

Lecture 16 071218

Il pdf delle lezioni puo' essere scaricato da
[http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/
cosmic_rays1819/](http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1819/)

Equazione di propagazione (1)

$$\frac{\partial N_i}{\partial t} - \vec{\nabla} \cdot (\hat{D}_i \vec{\nabla} N_i) + \frac{\partial}{\partial E} (b_i N_i) + m v \sigma_i N_i + \frac{N_i}{\tau_i} = q_i + \sum_{j < i} m v \sigma_{ij} N_j + \sum_j \frac{N_j}{\tau_{ji}}$$

- I CR non sono accelerati nell'ISM, sono accelerati da sorgenti puntiformi
- La loro potenza e distribuzione spazio-temp. e' descritta dalle funzioni $q_i(t, \vec{r}, E)$ (per la specie i)

- * $\hat{D}_i(\vec{r}, E)$ e' il tensore di diffusione
- * $b_i(\vec{r}, E)$ caratterizza le perdite continue di energia delle singole particelle
cosi' che $dE/dt = b_i$ ($\equiv$ ionizz. + brems + sincrotr + Compton inverso)
- * $\sigma_i(E)$ e' la sez. d'urto inelastica del nucleo i con i nuclei dell'ISM
- * $n(\vec{r})$ e' la densita' dell'ISM
- * σ_{ij} e' la sez. d'urto di prod. di nuclei di tipo i da nuclei piu' pesanti
- * τ_i e' la vita media rispetto a decad. radioatt.
- * N_i/τ_{ji} descrive l'apparizione di nuclei di tipo i a causa del decad. di altri nuclei

Formation and Interactions of CR's

Energy Supply: gravitational, nuclear, ELM,...

Provide energy for particles

Shock & Hydromagnetic waves, ELM Fields, Turbulent B fields,...

Store and transport energy

Processes transfer a fraction of E to particles: injection and acceleration

Relativistic particles = Cosmic rays

Particles interact with

Matter

"interstellar,
intergalactic medium"

B Fields

Photons

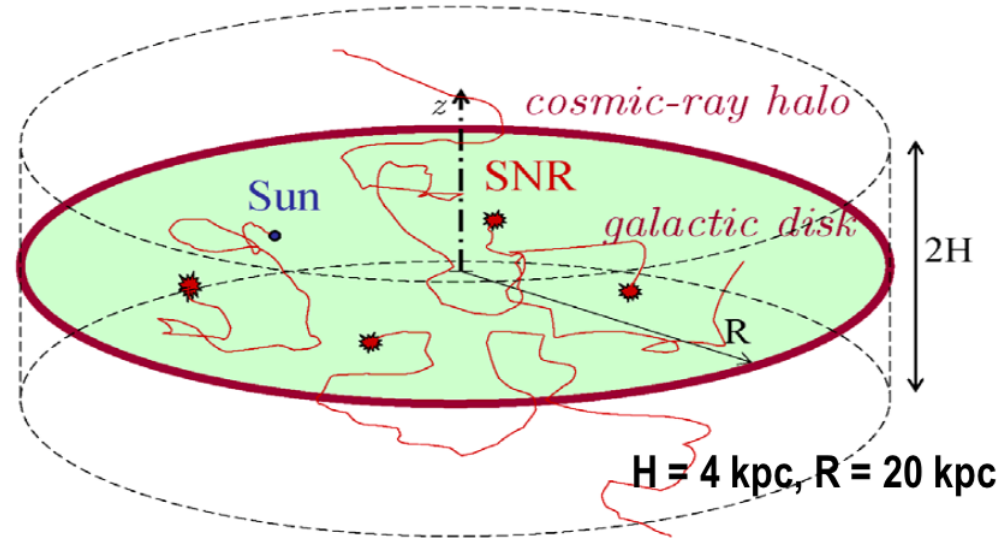
Ionization
Nuclear interactions
Bremsstrahlung

Synchrotron &
curvature radiation

Inverse Compton &
Thomson Scattering
Self-Absorption

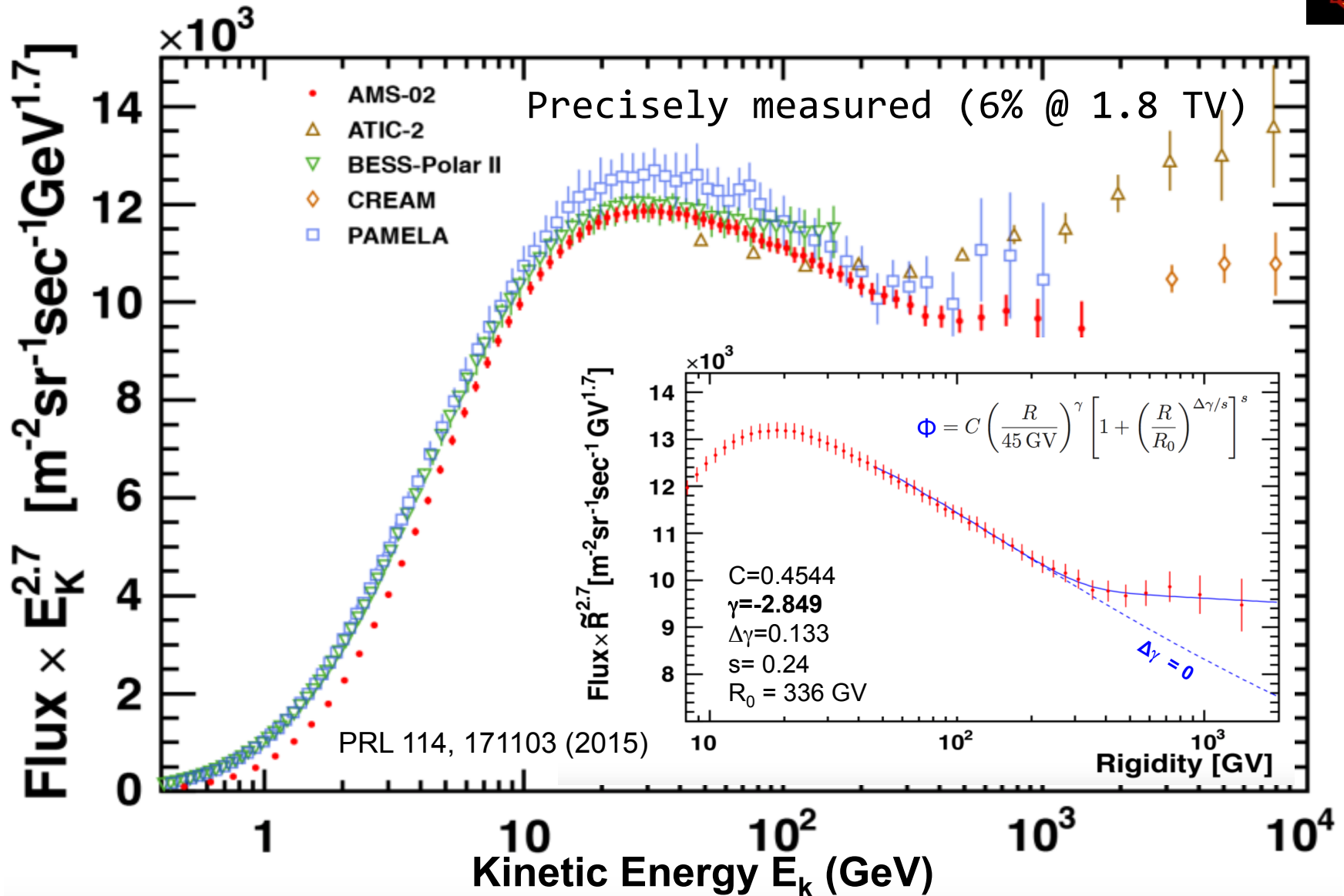
Formation and Interactions of CR's

- **Particles get accelerated at astrophysical sources**
- They leave sources and propagate in the ISM
- They interact with the ISM particles and/or decay producing new particles
- They loose energy in the ISM by elm processes (brems, IC, sync)
- They are bent randomly by ISM magnetic field
- Until they reach the galaxy border and escape the galaxy
- Therefore: Their spectrum and composition at the Earth location are not representative of the source spectrum





Proton flux

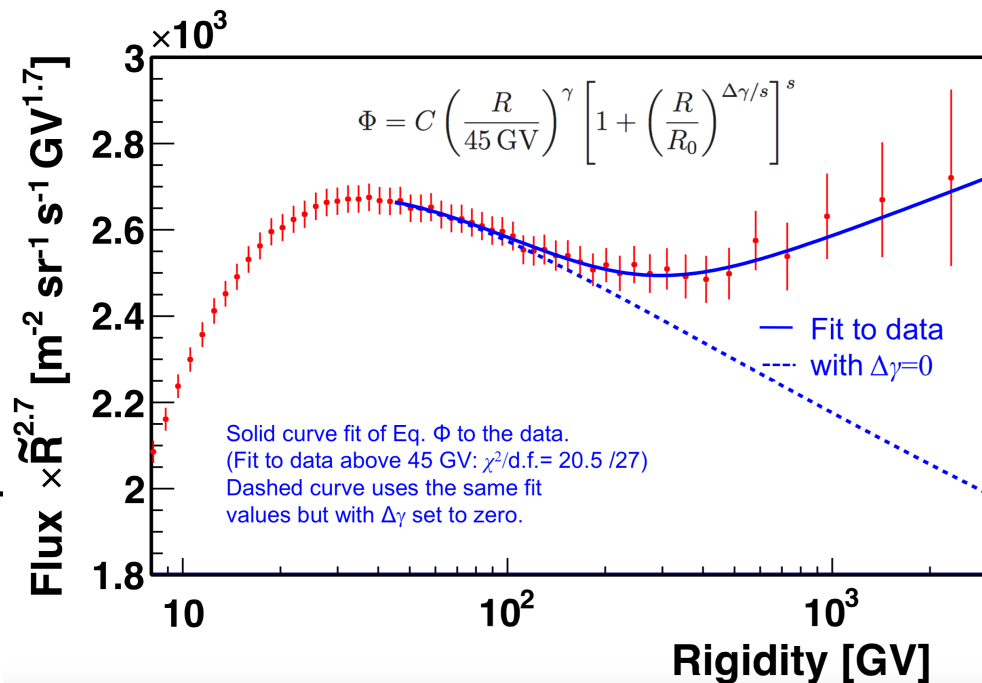
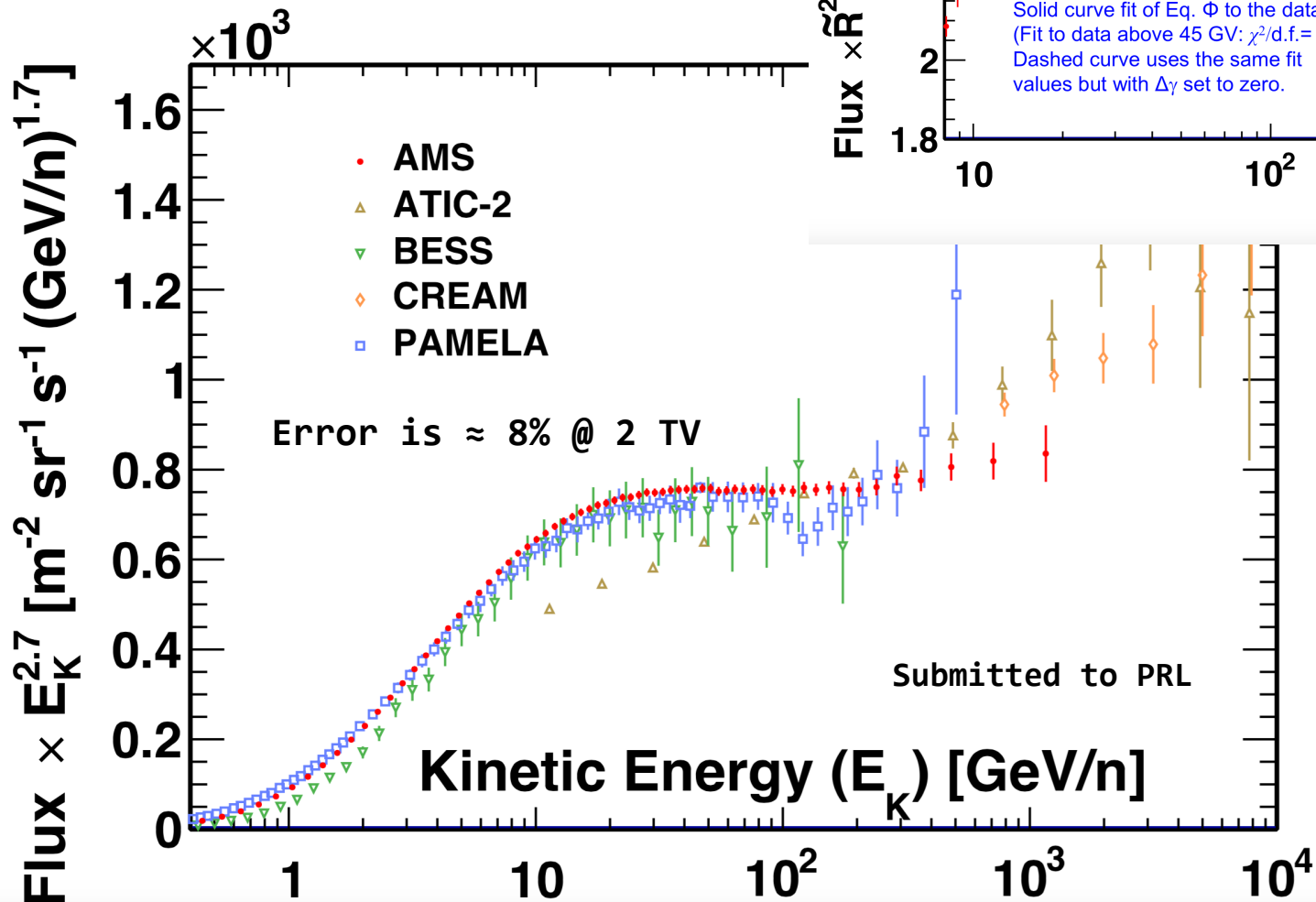


A change in the the spectral index is observed at 336 GV

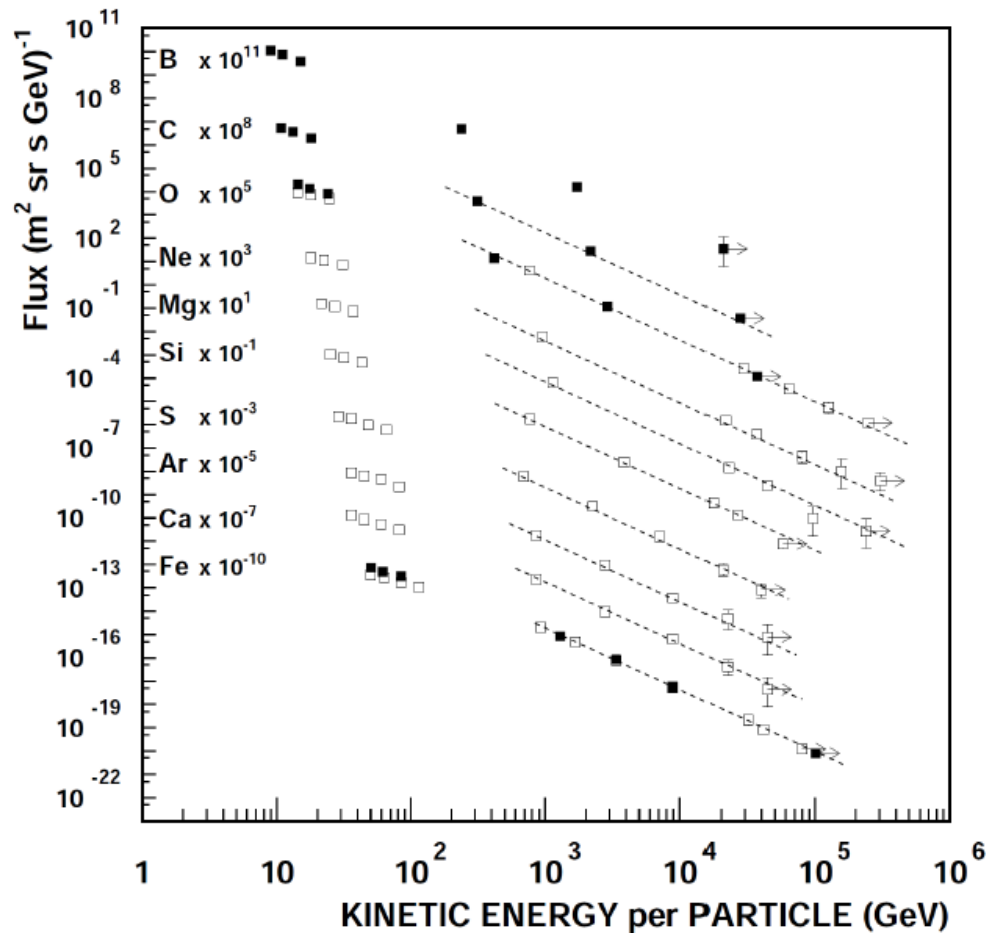


He Flux

Transition energy R_0 and smoothness parameter s are the same as for protons



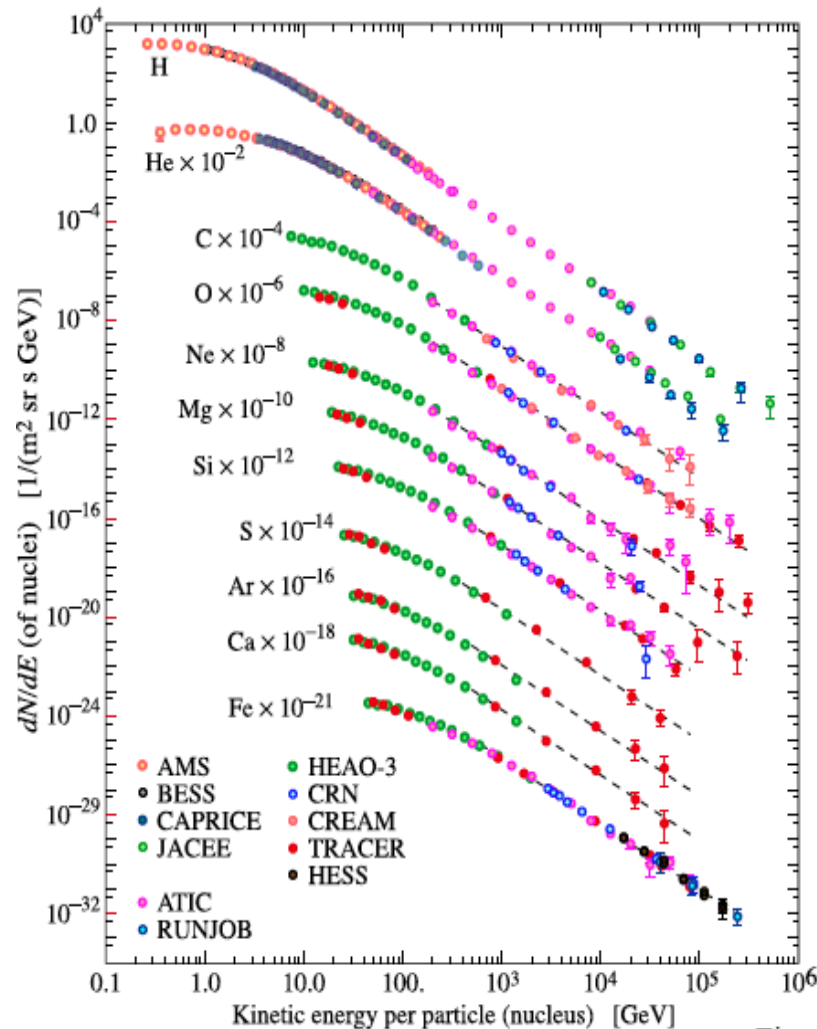
Nuclear species flux



TRACER 2 flights data:
POWER_LAW fit above 20 GeV/n:

$$\text{index } 2.65 \pm 0.05$$

Comparison between experimental results



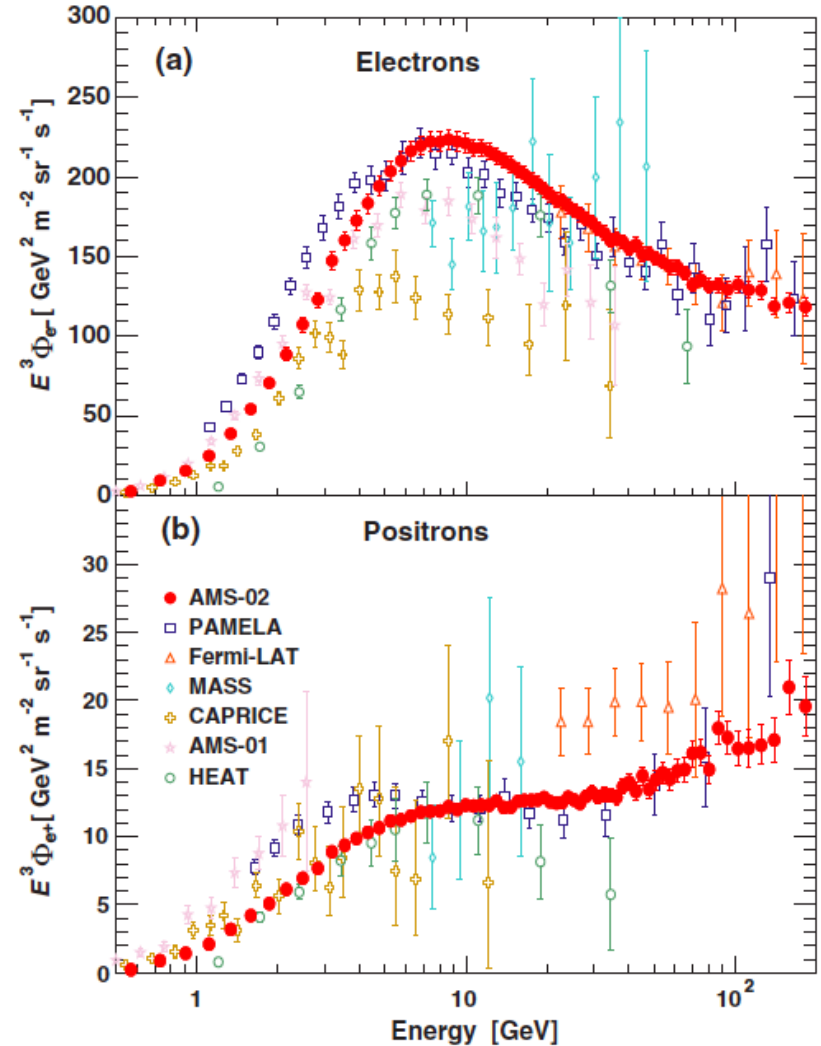
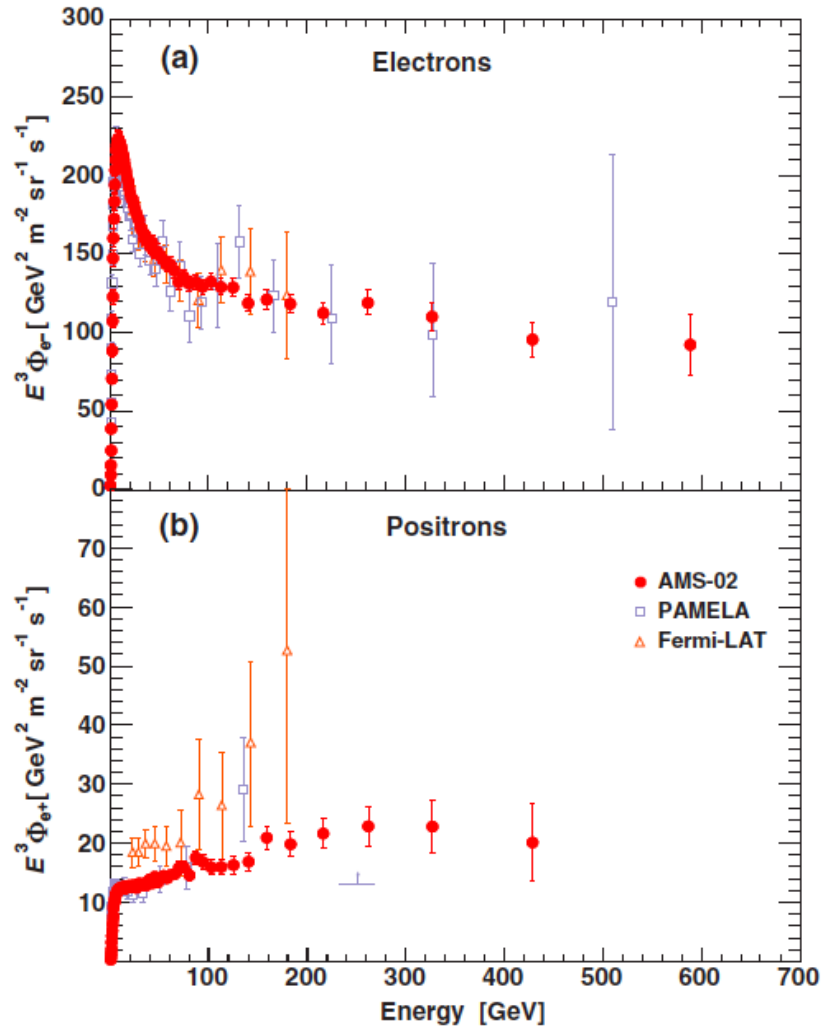
QUESTION:

Are all the hi-Z spectra simple power laws or is there beginning to be evidence for evolution (hardening), as now seems to be the case for H and He?

Figure courtesy of P.J. Boyle and D. Mueller

Electron/Positron fluxes:

No sharp structures



Accelerazione di particelle

- L'accelerazione di RC da parte di un qualche meccanismo Galattico (o extragalattico) deve tener conto dei seguenti fatti sperimentali:
 - Lo spettro di potenza (per tutti i RC) del tipo $dN/dE \sim E^{-\gamma}$ con $\gamma=2.7$ per p sino a $E \sim 4 \times 10^{15}$ eV e $\gamma \approx 3.1$ per e^+ ed e^-)
 - lo spettro alle sorgenti ha un indice spettrale prossimo a 2
 - L'energia massima misurata ($E \sim 10^{20}$ eV)
 - Le abbondanze relative tra gli elementi, tenendo conto degli effetti di propagazione

Confinamento:

$$r_{Larmor} = \frac{1}{300} \frac{E}{ZB} (eV / Gauss) [cm]$$

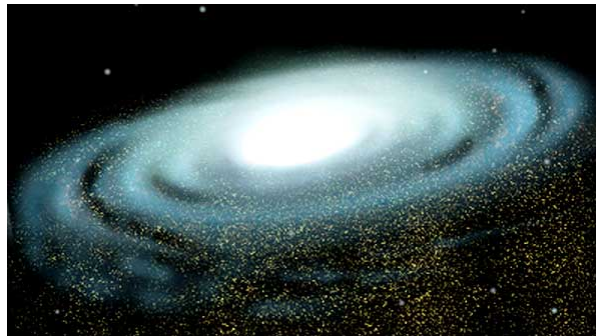
- Utilizziamo i valori tipici del campo B (3×10^{-6} G) galattico per protoni:

$$r_L = \begin{cases} (E = 10^{12} eV) & = 10^{15} cm = 3 \cdot 10^{-4} pc \\ (E = 10^{15} eV) & = 10^{18} cm = 0.3 pc \\ (E = 10^{18} eV) & = 10^{21} cm = 300 pc \end{cases}$$

- I p hanno un raggio di Larmor sempre minore dello spessore del disco galattico (300 pc) se $E < 10^{18}$ eV. Per questo motivo tutti i RC (meno quelli di energia estrema) sono *confinati* nel piano Galattico dal campo magnetico.

Isotropia dei RC

- I RC primari al di sotto di 10^{18} eV hanno una distribuzione di arrivo completamente isotropa sulla sommità della nostra atmosfera. **Qualè il motivo?**
 - Campi magnetici **galattici**:
 - $B \cong 3 \times 10^{-6}$ G
 - coerenti su scale di distanza 1-10 pc
 - NOTA: $1 \text{ pc} = 3 \times 10^{18} \text{ cm}$
 - **Galassia** $\equiv$ disco di raggio $R = 15 \text{ kpc}$, spessore $h = 300\text{-}1000 \text{ pc}$

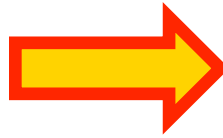


Richiamo: moto di un RC nel campo magnetico Galattico

$$mv^2 / r = pv / r = ZevB / c$$

$$r = pc / ZeB$$

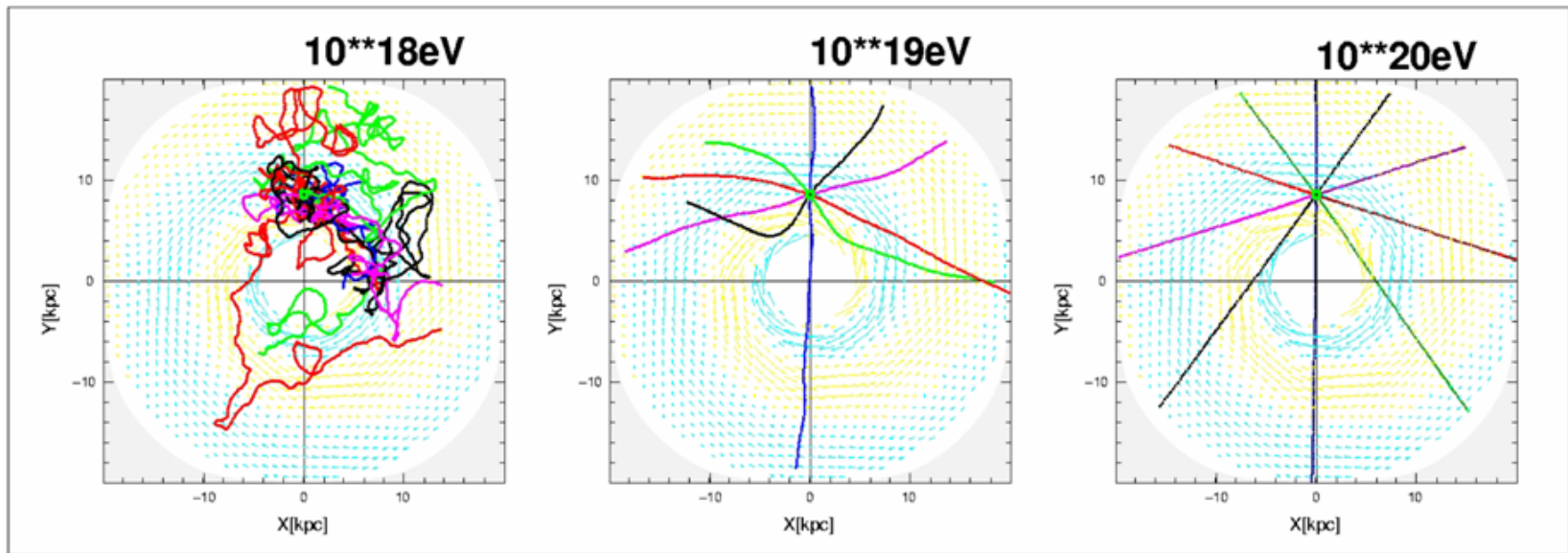
$$r(cm) = \frac{1}{300} \frac{E(eV)}{ZB(G)}$$



$$(10^{12} eV) = 10^{15} cm = 3 \times 10^{-4} pc$$

$$r = (10^{15} eV) = 10^{18} cm = 3 \times 10^{-1} pc$$

$$(10^{18} eV) = 10^{21} cm = 300 pc$$



Le particelle diffondono e vengono isotropizzate dalle irregolarita' del campo magnetico grazie a risonanze (es. di ciclotrone) quando $\rho_g \sim \lambda$ delle onde di alfvén con una lunghezza di diffusione $\lambda_{sc} = \rho_g (B_o / \delta B(k))^2$ con $\delta B(k) = \int w(k) dk$, $w(k) \sim k^{-\alpha}$ densita' spettrale di potenza

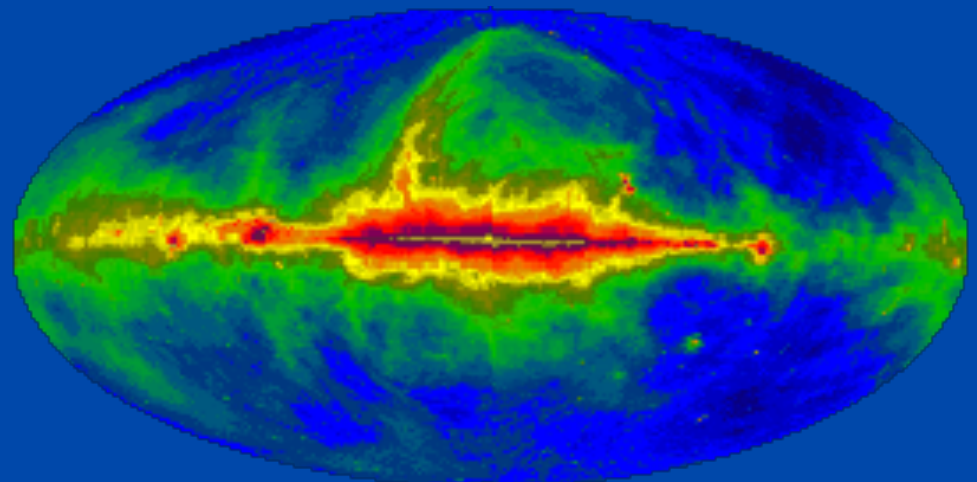
Il processo e' quindi diffusivo con $D = c\lambda_{sc}/3 \sim D_o (R/R_o)^{\alpha-2}$

Volume di confinamento

- Le particelle attraversano in media $\xi = 50 \text{ kgm}^{-2}$ di ISM
- $\xi = \rho c \tau \rightarrow$ otteniamo $\rho < 1 \text{ pcm}^{-3}$ tipico del disco galattico
- Questo implica che le part viaggino anche in regioni meno dense di quelle tipiche del disco (dove le abbondanze non cambiano) con un percorso casuale "tortuoso" per via delle irregolarita' del campo B galattico. Si puo' dire che esse non sono libere di sfuggire liberamente
- E' importante quindi stimare quanto e' grande il volume in cui i CR sono confinati

Volume di confinamento (2)

- Il volume puo' essere:
 - il disco della galassia, cioe' approx un disco di raggio 10-15 kpc e spessore 300-500 pc
 - galassia + alone, una regione meno densa di raggio ~ 15 kpc che circonda il disco di forma sferico-ellissoidale → favorito dalle misure sperimentali



Other evidence of galactic halo: 408 MHz map of the sky :
synchrotron emission
of few GeV electrons in the galactic magnetic field₁₅

Potenza delle sorgenti dei RC

- Il *confinamento* dei RC ci induce a sospettare che le sorgenti siano di origine Galattica (tranne che per i RC di energia estrema).
- Qual è *l'energetica* delle sorgenti? (necessaria per individuarle).
 - Il tempo di confinamento dei RC: $\tau = 3 \times 10^7$ y (calc. Fra poco)
 - Volume della galassia (con o senza alone) :

$$V_G = (15 \text{ kpc})^2 \cdot \pi \cdot 300 \text{ pc} \cong 6 \times 10^{66} \text{ cm}^3$$

$$V_G^{\text{Alone}} = 4/3 \cdot \pi (10 \text{ kpc})^3 \cong 10^{68} \text{ cm}^3$$

- Potenza necessaria per mantenere uno *stato stazionario* di RC:

$$W_{CR} = \frac{w_{CR}(\text{erg} / \text{cm}^3) \cdot V_G^A(\text{cm}^3)}{\tau(s)} = \frac{10^{68} \times 1.6 \cdot 10^{-12}(\text{erg})}{3 \cdot 10^7 \times (3.15 \cdot 10^7 \text{ s})} \cong 10^{41} \frac{\text{erg}}{\text{s}}$$

Esiste un meccanismo con una potenza tale da sostenere il flusso dei RC nella Galassia?

- Una esplosione di Supernova libera:

- $10^{51} \text{ erg/esplosione}$

- La stima della frequenza di SN nella nostra Galassia è

- $f_{\text{SN}} = 1/\tau_{\text{SN}} = 1/30 \text{ y}^{-1}$

- Si noti che $\tau_{\text{SN}} < \tau \approx 10^7 \text{ y}$. Le SN sono un fenomeno quasi continuo su scala dei tempi del confinamento dei RC.

- Potenza energetica liberata dalle SN:

$$W_{\text{SN}} = \frac{10^{51} \text{ erg}}{30 \times 3.15 \cdot 10^7 \text{ s}} \approx 10^{42} \text{ erg / s}$$

- Perché il quadro sia coerente, occorre trovare un meccanismo che trasferisca al più il 10% di energia dalle supernovae in energia cinetica di particelle (i RC) → **Meccanismo di Fermi**

Leaky Box Model

A useful approximation is the following:

- Assume there is no diffusion (eg $D=0$)
- CR propagate freely in the galaxy volume, uniformly filled with ISM and regular B, until they reach the “border” and escape
- A CR has a probability per unit of time to escape the galaxy $p = 1/\tau_{\text{esc}}$, where τ is the measured residence time of CR in the galaxy

Then

$$D\nabla^2 \mathcal{N} \rightarrow -\frac{\mathcal{N}}{\tau_{\text{esc}}}$$

As the diffusion coefficient D is energy dependent, also the characteristic escape time of CRs from the Galaxy $\tau_{\text{esc}} = \tau_{\text{esc}}(E)$ is energy dependent

Leaky Box Model

- The transport equation becomes

$$\frac{d\mathcal{N}_i}{dt} = -\frac{\mathcal{N}_i}{\tau_{\text{esc}}} + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i} \frac{P_{ji}}{\tau_j} \mathcal{N}_j.$$

- The leaky-box model provides the most common description of CR transport in the Galaxy at energies below $\sim 10^{17}$ eV. The model is based on particles injected by sources Q distributed uniformly over the galactic volume (the *box*) filled with a uniform distribution of matter and radiation fields. The particles get-away from this volume with an escape time independent of their position in the box. The escape time $\tau_{\text{esc}}(E)$ depends on the particle energy, charge, and mass number, but it does not depend on the spatial coordinates. Secondary nuclei are produced during the propagation as a function of the path length

Spettro dei RC alle sorgenti

- Il modello Leaky Box permette di collegare lo spettro osservato a Terra con quello alle sorgenti
- In assenza di tutti i processi e in uno stato stazionario, tranne che la fuga dalla galassia, il modello ci da

$$0 = Q(E) - \frac{N(E)}{\tau(E)}$$

$$N(E) = Q(E)\tau(E)$$

- In questa approssimazione, lo spettro a Terra e' quello delle sorgenti $Q(E)$ modificato dall'effetto della propagazione, descritta dal termine $\tau(E)$

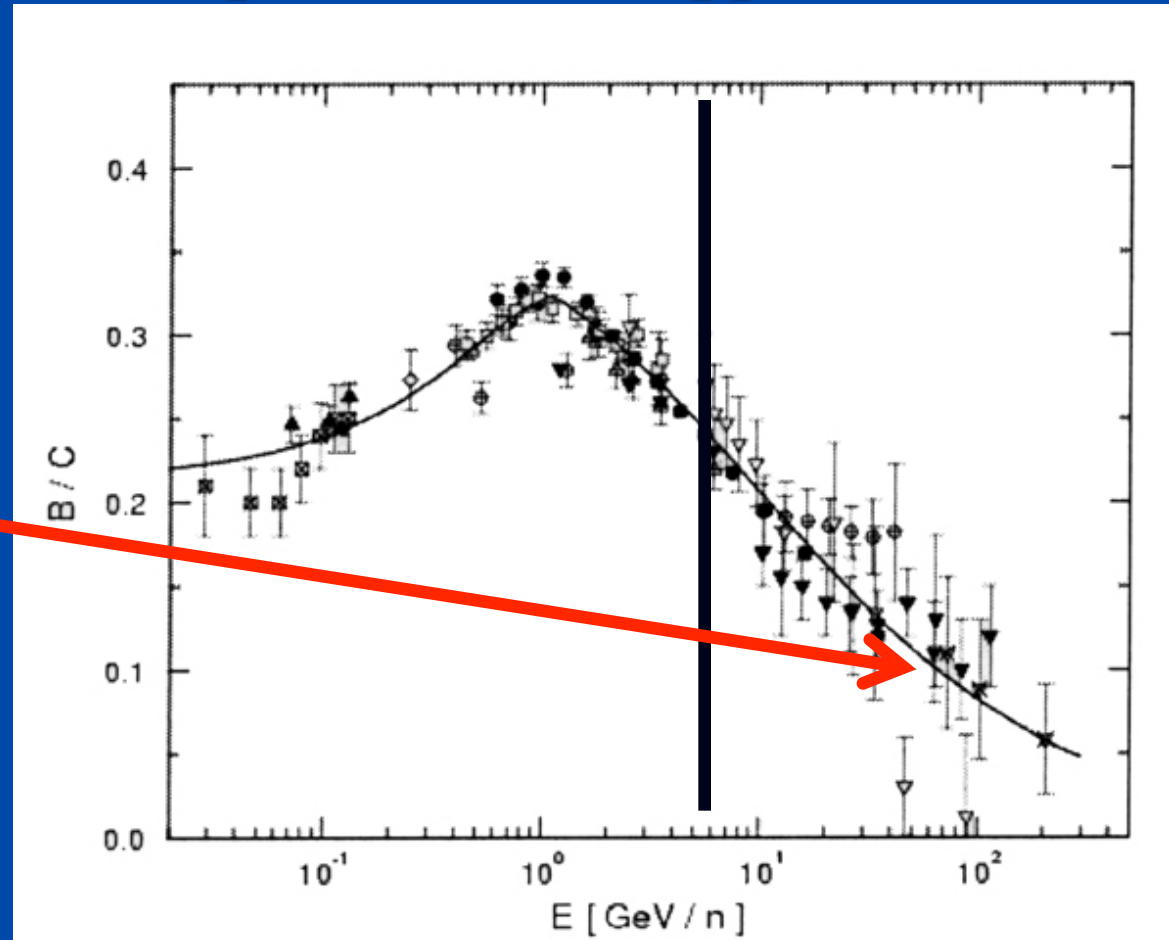
Dipendenza del rapporto r vs. E

- I dati sperimentali confermano questa ipotesi.
- In particolare, si ottiene che la probabilità di fuga dalla Galassia dipende dall'energia come:

$$\tau = \tau_o \left(\frac{R}{R_o} \right)^{-\delta}$$

- Ossia, poiché $\tau \sim \xi$

$$\xi = \xi_o \left(\frac{R}{R_o} \right)^{-\delta}$$



NB: $\delta = \alpha - 2$ e $D \sim H^2/\tau$, H spessore dell'alone, perciò la misura di δ da informazioni sulla natura della turbolenza del campo magnetico a queste scale di energia

Valori tipici $\xi_o = 11.8 \text{ gr/cm}_2$, $R_o = 5 \text{ GV/c}$, $\delta = 0.6$

L'indice spettrale δ ha un'incertezza piuttosto grande. Il suo valore è compreso in un intervallo 0.15-0.8 a seconda del modello. Per esempio un regime in cui D o τ dipendono dalla posizione, come modelli a doppia regione (disco+alone), effetti non lineari,...

Spettro dei RC alle sorgenti

- Il risultato appena ottenuto è estremamente importante, perché permette di avere informazioni sullo spettro energetico dei RC alle sorgenti.
- Poiché il flusso dei RC sulla Terra è stazionario, vi deve essere equilibrio tra:
 - Spettro energetico misurato: $\Phi(E) \propto E^{-2.7} (erg / cm^3 \cdot GeV)$
 - Spettro energetico alle Sorgenti: $Q(E) \propto E^{-?} (erg / s \cdot GeV)$
 - Probabilità di diffusione: $\tau(E) \propto E^{-0.6} (s)$

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

Spettro dei nuclei alle sorgenti

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

- Quindi, inserendo le dipendenze funzionali:

$$Q(E) = \frac{\Phi(E)}{\tau(E)} = \frac{E^{-2.7}}{E^{-0.6}} = E^{-2.1}$$

- Il modello che descrive le sorgenti di RC nella Galassia, dovrà prevedere una dipendenza con l'energia del tipo $\sim E^{-2}$.
- Occorre trovare un processo che produca uno spettro di questo tipo alla sorgente.
- Il modello di Fermi prevede proprio un andamento funzionale di questo tipo!

Elettroni e positroni

$$\frac{\partial N_i}{\partial t} - \vec{\nabla} \cdot (\hat{D}_i \vec{\nabla} N_i) + \frac{\partial}{\partial E} (b_i N_i) + m v r_i N_i + \frac{N_i}{\tau_i} = q_i + \sum_{j < i} m v b_{ij} N_j + \sum_j \frac{N_j}{\tau_{ji}}$$

σ_i e' la sez d'urto per il processo $p_{CR} p_{ISM} \rightarrow \pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$ ed n e' la densita' di p nell ISM, N_i e' la densita' di elettroni

Da dove vengono gli elettroni?

Da interazione dei RC con i protoni del mezzo interstellare

Da sorgenti primarie, cioe' siti di accelerazione

Da sorgenti esotiche(?)

Per gli elettroni, a differenza della componente nucleare, sono importanti i processi di perdita di energia durante la propagazione. Il termine che descrive le perdite di energia continue non puo' essere trascurato ma anzi diventa dominante

Toy Model

o Caso semplice di: - soluz. stazionarie $dN/dt = 0$

x C'è una distrib. infinita e uniforme di sorgenti che iniettano e^- con spettro di iniezione $Q(E) = k E^{-p}$

$\Rightarrow$ La diffusione non è importante $\Rightarrow$ L'equ. diventa
$$-\frac{d}{dE}[b(E)N(E)] = Q(E) \Rightarrow \int d[b(E)N(E)] = - \int Q(E) dE$$

x Assumiamo $N(E) \rightarrow 0$ per $E \rightarrow \infty \Rightarrow$

$$N(E) = \frac{kE^{-(p-1)}}{(p-1)b(E)} \quad \text{dove } b(E) = A_1 \left(\ln \frac{E}{mc^2} + 19.8 \right) + A_2 E + A_3 E^2$$

$\Rightarrow$ Se domina ionizz. $N(E) \propto E^{-(p-1)}$. spettro di E più piatto

Se " Brems $N(E) \propto E^{-p}$. lo spettro non cambia
(o adiab. loss)

Se IC o sincr. dominano $N(E) \propto E^{-(p+1)}$ spettro più ripido

■ La diffusione non è importante perché se c'è una distribuzione infinita e uniforme di sorgenti non ci sono gradienti di densità spaziale, a regime

Spettro degli e⁺,e⁻ alle sorgenti

$$\frac{4\pi}{c} \int \Phi(E) dE = \int \frac{Q(E) \cdot \tau(E)}{Volume} dE$$

- Quindi, inserendo le dipendenze funzionali:

$$Q(E) = \frac{\Phi(E)}{\tau_{loss}(E)} = \frac{E^{-3.1}}{E^{-1.0}} = E^{-2.1}$$

- Anche per gli elettroni di alta energia lo spettro alle sorgenti e' compatibile con -2

o Nel caso di diffusione, le particelle si spostano in media di $\lambda \approx (2D\tau)^{1/2}$

o In generale $D = D(E) \Rightarrow \lambda(E, E_0) = \left(\int_0^{\tau(E)} D(E') d\tau' \right)^{1/2} = \left(\int_{E_0}^E \frac{D(E') dE'}{b(E')} \right)^{1/2}$

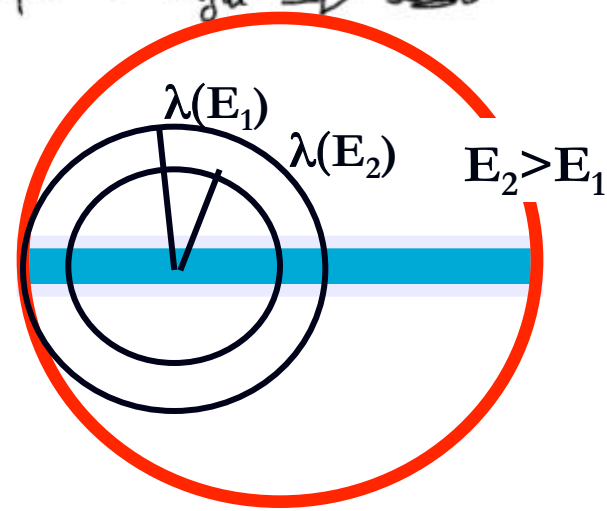
x $D \approx 10^{29} \text{ cm}^2 \text{ s}^{-1}$ (val. medio galattico) $\Rightarrow \lambda \approx 10 \text{ kpc}$ @ 1 GeV
(solo 10 + Sincro i.e. $\tau = \frac{1}{KE_0}$)

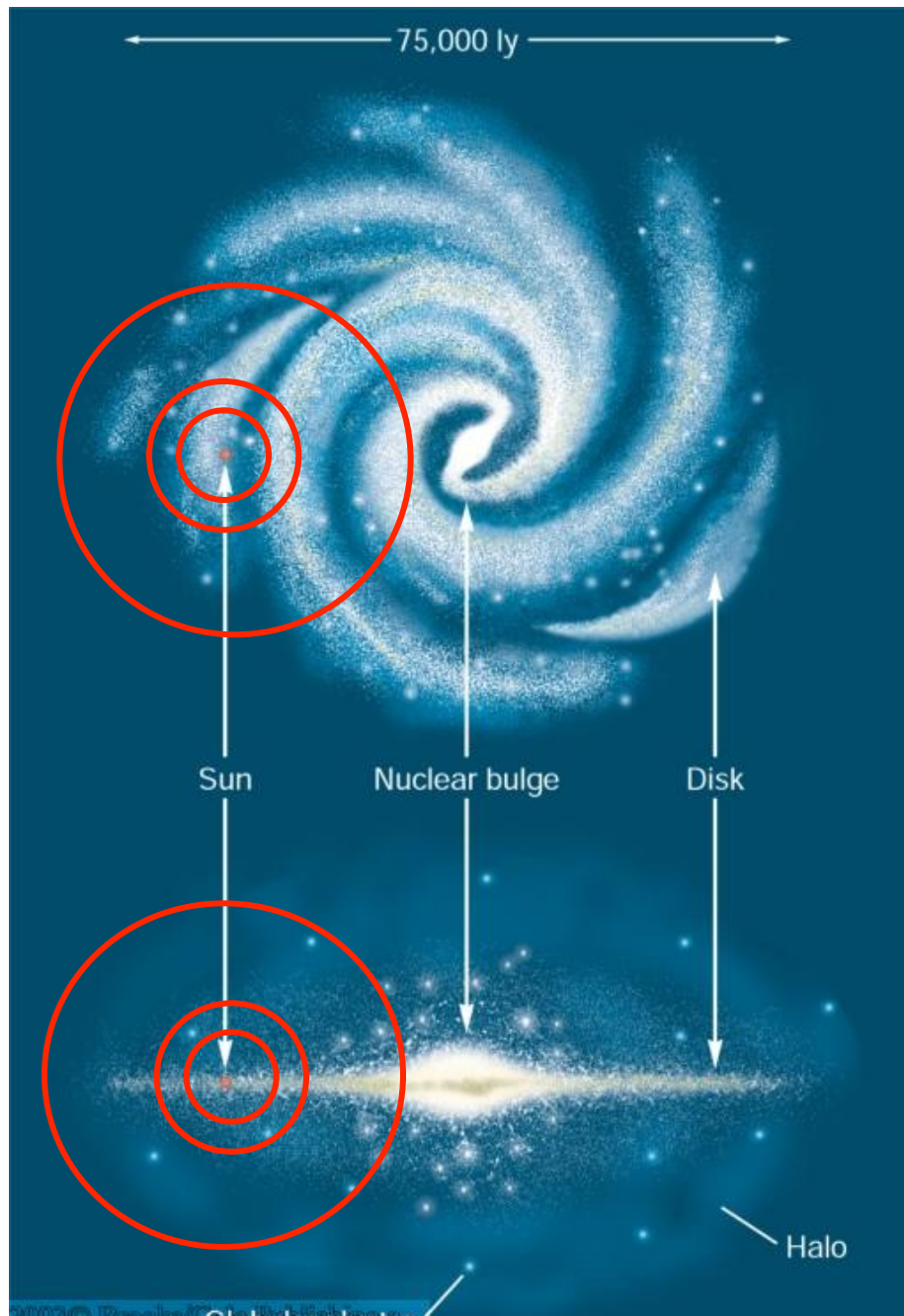
x Trascurando la dipend. di D da E $\lambda \propto E^{-1/2} \Rightarrow$
Lo spettro degli $e^\pm$ nei differenti intervalli di energia dipende alle condizioni medie di propagazione nell'ISM in una regione di raggio $R \sim \lambda(E)$

x Esempio: per e^- con $E > 1 \text{ GeV}$, $\lambda(E) \lesssim 10 \text{ kpc} = R_{\text{gal}} \Rightarrow$ ~~solo~~

$\Rightarrow$ Gli e^- sono galattici.

■ Quindi lo spettro osservato cambia parecchio durante la propagazione





$\approx 10^2$ ly @ 1 TeV

$\approx 3.16 \times 10^3$ ly @ 100 GeV

$\approx 10^4$ ly @ 10 GeV

$\approx 3 \times 10^4$ ly @ 1 GeV

Gli elettroni
vengono da vicino!

SNRs vicine

Vicino al noi ci sono numerose potenziali sorgenti di elettroni

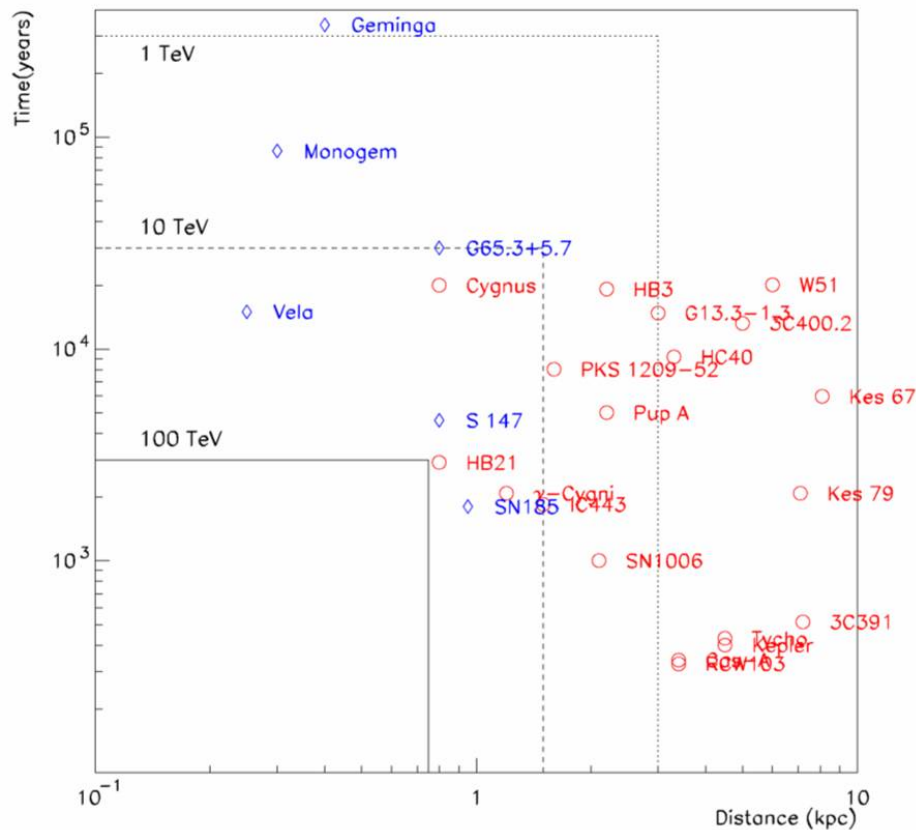


TABLE 1
LIST OF NEARBY SNRs

SNR	Distance (kpc)	Age (yr)	$E_{\max}^a$ (TeV)	Reference
SN 185.....	0.95	1.8×10^3	1.7×10^2	1
S147.....	0.80	4.6×10^3	63	2
HB 21.....	0.80	1.9×10^4	14	3, 4
G65.3+5.7.....	0.80	2.0×10^4	13	5
Cygnus Loop.....	0.44	2.0×10^4	13	6, 7
Vela.....	0.30	1.1×10^4	25	8
Monogem.....	0.30	8.6×10^4	2.8	9
Loop I.....	0.17	2.0×10^5	1.2	10
Geminga.....	0.4	3.4×10^5	0.67	11

^a Maximum energy limited by the propagation of electrons in the case of the prompt release after the explosion. The delay of the release time gives the larger value.

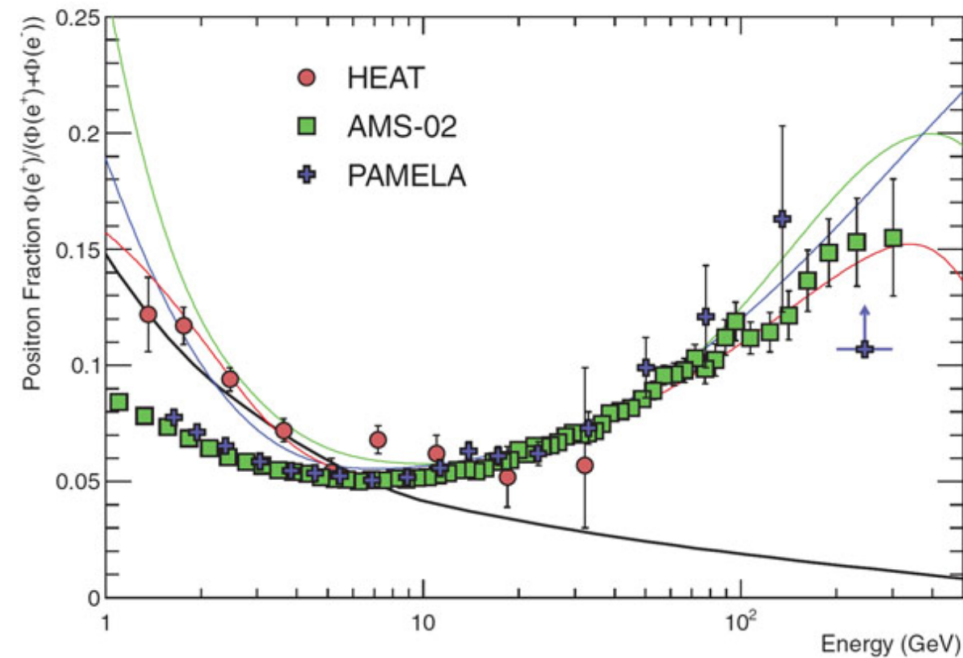
REFERENCES.—(1) Strom 1994; (2) Braun et al. 1989; (3) Tatematsu et al. 1990; (4) Leahy & Aschenbach 1996; (5) Green 1988; (6) Miyata et al. 1994; (7) Blair et al. 1999; (8) Caraveo et al. 2001; (9) Plucinsky et al. 1996; (10) Egger & Aschenbach 1995; (11) Caraveo et al. 1996.

Eccesso rispetto a cosa?

Produzione secondaria nell'ISM di e^+ ed e^-

Sorgenti primarie di e^- : $n_-(E) = K_{ep} N_{cr}(E) + \sigma_{pe^-} n_{ISM} c n_{cr}(E)$, $K_{ep} \approx 10^{-3}$

No sorgenti primarie di e^+ : $n_+(E) = \sigma_{pe^+} n_{ISM} c n_{cr}(E)$



Leaky box equilibrium density $n_{cr}(E) = N_{cr}(E) \tau_{esc}(E)$, $\tau_{esc}(E) = \tau_0 E^{-\delta}$

$$\frac{n_-(E)}{n_+(E)} = \frac{(K_{ep} N_{cr}(E) + \sigma_{pe^-} n_{ISM} c N_{cr}(E) \tau_{esc}(E))}{\sigma_{pe^+} n_{ISM} c N_{cr}(E) \tau_{esc}(E)} = \frac{\sigma_{pe^-}}{\sigma_{pe^+}} + \frac{K_{ep}}{\tau_{esc}(E)}$$

$$\frac{n_+(E)}{(n_+(E) + n_-(E))} = \frac{1}{(1 + n_-(E)/n_+(E))} = \frac{1}{(1 + \sigma_{pe^-}/\sigma_{pe^+} + K_{ep}/\tau_{esc}(E))}$$

$$\sigma_{pe^-}/\sigma_{pe^+} \approx 0.5 - 0.3 \rightarrow \frac{n_+(E)}{(n_+(E) + n_-(E))} \approx \frac{1}{(1.5 + (K_{ep}/\tau_0) E^\delta)}$$

→ Senza produzione primaria di e^+ alle sorgenti la frazione decresce con l'energia

Comparing our data with a minimal model, as an example.

In this model the e^+ and e^- fluxes, Φ_{e^+} and Φ_{e^-} , are parametrized as the sum of individual diffuse power law spectra and the contribution of a single common source of $e^\pm$:

$$\Phi_{e^+} = C_{e^+} E^{-\gamma_{e^+}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad \blacksquare \text{ pulsar source} \quad \text{Eq(1)}$$

$$\Phi_{e^-} = C_{e^-} E^{-\gamma_{e^-}} + C_s E^{-\gamma_s} e^{-E/E_s} \quad (E \text{ in GeV}) \quad \text{Eq(2)}$$

Coefficients C_{e^+} and C_{e^-} correspond to relative weights of diffuse spectra for positrons and electrons.

C_s is the weight of the source spectrum.

γ_{e^+} , γ_{e^-} and γ_s are the corresponding spectral indexes.

E_s is a characteristic cutoff energy for the source spectrum.

With this parametrization the positron fraction depends on 5 parameters.

Possiamo dare un quadro generale degli spettri alle sorgenti:

Per la componente nucleare $p \approx 2$

Per la componente e^+e^- $p \approx 2.1$

Fino ad almeno qualche centinaio di GeV

Gli spettri alla sorgente sono differenti per e^- e protoni (cum grano salis)

Per entrambe le componenti c'è evidenza sperimentale di hardening dello spettro a qualche $O(100 \text{ GeV})$

Per e^+ ed e^- l'evidenza è netta: c'è una sorgente che immette e^+ ed e^- di alta energia nell'ISM

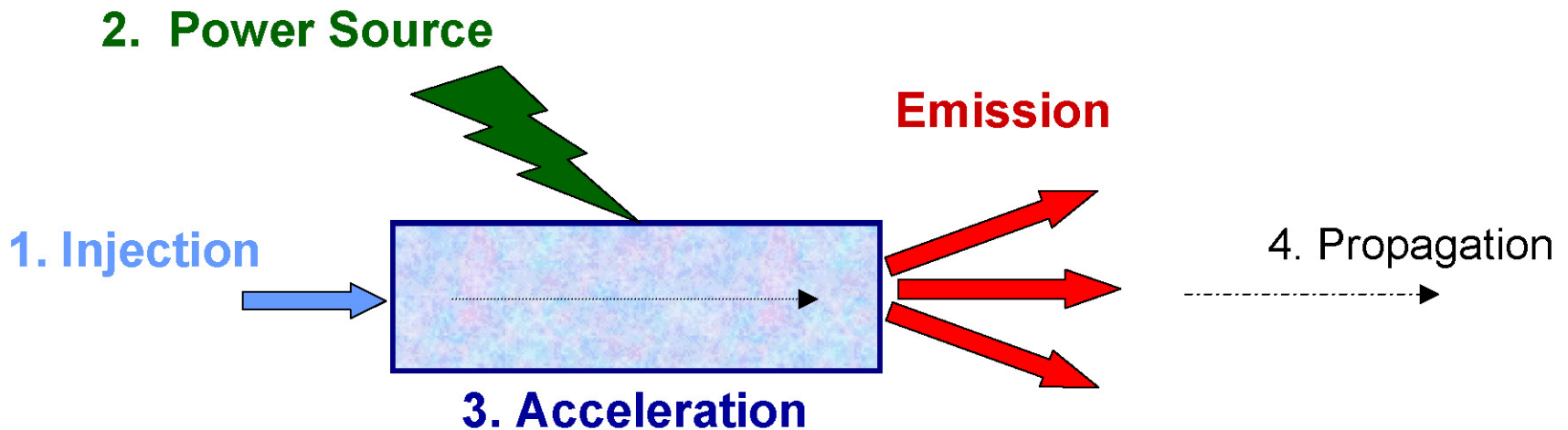
Per i nuclei la situazione è più incerta: nuova classe di sorgenti? Effetto di propagazione?

Servono più dati.

Le pulsar accelerano e^+ ed e^- ma non protoni (per esempio)

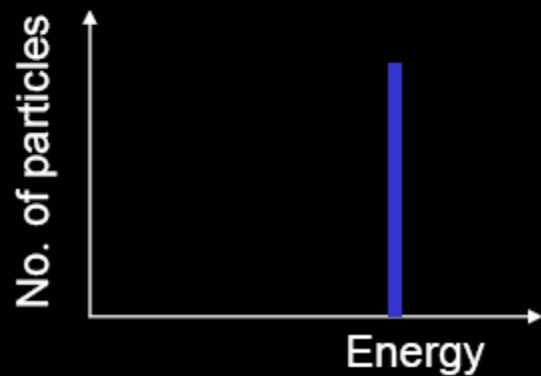
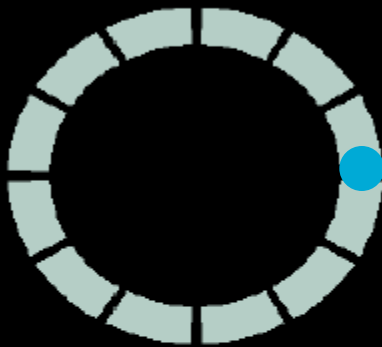
Accelerazione dei raggi cosmici

Per costruire un "acceleratore cosmico" abbiamo bisogno di:



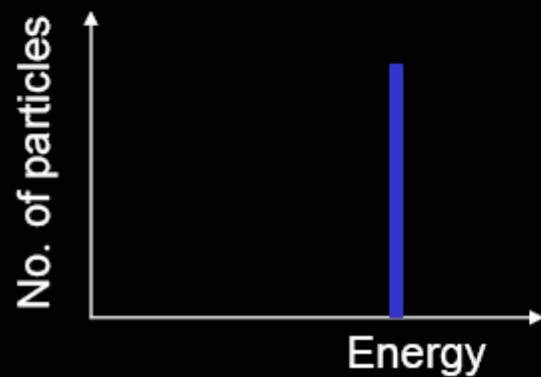
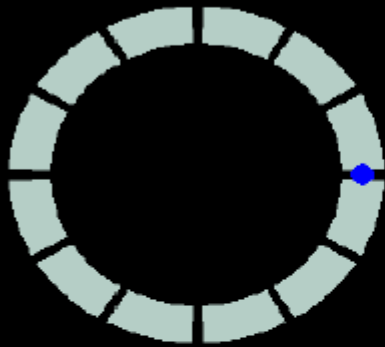
How might such cosmic accelerators work?

Man-made accelerators

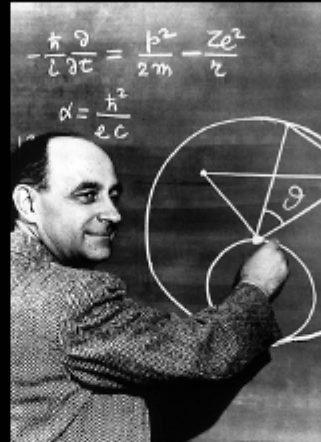


How might such cosmic accelerators work?

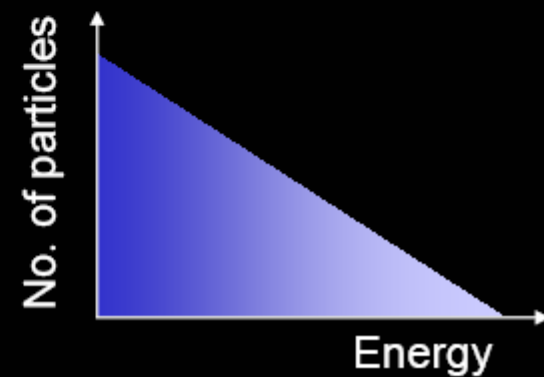
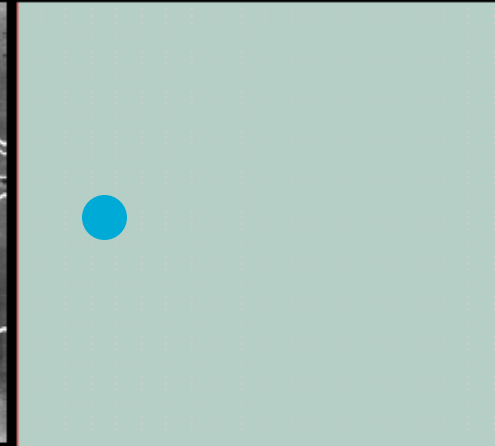
Man-made accelerators



Nature's accelerators



Enrico Fermi



In ambiente astrofisico (in presenza di particelle ionizzate, plasmi) *campi elettrostatici non possono essere mantenuti* a causa dell'alta conducibilità dei plasmi stessi

Sono possibili meccanismi in cui f.e.m. sono prodotte tramite
 $\nabla \times \mathbf{E} = \delta \mathbf{B} / \delta t$

Il meccanismo “idrodinamico” descrive accelerazione stocastica di RC da parte di ripetuti urti delle particelle con un'onda di shock, ad esempio emessa dall'esplosione di una SN.

Questo meccanismo venne utilizzato per i RC per la prima volta da parte di E. Fermi (1949), e prende per questo il suo nome.

Le particelle cariche sono riflesse da “specchi” magnetici dovute alla presenza dell'irregolare campo magnetico galattico.

Ad ogni riflessione, le particelle guadagnano (in media) energia

Il meccanismo predice il corretto andamento del flusso *vs.* E

Electromotive Acceleration

Fundamentally, Lorentz Force law $\Rightarrow$

$$\frac{d}{dt}(\gamma m \vec{v}) = e(\vec{E} + \vec{v} \wedge \vec{B})$$

But, on macroscopic scales, no electrostatic fields, i.e., $\langle \vec{E} \rangle = 0$.

Also, for a stationary $\vec{B}$, $\frac{d}{dt}|\vec{v}| = 0$.

Need “moving” magnetic fields, i.e., acceleration by electromotive force.

Expect, rate of energy gain by relativistic particles = work done by the electromotive force = $\frac{d\mathcal{E}}{dt} = \xi Z e c B . c$, with $\xi < 1$.

Fermi's original suggestion: Encounters of charged particles with moving, magnetized, interstellar clouds.

Electromotive Acceleration

The maximum energy supplied to a particle with charge Ze is

$$E_{\max} = \int Ze \varepsilon dx$$

If the energy is given by induction

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \quad \longrightarrow \quad \frac{\varepsilon}{L} \cong \frac{B \omega_0}{c}$$

ε =electrical field induced in a region of length L

We obtain

$$E_{\max} = \int Ze \cdot \varepsilon \cdot dx = Ze \cdot B \omega_0 L \cdot L / c$$

$$E_{\max} \cong Ze \frac{BL \omega_0}{c} L = Ze \beta_0 BL$$

$$E_{\max} (EeV) \cong Z \beta_0 B (\mu G) L (kpc)$$

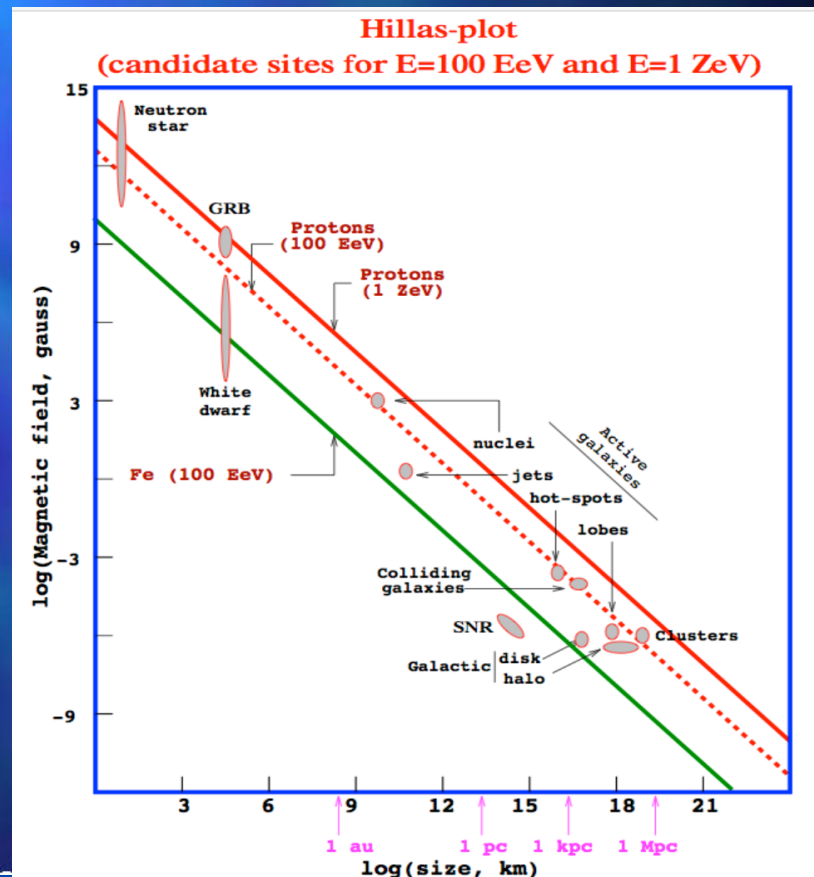
Hillas plot

Irrespective of the precise acceleration mechanism, there is a simple dimensional argument, given by Hillas, which allows one to restrict attention to only a few classes of astrophysical objects as possible sources capable of accelerating particles to a given energy.

In any statistical acceleration mechanism, there must be a magnetic field, B , to keep the particles confined within the acceleration site. The size R of the acceleration region must be larger than the diameter of the orbit of the particle. One gets the general condition

$$E(\text{EeV}) < Z \beta B(\mu\text{G}) R(\text{kpc})$$

The above condition also applies to direct acceleration scenarios in which the electric field arises due to a moving magnetic field.



Magnetic Confinement

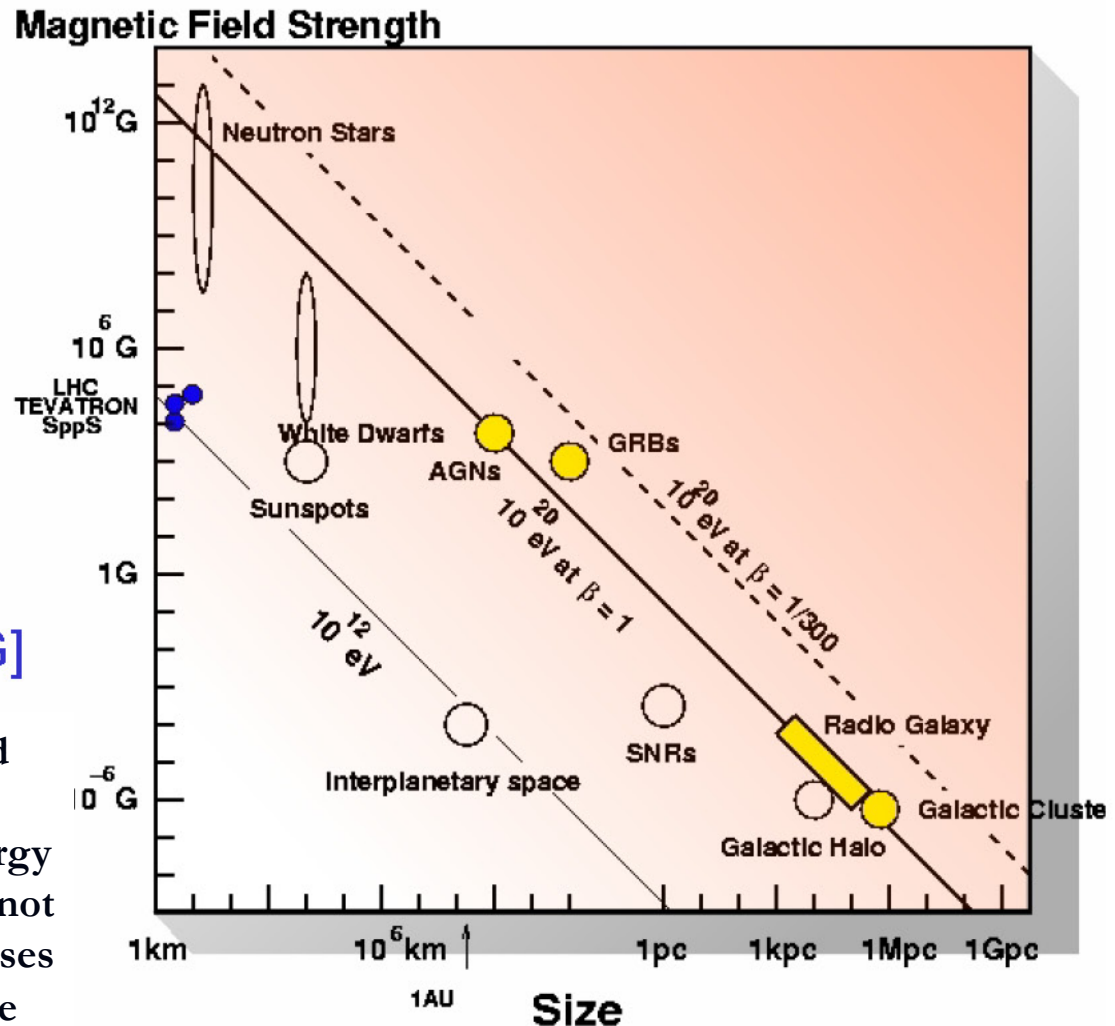
“Hillas Plot”

Minimum size of B field to contain particles being accelerated.

Achievable energy:

$$E [\text{EeV}] \sim Z R [\text{kpc}] B [\mu\text{G}]$$

The Hillas plot must be interpreted carefully. It represents a necessary condition to reach the maximum energy in a cosmic accelerator but it may be not sufficient since there are other processes that prevent to reach it, e.g. radiative losses, escape from acceleration region, age of the source...



■ Fermi Acceleration



- Stochastic Mechanism
- Charged particles collide with clouds in ISM and
 - are reflected from irregularities
- in galactic magnetic field
 - 2nd order

- Charged particles can be accelerated to high energies in astrophysical shock fronts
- 1st order acceleration

Tra i siti possibili di accelerazione dei raggi cosmici possiamo includere (ad energia crescente):

- i venti stellari

- le esplosioni di Supernovae

- le “remnants” di tali esplosioni: stelle di neutroni ruotanti, pulsar con nebulose, ...

- altri oggetti esotici, quali i “mini-black holes”, se esistono.

I raggi cosmici osservati con energie $E > 10^{19}$ eV, potrebbero essere stati accelerati da meccanismi extragalattici, quali jets di nuclei Galattici attivi o GRB

Il meccanismo di Fermi può essere attivo in molte di queste situazioni astrofisiche, e lo analizzeremo in qualche dettaglio per le esplosioni di Supernovae

Il fattore comune a queste classi di sorgenti è la presenza di onde di shock e/o zone di campo magnetico turbolento in moto ($\langle \Delta B \rangle = 0$, ma $\langle \Delta B^2 \rangle \neq 0$)

On the Origin of the Cosmic Radiation

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(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller¹ has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.² The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

I propose in the present note to discuss a hypothesis on the origin of cosmic rays which attempts to meet in part this objection, and according to which cosmic rays originate and are accelerated primarily in the interstellar space, although they are assumed to be prevented by magnetic fields from leaving the boundaries of the galaxy. The main process of acceleration is due to the interaction of cosmic particles with wandering magnetic fields which, according to Alfvén, occupy the interstellar spaces.

Such fields have a remarkably great stability because of their large dimensions (of the order of magnitude of light years), and of the relatively high electrical conductivity of the interstellar space. Indeed, the conductivity is so high that one might describe the magnetic lines of force as attached to the matter and partaking in its streaming motions. On the other hand, the magnetic field itself reacts on the hydrodynamics³ of the interstellar matter giving it properties which, according to Alfvén, can pictorially be described by saying that to each line of force one should attach a material density due to the mass of the matter to which the line of force is linked. Developing this point of view, Alfvén is able to calculate a simple formula for the velocity V of propagation of magneto-elastic waves:

$$V = H / (4\pi\rho)^{1/2}, \quad (1)$$

¹ Nuclear Physics Conference, Birmingham, 1948.

² Alfvén, Richtmyer, and Teller, *Phys. Rev.*, to be published.

³ H. Alfvén, *Arkiv Mat. f. Astr., o. Fys.* 29B, 2 (1943).

where H is the intensity of the magnetic field and ρ is the density of the interstellar matter.

One finds according to the present theory that a particle that is projected into the interstellar medium with energy above a certain injection threshold gains energy by collisions against the moving irregularities of the interstellar magnetic field. The rate of gain is very slow but appears capable of building up the energy to the maximum values observed. Indeed one finds quite naturally an inverse power law for the energy spectrum of the protons. The experimentally observed exponent of this law appears to be well within the range of the possibilities.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

II. THE MOTIONS OF THE INTERSTELLAR MEDIUM

It is currently assumed that the interstellar space of the galaxy is occupied by matter at extremely low density, corresponding to about one atom of hydrogen per cc, or to a density of about 10^{-24} g/cc. The evidence indicates, however, that this matter is not uniformly spread, but that there are condensations where the density may be as much as ten or a hundred times as large and which extend to average dimensions of the order of 10 parsec. (1 parsec. = 3.1×10^{18} cm = 3.3 light years.) From the measurements of Adams⁴ on the Doppler effect of the interstellar absorption lines one knows the radial velocity with respect to the sun of a sample of such clouds located at not too great distance from us. The root mean square of the radial velocity, corrected for the proper motion of the sun with respect to the neighboring stars, is about 15 km/sec. We may assume that the root-mean-square velocity

⁴ W. S. Adams, *A.p.J.* 97, 105 (1943).

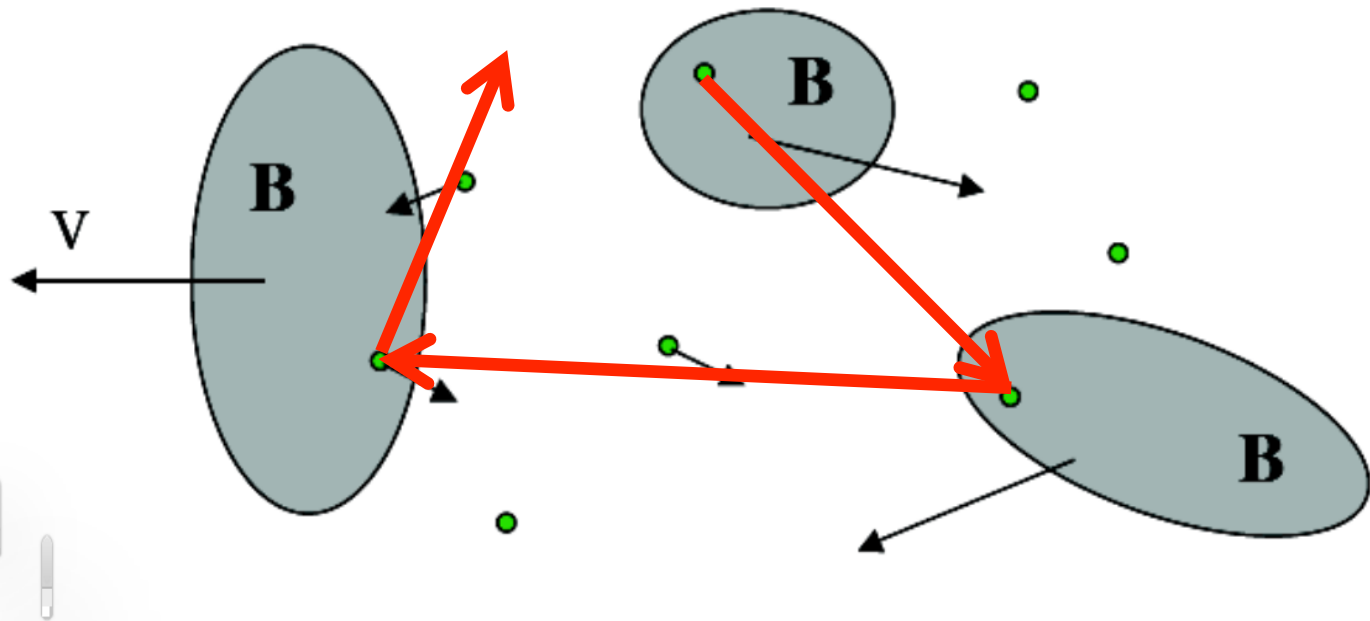
Il meccanismo di Fermi

Una “collisione” con una nube magnetica può causare un aumento dell’energia della particella. Un gran numero di collisioni possono far crescere l’energia fino a valori molto elevati. Guadagno di energia per collisione:

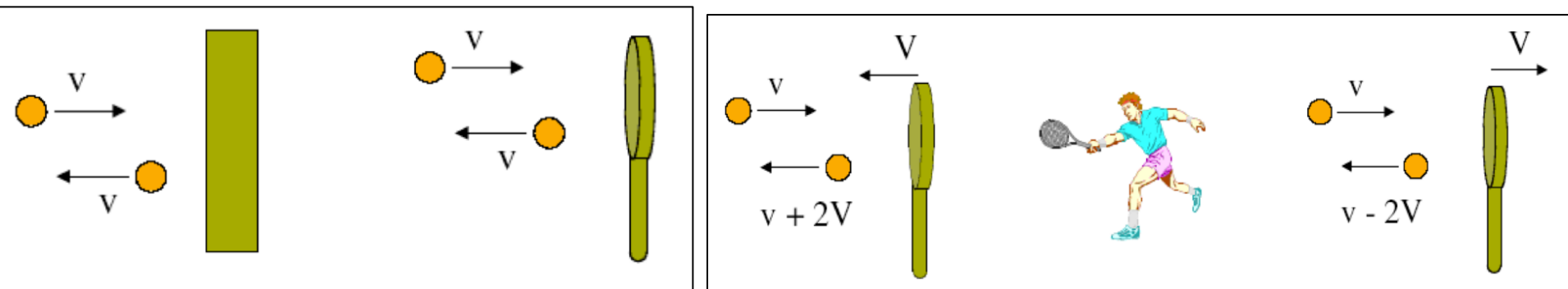
$$\Delta E/E = \varepsilon$$

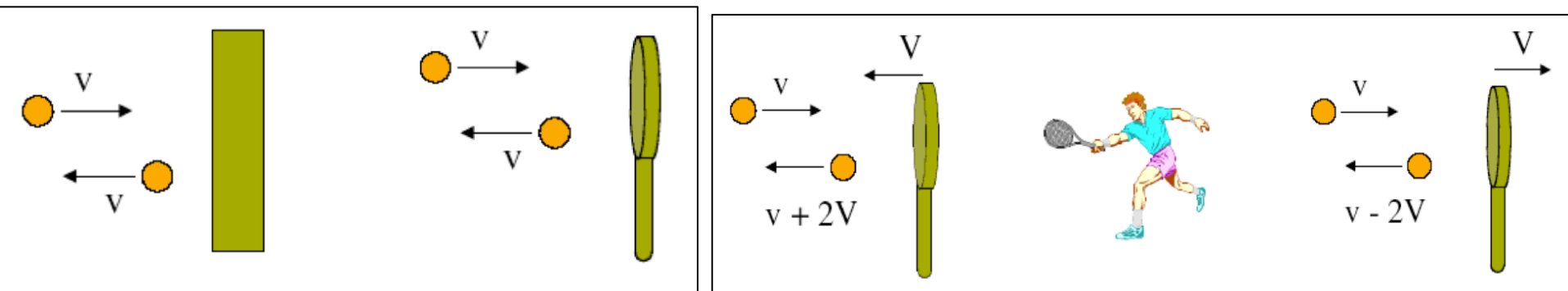
The original idea proposed by Fermi in 1949 for cosmic-ray acceleration was based on the fact that the interstellar medium is filled with "clouds" of ionized gas in movement with respect to the "Galactic frame". These clouds are carrying a magnetic field¹ and can in principle reflect the incoming charged particles (see Fig. 3.1). The acceleration mechanism based on moving "magnetic clouds" can be understood with a trivial but meaningful analogy with an idealized tennis game.

Magnetized clouds of velocity V in the interstellar medium



Let us assume a tennis ball is thrown, with a velocity v , on a steady racket. Ignoring the possible heat dissipation during the shock, the ball is simply reflected with the same velocity and no net energy gain, as it would be on a wall (see Fig. 3.2). Let us now assume that the racket is moving with a velocity V toward the ball (which still has a velocity v with respect to the tennis court). **In the racket frame**, the ball has a velocity $v + V$ and, assuming a perfectly elastic shock, the ball is reflected with the same (but opposite) velocity. **Back to the court frame** (adding V to the ball velocity in the racket frame), after the shock the ball has been accelerated to a velocity $v + 2V$ due to the head-on collision between the ball and the racket (see Fig. 3.3a). This result has been obtained with a double change of reference frame : court frame $\rightarrow$ racket frame $\rightarrow$ court frame. Let us now consider a *dropshot*, meaning that the racket is now going away from the ball with a velocity V with respect to the court frame (see Fig. 3.3b). With the exact same calculation we conclude that the ball has been decelerated to a velocity $v - 2V$, in the court frame after the shock.

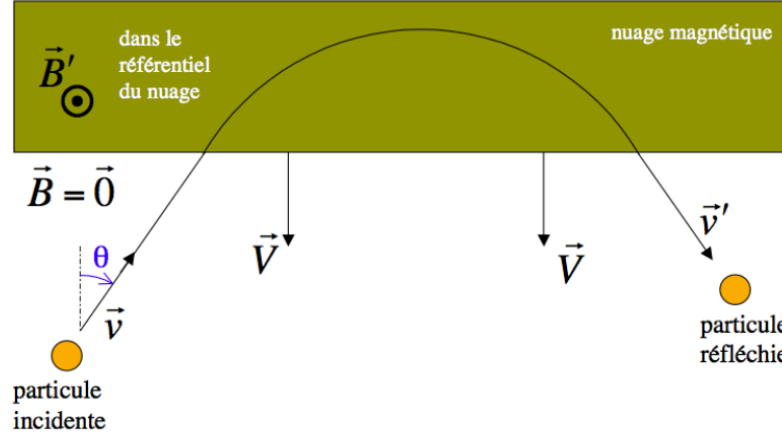




These simplistic analogies are enough to catch the essential origin idea behind Fermi's acceleration mechanism : one simply has to replace the tennis ball by a charged particle (a cosmic-ray) and the racket by a "magnetic cloud". Particles will be accelerated by each encounter with a magnetic cloud coming toward them and decelerated by the encounters with magnetic clouds going away from them. The energy gain (or loss) for each encounter can be calculated by a double change of reference frame, Galactic frame $\rightarrow$ cloud frame $\rightarrow$ Galactic frame.

Note that in this scheme the magnetic field is only the agent of the charged particles reflexion by the moving cloud. In the absence of magnetic field in the cloud the particles would just go through the moving cloud (as if the ball was going through the tennis racket) without any energy change (ignoring interactions with the cloud particles). We then expect the energy gain we will calculate in the following for some idealized cases to be independent of the magnetic field.

As mentioned above, another possible way of working out the acceleration the to calculate the electric field seen in the Galactic frame by Lorentz transformation of the pure B field seen in the cloud frame. Since the two approaches must be equivalent the acceleration and the energy gain of the particle must also be independent of the cloud magnetic field in this case. This result is however far less intuitive with this approach.



Let us consider the case of an idealized reflexion (for which the reflexion angle is equal to the incidence angle, see Figs. 3.1 and 3.4) of a particle (with a velocity $\vec{v}$) by a cloud coming toward it with a velocity $\vec{V}$, the incidence angle θ is then given by $\vec{V} \cdot \vec{v} = -\cos \theta$. Moreover for typical galactic magnetic cloud $V \ll c$ and $V \ll v$

We use as a convention, primed quantities for the cloud frame and unprimed for the Galactic frame. Passing from the Galactic frame to the cloud frame we have :

$$\left\{ \begin{array}{l} E'_{in} = \gamma_{cloud}(E_{in} - \vec{P}_{in} \cdot \vec{V}) = \gamma_{cloud}(E_{in} - P_{in}^{\parallel} V) \\ P_{in}^{\parallel} = \gamma_{cloud}(P_{in}^{\parallel} - \frac{V}{c^2} E_{in}) \end{array} \right. \quad (3.1)$$

where γ_{cloud} is the Lorentz factor of the cloud in the Galactic frame, the subscript *in* refers to the properties of the incoming particle and the parallel symbol "||" refers to the projection along the cloud velocity vector.

Inside the cloud, we assume (and it is in principle a good approximation) the particle is just reflected and does not loose or gain energy so we have $E'_{out} = E'_{in}$, where the subscript *out* refers to the properties of the outgoing particle. Moreover, we assumed the encounter with the cloud leads to a perfect reflexion so we have $P'_{out}^{\parallel} = -P'_{in}^{\parallel}$.

We now come back to the Galactic frame with the inverse Lorentz transformation :

$$\begin{cases} E_{out} = \gamma_{cloud}(E'_{out} + P'_{out}^{\parallel}.V) \\ P_{out}^{\parallel} = \gamma_{cloud}(P'_{out}^{\parallel} + \frac{V}{c^2}E'_{out}) \end{cases} \quad (3.2)$$

by substitution we get :

$$E_{out} = \gamma_{cloud}^2 \left[E_{in} \left(1 + \frac{V^2}{c^2} \right) - 2P_{in}^{\parallel}.V \right] \quad (3.3)$$

and since $P_{in}^{\parallel} = -\frac{E_{in}.v \cos \theta}{c^2}$ (the minus sign comes from the above definition of θ) to first order in V/c we get :

$$E_{out} = E_{in} \left(1 + \frac{2v.V \cos \theta}{c^2} \right) \Leftrightarrow \frac{\Delta E}{E} = \frac{E_{out} - E_{in}}{E_{in}} = -2 \frac{\vec{v}.\vec{V}}{c^2} \quad (3.4)$$

This final result leads to the following conclusions :

- The energy gain is proportional to the initial energy ($\Delta E/E$ is independent of E).
- The energy gain is positive for a head-on collision ($\vec{v}.\vec{V} < 0$).
- The energy gain is independent of B' , as anticipated, the magnetic field mediates the reflection but it does not appear in the Lorentz transformations.

Alternative approach using the induced E field

We place ourselves in the same configuration as before as schematized in Fig. 3.5. The electromagnetic tensor in the cloud frame can be transformed to the Galactic frame to obtain the electric field $\vec{E}$:

$$\vec{E}_{\perp} = \gamma_{cloud}(\vec{E}'_{\perp} - \vec{V} \times \vec{B}'_{\perp}) \quad (3.5)$$

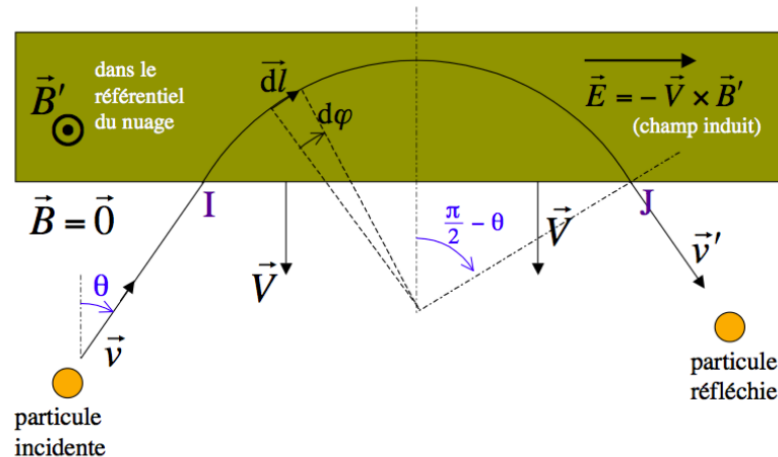
since $E' = 0$ by hypothesis and to first order in V/c we get :

$$\vec{E}_{\perp} = -\vec{V} \times \vec{B}'_{\perp} \quad (3.6)$$

with this calculation of the electric field in the Galactic frame we can calculate the energy gain during the encounter :

$$\Delta E = \int_{\text{inside the cloud}} \vec{F} \cdot d\vec{l} = q \vec{E}_{\perp} \cdot d\vec{l} = q E_{\perp} l_{IJ} \quad (3.7)$$

the segment IJ being the projection of the circular trajectory along the electric field vector (see Fig. 3.5) and $q = Ze$ the charge of the particle. We have :



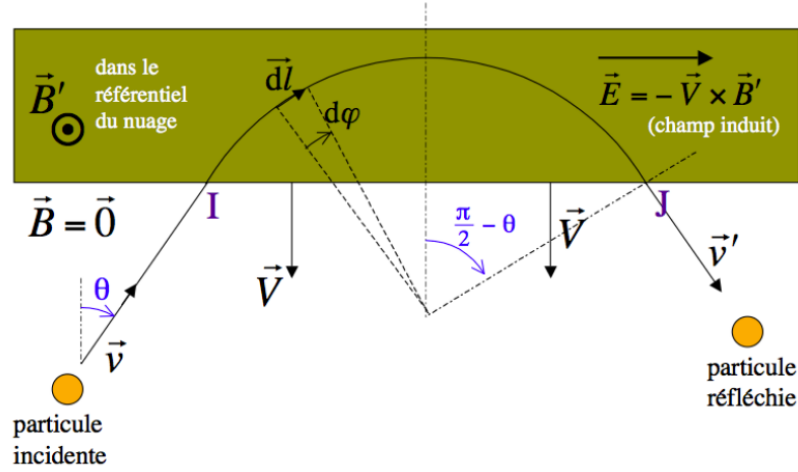


FIG. 3.5: Action of the induced electric field during the reflexion of a charged particle by a moving magnetic cloud.

$$l_{IJ} = 2r_L \sin\left(\frac{\pi}{2} - \theta\right) = 2r_L \cos \theta \quad (3.8)$$

where $r_L = \frac{P}{ZeB} = \frac{Ev}{ZeBc^2}$ is the Larmor radius or the particle with energy E , momentum P , velocity v and charge Ze in the magnetic field B .

We finally get :

$$\Delta E = ZeVB \times 2 \frac{Ev}{ZeBc^2} \cos \theta = \frac{2EVv \cos \theta}{c^2} = -2E \frac{\vec{v} \cdot \vec{V}}{c^2} \Leftrightarrow \frac{\Delta E}{E} = -2 \frac{\vec{v} \cdot \vec{V}}{c^2} \quad (3.9)$$

This results is equivalent to that obtained in Eq. 3.4, with the method of the double change of reference frame. We also find that the magnetic field B is eliminated from the equation which is a lot less obvious in principle than when the calculation is performed by double change of reference frame. In the following will only consider the latter method which leads to much simpler calculations although it takes no account of the induced electric field which is physically the true responsible for the particles energy gains.

So far we have studied single encounters with a cloud in the case of an idealized perfect reflexion. We saw that, indeed, a head-on collision was leading to an energy gain but also that energy can be lost if the cloud is moving away from the particle. The essence of Fermi's acceleration mechanism by interactions with moving Galactic magnetic clouds is that charged particles will suffer a series of encounter while propagating in the interstellar medium. Head-on collisions will lead to energy gains while energy will be lost when the cloud is moving away. All the point (which might not seem so intuitive at first sight) is that head-on collisions are **in average** more frequent so that in average particles do gain energy with this mechanism.

Why are head-on collisions more frequent? To understand that we can make a simple and familiar analogy with a car cruising on the highway. The car obviously crosses more cars coming toward than it overtakes cars going in the same direction. This is all a question of relative velocity and the same will be true with magnetic clouds. Moreover, on the highway the lower the velocity of the other cars the lower will be the difference between the number of car crossed and the number of cars overtaken. The same will be true for the magnetic clouds.

We now relax our simplifying hypothesis of a perfect reflexion and place ourselves in the case of Fig. 3.6a. We now assume that the magnetic field is turbulent so that charged particles are isotropized inside the cloud and that the angle of the particle escaping from the cloud θ'_{out} is random (the subscript "1" refers to our subscript "in" in Fig. 3.6a). To simplify our derivations we now assume that the particles are already ultra-relativistic, *i.e* $E \simeq Pc$. The double change of reference frame gives :

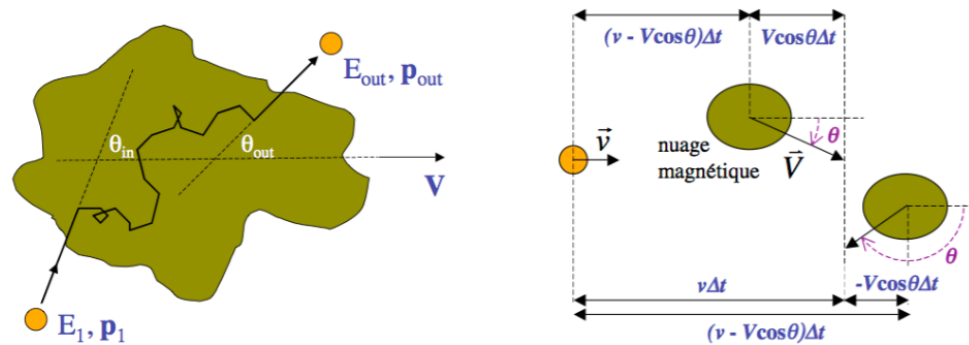


FIG. 3.6: Left : Schematic view of the "interaction" of a charged particle with a magnetic cloud. The particle enters the cloud and is isotropized by the magnetic turbulence. Right : Schematic view explaining the distribution of the incidence angle of particles with magnetic clouds : if the clouds velocity distribution is isotropic in the ISM and the density of cloud making an angle θ with the particle velocity vector is uniform the average number of clouds encountered during a time interval Δt is proportional to $(v - V \cos \theta)$.

The double change of frame gives

$$\begin{cases} E'_{in} = \gamma_{cloud} E_{in} (1 - \beta_{cloud} \cos \theta_{in}) \\ E_{out} = \gamma_{cloud} E'_{out} (1 + \beta_{cloud} \cos \theta'_{out}) \end{cases} \quad (3.10)$$

using $E'_{in} = E'_{out}$ and dropping the subscript "cloud" we get :

$$E_{out} = \gamma^2 E_{in} (1 - \beta \cos \theta_{in}) (1 + \beta \cos \theta'_{out}) \quad (3.11)$$

$$\Leftrightarrow \frac{\Delta E}{E} = \frac{E_{out} - E_{in}}{E_{in}} = \frac{\beta^2 - \beta \cos \theta_{in} + \beta \cos \theta'_{out} - \beta^2 \cos \theta_{in} \cos \theta'_{out}}{1 - \beta^2} \quad (3.12)$$

to get the average energy gain, we need to average the above expression. By hypothesis, the particles are isotropized in the cloud, hence $\langle \cos \theta'_{out} \rangle = 0$. We now need to calculate $\langle \cos \theta_{in} \rangle$, the probability to have an encounter with an incidence angle θ_{in} should be proportional to the relative velocity between the particles and the cloud (think of the car on the highway) in the case of uniformly distributed clouds. It gives (see Fig. 3.6b) : $P(\theta_{in}) \propto v - V \cos \theta_{in}$ ^{*} ($v \simeq c$ and V still being respectively the velocity of the particle and the cloud). We then have :

$$\langle \cos \theta_{in} \rangle = \frac{\int_{-1}^1 \cos \theta_{in} (v - V \cos \theta_{in}) d \cos \theta_{in}}{\int_{-1}^1 (v - V \cos \theta_{in}) d \cos \theta_{in}} = \frac{-2V/3}{2v} \simeq \frac{-2V/3}{2c} \simeq -\frac{\beta}{3} \quad (3.13)$$

substituting in Eq. 3.12 we finally get :

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{\beta^2 + \beta^2/3}{1 - \beta^2} \simeq \frac{4\beta^2}{3} \quad (3.14)$$

$$\langle \Delta E/E \rangle = \int_{\theta_1}^{\theta_2} P(\Omega) \frac{\Delta E}{E} (\cos \theta) d\Omega$$

* Relative velocity

$$v_{\text{rel}} = |\vec{v}_{\text{cloud}} - \vec{v}_{\text{particle}}|$$

$$= \sqrt{(c - v \cos \theta_i)^2 + v^2 \sin^2 \theta_i}$$

$$= c \sqrt{(1 - \beta \cos \theta_i)^2 + \beta^2 \sin^2 \theta_i}$$

$$= c \sqrt{1 + \beta^2 - 2\beta \cos \theta_i}$$

$$\simeq c \sqrt{1 - 2\beta \cos \theta_i}$$

$$\simeq c (1 - \beta \cos \theta_i)$$

$$\beta \ll 1$$

2nd order acceleration

We get an average energy gain which is indeed positive hence Fermi's mechanism is truly an acceleration mechanism for charged particles.

The average fractional energy gain is proportional to β^2 of the cloud so this mechanism is often called second order Fermi mechanism.

This mechanism is stochastic by nature, the average energy gain is positive but for a given particle, the energy gain can vary depending on the configuration of the series of encounters.

Acceleration time

The fact that the mechanism proposed by Fermi leads to a net energy gain in average does not mean that this mechanism is responsible for the production of the Galactic cosmic-rays we observe. In particular, to be a good candidate this mechanism must be sufficiently fast at accelerating particle to the very high energies measured by cosmic-ray observatories. To get a quantitative answer to this question, we need an approximate estimate of the particles acceleration times.

We start by giving the definition of the acceleration time t_{acc} :

$$t_{acc}(E) = \left(\frac{1}{E} \frac{dE}{dt} \right)^{-1} \quad (3.15)$$

Let L be the typical distance between two clouds and let us assume there is no magnetic field between two cloud (our estimate then becomes a lower limit), then the average time between two encounters will be $\langle t_{coll} \rangle = \frac{L}{c}$, neglecting the time spent by the particles inside the cloud we then get :

$$\frac{dE}{dt} \simeq \frac{\Delta E}{\langle t_{coll} \rangle} = \frac{4}{3} \frac{\beta^2 c E}{L} = \frac{E}{t_{acc}} \Leftrightarrow t_{acc} = \frac{3}{4} \frac{L}{c} \beta^{-2} \quad (3.16)$$

Power law spectrum

Let us consider the acceleration time t_{acc} , defined in Eq. 3.15, and the escape time t_{esc} . The latter would be the average time for the particles to leave the region of the Galaxy where the magnetic clouds are present. Let us assume, to simplify the calculation that t_{acc} and t_{esc} are both independent of the energy. It would make sense for our simple model since we have assumed that the medium was not magnetized and neglected the duration of the interaction between particles and clouds. Assuming particles are injected at $t = 0$ at an energy E_0 then :

$$E(t) = E_0 \exp\left(\frac{t}{t_{acc}}\right) \quad (3.17)$$

it means that particles with an energy E have been in the acceleration site for a time $t(E) = t_{acc} \ln(E/E_0)$ and that particles with energy between E and $E + dE$ have been injected at a time between $t(E)$ and $t(E + dE) = t(E) + dt$ earlier, with $dt = \frac{dt}{dE} dE = \frac{t_{acc}}{E} dE$ (using the definition of t_{acc}).

Moreover, during a time dt , the escape probability is $P_{esc} = \frac{dt}{t_{esc}}$. Let us call $\dot{N}_0$, the injection rate, the amount of particles injected during the time interval dt is $\dot{N}_0 dt$. Among these particles, only a fraction $f = \exp\left(-\frac{t(E)}{t_{esc}}\right)$ are still in the system after a time $t(E)$ and are able to reach an energy E . So the number of particles between E and $E + dE$ is :

$$n(E)dE = \dot{N}_0 dt \exp\left(-\frac{t(E)}{t_{esc}}\right) = \dot{N}_0 \frac{t_{acc}}{E} \exp\left(-\frac{t_{acc} \ln(E/E_0)}{t_{esc}}\right) dE \quad (3.18)$$

and we then get (taking the log of both sides)

$$n(E) = \frac{\dot{N}_0}{E_0} t_{acc} \left(\frac{E}{E_0}\right)^{-x} \quad (3.19)$$

with,

$$x = 1 + \frac{t_{acc}}{t_{esc}} \quad (3.20)$$

Power law spectrum

$$n(E)dE = \dot{N}_0 dt \exp\left(-\frac{t(E)}{t_{esc}}\right) dE = \dot{N}_0 \frac{t_{acc}}{E} \exp\left(-\frac{t_{acc} \ln(E/E_0)}{t_{esc}}\right) dE \quad (3.18)$$

and we then get

$$n(E) = \frac{\dot{N}_0}{E_0} t_{acc} \left(\frac{E}{E_0}\right)^{-x} \quad (3.19)$$

with,

$$x = 1 + \frac{t_{acc}}{t_{esc}} \quad (3.20)$$

We see that we indeed obtained a power law shape for the spectrum of accelerated particles, which is in principle a good point in favor of this mechanism. However, the exact shape of the spectrum depends on the t_{acc}/t_{esc} which cannot be predicted in principle since it will depend on the exact configuration of the region where one can find a large concentration of magnetic clouds. It is moreover likely that different regions in the Galaxy would accelerate cosmic-ray with different power law shape and the sum of the contribution would not be likely to give a global power law as observed for Galactic cosmic-rays detected on Earth.

Orders of magnitude

We can now apply our results to the case of the Galactic interstellar medium where Fermi's magnetic cloud are found. The typical velocity of a cloud is $\beta_{cloud} \simeq 10^{-4}$ which means $\beta_{cloud}^2 \simeq 10^{-8}$ and the typical distance $L \simeq 1pc$. Using these number one rapidly understands that with this acceleration mechanism it would take almost a billion year for a particle to double its energy. This is way too long to reach the very high energies observed for Galactic cosmic-rays. Moreover, we have neglected energy losses which might take place in the interstellar medium (such as ionization losses or spallation) and it turns out that for GeV nuclei, for instance, the energy loss time in the interstellar medium would be shorter than the acceleration time we just calculated.

We must conclude that the original acceleration mechanism presented by Fermi is far too slow to account for cosmic-ray acceleration (at least as the dominant process). We must not however forget the virtues of this pioneering idea, this mechanism indeed leads, in principle, in average to a net energy gain for the accelerated particle. Moreover although this scenario predicts a power law shape for the spectrum of accelerated particles, a robust prediction of the shape of the spectrum is very difficult to obtain. Despite its failure, Fermi mechanism is in fact the seed of most of the modern acceleration mechanisms which have been proposed since Fermi's pioneering work. Diffusive shock we will study in the next section, directly inherit of the basic ideas we discussed in this section, we will see that this mechanism manages to overcome quite elegantly most of the problems of the original Fermi mechanism.

Diffusive Shock Acceleration

Astrophysical shocks provide in principle such a mechanism, the so-called Diffusive Shock Acceleration (DSA)

Astrophysical shocks exist everywhere in the universe, from the solar system to more extreme objects such as supernova remnants, active galactic nuclei or gamma-ray bursts. We furthermore know particles are accelerated at these shocks as we detect radiation (from radio frequencies to γ -rays) from these object most often interpreted as the result of energy losses of accelerated electrons (see next chapter).

Astrophysical shock waves obey the same macroscopic rules as shock waves on Earth. They originate from outflows propagating with velocities larger than the local speed of sound. A shock front, through which physical quantities are discontinuous², forms. However, unlike "terrestrial shocks", the main difference is that astrophysical shock waves are in most cases *collisionless*, *i.e* the shock and the energy dissipation processes do not take place through particles collisions or coulombian interactions. The shock formation is due to the "interaction" of particles with the ambient magnetic field. Without the fields, supersonic outflows would just pass "unnoticed" through the ambient medium without the formation of any shock. Beyond these very simple considerations, the microphysics of astrophysical shocks is extremely complicated and far beyond the scope of this course.

Diffusive Shock Acceleration

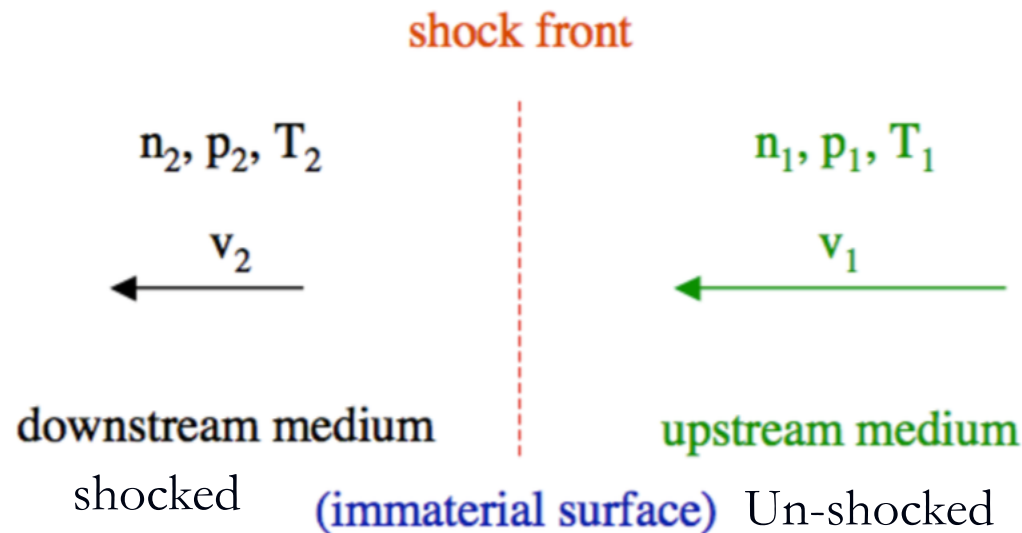


FIG. 3.7: Schematic view of a shock wave propagating in a medium, as seen in the shock rest frame. The shock is at rest and the upstream medium is coming toward it with a velocity v_1 while the downstream medium is going away with a velocity v_2 . Physical quantities are discontinuous through the (immaterial) surface of the shock.

Shock waves

$$[X] = X_2 - X_1$$

$$[\rho v_n] = 0$$

$$[p + \rho v_n^2 + \frac{1}{8\pi}(B_t^2 - B_n^2)] = 0 \quad [\rho v_n v_t + \frac{1}{4\pi}B_t B_n] = 0 \quad [B_n] = 0$$

$$[\rho v_n(v^2/2 + w) + \frac{1}{4\pi}(v_n B^2 - B_n(\vec{v} \cdot \vec{B}))] = 0 \quad [B_n v_t - B_t v_n] = 0$$

The analysis is simple in two limiting cases

The first is one in which the B field before the shock is normal to the shock surface, $B_{t1}=0$

This case is called "parallel" shock since the B field is parallel to shock normal direction

In such a case it is easy to show that $B_{t2}=0$ behind the shock

Since both B_n and B_t are continuous with $B_{n1}=B_{n2}$ and $B_{t1}=B_{t2}=0$, **it follows that the parallel shocks reduce to the pure hydrodynamic case, as if the B field is not present**

Shock waves

$$[\rho v_n] = 0$$

$$[p + \rho v_n^2 + \frac{1}{8\pi}(B_t^2 - B_n^2)] = 0 \quad [\rho v_n v_t + \frac{1}{4\pi} B_t B_n] = 0$$

$$[B_n] = 0$$

$$[\rho v_n (v^2/2 + w) + \frac{1}{4\pi} (v_n B^2 - B_n (\vec{v} \cdot \vec{B}))] = 0$$

$$[B_n v_t - B_t v_n] = 0$$

The analysis is simple in two limiting cases

The 2nd case is when the B field is parallel to the shock surface, ie perpendicular to the normal, $B_{n1} = 0$

In such a case, from 3rd equation we see that v_t is continuous $\rightarrow$ therefore we can choose a reference frame in which $v_t = 0$ and the shock is a normal shock

From $[B_n v_t - B_t v_n] = 0$ And $[\rho v_n] = 0$ We get

$$B_{t1}/B_{t2} = \rho_1/\rho_2 \quad \text{While the others reduce to}$$

$$[\rho v] = 0 \quad [p + \rho v^2 + \frac{B_t^2}{8\pi}] = 0 \quad [v^2/2 + w + \frac{B_t^2}{4\pi\rho}] = 0$$

Which are the hydrodynamics eqns with the additional magnetic terms is momentum and energy flux

Strong parallel shocks

For parallel strong shock the Rankine-Hugoniot read as

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)}{2/M_s^2 + (\gamma - 1)}$$



$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma + 1)}{(\gamma - 1)}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{(\gamma + 1)}$$



$$\frac{p_2}{p_1} \approx 2\gamma M_s^2$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma - 1)M_s^2}{(\gamma + 1)M_s^2}$$



$$\frac{V_2}{V_1} \approx \frac{\gamma - 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma - 1)M_s^2][2\gamma M_s^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_s^2}$$



$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2$$

$$M_{s2} = \frac{2 + (\gamma - 1)M_s^2}{2\gamma M_s^2 - \gamma + 1}$$



$$M_{s2} \approx \frac{\gamma - 1}{2\gamma}$$

Onda di shock

$v \cos \theta$

*Campi
magnetici*

v_{cl}

Scattering elastico

These processes are dominated by scattering in the irregularities downstream and upstream flux.

v_{cl}

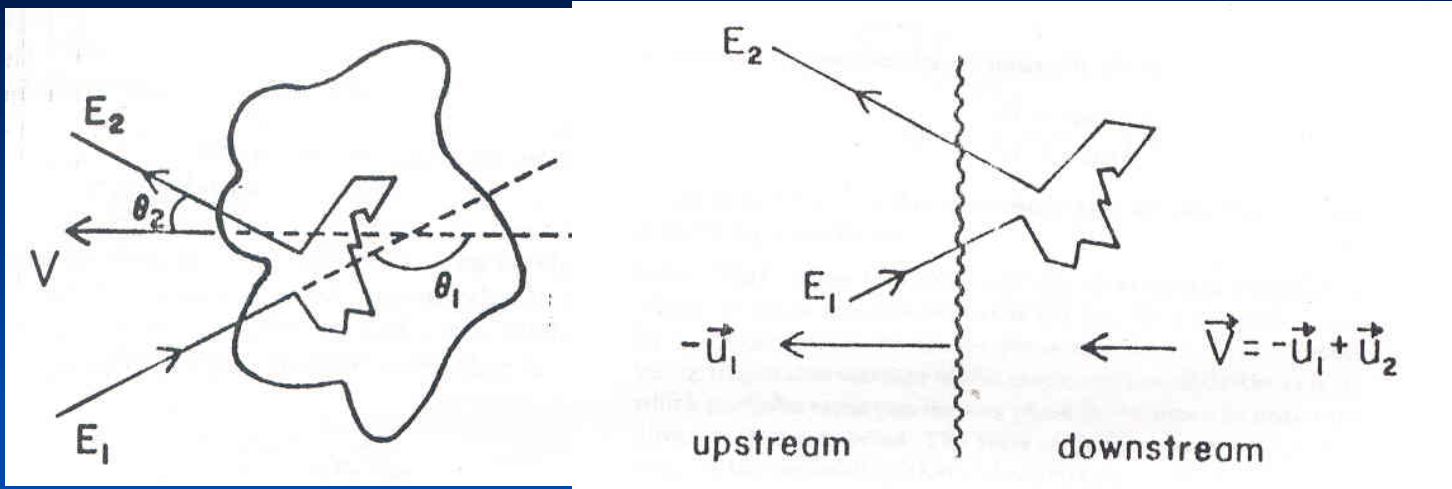
They act as “scattering centers” for collisionless interactions, frozen into the plasma flux. Because of this they may be approximated as “walls” of infinite mass on which particles are “reflected”

At shock waves, acceleration of particles occurs. The ingredients are:

- a physical mechanism to change energy at each collisionless interaction with the shock wave and
 - a statistical process which gives an average gain of energy with an enough high number of encounters with the shock wave front, i.e. suitable interaction time

The shock front does not play any direct role.

Isotropization



Particles in the cloud or around the shock front are scattered and isotropized by diffusion in the B field fluctuations, so that their average motion is the same of the cloud or of the fluid around the shock after few diffusion lengths. This means that the particles are in average at rest in the local fluid frame.

The process is called isotropization

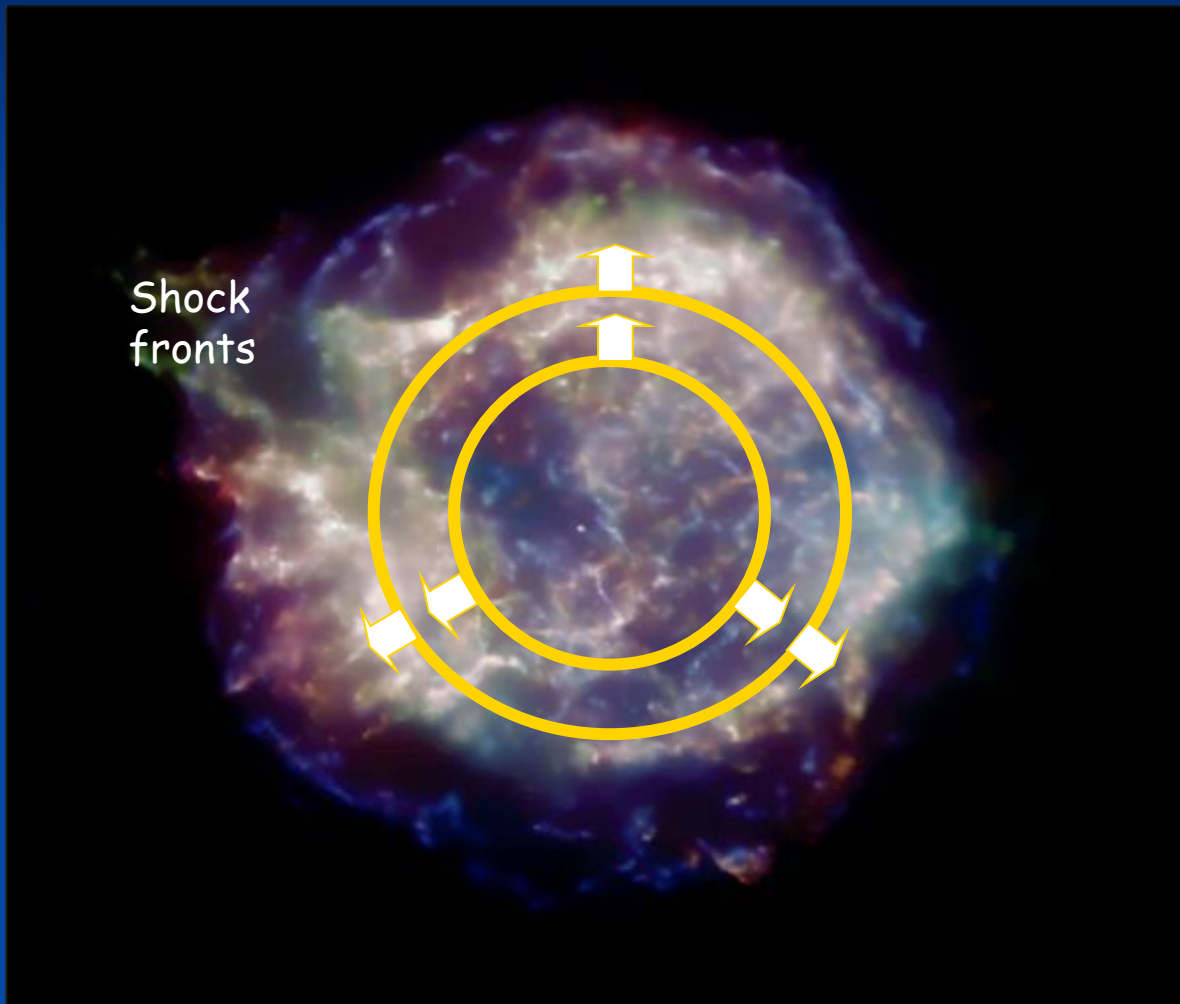
The process is collisionless therefore energy is not changed in the reference frame of the cloud or shock

Since the mass of particle is $\ll M_{\text{cloud}}$ or M_{shock} we can, in first approximation, consider the cloud or the shock as “walls” of infinite mass on which the particle “bounce” forth and back

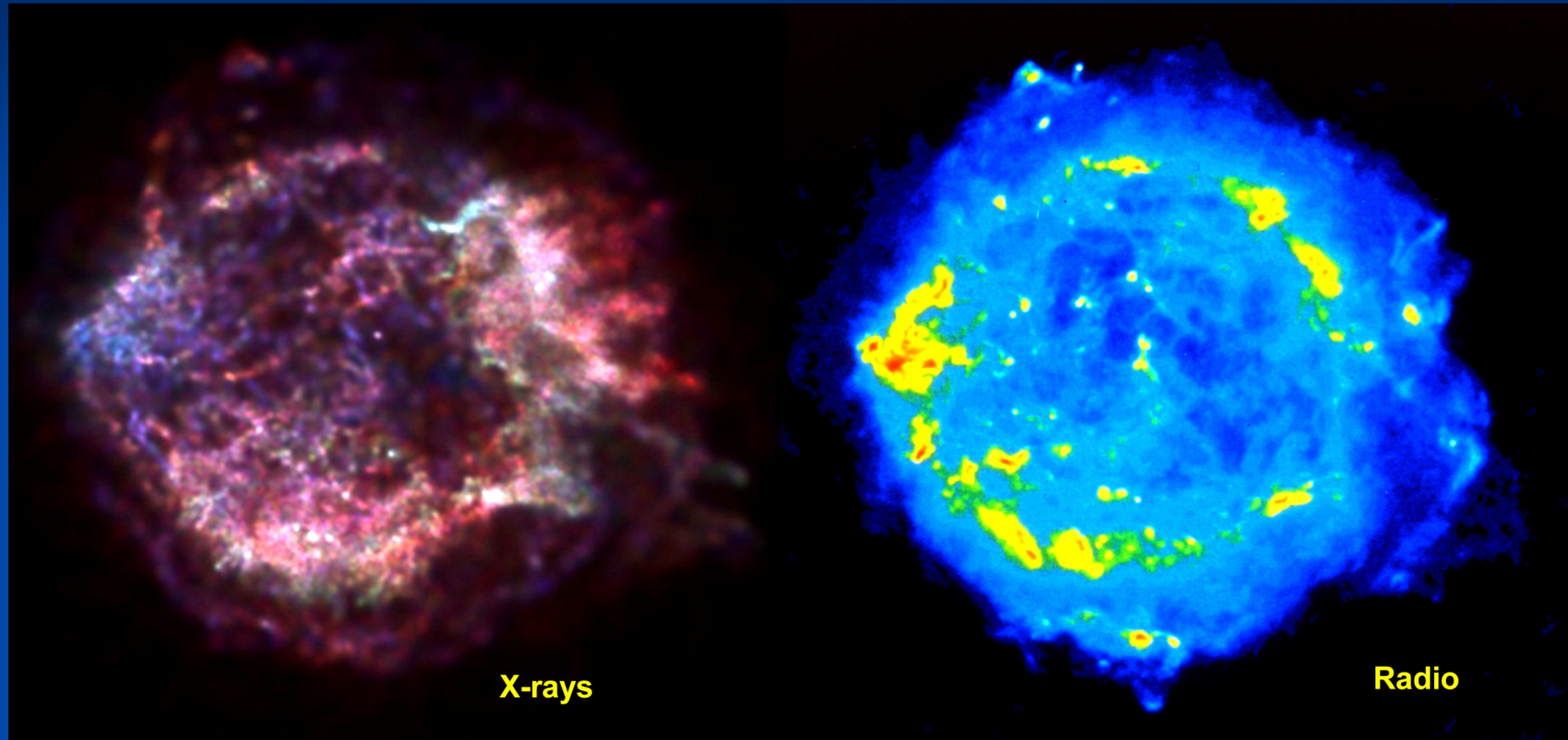
SuperNova Remnants (SNRs)

A supernova remnant (SNR) consists essentially of the stellar ejecta of the SN explosion embedded in a hot expanding bubble, preceded by swept-up interstellar material and an outer blast wave (strong shock) propagating into the interstellar medium

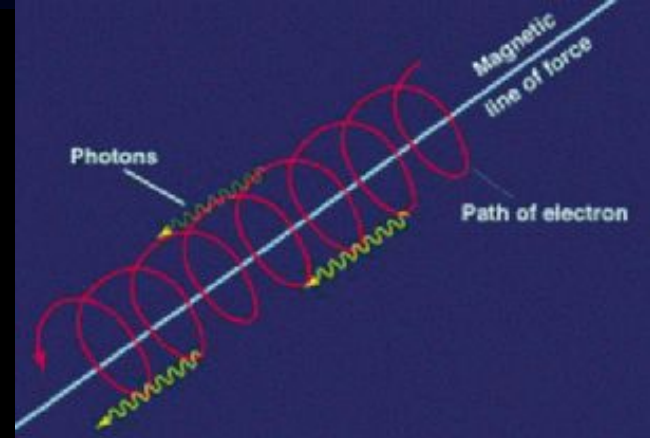
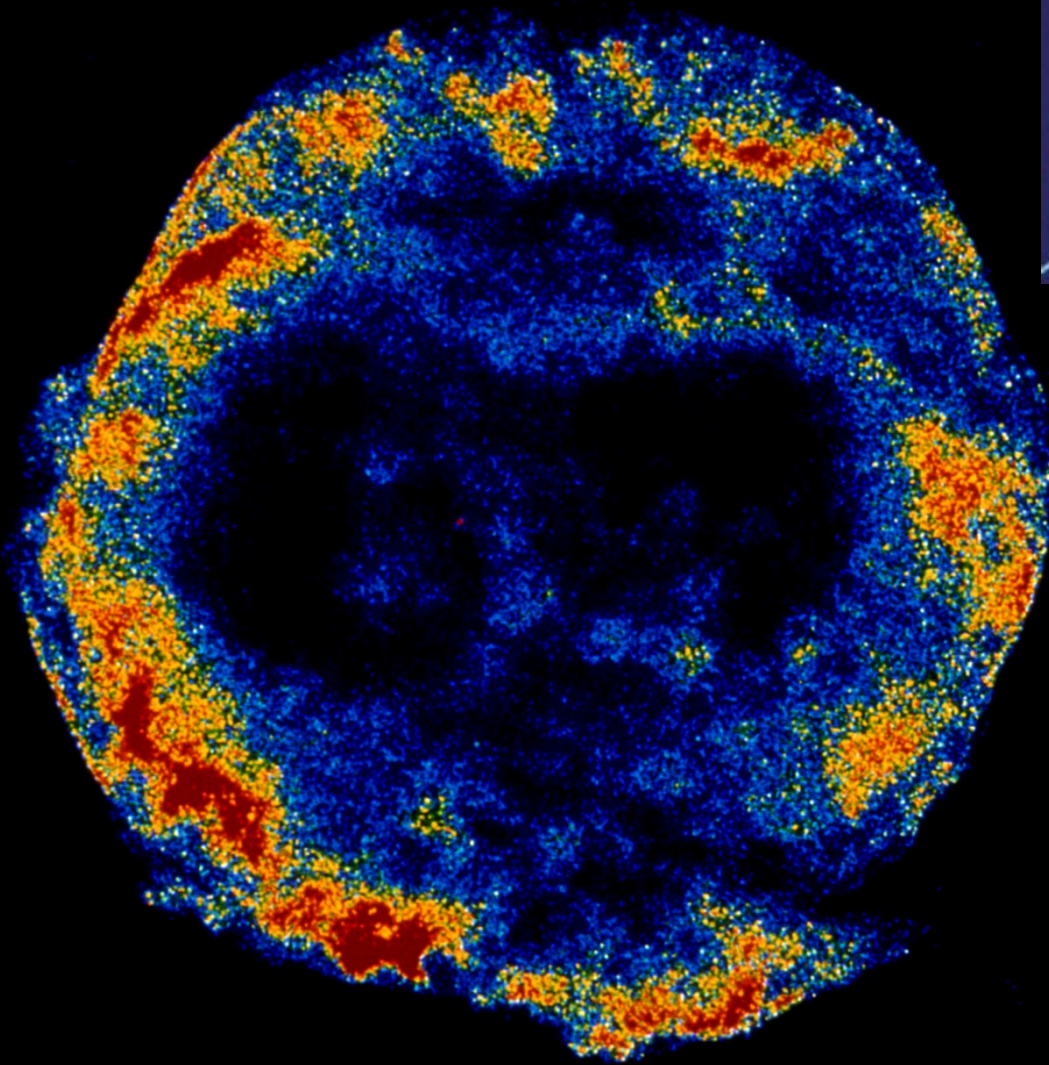
CasA Supernova Remnant in X-rays



Supernova Remnant Cassiopeia A



2.9 pc, exploded 350 years ago



■ Remnant of Tycho's supernova of 1572 AD

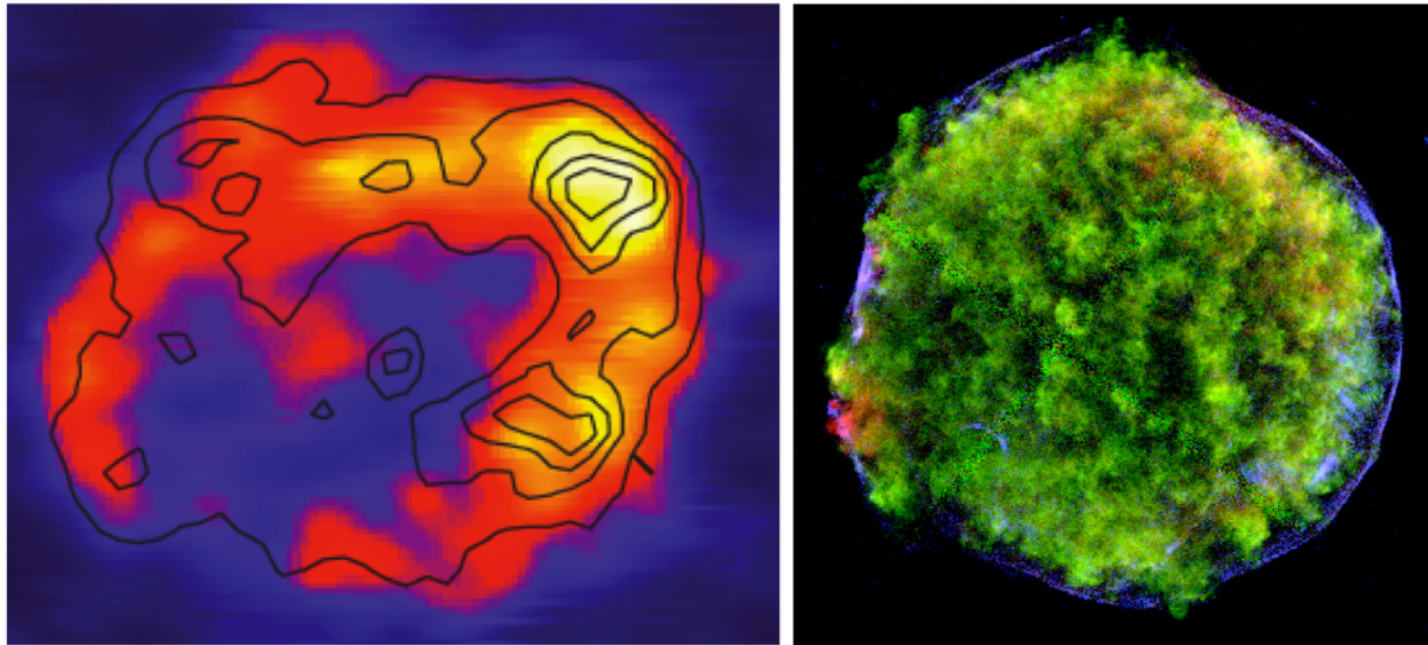
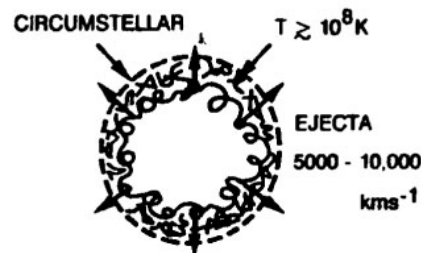
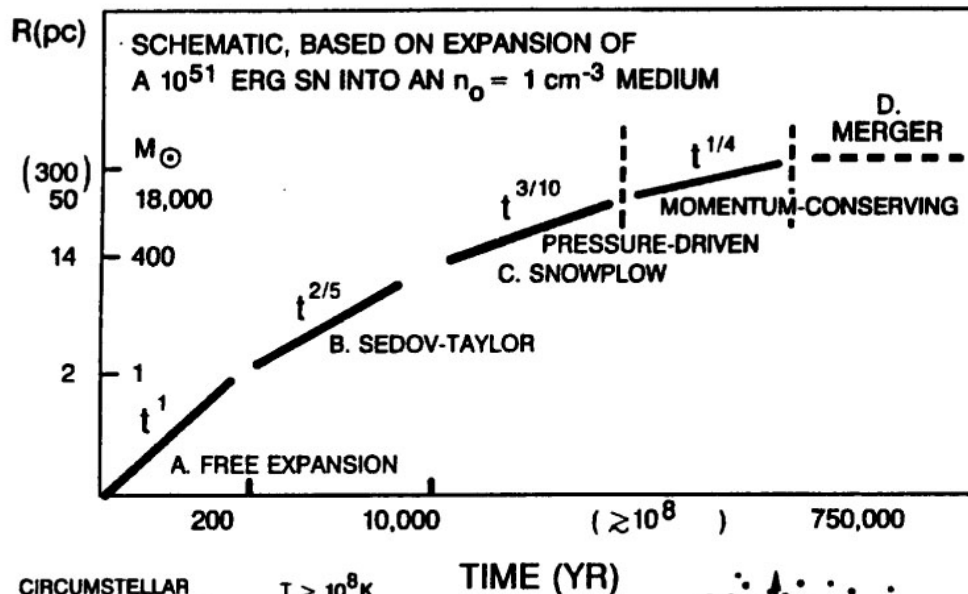


Fig. 4 *Left Panel:* Morphology of the RX J1713.7-3946. The colors illustrate the high energy gamma ray emission as measured by HESS (Aharonian et al, 2007), while the contours show the X-ray emission in the 1-3 keV band measured by ASCA (Uchiyama et al, 2002). *Right Panel:* Morphology of the Tycho SNR as measured with Chandra (Warren et al, 2005). The three colors refer to emission in the photon energy range 0.95 – 1.26 keV (red), 1.63 – 2.26 keV (green), and 4.1 – 6.1 keV (blue). The latter emission is very concentrated in a thin rim and is the result of synchrotron emission of very high energy electrons.

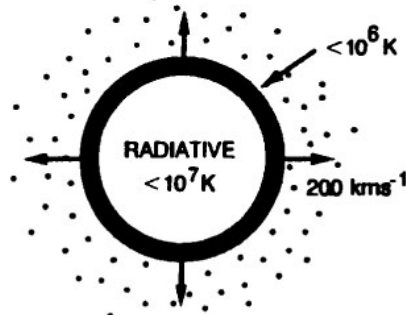
STANDARD SNR EVOLUTION



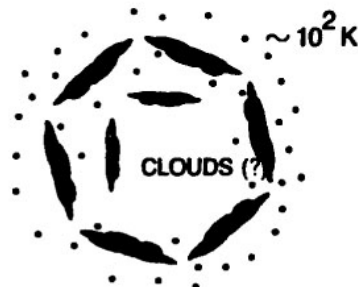
A. FREE EXPANSION



B. BLAST WAVE



C. SNOWPLOW



D. DEATH → COALESCENCE

Main properties:

Different expansion stages:

- Free expansion stage
($t < 1000 \text{ yr}$)

$$R \propto t$$

- Sedov-Taylor stage
($1000 \text{ yr} < t < 10,000 \text{ yr}$)

$$R \propto t^{2/5}$$

- Pressure-driven
snowplow ($10,000 \text{ yr} < t < 250,000 \text{ yr}$)

$$R \propto t^{3/10}$$

- Momentum-conserving
($250,000 < t < 750,000 \text{ yr}$)

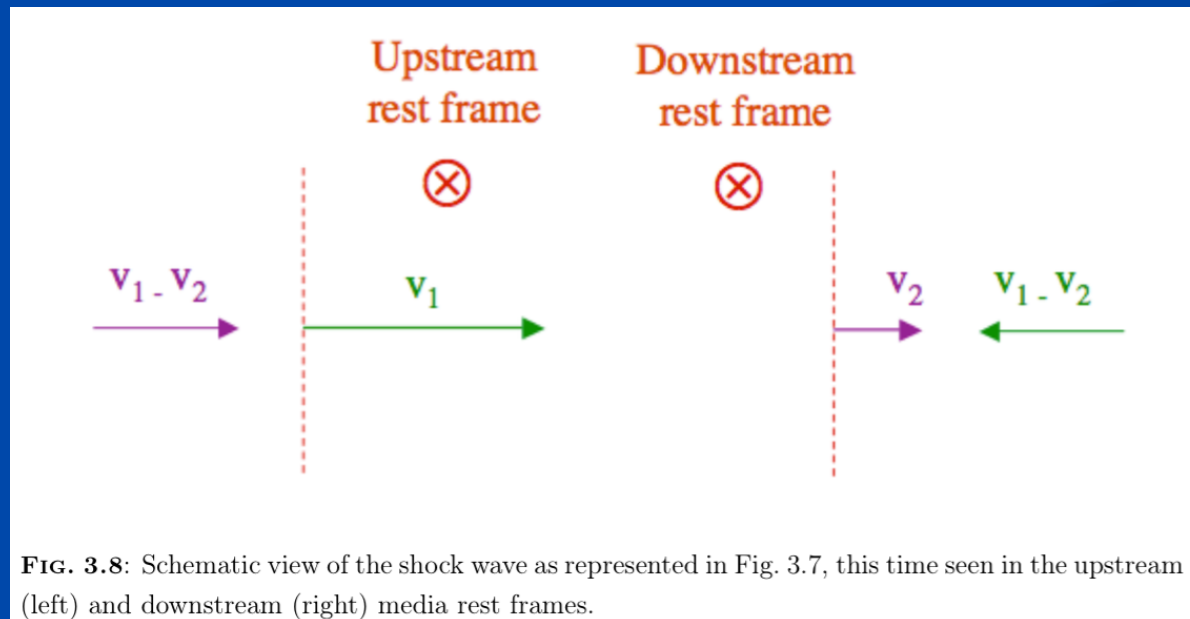
$$R \propto t^{1/4}$$

DSA

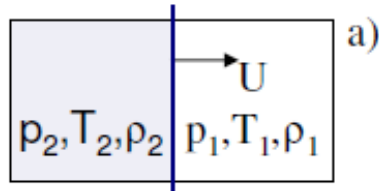
If we concentrate only on the velocity discontinuity, we can easily understand the interest of shock waves for particles acceleration. In the shock frame, the upstream medium is coming toward the shock with a velocity v_1 (note of course that $v_1 = v_{sh}$ where v_{sh} is the shock velocity). Passing through the shock the gas slows down and the downstream medium is moving away from the shock with a velocity $v_2 = v_1/r = v_{sh}/r$ (see Fig. 3.7).

Let us now consider an observer at rest in the upstream frame. He sees the shock approaching with a velocity $v_1 = v_{sh}$, he also sees the downstream medium approaching with a velocity³ $\Delta v = v_1 - v_2 = \left(\frac{r-1}{r}\right) v_{sh}$ (see Fig. 3.8a).

Now for an observer at rest with respect to the downstream fluid, the shock is going away with the velocity v_2 we obtained before, but the upstream medium is coming toward the observer, again with a velocity $\Delta v = v_1 - v_2 = \left(\frac{r-1}{r}\right) v_{sh}$ (see Fig. 3.8b).

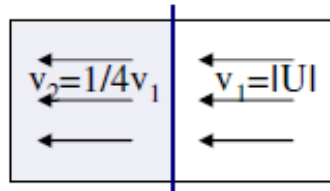


Infinite plane shock with a magnetic field perpendicular to it



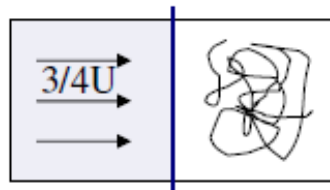
a)

$U \gg c_s$ through stationary interstellar gas
mass conservation: $\rho_1 v_1 = \rho_2 v_2$



b)

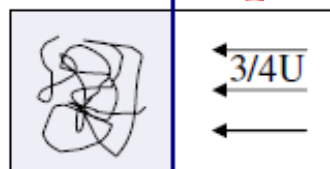
reference frame of the shock front at rest:
for monoatomic or fully ionised gas $\gamma=5/3$, $\rho_2/\rho_1=(\gamma+1)/(\gamma-1)$



c)

upstream gas stationary:
isotropic distribution of velocities
the gas behind the shock moves with velocity $V=-v_1+v_2$

Upstream



d)

downstream gas stationary:
isotropic distribution of velocities

**Up -> Down and vice versa : increase in energy,
increment of same order**

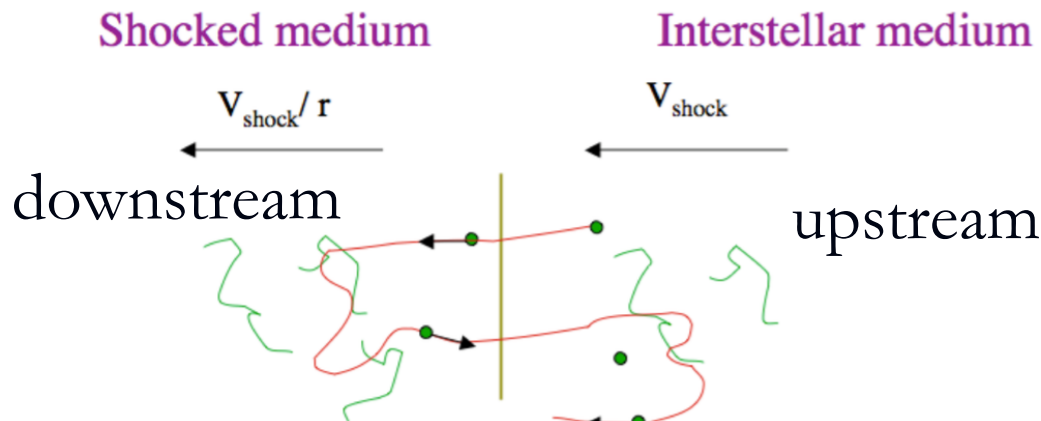
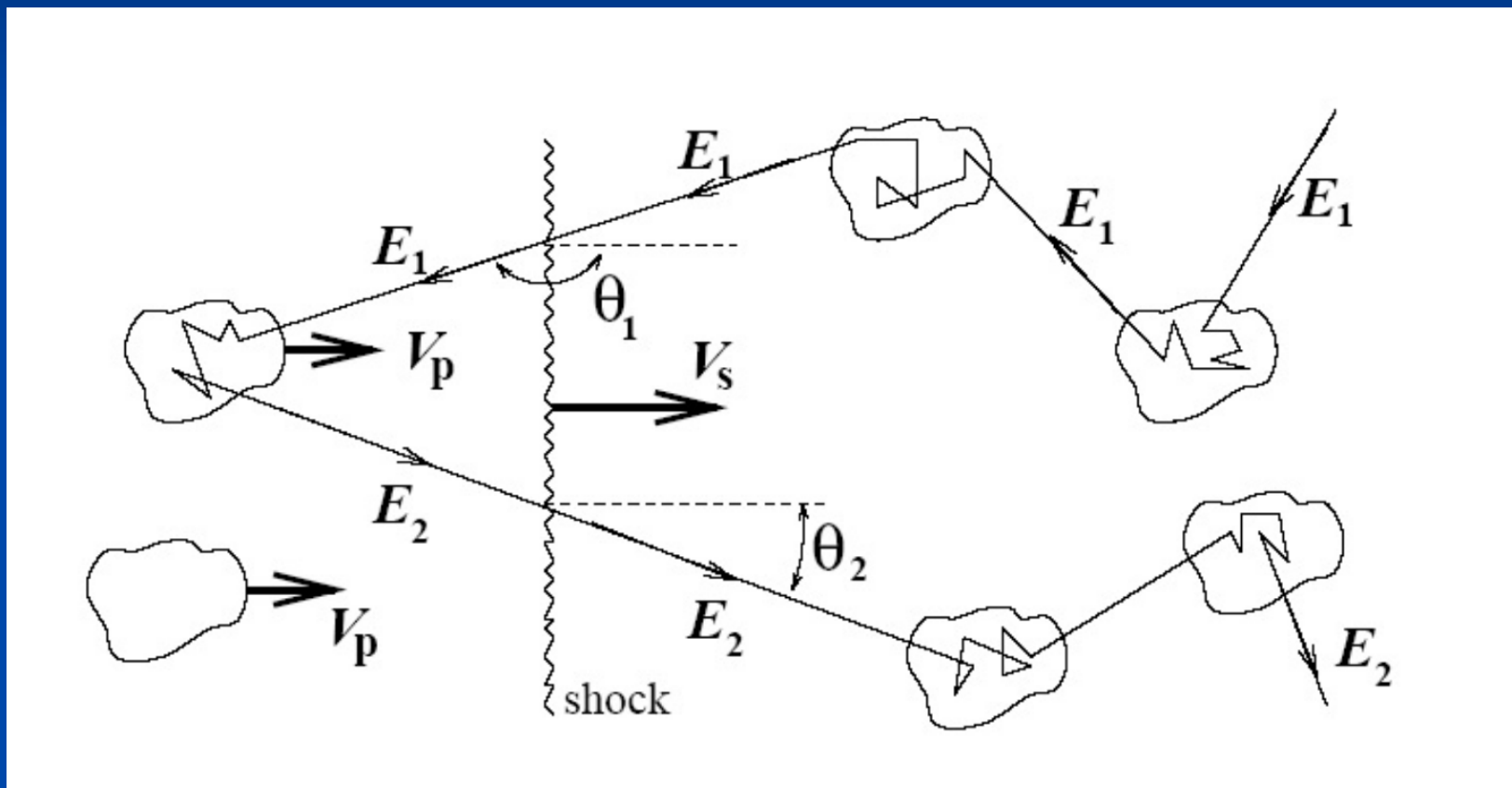


FIG. 3.9: Schematic view of a cycle as seen in the shock rest frame : the particle initially in the ISM (upstream medium) enters the shocked medium (downstream medium), it is then isotropized by the magnetic turbulence and reflected back to the upstream medium where the particle is isotropized again and eventually crossed the shock to start a new cycle.

Let us assume the both the upstream and downstream media are magnetized ⁴. We are then in a situation where a particle coming from the upstream medium and passing through the shock would see the downstream medium as a "magnetic cloud" coming toward it (a cloud with a velocity Δv with respect to the upstream fluid rest frame). Likewise a particle coming from the downstream medium and passing through the shock would see the upstream medium as a "magnetic cloud" coming toward it (with a velocity Δv with respect to the downstream fluid rest frame). We can then understand that a particle which would cross several times the shock for instance from upstream to downstream then back upstream could gain energy by interacting with moving "magnetic clouds". The critical difference with the original mechanism proposed by Fermi is that with this configuration, all the collisions would now be head-on. It is then likely that this mechanism involving charged particles cycling across a shock front will turn out to be much more efficient than the original mechanism proposed by Fermi. This prove this statement we need to perform the calculation of the energy variation experienced by a charged particle during a cycle *upstream* $\rightarrow$ *downstream* $\rightarrow$ *upstream*.

collisions on magnetic irregularities are always head-on



Energy gain in a cycle

Before calculating the mean energy gain experienced by a charged particles during a cycle, we need to set a series of physical hypotheses to define the framework of our calculation. The relevance of these hypothesis will be discussed later on, throughout this section :

- The shock is an infinite plane
- The media upstream and downstream of the shock have an infinite spatial extension
- There is no limitation in time, the relevant physical quantities are in a steady state
- There are magnetic field inhomogeneities, both upstream and downstream of the shock which "isotropize" energetic charged particles, the propagation of particles is diffusive in both media.
- The shock is non-relativistic $v_{sh} \ll c$, the charged particles are relativistic $v_{part} \simeq c \gg v_{sh}$.

Energy gain in a cycle

For this calculation we use unprimed quantities for the upstream frame and primed quantities for the downstream frame. Let θ_{in} be the angle between the particle velocity and the shock normal at the initial shock crossing in the upstream frame and θ'_{out} the angle of the particle with the shock normal in the downstream frame, when crossing the shock back to the upstream medium. On a cycle upstream $\rightarrow$ downstream $\rightarrow$ upstream, we have :

$$\begin{cases} E'_{in} = \gamma E_{in}(1 - \beta \cos \theta_{in}) \\ E_{out} = \gamma E'_{out}(1 + \beta \cos \theta'_{out}) \end{cases} \quad (3.29)$$

γ and β would correspond to γ_{cloud} and β_{cloud} in the original Fermi mechanism. In the case of a shock wave, they correspond to the velocity of the downstream medium in the upstream fluid frame (see above) and then we have $\beta = \Delta v/c$ and $\gamma = \frac{1}{\sqrt{1 - \frac{\Delta v^2}{c^2}}}$. Using $E'_{in} = E'_{out}$, we get the already familiar expression :

$$\frac{\Delta E}{E} = \frac{\beta^2 - \beta \cos \theta_{in} + \beta \cos \theta'_{out} - \beta^2 \cos \theta_{in} \cos \theta'_{out}}{1 - \beta^2} \quad (3.30)$$

again, the mean fractional energy gain is obtained by averaging $\langle \cos \theta_{in} \rangle$ and $\langle \cos \theta'_{out} \rangle$, the result will however differ from the original case with magnetic clouds.

Energy gain in a cycle

To obtain $\langle \cos \theta_{in} \rangle$ and $\langle \cos \theta'_{out} \rangle$, we need to know the probability of crossing the shock with an angle between θ and $\theta + d\theta$. Let us neglect the shock velocity since $v_{sh} \ll v_{part}$. The particle crosses the shock with a velocity $v_{part} \cos \theta$ and then assuming a particle density n_0 , the number of particles passing the shock with an angle between θ and $\theta + d\theta$, through a surface dS during a time dt is :

$$dN^4 = \frac{n_0}{4\pi} v_{part} \cos \theta d\Omega dS dt = \frac{n_0}{2} v_{part} \cos \theta \sin \theta d\theta dS dt \quad (3.31)$$

the probability of crossing the shock with an angle between θ and $\theta + d\theta$ is then proportional to $\cos \theta \sin \theta d\theta$. We then have :

$$\langle \cos \theta \rangle = \frac{\int_{\theta_{min}}^{\theta_{max}} \cos^2 \theta \sin \theta d\theta}{\int_{\theta_{min}}^{\theta_{max}} \cos \theta \sin \theta d\theta} = \frac{\left[\frac{1}{3} \cos^3 \theta \right]_{\theta_{min}}^{\theta_{max}}}{\left[\frac{1}{2} \cos^2 \theta \right]_{\theta_{min}}^{\theta_{max}}} \quad (3.32)$$

for the crossing from upstream to downstream we have $\theta_{min} = \frac{\pi}{2}$ and $\theta_{max} = \pi$ (particle and shock velocities are antiparallel) which give :

$$\langle \cos \theta_{in} \rangle = -\frac{2}{3} \quad (3.33)$$

for the crossing back from downstream to upstream we have $\theta_{min} = 0$ and $\theta_{max} = \frac{\pi}{2}$ which give $\langle \cos \theta'_{out} \rangle = \frac{2}{3}$

Energy gain in a cycle

Note that we implicitly got rid of some complications and of second order corrections by neglecting the shock velocity in this calculation. Since $v_{sh} \ll c$ we also have $\Delta v \ll c$ we can then neglect terms in β^2 in Eq. 3.30 calculating $\langle \Delta E/E \rangle$. We finally get :

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\beta = \frac{4}{3}\beta_{sh} \left(\frac{r-1}{r} \right) \quad (3.35)$$

where $\beta_{sh} = v_{sh}/c$ and r is the above-mentioned compression ratio.

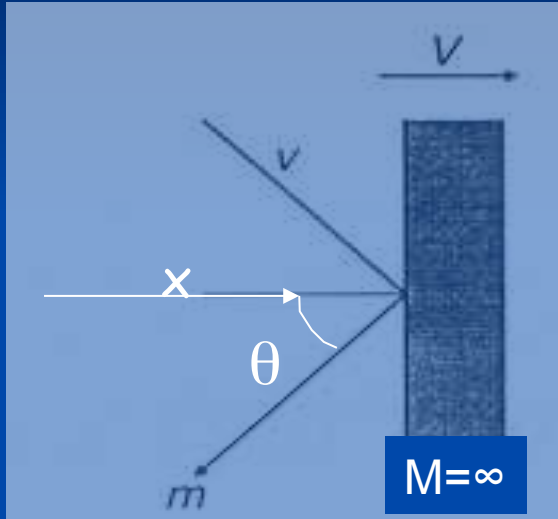
As anticipated, DSA is indeed an acceleration mechanism (since $\langle \frac{\Delta E}{E} \rangle$ is positive) with an energy gain proportional to β which is a very important step forward when comparing with the original Fermi mechanism. DSA is also called **first order Fermi mechanism** for obvious reasons.

Shock waves: Energy gain

The mechanism for energy gain is always the same

The gain is given by “elastic” collision in the wall rest frame and a relativistic boost to the lab.

- In the ref. frame of the walls, $E'_i = \gamma(E_i + Vp_{ix})$, $P'_{ix} = \gamma(p_{ix} + VE_i)$ ($c = 1$)



The collision is elastic (i.e. collisionless): $E'_f = E'_i$ and

$$p'_{fx} = -p'_{ix}$$

Going back to the lab frame

$$E_f = \gamma(E'_f - Vp'_{fx}) = \gamma(E'_i + Vp'_{ix})$$

$$p_{fx} = \gamma(p'_{fx} - VE'_f) = \gamma(-p'_{ix} - VE'_i)$$

$$\text{Therefore } E_f = \gamma(E'_i + Vp'_{ix}) = \gamma^2[(E_i + Vp_{ix}) + V(p_{ix} + VE_i)] = \gamma^2[E_i + 2Vp_{ix} + V^2E_i]$$

$$\text{But } p_{ix} = v_{ix}E_i \rightarrow E_f = \gamma^2[E_i + 2Vv_{ix}E_i + V^2E_i] = \gamma^2E_i[1 + 2Vv_{ix} + V^2]$$

→ the relative gain per each scattering is (put back c)

$$\Delta E/E_i = (E_f - E_i)/E_i = \gamma^2[1 + 2Vv_{ix} + V^2] - 1 = \gamma^2[1 + 2Vv \cos \theta / c^2 + V^2/c^2] - 1$$

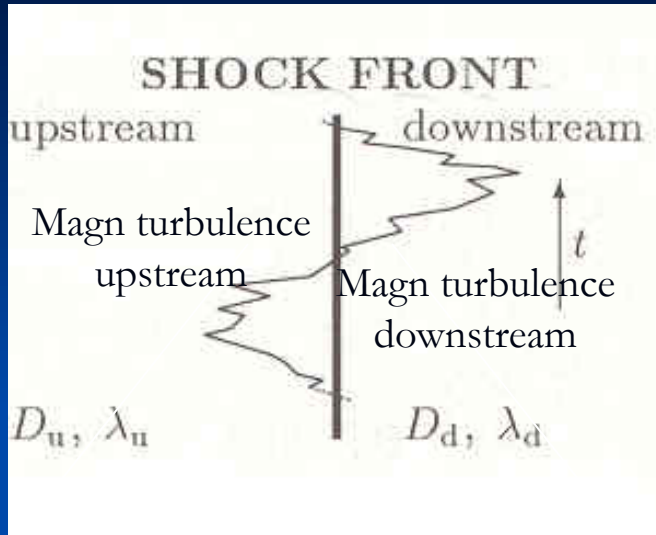
$$\gamma^2 = [1 - (V/c)^2]^{-1} \approx 1 + (V/c)^2 \text{ if } V \ll c$$

$$\Delta E/E_i \approx [1 + (V/c)^2][1 + 2Vv \cos \theta / c^2 + (V/c)^2] - 1$$

$$= 2Vv \cos \theta / c^2 + (V/c)^2 + (V/c)^2 + (V/c)^2[2Vv \cos \theta / c^2 + (V/c)^2] \approx 2Vv \cos \theta / c^2 + 2(V/c)^2$$

It is the gain for non-relativistic shock waves, i.e. $V \ll c$

Diffusive Shock Acceleration



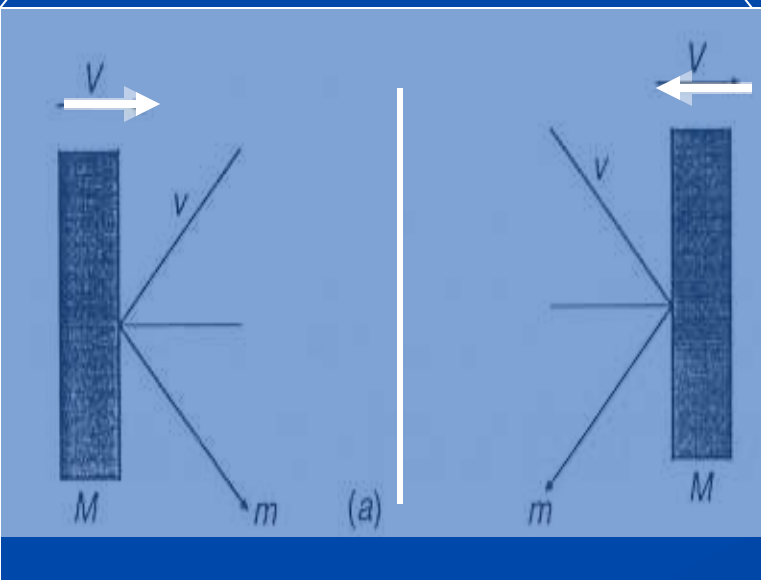
DSA is the dominant mech. for quasi-parallel shocks. In DSA particle scattering up- and down-stream is crucial $\rightarrow$ high level of turbulence and irregularities are requested on both sides of the shock.

The magnetic fields on both sides are turbulent, so that the resulting scattering is quantified by the diffusion coefficient D or by the mean free paths λ

The crucial point is that in the ref frame in which one of the fluxes is at rest, a particle always sees the other side moving toward it (in a symmetric way for the 2 sides), so that the “collision” is always head-on.

Since the scattering centers are frozen into the plasma, particle scatt. back and forth through the shock front can be understood as repeated reflections between converging scattering centers (i.e. “walls” with infinite mass).

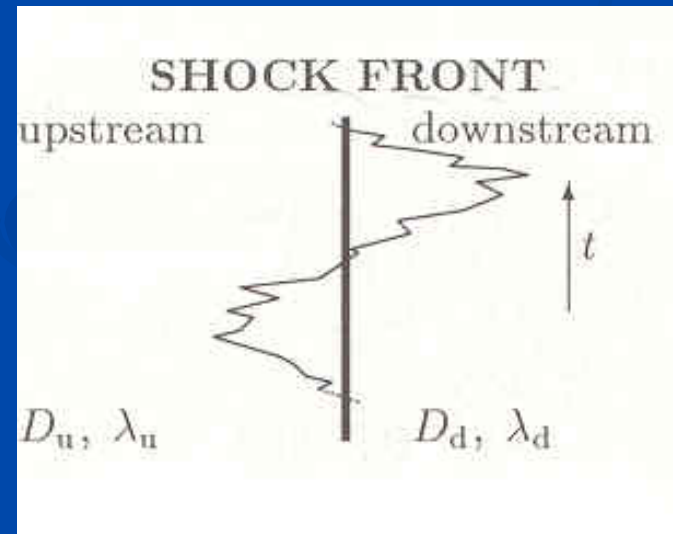
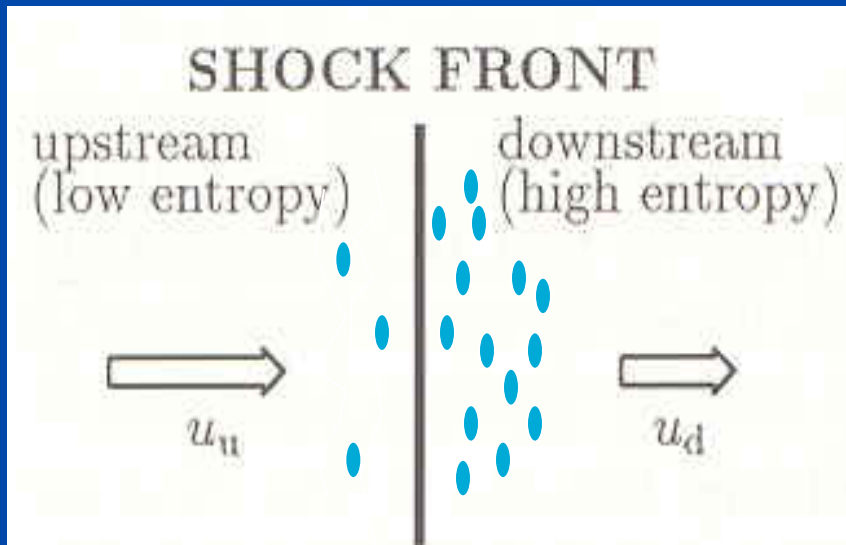
The B field fluctuations are frozen-in with the plasma and carried with the plasma flow



Shock waves: Energy gain

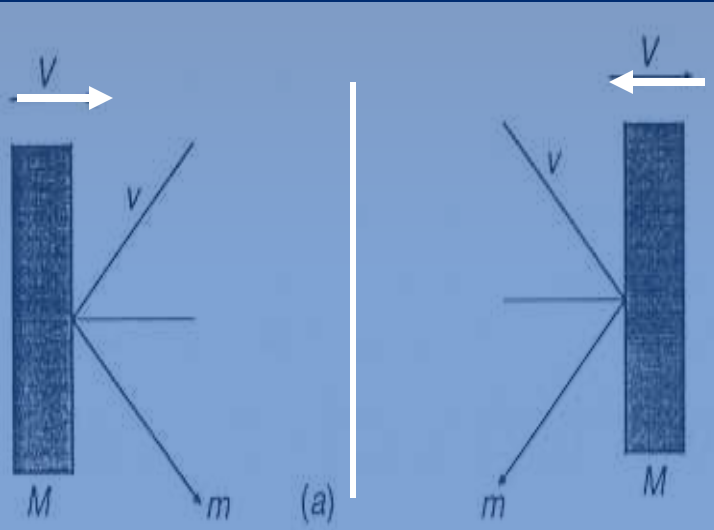
Because of the stochastic nature of the process we cannot follow a single particle, but we can work out the average energy gain.

The basic property is that when a particle enters in the scattering region, because of the random distribution of the scattering centers, also its direction gets very rapidly randomized –key point– so that the average speed of the test particle is the same as for the flow and an isotropic distribution of directions can be assumed in a suitable ref. frame (the shock rest frame).



$$\langle \Delta E/E \rangle = \int_{\theta_1}^{\theta_2} P(\Omega) \frac{\Delta E}{E} (\cos \theta) d\Omega$$

DSA: Energy gain (1st order Fermi mechanism)



TP(θ) is prop. to the normal comp. of the particle velocity along the normal to the shock, $v \cos \theta$:
 $P(\Omega) d\Omega = (\cos \theta / \pi) (2\pi \sin \theta d\theta) = 2 \cos \theta d(\cos \theta)$ with $0 < \theta < \pi/2$

$$\langle \Delta E / E \rangle \approx \int p(\Omega) d\Omega [2Vv \cos \theta / c^2 + 2(V/c)^2]$$

$$\langle \Delta E / E \rangle = \frac{4}{3} \left(\frac{V}{c} \right) \left(\frac{v}{c} \right) \quad V \ll c$$

V is NOT the shock speed, but relative speed $|u_u - u_d|$ of the flow speed upstream and downstream in the SRF (i.e. Standing shocks).

In the lab, $u_u = U$, the speed of shock front and downstream con $u_d = (\rho_u / \rho_d) U \rightarrow V = (1 - \rho_u / \rho_d) U$

For $V \ll c$, $\langle \Delta E / E \rangle$ is of 1st order in V/c and is maximal for relativistic particles $v \approx c$

$$\langle \Delta E / E \rangle = \frac{4}{3} \left(\frac{V}{c} \right)$$