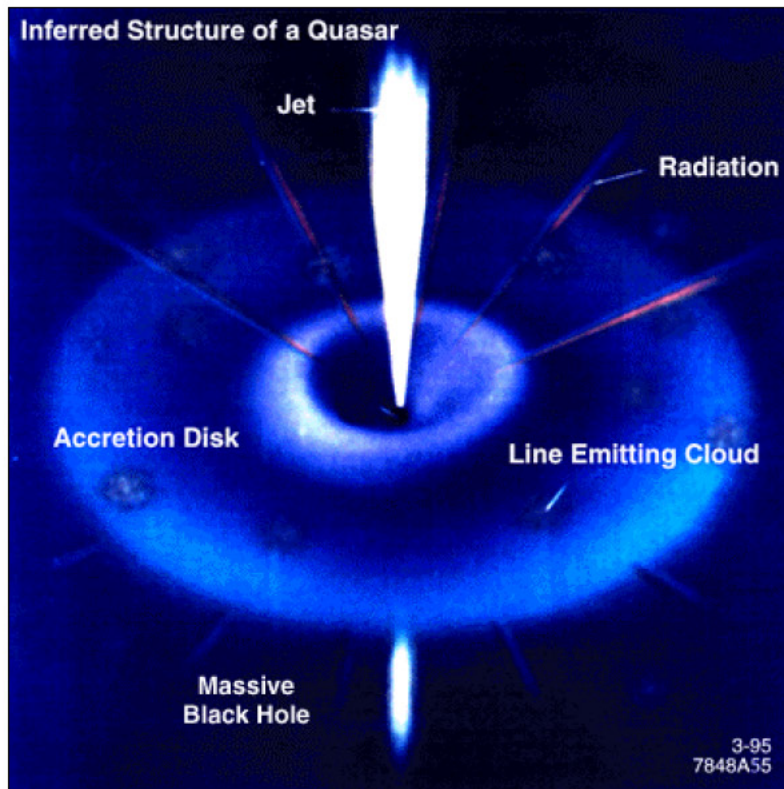


Lecture 5 171018

- Slides will be available at:

https://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1819

Power Source: BH Accretion



AGN model

- AGN are likely powered by accretion onto BH's of mass between $10^6 - 10^9$ solar masses.
- Matter falling in towards BH piles up in a rotating accretion disk.
- Released energy powers a broad spectrum of strong emission and jets of relativistic outflow.

Confinamento dei RC $E > 10^{19}$ eV

- Confinamento dovuto al campo magnetico Galattico (3×10^{-10} T).

Il raggio di curvatura di un nucleo di carica (Ze) in B:

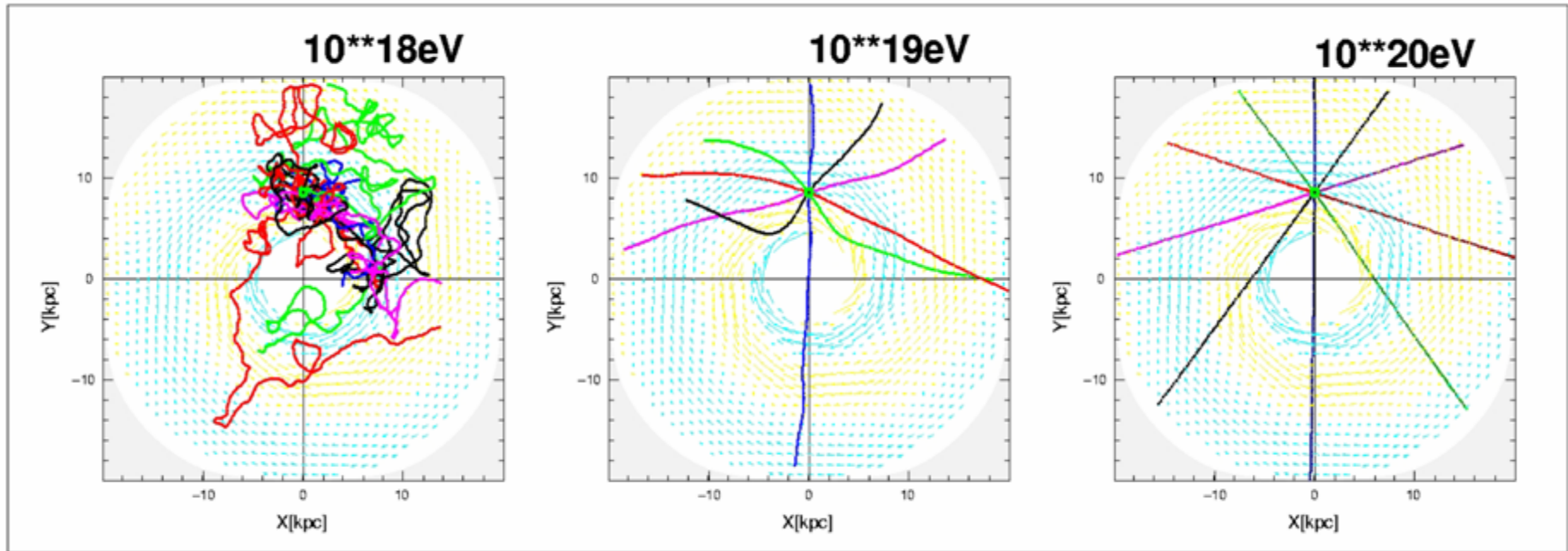
$$mv^2/r = pv/r = Ze v B / c$$

$$r = E / (ZeB)$$

$$r(kpc) \cong \frac{E(EeV)}{ZB(\mu G)}$$

- Per un protone di $E \sim 10^{19}$ eV, $r \sim 3.3$ kpc
(corrisponde a $\sim 10 \cdot$ spessore galattico)
- Un Fe potrebbe restare confinato a 10^{19} eV
 $r = 10(EeV) / 26 \cdot 3(\mu G) \approx 100$ pc

Confinamento 2



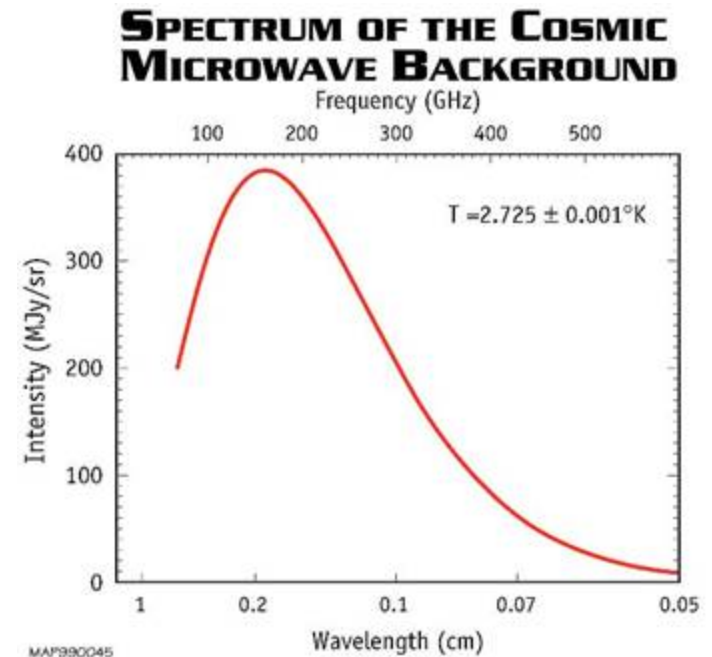
$\sim 10^{18}$ eV: RC ben confinati nella galassia

$\geq 10^{19}$ eV: sorgenti extragalattiche

$\sim 10^{20}$ eV la deviazione nella galassia è inferiore ad 1°

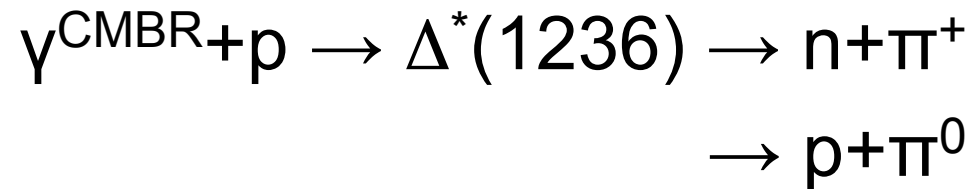
Volume di confinamento dei RC di origine extragalattico: il Cut-off di Greisen (GZK)

- L' universo è permeato dalla Radiazione Cosmica di Fondo a 3° K (CMBR) → 160.2 GHz
- CMBR: fotoni di energia
 $E_{\gamma}^{\text{CMBR}} = h\nu = 2\pi \times 6 \cdot 10^{-22} \text{ MeV} \cdot \text{s} \times 160.2 \cdot 10^9 \text{ Hz} = 6 \cdot 10^{-4} \text{ eV}$
- La densità dei fotoni di fondo è $\sim 400/\text{cm}^3$
- Il fondo di radiazione pone un limite sulla distanza massima da cui i RC possono provenire.



$G_{\text{reisen}} Z_{\text{atsep}} K_{\text{uzmin}}$ cutoff Soglia per reazioni di fotoproduzione

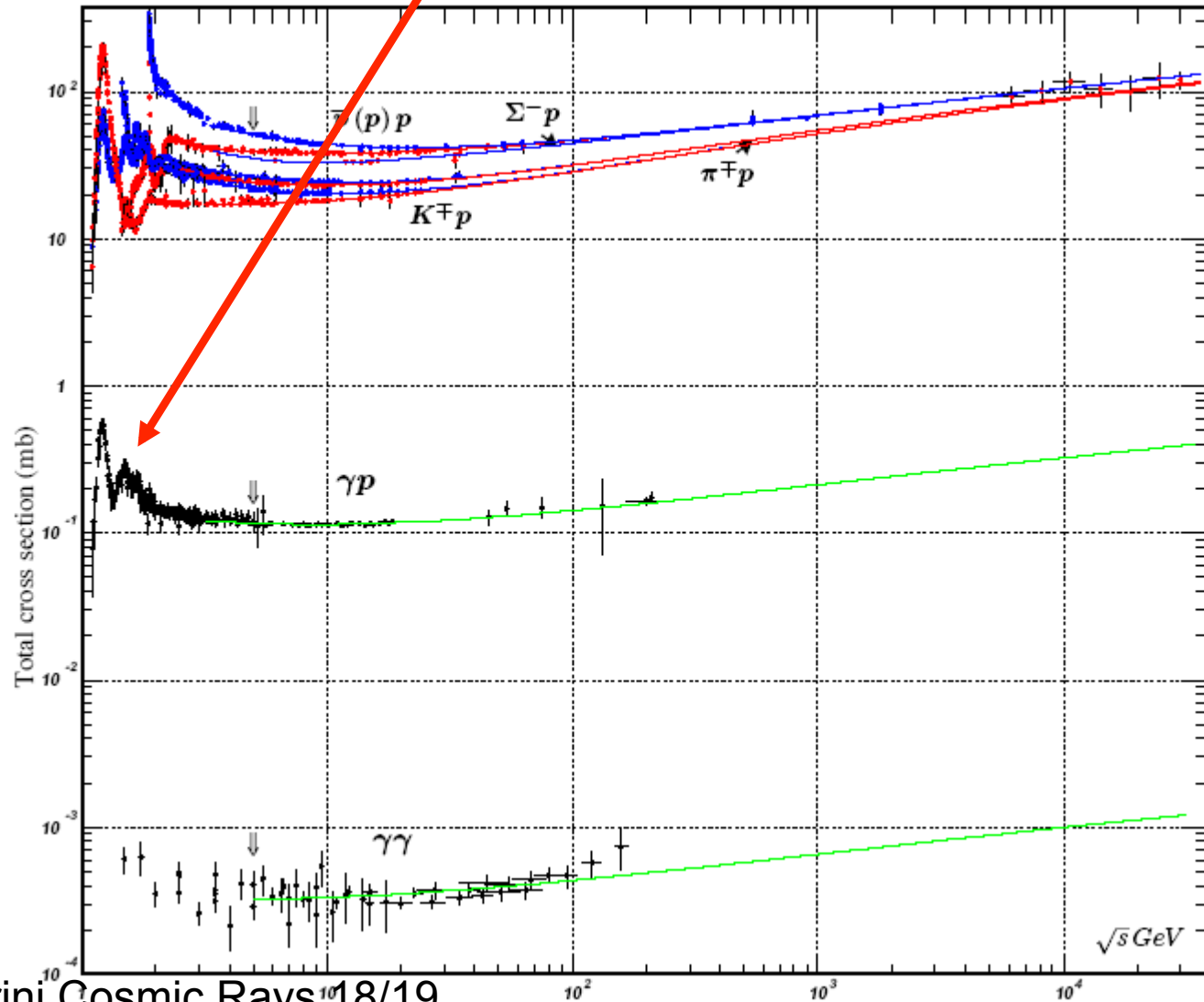
- **Fotoproduzione:** Protoni di alta energia possono interagire con fotoni, producendo un pione:



- È necessario essere sopra la soglia di fotoproduzione nel sistema di riposo del protone: $E_0^{\text{FP}} \approx 300 \text{ MeV}$
- Il processo ha una sezione d'urto in risonanza

$$\sigma_0^{\text{P}} \approx 250 \text{ } \mu\text{b}$$

Sezione d'urto per la Δ



Soglia per la fotoproduzione (FP)

SdR del CMB	SdR protone in quiete	
$p^\mu = (\gamma m_p \vec{v}_p c, \gamma m_p c^2)$	$\vec{p}'^\mu = (0, m_p c^2)$	
$k^\mu = (\frac{E_\gamma^{CMBR}}{c} c \hat{k}, E_\gamma^{CMBR})$	$\vec{k}'^\mu = (E_0^{FP} c \hat{k}' / c, E_0^{FP})$	

- Vogliamo determinare il valore di γ tale che si abbia un valore $E_0^{PF} > 300 \text{ MeV}$
- La trasformazione di Lorentz tra i due SdR:

$$\begin{aligned}
 E_0^{FP} &= \gamma \left(E_\gamma^{CMBR} + \frac{v_p}{c} E_\gamma^{CMBR} \right) = \\
 &= \gamma E_\gamma^{CMBR} (1 + v_p / c) \approx 2\gamma E_\gamma^{CMBR}
 \end{aligned}$$

**E_0^{FP} = Soglia
 fotoproduzione (~300 MeV)**
 E_γ^{CMBR} = Energia fotone = $h\nu$
 **γ = boost Lorentz del
 protone**

- Dalla relazione, si ricava il valore di γ necessario per la fotoproduzione nel sistema di riferimento del laboratorio

$$\gamma = \frac{E_0^{FP}}{2E_\gamma^{CMBR}} \approx \frac{300MeV}{2 \cdot 4.7 \cdot 10^{-4} eV} \approx 3 \cdot 10^{11}$$

- L'energia di soglia per i protoni per produrre π è quindi

$$E_0^{GZK} = \gamma(m_p c^2) \approx 3 \cdot 10^{20} eV$$

Se l'energia del protone supera E_0^{GZK} , si innesca la FP.

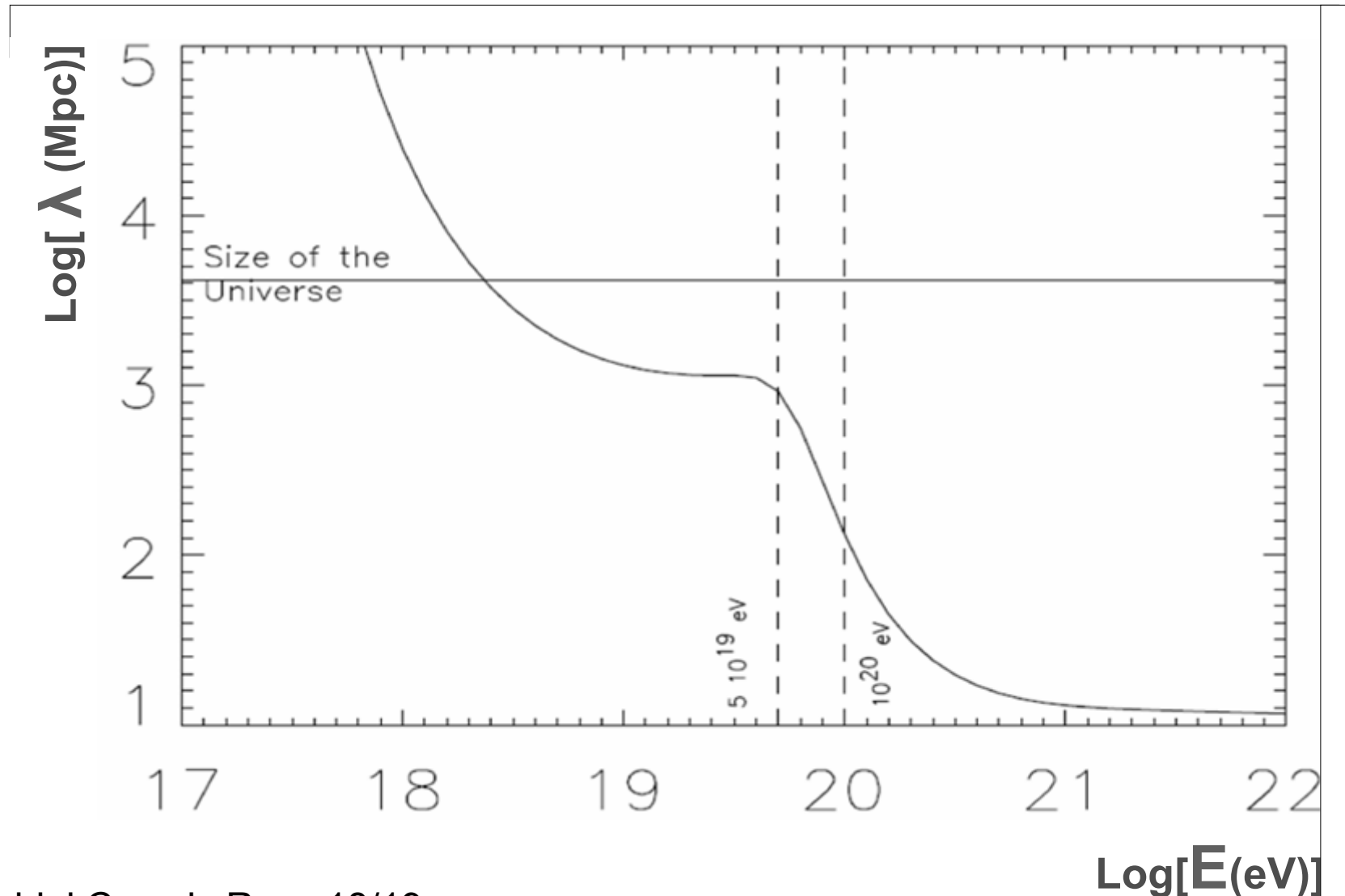
In ogni processo, il p perde circa 1/10 della sua energia

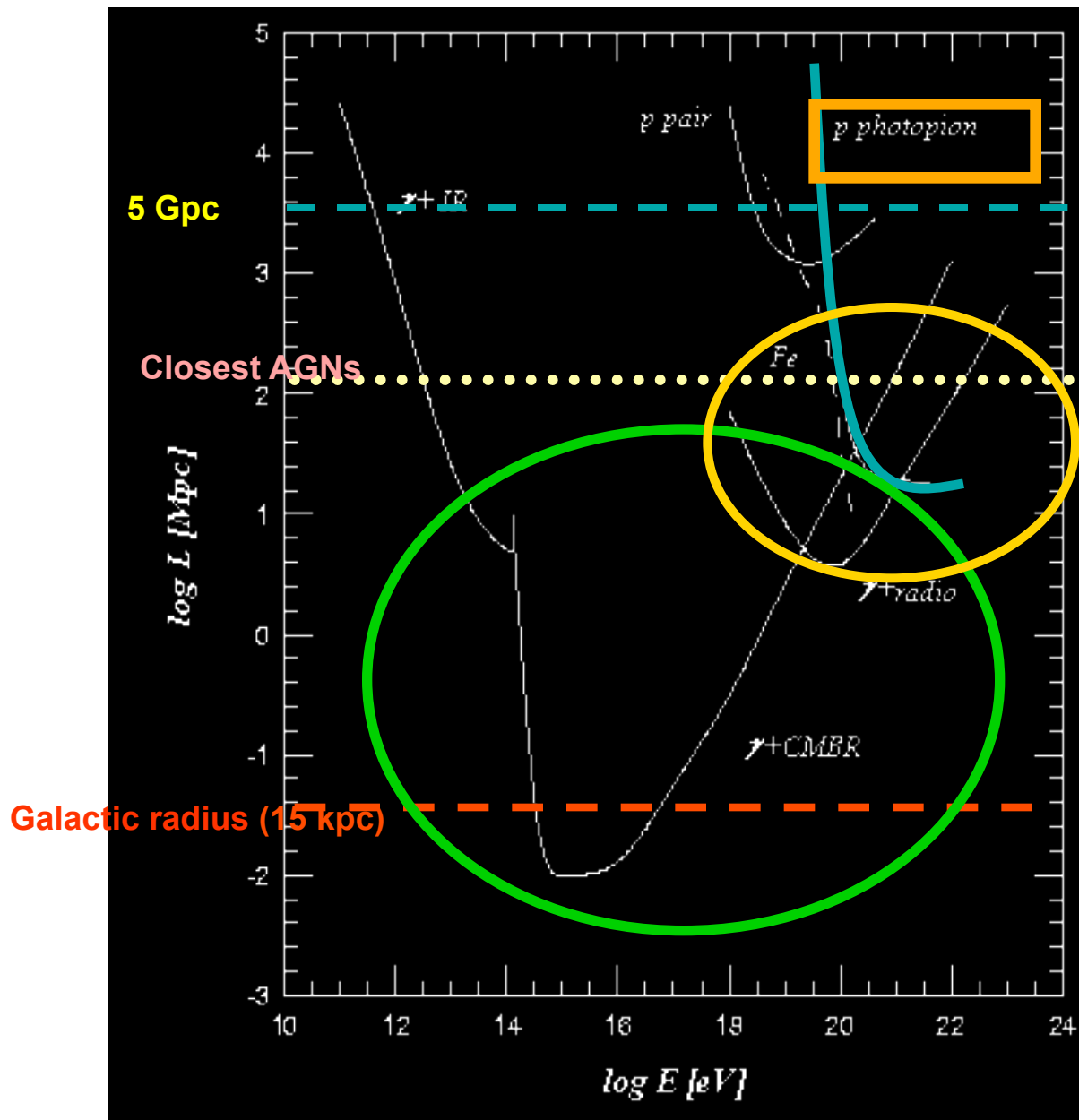
Nota la densità numerica della CMBR ($n_\gamma = 400 \text{ cm}^{-3}$), si stima il cammino libero medio del p:

$$\lambda = (\sigma_{p\gamma} n_\gamma)^{-1} = 10^{25} \text{ cm} = 3 \text{ Mpc}$$

Si può dunque stimare che i p NON possano giungere da distanze superiori a
 $10 \times 3 \text{ Mpc} = 30 \text{ Mpc}$

Figura: Risultato di calcoli dettagliati.





L'Universo NON è
trasparente ai protoni
di altissima energia

L'Universo NON è
trasparente ai fotoni
di alta energia

Ricerca delle sorgenti

- Nella reazione di fotoproduzione (responsabile del cutoff di GZK) sono prodotti $\pi^\pm$, π^0 che decadono:

$$\pi^+ \rightarrow \nu_\mu \mu^+ \rightarrow \nu_\mu \nu_\mu \nu_e e^+$$
$$\pi_0 \rightarrow \gamma\gamma$$

- Neutrini e fotoni di altissima energia possono quindi essere studiati per confermare il meccanismo GZK, e per localizzare le sorgenti di RC a $E > 10^{19}$ eV

Il RC di più alta energia osservato: $\gtrsim 3.2 \cdot 10^{20}$ eV
(?)

Se le sorgenti **non possono** essere troppo lontane (<30 Mpc), possiamo cercare di localizzarle tramite:

- Studi di anisotropia con esperimenti di RC
- Confronto con altre misure astronomiche

Osservabili

- ❑ Energy distribution → Flux and Spectrum
- ❑ Particle Type: p, He, Li, Be,.. → Composition
- ❑ Arrival Direction → Anisotropy

Spettro di energia

Lo spettro differenziale e' il numero di particelle che attraversano l'unita' di superficie per unita' di tempo, di angolo solido e di energia

$$\Phi(E) = dN/dEdSdtd\Omega \text{ [# part. cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{Gev}^{-1}]$$

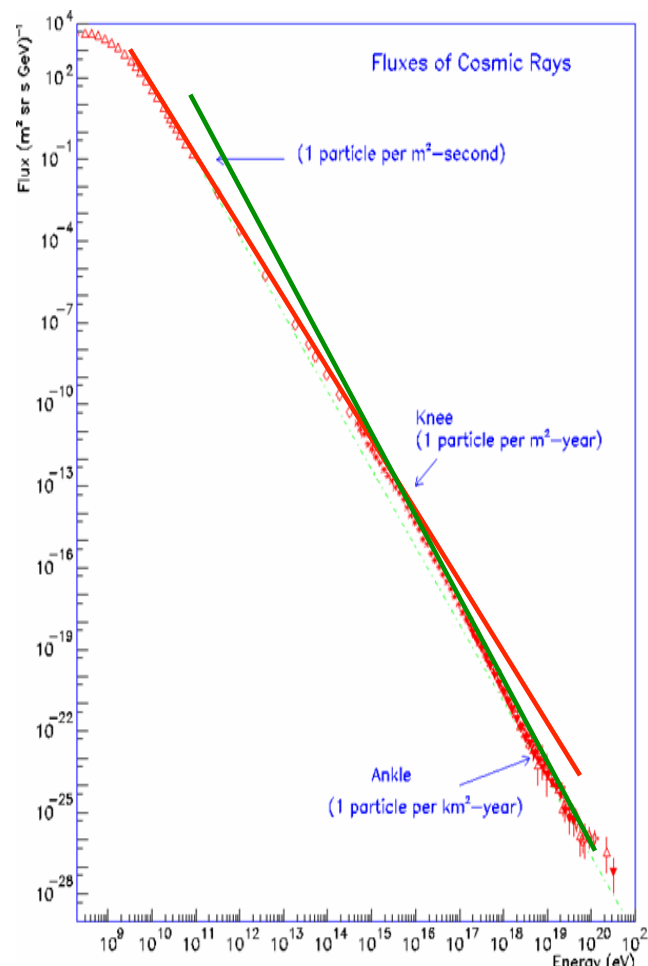
→ da informazioni sui processi di accelerazione alle sorgenti e di propagazione nel mezzo interstellare delle particelle

- Lo spettro energetico può essere descritto dalla legge **(flusso o spettro differenziale dei RC:)**

$$\phi(E) = k \cdot E^{-\gamma} \quad (cm^{-2} s^{-1} sr^{-1} GeV^{-1})$$



- Legge che descrive i dati sperimentali.
- I parametri k e γ vengono determinati dall'adattamento della curva coi dati
- Regolarità di un fenomeno fisico
- Legge di natura!
- Occorre scoprire quali sono i fenomeni naturali che producono tale legge di natura.



Spettro integrale

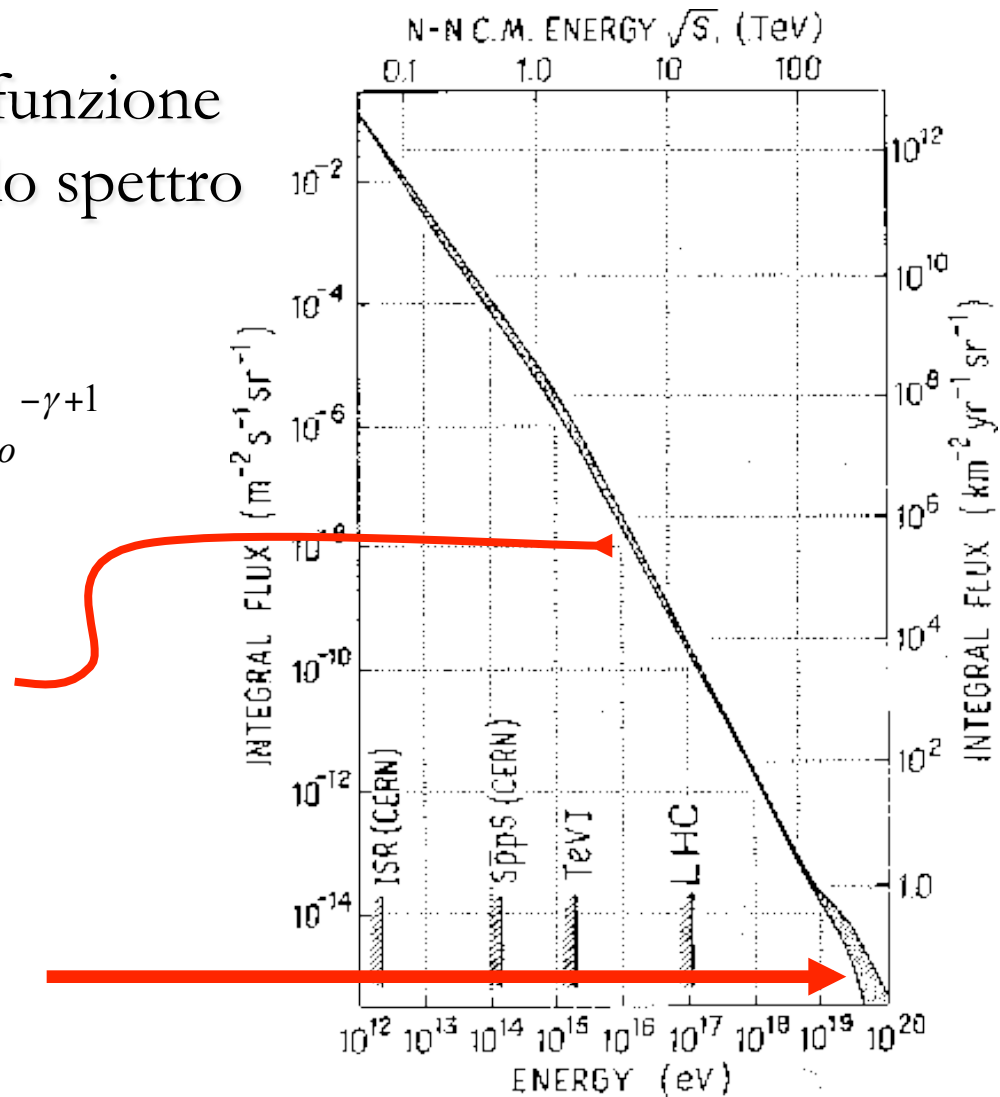
■ Dallo spettro differenziale (in funzione dell'energia), si può passare allo spettro integrale:

$$\Phi(> E_0) = \int_{E_0}^{\infty} \phi(E) dE = \left(\frac{k}{\gamma - 1} \right) E_0^{-\gamma+1}$$

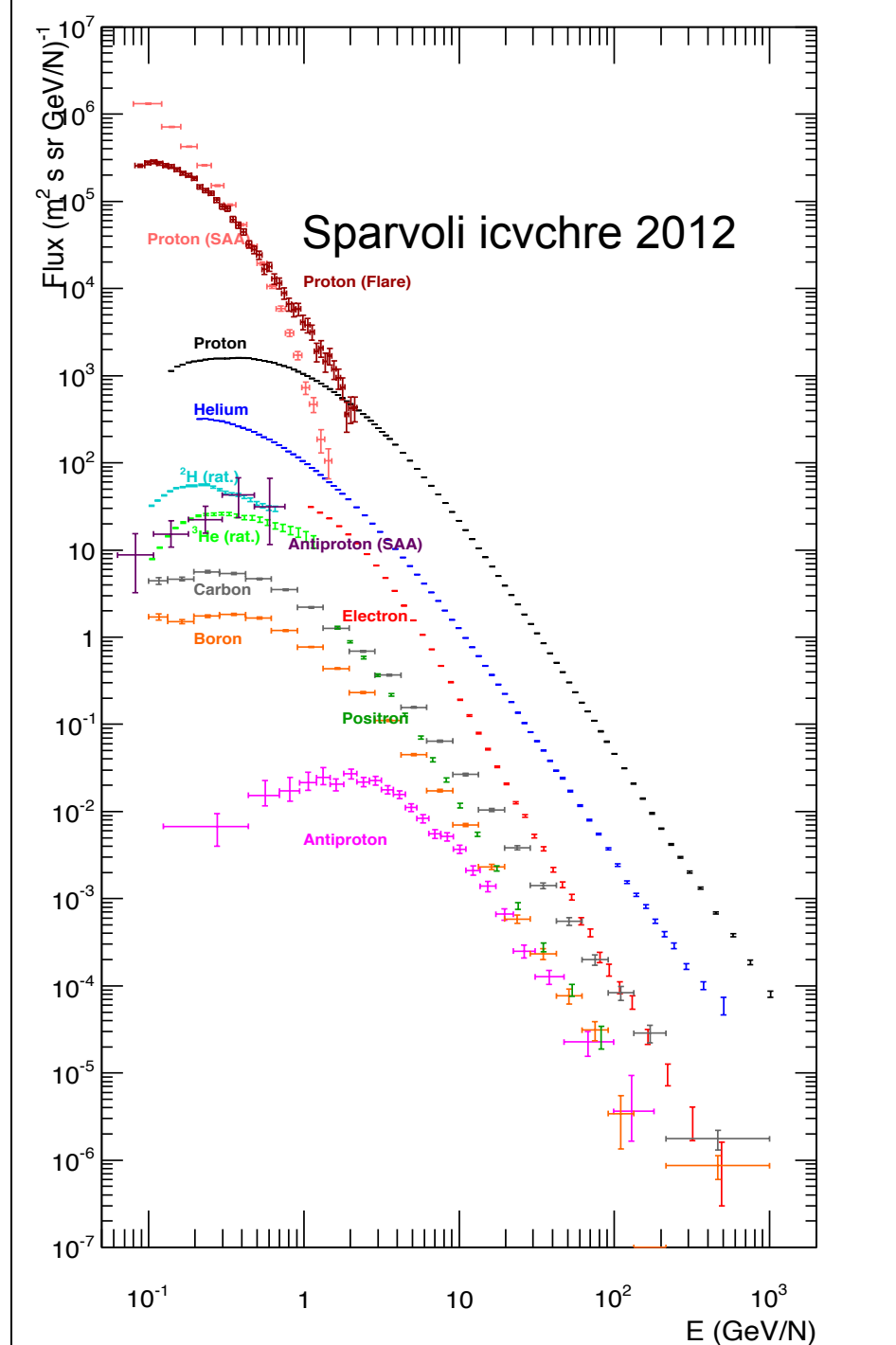
$$(m^{-2} s^{-1} sr^{-1})$$

■ 1 PC = 1 RC durante una partita di calcio = $3 \cdot 10^8 m^2 \cdot s \cdot sr$

■ 1 GRAnno = 1 RC entro il Grande Raccordo Anulare all'anno = $3 \cdot 10^2 km^2 \cdot y \cdot sr$



- ❑ Power law for all the particles
- ❑ May be not a single power law (eg spectrum hardening at high E)
- ❑ Non thermal spectra → acceleration mechanisms at work



Heavier elements: hardening

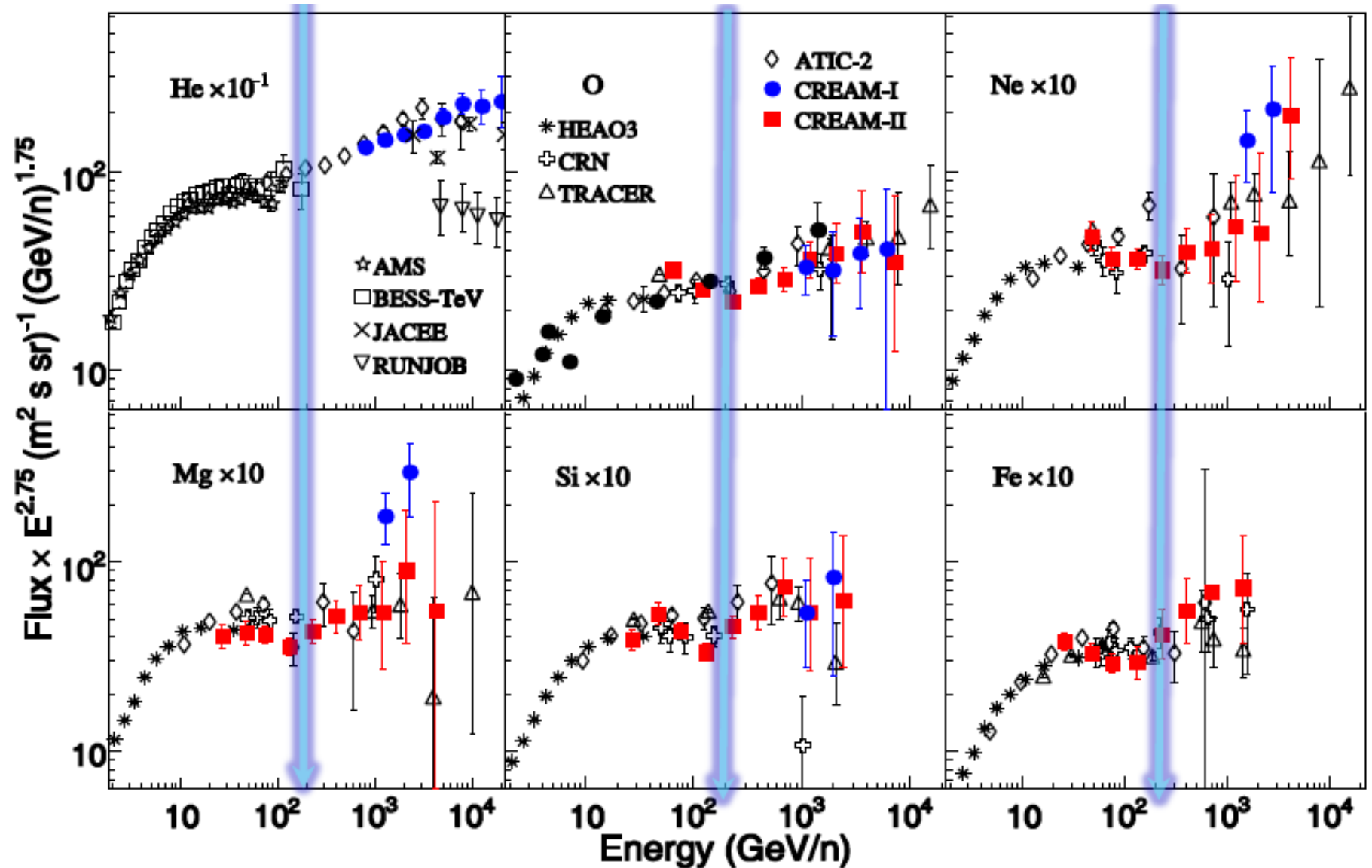
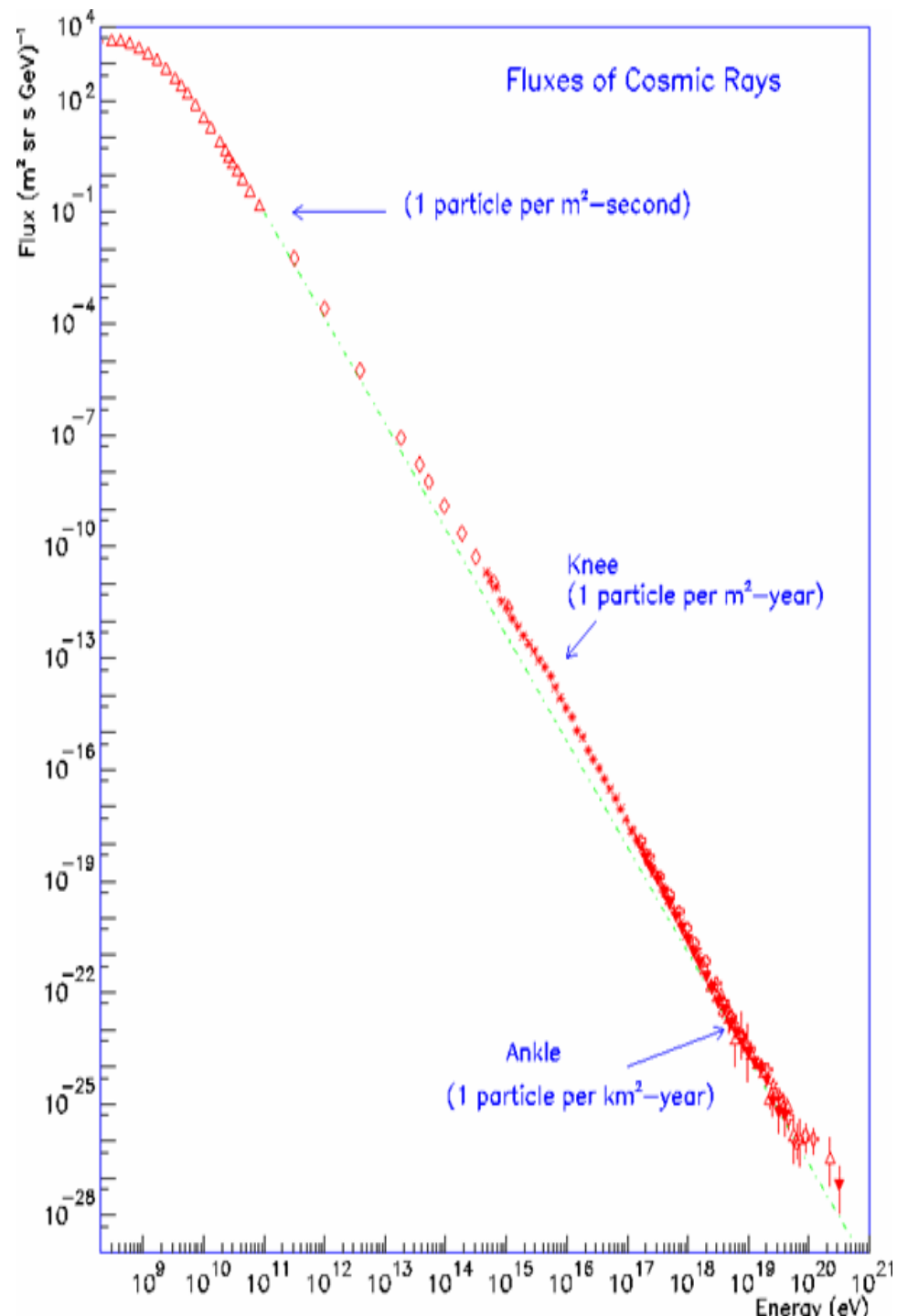


Fig. 1

- $\Phi_{\text{TOT}} \sim 10000 \text{ m}^{-2}\text{s}^{-2}\text{sr}^{-1}$
- Misure dirette: 85% p, 12% He, ~1% nuclei pesanti, ~2% $e^{\pm}$, antiprotoni
- Si estende per 13 ordini di grandezza in energia
- Per 32 ordini di grandezza in flusso
- Legge di potenza su tutto lo spettro, con almeno due cambi di pendenza



Densità numerica e di energia dei RC

- Il flusso di RC sulla terra: $\Phi(E) = k \cdot E^{-\gamma} \quad (cm^{-2} s^{-1} sr^{-1} GeV^{-1})$
 $k = 1.8, \gamma = 2.7 \quad 10 < E < 10^6 GeV$
- Per energie $E < \sim 10 GeV$ dominano i contributi dal sole;
- Per $E > 3 \cdot 10^6 GeV$, cambio di pendenza nello spettro
- Possiamo calcolare il flusso di RC integrando a partire da 1 GeV (per eliminare il contributo solare):

$$\Phi(> 1 GeV) = \int_{E_0}^{10^6} \phi(E) dE = \frac{k}{\gamma - 1} \left(E^{-\gamma+1} \right)_{3 \times 10^6 GeV}^{1 GeV} = \frac{1.8}{2.7 - 1} (1)^{-1.7} =$$

$$= 1 \quad (cm^{-2} s^{-1} sr^{-1})$$

- Dal flusso, è possibile ricavare la densità numerica dei RC:

$$N_{CR} = 4\pi \cdot \frac{\Phi(> E_0)}{c} \quad (cm^{-3})$$

Esercizio: dal *flusso* alla *densità*

An important quantity in astrophysics is the number density (# part cm^{-3}) of CRs moving with velocity v . The number density n corresponds to the number of particles present in a given volume at a given time.

Any volume is continuously crossed by CRs at given rate.

If we imagine of taking a snapshot of the particles present in the test volume at a given time and counting them, we could derive the number density of particles in the test volume.

Since the flux is isotropic for a spherical surface we have

$$\frac{dN}{dSdt} (\text{cm}^{-2} \text{s}^{-1}) = 4\pi(\text{sr})\Phi(\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1})$$

Therefore $dN = 4\pi(\text{sr})\Phi(\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1})dSdt$

If the particle move at speed v then $dt = dl/v$

$$dN = 4\pi(\text{sr})\Phi(\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1})dSdl / v = 4\pi(\text{sr})\Phi(\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1})dV / v$$

$$n(\text{cm}^{-3}) \equiv \frac{dN}{dV} = \frac{4\pi}{v} \cdot \Phi$$

- Densità numerica dei RC, stimata dalla misura del flusso:

$$n_{CR} = \frac{4\pi}{c} \cdot \Phi(> E_o) = \frac{4\pi \cdot 1}{3 \cdot 10^{10}} \approx 4 \cdot 10^{-10} (cm^{-3})$$

- Stima della densità di energia dei RC:

$$\begin{aligned} w_{CR} &= \int_0^{\infty} E \cdot n(E) \cdot dE = \int_0^{\infty} E \cdot \left(\frac{4\pi}{c} k E^{-\gamma} \right) \cdot dE = \\ &= \frac{4\pi}{c} \frac{k}{\gamma - 2} \left[E^{-\gamma+2} \right]_{3 \times 10^6 GeV}^{GeV} = \frac{4\pi}{c} \frac{1.8}{2.7 - 2} (1)^{-0.7} \cong 10^{-9} GeV / cm^3 \end{aligned}$$

$$w_{CR} \cong 1 \quad eV / cm^3$$

- **Domanda: Come possiamo confrontare questo numero?**
- **E' “grande” o “piccolo” su scala dei fenomeni astrofisici?**

- Densità di energia del campo magnetico galattico ($B=3 \times 10^{-6}$ G)

$$w_B = \frac{1}{8\pi} B^2 = \frac{1}{8\pi} (3 \cdot 10^{-6})^2 = 4 \cdot 10^{-13} \frac{\text{erg}}{\text{cm}^3} = 0.2 \frac{\text{eV}}{\text{cm}^3}$$

- Densità di energia della radiazione cosmica di fondo a 3 K

$$w_{CMB} = \left(500 \frac{\text{fotoni}}{\text{cm}^3} \right) \cdot 3kT = 500 \times 3 \times 3 \times 8 \cdot 10^{-5} (\text{eV} / K) = 0.4 \frac{\text{eV}}{\text{cm}^3}$$

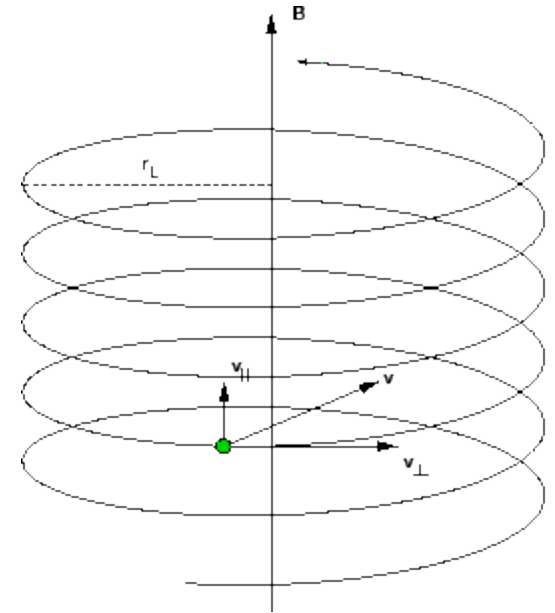
- Luce delle stelle (da misure fotometriche)

$$w_o = 10^{-2} \frac{\text{eV}}{\text{cm}^3}$$

- La densità di energia che compete ai RC ($1 \text{ eV}/\text{cm}^3$) è dunque importante su scala galattica.

Raggio di curvatura di una particella in moto in un campo magnetico

- Determiniamo il raggio di curvatura (denominato *raggio di Larmor*) di una particella con carica q ed energia E in moto in un campo magnetico $\mathbf{B}$.



$$m \frac{v^2}{r} = \frac{pv}{r} \stackrel{\text{Lorentz}}{=} Ze \cdot \frac{v}{c} \cdot B$$

$$r = \frac{pc}{ZeB} \cong \frac{E}{ZeB}$$

$$r_{Larmor} = \frac{1.6 \times 10^{-12} [\text{erg} / \text{eV}] \cdot E [\text{eV}]}{Z \cdot [4.8 \times 10^{-10} \text{ u.e.s.}] B [\text{Gauss}]} = \frac{1}{300} \frac{E}{ZB} (\text{eV} / \text{Gauss}) [\text{cm}]$$

Confinamento:

$$r_{Larmor} = \frac{1}{300} \frac{E}{ZB} (eV / Gauss) [cm]$$

- Utilizziamo i valori tipici del campo B (3×10^{-6} G) galattico per protoni:

$$r_L = \begin{cases} (E = 10^{12} eV) & = 10^{15} cm = 3 \cdot 10^{-4} pc \\ (E = 10^{15} eV) & = 10^{18} cm = 0.3 pc \\ (E = 10^{18} eV) & = 10^{21} cm = 300 pc \end{cases}$$

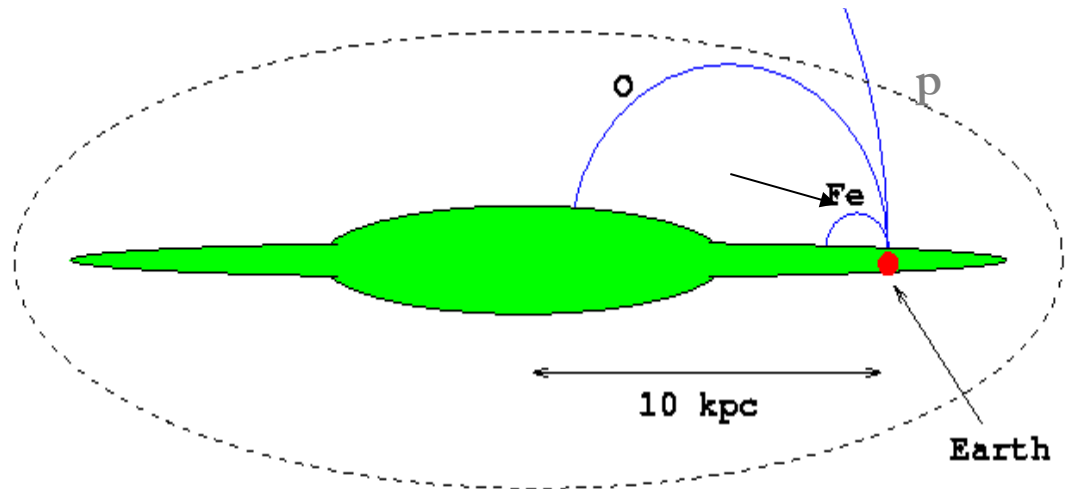
- I p hanno un raggio di Larmor sempre minore dello spessore del disco galattico (300 pc) se $E < 10^{18}$ eV. Per questo motivo tutti i RC (meno quelli di energia estrema) sono *confinati* nel piano Galattico dal campo magnetico.

Confinamento

→ for a proton with 10^{14} eV, $r \approx 3 \times 10^{17}$ cm ≈ 0.1 pc, which is $\ll$ of galactic disk (≈ 0.3 kpc) and galactic halo (≈ 10 kpc) and $\ll$ of all the likely sources of CR near Earth, like Crab nebula → low energy CR do not point sources (ie there are not potential sources within 1 pc), their directions are randomized by the magnetic field
AND
they are fully contained by galactic magnetic field → their origin must be galactic

$$r_{Larmor}(cm) = \frac{1}{300} \frac{E}{ZB} (eV / Gauss)$$

$$B \approx 10^{-6} \text{ G or } 10^{-10} \text{ T}$$



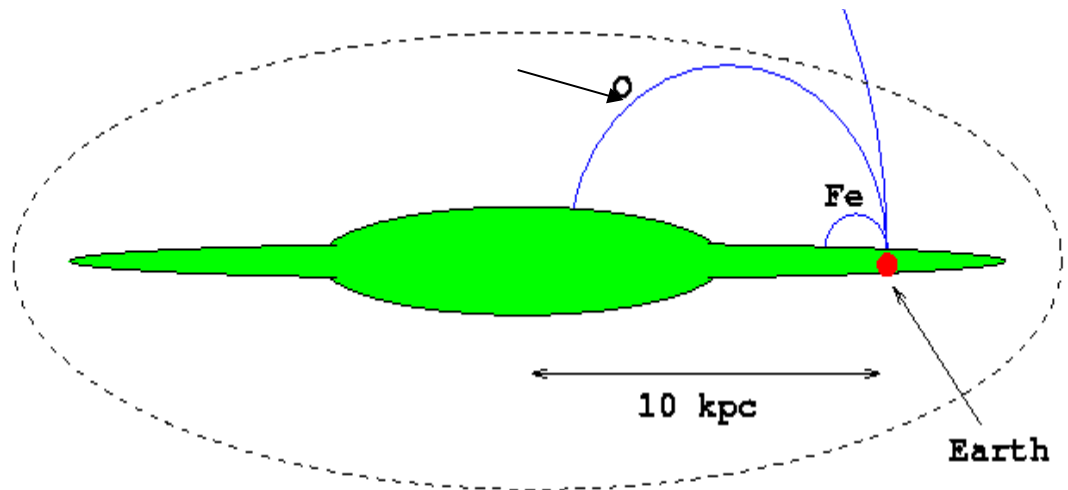
Confinamento

→ for a proton with 10^{18} eV, $r \approx 3 \times 10^{21}$ cm ≈ 300 pc of the same order of galactic disk thickness → the trapping by magnetic field starts to be inefficient → presumably CR with much higher E are extragalactic, unless they can be associated with a local source, which could be possible as the direction of such CR would point back to the source (on scales $<$ of the galactic radius the deflection would be small)

NB: no local sources found up to now

$$r_{Larmor}(cm) = \frac{1}{300} \frac{E}{ZB} (eV / Gauss)$$

$$B \approx 10^{-6} \text{ G or } 10^{-10} \text{ T}$$



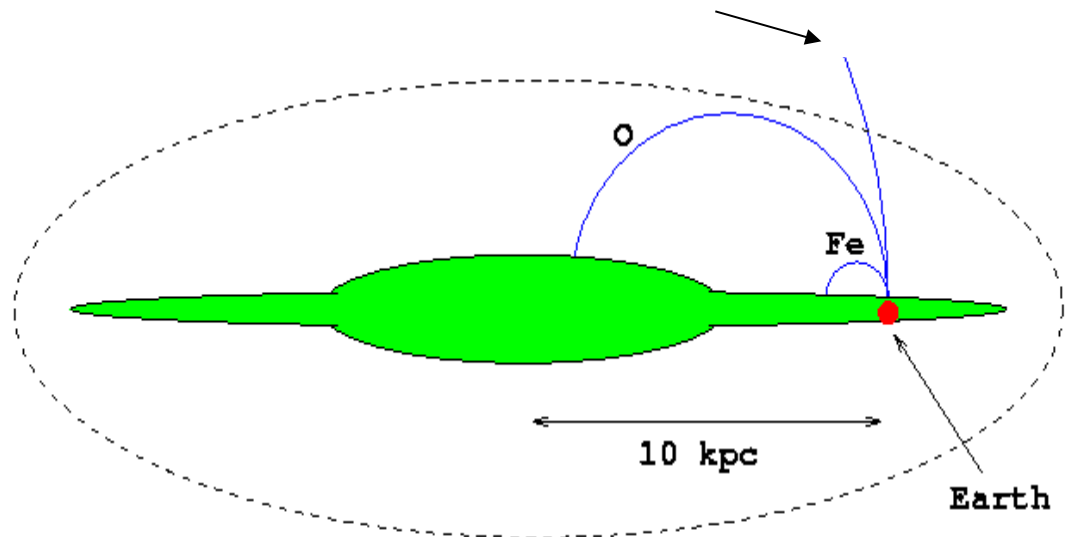
Confinamento

$$r_{Larmor}(cm) = \frac{1}{300} \frac{E}{ZB} (eV / Gauss)$$

$$B \approx 10^{-6} \text{ G or } 10^{-10} \text{ T}$$

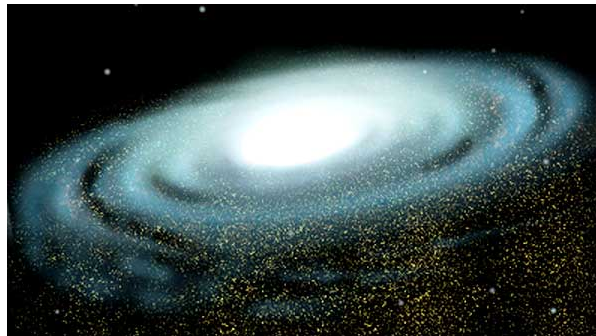
→ for a proton with 10^{20} eV, $r \approx 3 \times 10^{23}$ cm ≈ 100 kpc $\gg$ galactic radius → the B field can not trap the CR and would change the initial direction negligibly → they (would) point back to the sources, likely extragalactic, unless, as before local sources are identified (no one found yet)

If this is true one would expect a possible anisotropy in the angular distribution of CR arrival directions



Isotropia dei RC

- I RC primari al di sotto di 10^{18} eV hanno una distribuzione di arrivo completamente isotropa sulla sommità della nostra atmosfera. **Qualè il motivo?**
 - Campi magnetici **galattici**:
 - $B \cong 3 \times 10^{-6}$ G
 - coerenti su scale di distanza 1-10 pc
 - NOTA: $1 \text{ pc} = 3 \times 10^{18} \text{ cm}$
 - **Galassia** $\equiv$ disco di raggio $R = 15 \text{ kpc}$, spessore $h = 300\text{-}1000 \text{ pc}$



Cosmic Ray Isotropy

Anisotropies are often quoted in terms of the parameter δ :

$$\delta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Where $I_{\max}$ and $I_{\min}$ are the minimum and maximum intensities measured in all directions.

Isotropy

In case of ideal uniform distribution of sources, we would expect the same intensity from all directions in the sky

But the sources are not uniformly distributed and a variety of processes can alter the original direction of a cosmic ray, as for instance collision-less interactions with the irregular galactic magnetic field

Isotropy (cont.)

- So far, experimental results indicate only small amounts of anisotropy at low energies, with δ increasing with E .
- Below $E \sim 10^{10}$ eV, solar modulation hides the original directions.
 - For higher energies, direction of maximum excess is close to that of the Local Supercluster of Galaxies.

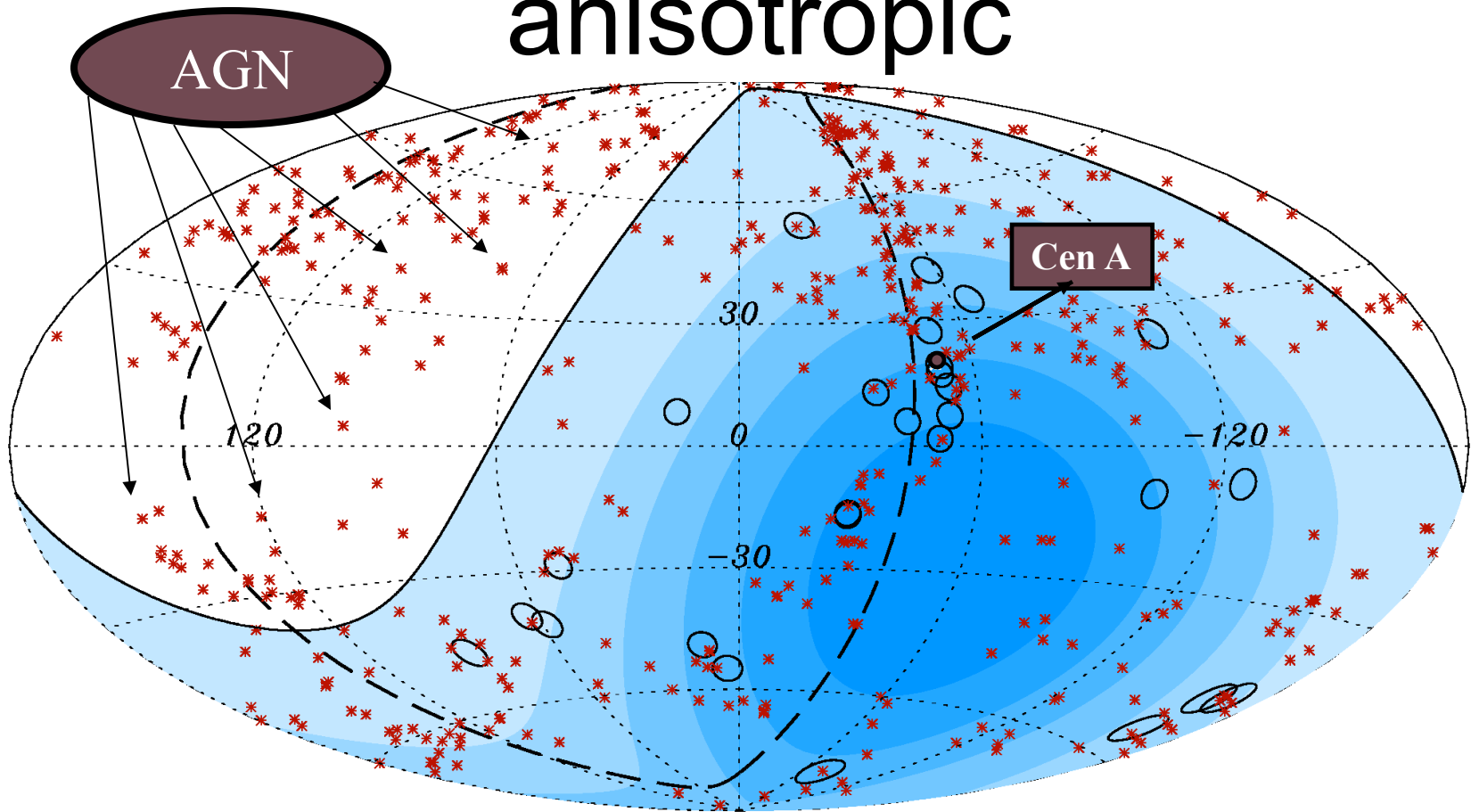
Isotropy Table

<u>Log E (eV)</u>	<u>$\delta(\%)$</u>
12	~0.05
14	~0.1
16	~0.6
18	~2
19-20	~20+

The direction of the maximum excess at the highest energies is close to that of the local supercluster of galaxies.

They would point the sources but they are so low in number that sources are not yet identified

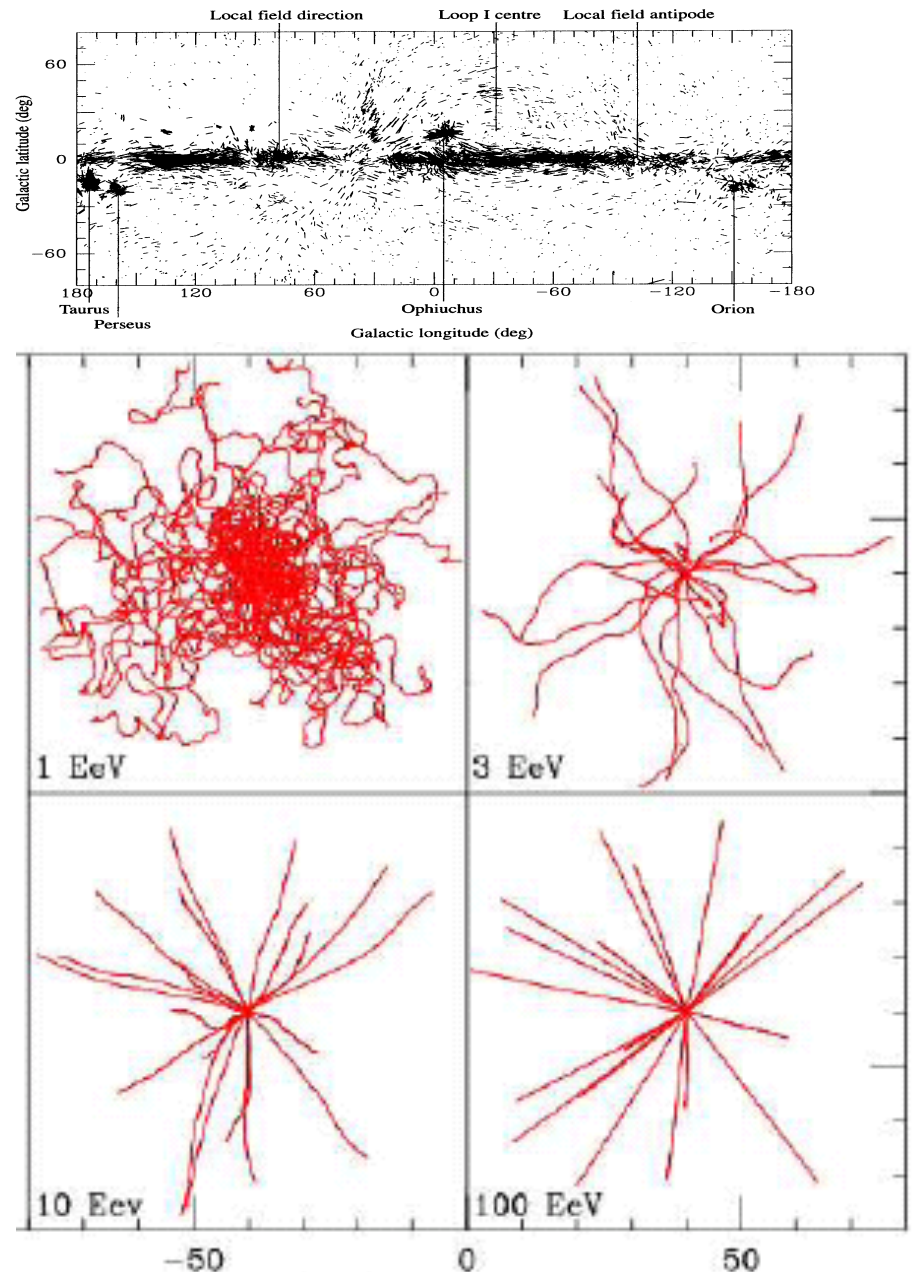
II. The UHECR Sky is anisotropic



Pierre Auger Collaboration 2007, Science, 318, 939
Pierre Auger Collaboration 2008, APh, 29, 188

B field direction

- Charged particle primaries are significantly deflected by magnetic fields
- Right: proton primaries propagating through random magnetic fields
 - B direction random
 - Cell size = 1 Mpc
 - $B = 10^{-9}$ gauss



EeV = 10^{18} eV

Diagram from Cronin (TAUP 2004),

Confinamento

Il cammino dei raggi cosmici sotto la caviglia NON e' "liscio" e continuo, ma e' assimilabile a un cammino casuale governato dalla distribuzione di radiazione, materia e campi magnetici nella galassia con cui essi interagiscono (cfr. diffusione)



La direzione di arrivo dei raggi cosmici carichi NON punta alla sorgente che li ha generati ma il loro flusso e' isotropo fino ad energie molto elevate

Potenza delle sorgenti dei RC

- Il *confinamento* dei RC ci induce a sospettare che le sorgenti siano di origine Galattica (tranne che per i RC di energia estrema).
- Qual è *l'energetica* delle sorgenti? (necessaria per individuarle).
 - Il tempo di confinamento dei RC: $\tau = 3 \times 10^7$ y (calc. Fra poco)
 - Volume della galassia (con o senza alone) :

$$V_G = (15 \text{ kpc})^2 \cdot \pi \cdot 300 \text{ pc} \cong 6 \times 10^{66} \text{ cm}^3$$

$$V_G^{\text{Alone}} = 4/3 \cdot \pi (10 \text{ kpc})^3 \cong 10^{68} \text{ cm}^3$$

- Potenza necessaria per mantenere uno *stato stazionario* di RC:

$$W_{CR} = \frac{w_{CR}(\text{erg} / \text{cm}^3) \cdot V_G^A(\text{cm}^3)}{\tau(s)} = \frac{10^{68} \times 1.6 \cdot 10^{-12}(\text{erg})}{3 \cdot 10^7 \times (3.15 \cdot 10^7 \text{ s})} \cong 10^{41} \frac{\text{erg}}{\text{s}}$$

Cosmic Rays from stars

- Ordinary stars *Too low!!!*
- Our Sun emits CR during flares but these have low energy particles (up to 10^{10} - 10^{11} eV)
- the power is only $\sim 10^{17}$ J/s,
- Integrated on all stars in galaxy (10^{11} stars in Galaxy) we get a total energy output of $\approx 10^{35}$ erg/s

Supernova Energetics

- Explosion powered by the collapse (death) of a massive core
- Energy source: Potential Energy from the collapse of the iron core down to a neutron star or black hole:

$$\text{Energy} = G M_{\text{core}}^2 / r_{\text{NS,BH}} - G M_{\text{core}}^2 / r_{\text{before collapse}} \\ > 10^{53} \text{ ergs}$$

$$M_{\text{core}} \sim 1.4\text{-}3 \text{ solar masses}$$

$$R_{\text{NS,BH}} \sim 10 \text{ km}$$

$$R_{\text{core}} \sim 10,000 \text{ km}$$

Only 1% of energy goes in kinetic energy of the particles, the rest is in neutrinos

Supernova energetics

Core Collapse Supernova Energetics

Liberated gravitational binding energy of neutron star:

$$E_b \approx 3 \times 10^{53} \text{ erg} \approx 17\% M_{\text{SUN}} c^2$$

This shows up as

99%	Neutrinos
1%	Kinetic energy of explosion (1% of this into cosmic rays)
0.01%	Photons (outshine host galaxy)

Neutrino luminosity

$$L_\nu \approx 3 \times 10^{53} \text{ erg} / 3 \text{ sec} \approx 3 \times 10^{19} L_{\text{SUN}}$$

While it lasts, outshines the photon
luminosity of the entire visible universe!

Esiste un meccanismo con una potenza tale da sostenere il flusso dei RC nella Galassia?

- Una esplosione di Supernova libera:

- $10^{51} \text{ erg/esplosione}$

- La stima della frequenza di SN nella nostra Galassia è

- $f_{\text{SN}} = 1/\tau_{\text{SN}} = 1/30 \text{ y}^{-1}$

- Si noti che $\tau_{\text{SN}} \ll \tau \approx 10^7 \text{ y}$. Le SN sono un fenomeno quasi continuo su scala dei tempi del confinamento dei RC.

- Potenza energetica liberata dalle SN:

$$W_{\text{SN}} = \frac{10^{51} \text{ erg}}{30 \times 3.15 \cdot 10^7 \text{ s}} \approx 10^{42} \text{ erg / s}$$

- Perché il quadro sia coerente, occorre trovare un meccanismo che trasferisca al più il 10% di energia dalle supernovae in energia cinetica di particelle (i RC) → **Meccanismo di Fermi**

Novae

Novae are also promising candidates for CR sources.

Novae are powered by the accretion onto a white dwarf: the matter is sucked from a companion star, perhaps a giant star or a main sequence star, under conditions that allow periodic runaway thermonuclear reactions on the white dwarf surface.

Novae outbursts occur with a frequency $\approx 100/\text{year}$ with an energy output of $\approx 10^{47}$ erg (10^{-4} of SN), thus the integrated novae output is similar to that of galactic SN

Formation and Interactions of CR's

Energy Supply: gravitational, nuclear, ELM,...

Provide energy for particles

Shock & Hydromagnetic waves, ELM Fields, Turbulent B fields,...

Store and transport energy

Processes transfer a fraction of E to particles at the sources : injection and acceleration

Particles escape into ISM from the acceleration sites

Relativistic particles = Cosmic rays

Particles interact with

Matter

"interstellar,
intergalactic medium"

B Fields

Photons

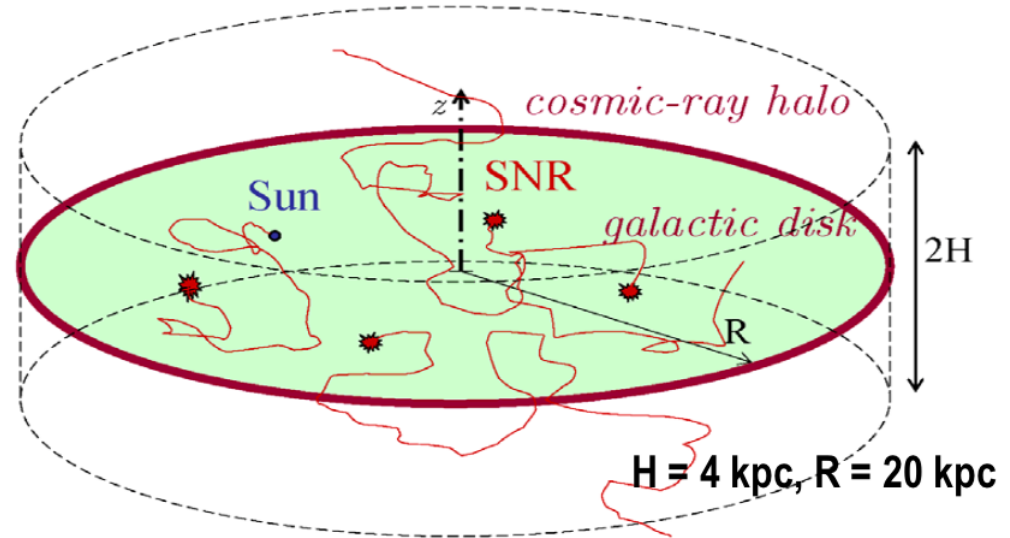
Ionization
Nuclear interactions
Bremsstrahlung

Synchrotron &
curvature radiation

Inverse Compton &
Thomson Scattering
Self-Absorption

Formation and Interactions of CR's

- Particles get accelerated at astrophysical sources
- They leave sources and propagate in the ISM
- They interact with the ISM particles and/or decay producing new particles
- They lose energy in the ISM by elm processes (brems, IC, sync)
- They are bent randomly by ISM magnetic field
- Until they reach the galaxy border and escape the galaxy
- Therefore: Their spectrum and composition at the Earth location are not representative



Densità media del mezzo Interstellare

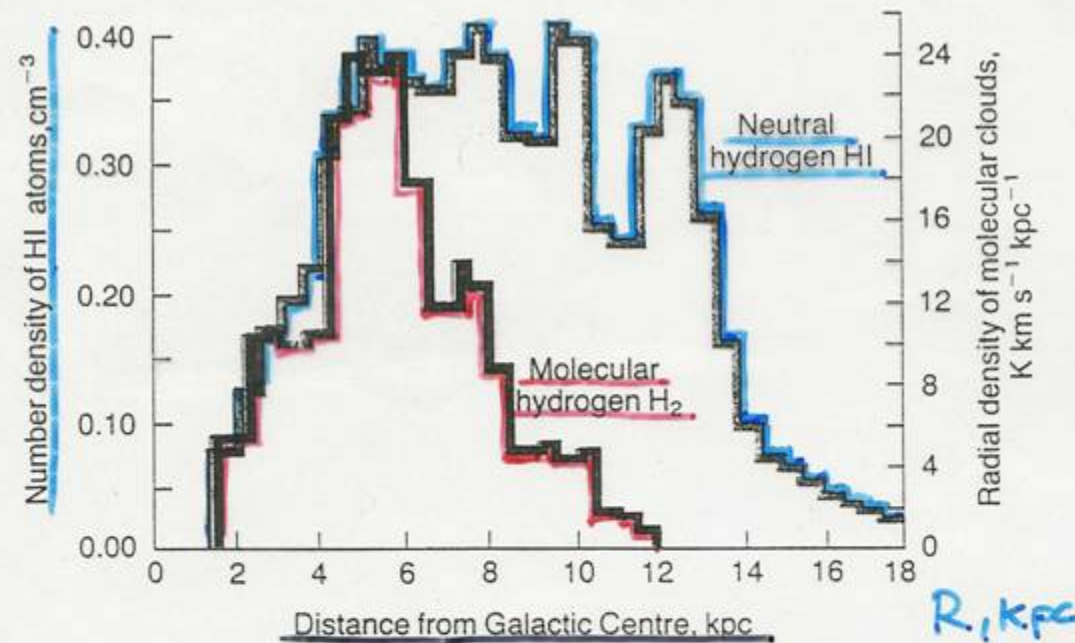


Figure 17.2. The radial distribution of atomic and molecular hydrogen as deduced from radio surveys of the Galaxy in the 21-cm line of atomic hydrogen and from millimetre surveys of the molecular emission lines of carbon monoxide, CO. (After D. Michalis and J. Binney (1981). *Galactic astronomy: structure and kinematics*, pp. 535, 554. San Francisco: W.H. Freeman and Co.)

$$\rho_{IG} = 1 \text{ p/cm}^3 = 1.67 \times 10^{-24} \text{ g/cm}^3$$

IL MEZZO INTERSTELLARE					
NOME	COSTITUENTI	Rivelati da...	% VOLUME e MASSA del Mezzo Int.	$N_{\text{cm}^{-3}}$	T (K)
MOLECOLARI NUBI	H_2, CO CS etc	Linee molecolari Emiss. Polveri	~ 0.5% 40%	1000	10
NUBI DI H NUBI DIFFUSE	H, C, O neutri	linee di 21 cm Linee Assorbim.	5% 40%	1-100	80
INTERNEBULE	H, H^+, e^- (ionizzati 40%)	21 cm + assorbim. Linee H	40% 20%	0.1-1	$\sim 10^4$
CORONE stellari	H^+, e^- O^{++}	soft X (0.1-2 keV)	~50% 0.1%	1000	10^6

Il campo magnetico galattico

- Si misura tramite la polarizzazione della luce delle stelle
- Intensità media: $(3 \pm 3) \mu\text{Gauss}$
- Coerenti su scale di 1-10 pc

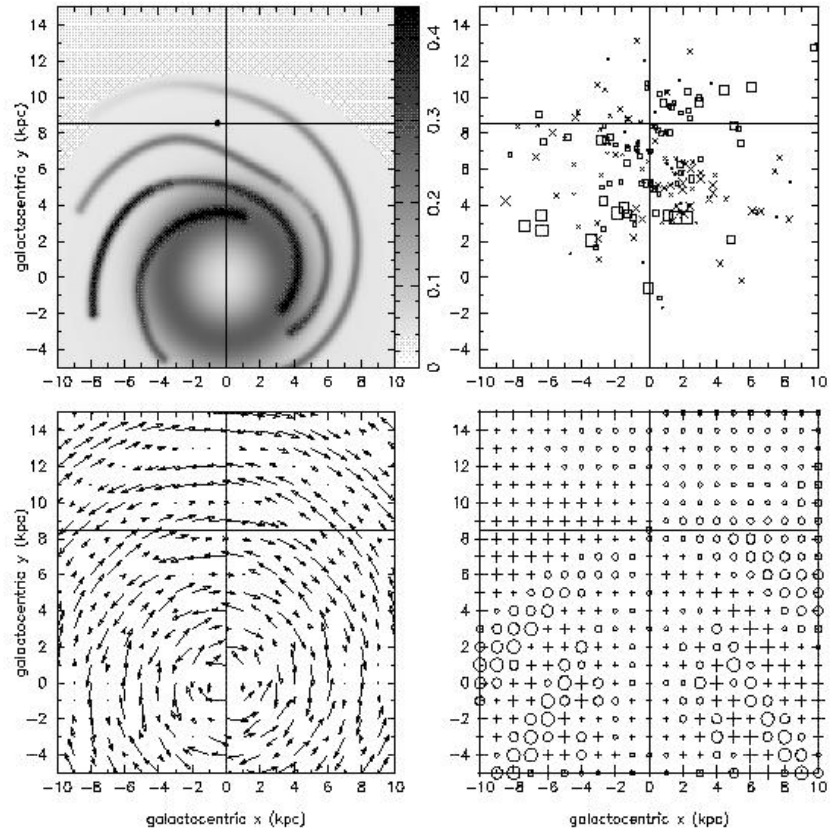
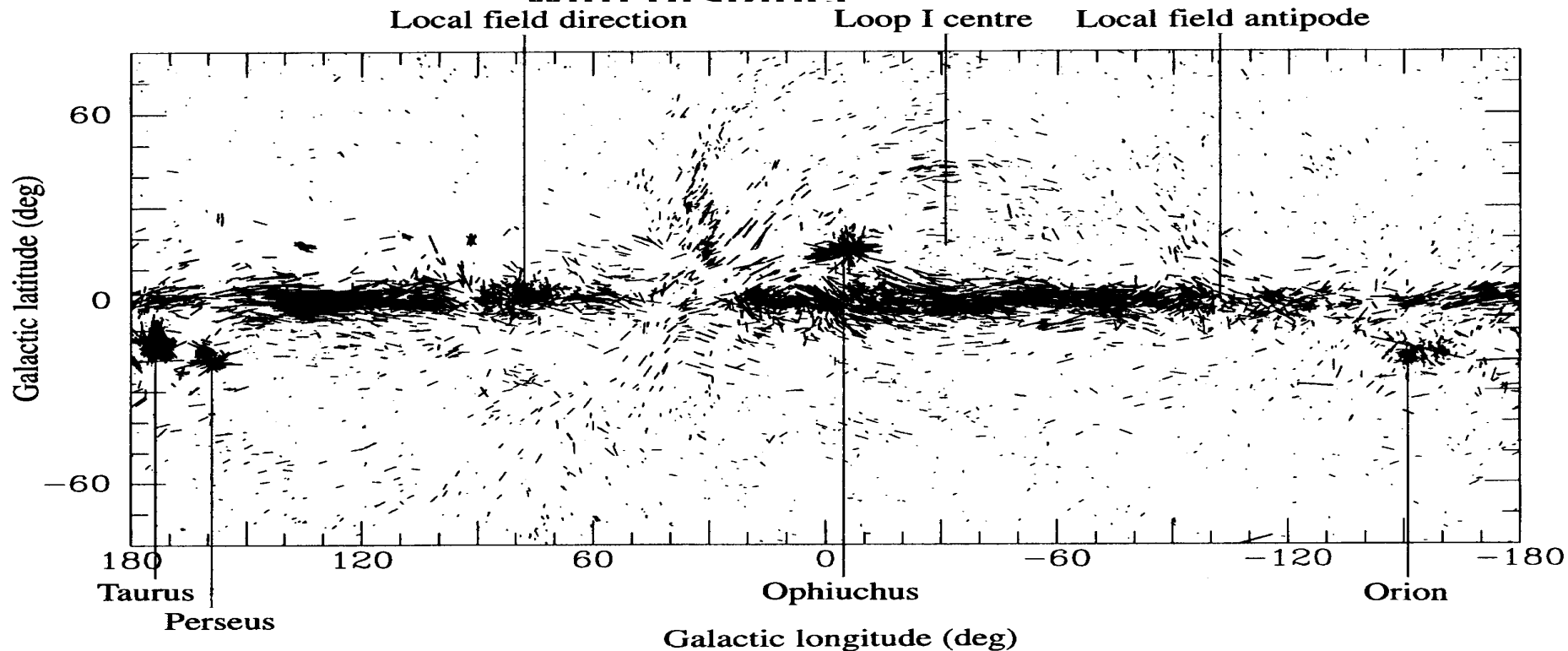
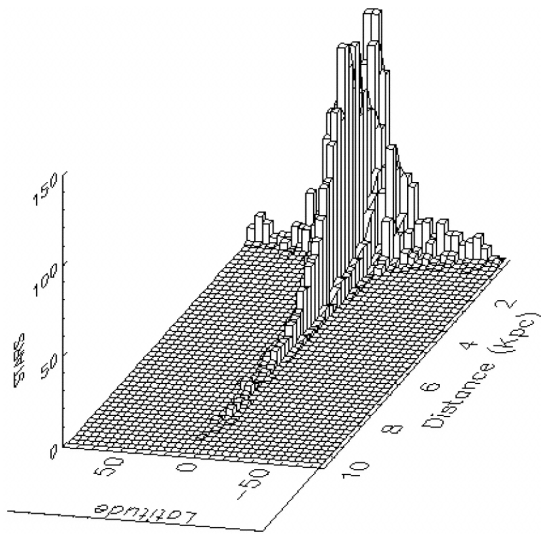


Figure 4. The large scale pattern of the magnetic field, using $p = -4.5^\circ$ and $R_o = 9.3$ kpc. The panels arrangement is the same as in figures 1 and 2.

Polarizzazione della luce delle stelle: *local field // arm*

- 9000 stars have polarization measured
- mostly nearby (1~2kpc)
- polarization percentage increases with distance



Il campo magnetico galattico

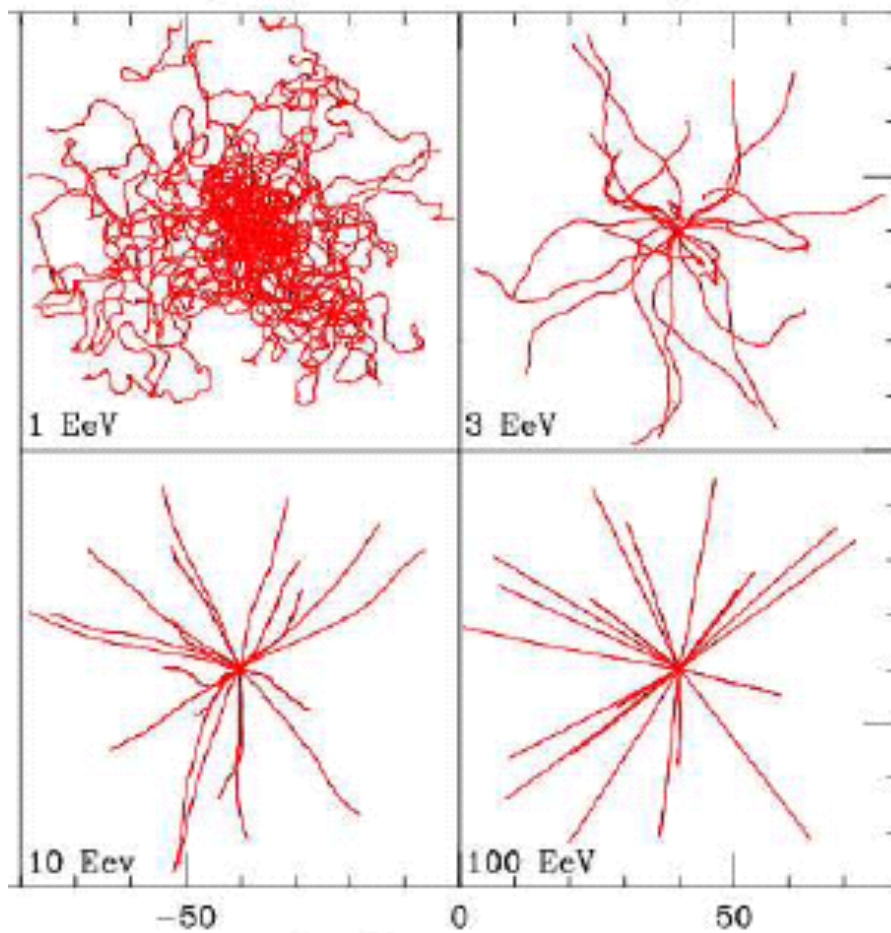
- The galactic magnetic field can be usually decomposed in two components:
 - Regular, large scale field
 - Turbulent, random irregularities
- Particles drift in the large scale field and scatter (collisionless) on the field irregularities

Isotropy

- So far, experimental results indicate only small amounts of anisotropy at low energies, with δ increasing with E .
- Below $E \sim 10^{10}$ eV, solar modulation hides the original directions.
 - For higher energies, direction of maximum excess is close to that of the Local Supercluster of Galaxies.

Isotropy

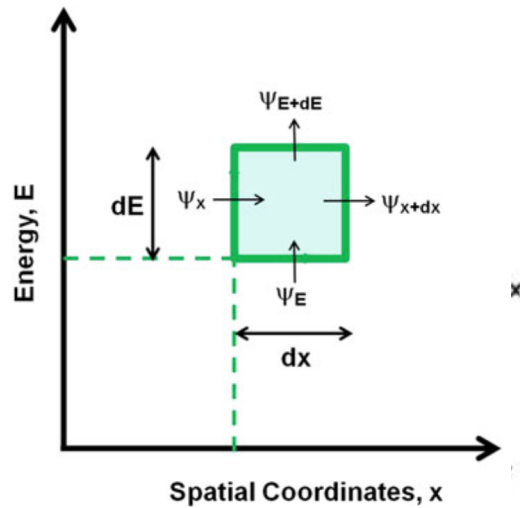
- This suggests that CR propagation may be described as a diffusion process in the ISM of galactic volume from injection sources, that is the CR trajectories are random walks on scattering centers (as we will see, CR scattering is collisionless) in the Milky Way, at least up to EeV until they reach the “border” where there is some probability to escape the galactic diffusion “volume” and ride into deep intergalactic space



Propagation equation

- CRs undergo a diffusion process through the interstellar medium from their sources until they exit the Galaxy. Occasionally, some CRs can be intercepted by detectors near the Earth.
- This propagation modifies also the CRs energy spectrum from the sources to the observer. As the solar system has nothing of peculiar with respect to any other point of our Galaxy, our observations are not influenced by the particular region where they are done. Particles having energies smaller than a few GeV, which are affected by the solar modulations, should not be considered.
- The galactic magnetic fields are the main factors which affect the CRs motion, as the Larmor radius for particles below the knee ($E < 10^{15}$ eV) is $\ll$ than the typical spatial dimension over which the magnetic fields are coherent. A random component in the motion is induced by the presence of irregularities, associated either with fluctuations in the fields or with the induction of instabilities due to the streaming motions of the charged particles themselves.
- During their diffusion, CRs are subject to energy-loss mechanisms and absorption.
- We give a “derivation” of the diffusion-loss equation which closely follows

Equazione di diffusione



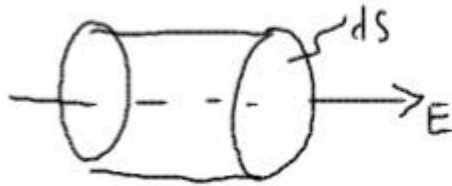
- Spazio delle fasi $(\bar{E}, x)$: in x diffonde, lungo E si "sposta" per guadagni e perdite
- Nel volume $dE dx$ ci sono $dm = N(E, x) dE dx$ particelle; possono entrare e uscire dal volume a causa dei flussi ϕ_x e ϕ_E

$$\circ \frac{dN}{dt} dE dx = [\phi_x(E, x, t) - \phi_x(E, x+dx, t)] dE + [\phi_E(E, x, t) - \phi_E(E+dE, x, t)] dx + Q(E, x, t) dE dx$$

$$\circ \frac{dN}{dt} = - \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_E}{\partial E} + Q$$

Sviluppo in serie di Taylor

$$\circ \text{Se } b(E) = -dE/dt \Rightarrow \phi_E = + N \frac{dE}{dt} = -b(E) N(E)$$



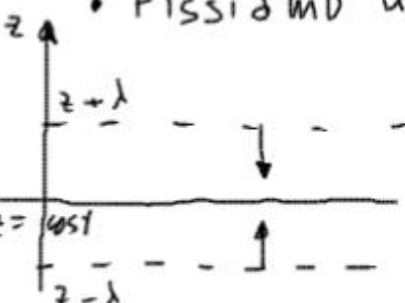
Flusso attraverso dS , lungo E .
tutte le part. nel volume $dS \cdot v_E dt$
attraversano $dS \Rightarrow \phi_E = \frac{N v_E dS dt}{dS dt}$

$v_E = \frac{dE}{dt}$ ("veloc. con cui la singola part
si sposta w , i.e. perde o guadagna $\bar{E}$)

Parentesi: moto diffusivo

- Il moto di diffusione si ha in presenza di gradienti di densità
 - Vale la legge di Fick: $\vec{J} = -D \vec{\nabla} n$
 - Il parametro D è il coeff. di diffusione: dipende dal mezzo in cui avviene la propagazione $[L^2 T^{-1}]$

Es. diffusione in un gas diluito

- 
- Fissiamo un piano arbitrario nel gas. Il flusso netto è dato dalla somma dei flussi da sopra e da sotto il piano, J_+ e J_-
 $J_{net} = J_+ + J_-$
 - Se l è il libero cammino medio, le particelle che attraversano il piano hanno subito l'ultimo urto in media a una dist. $\pm l$ dal piano, cioè a $z' = z \pm l$
 - Moto random $\Rightarrow$ il # di part in moto con vel. media $\langle v \rangle$ in una direzione è lo stesso in tutte le direzioni (isotropia) $\Rightarrow n_x = n_y = n_z = n/3$ si muovono con vel $\langle v \rangle$ lungo le 3 direzioni $\perp$ del rif. scelto
 - Di queste 1/2 si muovono verso $+z$ e 1/2 lungo $-z \Rightarrow$
 $J_{\pm} \approx J(z \pm l) = \pm \frac{\langle v \rangle n}{6} (z \pm l) \approx \pm \frac{\langle v \rangle}{6} \left[n(z) \pm \frac{\partial n}{\partial z} l \right]$

• quindi. $J_{nel} = \frac{\langle v \rangle}{6} \left[-\frac{\partial n}{\partial z} \cdot l \right] - \frac{\langle v \rangle}{6} \left[\frac{\partial n}{\partial z} \cdot l \right] = -\frac{l \langle v \rangle}{3} \left(\frac{\partial n}{\partial z} \right)$

$\Rightarrow D = \frac{l \langle v \rangle}{3}$

• Se le part. diffondono in un mezzo di densità $N \Rightarrow l \sim \frac{1}{N \sigma_i}$

σ_i = sez. d'urto di collisione fra le part. che diffondono e quelle del mezzo

$\Rightarrow D = \frac{\langle v \rangle}{3 N \sigma_i}$

* $\langle v \rangle \ll c \Rightarrow v \sim \left(\frac{kT}{m} \right)^{1/2}$

$p = N kT \Rightarrow \frac{1}{m} = \frac{kT}{p} \Rightarrow D \propto \frac{(kT)^{3/2}}{3 \sigma_i p}$

* $\langle v \rangle \approx c \Rightarrow D \approx \frac{c}{3 N \sigma_i} \Rightarrow D \propto \frac{c (kT)}{3 \sigma_i p}$

* Se il moto avviene in un campo magnetico, la dist. alla quale la part. ha cambiato "molto" la direzione è il raggio di Larmor $\Rightarrow$

$l \sim r_L = \frac{cp}{e \hbar B} \approx \frac{E}{\hbar \omega_B} \quad (\text{se } v \approx c) \Rightarrow D \approx \frac{\lambda_c}{3} = \frac{c E}{3 \hbar \omega_B} \Rightarrow D \propto E$

In tal caso il coefficiente di diffusione dipende da E

Equazione di diffusione

therefore

$$\frac{d\mathcal{N}_i}{dt} = D\nabla^2 \mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q$$

Additional terms can be added to this equation to include other physical effects, as the *escape probability*, the *radioactive decay*, and the *spallation* of nuclei during the propagation of cosmic ray nuclei from source

$$\frac{d\mathcal{N}_i}{dt} = D\nabla^2 \mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i} \frac{P_{ji}}{\tau_j} \mathcal{N}_j.$$

τ_i and τ_j are the lifetimes of particles of species i and j . For the spallation process, they correspond to $\tau_i = \lambda_i/c$ and $\tau_j = \lambda_j/c$, where $\lambda_{i,j}$ are their interaction lengths.

P_{ji} is the probability that, in an inelastic collision involving the destruction of the nucleus j , the nucleus i is produced. The decay of instable elements can also account for by $\frac{1}{\tau_i} = \frac{1}{\tau_{\text{decay}}} + \frac{c}{\lambda_i}$ giving that:

The equation is time dependent. Normally, one is interested in the steady-state solution, corresponding to $d\mathcal{N}_i/dt = 0$. Electrons, positrons, and antiproton propagation can be described as well by the diffusion equation. They constitute special cases, differing principally due to the energy losses and production rates of these particles

What D means?

- Random walk: after N steps of the same length l_0 , a particle starting from the O of a ref. Frame is at position $\mathbf{d} = \sum_N \mathbf{l}_i$
 - $\langle \mathbf{d} \rangle = 0$
 - $\langle d^2 \rangle = \mathbf{d} \cdot \mathbf{d} = \sum_i \sum_j \mathbf{l}_i \cdot \mathbf{l}_j = N l_0^2 + 2 l_0 \sum_{i \neq j} \cos \theta_{ij} = N l_0^2$
 - The solution of diffusion eqn (without E loss or any other process) is found to
- For $Q(t) = N_0 \delta(t)$, an instantaneous injection at $t = 0$ in the origin

What D means?

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

No convezione (per ora)

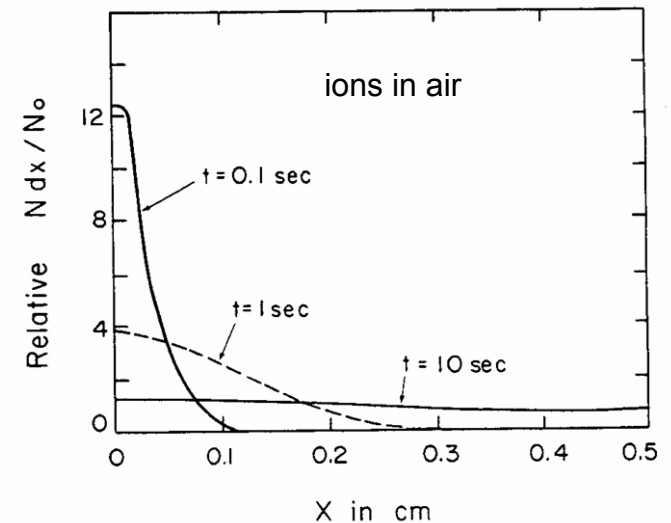
Se iniettiamo N particelle nell'origine con una distribuzione spaziale $N = N_0 \delta(x)$ a $t = 0$, l'evoluzione temporale e' data da

$$N(x, t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \quad (\text{calcolato alla lavagna?})$$

Il bunch di particelle rimane in media fermo nell'origine, ma si allarga nel tempo con una distribuzione gaussiana di larghezza crescente

$$\sigma_x = \sqrt{2Dt}$$

Una frazione della carica N/N_0 , inizialmente tutta in $x = 0$, la si ritrova ad una distanza x dopo un tempo t



What D means?

- After a time t , the particles is at a average quadratic distance form origin $d \sim (Dt)^{1/2}$
- Therefore $NI_o^2 = Dt \rightarrow D = NI_o^2/t$
- $NI_o = \text{total distance} \rightarrow NI_o/t = v$
- So that $D \sim v l_o$

What D means?

- An order of magnitude estimate for D can be determined from the residence times of CR in the galaxy
- If L is the characteristic scale of the system $D\nabla N \approx \frac{DN}{L^2}$
- If τ is the average residence time of CR in the galaxy $\frac{dN}{dt} \approx \frac{N}{\tau}$
- Therefore, by order of magnitude $\frac{N}{\tau} \approx \frac{DN}{L^2}$

$$D \approx \frac{L^2}{\tau}$$

- The scale L for the galaxy is the disk thickness. Assuming L = 300 pc = 9×10^{20} cm and a typical residence time of 10^7 years = 3×10^{14} s, $D \sim 3 \times 10^{27} \text{ cm}^2\text{s}^{-1}$

What D means?

- From $D = vl_o/3$, an estimate of the “step length” for a charged particle in the galaxy can be given
- $l_o = 3D/c \sim 3 \times 10^{17} \text{ cm} = 0.1 \text{ pc}$
- More refined models give D up to $3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$
- $\rightarrow l_o \sim 1 \text{ pc}$
- Also the halo thickness is uncertain. Estimates range up to 1000 pc (which, for example, results in a factor 10 for D)

What D means?

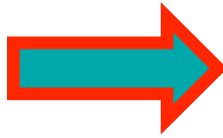
- $\rightarrow l_o \sim 0.1 - 1 \text{ pc}$
- Average particle density is too low for an efficient scattering by collisions
- This quantity can be interpreted as the typical scale of magnetic fields inhomogeneities of the magnetic field in the ISM
- On microscopic level, the diffusion of CR results from particle scatterings on magnetohydrodynamics waves and discontinuities (eg shock waves).

Richiamo: moto di un RC nel campo magnetico Galattico

$$mv^2 / r = pv / r = ZevB / c$$

$$r = pc / ZeB$$

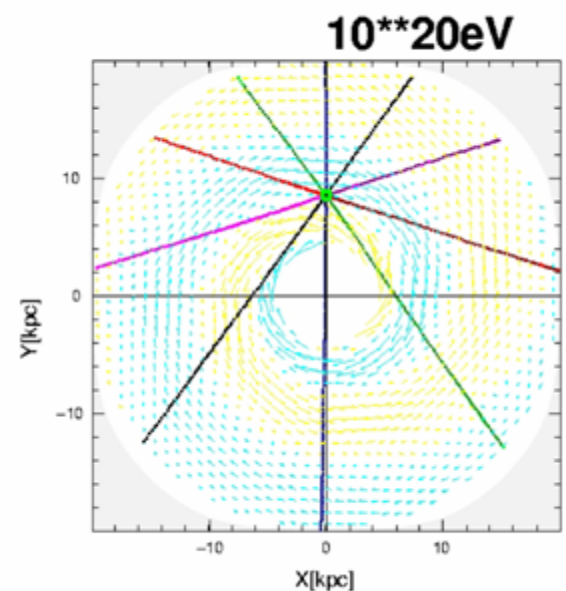
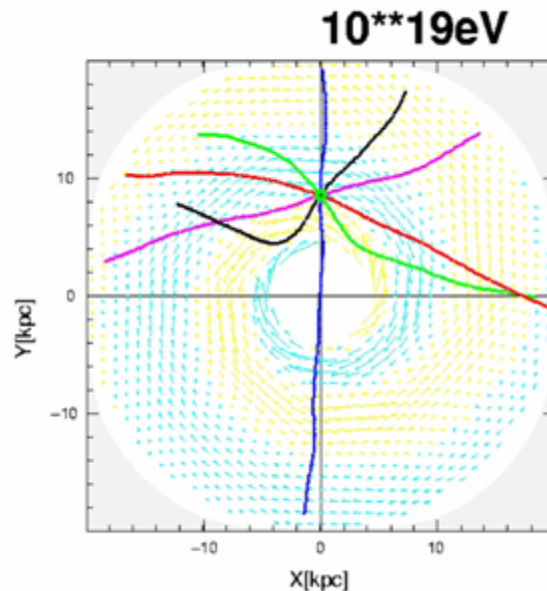
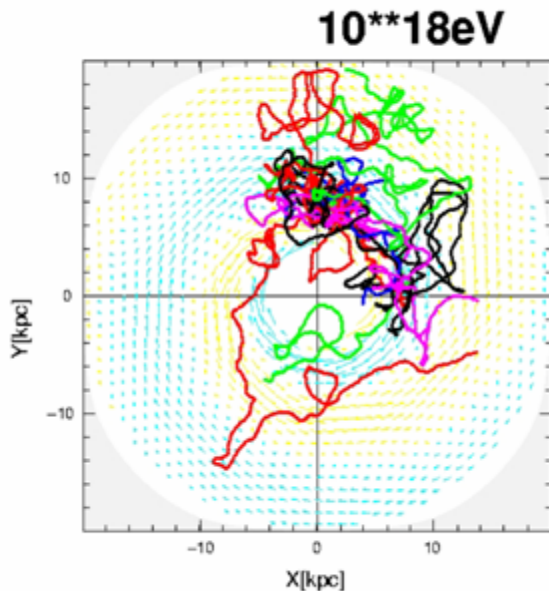
$$r(cm) = \frac{1}{300} \frac{E(eV)}{ZB(G)}$$



$$(10^{12} eV) = 10^{15} cm = 3 \times 10^{-4} pc$$

$$r = (10^{15} eV) = 10^{18} cm = 3 \times 10^{-1} pc$$

$$(10^{18} eV) = 10^{21} cm = 300 pc$$



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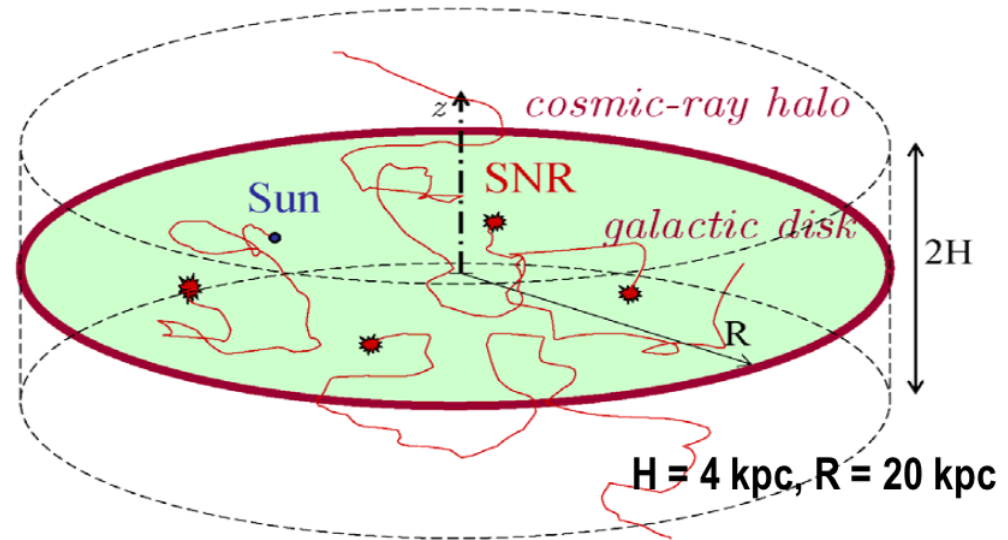
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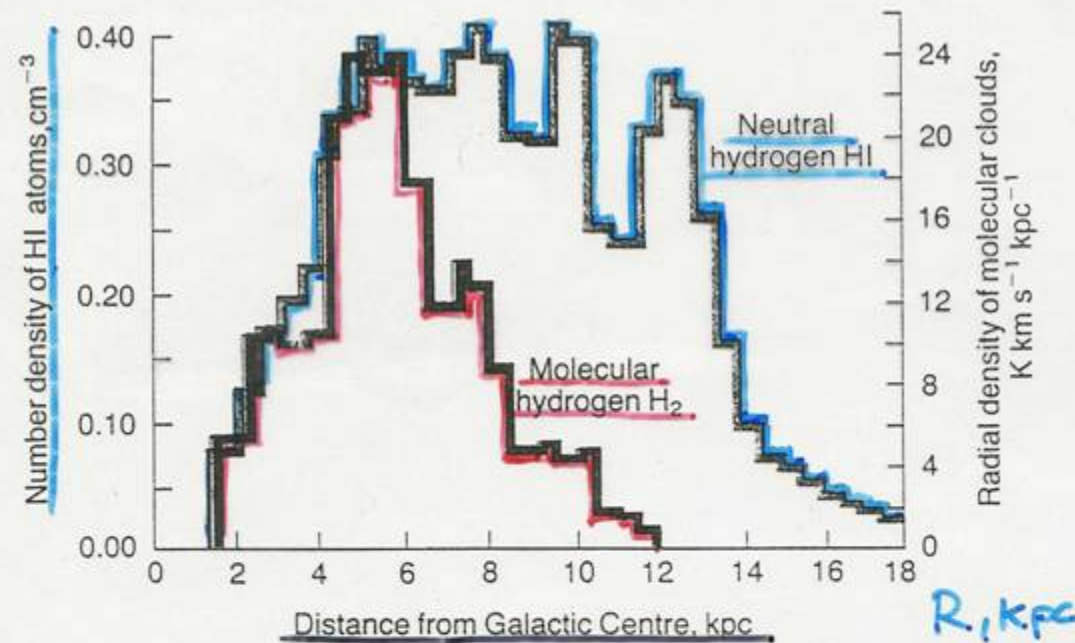


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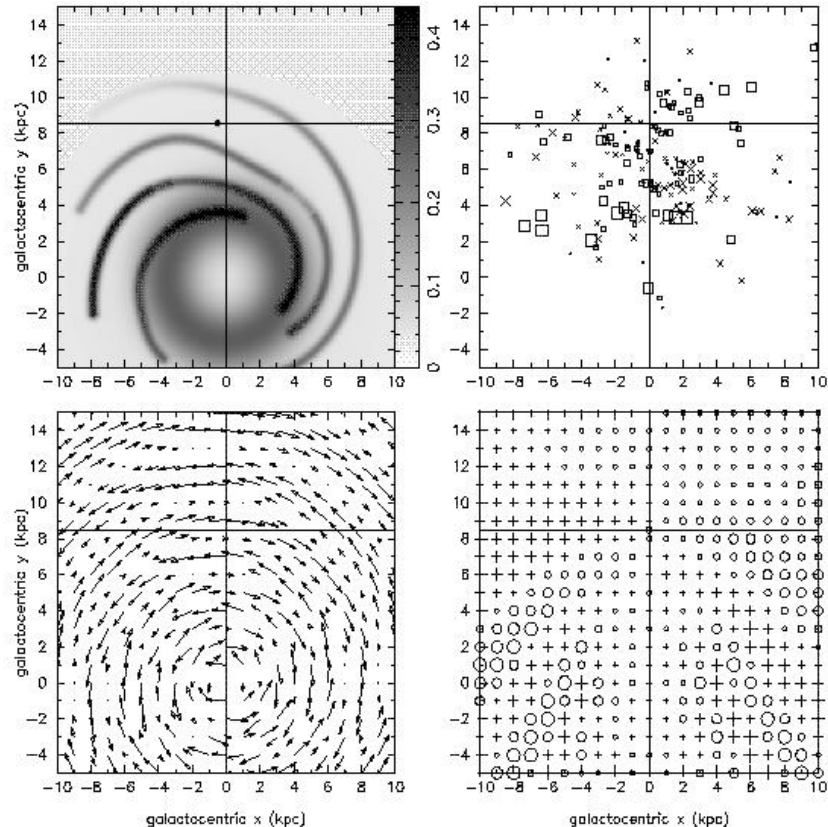
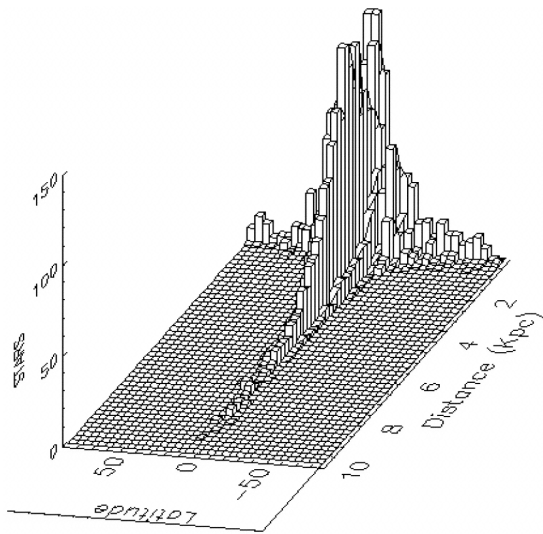
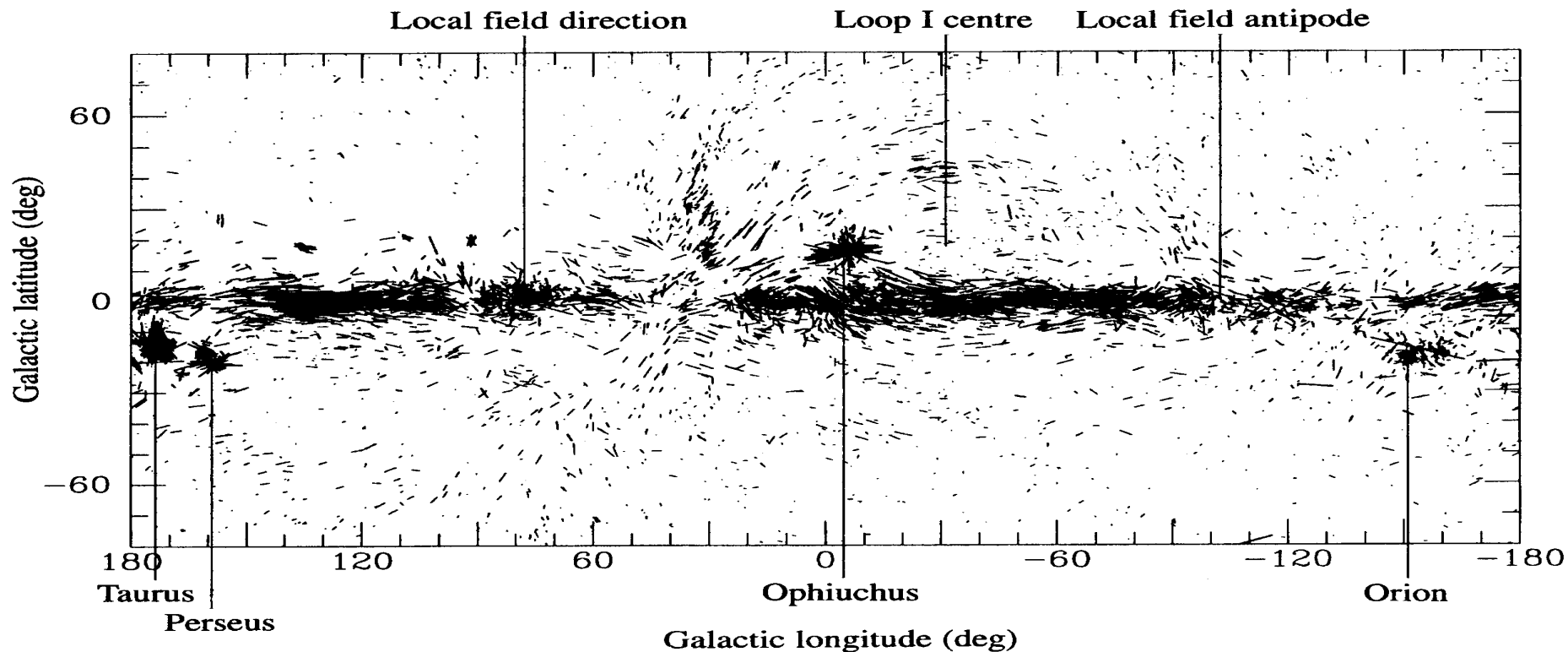


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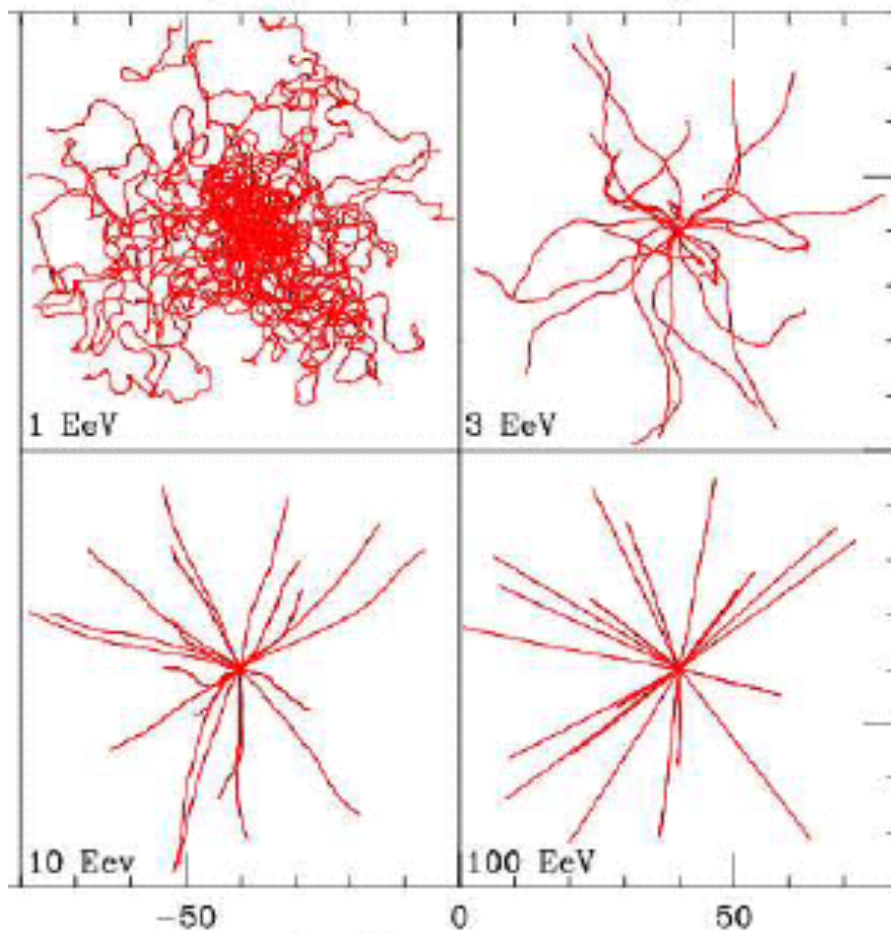
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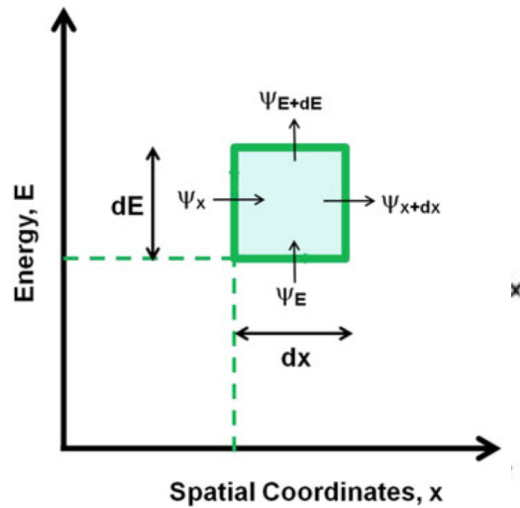
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Equazione di diffusione



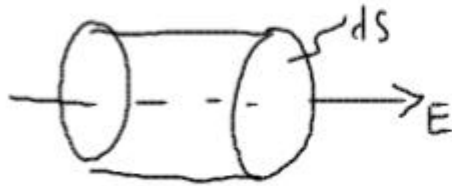
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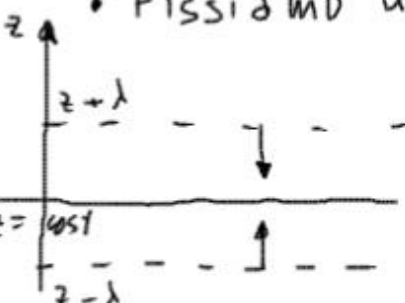
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 $J_{net} = J_+ + J_-$

- Se l è il libero cammino medio, le particelle che attraversano il piano hanno subito l'ultimo urto in media a una dist. $\pm l$ dal piano, cioè a $z' = z \pm l$.
- Moto random $\Rightarrow$ il # di part in moto con vel. media $\langle v \rangle$ in una direzione è lo stesso in tutte le direzioni (isotropia) $\Rightarrow n_x = n_y = n_z = n/3$ si muovono con vel $\langle v \rangle$ lungo le 3 direzioni $\perp$ del rif. scelto.
- Di queste $1/2$ si muovono verso $+z$ e $1/2$ lungo $-z \Rightarrow$
 $J_{\pm} \approx J(z \pm l) = \pm \frac{\langle v \rangle}{6} n(z \pm l) \approx \pm \frac{\langle v \rangle}{6} [n(z) \pm \frac{\partial n}{\partial z} l]$

• quindi. $J_{nel} = \frac{\langle v \rangle}{6} \left[-\frac{\partial n}{\partial z} \cdot l \right] - \frac{\langle v \rangle}{6} \left[\frac{\partial n}{\partial z} \cdot l \right] = -\frac{l \langle v \rangle}{3} \left(\frac{\partial n}{\partial z} \right)$

$$\Rightarrow D = \frac{l \langle v \rangle}{3}$$

• Se le part. diffondono in un mezzo di densità $N \Rightarrow l \sim \frac{1}{N \sigma_i}$

σ_i = sez. d'urto di collisione fra le part. che diffondono e quelle del mezzo

$$\Rightarrow D = \frac{\langle v \rangle}{3 N \sigma_i} \quad \times \langle v \rangle \ll c \Rightarrow v \sim \left(\frac{kT}{m} \right)^{1/2} \Rightarrow D \propto \frac{(kT)^{3/2}}{3 \sigma_i p}$$

$$p = N kT \Rightarrow \frac{1}{m} = \frac{kT}{p}$$

$$\times \langle v \rangle \approx c \Rightarrow D \approx \frac{c}{3 N \sigma_i} \Rightarrow D \propto \frac{c (kT)}{3 \sigma_i p}$$

* Se il moto avviene in un campo magnetico, la dist. alla quale la part. ha cambiato "molto" la direzione è il raggio di Larmor $\Rightarrow$

$$l \sim r_L = \frac{cp}{e \hbar B} \approx \frac{E}{\hbar \omega_B} \quad (\text{se } v \approx c) \Rightarrow D \approx \frac{\lambda_L}{3} = \frac{c E}{3 \hbar \omega_B} \Rightarrow D \propto E$$

In tal caso il coefficiente di diffusione dipende da E

Equazione di diffusione

therefore
$$\frac{d\mathcal{N}_i}{dt} = D\nabla^2 \mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q$$

Additional terms can be added to this equation to include other physical effects, as the *escape probability*, the *radioactive decay*, and the *spallation* of nuclei during the propagation of cosmic ray nuclei from sources to Earth.

$$\frac{d\mathcal{N}_i}{dt} = D\nabla^2 \mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i} \frac{P_{ji}}{\tau_j} \mathcal{N}_j.$$

τ_i and τ_j are the lifetimes of particles of species i and j . For the spallation process, they correspond to $\tau_i = \lambda_i/c$ and $\tau_j = \lambda_j/c$, where λ_{ij} are their interaction lengths.

P_{ji} is the probability that, in an inelastic collision involving the destruction of the nucleus j , the nucleus i is produced. The finite lifetime τ decay of instable elements can also account for by simply assuming that:
$$\frac{1}{\tau_i} = \frac{1}{\tau_{\text{decay}}} + \frac{c}{\lambda_i}$$

The equation is time dependent. Normally, one is interested in the steady-state solution, corresponding to $d\mathcal{N}_i/dt = 0$. Electrons, positrons, and antiproton propagation can be described as well by the diffusion equation. They constitute special cases, differing principally due to the energy losses and production rates of these particles

What D means?

- Random walk: after N steps of the same length l_0 , a particle starting from the O of a ref. Frame is at position $\mathbf{d} = \sum_N \mathbf{l}_i$
- $\langle d \rangle = 0$
- $\langle d^2 \rangle = \mathbf{d} \cdot \mathbf{d} = \sum_i \sum_j \mathbf{l}_i \cdot \mathbf{l}_j = N l_0^2 + 2 l_0 \sum_{i \neq j} \cos \theta_{ij} = N l_0^2$
- The solution of diffusion eqn $\frac{dN}{dt} - D \nabla^2 N = Q$
(without E loss or any other process) is found to be

$$N(r) = \frac{N_o}{(4\pi Dt)^{3/2}} e^{-\frac{r^2}{4Dt}}$$

For $Q(t) = N_o \delta(t)$, an instantaneous injection
at $t = 0$ in the origin

What D means?

$$\frac{\partial n}{\partial t} + \nabla \cdot (-D \nabla n + \vec{v} n) = R.$$

No convezione (per ora)

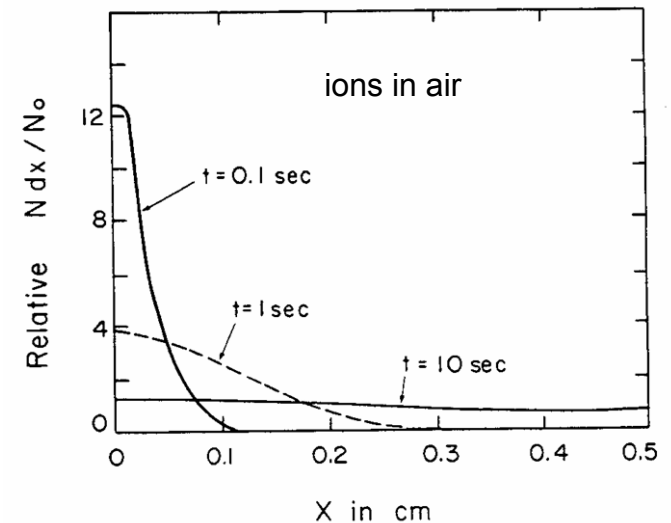
Se iniettiamo N particelle nell'origine con una distribuzione spaziale $N = N_0 \delta(x)$ a $t = 0$, l'evoluzione temporale e' data da

$$N(x, t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \quad (\text{calcolato alla lavagna?})$$

Il bunch di particelle rimane in media fermo nell'origine, ma si allarga nel tempo con una distribuzione gaussiana di larghezza crescente

$$\sigma_x = \sqrt{2Dt}$$

Una frazione della carica N/N_0 , inizialmente tutta in $x = 0$, la si ritrova ad una distanza x dopo un tempo t



What D means?

- After a time t , the particles is at a average quadratic distance form origin $d \sim (Dt)^{1/2}$
- Therefore $NI_o^2 = Dt \rightarrow D = NI_o^2/t$
- $NI_o = \text{total distance} \rightarrow NI_o/t = v$
- So that $D \sim vI_o$

What D means?

- An order of magnitude estimate for D can be determined from the residence times of CR in the galaxy
- If L is the characteristic scale of the system $D\nabla N \approx \frac{DN}{L^2}$
- If τ is the average residence time of CR in the galaxy $\frac{dN}{dt} \approx \frac{N}{\tau}$
- Therefore, by order of magnitude $\frac{N}{\tau} \approx \frac{DN}{L^2}$

$$D \approx \frac{L^2}{\tau}$$

- The scale L for the galaxy is the disk thickness. Assuming L = 300 pc = 9×10^{20} cm and a typical residence time of 10^7 years = 3×10^{14} s, $D \sim 3 \times 10^{27} \text{ cm}^2\text{s}^{-1}$

What D means?

- From $D = vl_o/3$, an estimate of the “step length” for a charged particle in the galaxy can be given
- $l_o = 3D/c \sim 3 \times 10^{17} \text{ cm} = 0.1 \text{ pc}$
- More refined models give D up to $3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$
- $\rightarrow l_o \sim 1 \text{ pc}$
- Also the halo thickness is uncertain. Estimates range up to 1000 pc (which, for example, results in a factor 10 for D)

What D means?

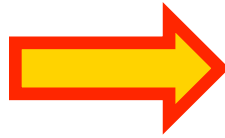
- $\rightarrow l_o \sim 0.1 - 1 \text{ pc}$
- Average particle density is too low for an efficient scattering by collisions
- This quantity can be interpreted as the typical scale of magnetic fields inhomogeneities of the magnetic field in the ISM
- On microscopic level, the diffusion of CR results from particle scatterings on magnetohydrodynamics waves and discontinuities (eg shock waves).

Richiamo: moto di un RC nel campo magnetico Galattico

$$mv^2 / r = pv / r = Ze v B / c$$

$$r = pc / ZeB$$

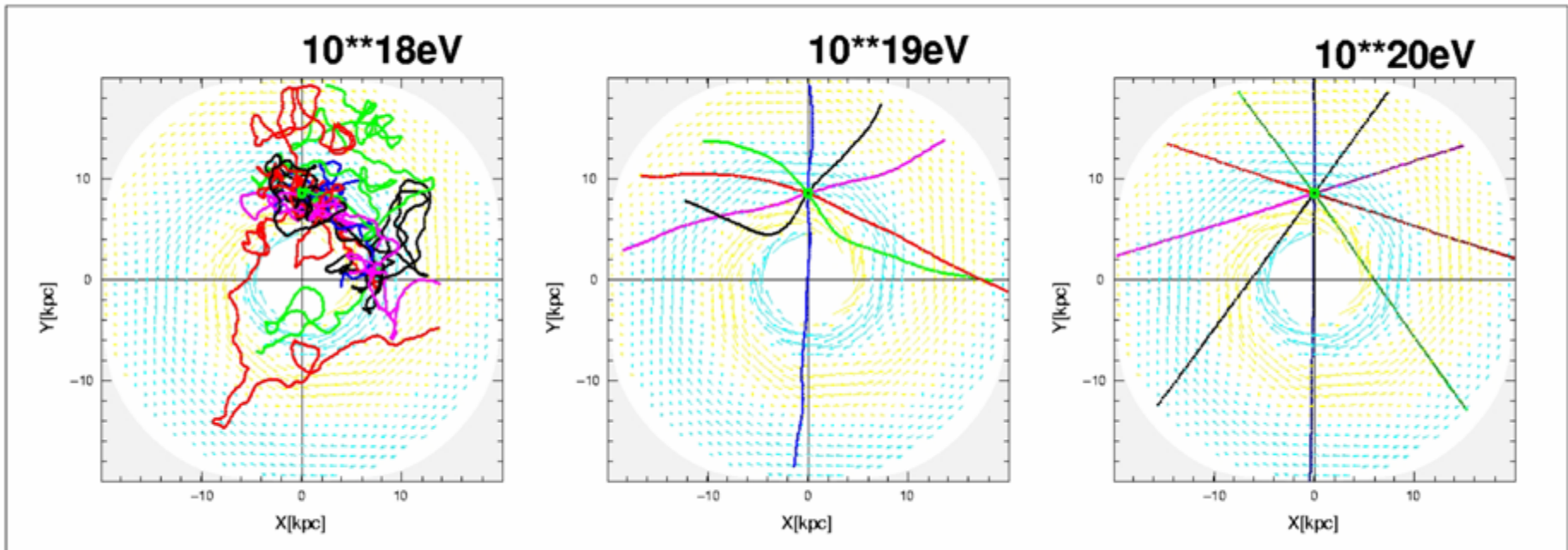
$$r(cm) = \frac{1}{300} \frac{E(eV)}{ZB(G)}$$



$$(10^{12} eV) = 10^{15} cm = 3 \times 10^{-4} pc$$

$$r = (10^{15} eV) = 10^{18} cm = 3 \times 10^{-1} pc$$

$$(10^{18} eV) = 10^{21} cm = 300 pc$$



Motion in magnetic fields

Uniform $\vec{B}$ field

$$\frac{d\vec{p}}{dt} = q \vec{v} \times \vec{B}$$

Scompongo in comp. $\parallel$ e $\perp$

$$i) \quad m \vec{a}_{\parallel} = q(\vec{v} \times \vec{B})_{\parallel} = 0$$

$$\Rightarrow a_{\parallel} = 0 \text{ e } \vec{v}_{\parallel} = \text{cost.}$$

Dato che $|\vec{v}| = \text{cost.}$ (B non fa lavoro)

$$\Rightarrow \alpha = \text{cost.} \text{ e } |\vec{v}_{\perp}| = \text{cost.}$$

$$ii) \quad m \vec{a}_{\perp} = q(\vec{v} \times \vec{B})_{\perp} \quad a_{\perp} = \frac{q v_{\perp} B}{m} = \text{cost.}$$

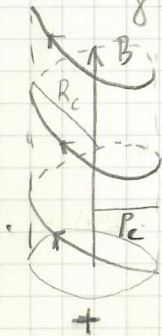
$\Rightarrow$ moto circolare unif. con

$$r_c = \frac{p_{\perp}}{qB} \quad \text{e} \quad \Omega_c = qB/m$$

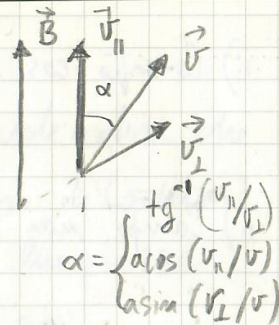
o anche

$$r_c = \frac{m_0 c}{q} \frac{B \gamma \sin \alpha}{B}$$

$$\Omega_c = \frac{qB}{\gamma m_0}$$



$$r_c = \frac{p}{qB \sin \alpha}$$



Uniform B field + External force $\vec{F}$

$$\frac{d\vec{p}_{\parallel}}{dt} = \vec{F}_{\parallel}$$

$$\frac{d\vec{p}_{\perp}}{dt} = \vec{F}_{\perp} + q \vec{v}_{\perp} \times \vec{B}$$

$$\Rightarrow v_{\parallel} = \left(\frac{F_{\parallel}}{m} \right) t \quad \text{accelera a causa di } F_{\parallel}$$

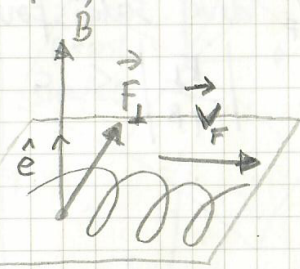
Assumo che $\vec{F}_{\parallel} = 0$ (per semplicità')

$$\frac{d\vec{p}_{\perp}}{dt} = \vec{F}_{\perp} + q \vec{v}_{\perp} \times \vec{B}$$

Nel piano $\perp \vec{B}$ la

particella drifta

con una vel costante $= \vec{v}_F$



Infatti, posso trovare un riferimento in cui il moto (istantaneo) è circolare: è quello in cui $\vec{F}_{\perp}$ è bilanciata dal campo $\vec{E}^*$ indotto dal moto di deriva $q \vec{E}^* = q \vec{v}_F \times \vec{B}$

$$q \vec{E}^* + \vec{F}_{\perp} = q \vec{v}_F \times \vec{B} + \vec{F}_{\perp} = 0$$

Moltiplico vettorialmente per $\hat{e}/qB$

$$\vec{v}_F = \frac{\vec{F}_{\perp} \times \hat{e}}{qB} = \frac{\vec{F} \times \hat{e}}{qB}$$

Se il moto della part. è un moto di ciclotrone + un moto di traslazione nella dir $\perp$ a $\vec{F}_{\perp}$ e $\vec{B}$

Motion in magnetic fields

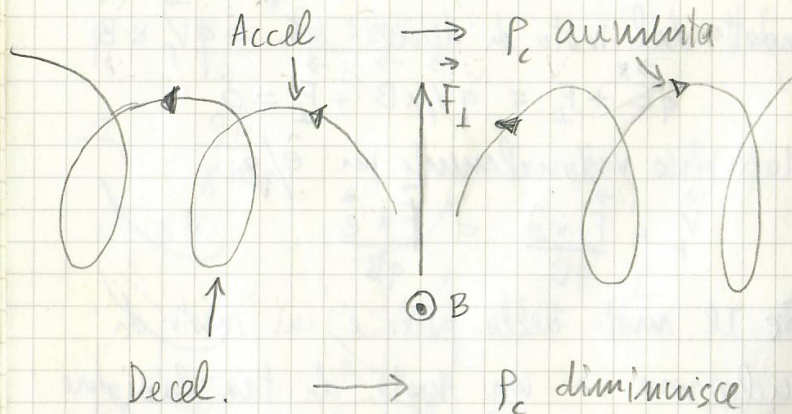
La particella "reagisce" perpendicolarmente alla forza esterna.

Non c'è lavoro netto sulla particella durante il moto di deriva.

La V_F deve soddisfare la condizione

$$\left| \frac{F_L}{qB} \right| < c$$

Intuitivamente si capisce la ragione fisica del drift



Campi $\vec{B}$ non uniformi

Un campo è inhomogeneo se $\partial B_i / \partial x_j \neq 0$ in qualche regione dello spazio

Esamineremo l'effetto dei due gradienti $\vec{\nabla}_\perp B$ e $\vec{\nabla}_\parallel B$

i) $\vec{\nabla}_\perp B$

Inietto una part. con $\alpha = \pi/2$

Dalla fig. si vede che c'è un drift.

Il campo B

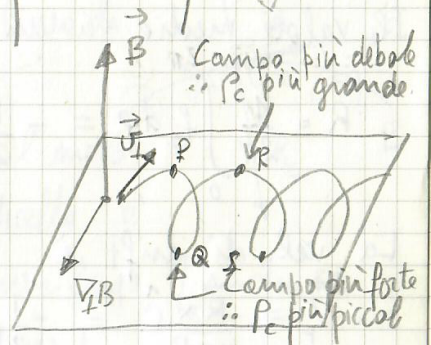
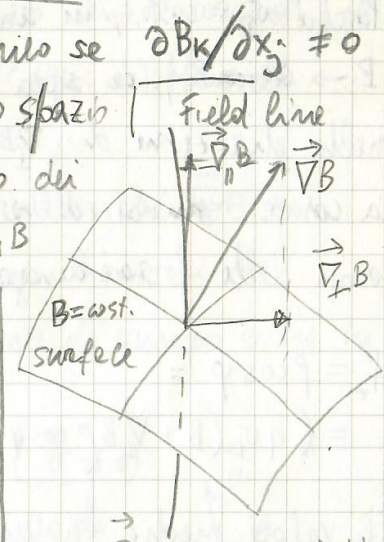
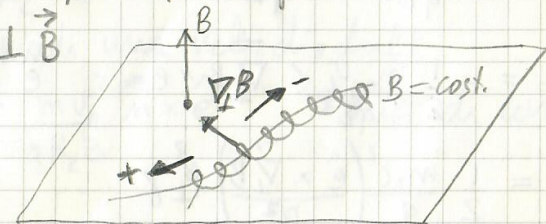
è più debole in P, R rispetto a quello in Q, S.

Cio' causa una variazione

"periodica" di P_c lungo l'orbita della particella

dato che $|\vec{v}_\perp| = \text{cost.}$

Una part. con $\alpha = \pi/2$ drifta lungo una linea $B = \text{cost.}$ in un piano $\perp \vec{B}$



Motion in magnetic fields

Poichè $v_{\perp} = \text{cost.}$, in una "rivoluzione" completa $P \rightarrow Q \rightarrow R$, ci sarà una forza media nella direzione di $\nabla_{\perp} B$.

La comp. media ist. delle comp. delle forze lungo z

$$F_z = F \cos \varphi = \frac{1}{2} q v_{\perp} (B + \nabla_{\perp} B \rho_c \cos \varphi) \cos \varphi$$

Il valore medio su un periodo di Larmor

$$\bar{F}_z = \frac{1}{2\pi} \int_0^{2\pi} F_z d\varphi = -\frac{1}{2} q v_{\perp} \rho_c \nabla_{\perp} B$$

La vel. di drift è

$$\vec{V}_F = \frac{\vec{R} \times \hat{e}}{qB} = -\frac{1}{2} \frac{q v_{\perp} \rho_c}{qB} \nabla_{\perp} B \times \hat{e}$$

Ma $\rho_c = \frac{p_{\perp}}{qB} = \frac{m_0 \gamma v_{\perp}}{qB}$ quindi

$$\vec{V}_F = -\frac{1}{2} \frac{m_0 v_{\perp}^2 \gamma}{qB^2} \nabla_{\perp} B \times \hat{e}$$

$$= \frac{1}{2} \frac{m_0 c^2}{q} \left(\frac{\hat{e} \times \nabla_{\perp} B}{B^2} \right) \beta_{\perp}^2 \gamma$$

Drift di gradiente

In una regione di spazio senza correnti, il campo $\vec{B}$ soddisfa le equ. di Maxwell

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{B} = 0$$

Così se $\nabla_{\perp} B \neq 0$, allora anche $\nabla_{\parallel} B \neq 0$ e le linee di campo sono curve.

La curvatura delle linee di campo entra in gioco se $\alpha \neq \pi/2$.

Assumiamo che $\alpha > 0$ ($v_{\perp} \neq 0$)

e che la part. segue la linea di campo.

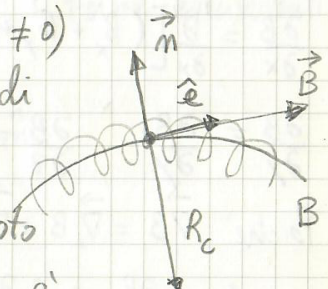
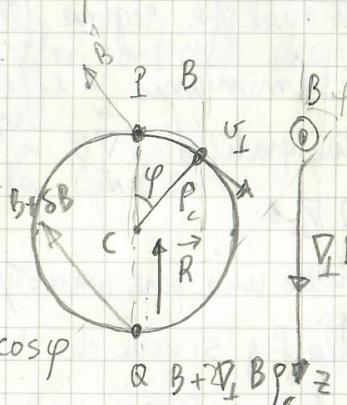
Nel sist. ~~istituito~~ in cui il moto è di girazione, la particella è soggetta ad una forza centrifuga [anche se $|v_{\parallel}| = \text{cost.}$, $\vec{v}_{\parallel}$ cambia diraz.]

$$\vec{F}_c = \frac{m v_{\parallel}^2}{R_c} \hat{m} \perp \vec{B}$$

Abbiamo quindi una vel. di drift

$$\vec{V}_c = \frac{m v_{\parallel}^2}{q R_c B} \hat{m} \times \hat{e} \quad \text{drift di curvatura}$$

Bisogna eliminare R_c dall'espressione ed esprimerlo in funzione di B e del suo gradiente perpendicolare



Motion in magnetic fields

Per campi $\vec{B}$ t.c. $\vec{\nabla} \times \vec{B} = 0$

$$\vec{\nabla}_{\perp} \vec{B} = -\frac{B}{R_c} \hat{n} = -B \frac{\vec{R}_c}{R_c^2} = B \frac{\partial \hat{e}}{\partial s}$$

In $\mathbb{R}$ $B_x = B_y = 0$ $B_z = B$

Da $\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \frac{\partial B_z}{\partial z} = 0$

Quindi $\vec{\nabla} \times \vec{B} = 0 \rightarrow \frac{\partial B_z}{\partial x} = 0$

Per le componenti di $\vec{\nabla} \vec{B}$

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x} \left[(B_x^2 + B_y^2 + B_z^2)^{\frac{1}{2}} \right] = \frac{\partial B_z}{\partial x} = 0$$

$$\frac{\partial B}{\partial y} = \frac{\partial B_z}{\partial y} \quad \frac{\partial B}{\partial z} = \frac{\partial B}{\partial s} = \nabla_{\parallel} B$$

Poiché $\vec{\nabla} \vec{B} = \vec{\nabla}_{\perp} \vec{B} + \vec{\nabla}_{\parallel} \vec{B}$, segue che $\vec{\nabla}_{\perp} \vec{B} = \frac{\partial \vec{B}}{\partial y}$

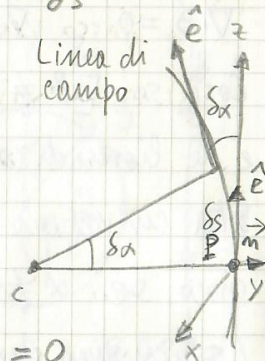
e che $\frac{\partial B_y}{\partial z} \approx B \frac{\partial \alpha}{\partial s} \approx \frac{B}{R_c}$ dove $R_c = \frac{\partial s}{\partial \alpha}$ raggio di curv. della linea

Infatti vettoriale $\vec{\nabla}_{\perp} \vec{B} = -\frac{B}{R_c} \hat{n} =$

Sostituiamo in $\vec{V}_c$

$$\vec{V}_c = \frac{m v_{\parallel}^2}{q B^2} \hat{e} \times \vec{\nabla}_{\perp} \vec{B}$$

Da questo significa che il centro di guida drifta dalla linea iniziale di campo



Drift di curvatura = 0 se

$v_{\parallel} = 0$ cioè se $\alpha = \pi/2$

Ha lo stesso verso di V_F

Drift di curvatura e gradiente

compaiono sempre insieme

$$V_{cg} = \frac{m}{2qB^2} (v_{\perp}^2 + 2v_{\parallel}^2) \hat{e} \times \vec{\nabla}_{\perp} \vec{B}$$

$$= \frac{m v^2}{2qB^2} (1 + \cos^2 \alpha) \hat{e} \times \vec{\nabla}_{\perp} \vec{B}$$

Forme equivalenti

$$= -\frac{m v^2}{2qB R_c} (2 - \sin^2 \alpha) \hat{e} \times \hat{n}$$

NB: la velocità di drift NON è quella della particella ma è quella media del centro di guida, cioè del centro di curvatura istantanea della particella in un periodo di Larmor.

Si ricordi infatti che la V è stata ricavata mediando il moto su periodo di Larmor

