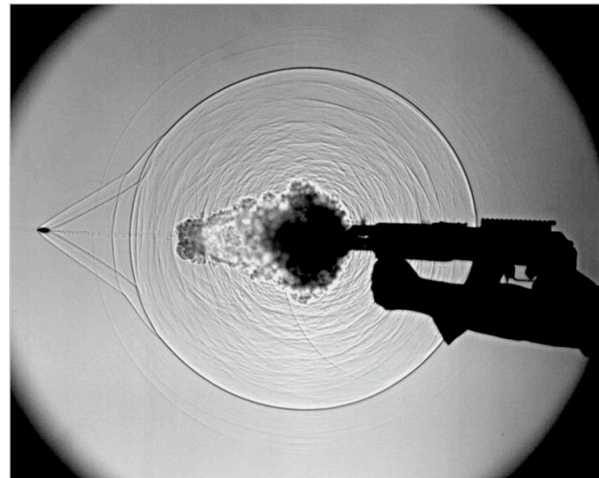


Lecture 14 231116

- Il pdf delle lezioni puo' essere scaricato da
- http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1617/



Blast shock waves

A blast shock wave is a shock wave formed by a hot gas bubble expanding supersonically in the ambient medium

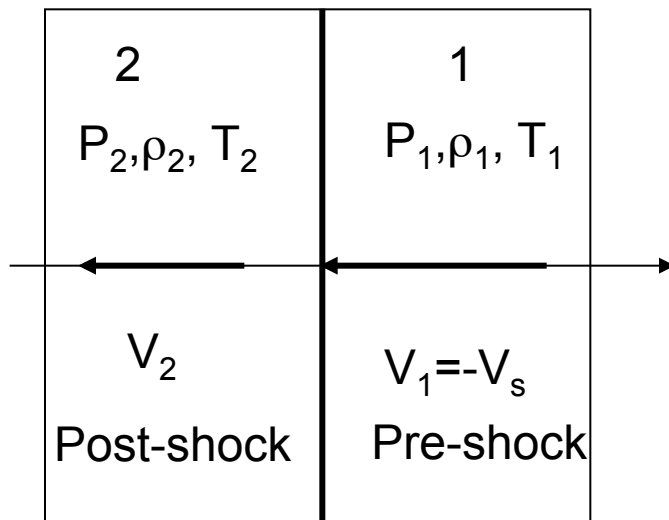
Let us assume that the expansion occurs in a uniform stationary polytropic medium with density ρ_0 and pressure p_0

how does the shock wave evolve in time?

First, since the surrounding medium is uniform, the expansion will have spherical symmetry

Blast shock waves

We worked out the physics of the (strong) shock in the shock reference frame, where it is stationary



Shock rest frame

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma + 1)}{(\gamma - 1)}$$

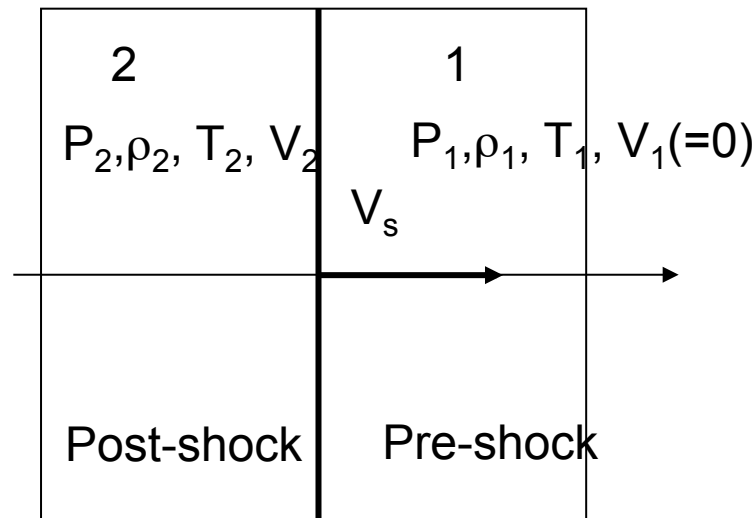
$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma + 1)}$$

$$\frac{V_2}{V_1} \approx \frac{\gamma - 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2$$

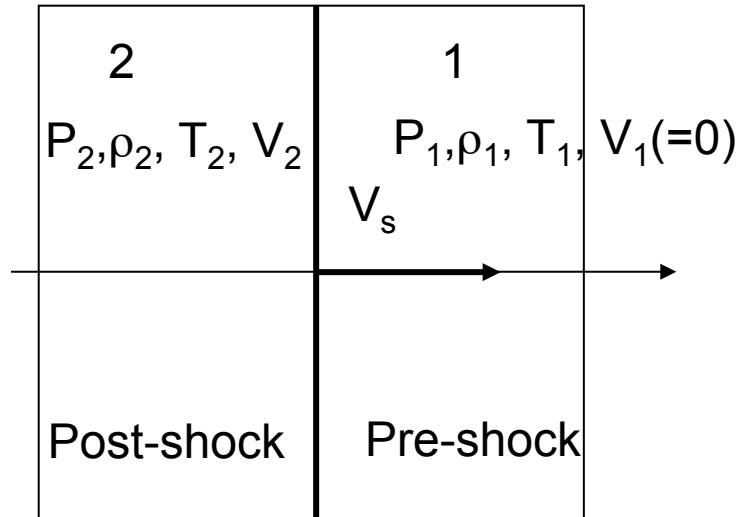
Blast shock waves

Now we have an (supersonic) expansion in a uniform, stationary ($V_1=0$) polytropic medium with density $\rho_1 = \rho_0$ and pressure $p_1 = p_0$ and a supersonic shock wave propagating to the right (+x dir) with speed V_s ahead the expanding gas, as in the case of the supersonic piston

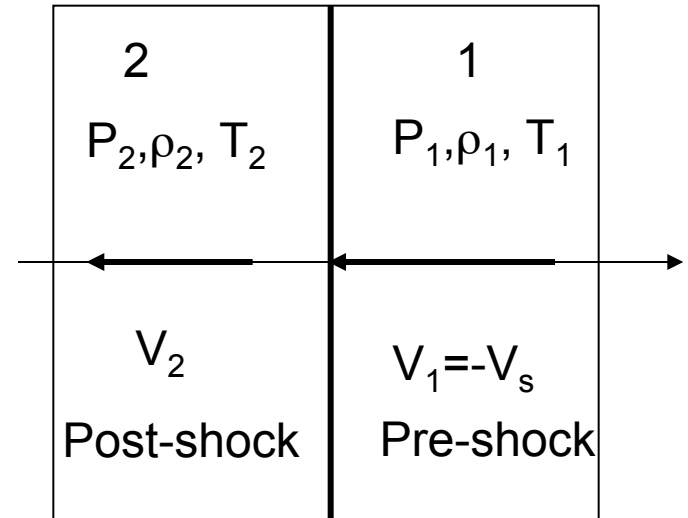


As in the supersonic piston example, the situation in a reference frame where the shock is traveling with speed V_s can be obtained, for non-relativistic shocks, from a simple galileian transformation

Blast shock waves



Observer frame



Shock rest frame

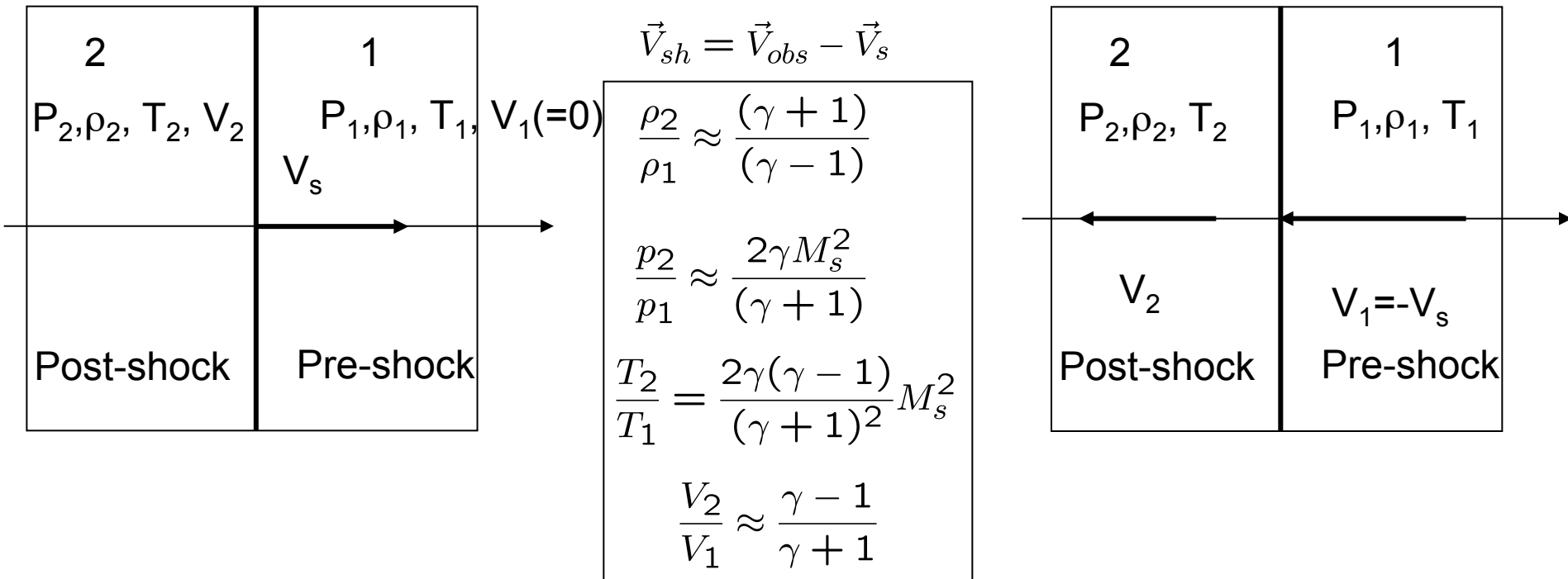
The RH conditions can be still applied, provided one interprets the speeds V_1 and V_2 as relative speeds with respect to the shock, ie apply a galileian velocity tranformation

$$\vec{V} \rightarrow \vec{V}_{rel} = \vec{V} - \vec{V}_s$$

For a shock propagating with velocity \mathbf{V}_s into a medium at rest, $\mathbf{V}=0$, one has $\mathbf{V}_1=-\mathbf{V}_s$ and $\theta_s=0$ (as for the supersonic piston) \rightarrow in this case any shock is a normal shock, with $\mathbf{V}_t=0$, even when the shock surface itself is not a plane!

\rightarrow spherical shocks are normal shocks

Blast shock waves



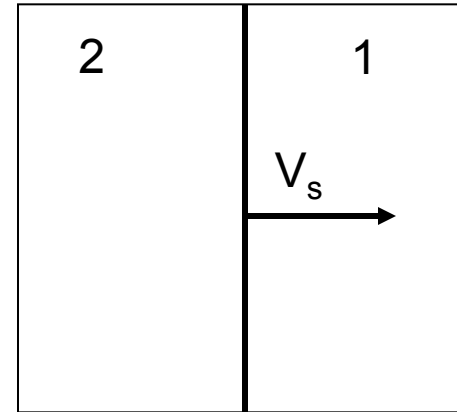
All the conditions may be directly applied, except the speed for which we need to make the substitution $V_{2sh} = V_2^{obs} - V_s$ and $V_1 = -V_s$

$$\frac{V_2}{V_1} \rightarrow \frac{V_2 - V_s}{-V_s} \approx \frac{\gamma - 1}{\gamma + 1} \quad \Rightarrow \quad V_2 \approx \frac{2V_s}{\gamma + 1}$$

Is the post shock speed in the obs frame

Blast shock waves

Let assume also that the explosion occurs in a uniform stationary polytropic medium with density $\rho_1 = \rho_o$ and pressure $p_1 = p_o$



The strong shock satisfies the relation $M_s^2 = \left(\frac{V_s}{c_s}\right)^2 = \frac{\rho_o V_s^2}{\gamma p_o} \gg 1$

The RH relations then give

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma + 1)}{(\gamma - 1)}$$

$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma + 1)}$$

$$V_2 \approx \frac{2V_s}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2$$

We can have the pressure p_2 immediately behind the shock

$$p_2 \approx \frac{2\gamma M_s^2}{(\gamma + 1)} p_o = \frac{2\rho_o V_s^2}{(\gamma + 1)}$$

Inverting this relation, one can calculate the shock speed as a function of the post-shock pressure and the pre-shock density

$$V_s = \left(\frac{\gamma + 1}{2}\right)^{1/2} \left(\frac{p_2}{\rho_o}\right)^{1/2}$$

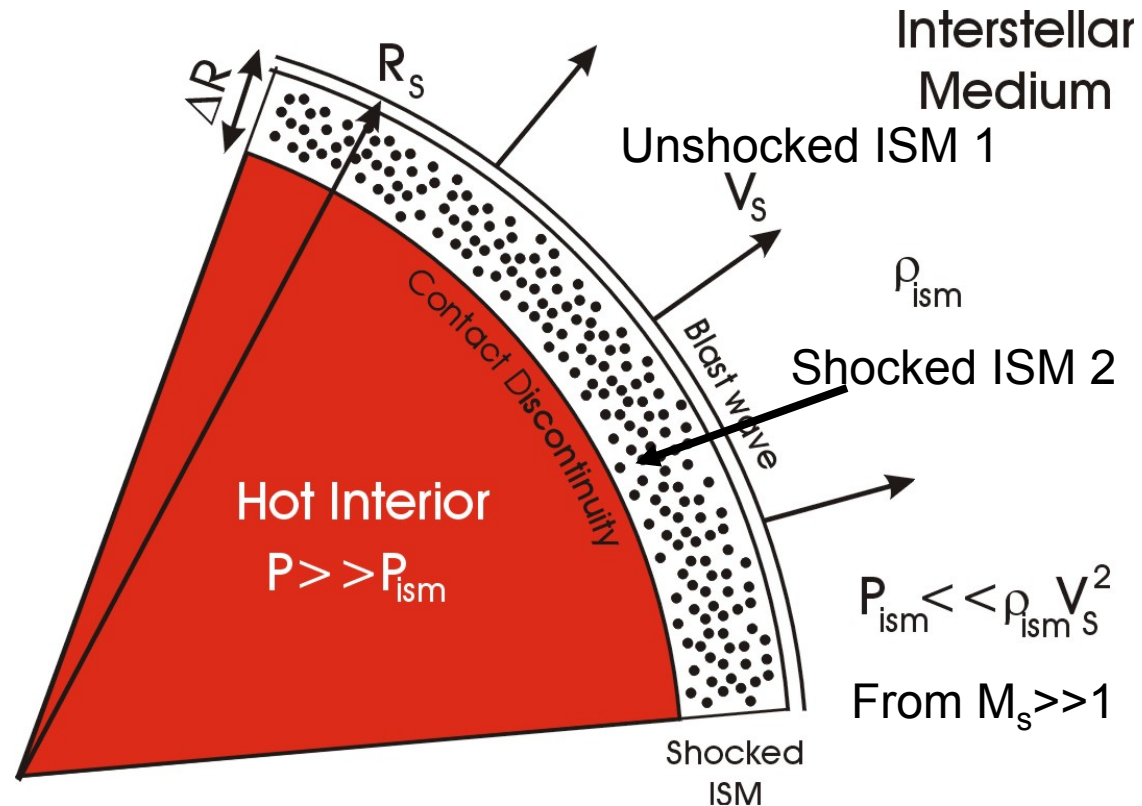
This result can be applied for the formation of high pressure bubbles in a stationary medium

As for instance SuperNova remnants (SNRs) and stellar wind bubbles in the interstellar medium

(...and to nuclear explosions too, unfortunately)

Blast shock waves

Consider a spherical bubble containing a low density, very hot gas with internal pressure p_i and density ρ_i embedded in a cold, dense stationary medium with low pressure p_o and a high density ρ_o (...we can have low pressure with high density at low T because $p=nkT$)



Because of the high pressure difference, the bubble will start to expand rapidly

Blast shock waves

If the difference between internal and external pressure is sufficiently large, the expansion speed will be supersonic with respect to sound speed of the surrounding medium

$$M_s^2 = \left(\frac{V_s}{c_s}\right)^2 = \frac{\rho_o V_s^2}{\gamma p_o}$$

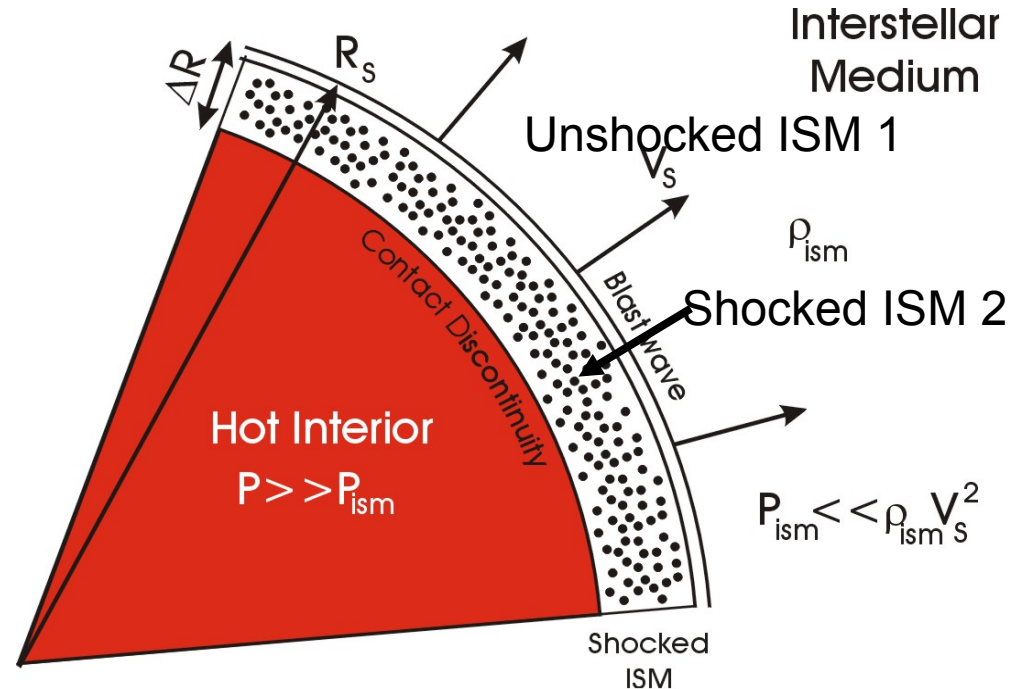
$$V_s = \left(\frac{\gamma + 1}{2}\right)^{1/2} \left(\frac{P_2}{\rho_o}\right)^{1/2}$$

Imply that $M_s > 1$ if $p_2/p_o > 2\gamma/(\gamma+1)$

For instance, the typical observed expansion speed of a supernova remnant is ~ 10000 km/s, while the sound speed in the ISM ranges 10-100 km/s

Because of supersonic speed, a shock will form at the outer edge of the bubble (which acts as a supersonic piston).

This shock is usually called blast wave



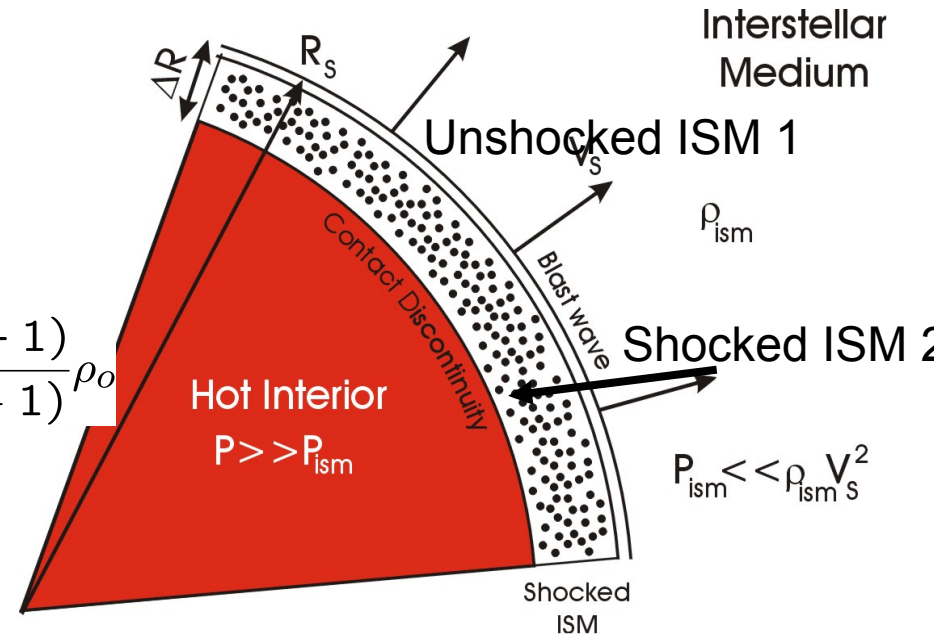
Blast shock waves

The mass that has been swept up by the expanding bubble will collect in a dense shell at its outer rim

If $M_s \gg 1$, the typical density of the shocked material in the shell is

$$\rho_{sh} \approx \frac{(\gamma + 1)}{(\gamma - 1)} \rho_o$$

This allows us to calculate the thickness of the shell



Neglecting, for now, the mass of the bubble (because $\rho_i \ll \rho_o$), a bubble with radius R has swept up a mass from surrounding ISM

$$M_{sw} = \frac{4}{3} \pi \rho_o R^3$$

This mass is now in a shell with thickness ΔR with density $\rho_{sh} \rightarrow$ if $\Delta R \ll R$

$$M_{sw} \approx 4\pi \rho_{sh} R^2 \Delta R$$

Combining we get $\Delta R \approx \frac{(\gamma - 1)}{3(\gamma + 1)} R$
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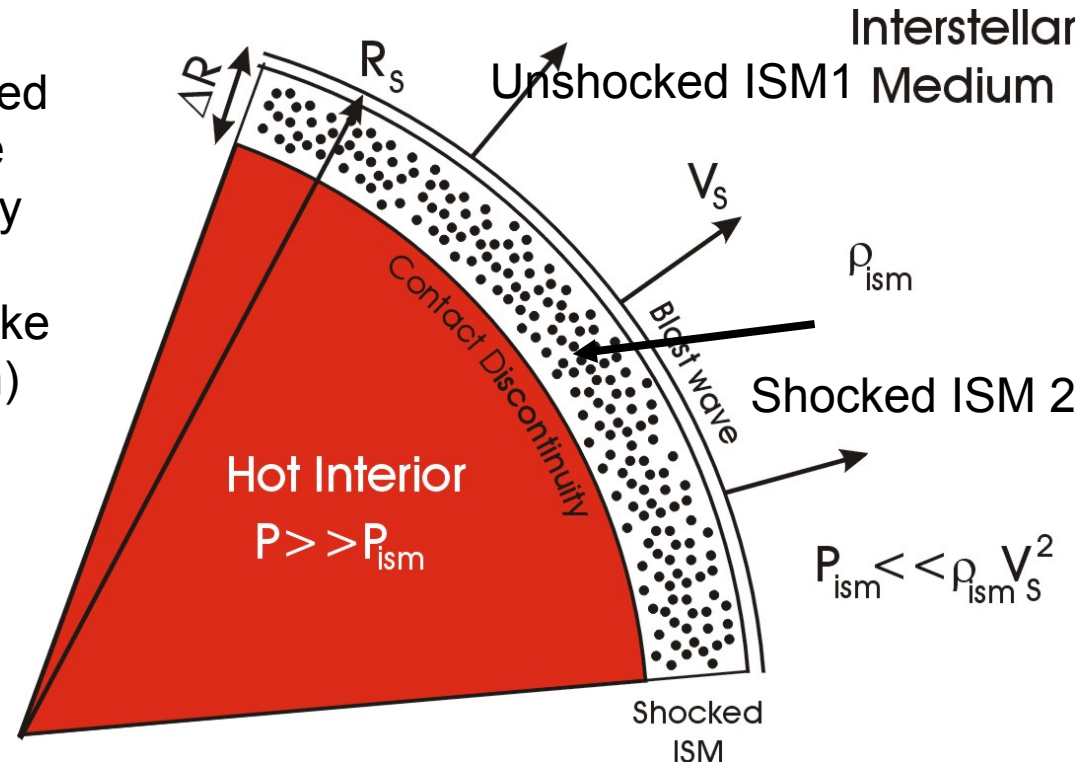
For $\gamma = 5/3$, $\Delta R = 0.083 R$
 \rightarrow thin shell approximation is fine

Blast shock waves

The swept up material is separated from the hot material inside the bubble by a contact discontinuity because the relative speed between the two media is zero (like in the case of supersonic piston)

The expansion speed is

$$\frac{dR}{dt} \approx V_s = \left(\frac{\gamma + 1}{2}\right)^{1/2} \left(\frac{p_2}{\rho_0}\right)^{1/2}$$

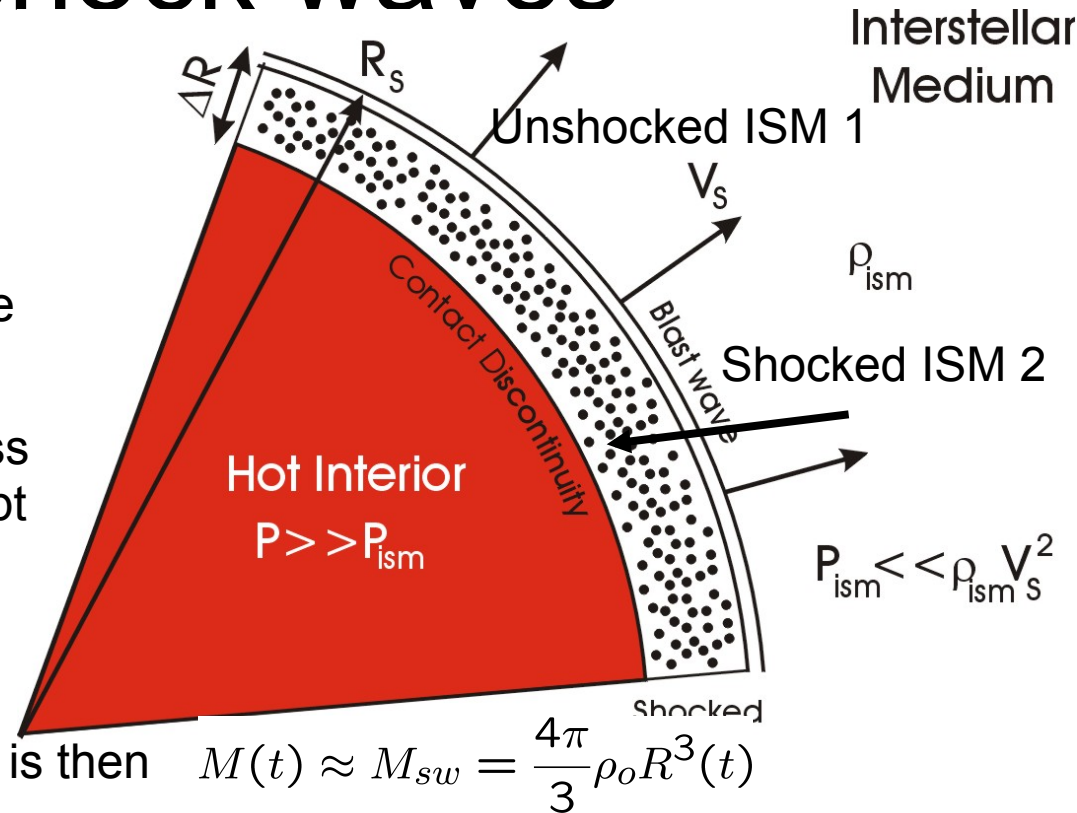


Blast shock waves

$$\frac{dR}{dt} \approx V_s = \left(\frac{\gamma + 1}{2}\right)^{1/2} \left(\frac{P_2}{\rho_o}\right)^{1/2}$$

We can get the time evolution of the shock radius

Let us assume that most of the mass of the expanding bubble is the swept up mass and which resides in the shocked shell of thickness $\Delta R \ll R$



→ the mass contained in the bubble is then $M(t) \approx M_{sw} = \frac{4\pi}{3} \rho_o R^3(t)$

The total energy of the bubble consists of the kinetic energy of the expanding massive shell and the internal energy of the hot tenous gas inside the bubble

$$E(t) = \frac{1}{2} M(t) \left(\frac{dR(t)}{dt} \right)^2 + \left(\frac{4\pi}{3} \rho_o R^3 \right) \left(\frac{p_i(t)}{\rho_o (\gamma - 1)} \right)$$

mistake: here ρ_o is ρ_i

Assuming uniform pressure p_i inside the bubble

Blast shock waves

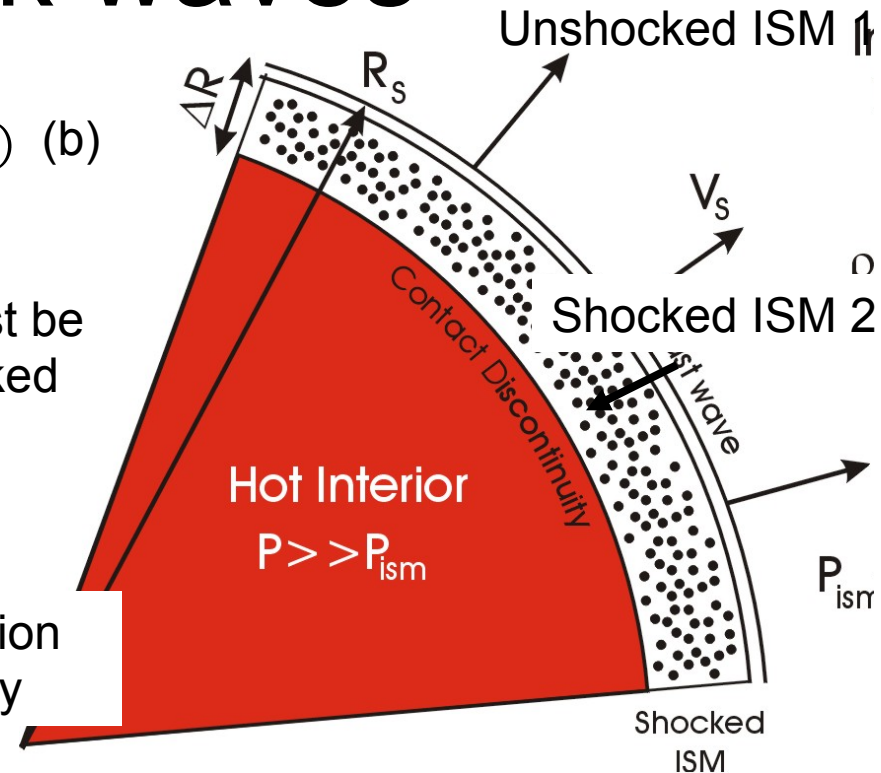
mistake: here ρ_o is ρ_i

$$E(t) = \frac{1}{2}M(t)\left(\frac{dR(t)}{dt}\right)^2 + \left(\frac{4\pi}{3}\rho_o R^3\right)\left(\frac{p_i(t)}{\rho_o(\gamma - 1)}\right) \quad (b)$$

The interior pressure in the hot bubble must be roughly equal to the pressure in the shocked material in the shell

$$p_i \approx p_2 \approx \frac{2}{\gamma + 1}\rho_o\left(\frac{dR}{dt}\right)^2 \quad (a)$$

Which is simply the pressure-balance condition which must hold at the contact discontinuity



Inserting (a) into (b) we get
$$E(t) = \frac{2\pi}{3}\rho_o R^3 \left(\frac{dR}{dt}\right)^2 \left[1 + \frac{4}{\gamma^2 - 1}\right]$$

→ the ratio of thermal energy and kinetic energy is a constant
$$E_{th}/E_{kin} = \frac{4}{\gamma^2 - 1}$$

For $\gamma=5/3$, $E_{th}/E_{kin}=9/4 \rightarrow E_T=E_{th}+E_{kin}=(1+9/4)E_{kin}=(13/4)E_{kin}$

→ $E_{kin} = (4/13)E_T \approx 31\% E_T$...most part of the initial energy is thermal energy 13

Blast shock waves

$$E(t) = \frac{2\pi}{3} \rho_o R^3 \left(\frac{dR}{dt} \right)^2 \left[1 + \frac{4}{\gamma^2 - 1} \right]$$

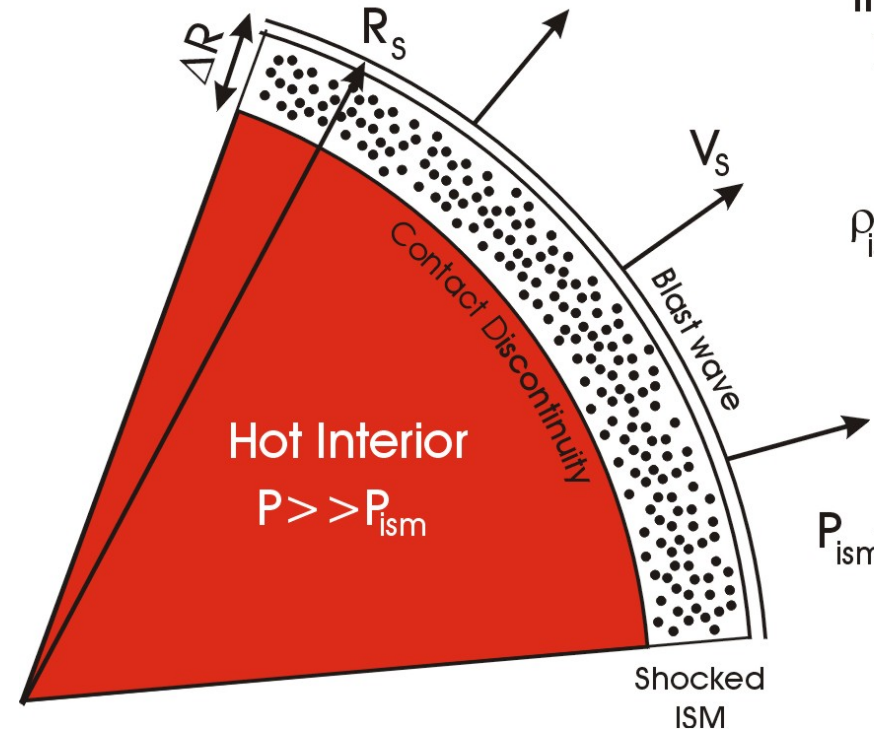
→ the total energy can be then written as

$$E(t) = C_\gamma M(t) \left(\frac{dR}{dt} \right)^2$$

With $C_\gamma = \frac{\gamma^2 + 3}{2(\gamma^2 - 1)}$

For an ideal monoatomic gas $C_\gamma = 1.625$

C_γ is approximate because of the various approximations made in the derivation. However, more exact treatments come at the same result with a somewhat smaller value



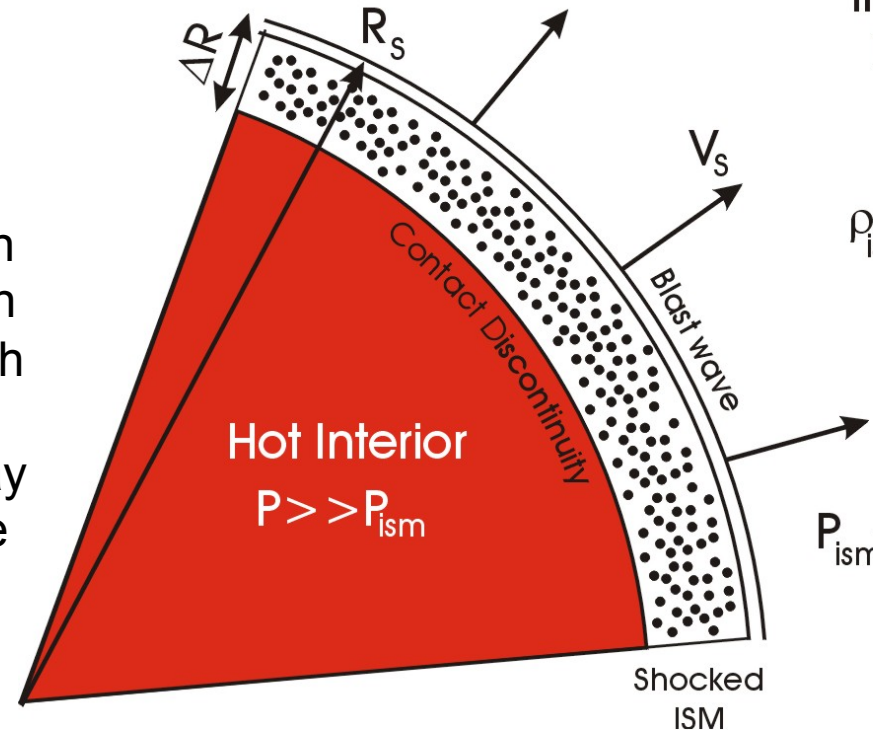
Blast shock waves

$$E(t) = C_\gamma M(t) \left(\frac{dR}{dt} \right)^2$$

The energy may depend on time, because in general there may be some energy source in the bubble which fuels the hot bubble through emission of power L , as for instance a star driving a strong stellar wind, or the SNRs may radiate away part or all their energy at some point of its evolution

Two cases:

- i) a point explosion where a fixed amount of energy is supplied impulsively at $t=0$ and where no energy losses occur afterwards (blast shock wave, SN explosions)
- li) a constant energy supply at some luminosity $L=dE/dt$ so that $E(t)=L \times t$, which can serve as a crude model of the energy of a bubble blown into the ISM by a strong stellar wind



Blast shock waves

Blast wave generated by a 4.8 kiloton explosion



Explosions (eg nuclear detonations) generate blast shock wave, very much like a supernova: the piston is the exploding material

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Blast shock waves



Most of the "experimental" knowledge comes from military nuclear tests during the cold war

For long time, and still now, most of the data on nuclear tests are classified top secret

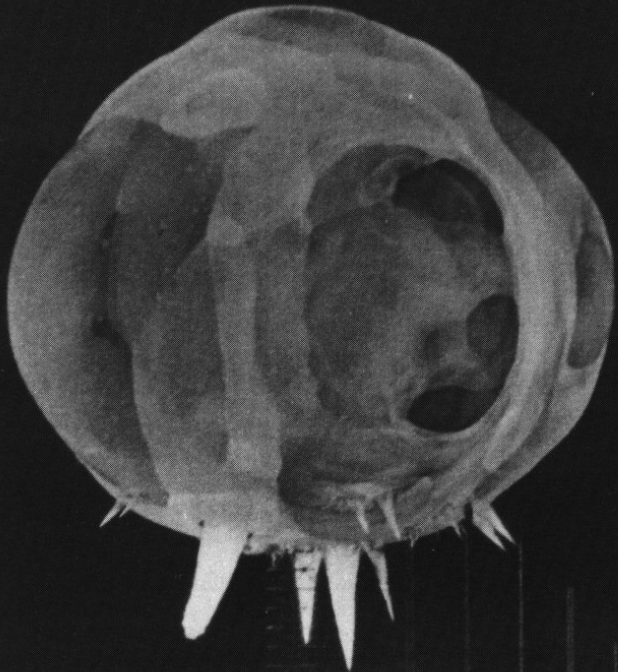
Whatever the mechanism by which initial energy is liberated, the subsequent blast wave evolution does not depend on it!

Blast shock waves

10 ms from detonation



1 ms from detonation



Trinity test



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Supernova Blast Waves

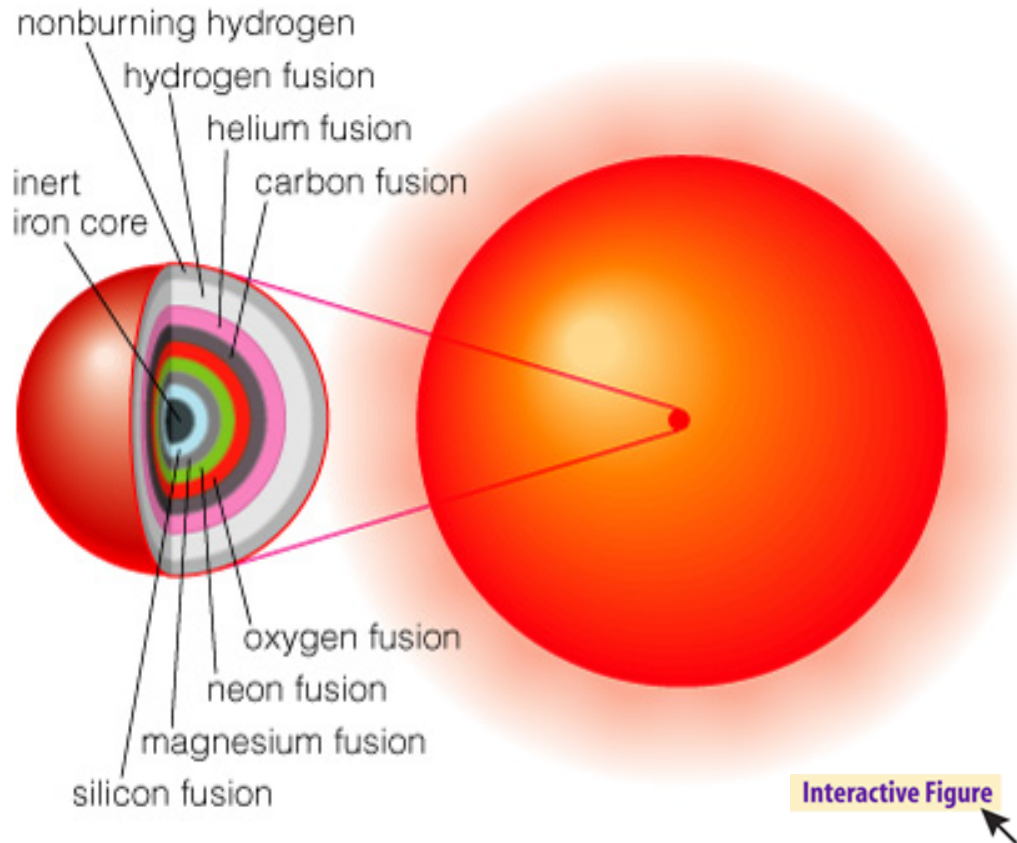
<u>Supernovae:</u>	
Type Ia	<u>Mechanism:</u> explosive carbon burning in a mass-accreting white dwarf
Type Ib-Ic & Type II	<u>Core collapse</u> of massive star

We may neglect the details of the explosive event: for us is a sudden, point-like release (a $\delta(x,t)$) of energy

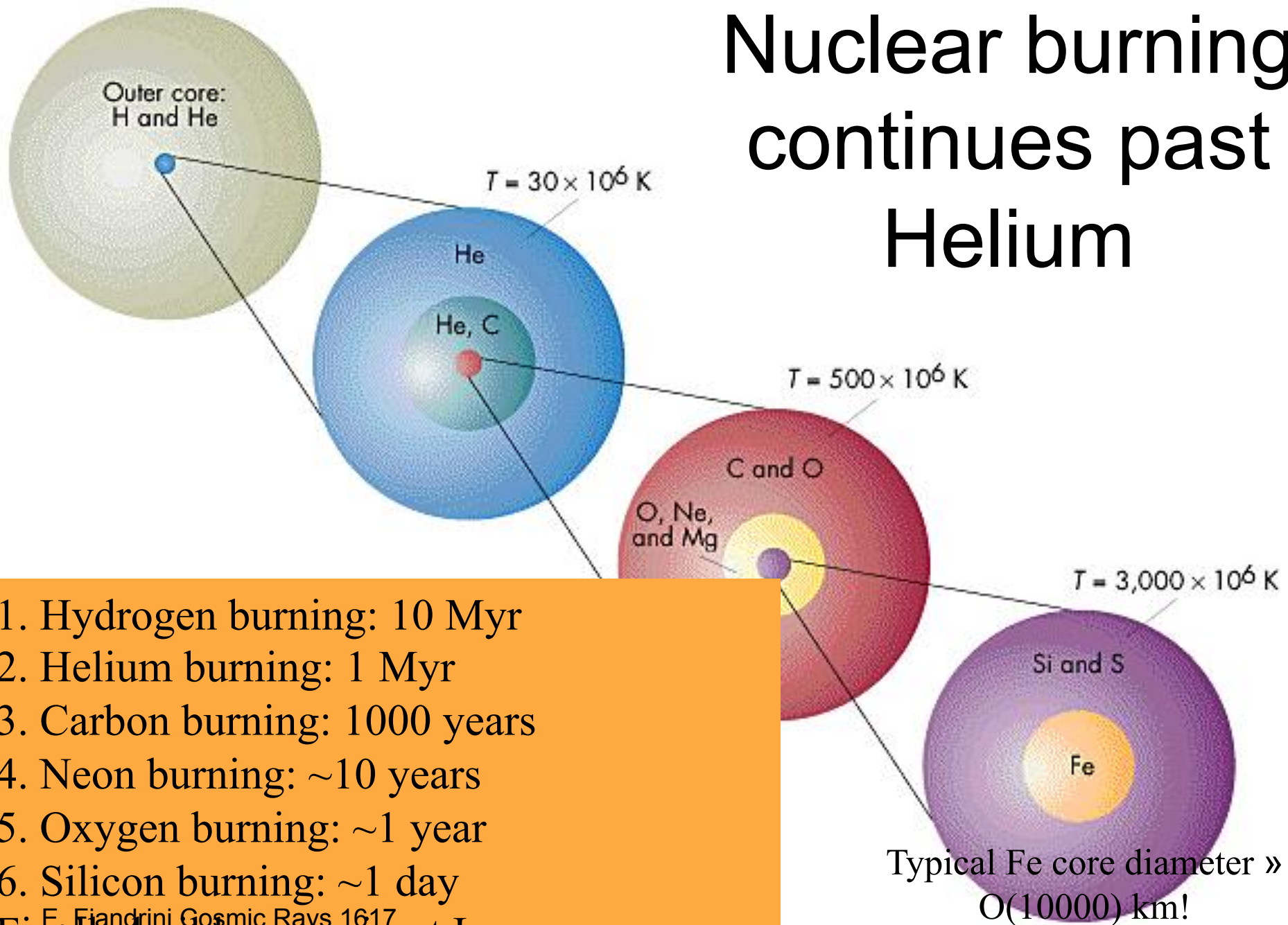
While the composition of the material of the SNR does depend on the mechanism (ie star composition) and on the surrounding ISM composition

Multiple Shell Burning

- Advanced nuclear burning proceeds in a series of nested shells, like an onion

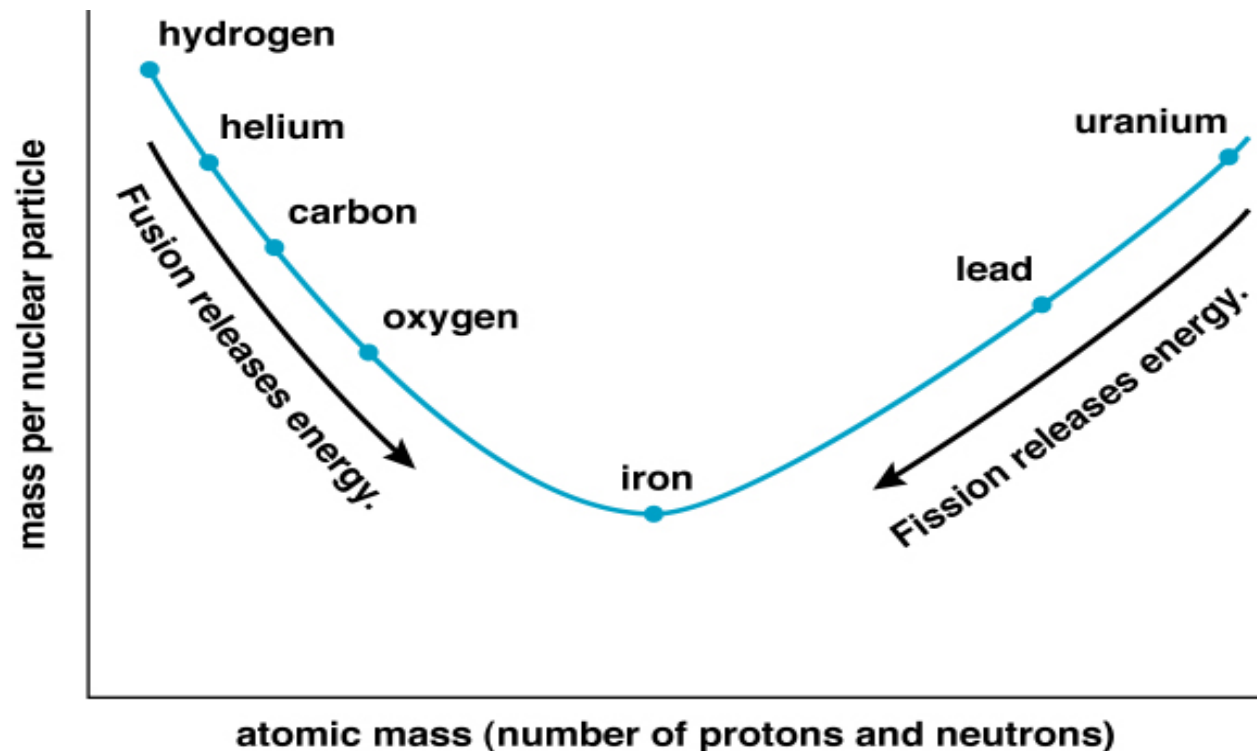


Nuclear burning continues past Helium

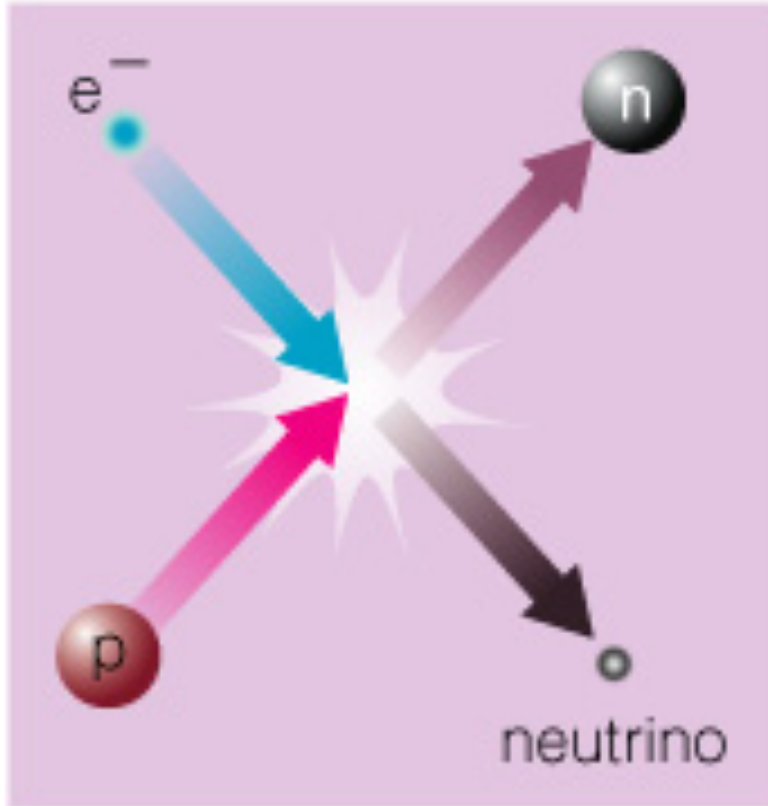


- The supergiant has an inert Fe core which collapses & heats
 - Fe can not fuse
 - It has the lowest mass per nuclear particle of any element
 - It can not fuse into another element without *creating* mass

So the Fe core continues to collapse until it is stopped by electron degeneracy.
(like a White Dwarf)



Supernova Explosion



- Core degeneracy pressure goes away because electrons combine with protons, making neutrons and neutrinos
$$p + e^- \rightarrow n + \nu_e$$
- Neutrons collapse to the center, forming a **neutron star**

Core collapse

- Iron core is degenerate
- Core grows until it is too heavy to support itself ($M_{\text{nuc}} > M_{\text{chandrasekhar}}$)
- Core collapses, density increases, normal iron nuclei are converted into neutrons with the emission of neutrinos
- Core collapse stops, neutron star is formed
- Rest of the star collapses on the core, but bounces off the new neutron star (also pushed outwards by the neutrinos)



Pre-supernova star



Collapse of the core



Interaction of shock
with collapsing envelope



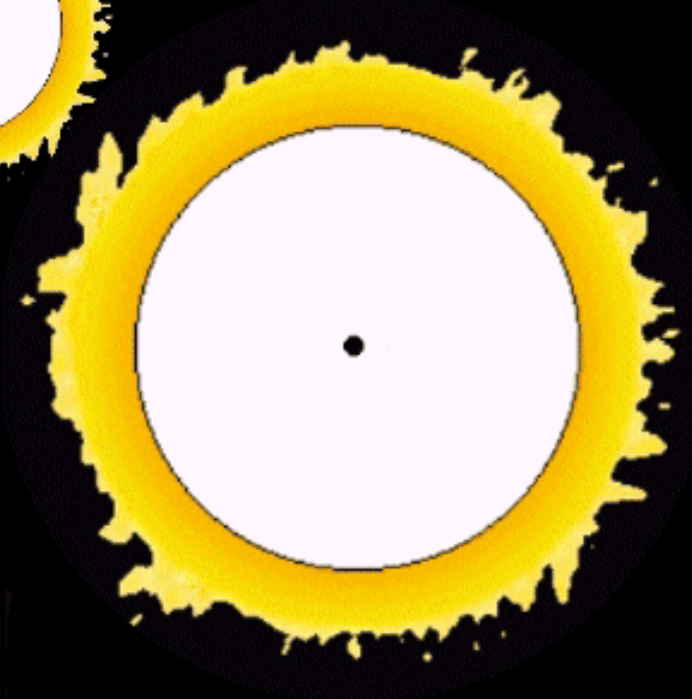
neutrinos emitted



light emitted



Explosive ejection of envelope



Expanding remnant emitting X-rays,
visible light, and radio waves.
The collapsed stellar remnant may be
observable as a pulsar.

Star brightens by $\sim 10^8$ times

Shock Waves in Supernovae

- Discontinuity in velocity, density and pressure in a flow of matter.
- Unlike a sound wave, it causes a permanent change in the medium
- Shock speed \gg sound speed - between 30,000 and 50,000 km/s.
- Shock wave may be *stalled* if energy goes into breaking-up nuclei into nucleons.
- This consumes a lot of energy, even though the pressure (nkT) increases because n is larger.

Supernova Energetics

Same source for supernovae (Ib/Ic and II),

- Explosion powered by the collapse (death) of a massive core
- Energy source: Potential Energy from the collapse of the iron core down to a neutron star or black hole:

$$\text{Energy} = G M_{\text{core}}^2 / r_{\text{NS,BH}} - G M_{\text{core}}^2 / r_{\text{before collapse}} \\ > 10^{53} \text{ ergs}$$

$$M_{\text{core}} \sim 1.4\text{-}3 \text{ solar masses}$$

$$R_{\text{NS,BH}} \sim 10 \text{ km}$$

$$R_{\text{core}} \sim 10,000 \text{ km}$$

Supernova energetics

Core Collapse Supernova Energetics

Liberated gravitational binding energy of neutron star:

$$E_b \approx 3 \times 10^{53} \text{ erg} \approx 17\% M_{\text{SUN}} c^2$$

This shows up as

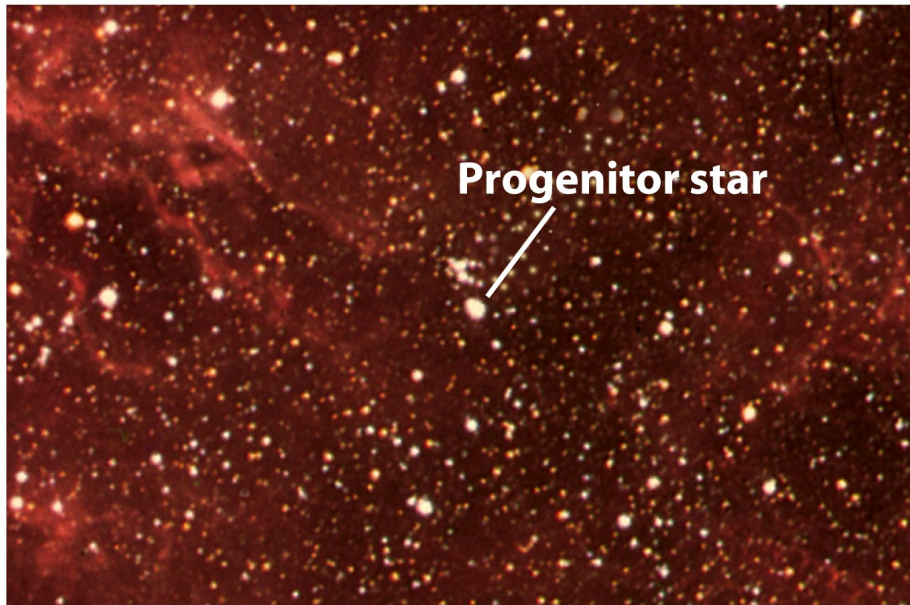
99%	Neutrinos
1%	Kinetic energy of explosion (1% of this into cosmic rays)
0.01%	Photons (outshine host galaxy)

Neutrino luminosity

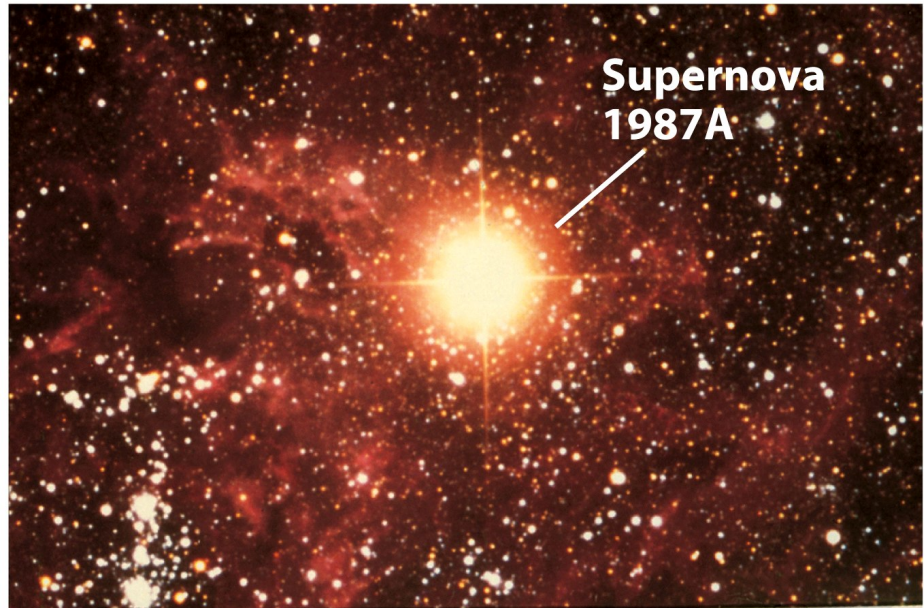
$$L_\nu \approx 3 \times 10^{53} \text{ erg} / 3 \text{ sec} \approx 3 \times 10^{19} L_{\text{SUN}}$$

While it lasts, outshines the photon
luminosity of the entire visible universe!

In 1987 a nearby supernova gave us a close-up look at the death of a massive star



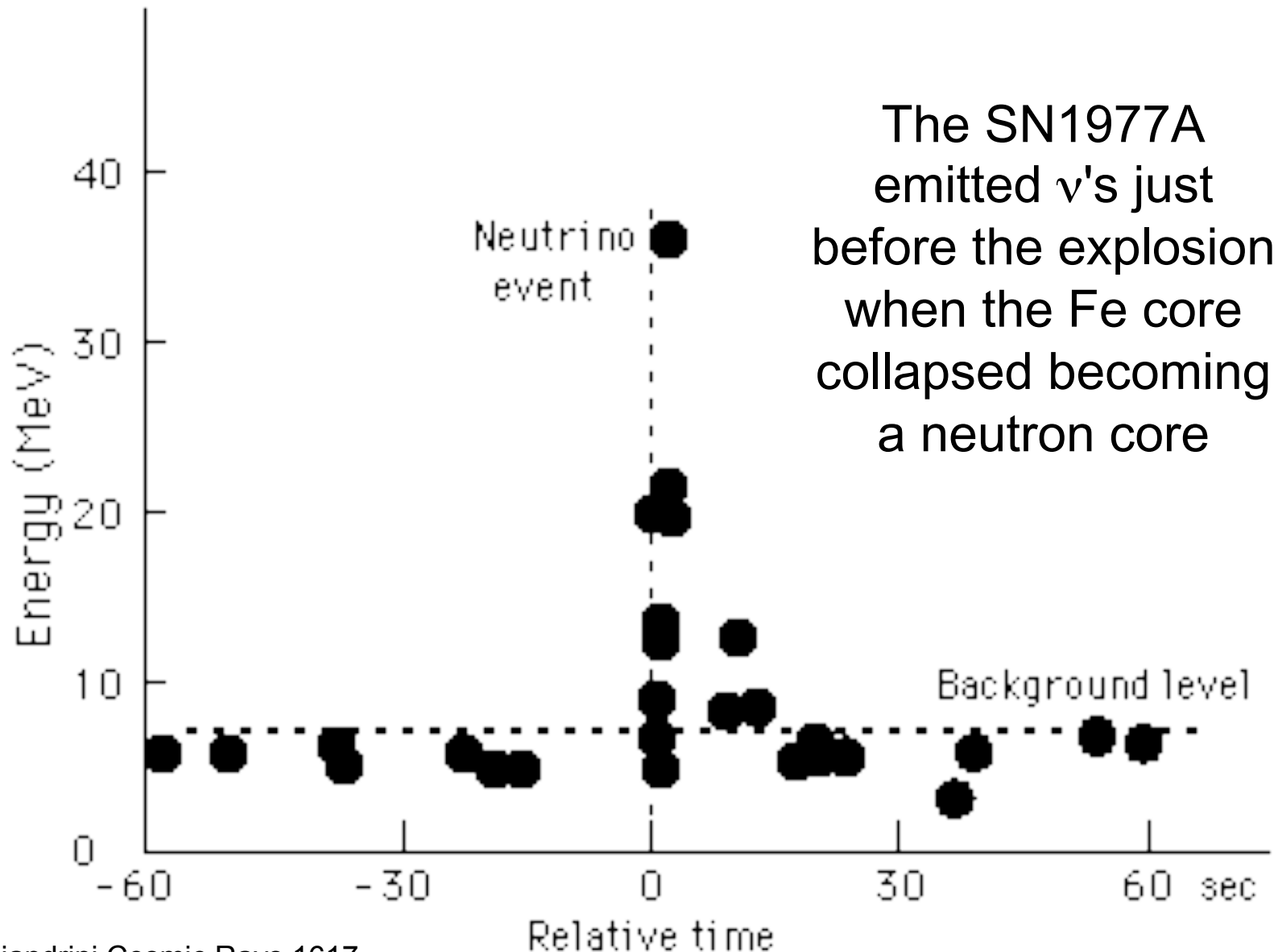
Before the star exploded



After the star exploded

At peak its luminosity was greater than host galaxy luminosity
(Large magellanic Cloud)

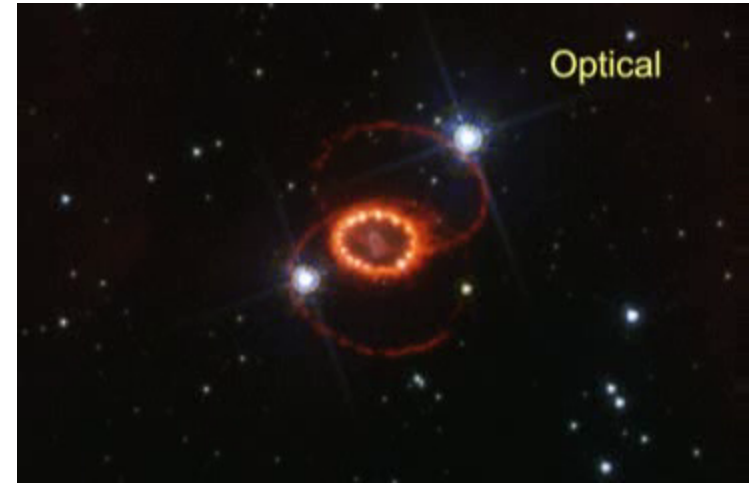
Neutrinos from SN1987A



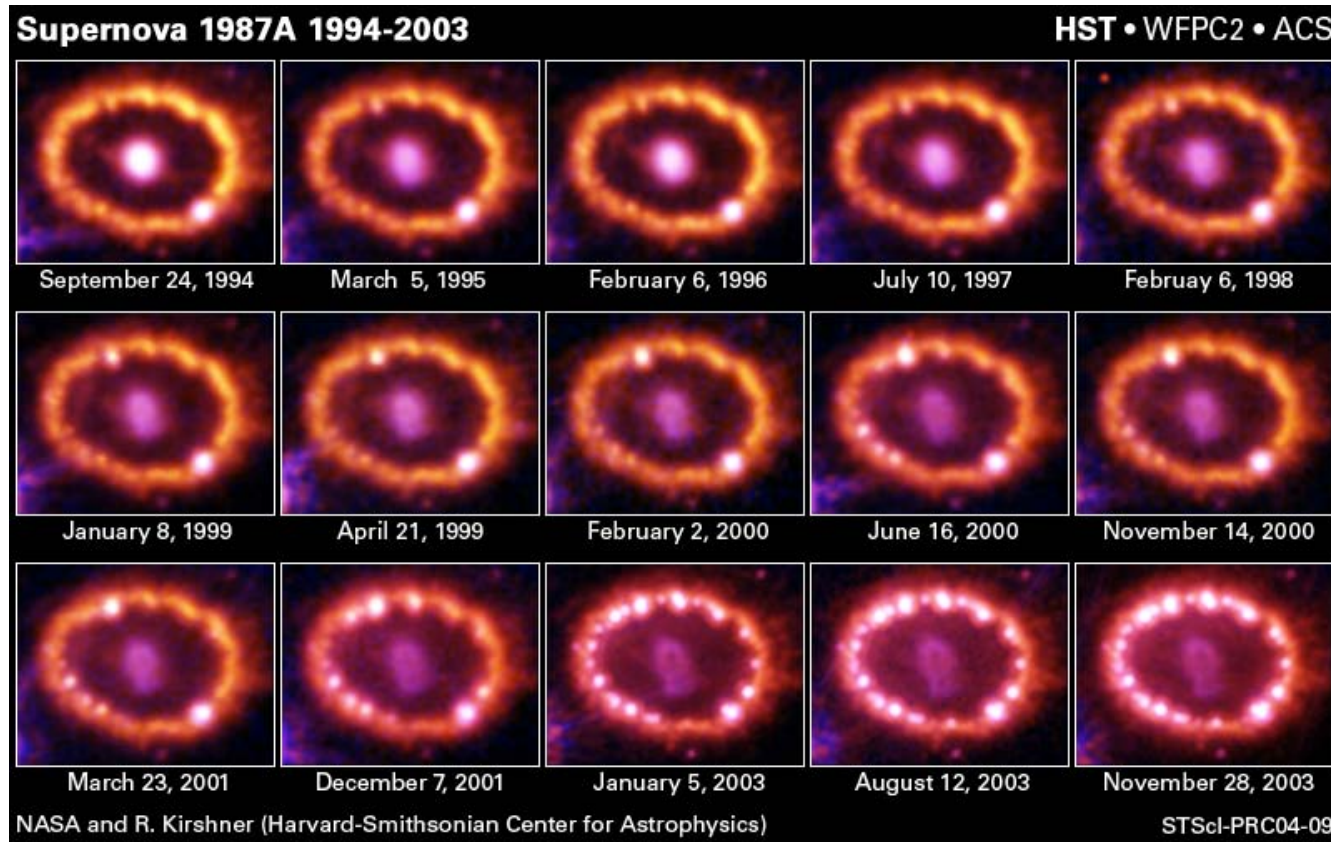
SN1987A – Blue Supergiant Supernova?

The progenitor of SN1987A was a blue giant with a mass of about $18 M_{\text{sun}}$.

- Probably, the high-mass progenitor of SN1987A lost most of its outer layer by a slow stellar wind long before the supernova explosion.
- Right before the supernova explosion, a fast wind pushes the envelop to make a cavity around the star. Making the outer layer of the star unusually thin and warm
- The outer gas cloud forms a ring.
- The shockwave from the supernova explosion was expected to hit the edge of the ring around 1999.
- Chandra X-ray images from 1999 to 2005 shows brightening of the ring.



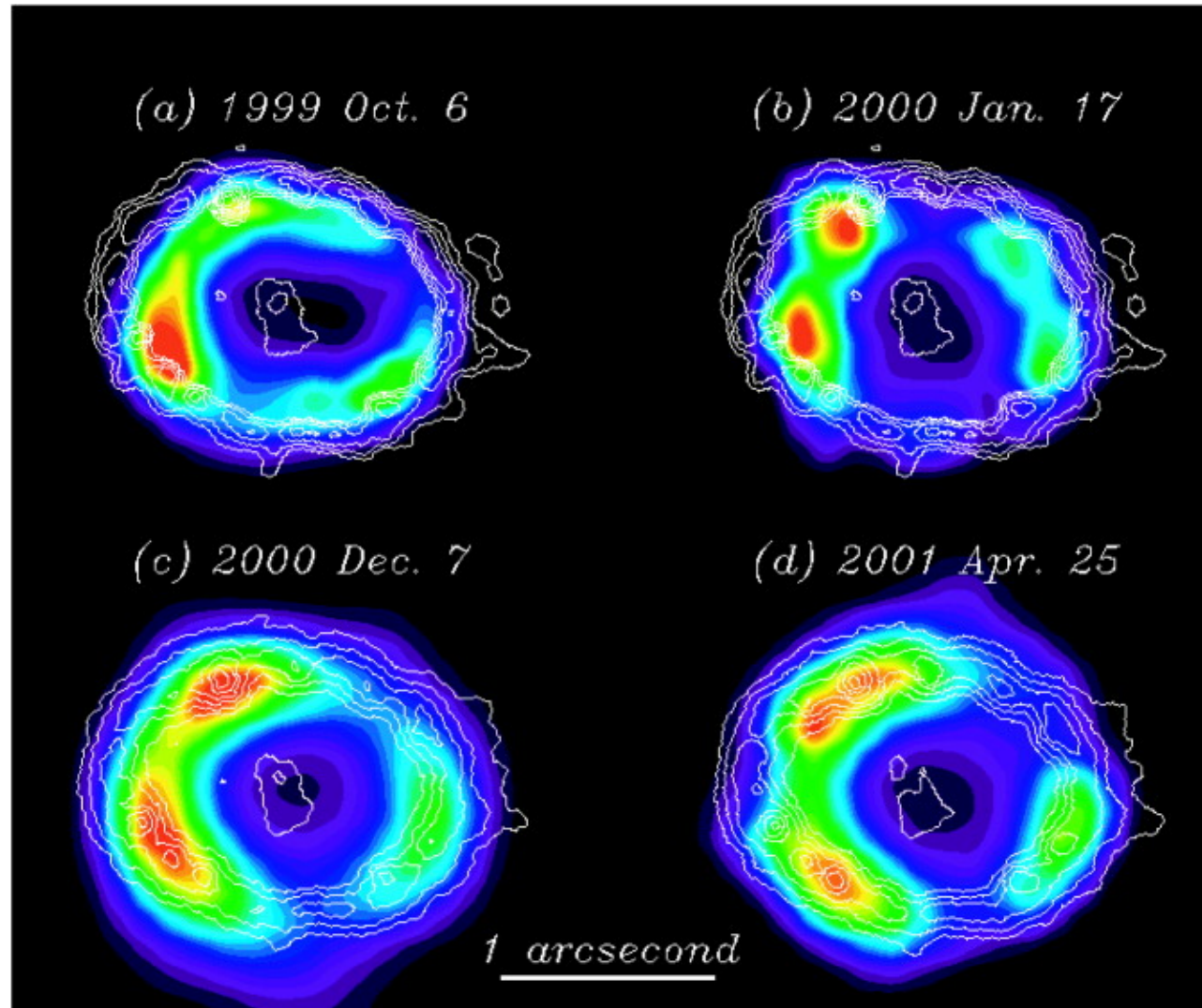
Shock hits inner ring



The shock has hit the inner ring at 20,000 km/s, lighting up knots of shocked, compressed and heated material in the ring which is 160 billion km wide.

Chandra X-ray Images of SN 1987A

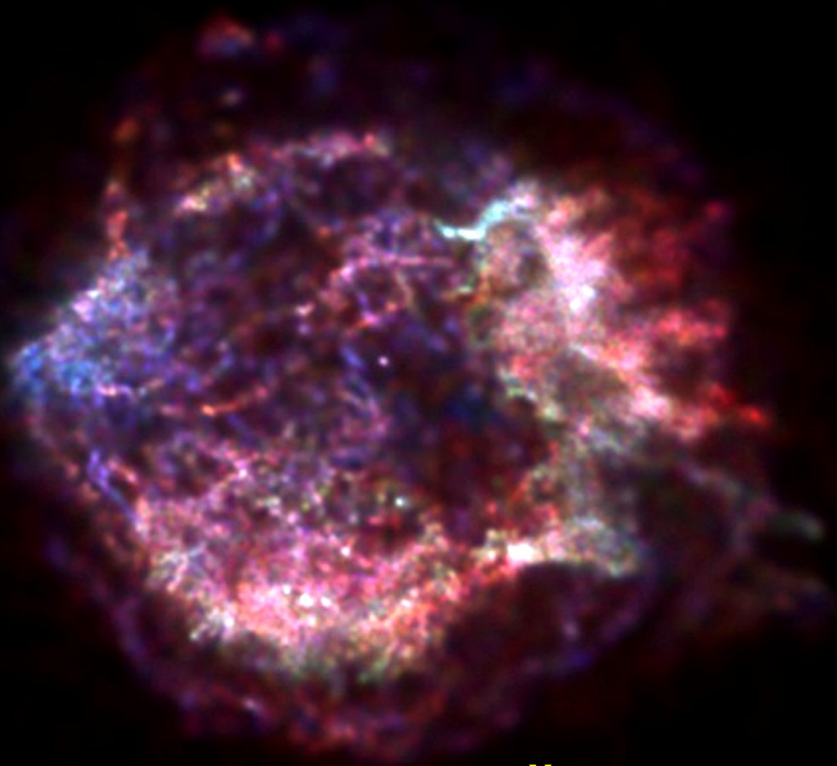
- X-ray intensities (0.5 – 8.0 keV) in colour with HST H α images as contours
- Low energy X-rays are well correlated with optical knots in ring – dense gas ejected by progenitor?
- Higher energy X-rays well correlated with radio emission – fast shock hitting circumstellar H II region?
- No evidence yet for emission from central pulsar



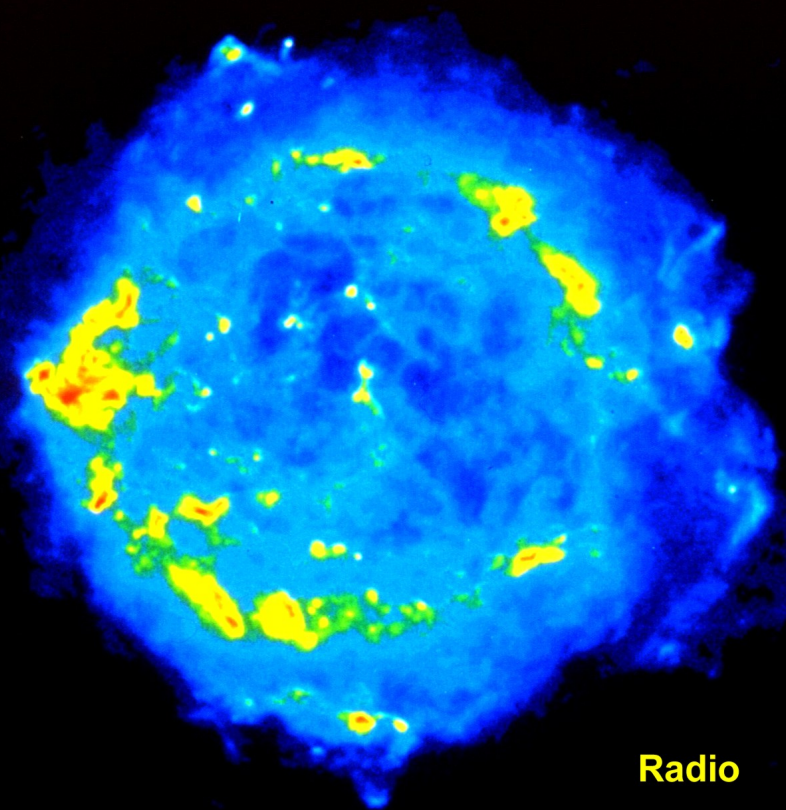
SuperNova Remnants (SNRs)

A supernova remnant (SNR) consists essentially of the stellar ejecta of the SN explosion embedded in a hot expanding bubble, preceded by swept-up interstellar material and an outer blast wave (strong shock) propagating into the interstellar medium

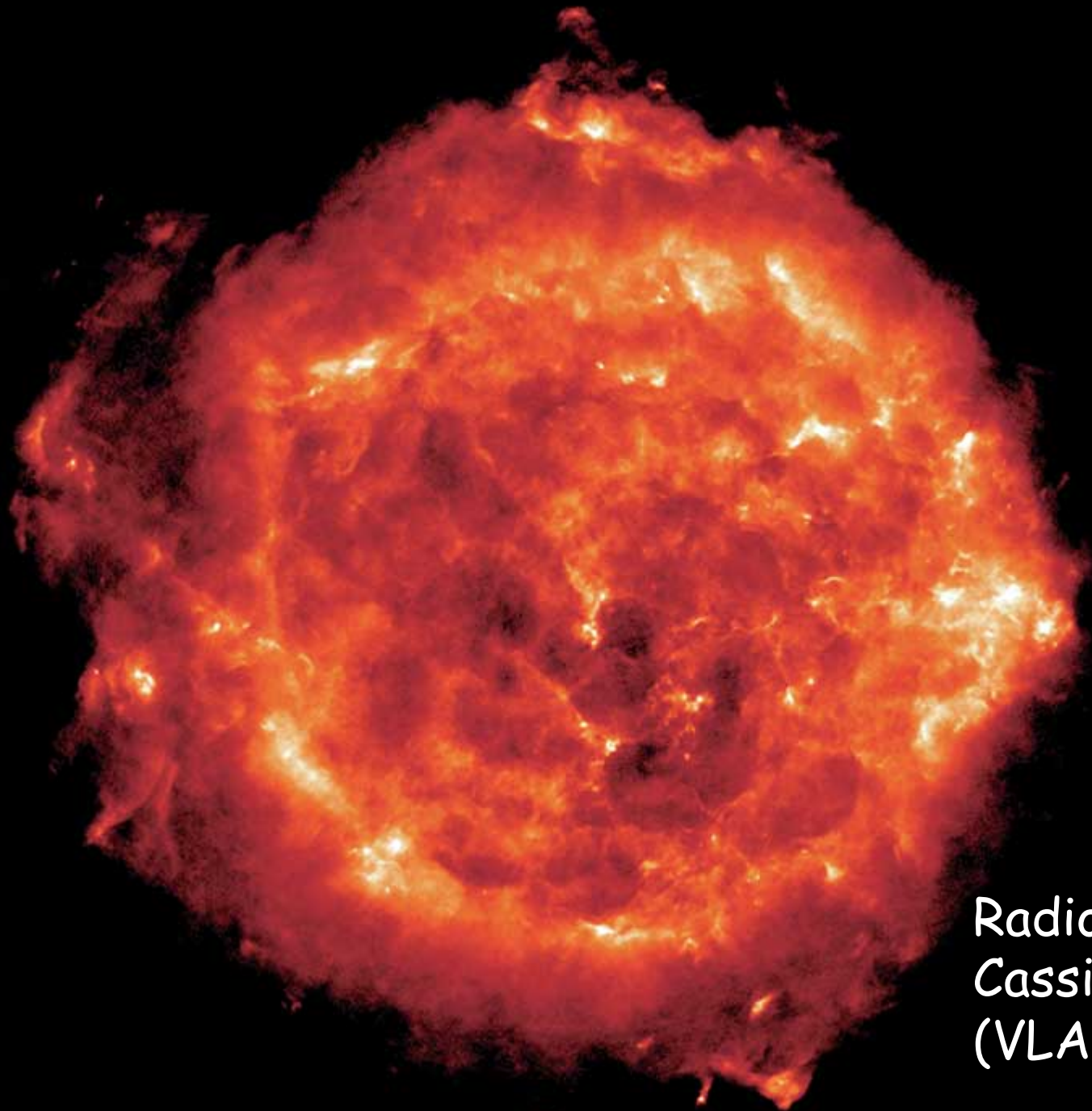
Supernova Remnant Cassiopeia A



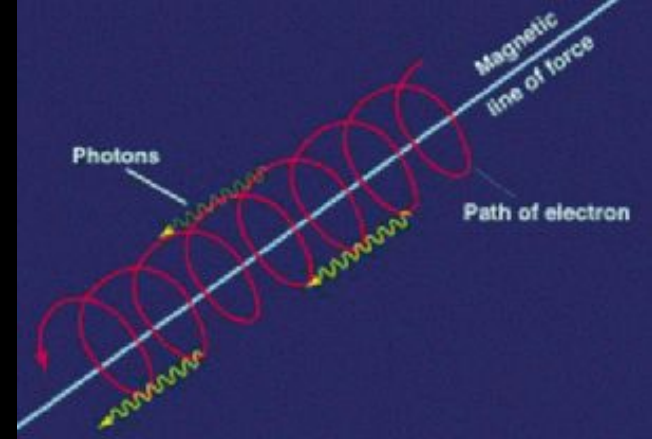
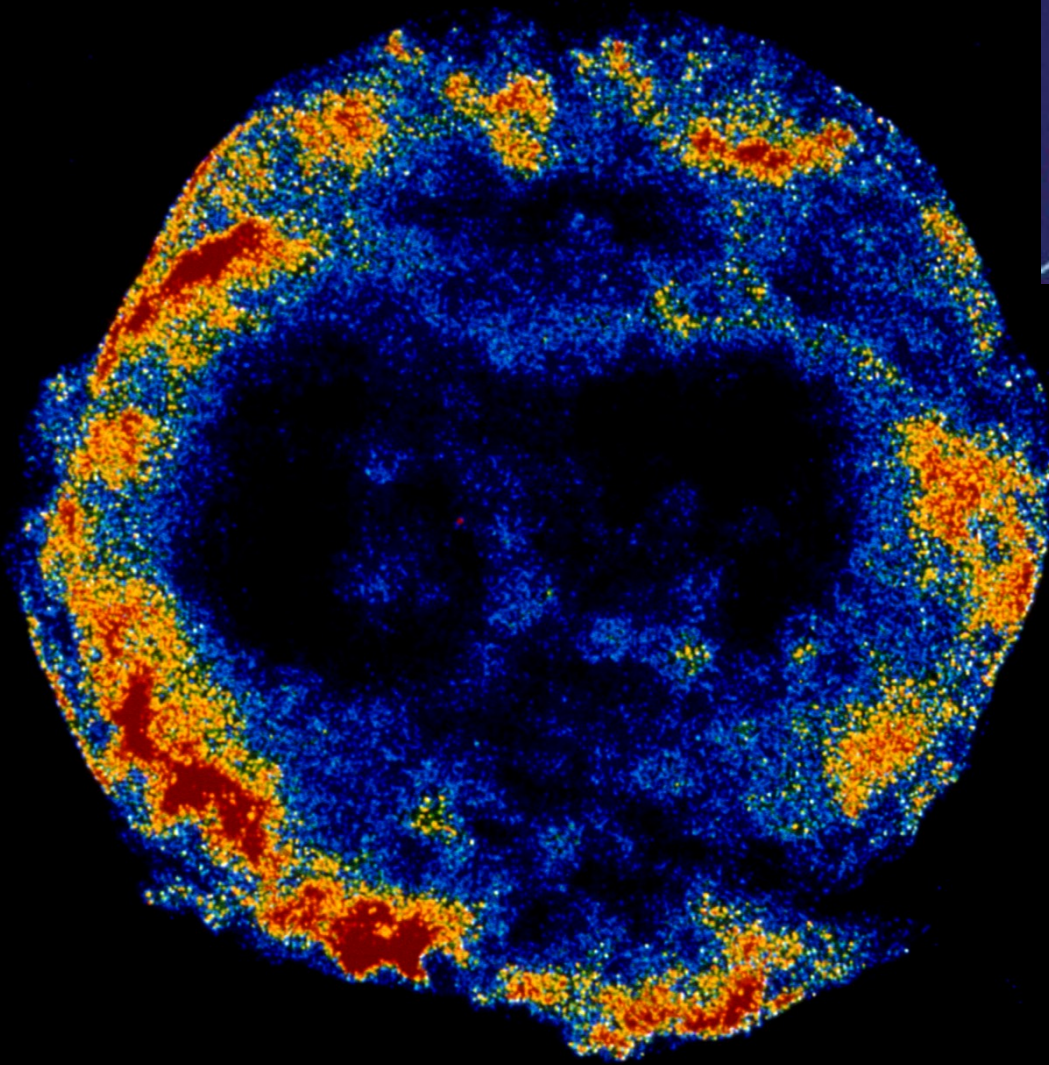
X-rays



Radio



Radio map
Cassiopeia A
(VLA)



Remnant of Tycho's supernova of 1572 AD

SNR evolution: energy budget

The mechanical energy of the ejecta is of the order $E_{\text{snr}} \sim 0.01 \times E_{\text{sn}} \sim 10^{51}$ erg

This is the energy that fuels the explosive event and ultimately creates a SNR

The typical speed V_s of this material can be estimated by energy conservation

Let us assume that all the mechanical energy is converted into kinetic energy of the remnant and that energy loss is negligible

$$E_{\text{snr}} = \frac{1}{2} M_{\text{snr}} V_s^2 \quad \longrightarrow \quad V_s = \left(\frac{2E_{\text{snr}}}{M_{\text{snr}}} \right)^{1/2}$$

The mass is $M_{\text{snr}} = M_{\text{ej}} + M_{\text{sw}}$, the sum of explosively ejected mass from the star at the time of explosion and the mass swept up added later as the remnant sweeps up more and more ISM material

SNR evolution: free expansion phase

If the density of the ISM is constant, then $M_{snr} = M_{ej} + \frac{4\pi}{3}\rho_o R_s^3$

Since we know the typical energy involved and that the mass of the remnant must be several solar masses, we can estimate the typical expansion speed

$$V_s \simeq 3000 \left(\frac{E_{snr}}{10^{51} \text{ erg}} \right)^{1/2} \left(\frac{M_{snr}}{10 M_{sun}} \right)^{-1/2} \text{ km/s}$$

Initially, the mass consists almost entirely of the ejecta mass $M_{snr} \sim M_{ej} \sim 2 - 10 M_{\odot}$

→ the expansion speed is almost constant $V_s \approx \left(\frac{2E_{snr}}{M_{ej}} \right)^{1/2} = 10000 \text{ km/s}$

This is \gg of sound speed (~ 10 - 100 km/s) → shock must form

The bubble expands as if there is not surrounding medium, therefore this phase is called **free expansion phase** and lasts few hundreds of years until the swept up mass increases enough to start the deceleration

Sedov-Taylor phase

As more and more ISM gas is swept up, the mass of remnant increases

After few hundreds of years, the mass is dominated by the swept up material so that

$$M_{snr} = M_{ej} + \frac{4\pi}{3}\rho_o R_s^3 \approx \frac{4\pi}{3}\rho_o R_s^3 \quad V_s \approx \left(\frac{2E_{snr}}{M_{ej}}\right)^{1/2} \approx \left(\frac{6E_{snr}}{4\pi\rho_o}\right)^{1/2} R_s^{-3/2}$$

Speed decreases as $R^{-3/2}$, result of the increasing remnant mass $\sim R^3$

This is the so called Sedov-taylor phase or energy-conserving phase

The typical expansion speed remains supersonic for a considerable time, typically 10000 years, so that the shock at the outer boundary persists in this evolutionary phase

Sedov-Taylor phase

The transition between the free expansion and the Sedov-taylor phases occurs gradually when the radius of the remnant reaches the deceleration radius, defined as the radius at which $M_{ej} = M_{sw}$

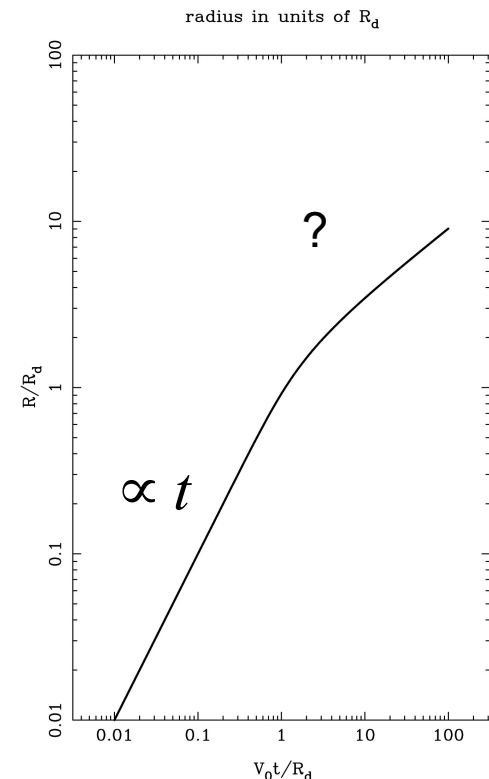
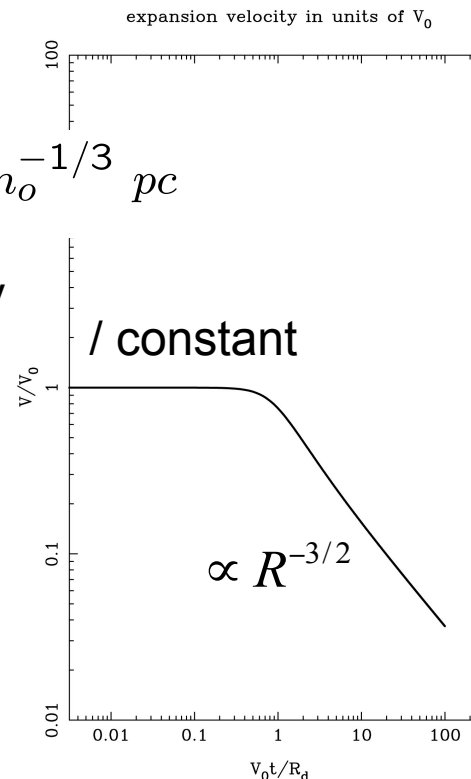
$$\frac{4\pi}{3}\rho_o R_d^3 = M_{ej} \quad \longrightarrow$$

$$R_d = \left(\frac{3M_{ej}}{4\pi\rho_o}\right)^{1/3} \approx 2.2\left(\frac{M_{ej}}{M_{sun}}\right)^{1/3} \times n_o^{-1/3} \text{ pc}$$

Here $n_o = \rho_o/m_p$ is the number density of ISM, which is typically $\sim 1 \text{ cm}^{-3}$

Assuming $M_{ej} = 5M_{sun}$ we have
 $R_d \sim 4 \text{ pc}$

A crude estimation of time spent during the free expansion phase is $\sim R_d/V_{free} \sim 400 \text{ years}$



Sedov-Taylor phase

In a SN explosion, the mechanical energy $E_0 \approx E_{\text{snr}}$ that drives the expansion is supplied impulsively in a point explosion at $t=0$

If no energy is lost, for instance through radiation losses, E remains constant for $t>0$

$$E(t) = C_\gamma M(t) \left(\frac{dR}{dt} \right)^2 = \text{constant} \qquad C_\gamma = \frac{\gamma^2 + 3}{2(\gamma^2 - 1)}$$

Once the remnant has expanded to a radius larger than R_d , the mass is

$$M(t) \approx M_{\text{sw}} = \frac{4\pi}{3} \rho_o R^3(t)$$

→ the energy equation can be written as

$$R^{3/2} \frac{dR}{dt} = \left(\frac{3E_{\text{snr}}}{4\pi C_\gamma \rho_o} \right)^{1/2} = \text{constant}$$

Sedov-Taylor phase

$$R^{3/2} \frac{dR}{dt} = \left(\frac{3E_{snr}}{4\pi C_\gamma \rho_o} \right)^{1/2} = \text{constant}$$

This relationship between the speed and the radius of the bubble is the same one as derived above using a simple conservation law for the kinetic energy but in this derivation we also take into account of the thermal energy of the hot bubble material

The integration is straightforward

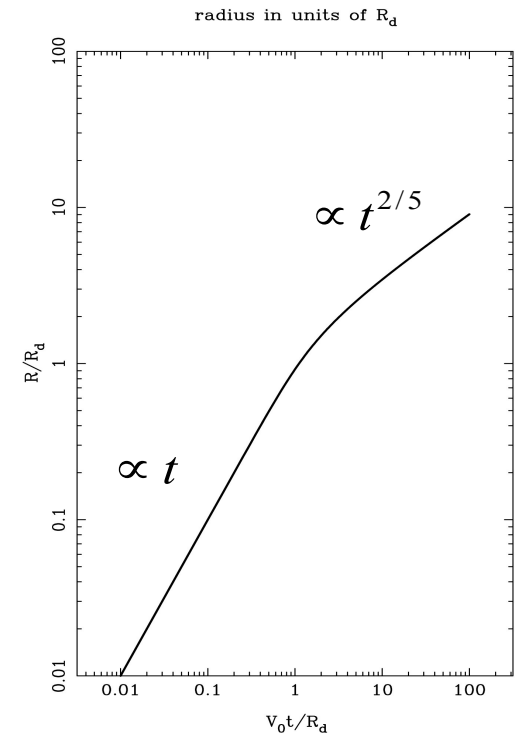
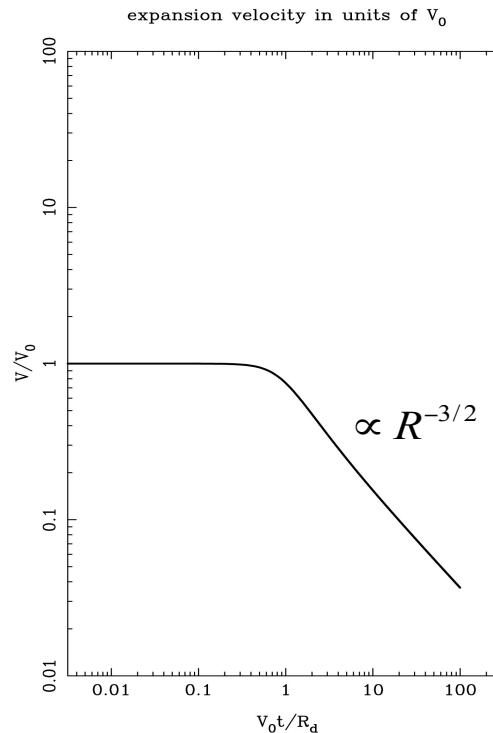
$$\frac{2}{5} R^{5/2} \approx \left(\frac{3E_{snr}}{4\pi C_\gamma \rho_o} \right)^{1/2} t$$

Assuming $R(0) = 0$

$$R = \left(\frac{75}{16\pi C_\gamma \rho_o} E_{snr} \right)^{1/5} t^{2/5}$$

$$V_s = (2/5) \left(\frac{75}{16\pi C_\gamma \rho_o} E_{snr} \right)^{1/5} t^{-3/5}$$

E. Fiandrini Cosmic Rays 1617



The solution interior to the shock obeys the equations

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} &= -\frac{1}{\rho} \frac{\partial P}{\partial r}, \\ \frac{\partial}{\partial t} \left[\rho \left(\mathcal{E} + \frac{U^2}{2} \right) \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \rho u \left(\mathcal{E} + \frac{P}{\rho} + \frac{U^2}{2} \right) \right] &= 0\end{aligned}$$

S-T phase:
exact
solution

with $\mathcal{E} = P/[\rho(\gamma - 1)]$, for an ideal gas.

Sedov recognised that the solution must be self-similar, *i.e.*, that at any time the pressure, density, velocity, *etc.* at all points interior to the shock can be expressed in terms of *single* similarity variable

$$\xi = r \left(\frac{\rho_0}{t^2 E} \right)^{1/5} = \xi_0 \frac{r}{r_s}.$$

Thus

$$\begin{aligned}\rho(r, t) &= \rho_2 \alpha(\xi), \\ u(r, t) &= u_2(t) \frac{r}{r_s} v(\xi), \\ P(r, t) &= P_2(t) \left(\frac{r}{r_s} \right)^2 p(\xi),\end{aligned}$$

where α , v & P are dimensionless *time independent* functions of the similarity variable ξ . All three functions are deliberately scaled so that they reach the value unity at $\xi = \xi_0$.

The value of the constant ξ_0 is determined by the condition that total energy is conserved, which can be written

$$\int_0^{r_s(t)} \rho \left(\mathcal{E} + \frac{U^2}{2} \right) 4\pi r^2 dr = E$$

or

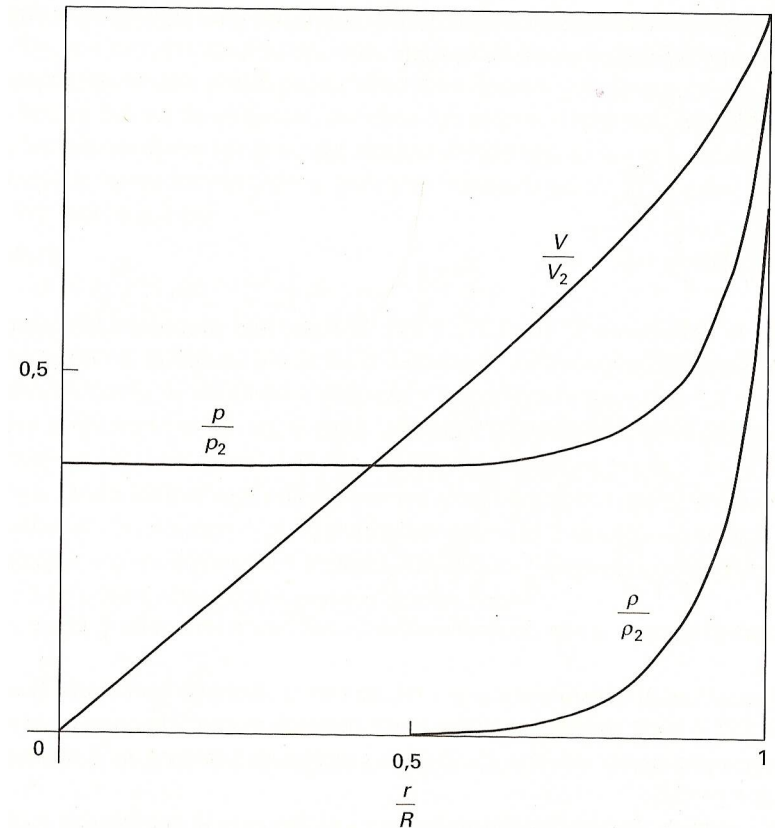
$$E. \text{Fiandrini Cosmic Rays 1617} \quad \frac{32\pi}{25(\gamma^2 - 1)} \int_0^{\xi_0} (p(\xi) + \alpha(\xi) v^2(\xi)) \xi^4 d\xi = 1.$$

Sedov-Taylor: ρ , p , V distribution in the bubble

Integration has to be made numerically

The solution is self-similar: at any stage of the expansion the functional form is the same

NB: the quantities P_2 , V_2 and ρ_2 are the values immediately post-shock as given by the RH conditions



Our assumption that all the swept up mass is in a thin layer of thickness $\delta R/R \sim 0.1$ behind the shock is well justified

Also the pressure is concentrated in the layer between the contact discontinuity and the blast wave

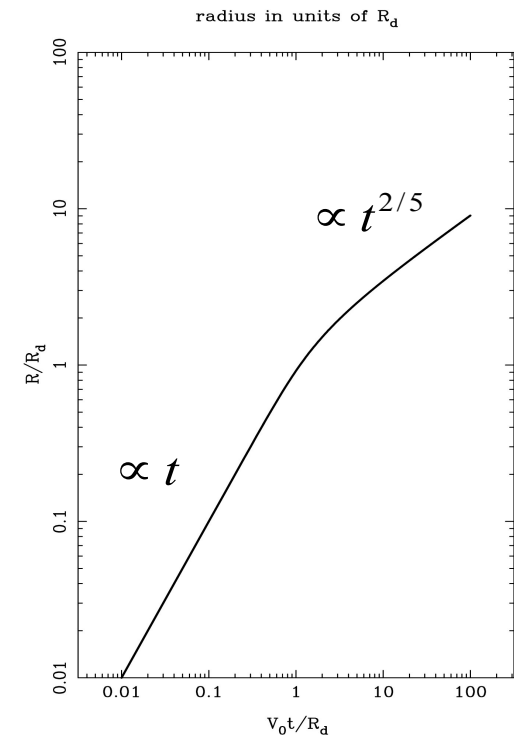
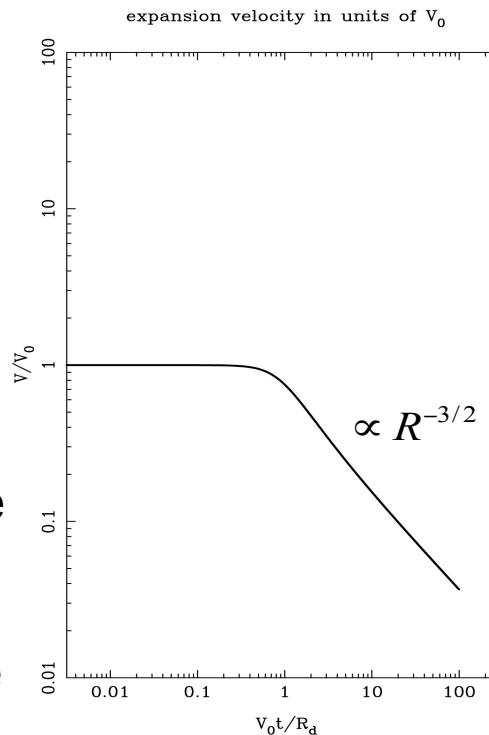
The pressure in the bubble decays as the bubble expands

From $p_i \approx p_2 = \frac{2\rho_o V_s^2}{(\gamma + 1)}$ One finds $p_i \sim t^{-6/5}$ or $p_i \sim R_s^{-3}$

This decay is simply an expansion loss as the internal pressure is converted into kinetic energy of the expanding shell

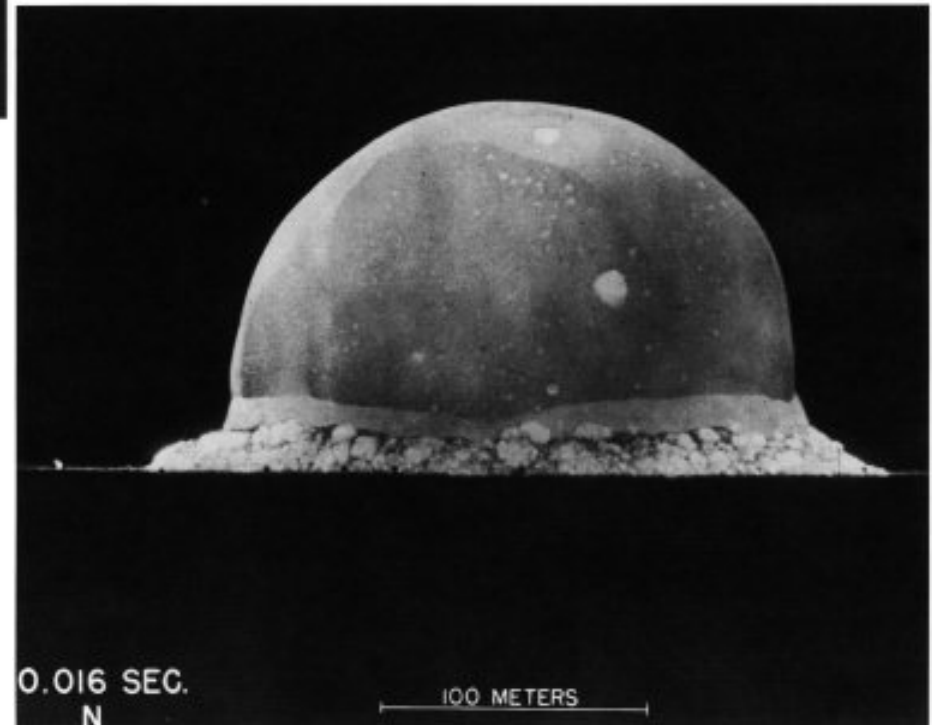
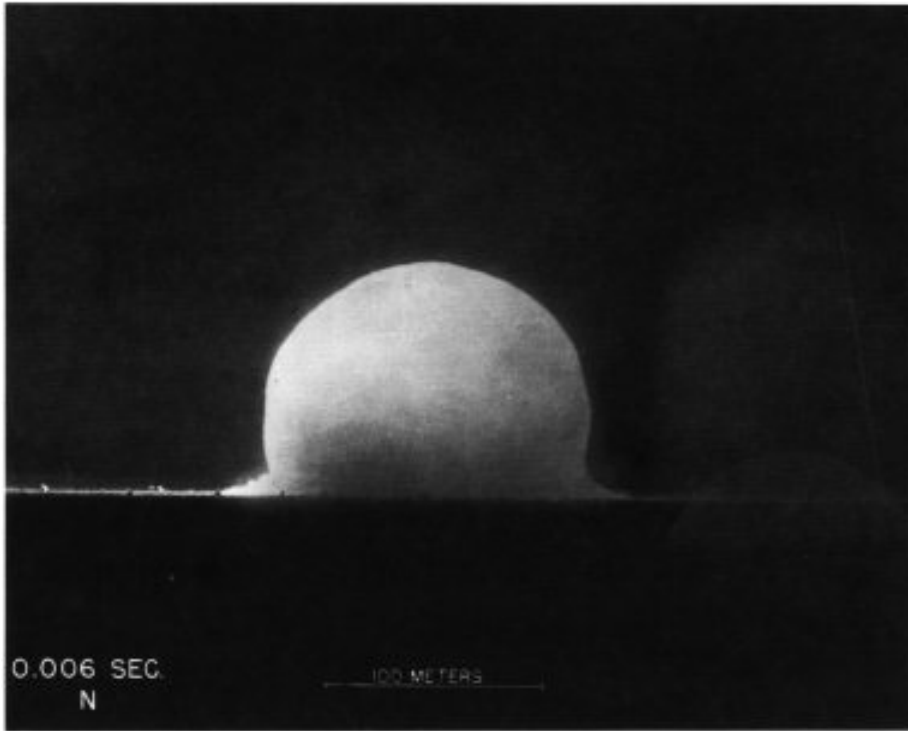
This energy conserving Sedov-taylor solution applies for $R > R_d$ and until the radiation losses become important

Radiative cooling makes the pressure inside the hot bubble decay faster and consequently the remnant loses energy and the expansion slows down more rapidly than in Sedov-taylor phase



The cooling dominated stages of the evolution set in after about 10000 years

Sedov & Taylor



Sedov & Taylor



Crab Nebula: SNR exploded in 1054 ad
Distance 2000 ± 500 pc
Diameter ~ 1.7 pc

Temperature

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_s^2 = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} \frac{V_s^2 \rho_o}{\gamma p_o} \quad T_2 = \frac{2(\gamma - 1)}{(\gamma + 1)^2} \frac{V_s^2 \rho_o T_1}{p_o} = \frac{2(\gamma - 1)}{(\gamma + 1)^2} V_s^2 \frac{\mu}{R_{gas}}$$

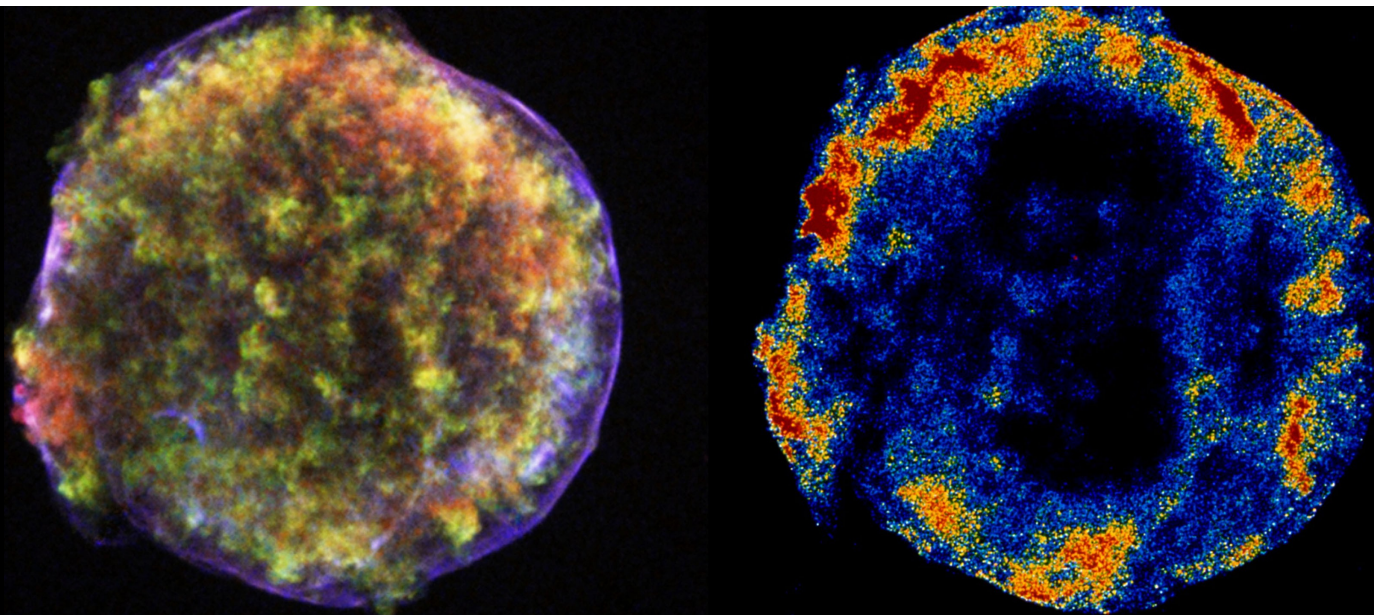
For typical values, T_2 is several million K ($>10^7$ K) after 10^4 years and the SNR is a bright X ray source

After 10^5 years the post-shock T decreases to $\sim 2 \times 10^5$ K, when radiative cooling becomes important and the energy conservation assumption breaks down

This occurs because at sufficiently low T , nuclei and electrons recombine so that radiation is no longer trapped in the shell and radiates away, cooling the shell

Two pictures of the remnant of Tycho's supernova (AD 1572), a picture in X-Rays (left) , made with the CHANDRA satellite, and a radio picture made with the Very Large Array radio synthesis telescope (right). The X-ray picture shows the hot ($T \sim 10^8$ K) gas in the remnant's interior in yellow. This is mostly line emission from excited nuclei. The blue radiation at the outer rim of the remnant is synchrotron continuum emission, caused by relativistic electrons moving in a weak magnetic field. The radio emission is also synchrotron radiation. It is believed that these relativistic electrons are accelerated at the outer shock.

This is a 'classical' remnant with a nearly perfect spherical shape. It is believed to be entering the Sedov-Taylor phase. Note the sharp outer edge of the remnant, which is believed to coincide with the position of the outer blast wave.



Tycho's Remnant (SN 1572AD)

X-Rays (CHANDRA Observatory)

Radio (21cm)

Snowplow phase

When the SNR becomes sufficiently old, radiative cooling becomes important and the total energy is no longer conserved

In the energy conserving Sedov-Taylor phase, pressure forces accelerate the swept up ISM converting thermal energy (which came from original explosion) into kinetic energy of the shell of swept up mass

Since radiative cooling depends on the particle density ($\sim n^2$), ie the higher is particle density, the higher is the radiation, and since the density in the shocked shell is much higher than in the bubble interior, most of the cooling occurs in the shocked ISM layer

Snowplow phase

In the snowplow approximation, it is assumed that all the energy in the shocked shell is radiated away, but that the hot interior does not cool because there radiative processes are much less effective due to the very low density (ie the time scale of radiative cooling in the hot interior is much longer than in the shocked shell)

In such a case the shell must collapse until it becomes very thin because the pressure in the shell is decreased together with temperature but the pressure equilibrium at the contact discontinuity must still hold and the pressure in bubble interior is not changed much due to lack of radiative cooling, so the shell is compressed until a new pressure equilibrium is reached

The hot interior can be therefore considered as adiabatic $\rightarrow p_i \sim \rho_i^\gamma$

Since the mass in the interior is conserved, one has $\rho_i = \frac{M_{ej}}{(4\pi/3)R_s^3}$

Combining the two relations one gets $p_i = \left(\frac{M_{ej}}{(4\pi/3)}\right)^\gamma R_s^{-3\gamma} \sim r_s^{-5}$ for $\gamma=5/3$

To be compared with the scaling law in the Sedov-Taylor phase $\sim R_s^{-3}$

Snowplow phase

$$p_i = \left(\frac{M_{ej}}{4\pi/3}\right)^\gamma R_s^{-3\gamma}$$

The motion of the collapsed shell, containing most of the mass, is driven by the pressure p_i of the hot interior (neglecting the ISM pressure, which is very low compared to p_i)

The motion equation of the expanding blast shock is then

$$\frac{d}{dt}\left(M(R_s)\frac{dR_s}{dt}\right) = 4\pi R_s^2 p_i(R_s) = 4\pi\left(\frac{3M_{ej}}{4\pi}\right)^\gamma R_s^{2-3\gamma} \equiv A R_s^{2-3\gamma}$$

Taking into account that $M_s(R_s) = \frac{4\pi}{3}\rho_o R_s^3$

$$\frac{4\pi}{3}\rho_o \frac{d}{dt}\left(R_s^3 \frac{dR_s}{dt}\right) = 4\pi\left(\frac{3M_{ej}}{4\pi}\right)^\gamma R_s^{2-3\gamma} \quad \longrightarrow \quad \frac{d}{dt}\left(R_s^3 \frac{dR_s}{dt}\right) = \frac{3}{\rho_o}\left(\frac{3M_{ej}}{4\pi}\right)^\gamma R_s^{2-3\gamma}$$

Snowplow phase

$$\frac{d}{dt}(R_s^3 \frac{dR_s}{dt}) = \frac{3}{\rho_o} (\frac{3M_{ej}}{4\pi})^\gamma R_s^{2-3\gamma}$$

We look for power law solutions $\mathbf{R_s(t) = Bt^\alpha}$, with B some constant and α to be determined

The index α is determined by substituting the trial solution into the equation

$$\alpha(4\alpha - 1)B^3 t^{(4\alpha-2)} = AB^\beta t^{(2-3\gamma)\alpha} \quad A = \frac{3}{\rho_o} (\frac{3M_{ej}}{4\pi})^\gamma$$

For the trial function to be solution, the two exponents must be equal

$$4\alpha - 2 = (2 - 3\gamma)\alpha \quad \alpha = \frac{2}{3\gamma + 2}$$

Which for $\gamma=5/3$ yields $\alpha = 2/7=0.286$

The actual value of the index of this pressure driven phase obtained by numerical, more accurate, simulations yields a value closer to $3/10=0.3$

We have assumed uniform pressure in the bubble, no mixing at the contact discontinuity between bubble and shocked ISM, and neglected radiative losses

Momentum driven phase

$$p_i = \left(\frac{M_{ej}}{4\pi/3}\right)^\gamma R_s^{-3\gamma}$$


$$\frac{d}{dt}(M(R_s)\frac{dR_s}{dt}) = 4\pi R_s^2 p_i(R_s)$$

As the SNR evolves, the pressure inside the bubble decreases until becomes negligible since the remnant radiates away all its internal energy

In this limit, the total momentum is conserved since the total force acting the bubble is zero (hence the name momentum-conserving phase)

$$\frac{d}{dt}(M(R_s)\frac{dR_s}{dt}) = 0$$

$$M_s(R_s) = \frac{4\pi}{3}\rho_o R_s^3$$

This implies $M(R_s)\frac{dR_s}{dt} = \text{constant}$  $\frac{dR_s}{dt} = \left(\frac{3}{4\pi\rho_o}\right)R_s^{-3}$

$$R_s^3 dR_s = \left(\frac{3}{4\pi\rho_o}\right)dt \quad \Rightarrow \quad (1/4)R_s^4 dR_s = \left(\frac{3}{4\pi\rho_o}\right)t \quad \Rightarrow \quad R_s = \left[4\left(\frac{3}{4\pi\rho_o}\right)\right]^{1/4} t^{1/4}$$

Coalescence phase

The momentum conserving phase lasts until the shock speed remains beyond sound speed in the ISM

The shock speed decreases as $R_s^{-3} \sim t^{-3/4}$, so at some point it approaches the sound speed and the shock itself disappear.

The material starts to straggle into the ISM (initially at sound speed) becoming part of it

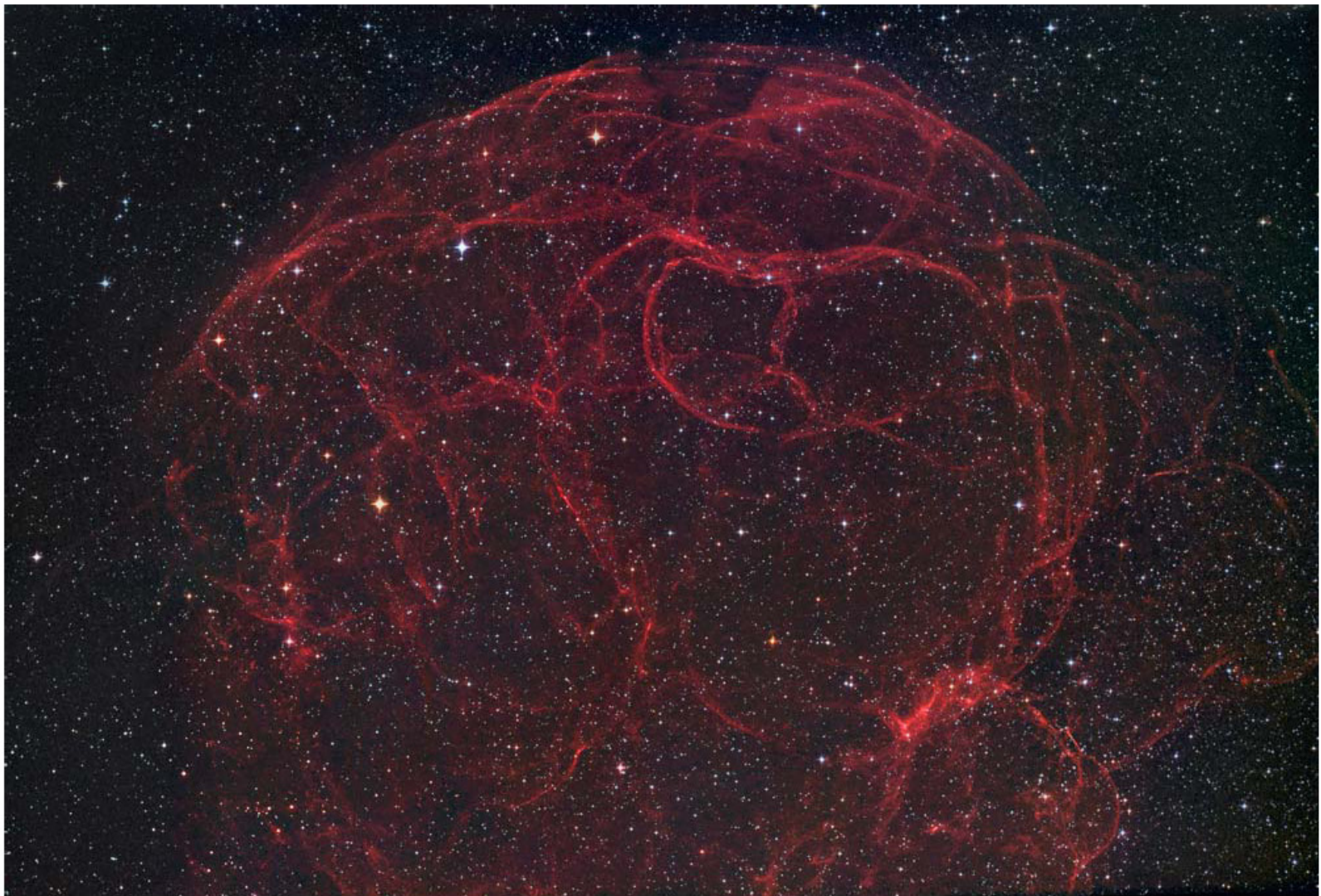


Figure 7.15: *The old supernova remnant S147, which is in the process of dissolving into the general interstellar medium.* Photo credit: Robert Gendler



An old supernova remnant
(age ~ 10,000 years)



