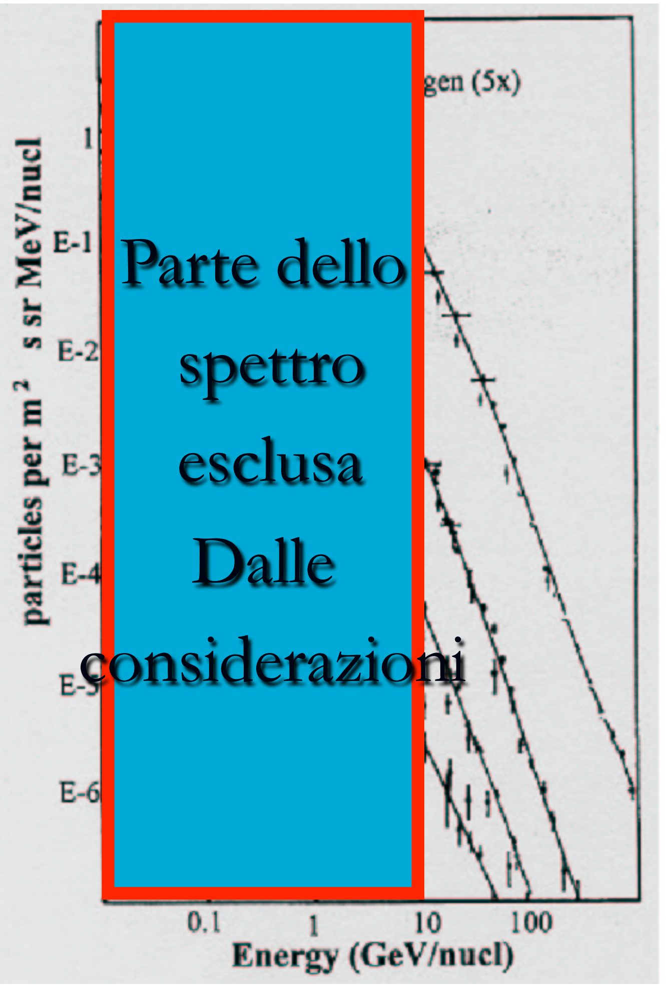
Lecture 6 061117

- Slides will be available at:
- https://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1718/

Modulazione dei RC di bassa energia dovuta al ciclo del Sole

- Le variazioni del ciclo solare hanno effetti per energie < 1 GeV
- RC con E > 10 GeV non affetti dal ciclo solare
- Flusso di RC di bassa energia (>1 GeV): ~ 1000 p/(m^{2}s sr).
- Pensateci prima di offrirvi volontari per una missione su Marte.



Raggi cosmici

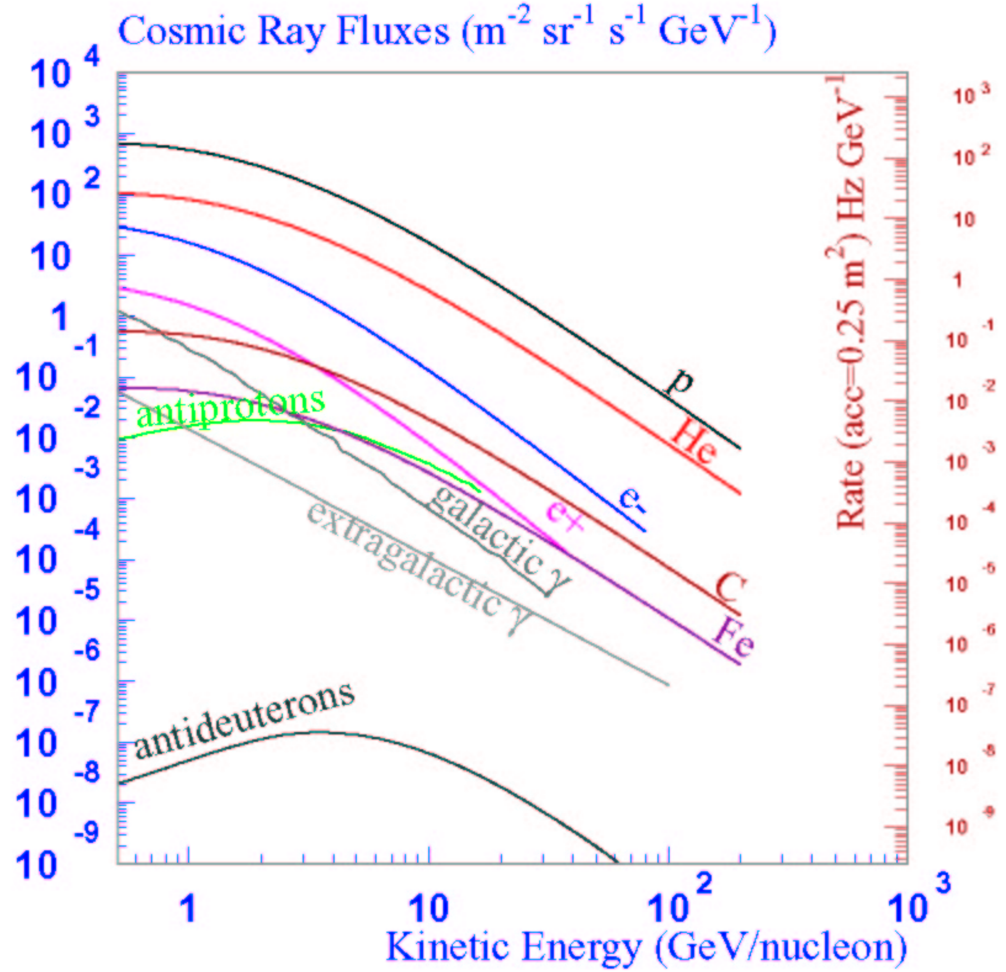
CR Composition

85% protons → $\bar{p}/p \sim O(10^{-4})$

12% He → $\overline{He}/He \sim 10^{-6}$

2% e → $e^+/e^- \sim 10^{-1}$

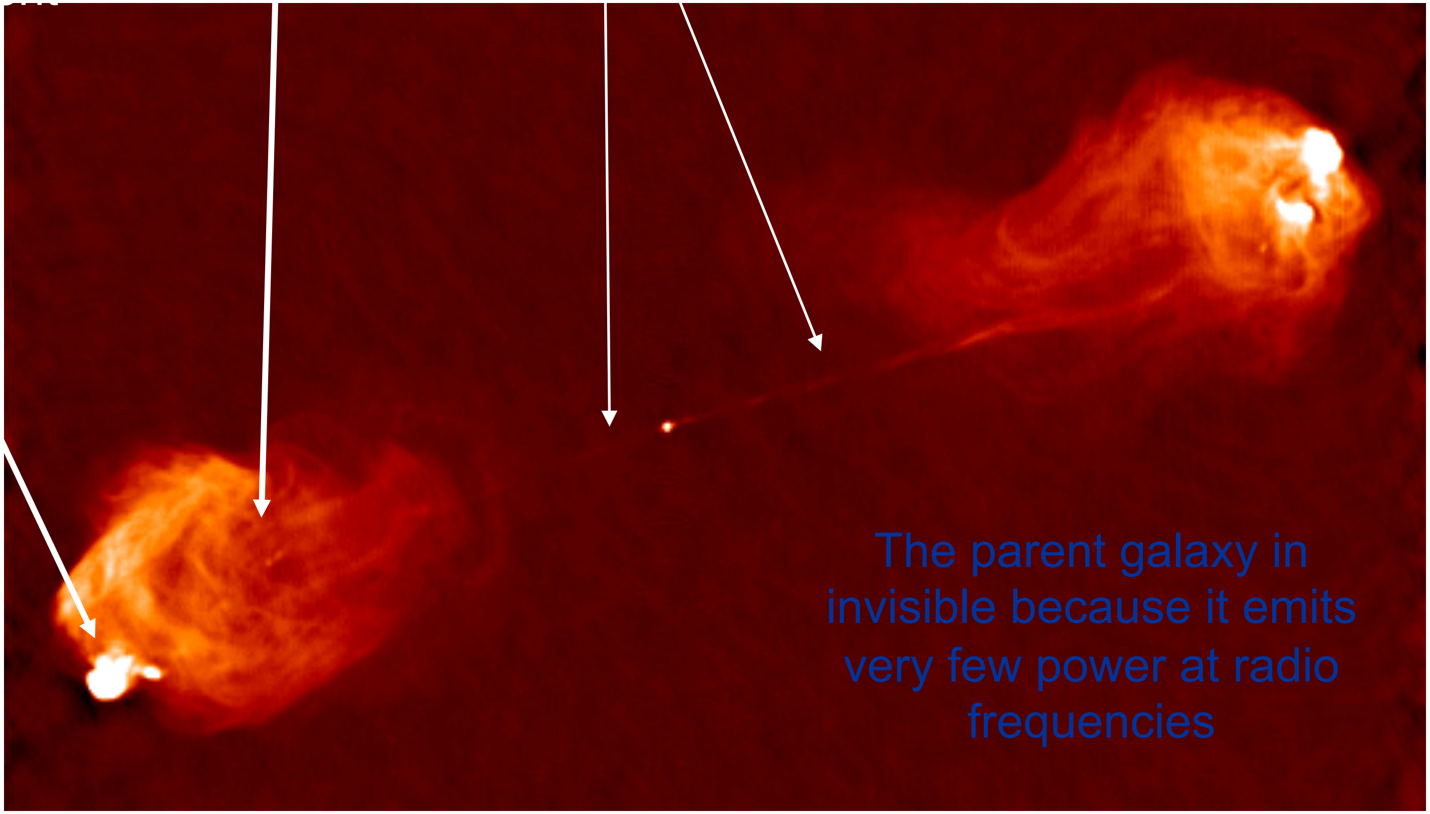
1% Nuclei $e^+/p \sim O(10^{-4})$



Rare components provide a plenty of info

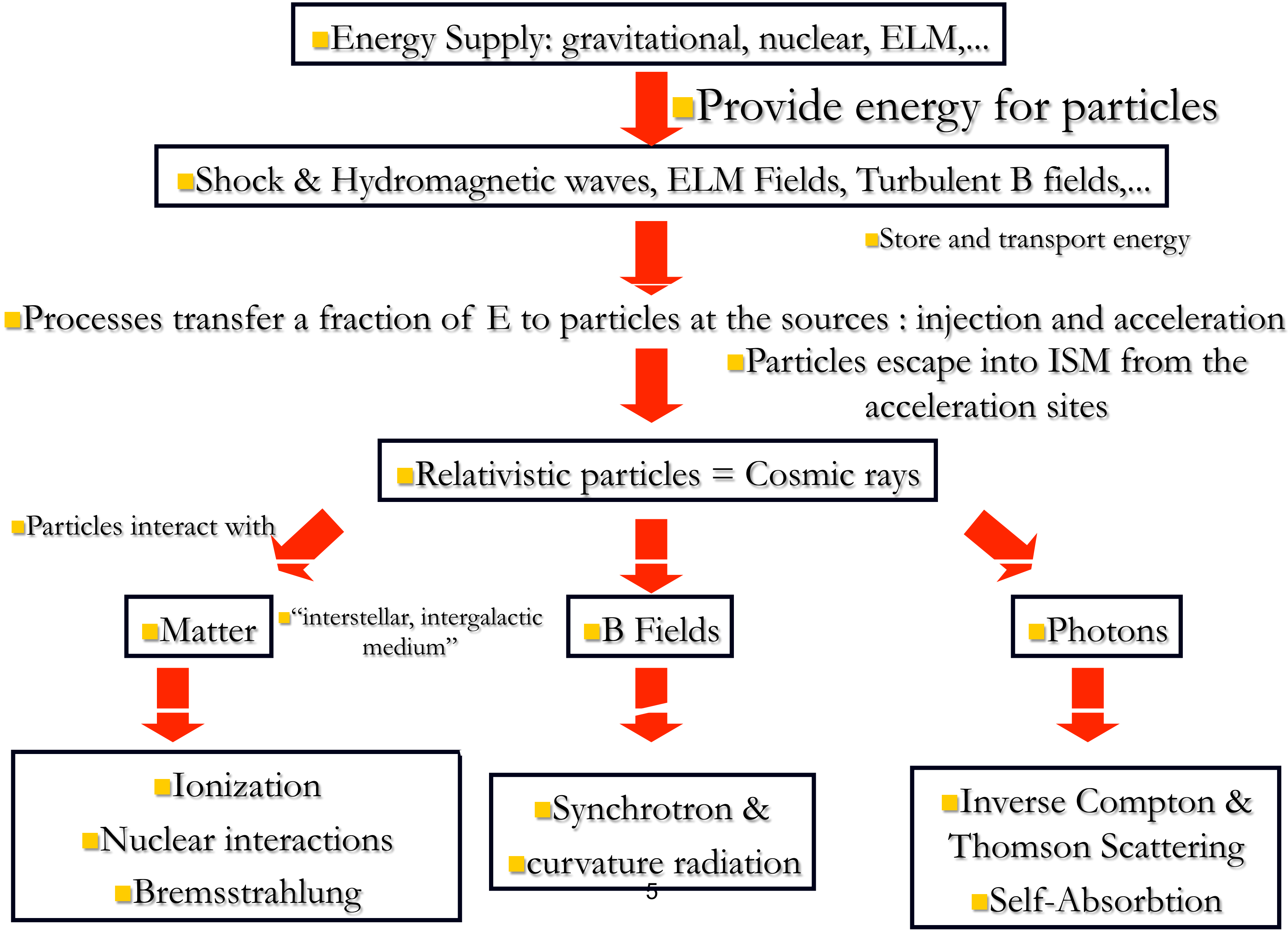
Electrons represents $O(10^{-2})$ of the charged CR flux but can reveal aspects of the source distribution and galactic propagation not observable in the nuclear CR component.

Radio galaxy Cignus A



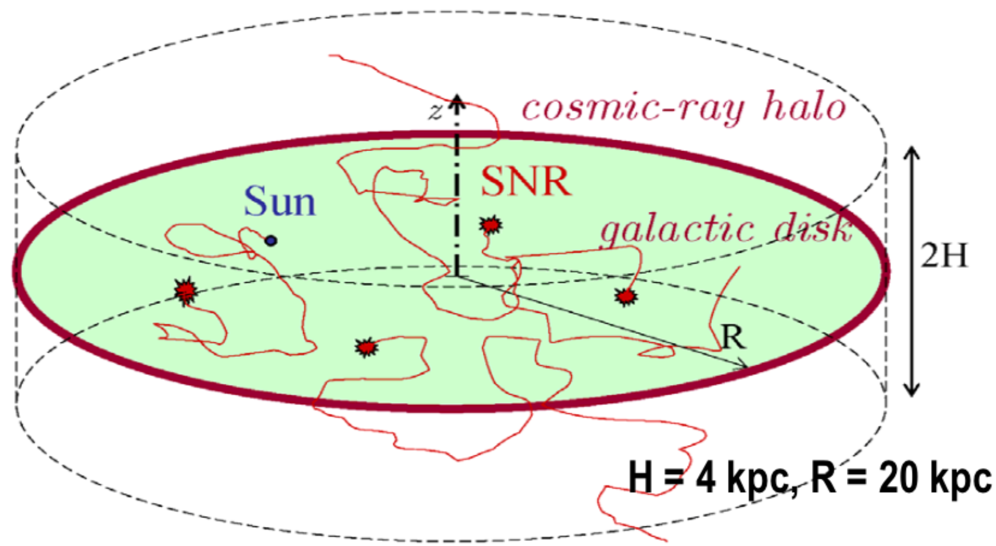
The parent galaxy in
invisible because it emits
very few power at radio
frequencies

Formation and Interactions of CR's



Formation and Interactions of CR's

- Particles get accelerated at astrophysical sources
- They leave sources and propagate in the ISM
- They interact with the ISM particles and/or decay producing new particles
- They loose energy in the ISM by elm processes (brems, IC, sync)
- They are bent randomly by ISM magnetic field
- Until they reach the galaxy border and escape the galaxy
- Therefore: Their spectum and composition at the Earth location are not representative of the source spectrum



Densità media del mezzo Interstellare

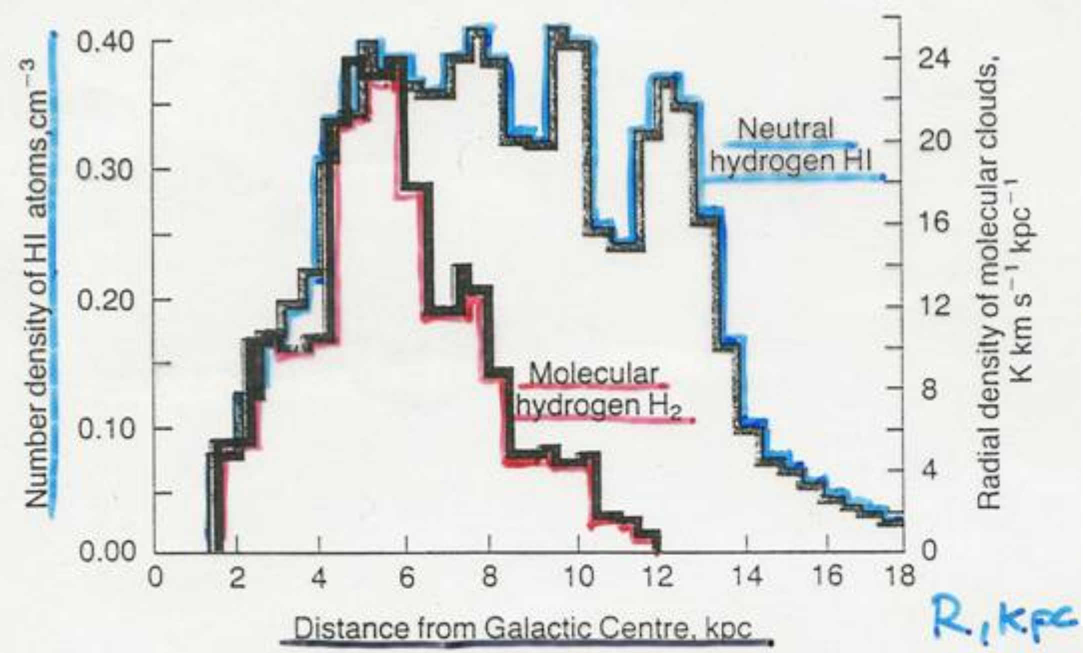


Figure 17.2. The radial distribution of atomic and molecular hydrogen as deduced from radio surveys of the Galaxy in the 21-cm line of atomic hydrogen and from millimetre surveys of the molecular emission lines of carbon monoxide, CO. (After D. Michalis and J. Binney (1981). *Galactic astronomy: structure and kinematics*, pp. 535, 554. San Francisco: W.H. Freeman and Co.)

$$\rho_{IG} = 1 \text{ p/cm}^3 = 1.67\times10^{-24} \text{ g/cm}^3$$

IL MEZZO INTERSTELLARE

NOME	COSTITUENTI	Rivelati da...	% VOLUME e MASSA del Mezzo Int.		N cm^{-3}	T (K)
MOLECOLARI NUBI	H_2, CO CS etc	Linee molecolari Emiss. Polveri	~0.5%	40%	1000	10
NUBI DI H NUBI DIFFUSE	H, C, O neutri	linea di 21 cm Linee Assorbim.	5%	40%	1-100	80
INTERNEBULE	H, H^+, e^- (ionizz. 10%)	21 cm + assorbim. Linee H	40%	20%	0.1-1	~10^4
CORONE stellari	H^+, e^-... O^{5+}	soft X (0.1-2 keV)	~50%	0.1%	1000	10^6

Il campo magnetico galattico

- Si misura tramite la polarizzazione della luce delle stelle
- Intensità media: (3 ±3) μGauss
- Coerenti su scale di 1-10 pc

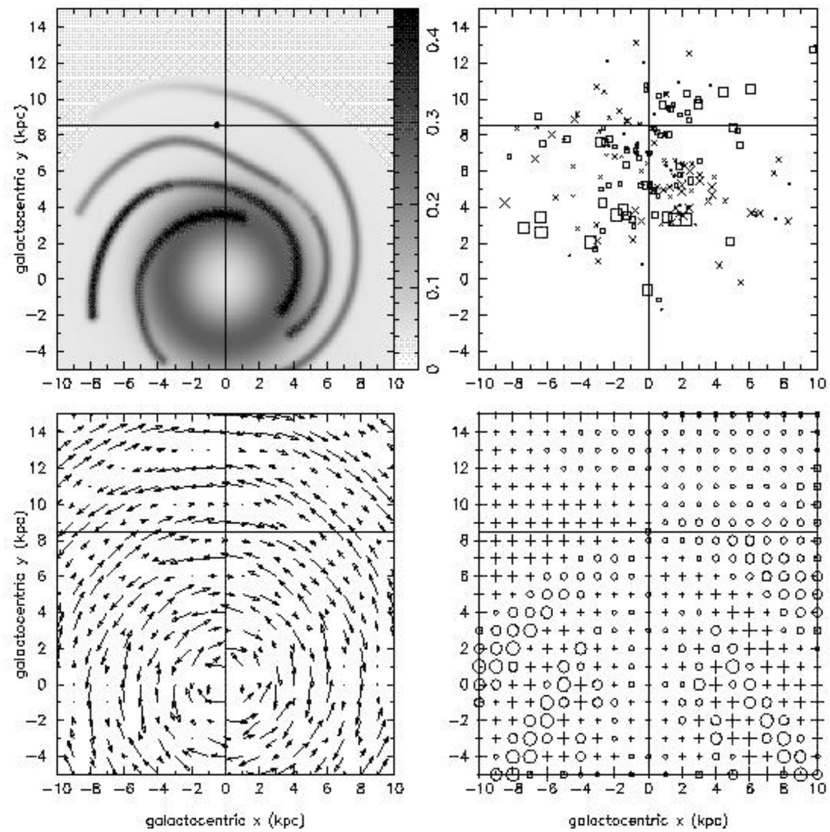
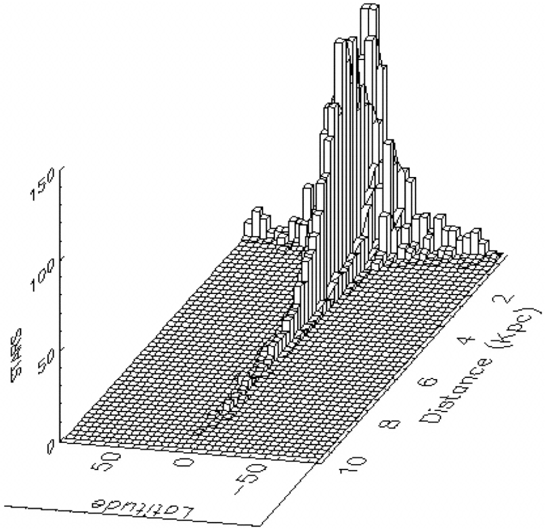
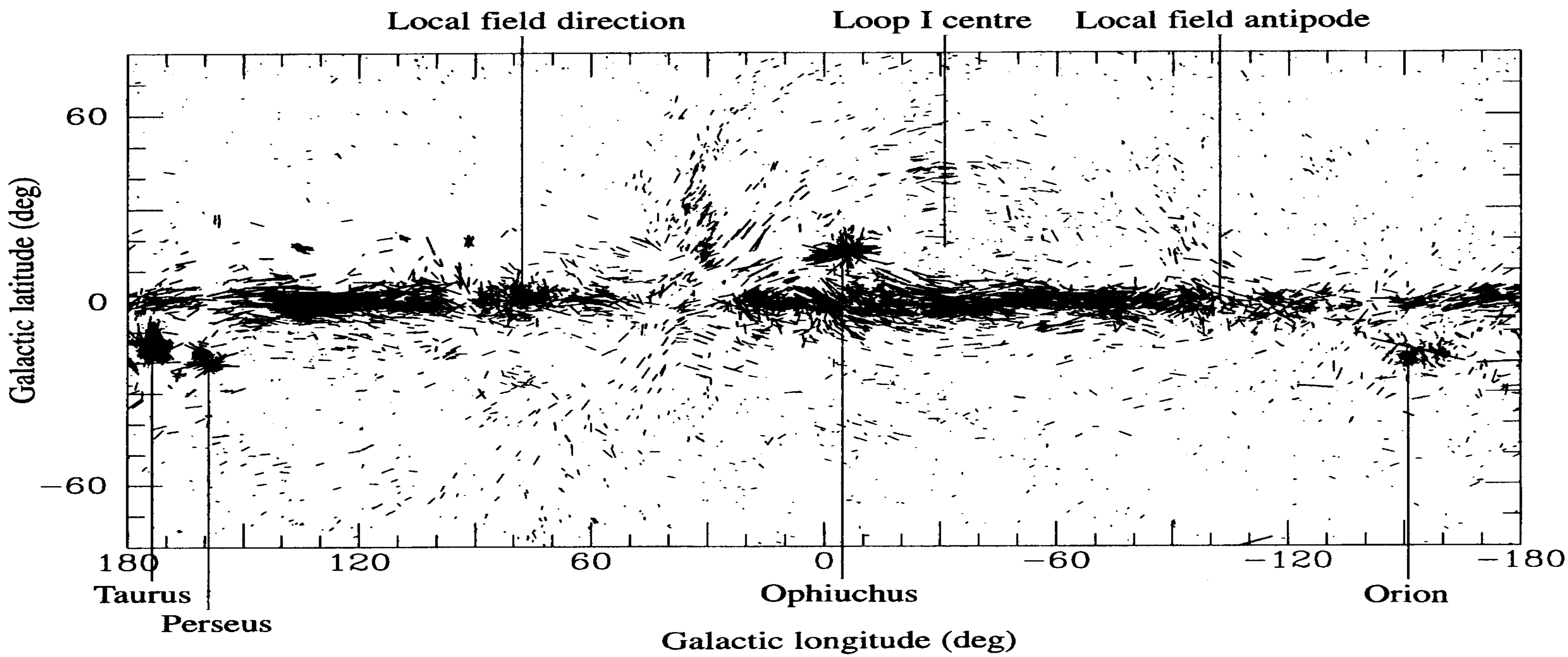


Figure 4. The large scale pattern of the magnetic field, using $p = -4.5^o$ and $R_o = 9.3$ kpc. The panels arrangement is the same as in figures 1 and 2.

Polarizzazione della luce delle stelle: *local field // arm*



- **9000 stars have polarization measured**
- **mostly nearby (1~2kpc)**
- **polarization percentage increases with distance**



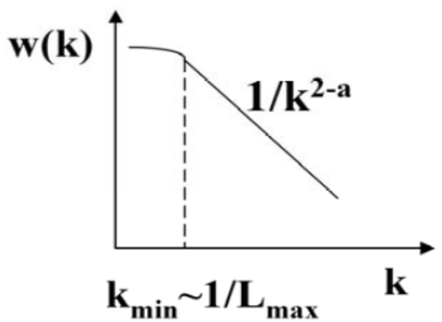
Il campo magnetico galattico

- The galactic magnetic field can is usually decomposed in two components:
  - Regular, large scale field
  - Turbulent, random irregularities
- Particles drift in the large scale field and scatter (collisionless) on the field irregularities

spectrum of turbulence

$$B_{res}^2 = \int_{k_{res}} w(k)dk$$

$w(k)dk \sim k^{-2+a}dk,$ $\quad D_{ll} \sim v r_g{}^a$



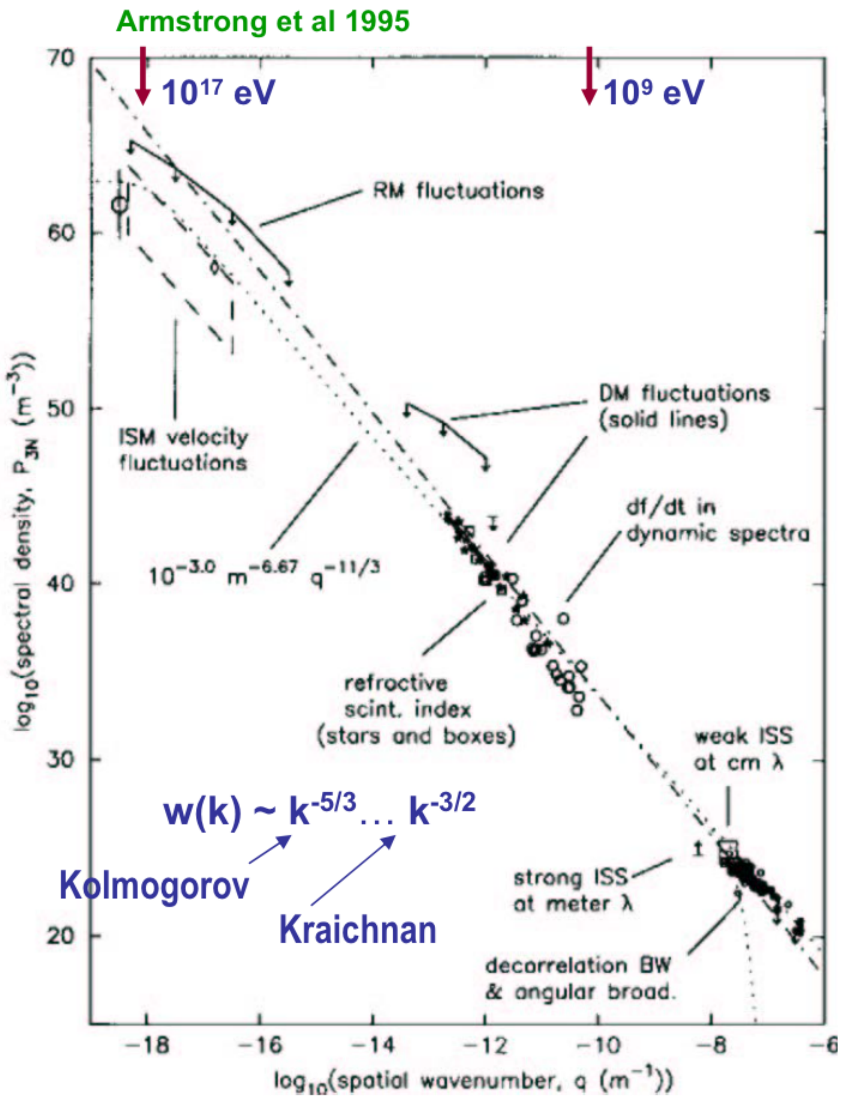
$a = 1/3$ Kolmogorov spectrum

$a = 1/2$ Kraichnan spectrum

$a = 0$ random discontinuities

$a = 1$ white noise (leads to Bohm diffusion scaling $D_B = v r_g/3$)

interstellar turbulence

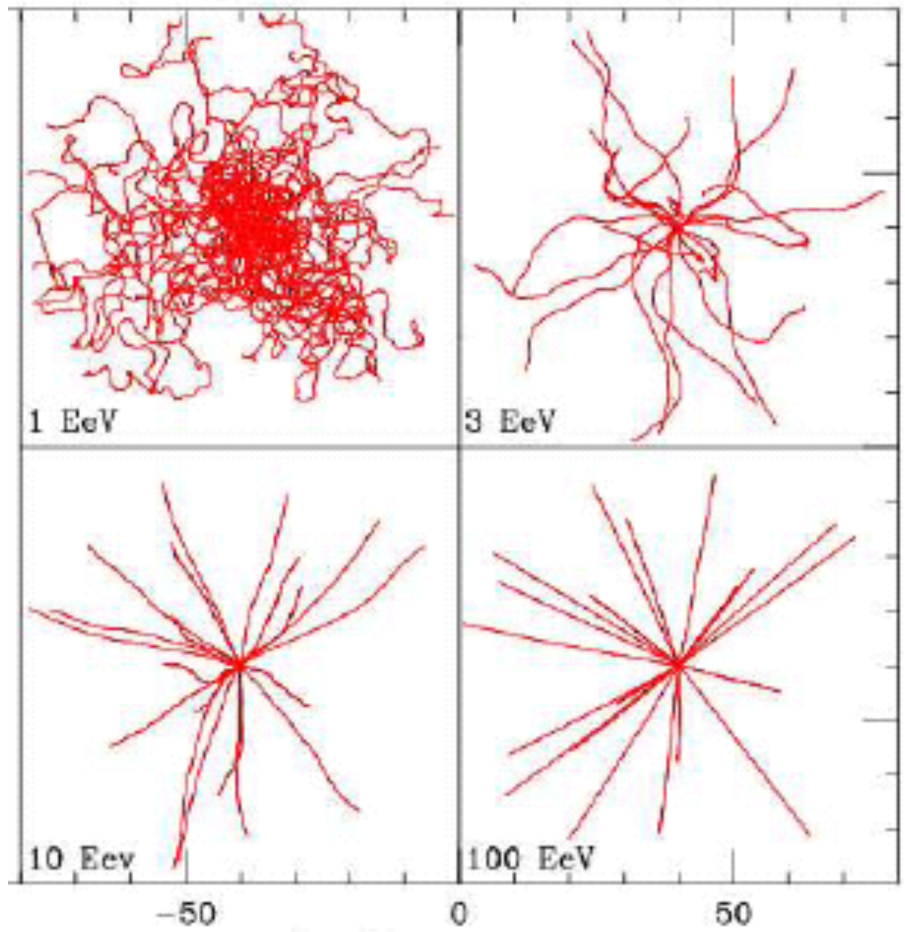


Isotropy

- So far, experimental results indicate only small amounts of anisotropy at low energies, with δ increasing with E.
- Below E~ 10^{10} eV, solar modulation hides the original directions.
- For higher energies, direction of maximum excess is close to that of the Local Supercluster of Galaxies.

Isotropy

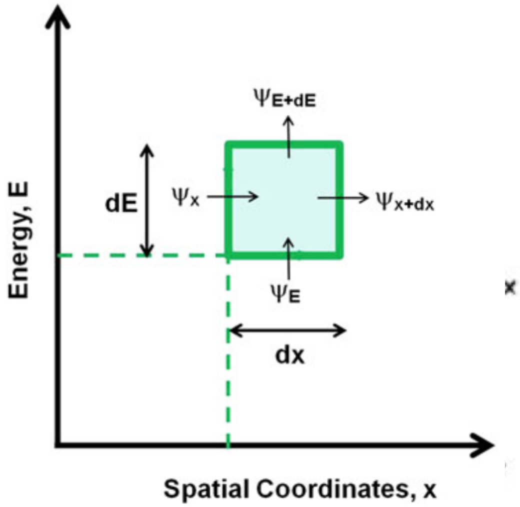
- This suggests that CR propagation may be described as a diffusion process in the ISM of galactic volume from injection sources, that is the CR trajectories are random walks on scattering centers (as we will see, CR scattering is collisionless) in the Milky Way, at least up to EeV until they reach the "border" where there is some probability to escape the galactic diffusion "volume" and ride into deep intergalactic space


1 EeV
3 EeV
10 Eev
100 EeV
-50
0
50

Propagation equation

- CRs undergo a diffusion process through the interstellar medium from their sources until they exit the Galaxy. Occasionally, some CRs can be intercepted by detectors near the Earth.
- This propagation modifies also the CRs energy spectrum from the sources to the observer. As the solar system has nothing of peculiar with respect to any other point of our Galaxy, our observations are not influenced by the particular region where they are done. Particles having energies smaller than a few GeV, which are affected by the solar modulations, should not be considered.
- The galactic magnetic fields are the main factors which affect the CRs motion, as the Larmor radius for particles below the knee ($E < 10^{15}$ eV) is << than the typical spatial dimension over which the magnetic fields are coherent. A random component in the motion is induced by the presence of irregularities, associated either with fluctuations in the fields or with the induction of instabilities due to the streaming motions of the charged particles themselves.
- During their diffusion, CRs are subject to energy-loss mechanisms and absorption.
- We give a "derivation" of the diffusion-loss equation which closely follows the so-called *coordinate space approach* (Longair, Spurio)

Equazione di diffusione

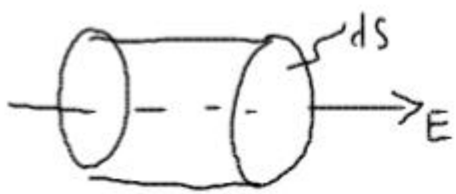


- Spazio delle fasi (E, x): in x diffonde, lungo E si "sposta" per guadagni e perdite
- Nel volume $dE\, d^3x$ ci sono $dn = N(E,x)\, dE\, dx$ particelle; possono entrare e uscire dal volume a causa dei flussi ϕ_x e ϕ_E

- $$\frac{dN}{dt}\, dE\, dx\, ds = \left[\phi_x(E,x,t) - \phi_{x+dx}(E,x+dx,t)\right] dE\, ds + \left[\phi_E(E,x,t) - \phi_{E+dE}(E+dE,x,t)\right] dx\, ds + Q(E,x,t)\, dE\, dx\, ds$$

- $$\frac{dN}{dt} = -\frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_E}{\partial E} + Q$$

- Se $b(E) = -dE/dt \Rightarrow \phi_E = +N \frac{dE}{dt} = -b(E)\, N(E)$



Flusso attraverso dS, lungo E.
tutte le part. nel volume $dS \cdot v_E\, dt$ attraversano $dS \Rightarrow \phi_E = \frac{N v_E\, dS\, dt}{dS\, dt}$

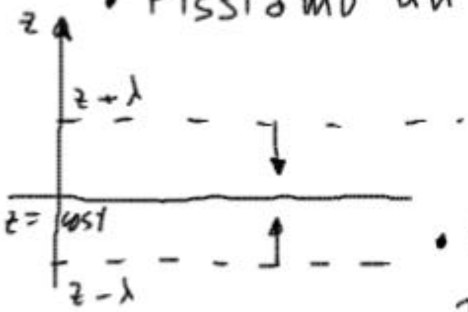
$v_E = \frac{dE}{dt}$ ("veloc. con cui la singola part si sposta, i.e. perde o guadagna E)

Parentesi: moto diffusivo

- Il moto di diffusione si ha in presenza di gradienti di densità
  - Vale la legge di Fick: $\vec{J} = -D\vec{\nabla}n$
    Il parametro D e' il coeff. di diffusione: dipende dal mezzo in cui avviene la propagazione $[L^2 T^{-1}]$

Es. diffusione in un gas diluito

- Fissiamo un piano arbitrario nel gas il flusso netto e' dato dalla somma dei flussi da sopra e da sotto il piano, J_+ e J_-



$J_{net} = J_+ + J_-$

- Se ℓ e' il libero cammino medio, le particelle che attraversano il piano hanno subito l'ultimo urto in media a una dist. $\pm\ell$ dal piano, cioe a $z' = z \pm \ell$
- Moto random ⇒ il # di part in moto con vel. media $\langle v\rangle$ in una direzione e' lo stesso in tutte direzioni (isotropia) ⇒ $n_x = n_y = n_z = n/3$ si muovono con vel $\langle v\rangle$ lungo le 3 direzioni ⊥ del rif. scelto
- Di queste 1/2 si muovono verso +z e 1/2 lungo −z ⇒

$$J_\pm \approx J(z\pm\ell) = \pm\frac{\langle v\rangle n(z\pm\ell)}{6} \approx \pm\frac{\langle v\rangle}{6}\left[n(z) \pm \frac{\partial n}{\partial z}\cdot\ell\right]$$

• Quind. $J_{nel} = \frac{\langle v \rangle}{6}\left[-\frac{\partial n}{\partial z} \cdot \ell\right] - \frac{\langle v \rangle}{6}\left[\frac{\partial n}{\partial z} \cdot \ell\right] = -\frac{\ell \langle v \rangle}{3}\left(\frac{\partial n}{\partial z}\right)$

$\Rightarrow D = \frac{\ell \langle v \rangle}{3}$

• Se le part. diffondono in un mezzo di densità $N \Rightarrow \ell \sim \frac{1}{N \sigma_t}$

σ_t = sez. d'urto di collisione fra le part. che diffondono e quelle del mezzo

$\Rightarrow D = \frac{\langle v \rangle}{3 N \sigma_t}$

× $\langle v \rangle \ll c \Rightarrow v \sim \left(\frac{kT}{m}\right)^{1/2}$
$p = NkT \Rightarrow \frac{1}{N} = \frac{kT}{p}$
$\Rightarrow D \propto \frac{(kT)^{3/2}}{3\sigma_t p}$

× $\langle v \rangle \simeq c \Rightarrow D \simeq \frac{c}{3 N \sigma_t} \Rightarrow D \propto \frac{c(kT)}{3\sigma_t p}$

∗ Se il moto avviene in un campo magnetico, la dist. alla quale la part. ha cambiato "molto" la direzione è il raggio di Larmor $\Rightarrow$

$\ell \sim r_L = \frac{cp}{eZB} \simeq \frac{E}{ZeB}$ (se $v \simeq c$) $\Rightarrow D \simeq \frac{\lambda_L}{3} = \frac{cE}{3ZeB} \Rightarrow D \propto E$

In tal caso il coefficiente di diffusione dipende da E

Equazione di diffusione

- Quindi $$\frac{d\mathcal{N}_i}{dt} = D\nabla^2\mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q$$

Additional terms can be added to this equation to include other physical effects, as the *escape probability*, the *radioactive decay*, and the *spallation* of nuclei during the propagation of cosmic ray nuclei from sources to Earth.

$$\frac{d\mathcal{N}_i}{dt} = D\nabla^2\mathcal{N}_i + \frac{\partial}{\partial E}[b(E)\mathcal{N}_i(E)] + Q - \frac{\mathcal{N}_i}{\tau_i} + \sum_{j>i}\frac{P_{ji}}{\tau_j}\mathcal{N}_j.$$

τ_i and τ_j are the lifetimes of particles of species *i* and *j*. For the spallation process, they correspond to $\tau_i = \lambda_i/c$ and $\tau_j = \lambda_j/c$, where $\lambda_{i,j}$ are their interaction lengths.

Pji is the probability that, in an inelastic collision involving the destruction of the nucleus *j*, the nucleus *i* is produced. The finite lifetime τ decay of instable elements can also account for by simply assuming that: $$\frac{1}{\tau_i} = \frac{1}{\tau_{\text{decay}}} + \frac{c}{\lambda_i}$$

The equation is time dependent. Normally, one is interested in the steady-state solution, corresponding to $dN_i/dt = 0$. Electrons, positrons, and antiproton propagation can be described as well by the diffusion equation. They constitute special cases, differing principally due to the energy losses and production rates of these particles

What D means?

- Random walk: after N steps of the same length l_0, a particle starting from the O of a ref. Frame is at position $\mathbf{d}=\sum_N \mathbf{l}_i$
- $<d> = 0$
- $<d^2> = d \bullet d = \sum_i \sum_j l_i \bullet l_j = N l_0^2 + 2 l_0 \sum_{i \neq j} \cos\theta_{ij} = N l_0^2$
- The solution of diffusion eqn $\frac{dN}{dt} - D\nabla N = Q$

(without E loss or any other process) is found to be

$$N(r) = \frac{N_o}{(4\pi D t)^{3/2}} e^{-\frac{r^2}{4Dt}}$$

For $Q(t) = N_o \delta(t)$, an istantaneous injection at t= 0 in the origin

What D means?

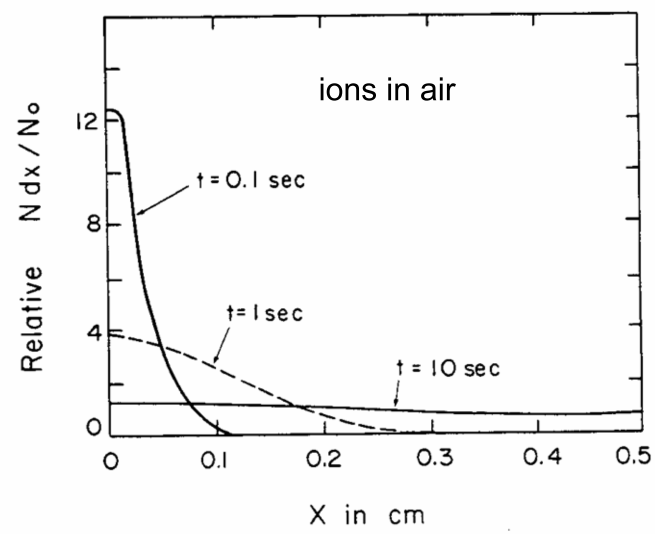
$$\frac{\partial n}{\partial t} + \nabla \cdot \left(-D \vec{\nabla} n + \vec{v} n\right) = R.$$

No convezione (per ora)

Se iniettiamo N particelle nell'origine con una distribuzione spaziale N = $N_0\delta(x)$ a t = 0, l'evoluzione temporale e' data da

$$N(x,t) = \frac{N_0}{2\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}}$$

(calcoletto alla lavagna?)



Il bunch di particelle rimane in media fermo nell'origine, ma si allarga nel tempo con una distribuzione gaussiana di larghezza crescente

$$\sigma_x = \sqrt{2Dt}$$

Una frazione della carica N/N_0, inizialmente tutta in x = 0, la si ritrova ad una distanza x dopo un tempo t

What D means?

- After a time t, the particles is at a average quadratic distance form origin $d \sim (Dt)^{1/2}$
- Therefore $Nl_o^2 = Dt \rightarrow D = Nl_o^2/t$
- Nl_o = total distance $\rightarrow Nl_o/t = v$
- So that $D \sim vl_o$

What D means?

- An order of magnitude estimate for D can be determined from the residence times of CR in the galaxy
- If L is the characteristic scale of the system $D\nabla N \approx \frac{DN}{L^2}$
- If τ is the average residence time of CR in the galaxy $\frac{dN}{dt} \approx \frac{N}{\tau}$
- Therefore, by order of magnitude $\frac{N}{\tau} \approx \frac{DN}{L^2}$

$$D \approx \frac{L^2}{\tau}$$

- The scale L for the galaxy is the disk thickness. Assuming L = 300 pc = 9×10^{20} cm and a typical residence time of 10^7 years = 3×10^{14} s, D ~ 3×10^{27} $cm^2 s^{-1}$

What D means?

- From $D = vl_o/3$, an estimate of the “step length” for a charged particle in the galaxy can be given
- $l_o = 3D/c \sim 3 \times 10^{17}$ cm = 0.1 pc
- More refined models give D up to 3×10^{28} cm^2 s^{-1}
- → $l_o \sim 1$ pc
- Also the halo thickness is uncertain. Estimates range up to 1000 pc (which, for example, results in a factor 10 for D)

What D means?

- → l_o ~ 0.1 - 1 pc
- Average particle density is too low for an efficient scattering by collisions
- This quantity can be interpreted as the typical scale of magnetic fields inhomogeneities of the magnetic field in the ISM
- On microscopic level, the diffusion of CR results from particle scatterings on magnetohydrodynamics waves and discontinuities (eg shock waves).