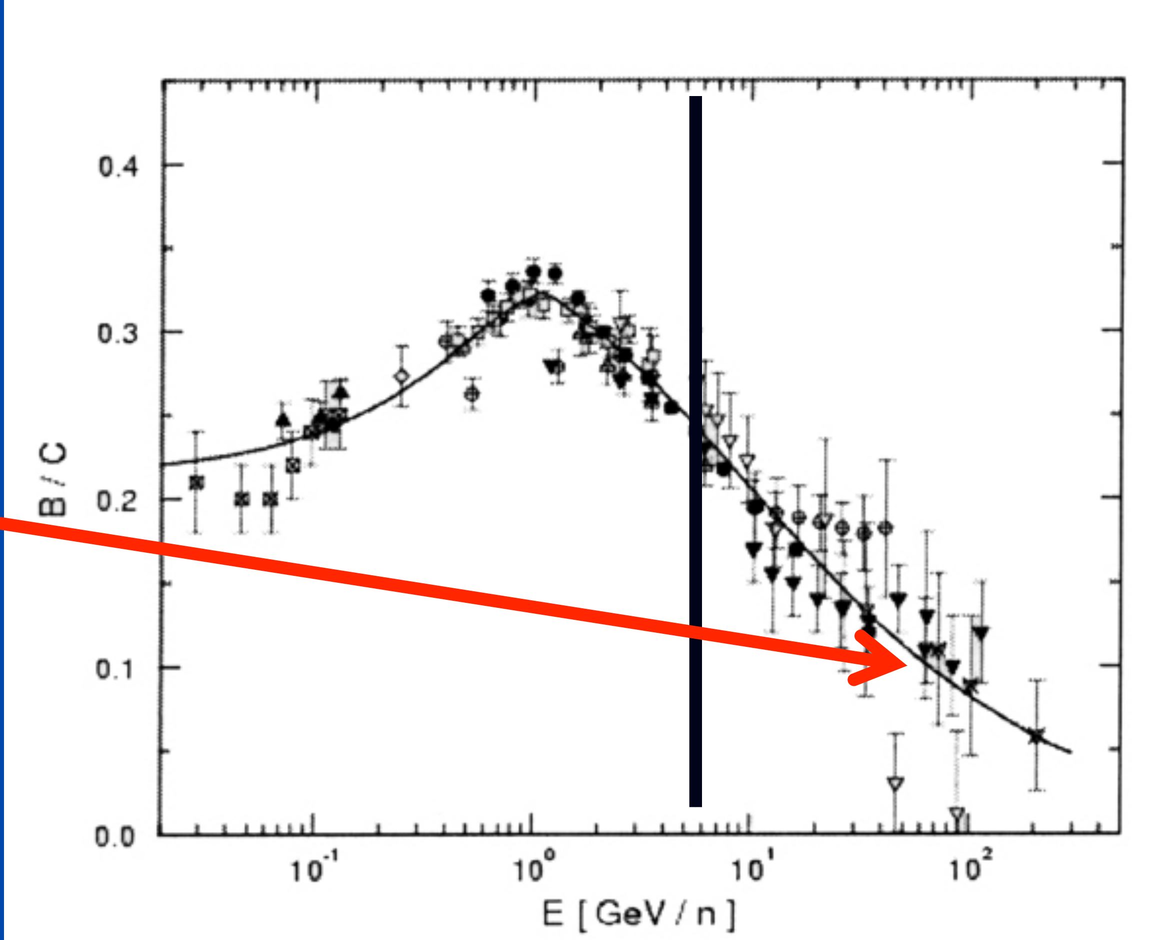
Dipendenza del rapporto r vs. E

- I dati sperimentali confermano questa ipotesi.
- In particolare, si ottiene che la probabilità di fuga dalla Galassia dipende dall'energia come:

$$\tau = \tau_o \left(\frac{R}{R_0} \right)^{-\delta}$$

- Ossia, poiché τ~ ξ

$$\xi = \xi_o \left(\frac{R}{R_o} \right)^{-\delta}$$



NB: $\delta = \alpha - 2$ e $D \sim H^2/\tau$, H spessore dell'alone, percio' la misura di δ da informazioni sulla natura della turbolenza del campo magnetico al queste scale di energia

Valori tipici $\xi_o = 11.8$ gr/cm_2, $R_0 = 5$ GV/c, $\delta = 0.6$

L'indice spettrale δ ha un'incertezza piuttosto grande. Il suo valore e' compreso in un intervallo 0.15-0.8 a seconda del modello. Per esempio un regime in cui D o τ dipendono dalla posizione, come modelli a doppia regione (disco+alone), effetti non lineari,…

Spettro dei RC alle sorgenti

- Il risultato appena ottenuto è estremamente importante, perché permette di avere informazioni sullo spettro energetico dei RC alle sorgenti.
- Poiché il flusso dei RC sulla Terra è stazionario, vi deve essere equilibrio tra:
  - Spettro energetico misurato: $\Phi(E) \propto E^{-2.7} (erg / cm^3 \cdot GeV)$
  - Spettro energetico alle Sorgenti: $Q(E) \propto E^{-?} (erg / s \cdot GeV)$
  - Probabilità di diffusione: $\tau(E) \propto E^{-0.6} (s)$

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

Spettro dei nuclei alle sorgenti

$$\frac{4\pi}{c}\int \Phi(E)dE = \int \frac{Q(E)\cdot \tau(E)}{Volume}dE$$

- Quindi, inserendo le dipendenze funzionali:

$$Q(E) = \frac{\Phi(E)}{\tau(E)} = \frac{E^{-2.7}}{E^{-0.6}} = E^{-2.1}$$

- Il modello che descrive le sorgenti di RC nella Galassia, dovrà prevedere una dipendenza con l'energia del tipo $\sim E^{-2}$.
- Occorre trovare un processo che produca uno spettro di questo tipo alla sorgente.
- Il modello di Fermi prevede proprio un andamento funzionale di questo tipo!

Elettroni e positroni

$$\frac{\partial N_i}{\partial t} - \vec{\nabla}\cdot\left(\hat{D}_i \vec{\nabla} N_i\right) + \frac{\partial}{\partial E}\left(b_i N_i\right) + n v \sigma_i N_i + \cancel{\frac{N_i}{\tau_i}} = q_i + \cancel{\sum_{j<i} n v \sigma_{ij} N_j + \sum_j \frac{N_j}{\tau_{ij}}}$$

σ_i e' la sez d'urto per il processo $p_{CR}p_{ISM} \rightarrow \pi^{\pm} \rightarrow \mu^{\pm} \rightarrow e^{\pm}$ ed n e' la densita' di p nell ISM, N_i e' la densita' di elettroni

Da dove vengono gli elettroni?

- Da interazione dei RC con i protoni del mezzo interstellare
- Da sorgenti primarie, cioe' siti di accelerazione
- Da sorgenti esotiche(?)

Per gli elettroni, a differenza della componente nucleare, sono importanti i processi di perdita di energia durante la propagazione. Il termine che descrive le perdite di energia continue non puo' essere trascurato ma anzi diventa dominante

Toy Model

- Caso semplice di: – soluz. stazionarie $dN/dt = 0$

× C'e' una distrib infinita e uniforme di sorgenti che iniettano e^- con spettro di iniezione $Q(E) = k\,E^{-p}$

⇒ La diffusione non e' importante ⇒ L'equ. diventa

$$-\frac{d}{dE}\left[b(E)N(E)\right] = Q(E) \quad \Rightarrow \quad \int d\left[b(E)N(E)\right] = -\int Q(E)\,dE$$

× Assumiamo $N(E) \rightarrow 0$ per $E \rightarrow \infty$ ⇒

$$N(E) = \frac{kE^{-(p-1)}}{(p-1)\,b(E)} \qquad \text{dove} \quad b(E) = A_1\left(\ln\frac{E}{mc^2} + 19.8\right) + A_2 E + A_3 E^2$$

⇒ Se domina ionizz. $N(E) \propto E^{-(p-1)}$. spettro di E più piatto

Se " Brems (o adiab. loss) $N(E) \propto E^{-p}$. lo spettro non cambia

Se IC o sincr dominano $N(E) \propto E^{-(p+1)}$ spettro più ripido

- La diffusione non e' importante perche' se c'e' una distribuzione infinita e uniforme di sorgenti non ci sono gradienti di densita' spaziale, a regime

Spettro degli e+,e- alle sorgenti

$$\frac{4\pi}{c}\int \Phi(E)dE = \int \frac{Q(E)\cdot\tau(E)}{Volume}dE$$

- Quindi, inserendo le dipendenze funzionali:

$$Q(E) = \frac{\Phi(E)}{\tau_{loss}(E)} = \frac{E^{-3.1}}{E^{-1.0}} = E^{-2.1}$$

- Anche per gli elettroni di alta energia lo spettro alle sorgenti e' compatibile con -2

- Nel caso di diffusione, le particelle si spostano in media di $\lambda \approx (2D\tau)^{1/2}$
- In generale $D = D(E) \Rightarrow \lambda(E, E_0) = \left(\int_0^{\tau(E)} D(E')\,d\tau' \right)^{1/2} = \left(\int_{E_0}^{E} \frac{D(E')\,dE'}{b(E')} \right)^{1/2}$

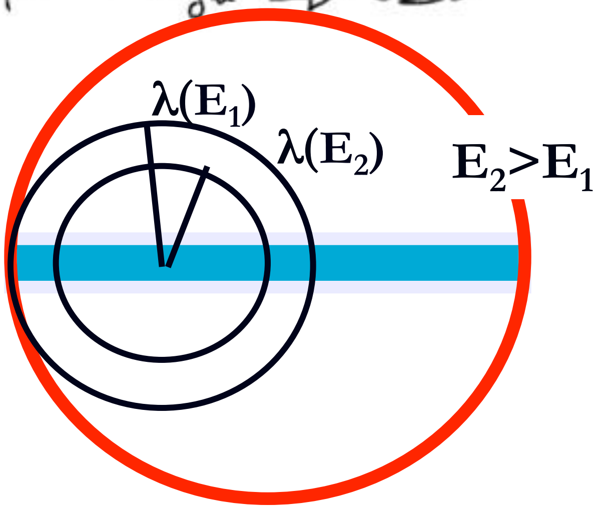
× $D \approx 10^{29}\ cm^2 s^{-1}$ (val. medio galattico) $\Rightarrow \lambda \approx 10$ kpc @ 1 GeV
(solo IC + Sincr. i.e. $\tau = \frac{1}{kE_0}$)

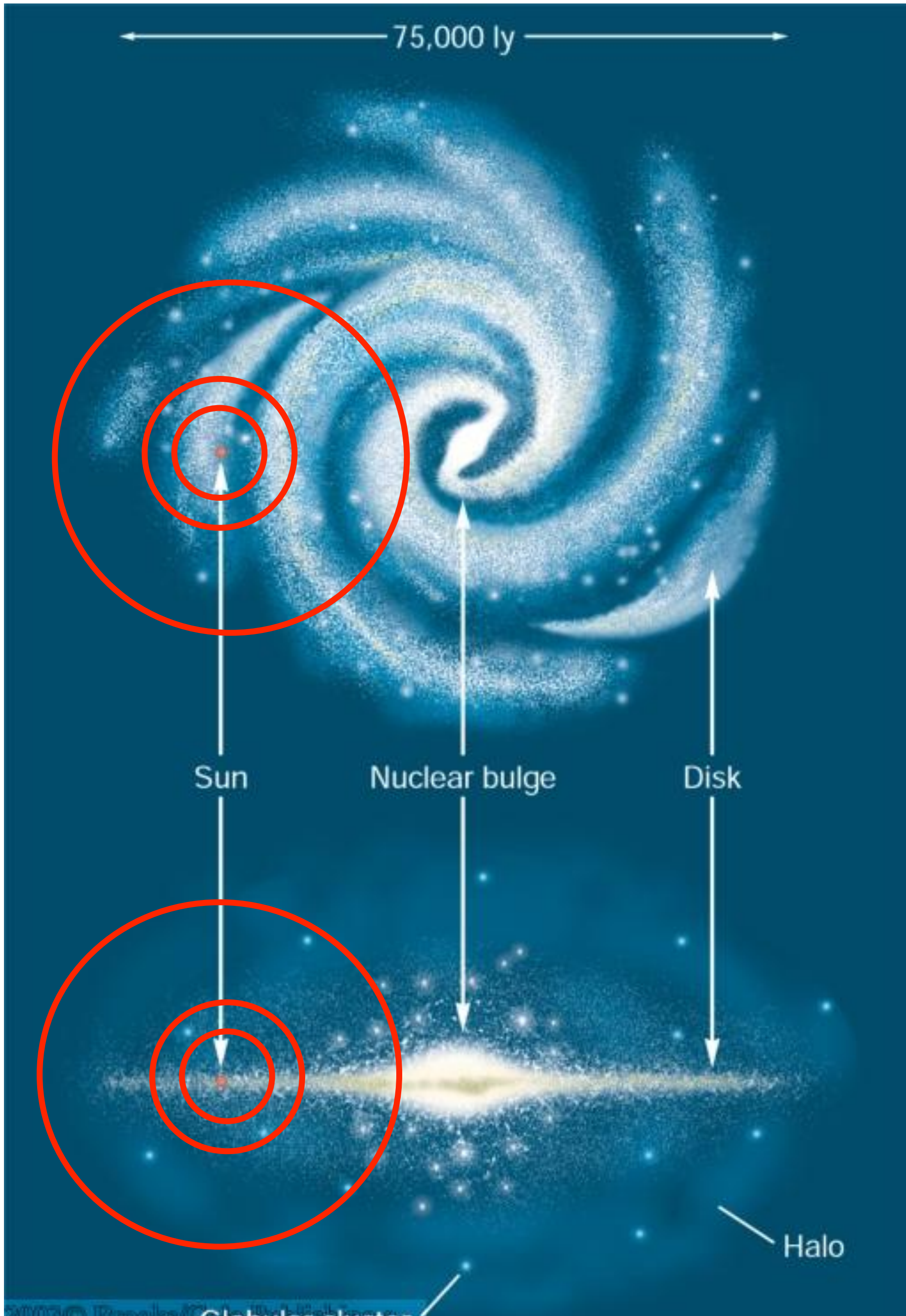
× Trascurando la dipend. di D da E $\lambda \propto E^{-1/2} \Rightarrow$
Lo spettro degli $e^{\pm}$ nei differenti intervalli di energia dipende alle condizioni medie di propagazione nell'ISM in una regione di raggio $R \sim \lambda(E)$

× Esempio: per e^- con $E > 1$ GeV, $\lambda(E) \lesssim 10$ kpc $= R_{gal} \Rightarrow$ ~~solo~~

$\Rightarrow$ Gli e^- sono galattici

- Quindi lo spettro osservato cambia parecchio durante la propagazione





≈10^2 ly @ 1 TeV

≈$3.16x10^3$ ly @ 100 GeV

≈10^4 ly @ 10 GeV

≈$3x10^4$ ly @ 1 GeV

Gli elettroni vengono da vicino!

SNRs vicine

Vicino al noi ci sono numerose potenziali sorgenti di elettroni

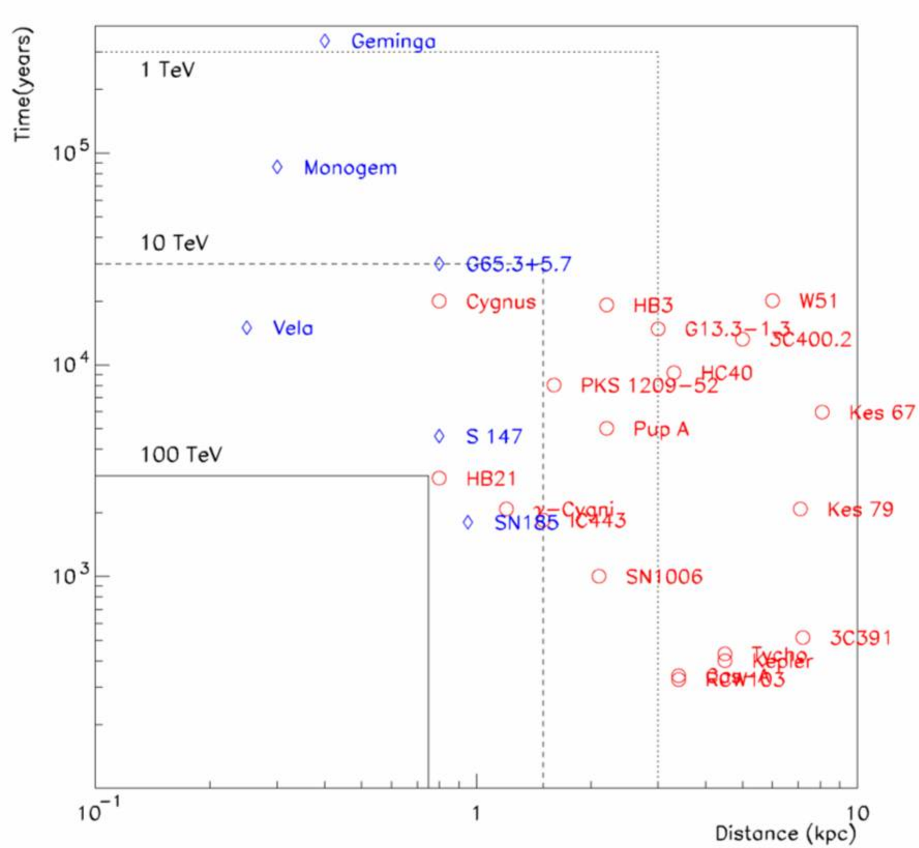


TABLE 1
List of Nearby SNRs

SNR	Distance (kpc)	Age (yr)	E_{max} [a] (TeV)	Reference
SN 185	0.95	1.8×10^3	1.7×10^2	1
S147	0.80	4.6×10^3	63	2
HB 21	0.80	1.9×10^4	14	3, 4
G65.3+5.7	0.80	2.0×10^4	13	5
Cygnus Loop	0.44	2.0×10^4	13	6, 7
Vela	0.30	1.1×10^4	25	8
Monogem	0.30	8.6×10^4	2.8	9
Loop1	0.17	2.0×10^5	1.2	10
Geminga	0.4	3.4×10^5	0.67	11

[a] Maximum energy limited by the propagation of electrons in the case of the prompt release after the explosion. The delay of the release time gives the larger value.

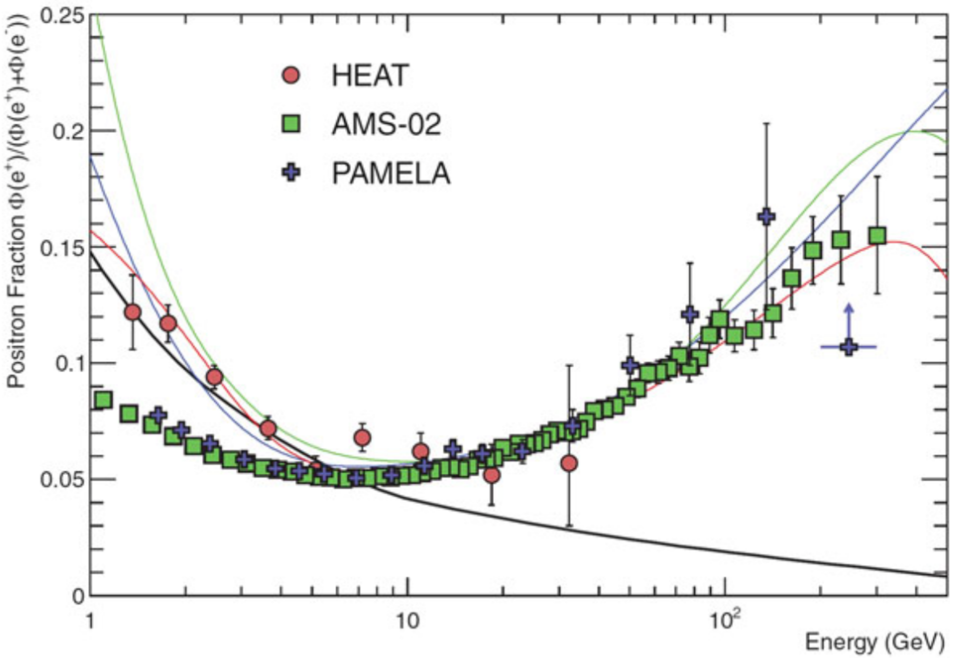
References.—(1) Strom 1994; (2) Braun et al. 1989; (3) Tatematsu et al. 1990; (4) Leahy & Aschenbach 1996; (5) Green 1988; (6) Miyata et al. 1994; (7) Blair et al. 1999; (8) Caraveo et al. 2001; (9) Plucinsky et al. 1996; (10) Egger & Aschenbach 1995; (11) Caraveo et al. 1996.

Eccesso rispetto a cosa?

Produzione secondaria nell'ISM di e+ ed e-

Sorgenti primarie di e-: $n_-(E)=K_{ep} N_{cr}(E) + \sigma_{pe-} n_{ISM} c\, n_{cr}(E)$, $K_{ep} \approx 10^{-3}$

No sorgenti primarie di e+: $n_+(E) = \sigma_{pe+} n_{ISM} c\, n_{cr}(E)$



Leaky box equilibrium density $n_{cr}(E) = N_{cr}(E)\tau_{esc}(E)$, $\tau_{esc}(E) = \tau_0 E^{-\delta}$

$$n_-(E)/n_+(E) = (K_{ep} N_{cr}(E) + \sigma_{pe-} n_{ISM} c\, N_{cr}(E)\tau_{esc}(E))/\sigma_{pe+} n_{ISM} c\, N_{cr}(E)\tau_{esc}(E) =$$

$$= \sigma_{pe-}/\sigma_{pe+} + K_{ep}/\tau_{esc}(E)$$

$$n_+(E)/(n_+(E) + n_-(E)) = 1/(1 + n_-(E)/n_+(E)) = 1/(1 + \sigma_{pe-}/\sigma_{pe+} + K_{ep}/\tau_{esc}(E))$$

$\sigma_{pe-}/\sigma_{pe+} \approx 0.5 - 0.3$ ➔ $n_+(E)/(n_+(E) + n_-(E)) \approx 1/(1.5 + (K_{ep}/\tau_0)E^{\delta})$

➔ **Senza produzione primaria di e+ alle sorgenti la frazione decresce con l'energia**

Comparing our data with a minimal model, as an example.

In this model the e^+ and e^- fluxes, Φ_{e+} and Φ_{e-}, are parametrized as the sum of individual diffuse power law spectra and the contribution of a single common source of $e^{\pm}$:

$$\Phi_{e^+} = C_{e^+} E^{-\gamma_{e^+}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad \text{Eq(1)}$$

- pulsar source
- Dark matter source

$$\Phi_{e^-} = C_{e^-} E^{-\gamma_{e^-}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad (E \text{ in GeV}) \quad \text{Eq(2)}$$

Coefficients C_{e+} and C_{e-} correspond to relative weights of diffuse spectra for positrons and electrons.

C_s is the weight of the source spectrum.

γ_{e+}, γ_{e-} and γ_s are the corresponding spectral indexes.

E_s is a characteristic cutoff energy for the source spectrum.

With this parametrization the positron fraction depends on 5 parameters.

Possiamo dare un quadro generale degli spettri alle sorgenti:

- Per la componente nucleare $p \approx 2$
- Per la componente e^{+-} $p \approx 2.1$

Fino ad almeno qualche centinaio di GeV

Gli spettri alla sorgente sono differenti per e- e protoni (cum grano salis)

Per entrambe le componenti c’e’ evidenza sperimentale di hardening dello spettro a qualche O(100 GeV)

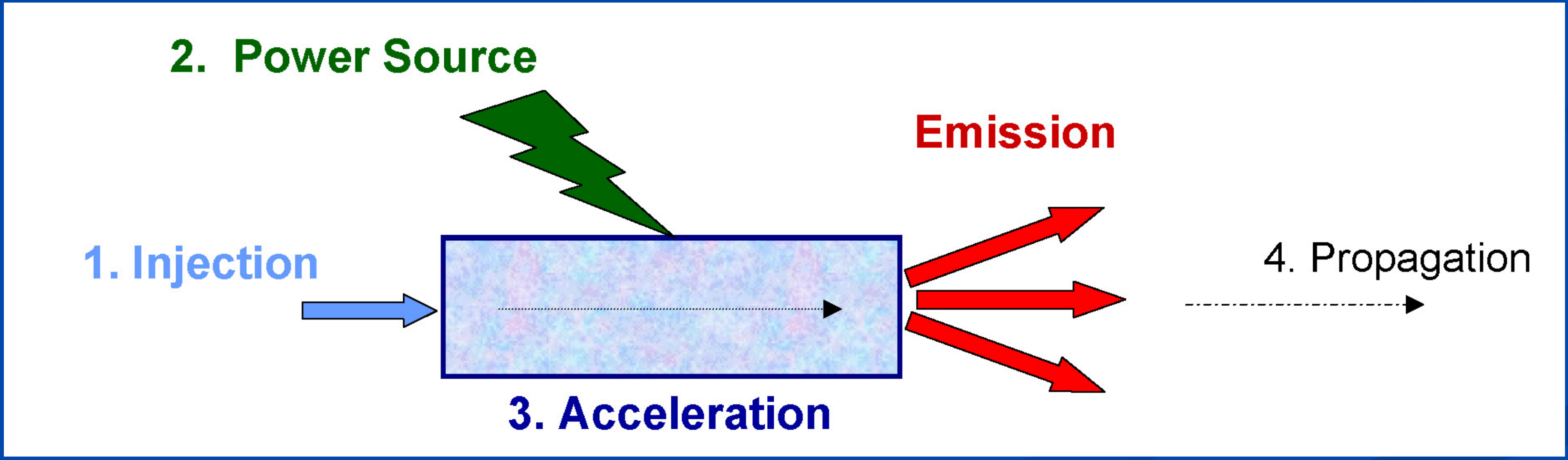
Per e+ ed e- l’evidenza e’ netta: c’e’ una sorgente che immette e+ ed e- di alta energia nell’ISM

Per i nuclei la situazione e’ piu’ incerta: nuova classe di sorgenti? Effetto di propagazione? Servono piu’ dati.

Le pulsar accelerano e+ ed – ma non protoni (per esempio)

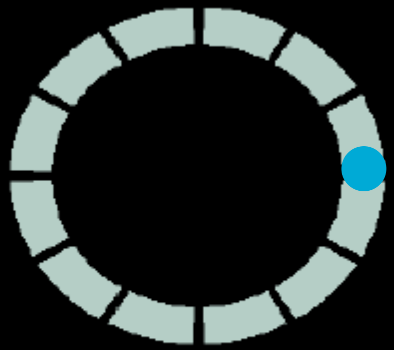
Accelerazione dei raggi cosmici

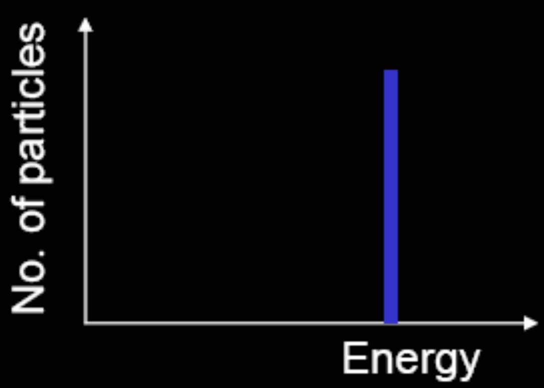
Per costruire un "acceleratore cosmico" abbiamo bisogno di:



How might such cosmic accelerators work?

Man-made accelerators





How might such cosmic accelerators work?

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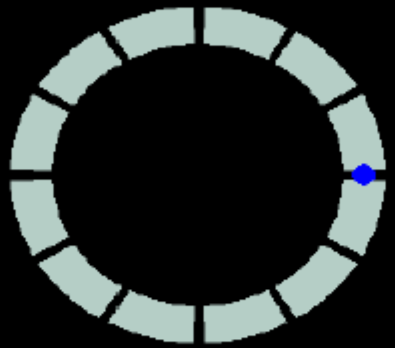
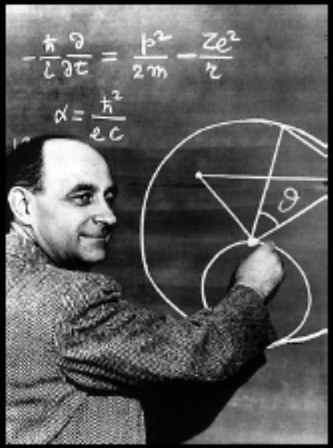
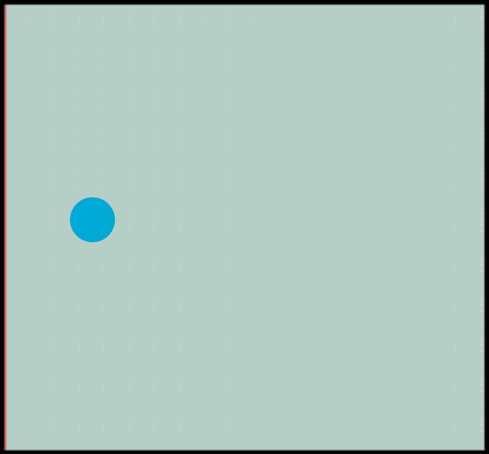
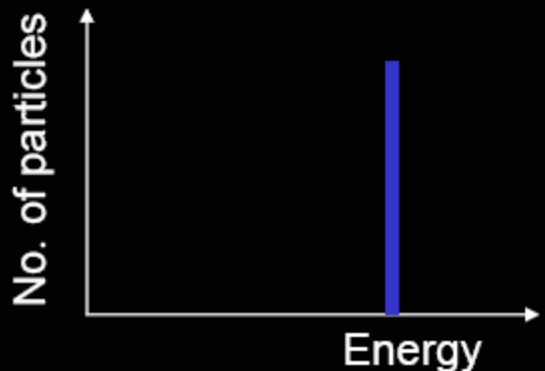


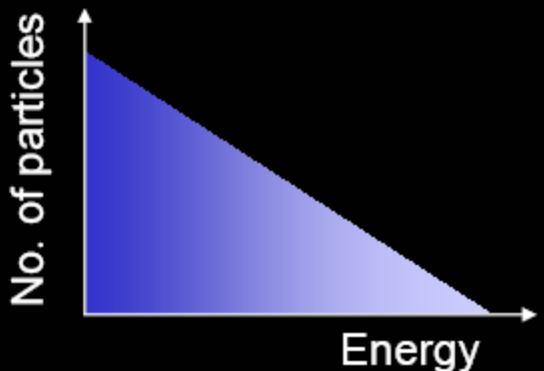
Nature's accelerators



Enrico
Fermi







- In ambiente astrofisico (in presenza di particelle ionizzate, plasmi) *campi elettrostatici non possono essere mantenuti* a causa dell'alta conducibilità dei plasmi stessi
- Sono possibili meccanismi in cui f.e.m. sono prodotte tramite $\nabla \times E = \delta \mathbf{B} / \delta t$
- Il meccanismo "idrodinamico" descrive accelerazione stocastica di RC da parte di ripetuti urti delle particelle con un'onda di shock, ad esempio emessa dall'esplosione di una SN.
- Questo meccanismo venne utilizzato per i RC per la prima volta da parte di E. Fermi (1949), e prende per questo il suo nome.
- Le particelle cariche sono riflesse da "specchi" magnetici dovute alla presenza dell'irregolare campo magnetico galattico.
- Ad ogni riflessione, le particelle guadagnano (in media) energia
- Il meccanismo predice il corretto andamento del flusso *vs.* E

Electromotive Acceleration

Fundamentally, Lorentz Force law $\Rightarrow$
$\frac{d}{dt}(\gamma m v) = e(\vec{E} + \vec{v} \wedge \vec{B})$

But, on macroscopic scales, no electrostatic fields, i.e., $\langle \vec{E} \rangle = 0$.

Also, for a stationary $\vec{B}$, $\frac{d}{dt}|\vec{v}| = 0$.

Need "moving" magnetic fields, i.e., acceleration by electromotive force.

Expect, rate of energy gain by relativistic particles = work done by the electromotive force $= \frac{d\mathcal{E}}{dt} = \xi ZecB.c$, with $\xi < 1$.

Fermi's original suggestion: Encounters of charged particles with moving, magnetized, interstellar clouds.

Electromotive Acceleration

The maximum energy supplied to a particle with charge Ze is

$$E_{\max} = \int Ze\,\varepsilon\,dx$$

If the energy is given by induction

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \Rightarrow \quad \frac{\varepsilon}{L} \cong \frac{B\omega_0}{c}$$

ε=electrical field induced in a region of length L

We obtain

$$E_{Max} = \int Ze\cdot\varepsilon\cdot dx = Ze\cdot B\omega_o L\cdot L/c$$

$$E_{\max} \cong Ze\frac{BL\omega_0}{c}L = Ze\beta_0 BL$$

$$E_{\max}(EeV) \cong Z\beta_0 B(\mu G)L(kpc)$$

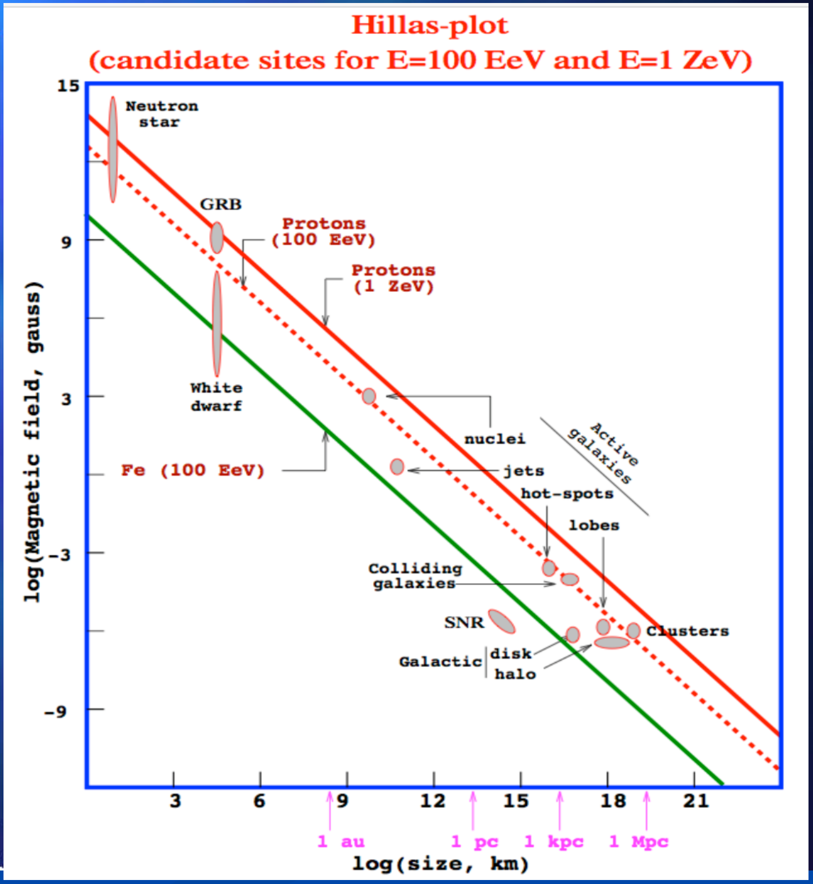
Hillas plot

Irrespective of the precise acceleration mechanism, there is a simple dimensional argument, given by Hillas, which allows one to restrict attention to only a few classe of astrophysical objects as possible sources capable of accelerating particles to a given energy.

In any statistical acceleration mechanism, there must be a magnetic field, B, to keep the particles confined within the acceleration site. The size R of the acceleration region must be larger than the diameter of the orbit of the particle. One gets the general condition

$$E(EeV) < Z\,\beta\,B(\mu G)R(kpc)$$

The above condition also applies to direct acceleration scenarios in which the electric field arises due to a moving magnetic field.



Magnetic Confinement

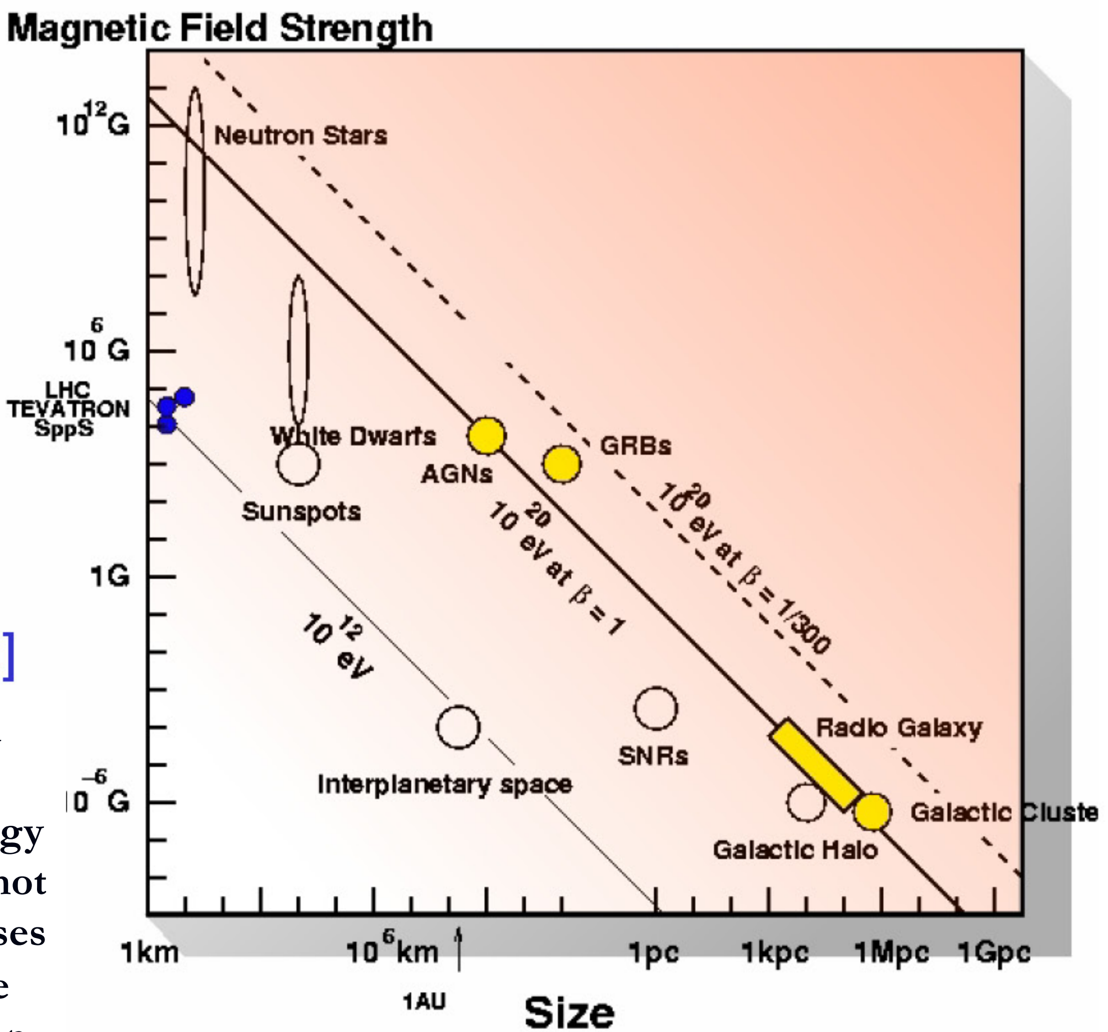
"Hillas Plot"

Minimum size of B field to contain particles being accelerated.

Achievable energy:

E [EeV] ~ Z R [kpc] B [μG]

The Hillas plot must be interpreted carefully. It represents a necessary condition to reach the maximum energy in a cosmic accelerator but it may be not sufficient since there are other processes that prevent to reach it, e.g. radiative losses, escape form acceleration region, age of the source…



Tra i siti possibili di accelerazione dei raggi cosmici possiamo includere (ad energia crescente):

- i venti stellari
- le esplosioni di Supernovae
- le "remnants" di tali esplosioni: stelle di neutroni ruotanti, pulsar con nebulose, …
- altri oggetti esotici, quali i "mini-black holes", se esistono.
- I raggi cosmici osservati con energie $E>10^{19}$ eV, potrebbero essere stati accelerati da meccanismi extragalattici, quali jets di nuclei Galattici attivi o GRB

Il meccanismo di Fermi può essere attivo in molte di queste situazioni astrofisiche, e lo analizzeremo in qualche dettaglio per le esplosioni di Supernovae

Il fattore comune a queste classi di sorgenti e' la presenza di onde di shock e/o zone di campo magnetico turbolento in moto ($\langle \Delta B \rangle = 0$, ma $\langle \Delta B^2 \rangle \neq 0$)

Il meccanismo di Fermi

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On the Origin of the Cosmic Radiation

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(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller[1] has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.[2] The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

I propose in the present note to discuss a hypothesis on the origin of cosmic rays which attempts to meet in part this objection, and according to which cosmic rays originate and are accelerated primarily in the interstellar space, although they are assumed to be prevented by magnetic fields from leaving the boundaries of the galaxy. The main process of acceleration is due to the interaction of cosmic particles with wandering magnetic fields which, according to Alfvén, occupy the interstellar spaces.

Such fields have a remarkably great stability because of their large dimensions (of the order of magnitude of light years), and of the relatively high electrical conductivity of the interstellar space. Indeed, the conductivity is so high that one might describe the magnetic lines of force as attached to the matter and partaking in its streaming motions. On the other hand, the magnetic field itself reacts on the hydrodynamics[3] of the interstellar matter giving it properties which, according to Alfvén, can pictorially be described by saying that to each line of force one should attach a material density due to the mass of the matter to which the line of force is linked. Developing this point of view, Alfvén is able to calculate a simple formula for the velocity V of propagation of magneto-elastic waves:

$$V = H/(4\pi\rho)^{\frac{1}{2}}, \qquad (1)$$

where H is the intensity of the magnetic field and ρ is the density of the interstellar matter.

One finds according to the present theory that a particle that is projected into the interstellar medium with energy above a certain injection threshold gains energy by collisions against the moving irregularities of the interstellar magnetic field. The rate of gain is very slow but appears capable of building up the energy to the maximum values observed. Indeed one finds quite naturally an inverse power law for the energy spectrum of the protons. The experimentally observed exponent of this law appears to be well within the range of the possibilities.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

II. THE MOTIONS OF THE INTERSTELLAR MEDIUM

It is currently assumed that the interstellar space of the galaxy is occupied by matter at extremely low density, corresponding to about one atom of hydrogen per cc, or to a density of about 10^{-24} g/cc. The evidence indicates, however, that this matter is not uniformly spread, but that there are condensations where the density may be as much as ten or a hundred times as large and which extend to average dimensions of the order of 10 parsec. (1 parsec. $= 3.1\times10^{18}$ cm $= 3.3$ light years.) From the measurements of Adams[4] on the Doppler effect of the interstellar absorption lines one knows the radial velocity with respect to the sun of a sample of such clouds located at not too great distance from us. The root mean square of the radial velocity, corrected for the proper motion of the sun with respect to the neighboring stars, is about 15 km/sec. We may assume that the root-mean-square velocity

[1] Nuclear Physics Conference, Birmingham, 1948.
[2] Alfvén, Richtmyer, and Teller, Phys. Rev., to be published.
[3] H. Alfvén, Arkiv Mat. f. Astr., o. Fys. 29B, 2 (1943).
[4] W. S. Adams, A.p.J. 97, 105 (1943).

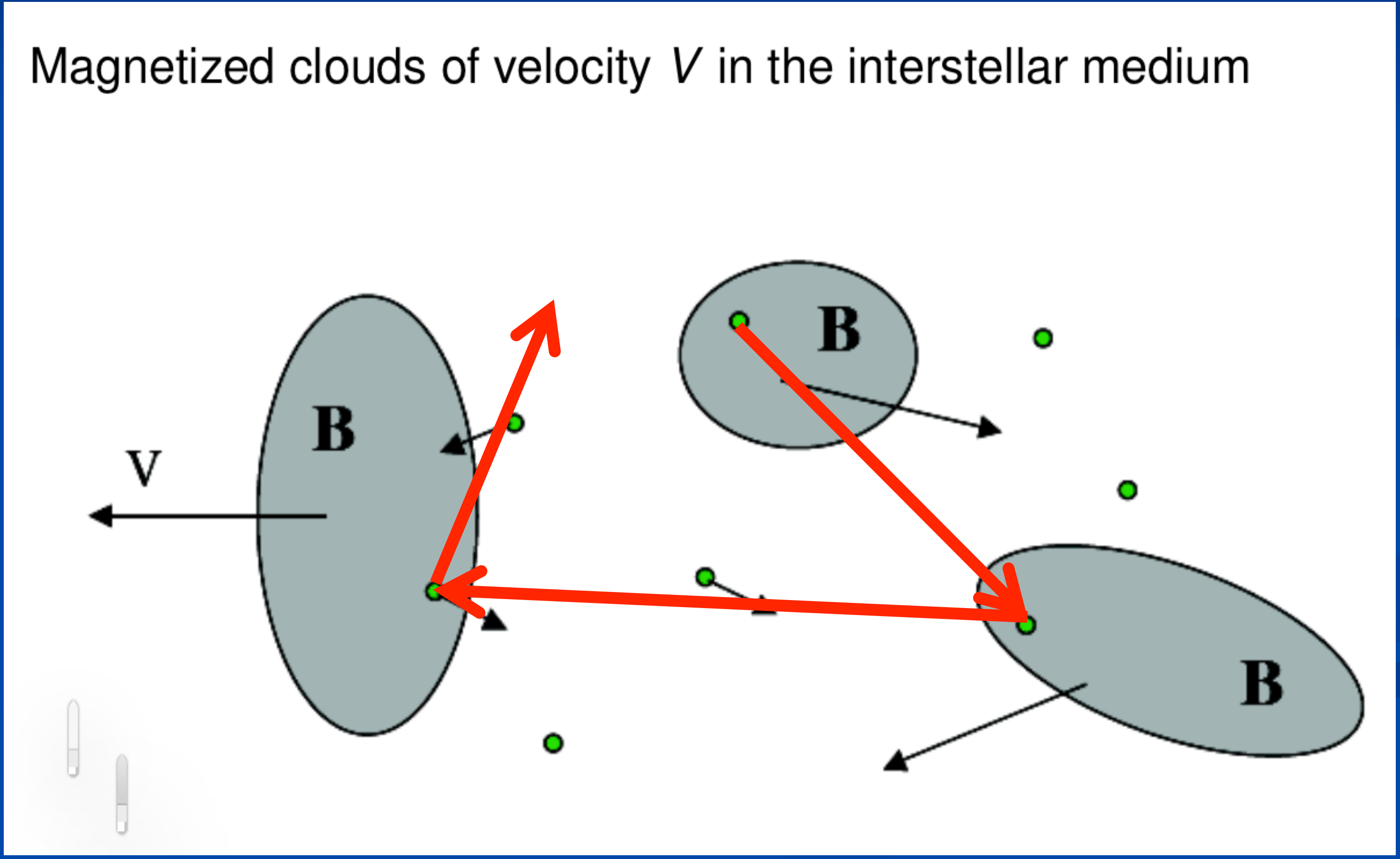
Una "collisione" con una nube magnetica puo` causare un aumento dell' energia della particella. Un gran numero di collisioni possono far crescere l' energia fino a valori molto elevati. Guadagno di energia per collisione:

$$\Delta E/E = \varepsilon$$

The most likely hypothesis is that this acceleration is due to electromagnetic fields present in astrophysical sources or in the interstellar medium. In the interstellar medium however the mean electric field $\langle E \rangle = 0$ since the interstellar ionized gas is almost perfectly conductor and globally neutral. Transient electric fields can be found for instance in solar flares (due to very complex magnetic reconnection phenomena) or as a result of locally varying magnetic fields. Finally, strong and lost lasting electric field are mostly found in the vicinity of neutron stars.

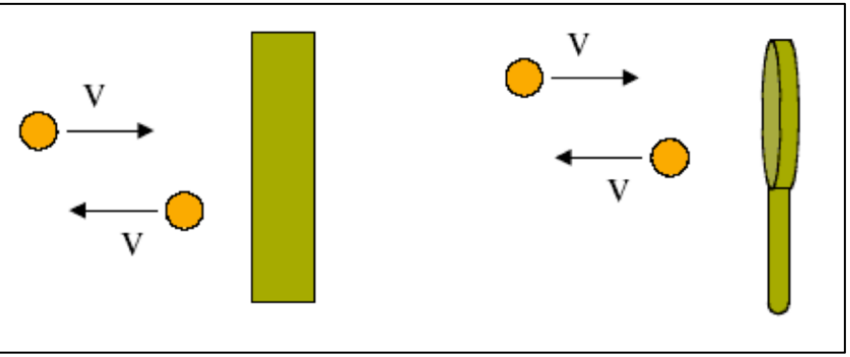
On the other hand, magnetic fields are found in all high-energy astrophysics sources as well as in the interstellar medium and are generally invoked in the most popular theoretical scenarios for cosmic-ray acceleration. This statement might look puzzling at first sight. Since the Lorentz force $\vec{F} = q\,\vec{v} \times \vec{B}$ does no work and then can in principle not be invoked to accelerate particles. However, as recalled above a time varying magnetic field induces an electric field as formalized in Maxwell's equation $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$. Moreover, a pure magnetic field $\vec{B}'$ in a given reference frame is seen as a magnetic field $\vec{B}$ and an electric field $\vec{E}$ in another reference frame moving relative to it, as implied in electrodynamics by the Lorentz transformation of the electromagnetic tensor $F^{\mu\nu}$.

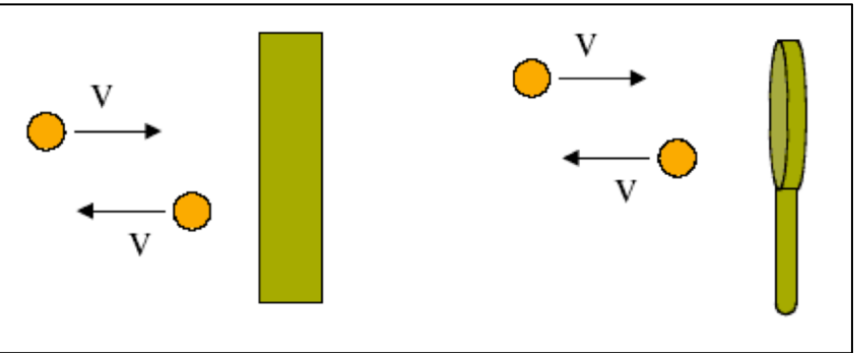
The original idea proposed by Fermi in 1949 for cosmic-ray acceleration was based on the fact that the interstellar medium is filled with "clouds" of ionized gas in movement with respect to the "Galactic frame". These clouds are carrying a magnetic field[1] and can in principle reflect the incoming charged particles (see Fig. 3.1). The acceleration mechanism based on moving "magnetic clouds" can be understood with a trivial but meaningful analogy with an idealized tennis game.

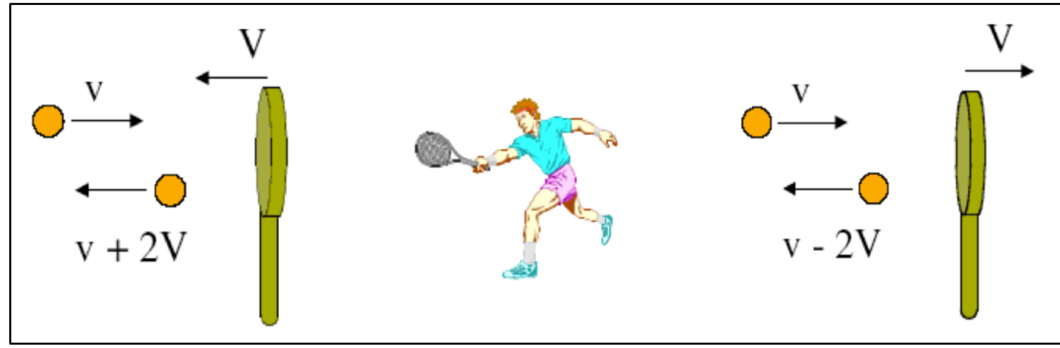


Let us assume a tennis ball is thrown, with a velocity v, on a steady racket. Ignoring the possible heat dissipation during the shock, the ball is simply reflected with the same velocity and no net energy gain, as it would be on a wall (see Fig. 3.2). Let us now assume that the racket is moving with a velocity V toward the ball (which still has a velocity v with respect to the tennis court). **In the racket frame**, the ball has a velocity $v+V$ and, assuming a perfectly elastic shock, the ball is reflected with the same (but opposite) velocity. **Back to the court frame** (adding V to the ball velocity in the racket frame), after the shock the ball has been accelerated to a velocity $v+2V$ due to the head-on collision between the ball and the racket (see Fig. 3.3a). This result has been obtained with a double change of reference frame : court frame $\rightarrow$ racket frame $\rightarrow$ court frame.

Let us now consider a *dropshot*, meaning that the racket is now going away from the ball with a velocity V with respect to the court frame (see Fig. 3.3b). With the exact same calculation we conclude that the ball has been decelerated to a velocity $v-2V$, in the court frame after the shock.



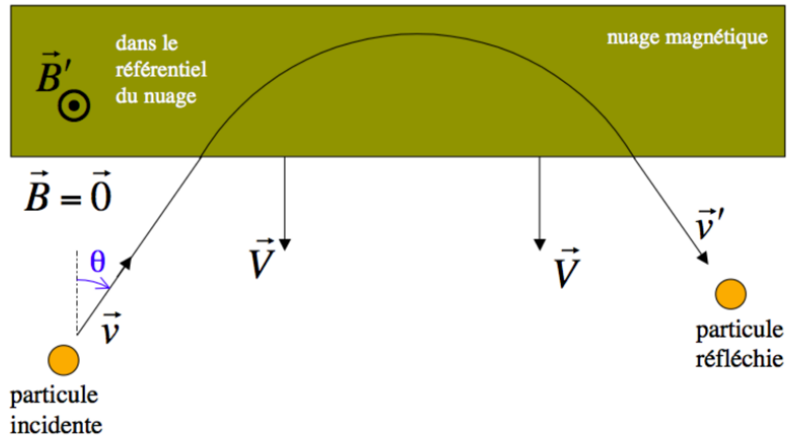




These simplistic analogies are enough to catch the essential origin idea behind Fermi's acceleration mechanism : one simply has to replace the tennis ball by a charged particle (a cosmic-ray) and the racket by a "magnetic cloud". Particles will be accelerated by each encounter with a magnetic cloud coming toward them and decelerated by the encounters with magnetic clouds going away from them. The energy gain (or loss) for each encounter can be calculated by a double change of reference frame, Galactic frame → cloud frame → Galactic frame.

Note that in this scheme the magnetic field is only the agent of the charged particles reflexion by the moving cloud. In the absence of magnetic field in the cloud the particles would just go through the moving cloud (as if the ball was going through the tennis racket) without any energy change (ignoring interactions with the cloud particles). We then expect the energy gain we will calculate in the following for some idealized cases to be independent of the magnetic field.

As mentioned above, another possible way of working out the acceleration the to calculate the electric field seen in the Galactic frame by Lorentz transformation of the pure B field seen in the cloud frame. Since the two approaches must be equivalent the acceleration and the energy gain of the particle must also be independent of the cloud magnetic field in this case. This result is however far less intuitive with this approach.



Let us consider the case of an idealized reflexion (for which the reflexion angle is equal to the incidence angle, see Figs. 3.1 and 3.4) of a particle (with a velocity $\vec{v}$) by a cloud coming toward it with a velocity $\vec{V}$, the incidence angle θ is then given by $\vec{V}.\vec{v} = -\cos\theta$. Moreover for typical galactic magnetic cloud V <<c and V<<v

We use as a convention, primed quantities for the cloud frame and unprimed for the Galactic frame. Passing from the Galactic frame to the cloud frame we have :

$$\begin{cases} E'_{in} = \gamma_{cloud}(E_{in} - \vec{P_{in}}.\vec{V}) = \gamma_{cloud}(E_{in} - P^{\parallel}_{in}.V) \\ P'^{\parallel}_{in} = \gamma_{cloud}(P^{\parallel}_{in} - \frac{V}{c^2}E_{in}) \end{cases} \quad (3.1)$$

where γ_{cloud} is the Lorentz factor of the cloud in the Galactic frame, the subscript *in* refers to the properties of the incoming particle and the parallel symbol "$\|$" refers to the projection along the cloud velocity vector.

Inside the cloud, we assume (and it is in principle a good approximation) the particle is just reflected and does not loose or gain energy so we have $E'_{out} = E'_{in}$, where the subscript *out* refers to the properties of the outgoing particle. Moreover, we assumed the encounter with the cloud leads to a perfect reflexion so we have $P'^{\|}_{out} = -P'^{\|}_{in}$.

We now come back to the Galactic frame with the inverse Lorentz transformation :

$$\left\{ \begin{array}{l} E_{out} = \gamma_{cloud}(E'_{out} + P'^{\|}_{out}.V) \\ P^{\|}_{out} = \gamma_{cloud}(P'^{\|}_{out} + \frac{V}{c^2}E'_{out}) \end{array} \right. \tag{3.2}$$

by substitution we get :

$$E_{out} = \gamma^2_{cloud}\left[E_{in}(1 + \frac{V^2}{c^2}) - 2P^{\|}_{in}.V\right] \tag{3.3}$$

and since $P^{\|}_{in} = -\frac{E_{in}.v\cos\theta}{c^2}$ (the minus sign comes from the above definition of θ) to first order in V/c we get :

$$E_{out} = E_{in}\left(1 + \frac{2v.V\cos\theta}{c^2}\right) \Leftrightarrow \frac{\Delta E}{E} = \frac{E_{out} - E_{in}}{E_{in}} = -2\frac{\vec{v}.\vec{V}}{c^2} \tag{3.4}$$

This final result leads to the following conclusions :

- The energy gain is proportional to the initial energy ($\Delta E/E$ is independent of E).
- The energy gain is positive for a head-on collision ($\vec{v}.\vec{V} < 0$).
- The energy gain is independent of B', as anticipated, the magnetic field mediates the reflection but it does not appear in the Lorentz transformations.

Alternative approach using the induced E field

We place ourselves in the same configuration as before as schematized in Fig. 3.5. The electromagnetic tensor in the cloud frame can be transformed to the Galactic frame to obtain the electric field $\vec{E}$:

$$\vec{E_\perp} = \gamma_{cloud}(\vec{E'_\perp} - \vec{V} \times \vec{B'_\perp}) \tag{3.5}$$

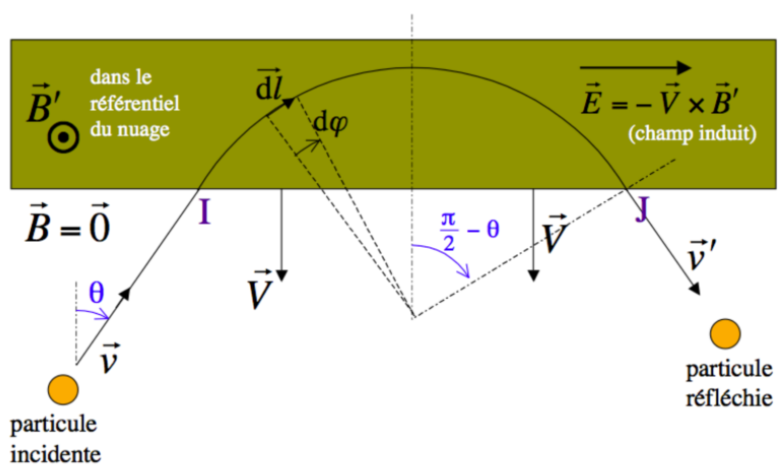
since $E' = 0$ by hypothesis and to first order in V/c we get :

$$\vec{E_\perp} = -\vec{V} \times \vec{B'_\perp} \tag{3.6}$$

with this calculation of the electric field in the Galactic frame we can calculate the energy gain during the encounter :

$$\Delta E = \int_{\text{inside the cloud}} \vec{F}.\vec{dl} = q\vec{E_\perp}.\vec{dl} = qE_\perp l_{IJ} \tag{3.7}$$

the segment IJ being the projection of the circular trajectory along the electric field vector (see Fig. 3.5) and $q = Ze$ the charge of the particle. We have :



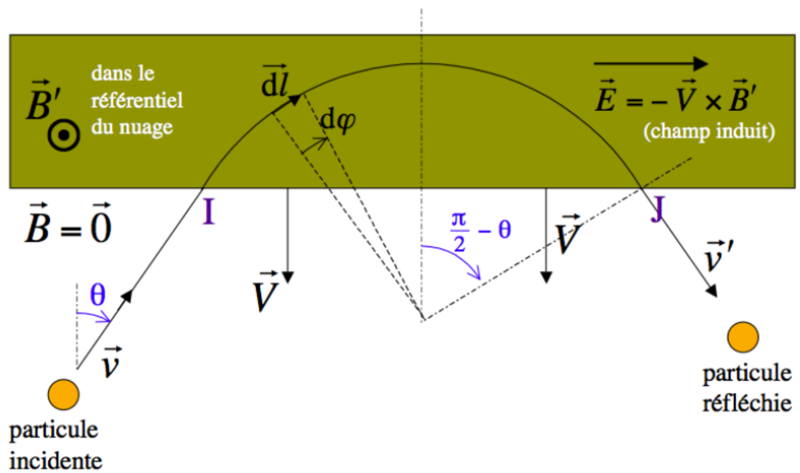


FIG. 3.5: Action of the induced electric field during the reflexion of a charged particle by a moving magnetic cloud.

$$l_{IJ} = 2r_L \sin(\frac{\pi}{2} - \theta) = 2r_L \cos\theta \tag{3.8}$$

where $r_L = \frac{P}{ZeB} = \frac{E.v}{ZeBc^2}$ is the Larmor radius or the particle with energie E, momentum P, velocity v and charge Ze in the magnetic field B.

We finally get :

$$\Delta E = ZeVB \times 2\frac{Ev}{ZeBc^2}\cos\theta = \frac{2EVv\cos\theta}{c^2} = -2E\frac{\vec{v}.\vec{V}}{c^2} \Leftrightarrow \frac{\Delta E}{E} = -2\frac{\vec{v}.\vec{V}}{c^2} \tag{3.9}$$

This results is equivalent to that obtained in Eq. 3.4, with the method of the double change of reference frame. We also find that the magnetic field B is eliminated from the equation which is a lot less obvious in principle than when the calculation is performed by double change of reference frame. In the following will only consider the latter method which leads to much simpler calculations although it takes no account of the induced electric field which is physically the true responsible for the particles energy gains.

So far we have studied single encounters with a cloud in the case of an idealized perfect reflexion. We saw that, indeed, a head-on collision was leading to an energy gain but also that energy can be lost if the cloud is moving away from the particle. The essence of Fermi's acceleration mechanism by interactions with moving Galactic magnetic clouds is that charged particles will suffer a series of encounter while propagating in the interstellar medium. Head-on collisions will lead to energy gains while energy will be lost when the cloud is moving away. All the point (which might not seem so intuitive at first sight) is that head-on collisions are **in average** more frequent so that in average particles do gain energy with this mechanism.

Why are head-on collisions more frequent? To understand that we can make a simple and familiar analogy with a car cruising on the highway. The car obviously crosses more cars coming toward than it overtakes cars going in the same direction. This is all a question of relative velocity and the same will be true with magnetic clouds. Moreover, on the highway the lower the velocity of the other cars the lower will be the difference between the number of car crossed and the number of cars overtaken. The same will be true for the magnetic clouds.

We now relax our simplifying hypothesis of a perfect reflexion and place ourselves in the case of Fig. 3.6a. We now assume that the magnetic field is turbulent so that charged particles are isotropized inside the cloud and that the angle of the particle escaping from the cloud θ'_{out} is random (the subscript "1" refers to our subscript "in" in Fig. 3.6a). To simplify our derivations we now assume that the particles are already ultra-relativistic, *i.e* $E \simeq Pc$. The double change of reference frame gives :

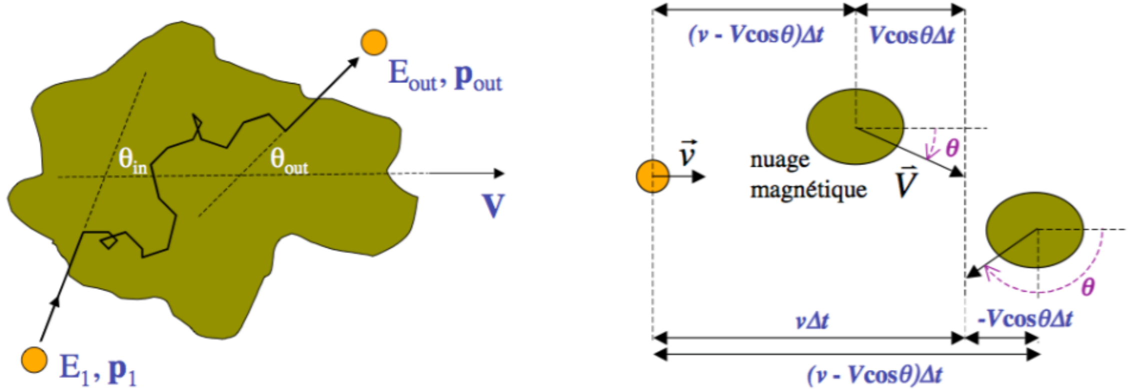


FIG. 3.6: Left : Schematic view of the "interaction" of a charged particle with a magnetic cloud. The particle enters the cloud and is isotropized by the magnetic turbulence. Right : Schematic view explaining the distribution of the incidence angle of particles with magnetic clouds : if the clouds velocity distribution is isotropic in the ISM and the density of cloud making an angle θ with the particle velocity vector is uniform the average number of clouds encountered during a time interval Δt is proportional to $(v - V\cos\theta)$.

The double change of frame gives

$$\begin{cases} E'_{in} = \gamma_{cloud} E_{in} (1 - \beta_{cloud} \cos\theta_{in}) \\ E_{out} = \gamma_{cloud} E'_{out} (1 + \beta_{cloud} \cos\theta'_{out}) \end{cases} \tag{3.10}$$

using $E'_{in} = E'_{out}$ and dropping the subscript "cloud" we get :

$$E_{out} = \gamma^2 E_{in} (1 - \beta \cos\theta_{in})(1 + \beta \cos\theta'_{out}) \tag{3.11}$$

$$\Leftrightarrow \frac{\Delta E}{E} = \frac{E_{out} - E_{in}}{E_{in}} = \frac{\beta^2 - \beta\cos\theta_{in} + \beta\cos\theta'_{out} - \beta^2\cos\theta_{in}\cos\theta'_{out}}{1 - \beta^2} \tag{3.12}$$

to get the average energy gain, we need to average the above expression. By hypothesis, the particles are isotropized in the cloud, hence $\langle \cos\theta'_{out} \rangle = 0$. We now need to calculate $\langle \cos\theta_{in} \rangle$, the probability to have an encounter with an incidence angle θ_{in} should be proportional to the relative velocity between the particles and the cloud (think of the car on the highway) in the case of uniformly distributed clouds. It gives (see Fig. 3.6b) : $P(\theta_{in}) \propto v - V\cos\theta_{in}$ * ($v \simeq c$ and V still being respectively the velocity of the particle and the cloud). We then have :

$$\langle \cos\theta_{in} \rangle = \frac{\int_{-1}^{1} \cos\theta_{in} (v - V\cos\theta_{in}) d\cos\theta_{in}}{\int_{-1}^{1} (v - V\cos\theta_{in}) d\cos\theta_{in}} = \frac{-2V/3}{2v} \simeq \frac{-2V/3}{2c} \simeq -\frac{\beta}{3} \tag{3.13}$$

substituting in Eq. 3.12 we finally get :

$$\langle \Delta E/E \rangle = \int_{\theta_1}^{\theta_2} P(\Omega) \frac{\Delta E}{E} (\cos\theta) \; d\Omega$$

$$\langle \frac{\Delta E}{E} \rangle = \frac{\beta^2 + \beta^2/3}{1 - \beta^2} \simeq \frac{4\beta^2}{3} \tag{3.14}$$

* Relative velocity

$$v_{\text{rel}} = |\vec{v}_{\text{cloud}} - \vec{v}_{\text{particle}}|$$

$$= \sqrt{(c - v\ \cos\theta_i)^2 + v^2\ \sin^2\theta_i}$$

$$= c\sqrt{(1 - \beta\cos\theta_i)^2 + \beta^2\ \sin^2\theta_i}$$

$$= c\sqrt{1 + \beta^2 - 2\beta\ \cos\theta_i}$$

$$\simeq c\sqrt{(1 - 2\beta\ \cos\theta_i}$$

Taylor expansion to 1st order

$$\simeq c\,(1 - \beta\ \cos\theta_i)$$

$\beta \ll 1$

2nd order acceleration

We get an average energy gain which is indeed positive hence Fermi's mechanism is truly an acceleration mechanism for charged particles.

The average fractional energy gain is proportional to $ß^2$ of the cloud so this mechanism is often called second order Fermi mechanism.

This mechanism is stochastic by nature, the average energy gain is positive but for a given particle, the energy gain can vary depending on the configuration of the series of encounters.

Acceleration time

The fact that the mechanism proposed by Fermi leads to a net energy gain in average does not mean that this mechanism is responsible for the production of the Galactic cosmic-rays we observe. In particular, to be a good candidate this mechanism must be sufficiently fast at accelerating particle to the very high energies measured by cosmic-ray observatories. To get a quantitative answer to this question, we need an approximate estimate of the particles acceleration times.

We start by giving the definition of the acceleration time t_{acc} :

$$t_{acc}(E) = \left(\frac{1}{E}\frac{dE}{dt}\right)^{-1} \quad (3.15)$$

Let L be the typical distance between two clouds and let us assume there is no magnetic field between two cloud (our estimate then becomes a lower limit), then the average time between two encounters will be $\langle t_{coll} \rangle = \frac{L}{c}$, neglecting the time spent by the particles inside the cloud we then get :

$$\frac{dE}{dt} \simeq \frac{\Delta E}{\langle t_{coll} \rangle} = \frac{4}{3}\frac{\beta^2 cE}{L} = \frac{E}{t_{acc}} \Leftrightarrow t_{acc} = \frac{3}{4}\frac{L}{c}\beta^{-2} \quad (3.16)$$

Power law spectrum

Let us consider the acceleration time t_{acc}, defined in Eq. 3.15, and the escape time t_{esc}. The latter would be the average time for the particles to leave the region of the Galaxy where the magnetic clouds are present. Let us assume, to simplify the calculation that t_{acc} and t_{esc} are both independent of the energy. It would make sense for our simple model since we have assumed that the medium was not magnetized and neglected the duration of the interaction between particles and clouds. Assuming particles are injected at $t = 0$ at an energy E_0 then :

$$E(t) = E_0 \exp\left(\frac{t}{t_{acc}}\right) \tag{3.17}$$

it means that particles with an energy E have been in the acceleration site for a time $t(E) = t_{acc}\ln(E/E_0)$ and that particles with energy between E and $E + dE$ have been injected at a time between $t(E)$ and $t(E + dE) = t(E) + dt$ earlier, with $dt = \frac{dt}{dE}dE = \frac{t_{acc}}{E}dE$ (using the definition of t_{acc}).

Moreover, during a time dt, the escape probability is $P_{esc} = \frac{dt}{t_{esc}}$. Let us call $\dot{N}_0$, the injection rate, the amount of particles injected during the time interval dt is $\dot{N}_0 dt$. Among these particles, only a fraction $f = \exp\left(-\frac{t(E)}{t_{esc}}\right)$ are still in the system after a time $t(E)$ and are able to reach an energy E. So the number of particles between E and $E + dE$ is :

$$n(E)dE = \dot{N}_0 dt \exp\left(-\frac{t(E)}{t_{esc}}\right) = \dot{N}_0 \frac{t_{acc}}{E} \exp\left(-\frac{t_{acc}\ln(E/E_0)}{t_{esc}}\right) dE \tag{3.18}$$

and we then get (taking the log of both sides)

$$n(E) = \frac{\dot{N}_0}{E_0} t_{acc} \left(\frac{E}{E_0}\right)^{-x} \tag{3.19}$$

with,

$$x = 1 + \frac{t_{acc}}{t_{esc}} \tag{3.20}$$

Power law spectrum

$$n(E)dE = \dot{N}_0 dt \exp\left(-\frac{t(E)}{t_{esc}}\right) dE = \dot{N}_0 \frac{t_{acc}}{E} \exp\left(-\frac{t_{acc} \ln(E/E_0)}{t_{esc}}\right) dE \quad (3.18)$$

and we then get

$$n(E) = \frac{\dot{N}_0}{E_0} t_{acc} \left(\frac{E}{E_0}\right)^{-x} \quad (3.19)$$

with,

$$x = 1 + \frac{t_{acc}}{t_{esc}} \quad (3.20)$$

We see that we indeed obtained a power law shape for the spectrum of accelerated particles, which is in principle a good point in favor of this mechanism. However, the exact shape of the spectrum depends on the t_{acc}/t_{esc} which cannot be predicted in principle since it will depend on the exact configuration of the region where one can find a large concentration of magnetic clouds. It is moreover likely that different regions in the Galaxy would accelerate cosmic-ray with different power law shape and the sum of the contribution would not be likely to give a global power law as observed for Galactic cosmic-rays detected on Earth.

Orders of magnitude

We can now apply our results to the case of the Galactic interstellar medium where Fermi's magnetic cloud are found. The typical velocity of a cloud is $\beta_{cloud} \simeq 10^{-4}$ which means $\beta^2_{cloud} \simeq 10^{-8}$ and the typical distance $L \simeq 1pc$. Using these number one rapidly understands that with this acceleration mechanism it would take almost a billion year for a particle to double its energy. This is way too long to reach the very high energies observed for Galactic cosmic-rays. Moreover, we have neglected energy losses which might take place in the interstellar medium (such as ionization losses or spallation) and it turns out that for GeV nuclei, for instance, the energy loss time in the interstellar medium would be shorter than the acceleration time we just calculated.

We must conclude that the original acceleration mechanism presented by Fermi is far too slow to account for cosmic-ray acceleration (at least as the dominant process). We must not however forget the vertues of this pioneering idea, this mechanism indeed leads, in principle, in average to a net energy gain for the accelerated particle. Moreover although this scenario predicts a power law shape for the spectrum of accelerated particles, a robust prediction of the shape of the spectrum is very difficult to obtain. Despite its failure, Fermi mechanism is in fact the seed of most of the modern acceleration mechanisms which have been proposed since Fermi's pioneering work. Diffusive shock we will study in the next section, directly inherit of the basic ideas we discussed in this section, we will see that this mechanism manages to overcome quite elegantly most of the problems of the original Fermi mechanism.

Diffusive Shock Acceleration

Astrophysical shocks provide in principle such a mechanism, the so-called Diffusive Shock Acceleration (DSA)

Astrophysical shocks exist everywhere in the universe, from the solar system to more extreme objects such as supernova remnants, active galactic nuclei or gamma-ray bursts. We furthermore know particles are accelerated at these shocks as we detect radiation (from radio frequencies to γ-rays) from these object most often interpreted as the result of energy losses of accelerated electrons (see next chapter).

Astrophysical shock waves obey the same macroscopic rules as shock waves on Earth. They originate from outflows propagating with velocities larger than the local speed of sound. A shock front, through which physical quantities are discontinuous[2], forms. However, unlike "terrestrial shocks", the main difference is that astrophysical shock waves are in most cases *collisionless*, *i.e* the shock and the energy dissipation processes do not take place through particles collisions or coulombian interactions. The shock formation is due to the "interaction" of particles with the ambient magnetic field. Without the fields, supersonic outflows would just pass "unnoticed" through the ambient medium without the formation of any shock. Beyond these very simple considerations, the microphysics of astrophysical shocks is extremely complicated and far beyond the scope of this course.

Diffusive Shock Acceleration

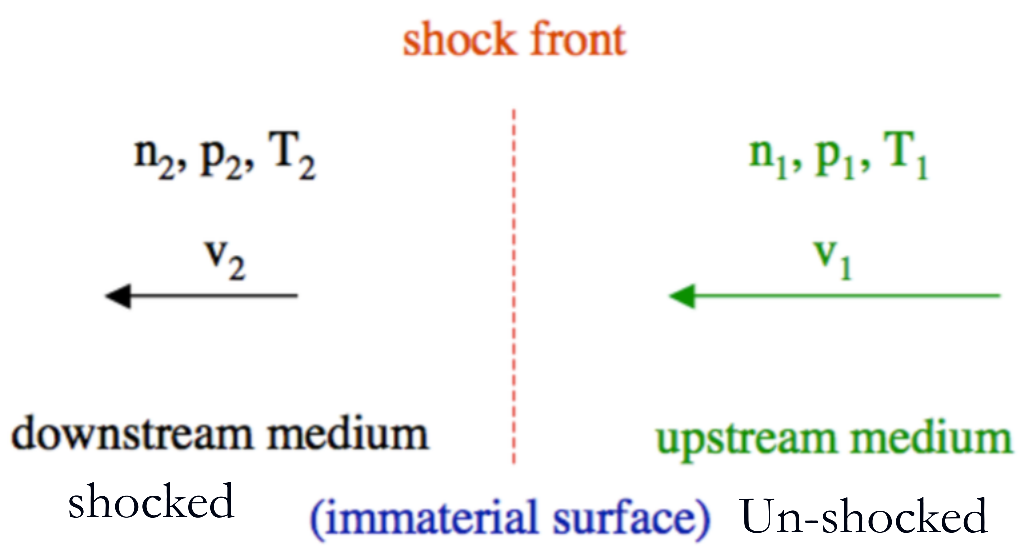


FIG. 3.7: Schematic view of a shock wave propagating in a medium, as seen in the shock rest frame. The shock is at rest and the upstream medium is coming toward it with a velocity v_1 while the downstream medium is going away with a velocity v_2. Physical quantities are discontinuous through the (immaterial) surface of the shock.