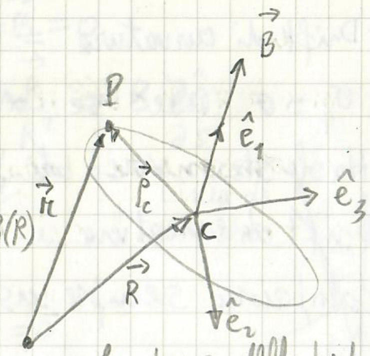
Lecture 5 231019

- Il pdf delle lezioni puo’ essere scaricato da
- http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1920/

Motion in magnetic fields

ii) $\vec{\nabla}_\parallel B$

$\vec{r} = \vec{\rho}_c + \vec{R}$ *



Sviluppo il campo $\vec{B}$ in serie $\vec{B}(\vec{r}) = \vec{B}(\vec{R}) + \vec{\rho}_c \cdot \vec{\nabla} B(\vec{R})$

Sost. nell'equ.

$m\ddot{\vec{r}} = q\vec{v} \times \vec{B}$

* la posiz. della part. è espresso in termini della posizione di C e del raggio ist. di curvatura rispetto a C

$m(\ddot{\vec{R}} + \ddot{\vec{\rho}}_c) =$

$= q(\dot{\vec{R}} + \dot{\vec{\rho}}) \times [\vec{B}(\vec{R}) + \vec{\rho}_c \cdot \vec{\nabla}\vec{B}(\vec{R})]$

con la condizione che $|\rho \cdot \nabla B| \ll 1$

Prendo il val. medio in un periodo di Larmor

Si fa vedere che $\langle \rho \rangle = \langle \dot{\rho} \rangle = \langle \ddot{\rho} \rangle = 0$ (*) (*) Vedi avanti

L'unico termine che contribuisce $\neq 0$ è

$\dot{\vec{\rho}} \times (\vec{\rho} \cdot \vec{\nabla}) B = \frac{1}{2}\Omega \rho^2 [\hat{e}_2 \times (\hat{e}_3 \cdot \vec{\nabla}) - \vec{e}_3 \times (\hat{e}_2 \cdot \vec{\nabla})]\hat{e}_1 B$

$= \frac{1}{2}\omega\rho^2 [(\hat{e}_3 \cdot \vec{\nabla}B)(\vec{e}_2 \times \vec{e}_1) - (\vec{e}_2 \cdot \vec{\nabla}B)(\hat{e}_3 \times \hat{e}_1)]$

$= \frac{1}{2}\omega\rho^2 [-(\hat{e}_3 \cdot \vec{\nabla}B)\hat{e}_3 - (\hat{e}_2 \cdot \vec{\nabla}B)\hat{e}_2]$

$= -\frac{1}{2}\omega\rho^2 q \vec{\nabla} B$

Quindi

$m\ddot{\vec{R}} = q[\dot{\vec{R}} \times \vec{B}(\vec{R})] - \frac{1}{2}\omega^2\rho_c^2 \vec{\nabla} B + \cdots$

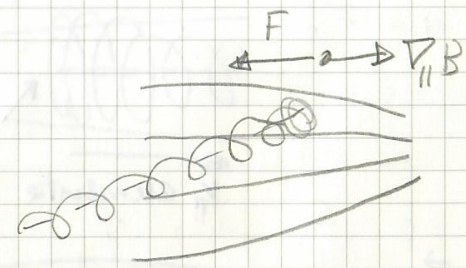
La componente parallela di $\ddot{\vec{R}}$

$m\ddot{\vec{R}} \cdot \hat{e}_1 = \underbrace{q[\dot{\vec{R}} \times \vec{B}(\vec{R})]\cdot\hat{e}_1}_{=0 \text{ perché } \perp} - \frac{1}{2}\omega^2\rho_c \underbrace{(\vec{\nabla}B)\cdot\hat{e}_1}_{\vec{\nabla}_\parallel B}$

$\Rightarrow \boxed{m\frac{d\vec{v}_\parallel}{dt} = -\frac{1}{2}\omega^2\rho_c \vec{\nabla}_\parallel B = -\frac{1}{2}\frac{v_\perp^2}{B}(\vec{\nabla}_\parallel B)}$

Il centro di guida è accelerato nel verso opposto al gradiente del campo.

Se si muove verso regioni con campo più forte, verrà respinta*, indipendentemente dal segno della carica o dalla direz. di B.



*più precisamente decelerata

Motion in magnetic fields

Come conseguenze si ha inversione del moto e variazione di angolo di pitch α

Infatti $v^2 = v_\parallel^2 + v_\perp^2 =$ cost. perché l'unica forza che agisce è quella di Lorentz

Quindi ~~sin~~ $\cos\alpha = \frac{v_\parallel}{v} \to 0$

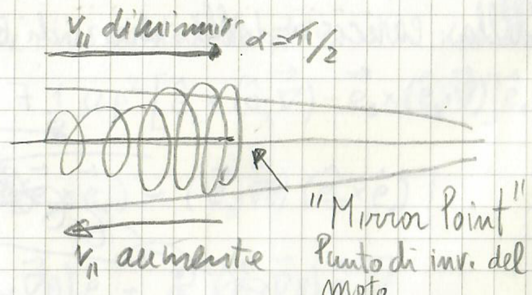
quando $v_\parallel \to 0$

Man mano che la particella avanza nel grad. $v_\parallel$ diminuisce e α aumenta

Quando $v_\parallel = 0 \to v_\perp = v$ e $\alpha = \pi/2$ e la forza

F è $F = -\frac{1}{2} v^2 \left(\frac{\nabla_\parallel B}{B}\right)$ max

La particella inverte il moto



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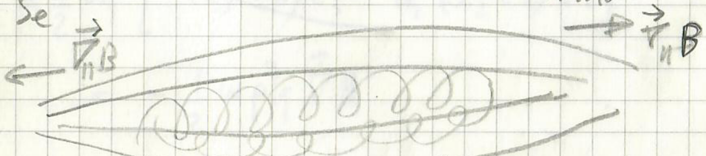


si può avere una trappola magnetica

(*) Infatti nel piano $\perp$ a B, ~~...~~
(*) nel rif. del centro di guida con assi $\hat e_1, \hat e_2, \hat e_3$

$$\vec\rho_c = \rho(\hat e_2 \sin\Omega t + \hat e_3 \cos\Omega t)$$

e quindi

$$\dot{\vec\rho} = \Omega\rho(\hat e_2\cos\Omega t - \hat e_3\sin\Omega t) + \sin\Omega t\,\frac{d}{dt}(\rho\hat e_2) + \cos\Omega\,\frac{d}{dt}(\rho\hat e_3)$$

$$\ddot{\vec\rho} = \Omega^2\rho(-\hat e_2\sin\Omega t - \hat e_3\cos\Omega t) + \dot\Omega\rho(\hat e_2\cos\Omega t - \hat e_3\sin\Omega t) + 2\Omega\cos\Omega t\,\frac{d}{dt}(\rho\hat e_2) - 2\Omega\sin\Omega t\,\frac{d}{dt}(\rho\hat e_3) + \sin\Omega t\,\frac{d^2}{dt^2}(\rho\hat e_2) + \cos\Omega t\,\frac{d^2}{dt^2}(\rho\hat e_3)$$

Dato che $\langle\sin\Omega t\rangle = \langle\cos\Omega t\rangle = 0 \Rightarrow \langle\rho\rangle = \langle\dot\rho\rangle = \langle\ddot\rho\rangle = 0$

~~I~~ Termini con $\ddot{\vec\rho}\times(\vec\rho\cdot\vec\nabla)$ contengono si termini con $\sin^2\Omega t$ e $\cos^2\Omega t$ che danno $\langle\ \rangle \neq 0$

Motion in magnetic fields

Pitch angle scattering

Quando il campo presenta irregolarità, gradienti con una distribuzione casuale, può essere rappresentato come

$$B(\vec{r},t) = \vec{B}_0 + \delta\vec{B}(\vec{r},t)$$

Campo medio — Irregolarità random in $\vec{r}$ e t

La densità di energia è $u = \frac{B^2}{8\pi}$

Siamo interessati ai valori medi (p. es. nel tempo) o su tutto

$$\langle u \rangle = \frac{\langle B^2 \rangle}{8\pi}$$

$$B^2 = \vec{B}\cdot\vec{B} = B_0^2 + \delta B^2 + 2\vec{B}_0\cdot\delta\vec{B}$$

$$\langle B^2 \rangle = \langle B_0 \rangle + \langle \delta B^2 \rangle + \underbrace{2\vec{B}_0 \cdot \langle \delta\vec{B} \rangle}_{=0}$$

Scriviamo il campo $\delta\vec{B}$ come sovrapposiz. di onde piane

$$\delta\vec{B} = \frac{1}{(2\pi)^{3/2}} \int d^3k\, \delta\tilde{B}(k)\, e^{i\vec{k}\cdot\vec{r}}$$

Perciò

$$\delta B^2 = \frac{1}{(2\pi)^3} \int d^3k\, d^3k'\, \delta\tilde{B}(k)\,\delta\tilde{B}(k')\, e^{i\vec{r}\cdot(\vec{k}-\vec{k}')}$$

L'energia totale del campo è

$$E_T = \int d^3r\, u = \frac{1}{8\pi} \int d^3r\, \delta B^2 =$$

$$= \frac{1}{(2\pi)^3} \int d^3r\, d^3k\, d^3k'\, \frac{\delta B_k\, \delta B_{k'}}{8\pi}\, e^{i\vec{r}\cdot(\vec{k}-\vec{k}')}$$

L'integr. su d^3r dà $\delta^3(k-k')(2\pi)^3$

$$E_T = \frac{1}{8\pi} \int \delta B^2(\vec{k})\, d^3k = \frac{1}{8\pi} \int k^2 \delta B^2(k)\, dk\, d\Omega$$

$$w(k) = \frac{1}{8\pi} \int k^2 \delta B^2(\vec{k})\, d\Omega \qquad [T^2/m^{-1}]$$

è la densità spettrale di potenza

Quindi $E_T ~~\delta B^2~~ = \int w(k)\, dk$

NB: alla vett. k è associata $\omega = kV$ con V = vel. di propag. dell'onda e $K = 2\pi/\lambda$

Motion in magnetic fields

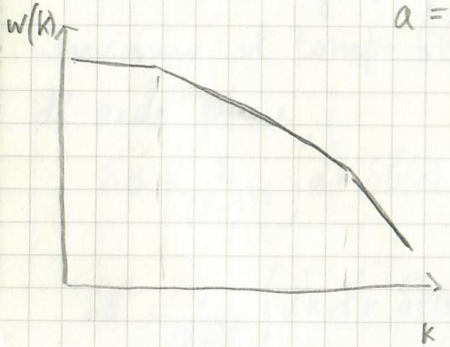
- $W(k)$ da l'energia associata a ciascuna comp. di Fourier

- A ciascuna componente k è associata una λ che definisca l'ampiezza della regione spaziale su cui l'onda agisca, i.e. la "scala" tipica della onda componente

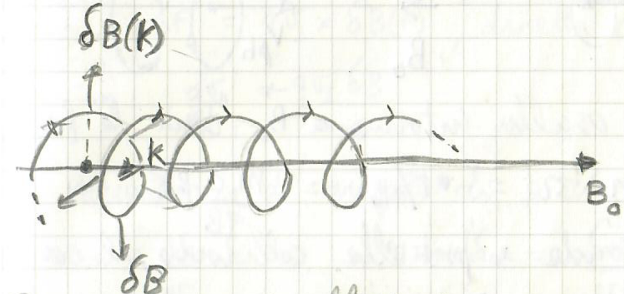
Di solito $W(k) \propto k^{\alpha}$ con l'indice α che in generale dipende da k _

La distrib. $W(k)$ dipende dal tipo di turbolenza MHD che determina la distribuzione delle irregolarità

$$\alpha = a - 2 \qquad a = \begin{cases} 1/3 & \text{Kolmogorov} \\ 1/2 & \text{Kraichnan} \\ 0 & \text{Random disc.} \\ 1 & \text{white noise} \end{cases}$$



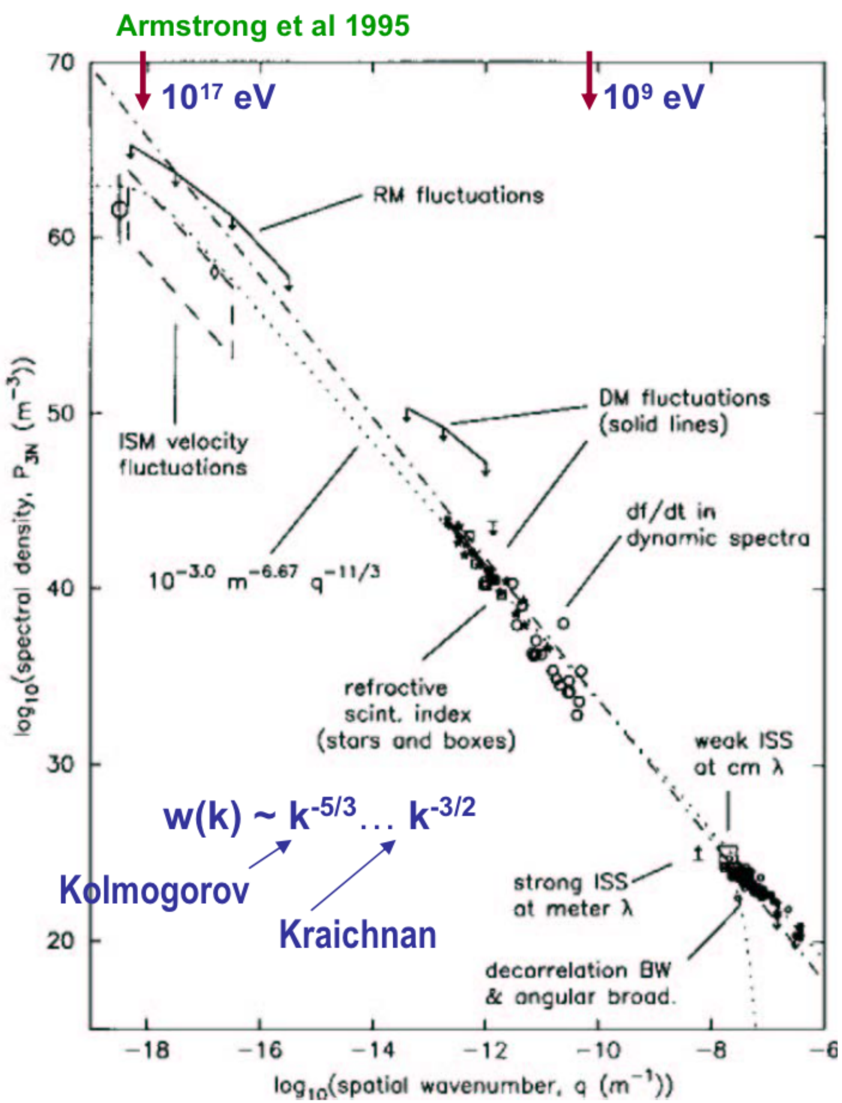
Il campo è quindi rappresentato come la sovrappos. di onde con numero d'onda k + freq. $\omega = \omega(k)$, ciascuna delle quali trasporta una energia $\sim W(k)\,dk$ _



Possiamo, a scopo illustrativo, considerare una $\vec{\delta B} \perp B_0$ che si propaga lungo una linea del campo medio $\vec{B_0}$ _

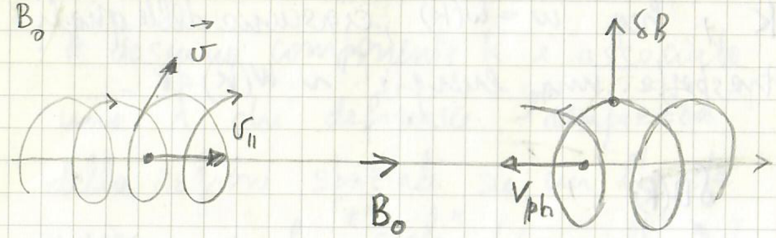
In tal caso δB "ruota" intorno alla direzione di B_0 con frequenza ω

interstellar turbulence



Motion in magnetic fields

Supponiamo ora che una particella viaggi con velocità $\vec{v}$ t.c. abbia una comp. $v_\parallel$ lungo B_0



La particella oscilla intorno a B_0 con la frequ. di Larmor $\Omega = p/qB = \frac{m_0 c}{qB}\beta\gamma$

Quando onda e particella collidono, il campo $\vec{\delta B}$ esercita una forza di Lorentz sulla particella

Dato che part. e onda si muovono l'una verso l'altra, la frequ. dell'onda "vista" dalla part. subisce uno spost. Doppler $\omega' = \gamma(\omega - \vec{k}\cdot\vec{v})$

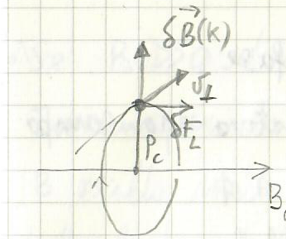
∴ Per semplicità supponiamo $v \ll c \Rightarrow \gamma \simeq 1$

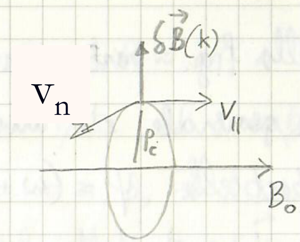
Dato che $\vec{k}$ e $\vec{v}$ hanno versi opposti

$\omega' \approx \omega + k v_\parallel$

Per facilitare la visualizz., separiamo in comp $V_\perp$ e $V_\parallel$ della vel. della particella

Nel caso di angoli di pitch arbitrari l'interazione è una combinazione delle due componenti





Prendiamo il caso di $v_\perp$.

Forza $\vec{\delta F}_L = q\vec{v}_\perp \times \vec{\delta B}(k)$, diretta come $\vec{B}_0$

Per cui $\frac{dv_\parallel}{dt} = \frac{q v_\perp \delta B}{m}$

La variazione di angolo di pitch

$\cos\alpha = \frac{v_\parallel}{v} \Rightarrow -\sin\alpha\,\delta\alpha = \frac{\delta v_\parallel}{v}$ (v=cost.)

$\Rightarrow \delta\alpha = -\frac{\delta v_\parallel}{v\sin\alpha} = -\frac{\delta v_\parallel}{v_\perp}$

Perciò $\delta\alpha = -\frac{q v_\perp \delta B\,\delta t}{m v_\perp} = -\left(\frac{qB_0}{m}\right)\left(\frac{\delta B}{B_0}\right)\delta t$

$\Omega = \frac{qB_0}{m}$ freq. di Larmor

$$\delta\alpha = -\Omega\,\delta t\left(\frac{\delta B}{B_0}\right)$$

δt è l'intervallo di tempo in cui particella e campo δB sono in fase e $\Omega\delta t$ è la fraz. di tempo che sono in fase in un periodo di Larmor

Motion in magnetic fields

Nella fig. part. e onde sono in fase.

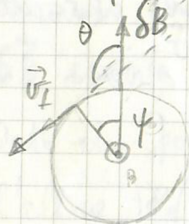
In generale c'è una fase relativa tra campo e particelle $\psi = (\omega + k v_\parallel - \Omega)t + \phi$

che dipende dalla ~~fase~~ frequ. relativa

$$R = \omega + k v_\parallel - \Omega$$

Se $R \neq 0$ $\quad \psi = \psi(t)$, la diff. di fase varia nel tempo

Se $R = 0$ $\quad \psi = \phi = \text{cost.}$, la diff. di fase non dipende dal tempo



La forza di Lorentz è

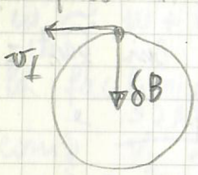
$$\delta \vec F = q \vec v_\perp \times \delta \vec B \quad \text{cioè}$$

$$\delta F = q v_\perp \delta B \sin\theta$$

Ma $\theta = \psi + \pi/2 \Rightarrow$

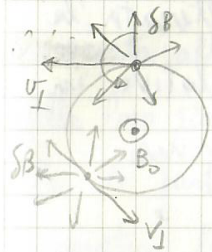
$$\delta F = q v_\perp \delta B \cos\psi$$

Se $\psi = \pi$ si ottiene una diffusione opposta a quella in cui $\psi = 0$ perché la forza di Lorentz ha verso opposto al precedente



• Se $R \gg 0$ $\quad \omega' = \omega + k v_\parallel \gg \Omega$. $R \approx \omega'$

In un periodo di Larmor $1/\Omega$, il campo B oscilla molte volte intorno a B_0 ⇒ Assume tutte le possibili orientazioni rispetto a $\vec v_\perp$ per cui $\langle \delta F \rangle = 0$.



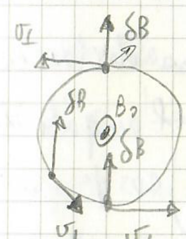
La particella vede un campo alla freq. ω'; dato che $\omega' \gg \Omega$ il valore medio di $\cos\psi(t)$ in un periodo di Larmor è

$$\langle \cos\psi \rangle_{Larmor} = \frac{1}{T}\int_0^T \cos\omega' t \, dt \approx 0$$

• Se $R \ll 0$ $\quad \omega' \ll \Omega$ e $R \approx -\Omega$

In tal caso, in un periodo di Larmor, la fase dell'onda varia di poco, cioè la dir. di $\vec{\delta B}$ è ≈ fissa mentre $\vec v_\perp$ le ruota intorno

⇒ $v_\perp$ assume tutte le possibili orientazioni rispetto a δB



In tal caso

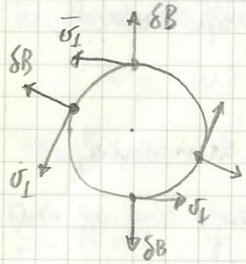
$$\langle \cos\psi \rangle_{Larmor} \approx \frac{1}{T}\int_0^T \cos\Omega t \, dt$$

$$= \Omega \int_0^T \cos\Omega t \, dt = 0$$

Motion in magnetic fields

Perciò se $|R| \gg 0$, in un periodo la forza netta è ≈ 0 e non c'è deflessione in α

$\therefore$ $R = \omega - k v_{\parallel} - \Omega = 0$



$\Rightarrow \psi(t) = \phi = \text{cost.}$

$\vec{\delta B}$ e particelle sono in fase, mantengono inalterata le loro orientaz. relative in un periodo di Larmor

Allora $\delta F = q v_{\perp} \delta B \cos\phi = \text{cost.}$ e

$\delta\alpha = -\Omega \delta t \left(\frac{\delta B}{B_0}\right) \cos\phi$ in un tempo δt

$$\delta\alpha_{Larmor} = \int \delta\alpha = -\int_0^{1/\Omega} \Omega \delta t \left(\frac{\delta B}{B_0}\right) \cos\phi$$

$$= \left(\frac{\delta B}{B_0}\right) \cos\Phi$$

In un periodo di Larmor o su una distanza λ $k = 2\pi/\lambda$, la particella cambia l'angolo di pitch di una quantità netta $\left(\frac{\delta B}{B_0}\right)\cos\phi$

La fase relativa ϕ determina l'intensità della deflessione. Se ϕ è random, anche $\delta\alpha$ lo è

- Dal dominio della frequ. si può passare a quello "spaziale" dei numeri d'onda $k = 2\pi/\lambda$ utilizzando $\omega = k V_{ph}$ $\Rightarrow$ $\lambda = \frac{2\pi V_{ph}}{\omega}$

che definisce la scala fisica su cui agisce la comp $\delta B(k)$.

Lo spettro di potenza $W(k)$ fornisce la energia associata alle irregolarità magnetiche in funzione della loro scala fisica

- Per le particelle che si propagano nel campo magnetico, la scala associata alla frequ. di Larmor è il raggio di Larmor ρ_g.

La condizione di risonanza di ciclotrone si traduce nella condizione $\rho_g \sim \lambda$ per avere interazione significativa tra onda e particella.

$\rho_g = \frac{p}{m\omega}$

Se $\omega \ll \Omega \rightarrow \rho_g \gg \lambda$ la particella non interagisce in media con le irregolarità e segue il campo medio $\vec{B}_0$

Motion in magnetic fields

Se $\omega' \gg \Omega$ cioè $\lambda \gg \rho_g$, non si ha deflessione da risonanza. Tuttavia in questo limite invece si applica l'approx del centro di guida poiché $\frac{\rho_g}{\lambda} \sim \rho \frac{\nabla B}{B} \ll 1$

In tal caso si può dimostrare che la quantità

$$\frac{p_\perp^2}{B} \approx \text{cost.}$$

cioè $\quad p^2 \frac{\sin^2\alpha}{B} \approx \text{cost.} \quad$ ovvero $\quad \frac{\sin^2\alpha}{B} = \text{cost.}$

Quindi

$$\frac{2\sin\alpha\cos\alpha\, d\alpha}{B} - \frac{\sin^2\alpha\, dB}{B^2} = 0$$

$$\frac{d\alpha}{\operatorname{tg}\alpha} = 2\frac{dB}{B} \qquad \delta\alpha = 2\operatorname{tg}\alpha \frac{\delta B}{B}$$

Nel caso $\rho_g \sim \lambda$, l'angolo di pitch può cambiare in modo significativo in un girorodio. Se la particella interagisce con "parecchie" onde che hanno una fase random si può avere una deviazione in angolo significativa su una lunghezza caratteristica, quella di scattering λ_{sc}

Dato che $\rho_g = \frac{cp}{qB_0} = \frac{R}{B_0}$ $\qquad R = \frac{cp}{q}$ = rigidità magn.

e $\rho_g \sim \lambda$, possiamo associare una rigidità magnetica con ciascuna scala fisica nello spettro di potenza: $\lambda \sim R/B_0$

Sono queste irregolarità che forniscono i centri di scattering più efficienti in angolo di pitch

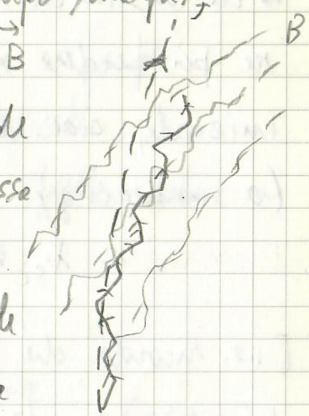
Motion in magnetic fields

Possiamo immaginare il campo nell'ISM come un campo medio $\vec{B}_0$ con sovrapposti treni d'onda monocromatici che si propagano in tutte le direzioni con fasi casuali.

Possiamo immaginare che ciascuna particella subisca l'azione di una particolare comp. del campo solo per una lunghezza d'onda λ prima di incontrare un'altra onda con la stessa λ ma con fase arbitraria rispetto all'onda precedente.

Così le particelle interagiscono successivamente con "molte" onde di lungh. d'onda λ con fasi relative casuali (e quindi casuale rispetto alla fase delle particelle) viaggiando rapidamente nel campo finché la deviazione cumulativa dell'angolo di pitch diventa grande e le particelle iniziano a interagire con un altro treno d'onde

La variazione $\delta\alpha$ in un giroradio implica uno spostamento casuale del centro di guida $\delta r_{cg} \approx \rho_g \delta\alpha$; c. le particelle si spostano in modo casuale attraverso le linee di campo magn., cioè diffondono nel campo $\vec{B}$



Dopo aver interagito con N onde della stessa λ cioè con la stessa densità di energia ma fasi casuali, in ciascuna delle quali subisce una deviazione $\delta\alpha \approx \left(\frac{\delta B}{B_0}\right)$, la deviazione media complessiva è

$$\delta\phi = \sum_1^N \delta\alpha_i \qquad \text{con } \langle \delta\phi \rangle = 0$$

e

$$\langle \delta\phi^2 \rangle = \sum_1^N (\delta\alpha_i)^2 = N\,\delta\alpha^2 = N\left(\frac{\delta B}{B_0}\right)^2$$

$$\Rightarrow \langle \delta\phi \rangle = \sqrt{\langle \delta\phi^2 \rangle} = \left(\frac{\delta B}{B_0}\right)\sqrt{N}$$

Perciò per avere una deviazione di $\pi/2$ sono necessarie $N = \frac{\pi^2}{4}\left(\frac{B_0}{\delta B}\right)^2$ interazioni o "collisioni" con le onde

Motion in magnetic fields

$N = \frac{\pi^2}{4}\left(\frac{B_0}{\delta B}\right)^2$ è il # di collisioni necessarie affinché la particella "perda memoria" della sua direzione iniziale.

La distanza che la particella deve percorrere per perdere memoria della sua direzione iniziale, cioè la lunghezza di diffusione (o scattering) è

$$\lambda_{sc} = N\lambda \approx \frac{\pi^2}{4}\rho_g\left[\frac{B_0}{\delta B(k)}\right]^2$$

[i.e. ricorda che alla risonanza $\lambda \propto \rho_g$]

λ_{sc} dipende da $\delta B^2(k)$, la densità di E associata alla scala fisica $2\pi/k = \lambda$.

Lo spettro di potenza del campo è dato in T^2/m^{-1}, cioè $\delta B^2(k) = W(k)dk \sim W(k)k$

$$\Rightarrow \lambda_{sc} \approx \frac{\pi^2}{4}\rho_g \frac{B_0^2}{W(k)k}$$

$$k = \frac{2\pi}{\rho_g} \Rightarrow \lambda_{sc} \approx \frac{\pi}{8}\rho_g^2\frac{B_0^2}{W(k)}$$

$$\rho_g = \frac{cp}{qB_0} = \frac{R}{B_0} \Rightarrow \lambda_{sc} \approx \frac{\pi}{8}\frac{R^2}{W(k)}$$

Dato che $W(k) \propto k^{-\alpha} \propto \rho_g^{+\alpha}$ $\quad \boxed{a = \alpha - 2}$ vedi 16. p dietro

$$\lambda_{sc} \propto \rho_g^{2+\alpha} = \rho^{+a}$$

cioè $\lambda_{sc} \propto R^{+a}$

La lunghezza di diffusione dipende dalla rigidità R della particella.

L'indice α dipende dallo spettro di potenza delle irregolarità δB.

Il coeff. di diffusione D

$$D = \frac{\lambda v}{3} \propto c R^{+a} \quad (v \approx c)$$

dipende dalla rigidità magnetica.

Di conseguenza anche il tempo di residenza dei RC dipende dalla rigidità, dato che

$\tau \approx \frac{H^2}{D}$ con H spessore del disco o dell'alone

$$\Rightarrow \tau \sim R^{-a}$$

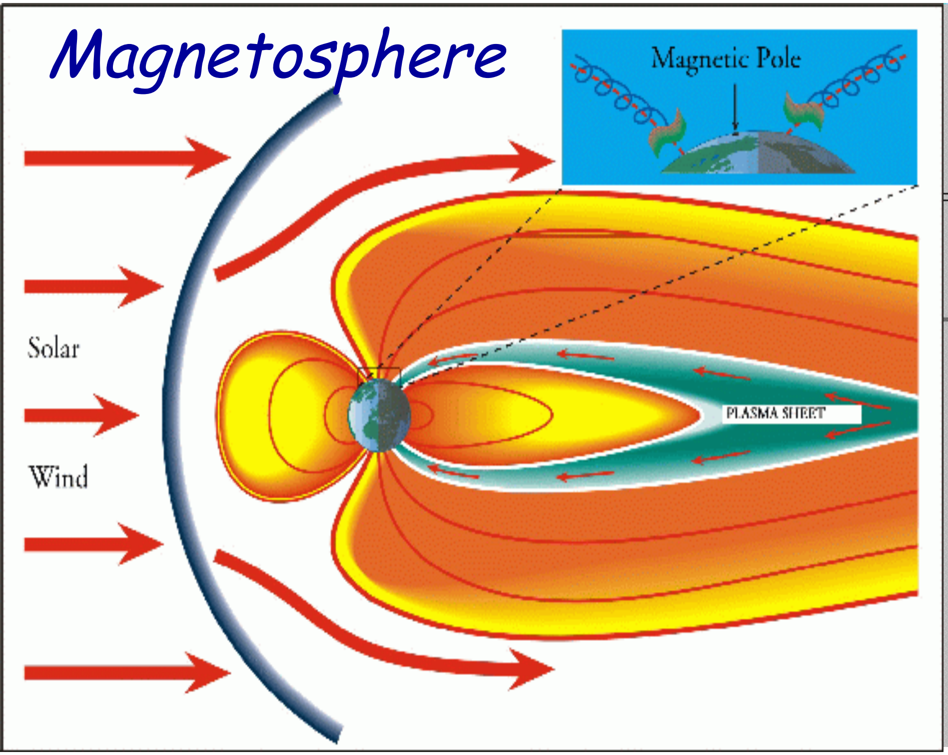
Cenni sulle fasce di van allen

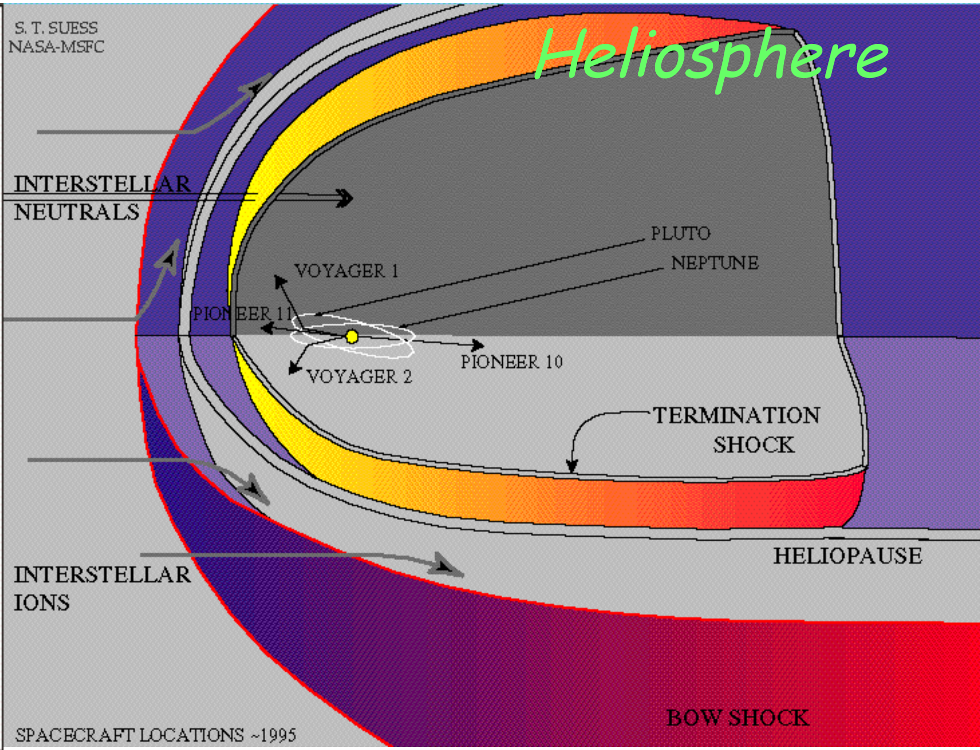
Le fasce di Van Allen nel campo geomagnetcio sono un esempio in cui drift e diffusione in campi magnetici giocano un ruolo essenziale.

Down to the magnetosphere

After a GCR has crossed the heliosphere or a solar particle reached the earth orbit, there is another obstacle they have to pass through before the detection around the Earth becomes possible.
This is created by the magnetic field of the Earth against the streaming solar wind from the Sun in a very similar way as it happens for the heliosphere in the interstellar wind.
The relevant difference is that for HE the pressure against the surrounding ISM medium is given by the kinetic (or ram) pressure of the solar wind (nmV^2), while around the earth the energy pressure is provided by the magnetic energy density ($B^2/2\mu$) of the earth's magnetic field against the SW ram pressure.

Local Environment





A *void* in the heliosphere where Earth's B field dominates

A *void* in the local interstellar medium where Sun B field dominates

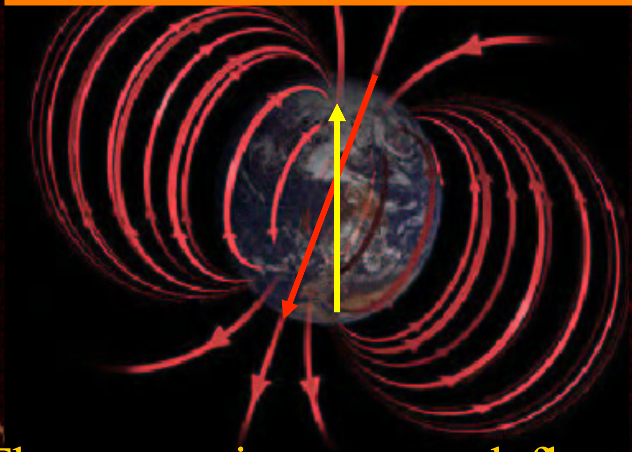
Different Scales, Similar Structure

Why is magnetosphere important?

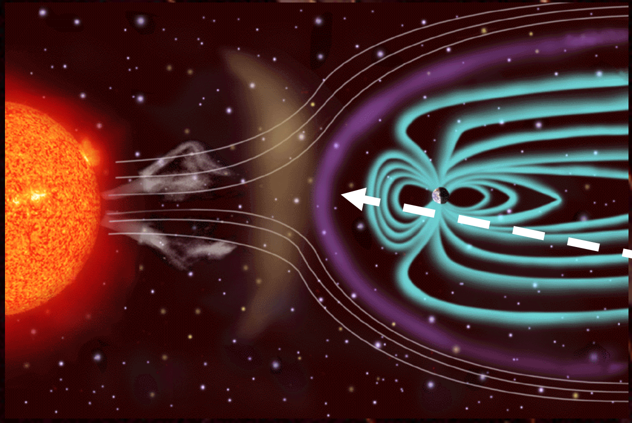
- →Despite the ‘*simple*’ physics, we still lack a complete knowledge of the mechanisms playing a role in filling and depleting belts
- →Very high particles flux
  - →hazards for manned satellites
  - →potential damage to electronic devices
- →Background to measurements @ satellite altitudes
- →On different scales, magnetospheres are present everywhere in Universe

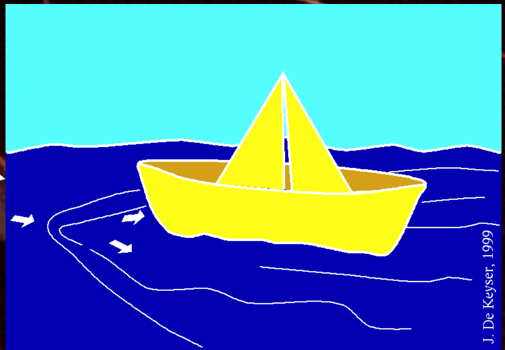
Earth’s Magnetosphere

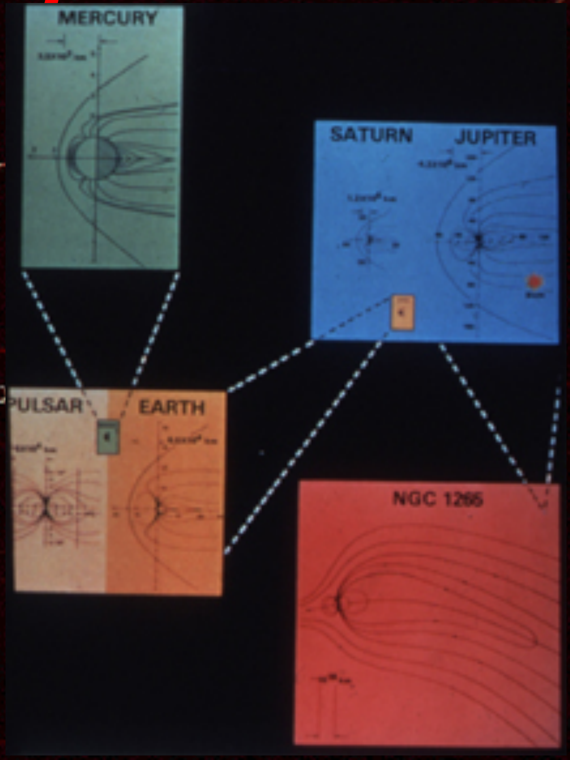
The source of the magnetosphere is Earth’s magnetic field



The magnetic pressure deflects solar wind much like the bow wave in front of a fast boat, shaping a cavity in the solar wind where earth B field dominates



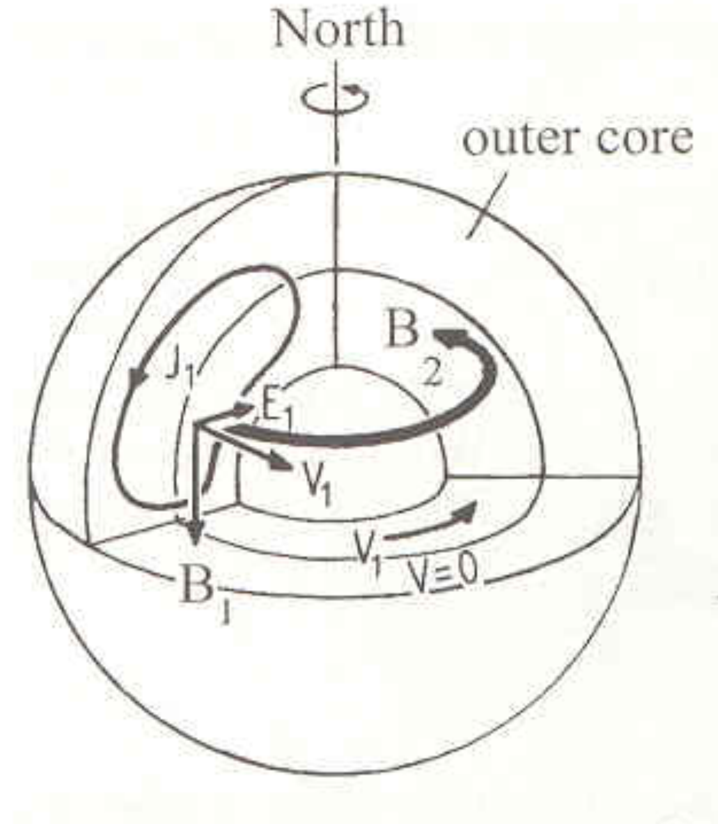


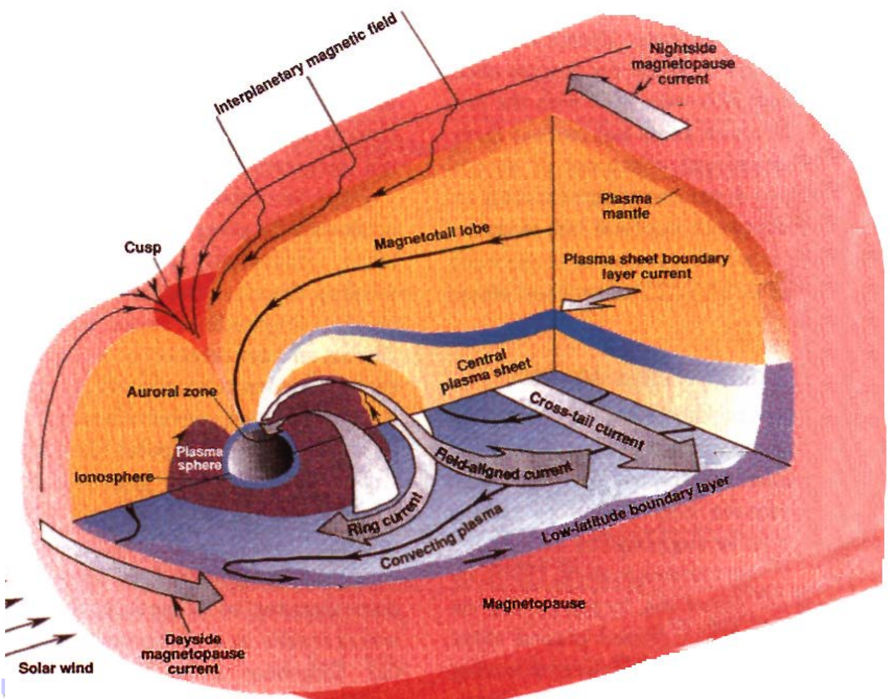


The universe is divided by magnetic bubbles within bubbles ranging from the planetary, stellar, and perhaps galactic and local group scale.

Earth's Magnetosphere

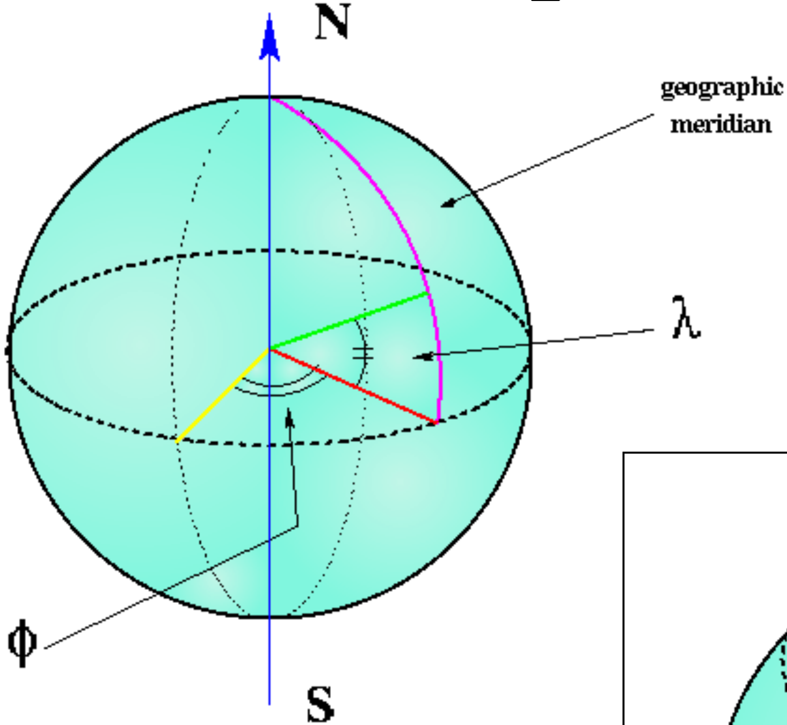
2 sources for Earth's field:
- inner dinamo due to the rotation in the outer core→ inner field





- external ring currents circulating in the magnetosphere→ external field

Geomagnetic Field: coordinates



Geographic Coordinates
of the
EARTH

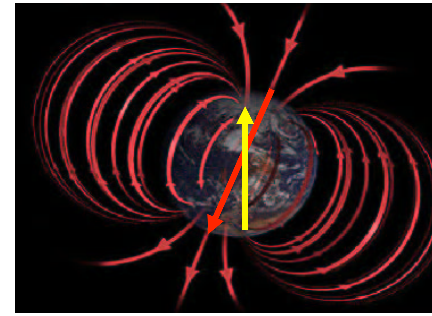
ϕ = geographic longitude

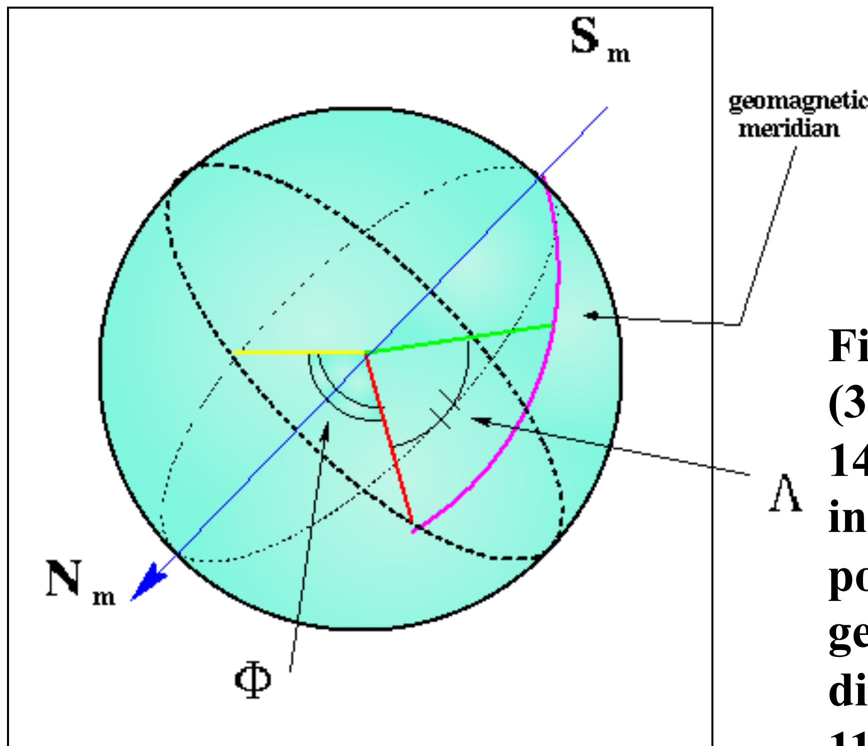
λ = geographic latitude

Φ = geomagnetic longitude

Λ = geomagnetic latitude

Geomagnetic Coordinates
of the
EARTH





Field center is at (320 km, 21.6 N, 144.3 E) in GTOD mag poles inverted wrt geographic ones, dip axis is tilted of 11.3° wrt the rotation axis

Inner Field

The inner field is given by a scalar potential V, because of the absence of free currents in the nearby space around the Earth.
In such a case, ∇xB=μj=0 and therefore, a potential V can be found such that

$$\vec{B} = -\vec{\nabla} V$$

From ∇•B=0, it follows that -∇•(∇V)=0 →

$$\Delta V = 0$$

A Laplace equation

Real Geomagnetic Field

The general solution for the Inner Field is described by a shifted multipole expansion

$$V = \sum_{n=1}^{\infty} \frac{1}{r^{n+1}} \sum_{m=0}^{n} [g_n^m \cos(m\varphi) + h_n^m \sin(m\varphi)] P_n^m(\cos(\theta))$$

g and h normalization coeff, P Legendre coeff and θ, ϕ are geomag coords, n gives the order of the multipole, and m the harmonis

Field center is at (320 km, 21.6 N, 144.3 E) in GTOD (geographic coord.)

Mag poles inverted wrt geographic ones, dip axis is tilted of 11.3°

International Geomag Ref Field (IGRF) model takes into account n=10 terms for field calculation

Accuracy is ~0.5% close to Earth's surface, ~6% at 3 R_E

Earth's Magnetosphere: dipole equations

At lowest order (n=1, m=0), the approximation is dipolar and $V=(1/r^2)g^0{}_1\cos\theta$

To first order, the earth field can be described as a sphere magnetized uniformly along its dipole axis

In spherical coordinates:

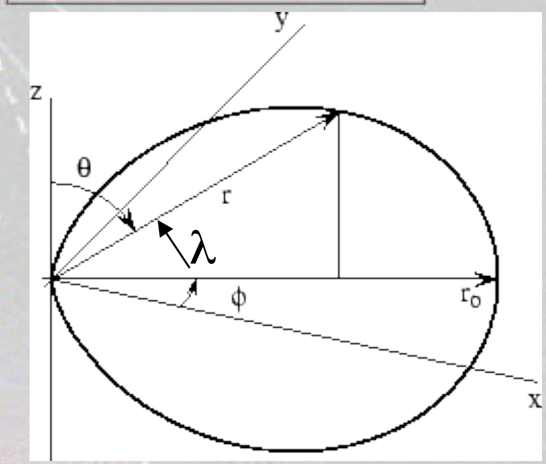
$$\boxed{B\,(r,\lambda) = M(1+3\sin^2\lambda)^{1/2}/r^3}$$

$$\left\{\boxed{\begin{array}{l} B_r = -2M\sin\lambda/r^3 \\ B_\lambda = M\cos\lambda/r^3 \end{array}}\right.$$

where M is the magnetic dipole moment and λ the magnetic latitude. M~ 8.1×10^{25} Gauss cm^3 and thus $B(R_E)$ ~ 0.31 Gauss.

The field lines have this form:

$$\boxed{r = r_0 \cos^2\lambda}$$



The module of the field B along the line has its minimum for λ=0. If λ=0, r= r_0 and this is the radial distance to the field line over the equator. Adopting $R=r/R_E$, in Earth-radii, the field line equation becomes:

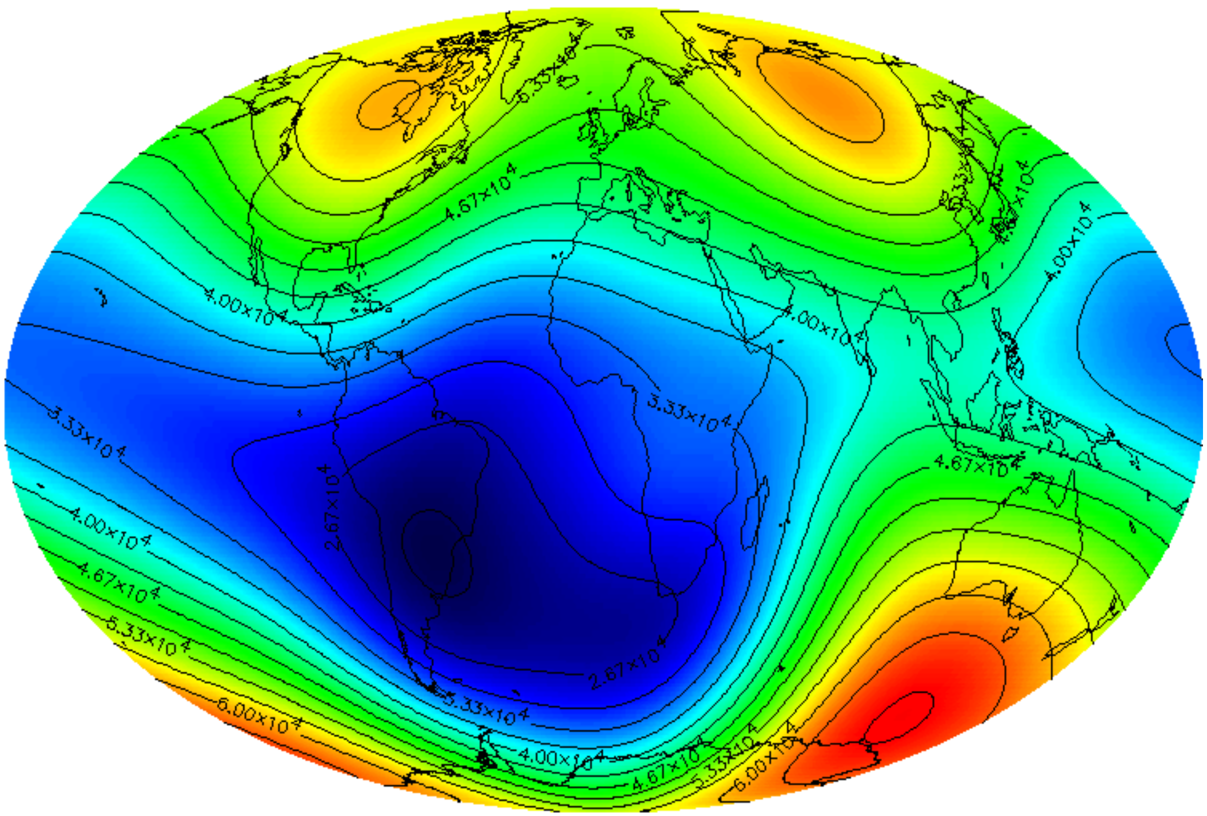
$$\boxed{R = R_0 \cos^2\lambda}$$

Earth's Magnetosphere: inner field

Sadly, Inner Earth's field is not dipolar and the approach is only a rough approximation at few Earth radii

Inner Field is described by the shifted multipole expansion

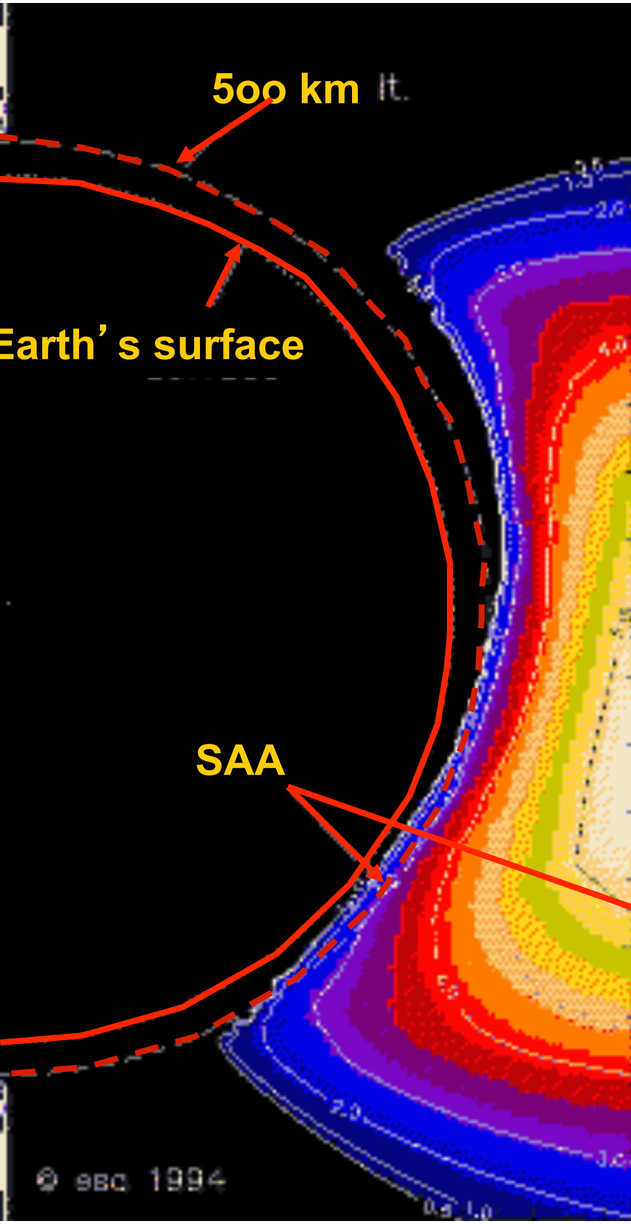
$$V = \sum_{n=1}^{N} \frac{r_E}{r^{n+1}} \sum_{m=0}^{n} [g_n^m \cos(m\varphi) + h_n^m \sin(m\varphi)] P_n^m(\cos(\theta))$$



International Geomagnetic Reference Field (IGRF) model takes into account N=10 terms for field calculation and it is the STANDARD model

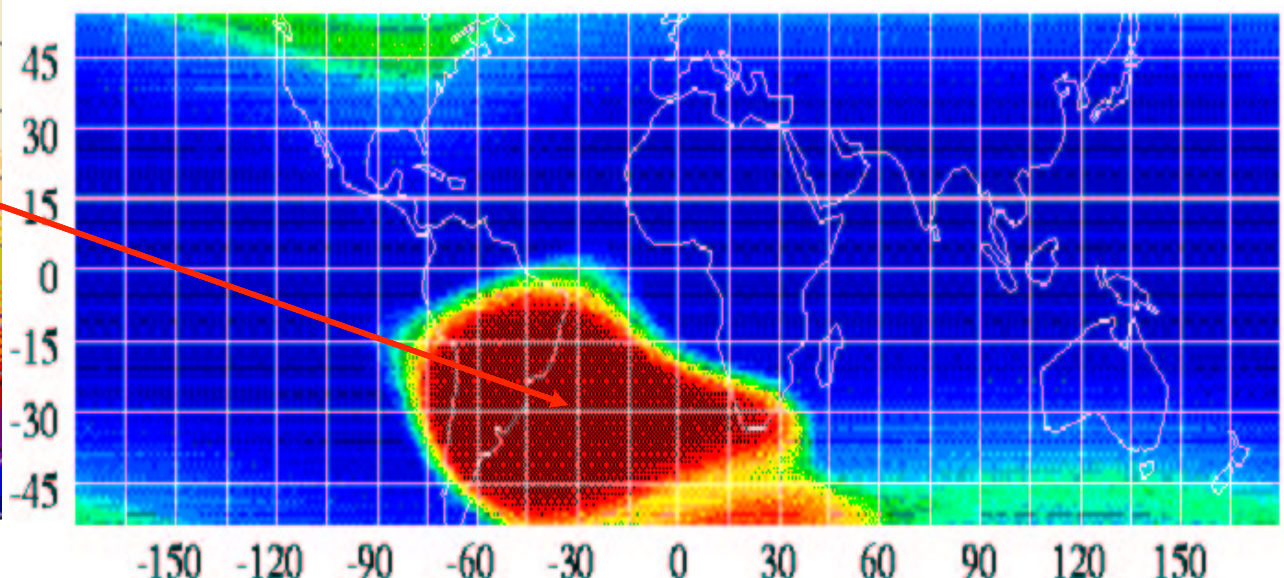
Accuracy is ~0.5% close to Earth's surface, ~6% at 3 R_E

Real Geomag Field: dipole shift effect (SAA)



Dipole center shift ➔B equipotential surfaces are not at constant height wrt Earth's surface

Tilt➔ region where a given B field is found at minimum height is above Brazilian coasts in south hemisphere. In this region plasma particles can reach very low altitudes.The region is called ***South Atlantic Anomaly (SAA)***



Earth's Magnetosphere: Inner Field

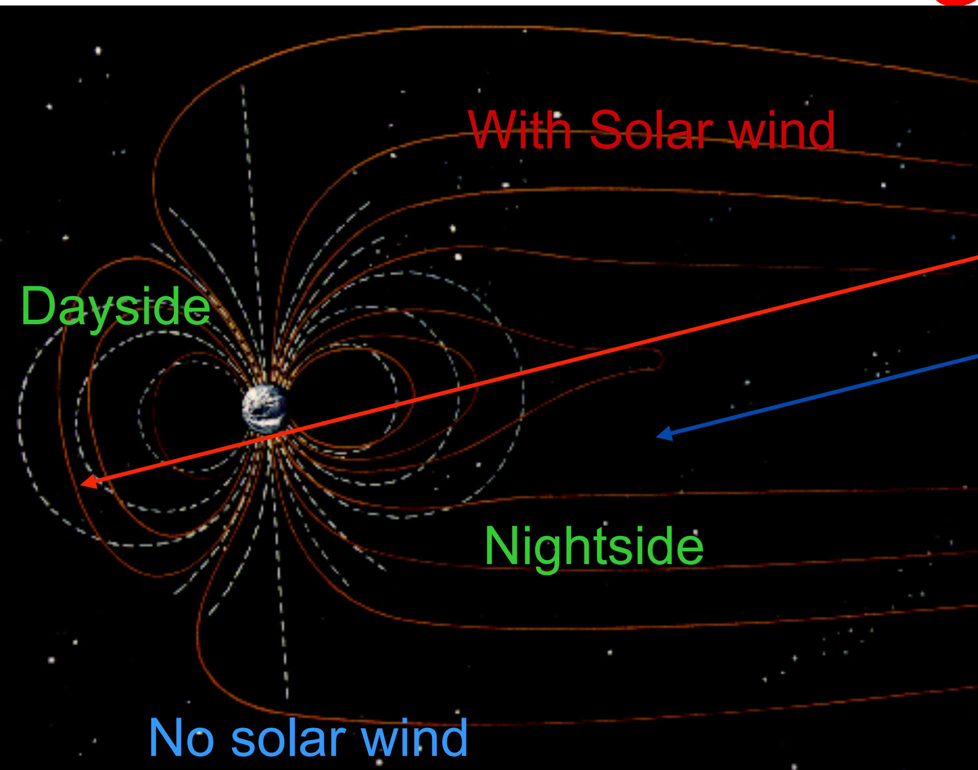
South Magn pole

Field changes slowly over the years, producing a secular drift of the magnetic poles

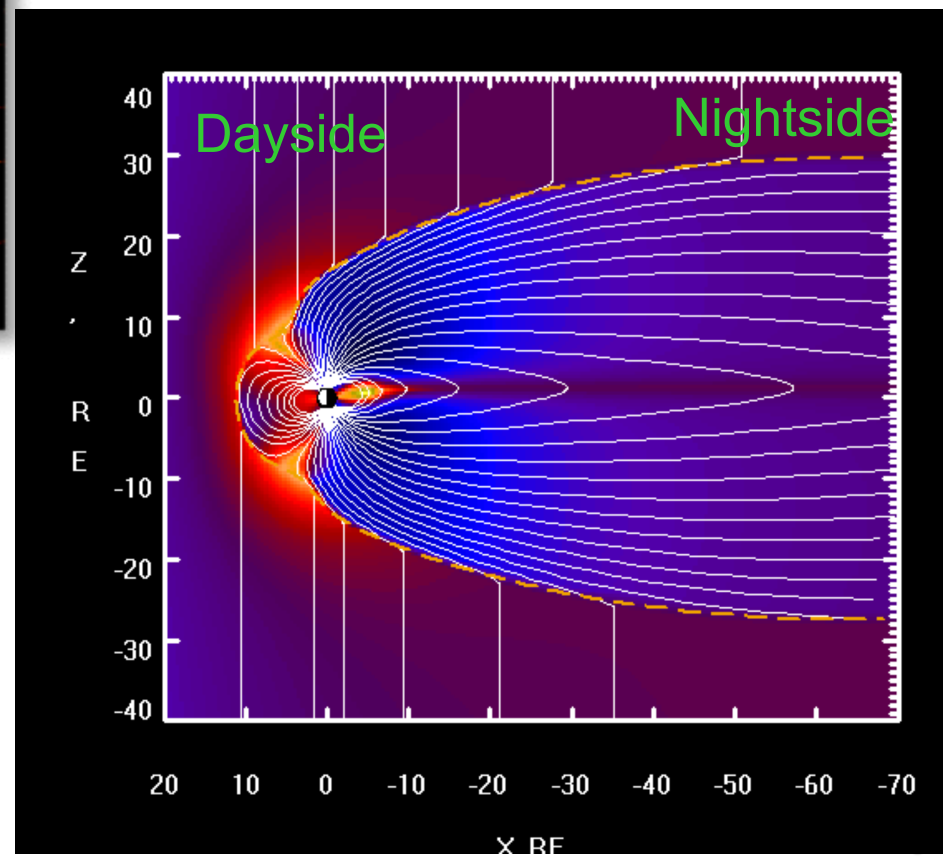
The magnetic moment M of the field is 8 x 10^{25} Gcm^3, resulting in a field intensity of ~0.3 G at equator and ~0.6 G at poles.

There are geological evidences of intensity variations of the dipole moment (it was less than half 0.5 Myear ago) and field polarity reversal with a period of ~ 0.5 Myear with shorter reversal on shorter time scales (from thousands to 200.000 years). Last polarity reversal occurred about 30.000 years ago.

Earth's Magnetosphere



The most evident effect of the solar wind pressure is a compression of field lines on the dayside and a drag effect in the nightside

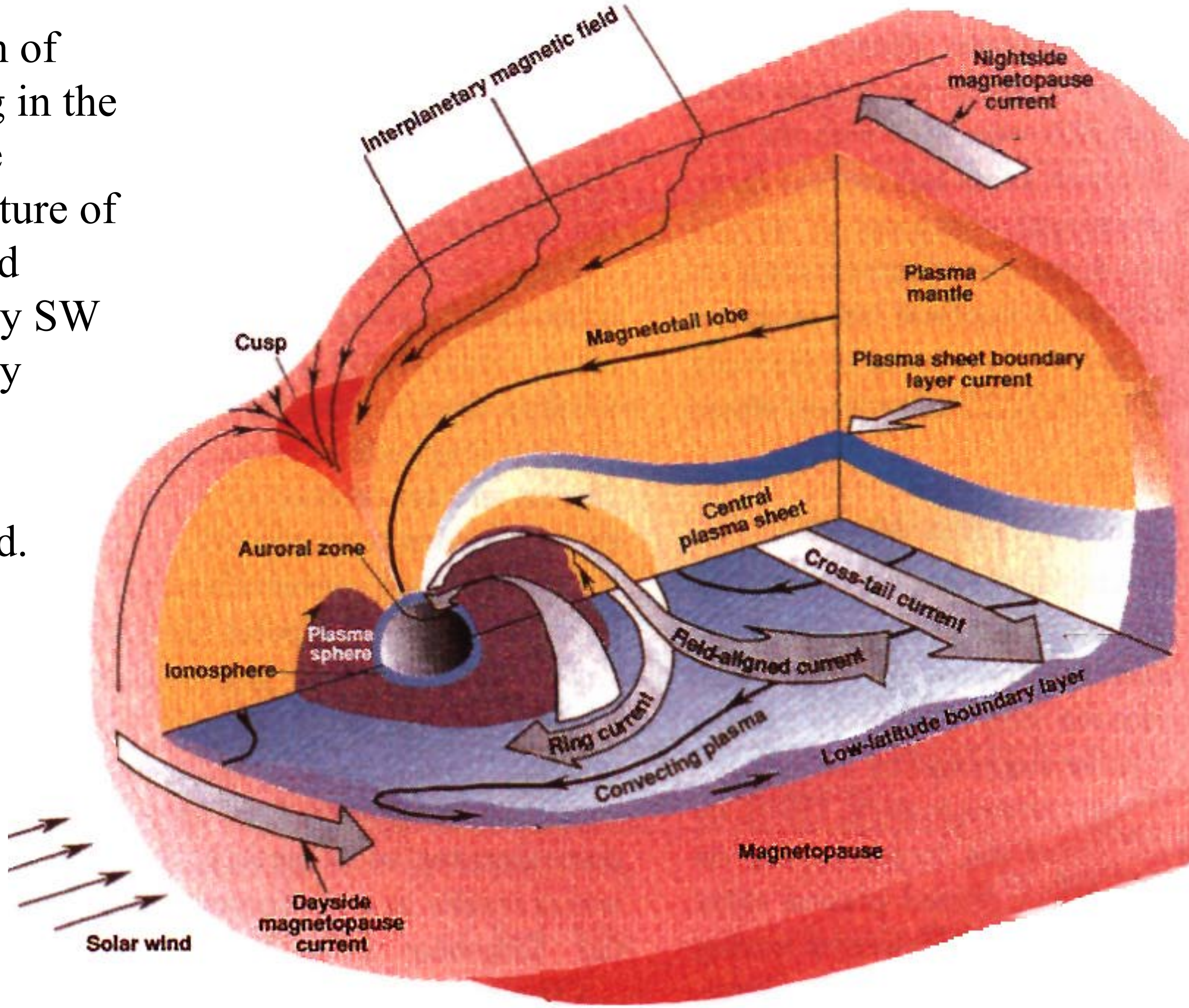


The inner field of the earth dynamo is shaped by the interaction with solar wind, which –as we will see- injects continously particles in the magnetosphere. The circulation in field leads to currents which contribute to the total field of the Earth above few R_E's

External Field

A complex system of currents is circulating in the magnetosphere determining the structure of the external field heavily influenced by SW and solar activity

It is directly due to the action of the solar wind. Its representation is based on the modular principle, according to which, B_{ext} is given by the sum of the contributions of the major magnetospheric current systems



Magnetosphere: magnetic trap

The main feature of the magnetosphere is that a characteristic particle population is present in the inner part from atmosphere limit to the magnetopause.

These particles are in a magnetic trap created by the Earth's field with trapping times from few seconds to years:

The Van Allen radiation belts

Why do radiation belts exist?

Necessary ingredients:

•Existence of a non-uniform B field

→Cutoff Energy

→Allowed/forbidden trajectories

•Dipole-like B field and closed field lines:

→Periodic motion

→ Separability of motion components

•Population:

→"Equilibrium" of physical processes for plasma injection and loss

Rigidity Cut-off: allowed/forbidden trajectories

Driving force is the Lorenz force (in quiet sun conditions)

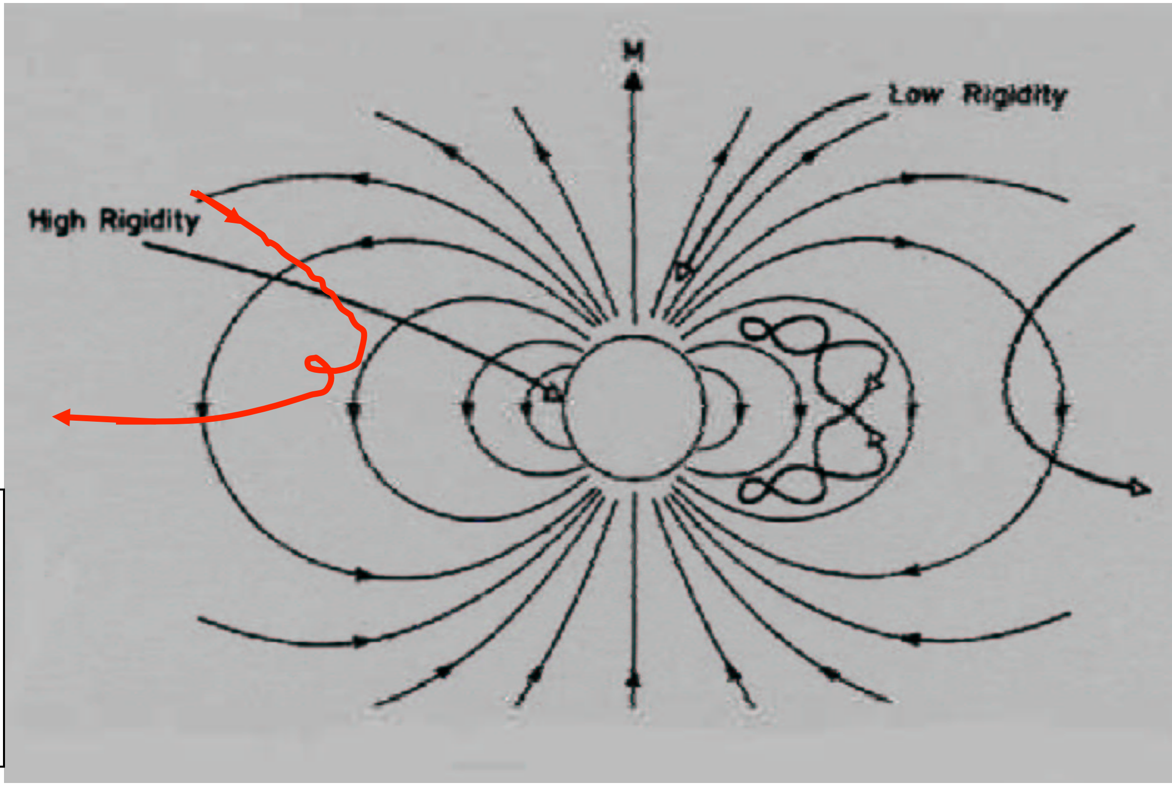
$d\underline{P}/dt=(q/c)(\underline{E}+\underline{v}x\underline{B})$

Typically $\underline{E}=0$ and given R=P/q, the magnetic rigidity ➔ $d\underline{R}/dt=\underline{\beta}x\underline{B}$

All particles with same R and β have same motion

<u>Only particles with rigidity $\vec{R}$ greater than a minimum rigidity R_c can reach a given position from outside magnetosphere,</u> because B field gradients bend particles trajectory as they approach earth.

R_c is a complex function of the position in the field and of particle momentum direction: R_c(r,Q,F,p/p)



Rigidity Cut-off: Stormer Cut-Off

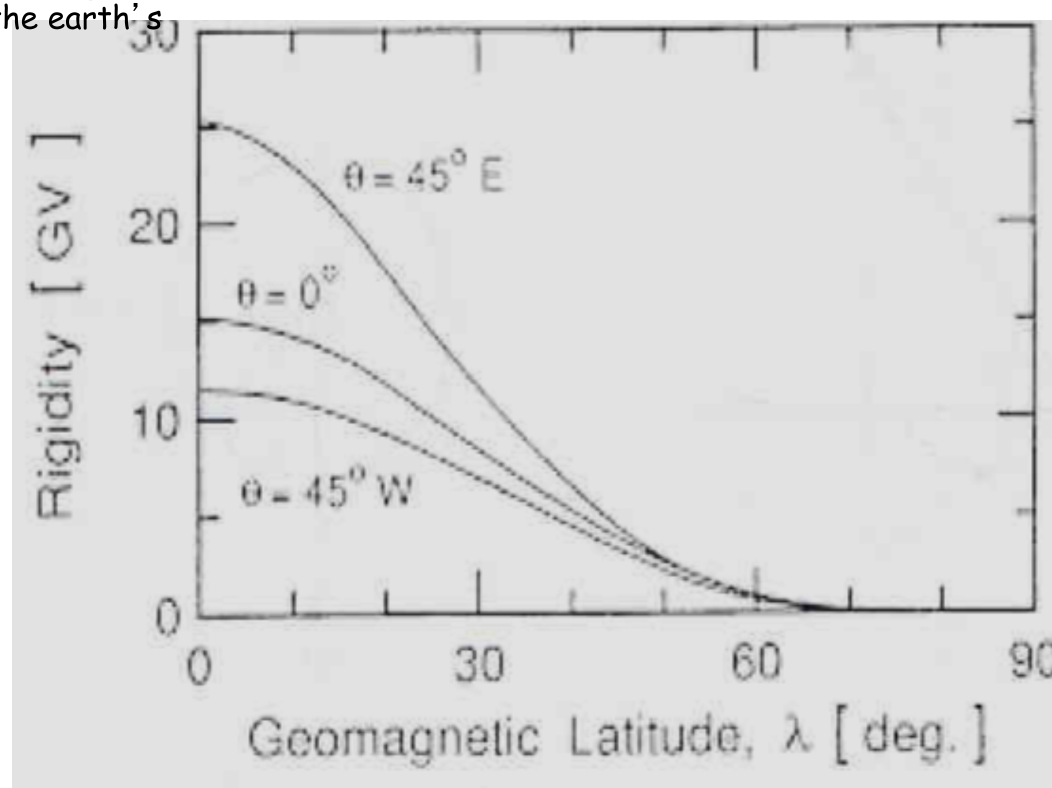
In a dipole field R_c is given by the classical Stormer cut-off:

R_c ~ 15 GV at equator,
~ 0.2 GV at poles

$$R_C = \frac{M_E}{r^2}\frac{\cos^4\Theta_{mag}}{[1+(1-Q\sin\vartheta\sin\varphi\cos^3\Theta_{mag})^{1/2}]^2}$$

being Q is the particle charge, θ, ϕ the polar and azimuthal angles wrt the local zenith, Θ_{mag} the geomagn latitude and M_E the earth's magnetic moment

The main features are that for any position in the field there a minimum threshold rigidity, below which a particle from outside can not reach that position.
→It depends on the arrival direction of the particle θ, ϕ
→Because of the factor Q$\sin\phi$, a charged particle from ϕ has a lower cut-off wrt a particle coming from $\phi+\pi$ and a charge Q has a lower cut-off wrt a charge –Q from the same direction (East-West effect)



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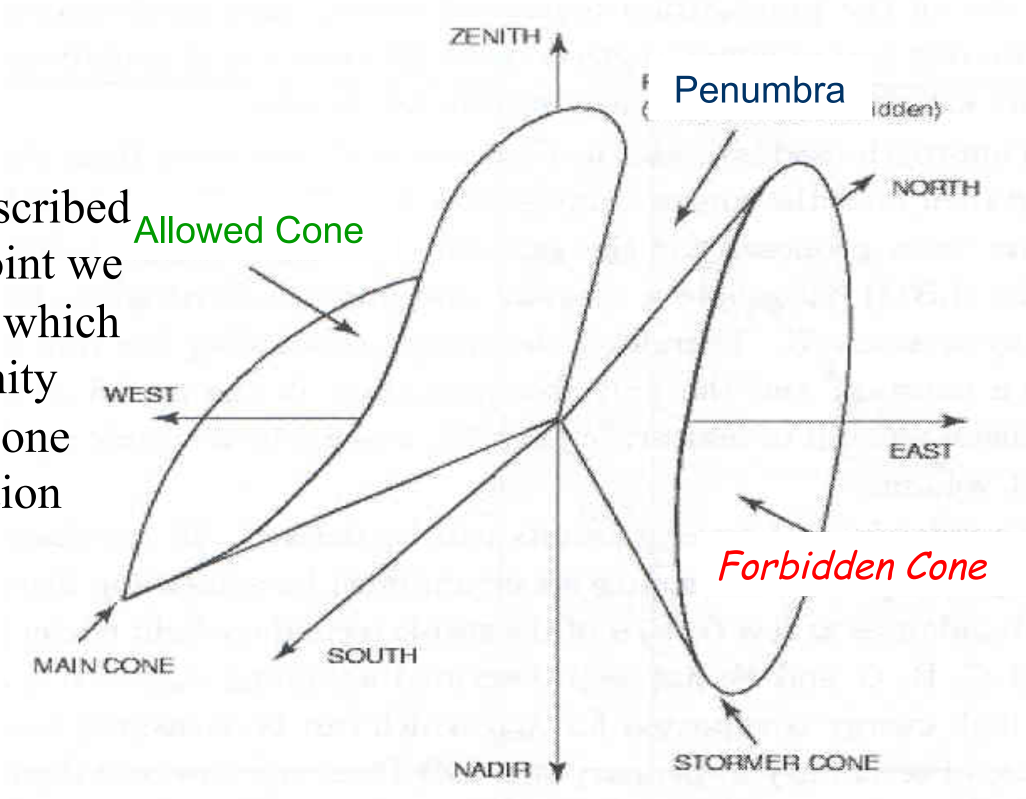
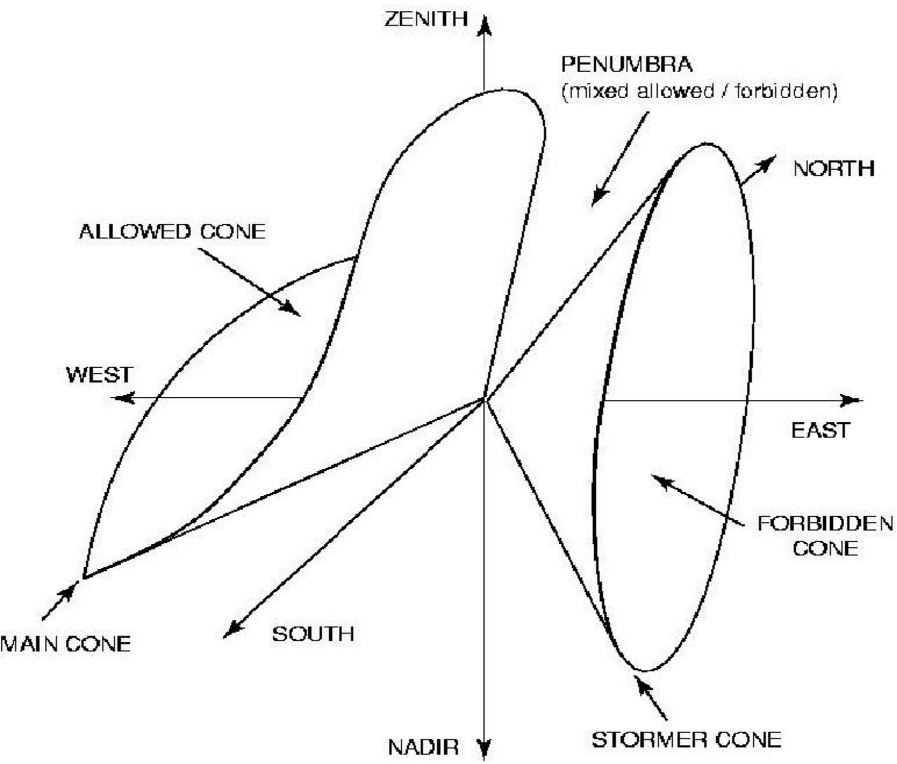
The access from infinity to any pos is described by a set directions of approach: at any point we can define a cone dependent on rigidity- which encloses all the trajectories from infinity reaching the point (**allowed**) and the cone enclosing those not reaching the position (**forbidden**) for the given rigidity



Rigidity Cut-off: Trapping



The forbidden/allowed cone explains the existence of trapped particles. Particle from outside cannot approach the position in the forbidden cone, but it is also true that a particle already within one of those regions couldn't get out if R<Rc. What were forbidden regions for particles approaching from the outside were trapping regions for some particles already there.

Then allowed/forbidden cones are relative to the particle direction of approach and particles with R<Rc cannot go from one cone to the other

No one realized that these trapping regions might well be filled with trapped radiations forming a radiation belt around the earth, i.e. biased to particles coming from outside. No one paid any attention to this possibility until Van Allen's discovery in 1958.

This is an excellent example of how initial orientation can markedly bias the investigator's conclusions

Trapping

A *necessary* condition to have trapping is then to have rigidity below the cutoff rigidity R_c but…
it is not sufficient for *stable* trapping
because it states only the impossibility for a particle to reach infinite distance from source

What is needed is a suitable field configuration which allows at least quasi-periodicity of particle motion

Motion in B fields: classical approach

Guiding center decomposition:

Parallel and normal components to the field line: $\mathbf{V}=\mathbf{V}_p + \mathbf{V}_n$ and

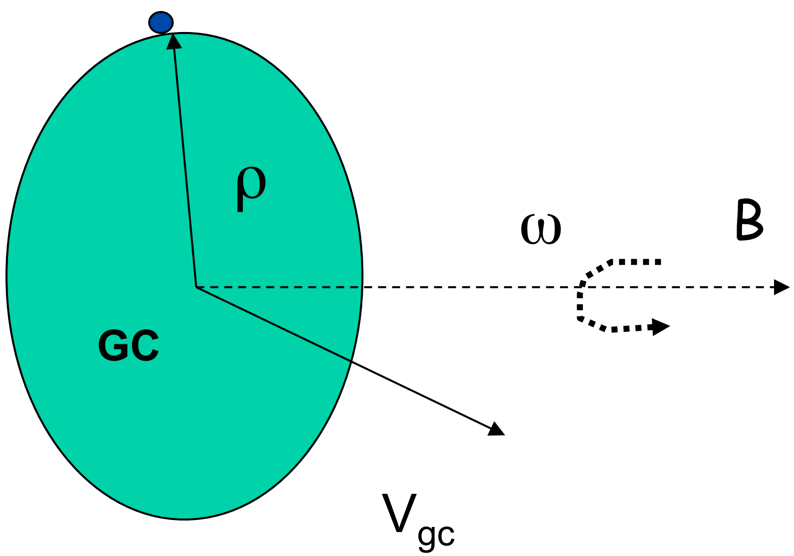
$\mathbf{V}_n$ is decomposed in a drift and a gyration with Larmor radius $\boldsymbol{\rho}=\mathbf{P}_n/\mathbf{Bq}$ and frequency

$$\omega=qB/m \rightarrow V= V_p + V_D + \omega x\rho = V_{gc} + \omega x\rho$$

The motion is then described by a traslation of a point, the Guiding Center, plus a gyration around GC normal to B

<u>Parallel and normal components are decoupled</u>

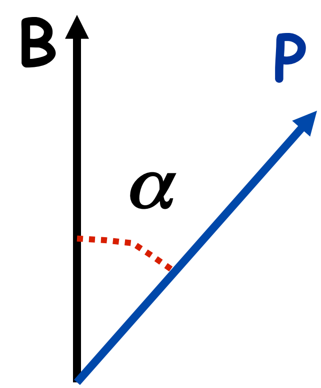
If $dB/Bdt \ll \omega/2\pi$



$V_n = V\sin\alpha$

$V_p = V\cos\alpha$

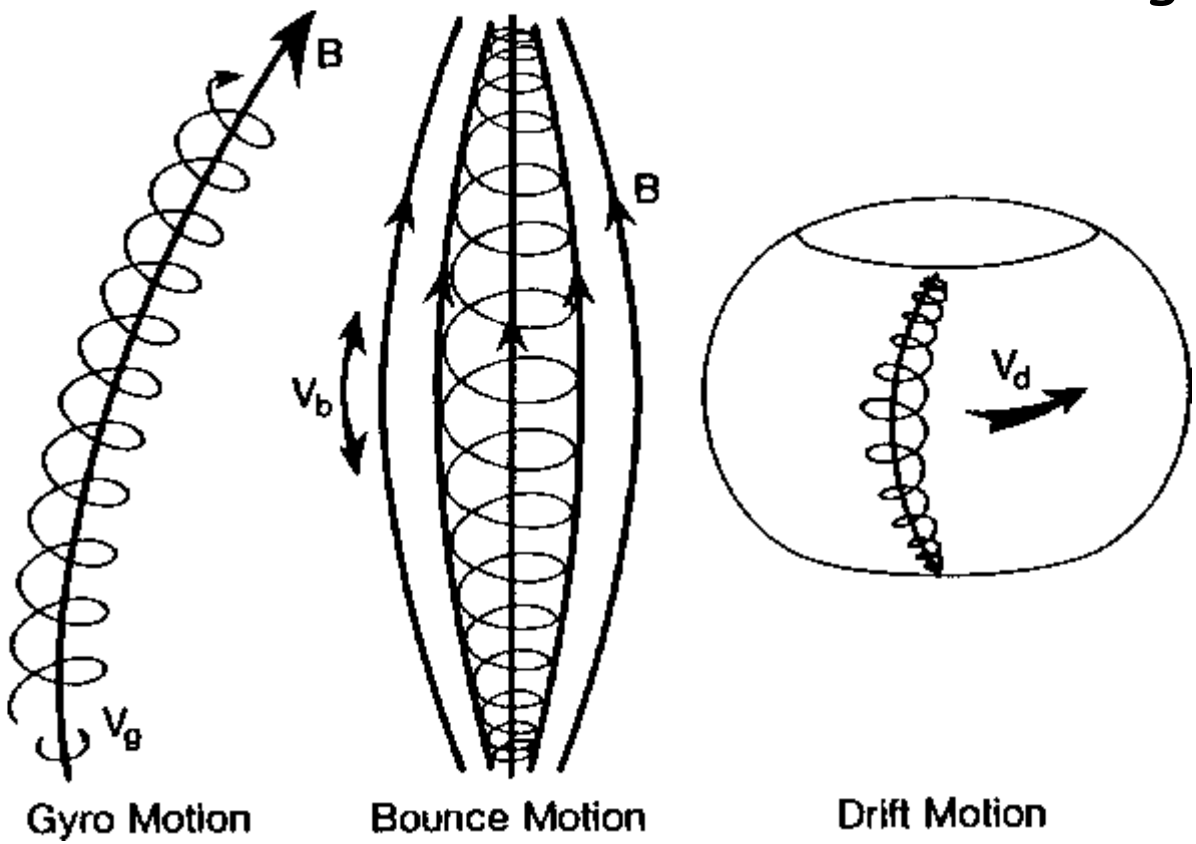
The “Pitch“ Angle



Motion in B fields: classical approach

As a consequence of the decoupling, the motion can be decomposed in 3 quasi-periodic components:

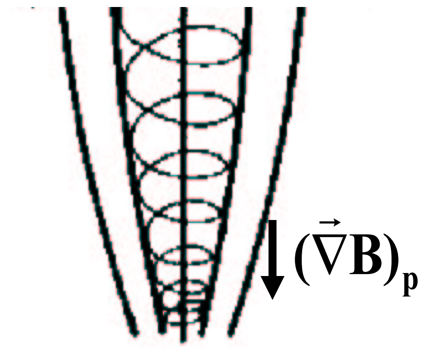
- gyration around the field line
- bouncing between the mirror points along the field line
- drifting normal to the field line and to the field gradient



Motion in B fields: classical approach (2)

The parallel motion is described by

Force is independent on charge and opposite to parallel field gradient

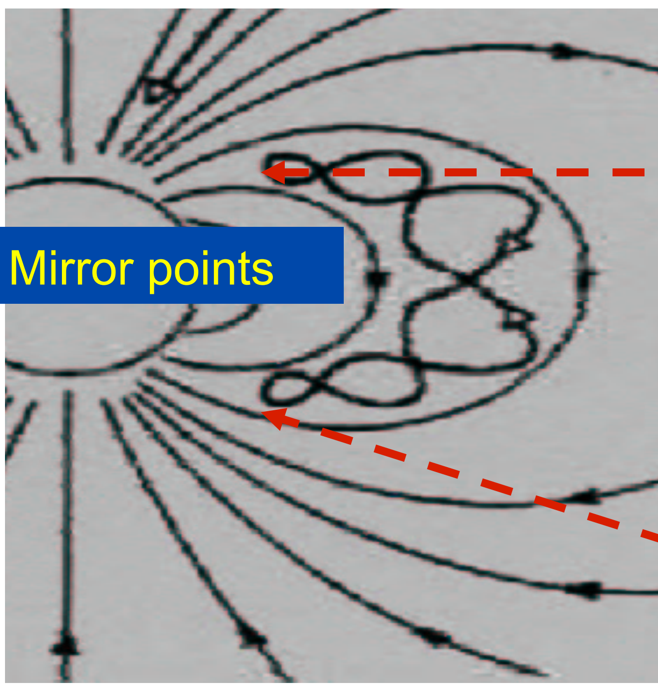


$$\frac{d\vec{v}_p}{dt} \approx -\frac{v_n^2}{2B}(\vec{\nabla}B)_p = -\frac{v^2 \sin^2 \alpha}{2B}(\vec{\nabla}B)_p$$

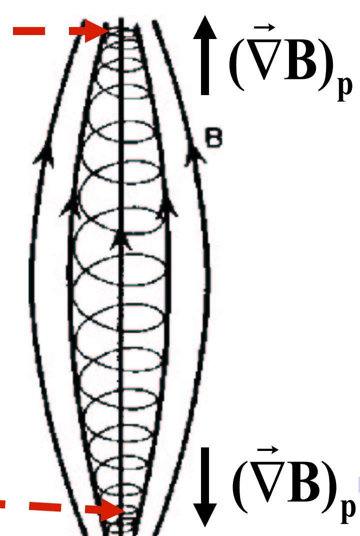
In a B field,
V is a constant →
$V^2 = v_p^2 + v_n^2 =$ const

$v_p \rightarrow 0$ → at some point and the particle is reflected back
$\alpha = 90^0$ at reflection

If we have a field config with 2 oppositely directed parallel gradients, there is periodic motion parallel to the field lines, because particles are reflected forth and back between the 2 gradients
→ magnetic bottle



Earth field has this feature due to the dipole-like structure of the field up to latitudes of 78^0

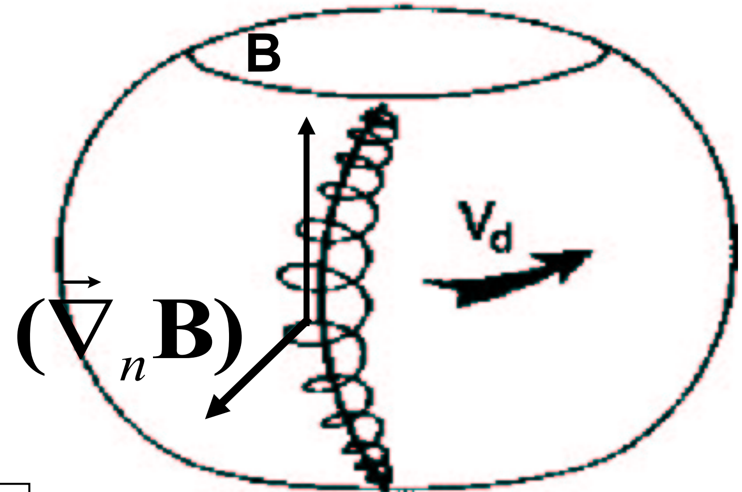


Motion in B fields: classical approach (3)

Normal motion is given by:

→ Gradient drift: v const. but ρ increases/decreases with decr./incr. field strength

→Curvature drift:
due to centrifugal force
on GC following curved B line



Gradient and curvature drifts appear always together

$$\vec{V}_{GC} = \frac{m}{2qB^3}(v_n^2 + 2v_p^2)\vec{B}\times\vec{\nabla}_n B = \frac{mv^2}{2qB^3}(1+2\cos^2\alpha)\vec{B}\times\vec{\nabla}_n B$$

V_{gc} always perp to B and B gradient→periodic drift motion if drift path is closed following const. B contours

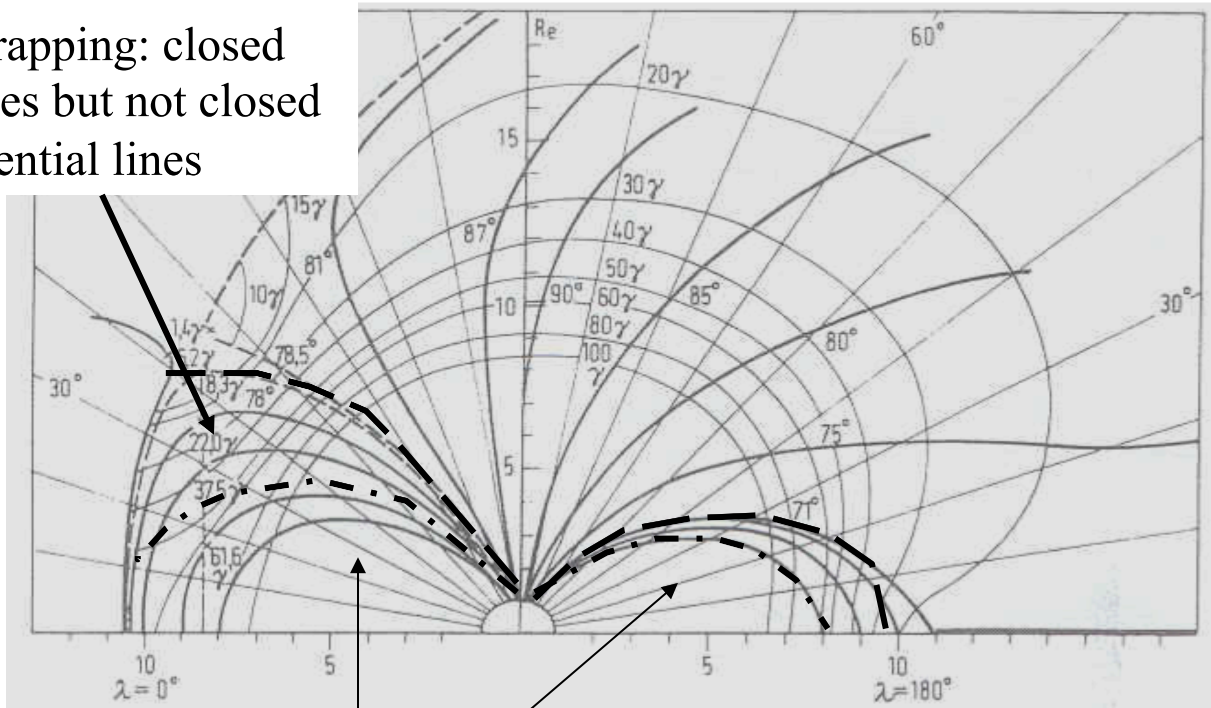
Drifts depends on q → oppositely charges drift in opposite directions

<u>Again Earth's field has the requested features</u>

Important implication: if particles are trapped inside a torus field, the best and finest adjustments in temp and field cannot prevent particles to drift across field lines and out of torus sooner or later→nuclear fusion problem

Motion in B fields: trapping limits

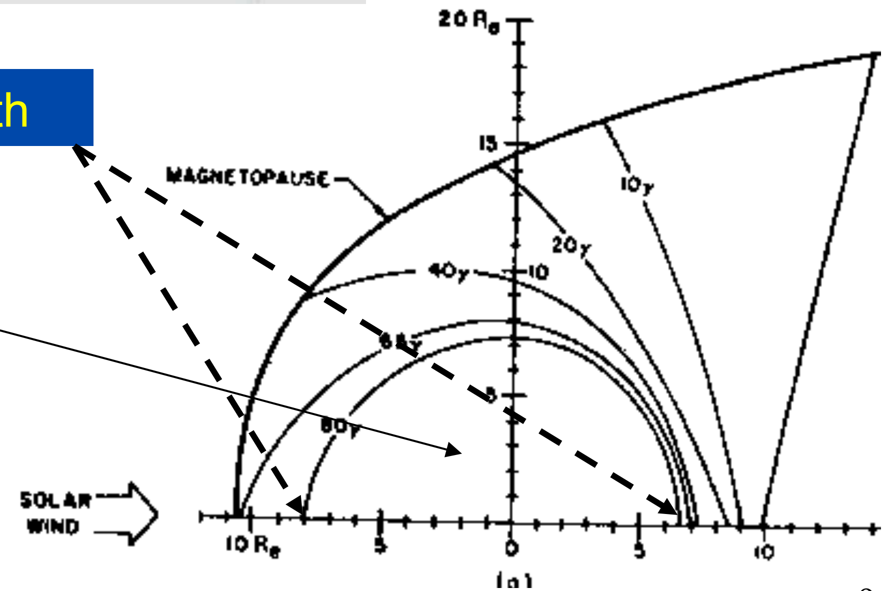
Quasi-trapping: closed field lines but not closed equipotential lines



Below 78[0] degrees, the field lines are closed on earth. Above this limits the polar cusps starts and the field lines are connected to the magnetopause, i.e. open field lines→ no trapping possible

Last closed path

<u>Within 7-8 R_E at equ. plane, all the drift paths are closed</u>→In this region a particle remains thus trapped in the earth's field forever, if there are no external perturbations: <u>stable trapping</u>



A more powerful approach: adiabatic invariants in B fields (1)

Guiding center equations are an enormous improvement wrt the Lorenz equation but drift and mirroring equations do not allow long-range predictions of particle location, if no axial simmetry is present

What is missing? The "constants of motion", analogous to the conservation of E, P, and angular momentum

Fortunately,in mechanical systems undergoing to periodic motion in which the force changes slightly over a period, approximate constants do exist➔ the adiabatic invariants

A more powerful approach: adiabatic invariants in B fields

The classical Hamilton-Jacobi theory defines adiabatic invariants for periodic motion: the action-angle variables

$$\mathbf{J_i = \oint p_i dq_i}$$

With p_i and q_i action angle variables canonically conjugated and the integral is taken over a full period of motion

dJ/dt~0 provided that changes in the variables occur slowly compared to the relevant periods of the system and the rate of change is constant

Because there are 3 periodic motions, 3 adiabatic invariants can be defined

For a charged particle in a magnetic field, the conjugate momentum is **P**=**p** + **qA** , with **A** vector potential of magn field

Simple example→ Mechanical pendulus: if the lenght increases only weakly during one swing, then Energy x Period, E•T, is a quasi-constant of motion, i.e. an adiabatic invariant