Lecture 13 211116

- Il pdf delle lezioni puo’ essere scaricato da
- http://www.fisgeo.unipg.it/~fiandrin/didattica_fisica/cosmic_rays1617/

Shocks: non-linear fluid structures

One of the peculiarities of the hydrodynamics is that it admits discontinous solutions, that is such that on some special surfaces, called discontinuity surfaces, all the physical observables that characterize the state of the fluid (ρ, p, V, T,...) are discontinuous

From a mathematical point of view, these solutions have true steps, while from a physical point of view the discontinuity is not sharp but has a finite thickness, very small with respect to all the other dimensions of the system

Shocks waves

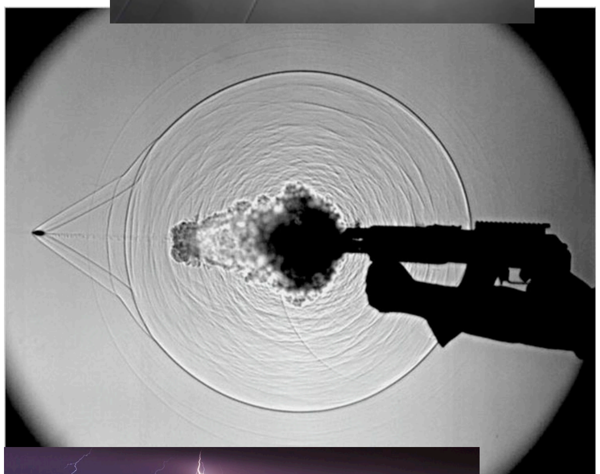
In the limit of small disturbances, where the non-linear term **(V•grad)V** can be neglected, we got the wave propagation for them with sound speed $c_s = (\gamma p/\rho)^{1/2}$

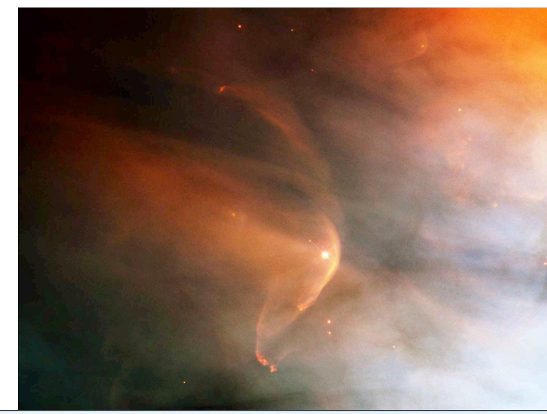
But when the approximation breaks down, the behavior of the fluid changes rapidly because intrinsic non linearities play an essential role

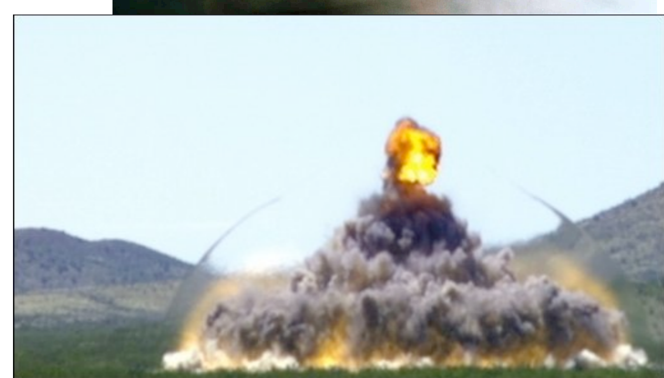
These lead to shock formation in a natural way: in practice, they are unavoidable if the perturbations to the fluid are not infinitesimal

Shock waves have an enormous importance is astrophysics because they are present everywhere and because the matter immediately after the transit of the shock wave emits much more than before, making it easily detectable by us

Shocks occur whenever a flow hits an obstacle at a speed larger than the (adiabatic) sound speed
The obstacle does not need to be a solid body: if the relative speed of two gases or plasmas is supersonic shock waves can develop.
A blast shock wave can result from the sudden release of thermal energy in a "small" volume, i.e. an explosion







Wave Breaking

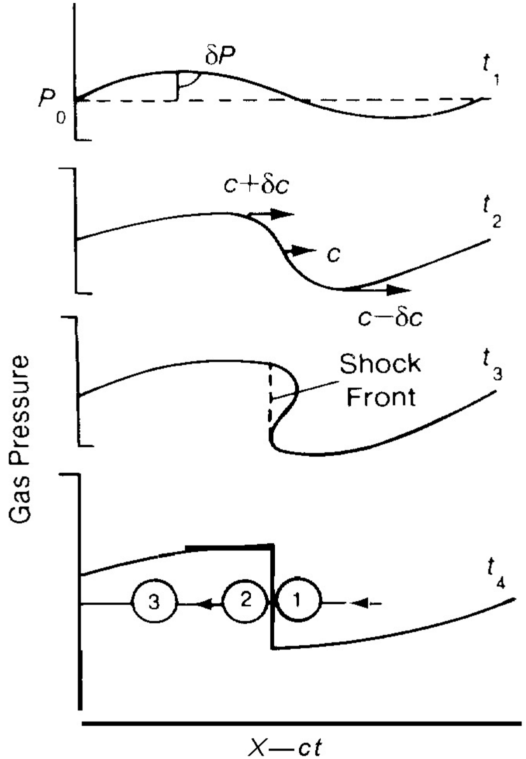


Fig. A. Self-steepening of a finite-amplitude sound wave. In the region where the state variables of the wave (here, pressure) would become multi-valued, irreversible processes dominate to create a steep, single-valued shock front (vertical dashed line).

The formation of a shock wave depends on the non-linearity of the motion equations

Let consider a perturbation with finite amplitude in a fluid otherwise homogeneus

It is possible to show that the propagation speed is higher where there is an overdensity and lower where there is an underdensity: so the wave crests will move faster than the ventral part

Shock <u>must</u> form

$$u = \frac{2c_{s0}}{\gamma - 1}\left[\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right]$$

High-pressure/density regions move faster

$$\approx c_{s0}\left(\frac{\Delta\rho}{\rho}\right)$$

Therefore the crest will reach the the ventral in a finite time, forming a discontinuity surface

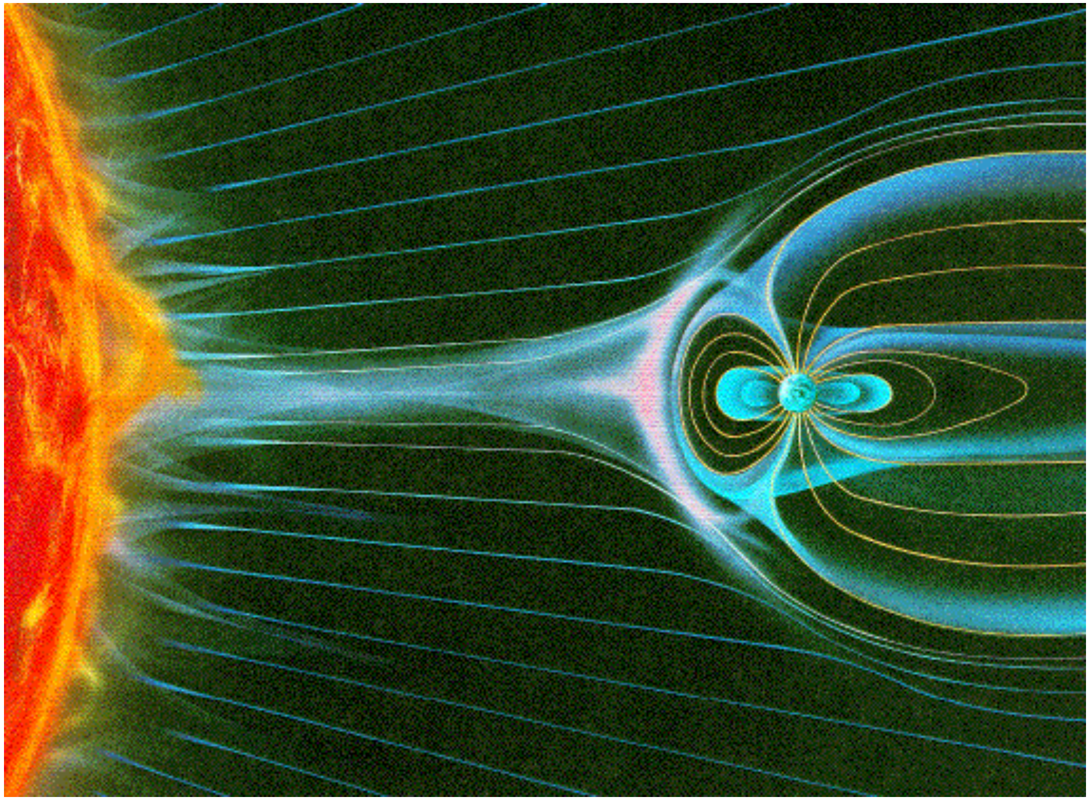
So, unless ρ is always constant, every perturbation of finite amplitude evolves toward a discontinuity (nb: a discontinuity DOES NOT imply a shock wave) if it can travel enough. In practice, this does not occur always because of the damping due to viscous dissipation and heat conduction

Shock formation

Shock waves are a feature of supersonic flows with a Mach number exceeding the unity $M_s = |V|/c_s > 1$

They occur when a supersonic flow encounters an obstacle which forces it to change its speed

For istance a bow shock forms around the Earth in the tenous supersonic solar wind when the ionized wind material "hits" the strongly magnetized earth's magnetosphere



Shock formation

We have already seen that small-amplitude sound waves in a flow propagate with a velocity

$$\vec{V_w} = \vec{V} + c_s\vec{k}$$

k is the direction of propagation

Sound waves act as "messengers": they carry density and pressure fluctuations that in some sense alert the incoming flux when an obstacle is present

For low mach number $M_s<1$, waves can propagate against the flow, getting ahead of the obstacle

However, in a supersonic flow with $M_s>1$, the *net* speed of the waves is always directed downstream and no waves can reach the flow upstream from the obstacle → in these conditions a shock forms, ie sudden transition of density, pressure, temperature and speed appears

Behind the shock, T is so high that the component normal to the shock surface of the flow becomes sub-sonic, so that sound waves are once again able to communicate the presence of an obstacle to the flow so that pressure forces can deflect the flow, steering it around the obstacle

In every-day life we are used to disturbances propagating at *sub-sonic* speeds. This means that sound waves from nearly all every-day "noise makers" – vehicles in traffic, for example – propagate to all directions faster than the source of the waves itself. There are only a couple of every-day examples of *super-sonic* sources of sound waves, the jet airplanes probably being the most well-known ones. In this case, information about the upcoming disturbance (the airplane) is transmitted by sound waves at sound speed

$$c_s = \sqrt{\frac{\gamma P}{\rho}} \propto T^{1/2},$$

where P and ρ are the pressure and density of the gas related to each other by an *equation of state*, $P\rho^{-\gamma} = \text{const.}$, and $T \propto P/\rho$ is the gas temperature. Thus, the information can not propagate ahead of the disturbance, because the disturbance itself moves faster than sound at speed $u > c_s$. The sound waves propagate into a cone of half-angle $\alpha_s = \arcsin(c_s/u) \equiv \arcsin M_s^{-1}$ trailing the (point-like) disturbance, see Fig. 1.1. The angle α_s is called the Mach angle and $M_s = u/c_s$ the (sonic) Mach number of the disturbance. When viewed in the coordinate system of the disturbance, the gas flows at velocity $-\boldsymbol{u}$ (to the right in Fig. 1.1). The component $u_n = u \sin \alpha_s$, which is normal to the surface of the cone, therefore satisfies $u_n = c_s$.

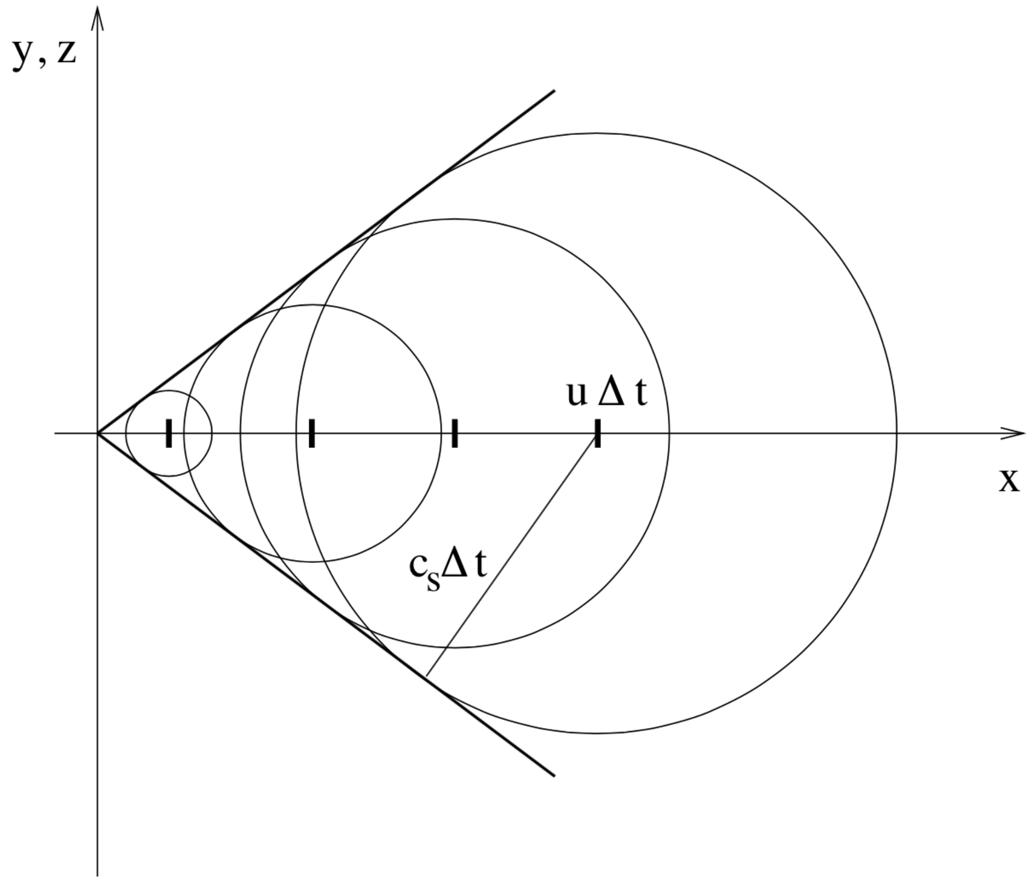
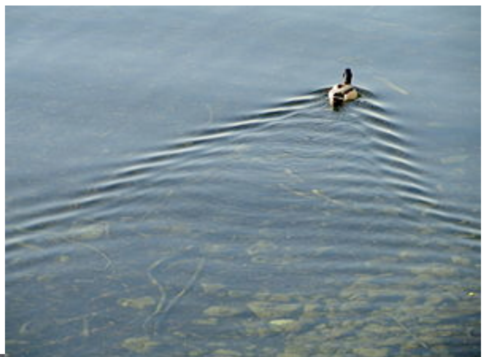
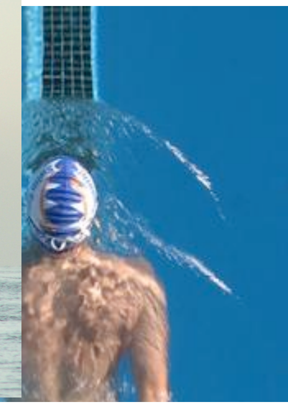
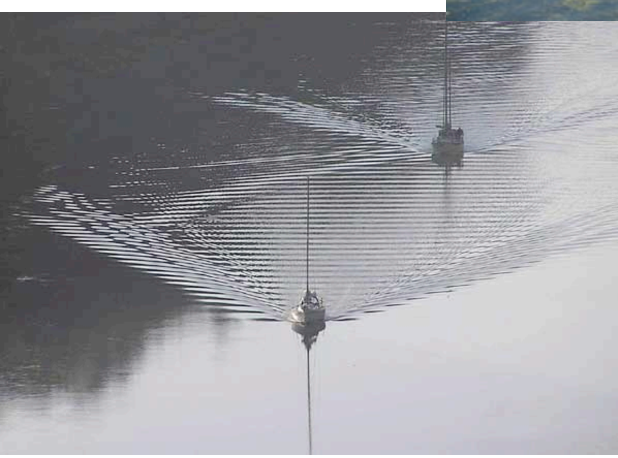


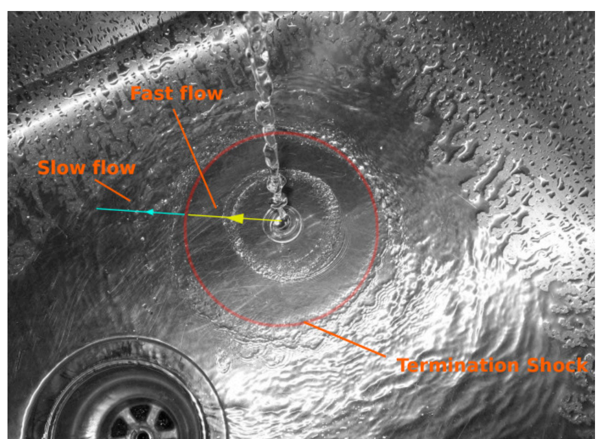
Figure 1.1: Propagation of a supersonic, point-like disturbance. The coordinate system is co-moving with the disturbance in the origin. Thus, the sound-wave crests emitted at time Δt before the present are circles (actually spheres) with their central point on the x-axis at $x = u\,\Delta t$ and with a radius $r = c_s\,\Delta t$. Thus, information about the disturbance is obtained only in the cone $\sqrt{y^2 + z^2} \leq x \tan \alpha_s$, where $\sin \alpha_s = c_s/u$ defines the Mach angle.

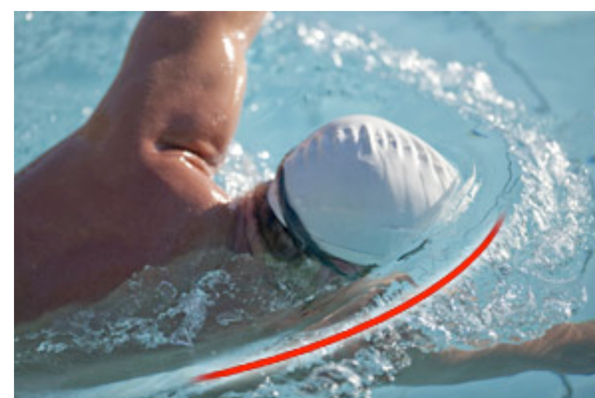
Similar phenomenon occurs also in case of water surface waves when a disturbance, e.g., a boat, moves faster than the speed of the waves, or the water itself flows past some obstacles at a speed greater than the speed of the surface waves. (Water waves have a phase speed c_p given by $c_p^2 = (g\lambda/2\pi)\tanh(2\pi h/\lambda)$, where h is the depth of the water, $g \approx 9.81\ \mathrm{m\,s^{-2}}$ is the gravitational acceleration and λ is the wave length.)











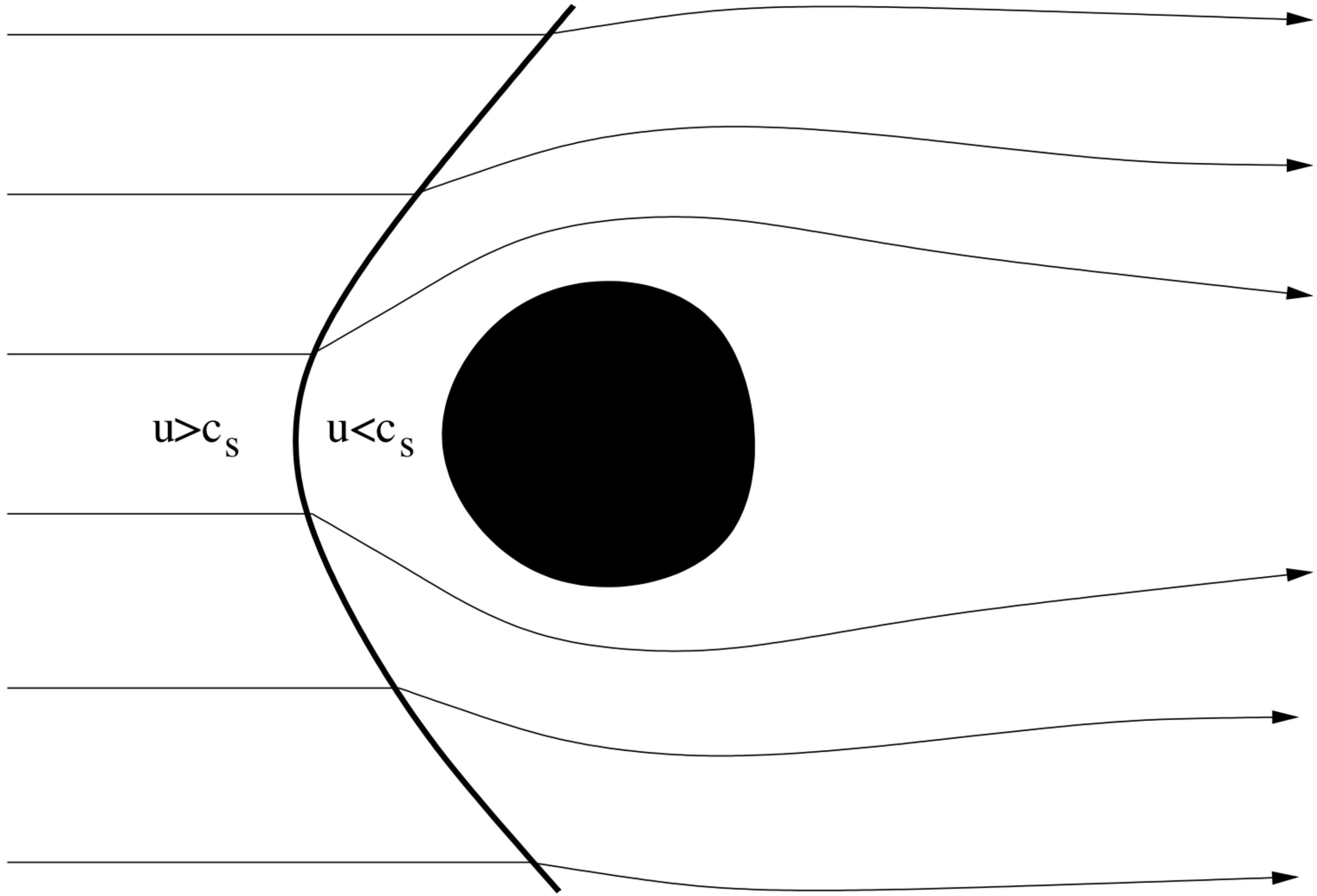


Figure 1.2: Super-sonic flow past an obstacle. A shock wave (thick curve) forms ahead of the obstacle, where the gas flow speed changes from super-sonic to sub-sonic. Thin arrow-headed curves denote streamlines. Far from the obstacle, the shock wave degenerates into the conical sound wave depicted in Fig. 1.1.

Shock waves are common phenomena in *super-sonic* streams of any fluid, gas or plasma. Consider a gas flowing toward a fixed obstacle at a *super-sonic* speed. The gas ahead of the obstacle has no way of obtaining information of the upcoming disturbance. If the obstacle is not point-like and does not absorb the particles incident on its surface, the information has to be passed somehow to the gas to give it a possibility to be deflected around it. The gas solves this problem by developing a *shock wave* ahead of the obstacle (Fig. 1.2). Between the obstacle and the shock wave, the gas is slowed down and heated enough to make the flow sub-sonic in this region, as viewed from the coordinate system attached to the obstacle. This way the information of the upcoming disturbance can propagate against the flow and the gas has a way of getting past it. Far from the obstacle (i.e. at $r \gg d$, where d is the dimension of the obstacle), the disturbance looks more and more point-like and the shock wave becomes weak. Thus, it degenerates into the conical sound wave crest propagating at Mach angle relative to the upstream flow, as depicted Fig. 1.1. The flow speed component in the wave-normal direction far from the obstacle is again $u_n = c_s$.

Shock waves

Generally speaking shock waves are abrupt transitions from supersonic to sub-sonic flow speed, accompanied by compression and dissipation.
Roughly, two different types of discontinuity exist:

i) tangential discontinuity, when two adjacent fluids are in relative motion without mass flow through the discontinuity. This discontinuity can occur at any speed (as in the case of slender jets)

ii) shock wave in which there is a discontinuity between two fluids with distinct properties, but with a flow of mass, momentum and energy across the surface

Shock properties

1. Shocks are <u>sudden</u> transitions in flow properties such as density, velocity and pressure;

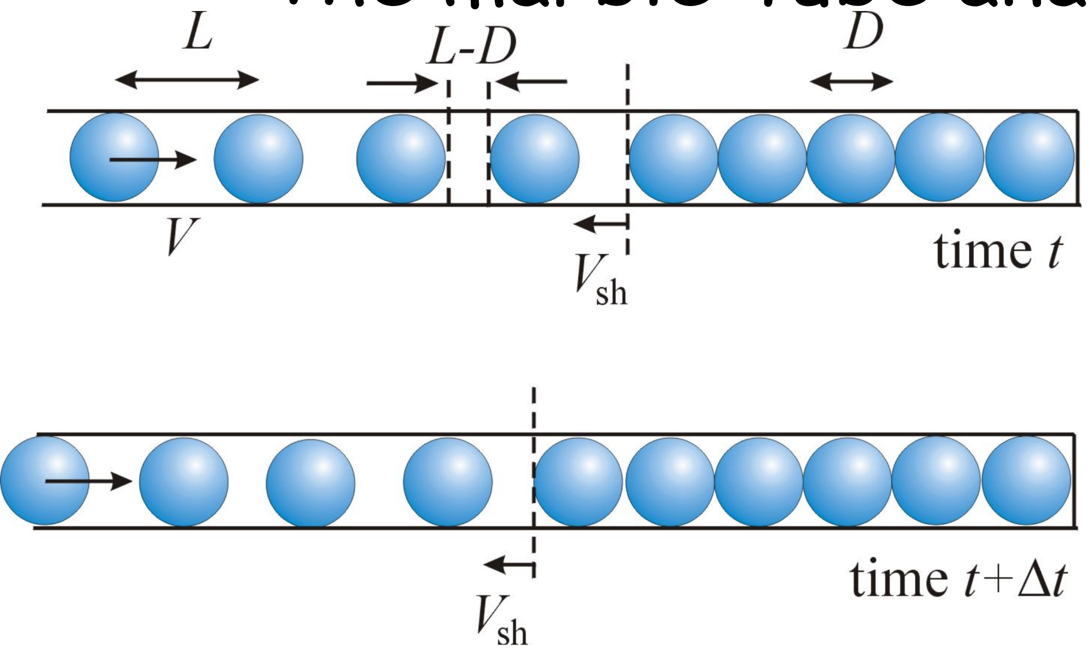
2. In shocks the kinetic energy of the flow is converted into <u>heat</u>, (pressure);

3. Shocks are <u>inevitable</u> if sound waves propagate over long distances;

4. Shocks always occur when a flow hits an obstacle <u>supersonically</u>

5. In shocks, the flow speed along the shock normal changes from <u>supersonic</u> to <u>subsonic</u>

The marble-tube analogy for shocks



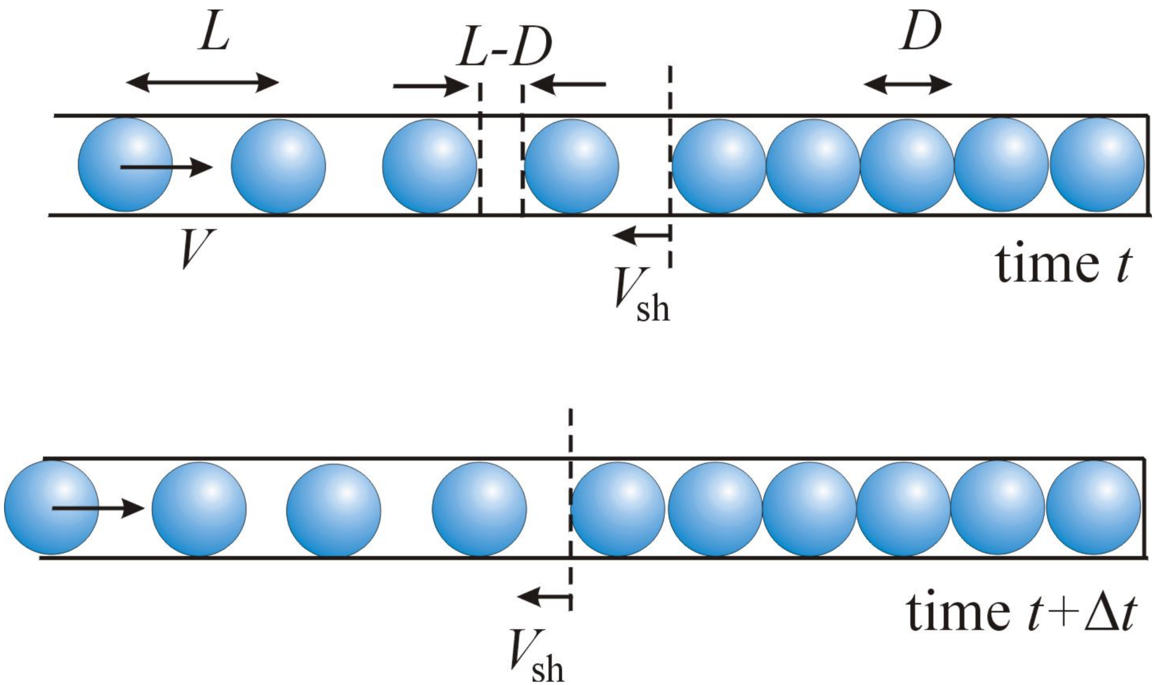
As a simple mechanical model for shock formation, consider in a hollow (semi-infinite) tube spherical marbles with diameter D, separated by a distance L>D which roll with speed V

The end of the tube is plugged, forming an obstacle that prevents the marbles from continuing onward

As a result, the marbles collide, loose their speed and accumulate in a stack at the plugged end of the tube. The transition between freely moving and stationary marbles is the analogue of a shock surface

Far ahead of the obstacle, where the marbles still move freely, the line density is $\mathbf{n_1=1/L}$

In the stack, the density is instead $\mathbf{n_2=1/D>n_1}$ → the density increases when the marbles are added to the stack

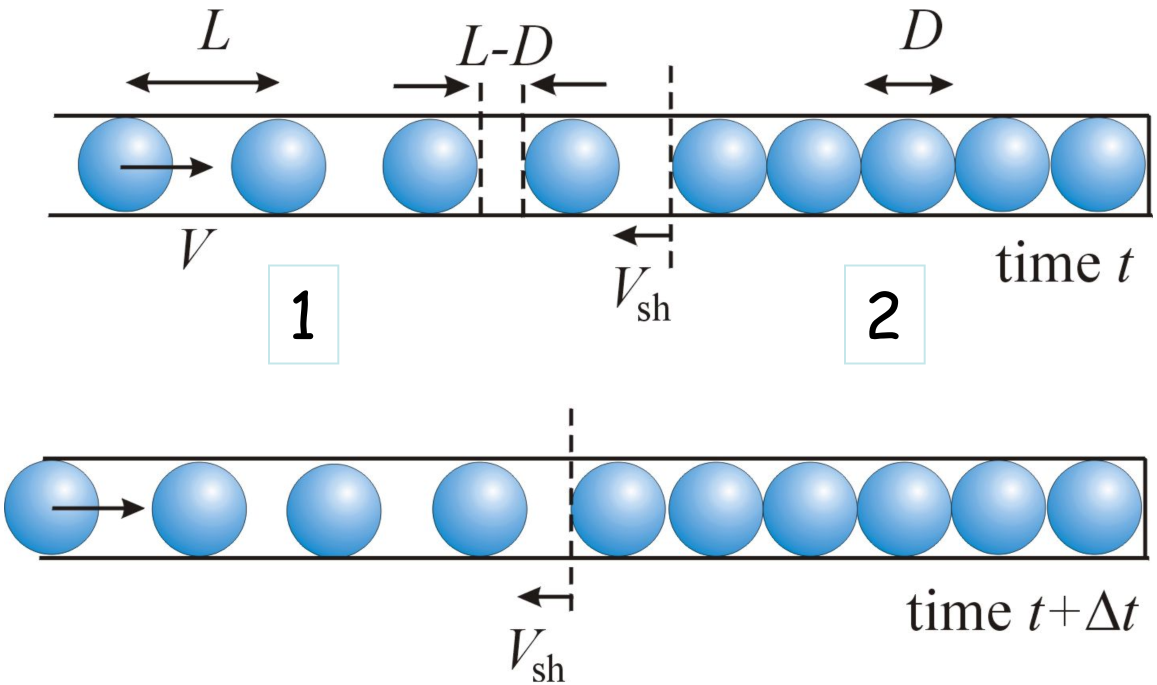


The growth of the stack is easily calculated: in order to collide, two adjacent marbles have to close the separation distance ΔD=L-D between the surfaces → the time between two collisions is
ΔT = (L-D)/V

At every collision, one marble is added to the stack → the length of the stack increases by D → the average speed with which the length of stack increases is

$$V_{sh} = -\frac{D}{\Delta T} = -\frac{D}{L-D}V$$ `Shock speed' = growth velocity of the stack.

The imaginary surface at the front-end of the stack, that separates a region of "low" density n_1 from a region of "high" density n_2 of marbles, is the analogue of a hydrodynamical shock wave



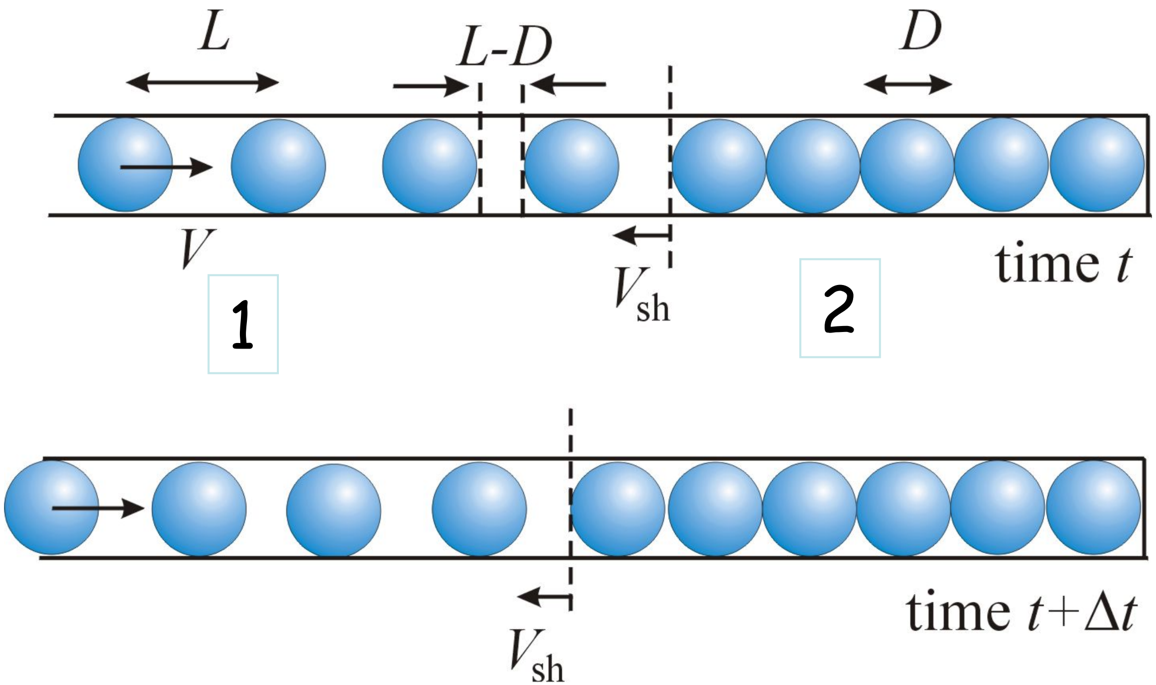
Let transform to a reference where the shock is stationary and neglect the fact that the stack grows impulsively each time a marble is added

In this referenc frame the incoming marbles have a speed:

$$V_1 = V - V_{sh} = V\left(\frac{L}{L-D}\right)$$

Marbles in stack, stationary in lab move away from the shock with speed:

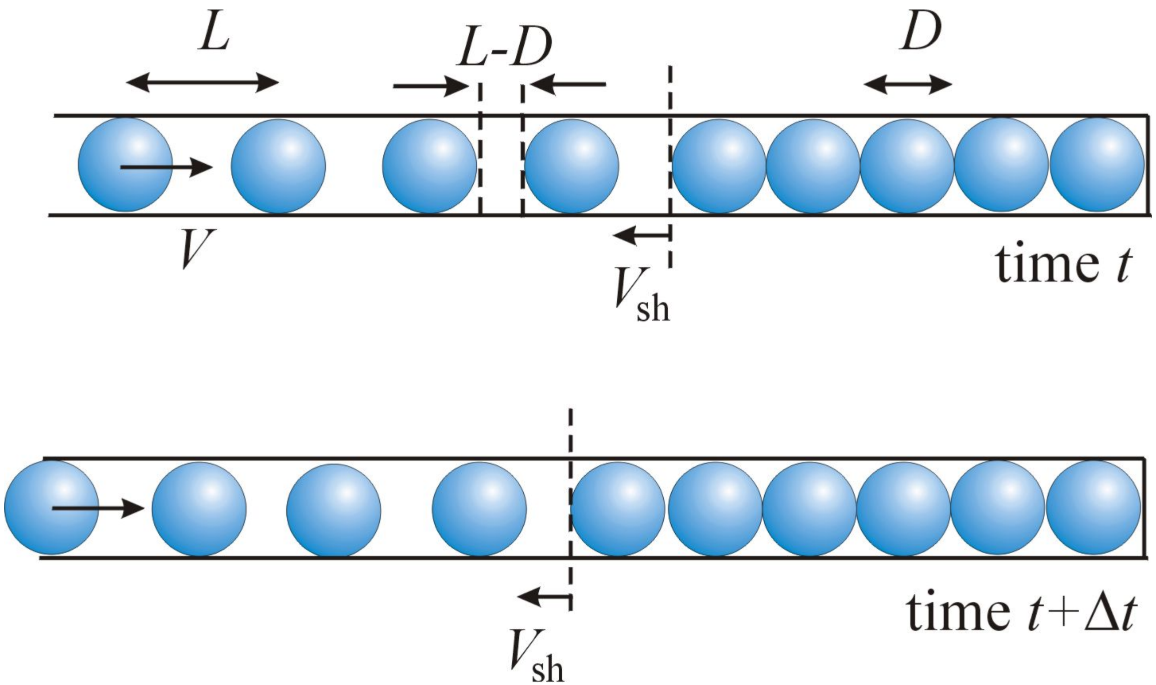
$$V_2 = -V_{sh} = V\left(\frac{D}{L-D}\right)$$



In any frame the flux is Flux = density x velocity

Incoming flux:
$$F_1 = n_1 \times V_1 = \left(\frac{1}{L}\right) \times V\left(\frac{L}{L-D}\right) = \frac{V}{L-D}$$

Outgoing flux:
$$F_2 = n_2 \times V_2 = \left(\frac{1}{D}\right) \times V\left(\frac{D}{L-D}\right) = \frac{V}{L-D}$$



Conclusions:

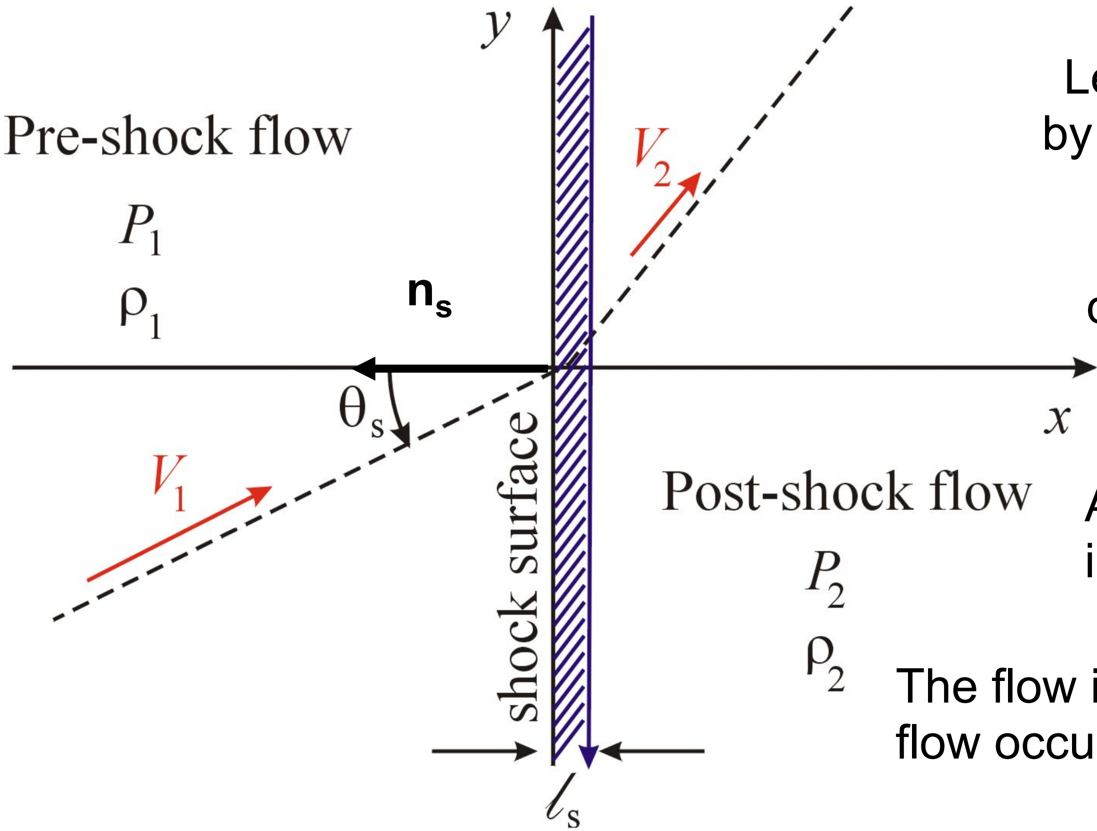
1. The density increases across the shock
2. The flux of incoming marbles equals the flux of outgoing marbles in the shock rest frame:

$$\boxed{F_1 = F_2}$$

This equality has a simple interpretation: the nbr of marbles crossing the shock surface in ΔT is $\Delta N = F\Delta t$

Since an infinitely thin surface has no volume, the nbr of marbles entering the surface at the front must exactly equal the nbr that leaves in the back...nothing can be "stored" in the shock! → $\Delta N_{in} = \Delta N_{out}$

Many of the concepts introduced here can be immediately traslated to the physics of a shock in a gas, in particular the flux conservation



Let consider a simple fluid described by a polytropic relation $\mathbf{P=a}\rho^{\gamma}$ on either side of the shock, but not with the same constant **a** as a result of dissipation in the shock (as we will see)

Assume the shock is planar, located in a <u>fixed position</u> in the y-z plane

The flow is from left-to-right, so that pre-shock flow occurs at x<0 and post shock at x>0

The normal $\mathbf{n_s}$ to the shock front coincides with x-axis toward negative x, $\mathbf{n_s}=-\mathbf{e_x}$

Let assume that flow properties (speed, density,...) depend only on the x coord: $\partial/\partial y = \partial/\partial z = 0$ and that the speed lies in the x-z plane (always possible with suitable choice of reference frame)

$$\vec{V} = V_n \vec{e}_x + V_t \vec{e}_z$$

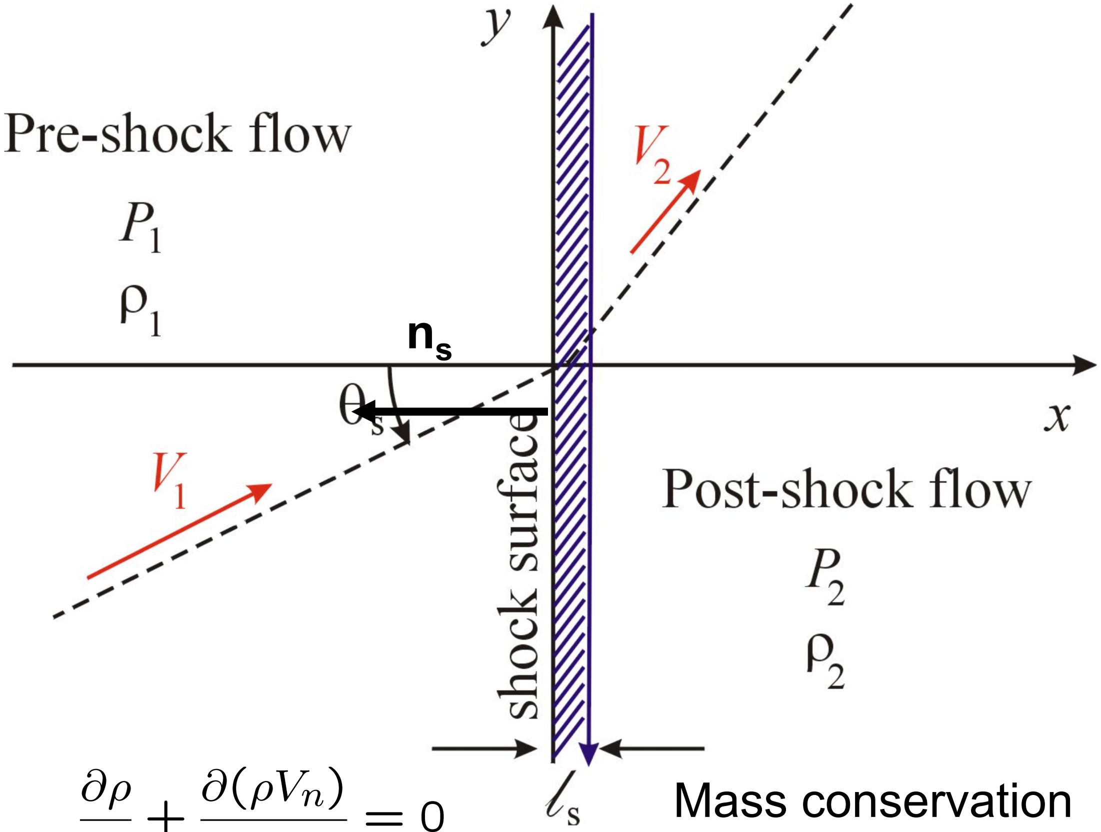
$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho\, \boldsymbol{V}) = 0$$

$$\frac{\partial (\rho \boldsymbol{V})}{\partial t} + \boldsymbol{\nabla} \cdot \left(\rho \boldsymbol{V} \otimes \boldsymbol{V} + P\, \boldsymbol{I}\right) = -\rho \boldsymbol{\nabla} \Phi$$

$R_{ik} = \rho V_i V_k + p\delta_{ik}$ is the momentum flux

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho\, V^2 + \rho e + \rho\Phi\right) + \boldsymbol{\nabla} \cdot \left[\rho \boldsymbol{V} \left(\tfrac{1}{2} V^2 + h + \Phi\right)\right] = \mathcal{H}_{\mathrm{eff}}$$

$$\mathcal{H}_{\mathrm{eff}} \equiv \mathcal{H} + \rho\, \frac{\partial \Phi}{\partial t}$$



$$\vec{V} = V_n \vec{e}_x + V_t \vec{e}_z$$

Neglecting the effect of gravity and dissipation in the flow the equations describing the flux are the mass, momentum and energy conservation, which reduce to

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_n)}{\partial x} = 0$$ Mass conservation

$$\frac{\partial(\rho V_n)}{\partial t} + \frac{\partial(\rho V_n^2 + p)}{\partial x} = 0$$ Momentum conservation perp shock front

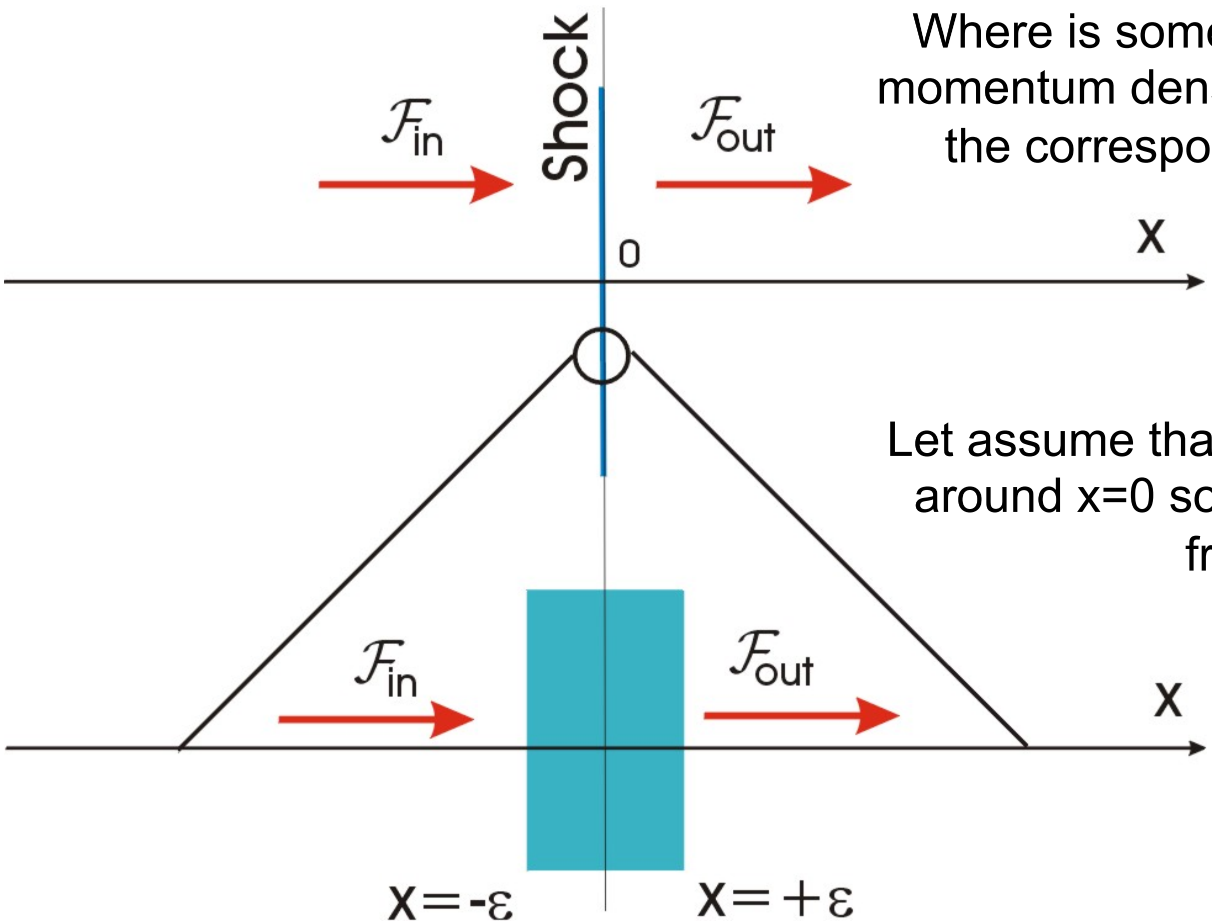
$$\frac{\partial(\rho V_t)}{\partial t} + \frac{\partial(\rho V_n V_t)}{\partial x} = 0$$ Momentum conservation || shock front

$$\frac{\partial}{\partial t}[\rho(\frac{V^2}{2} + e)] + \frac{\partial}{\partial x}[\rho V_n(\frac{V^2}{2} + h)] = 0$$ Energy conservation

With $e = \frac{p}{(\gamma - 1)\rho}$ $\quad h = \frac{\gamma p}{(\gamma - 1)\rho}$ Specific internal energy and enthalpy

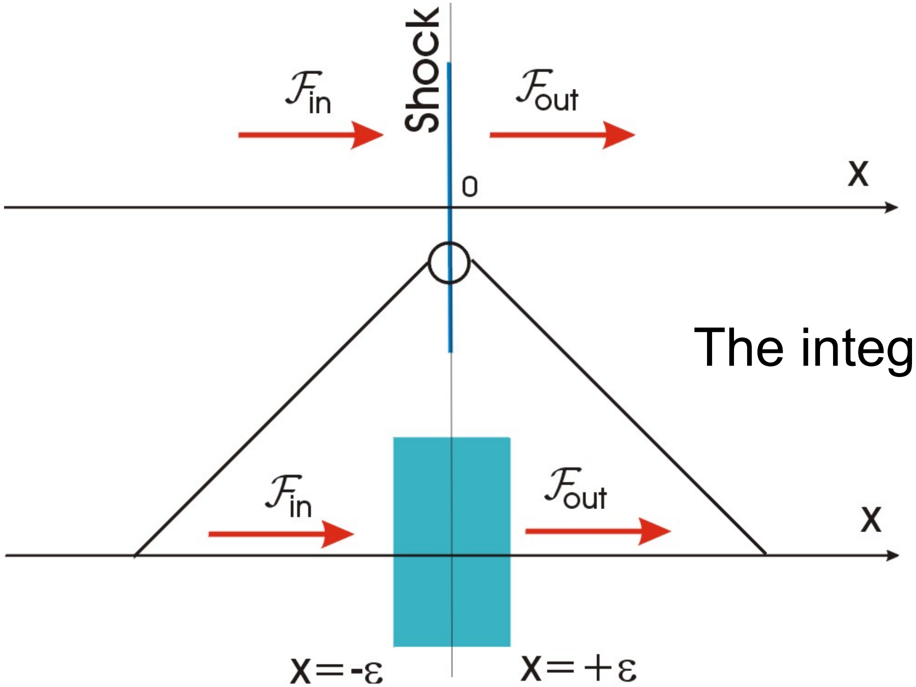
Effect of a sudden transition on the conservation law

All the equations have the form $$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = 0$$



Where is some quantity, like mass density, momentum density or energy density and F is the corresponding flux in the x direction

Let assume that the shock has a thickness ε around x=0 so that it extends in the range from -ε/2 to +ε/2



The integrated version of conservation law reads

$$\int_{-\varepsilon}^{+\varepsilon} dx \left(\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} \right) = 0 \Leftrightarrow$$

$$\frac{\partial}{\partial t} \left(\int_{-\varepsilon}^{+\varepsilon} dx \; Q \right) = F \; (x = -\varepsilon) - F \; (x = \varepsilon)$$

$$= F_{in} \quad - \quad F_{out}$$

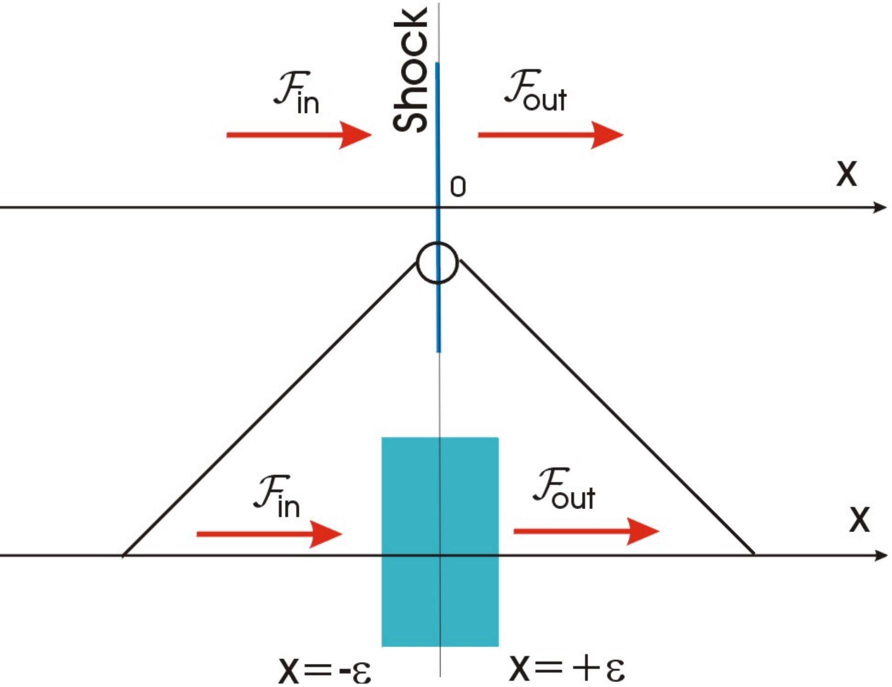
Change of amount in layer

flux in - flux out

If the shock thickness is small, one can estimate the integral using the mean value of ∂Q/∂t

$$-\Delta F = \int_{\epsilon/2}^{\epsilon/2} dx \frac{\partial Q}{\partial t} \approx \frac{\epsilon}{2}\left(\frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t}\right)$$

Infinitely thin layer:



Formal proof: limiting process

$$\lim_{\varepsilon\to 0}\left[\frac{\partial}{\partial t}\left(\int_{-\varepsilon}^{+\varepsilon} dx\, \mathrm{Q}\right) = \mathrm{F}\ (x=-\varepsilon) - \mathrm{F}\ (x=\varepsilon)\right]$$

$$\Leftrightarrow$$

$$0 = \mathrm{F}_{in} - \mathrm{F}_{out}$$

Flux in = Flux out

$$-\Delta F = \int_{\epsilon/2}^{\epsilon/2} dx \frac{\partial Q}{\partial t} \approx \frac{\epsilon}{2}\left(\frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t}\right)$$ → 0 when ε→0

What goes in must come out!

NB: in the case of steady flux (∂/∂t =0), the integral is identically zero

Rankine-Hugoniot relations

The result is that the fluxes across the shock front are conserved: this is the only condition that hydrodynamical equations impose to the fluid at the shock

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

1) Mass flux

$$\rho_1 V_{n1}^2 + p_1 = \rho_2 V_{n2}^2 + p_2$$

2) Momentum flux comp perp to shock front

$$\rho_1 V_{n1} V_{t1} = \rho_2 V_{n2} V_{t2}$$

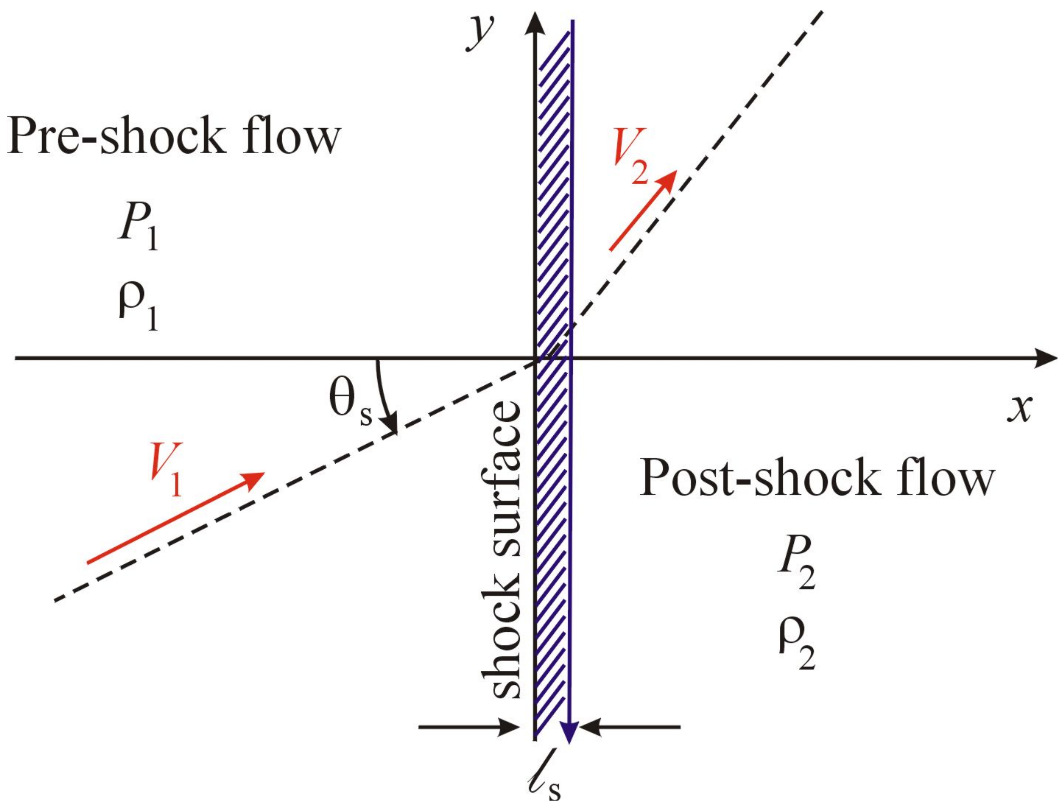
3) Momentum flux comp || to shock front

$$\rho_1 V_{n1}\left(\frac{V_1^2}{2} + h_1\right) = \rho_2 V_{n2}\left(\frac{V_2^2}{2} + h_2\right)$$

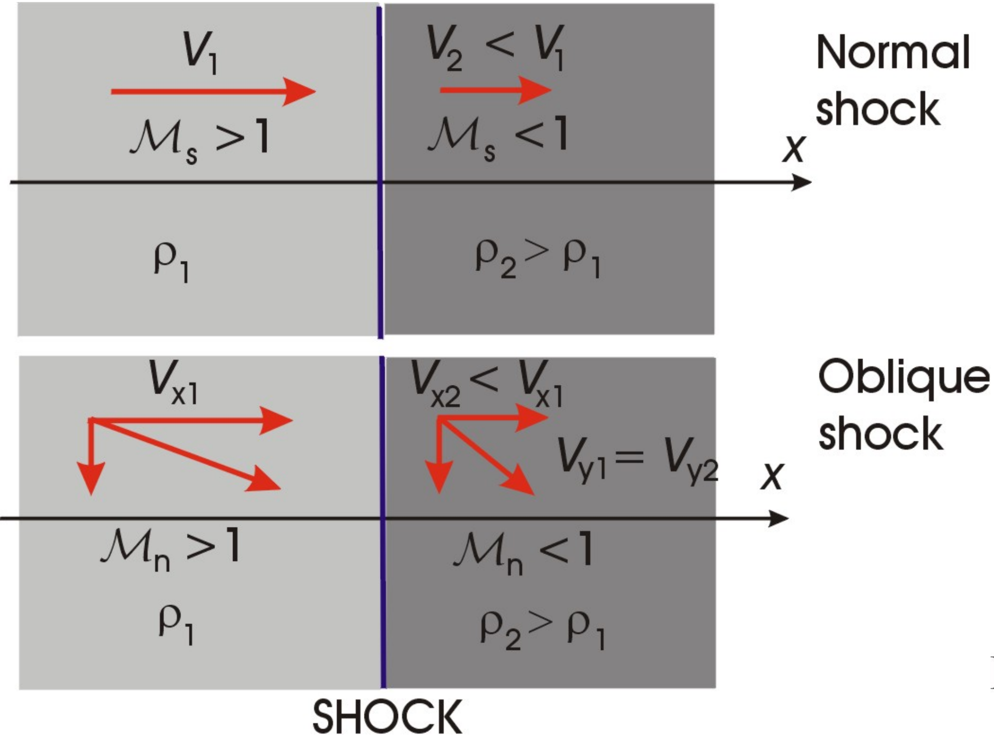
4) Energy flux

Known as Rankine-Hugoniot conditions

Fom 1) and 3) it follows that $V_{t1} = V_{t2}$ → the speed component of V || to the shock is conserved

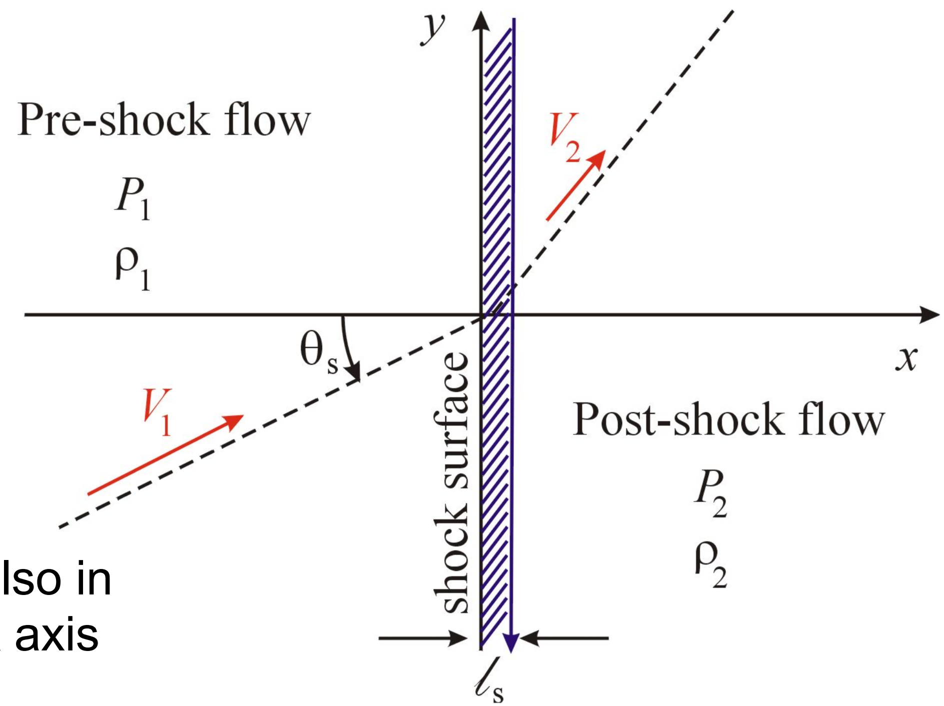


Shock waves



This means that we can *always* transform an oblique shock into a normal shock, simply going in a reference frame moving with V_t and viceversa

Therefore let work out the equations for a <u>normal shock</u>→ equations reduce to three in three unknowns (V_t eqn is trivially satisfied) and the problem is 1-dimensional (only x coord is concerned)



Momentum flux conservation tells us that also in the post-shock region it is directed along x axis

From normal shock to oblique shocks:

All relations remain the same if one makes the replacement:

$$V_1 \Rightarrow V_{\mathrm{n}1} = V_1 \cos\theta_1 \;,\; \mathrm{M}_S \Rightarrow \mathrm{M}_{\mathrm{n}} = V_{\mathrm{n}1} / c_{s1} = \mathrm{M}_S \cos\theta_1$$

θ is the angle between upstream velocity and normal on shock surface

Tangential velocity along shock surface is <u>unchanged</u>

$$V_{\mathrm{t}1} = V_1 \sin\theta_1 = V_{\mathrm{t}2} = V_2 \sin\theta_2$$

Shock conditions: what goes in must come out! (1 = in front of shock, 2= behind shock)

Three conservation laws means three conserved fluxes!

$$\rho_1 V_1 = \rho_2 V_2$$ Mass flux

$$\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2$$ Momentum flux

$$\rho_1 V_1 (\frac{V_1^2}{2} + h_1) = \rho_2 V_2 (\frac{V_2^2}{2} + h_2) \qquad h = \frac{\gamma p}{(\gamma - 1)\rho}$$ Energy flux

Three ordinary equations for three unknowns: post-shock state (2) is uniquely determined by pre-shock (1) state!

Shock jump relations

If we eliminate p_2 and v_2, then we find after little algebra

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{2+(\gamma-1)M_s^2} = \frac{(\gamma+1)}{2/M_s^2+(\gamma-1)} \quad \text{with} \quad M_s = \frac{v_1}{(\gamma p_1/\rho_1)^{1/2}} = \frac{v_1}{c_{s1}}$$

Eliminating ρ_2 and v_2 we get

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma-1)}{(\gamma+1)}$$

From mass flux conservation we get the speed

$$\frac{V_2}{V_1} = \frac{2+(\gamma-1)M_s^2}{(\gamma+1)M_s^2}$$

From equation of state $p=\rho RT/\mu$ we get temperature too

$$\frac{T_2}{T_1} = (\frac{\rho_1}{\rho_2})(\frac{p_2}{p_1}) = \frac{[2+(\gamma+1)^2 M_s^2][2\gamma M_s^2-(\gamma-1)]}{(\gamma+1)^2 M_s^2}$$

$$M_{s2} = \frac{2+(\gamma-1)M_s^2}{2\gamma M_s^2-\gamma+1}$$

Is the Mach number of post shock flow

Alternative form of conservation law

New variables: specific volume $v_1 = 1/\rho_1 \quad v_2 = 1/\rho_2$

The three conserved fluxes:

$$\rho_1 v_1 = \rho_2 v_2 \equiv J$$ Mass flux

$$\rho_1 V_1^2 + p_1 = \rho_2 V_2^2 + p_2 \Longrightarrow J^2 v_1 + p_1 = J^2 v_2 + p_2 \equiv F$$ Momentum flux

Energy flux

$$\frac{V_1^2}{2} + \frac{\gamma p_1}{(\gamma-1)\rho_1} = \frac{V_2^2}{2} + \frac{\gamma p_2}{(\gamma-1)\rho_2} \Longrightarrow \frac{J^2 v_1^2}{2} + \frac{\gamma p_1 v_1}{(\gamma-1)} = \frac{J^2 v_2^2}{2} + \frac{\gamma p_2 v_2}{(\gamma-1)} \equiv E$$

From momentum conservation:

$$J^2 = \frac{p_2 - p_1}{v_1 - v_2}$$

From energy conservation:

$$J^2(v_1^2 - v_2^2) = \frac{2\gamma}{\gamma - 1}(p_2 v_2 - p_1 v_1)$$

You can combine these two relations! (ie eliminate J^2)

$$\frac{\gamma}{\gamma - 1}(p_2 v_2 - p_1 v_1) = \frac{1}{2}(v_2 - v_1)(p_2 - p_1)$$

'Shock Adiabat'

Contact (tangential) discontinuity

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

$$\rho_1 V_{n1}^2 + p_1 = \rho_2 V_{n2}^2 + p_2$$

$$\rho_1 V_{n1} V_{t1} = \rho_2 V_{n2} V_{t2}$$

$$\rho_1 V_{n1} \left(\frac{V_1^2}{2} + h_1\right) = \rho_2 V_{n2} \left(\frac{V_2^2}{2} + h_2\right)$$

The jump conditions have another solution: let us assume that no mass crosses surface

From mass conservation we have $\rho_1 V_{n1} = \rho_2 V_{n2} = 0$

Since ρ_1 and ρ_2 cannot be zero this implies $V_{n1} = V_{n2} = 0$

In such a case || momentum and energy conservation are both trivially satisfied, while the perp-momentum implies that $\mathbf{p_1 = p_2}$

In particular one can have a situation where $\rho_1 \neq \rho_2$ and the velocity, which in this case is entirely along the discontinuity surface, is unconstrained

At a contact discontinuity it is allowed that $\mathbf{V}_{t1} \neq \mathbf{V}_{t2}$, like in the Blandford-Rees jet model

Shock compression ratio

$$r \equiv \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

Definition compression ratio

Substituting $v_1 = r v_2$ in $$\frac{\gamma}{\gamma - 1}(p_2 v_2 - p_1 v_1) = \frac{1}{2}(v_2 - v_1)(p_2 - p_1)$$ (cfr. pag 233)

We can express the compression ratio as a function of pressures pre- and post-shock

$$r = \frac{\rho_2}{\rho_1} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

Shock jump condition

Summary

$$r \equiv \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

$$r = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{2+(\gamma-1)M_s^2} = \frac{(\gamma+1)}{2/M_s^2+(\gamma-1)}$$

$$M_s = \frac{v_1}{(\gamma p_1/\rho_1)^{1/2}} = \frac{v_1}{c_{s1}} > 1$$

$$\frac{V_2}{V_1} = \frac{2+(\gamma-1)M_s^2}{(\gamma+1)M_s^2}$$

$$r = \frac{\rho_2}{\rho_1} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma-1)}{(\gamma+1)}$$

i) $r>1 \rightarrow \rho_2>\rho_1$ but limited at $(\gamma+1)/(\gamma-1)$ in the limit $M_s \rightarrow \infty$ so shock waves compress moderately the fluid

ii) $r>1 \rightarrow V_2<V_1 \approx (\gamma-1)/(\gamma+1)$ when $M_s \rightarrow \infty$

iii) $p_2>p_1$ and $p_2/p_1 \sim M_s^2 \rightarrow$ large compression of the fluid

iv) $T_2>T_1$ and $T_2/T_1 \sim M_s^2 \rightarrow$ large heating of the fluid

$$\frac{T_2}{T_1} = (\frac{\rho_1}{\rho_2})(\frac{p_2}{p_1}) = \frac{[2+(\gamma+1)^2M_s^2][2\gamma M_s^2-(\gamma-1)]}{(\gamma+1)^2M_s^2}$$

Physical interpretation

i) $\rho_2 > \rho_1$
ii) $V_2 < V_1$
iii) $p_2 > p_1$
iv) $T_2 > T_1$

Shocks decelerate the bulk flow speed (ii) and at same time heat and compress (i,iii) the fluid immediately after the shock (iv).
It is clear that the energy to make the heating must come from kinetic energy of the fluid: infact the speed is decreased by the same factor r by which density is increased

→ <u>shocks transform kinetic energy in internal (thermal) energy</u>

Physical interpretation

But this is not all the history...entropy change is also involved at the shock front

First, note that, when M_s=1, all the ratios are =1: in this case the shock does not make any transformation on the fluid

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{2+(\gamma-1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2+(\gamma-1)M_s^2}{(\gamma+1)M_s^2}$$

What does happen if M_s<1?

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2-(\gamma-1)}{(\gamma+1)}$$

The Rankine-Hugoniot eqns tell us that V_1<V_2 and T_1<T_2: the fluid is accelerated and cooled!

$$\frac{T_2}{T_1} = \frac{[2+(\gamma+1)^2M_s^2][2\gamma M_s^2-(\gamma-1)]}{(\gamma+1)^2M_s^2}$$

This means that a fraction of the internal energy is transformed in kinetic energy, without having done anything else

BUT...

Physical interpretation

We have seen that in an ideal polytropic gas the specific entropy is $s = c_v \ln(p\rho^{-\gamma})$

Since the have neglected dissipation in deriving the equations (ie adiabatic transformations), the entropy is constant on either sides of the shock:
s(x<0)=const.= s_1, s(x>0)=const.= s_2

However we can calculate the entropy jump at the shock

$$\Delta s = s_2 - s_1 = c_v \ln[(\frac{p_2}{p_1})(\frac{\rho_1}{\rho_2})^{\gamma}]$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{2+(\gamma-1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2+(\gamma-1)M_s^2}{(\gamma+1)M_s^2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2-(\gamma-1)}{(\gamma+1)}$$

It is clear that:
i) Δs>0, provided that is M_s>1
ii) Δs =0 when M_s=1 → again the fluid properties do not change across the shock

i) clearly indicates that some form of dissipation occurs at the shock: kinetic energy of upstream flow is irreversibly dissipated into thermal (internal) energy of the shock-heated gas downstream. Neverthless, the details of the dissipation mechanism do not enter into the final equations

Physical interpretation

$$\Delta s = s_2 - s_1 = c_v \ln[(\frac{p_2}{p_1})(\frac{\rho_1}{\rho_2})^\gamma]$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{2+(\gamma-1)M_s^2}$$

$$\frac{V_2}{V_1} = \frac{2+(\gamma-1)M_s^2}{(\gamma+1)M_s^2}$$

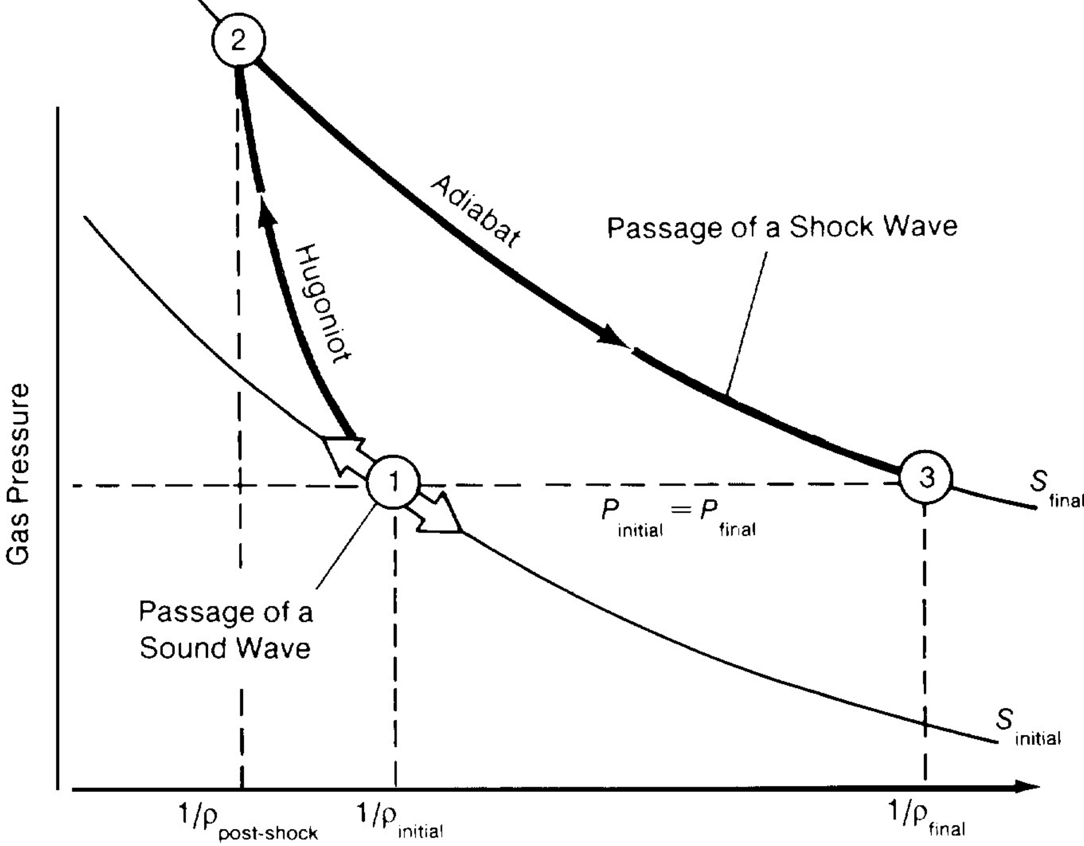
$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2-(\gamma-1)}{(\gamma+1)}$$

If $M_s<1$, then $\rho_2<\rho_1 \rightarrow$ we have $\Delta s<0$!

This is impossible: 2nd law of thermodynamics states that the only possible transformations are those for which $\Delta s \geq 0$

Therefore, we CANNOT have shock waves with $M_s<1$

Shocks occurs ONLY for supersonic speeds



Inverse Density

Fig. B, Effects of the passage of a sound wave and of a shock wave. As a sound wave passes through a gas, the pressure and density of the gas oscillates back and forth along an adiabat (a line of constant entropy], which is a reversible path. In contrast, the passage of a shock front causes the state of the gas to jump along an irreversible path from point 1 to point 2, that is, to a higher pressure, density, **and** ***entropy. The curve connecting these two points is called a Hugoniot, for it was Hugoniot (and simultaneously Rankine) who derived, from the conservation laws, the jump conditions for the state variables across a shock front, After passage of the shock, the gas relaxes back to point 3 along an adiabat, returning to its original pressure but to a higher temperature and entropy and a lower density. The shock has caused an irreversible change in the gas.***

Shock thickness

We have seen that shock waves transform inflow kinetic energy into thermal (internal) energy of the outflow flux with entropy creation and we have assumed infinitely thin shock wave

The question is: physically it is impossible to have a mathematical step, so how good is the infinitely thin layer approximation?

The mechanism of entropy generation is given by the collisions among particles

It follows then that the shock wave must be thick enough to allow the particles to undergo to some collisions so that ordered bulk kinetic energy (that is directed along the same direction of all the other particles) is transformed in internal disordered kinetic energy, ie thermal

Therefore the shock thickness will be given, to order-of-magnitude, by the free mean path λ of the particles, since is over such scale length that the speeds are changed (in direction)

Shock thickness

We know that $\lambda = 1/n\sigma$, with n particle density and σ diffusion cross section

For istance in standard atmospheric conditions in which $n \sim 10^{23}$ cm^{-3} and $\sigma = \pi a_B^2$ with $a_B \sim 10^{-8}$ cm, atomic Bohr radius, we get $\lambda \sim 10^{-7}$ cm, so that the infinitely thin layer approximation is very good

This type of shock in which the collisions between particles are responsible for the transformation of ordered kinetic energy into disordered (thermal) kinetic energy, is said "collisional" shock

All the shocks studied in laboratories or in atmosphere are collisional

Shock thickness

When we try to apply the relation to the astrophysical situations, we find a paradox

First, most of the particles are ionized, so that they have not the dimensions of Bohr radius, but are much smaller ($\sim 10^{-13}$ cm) and, more importantly, the typical densities are much smaller, $n \sim 1$ cm^{-3}

This means that the shock thickness would be macroscopic ($\sim 10^{16}$ cm)

But most of the astrophysical matter is ionized and electrons and nuclei undergo to acceleration by electric and magnetic fields

It is then generally believed that a combination of electric and magnetic fields, partly transient, is responsible of the kinetic energy isotropization of the particles

Shock thickness

In such a case, the thickness of the shock must be comparable to the Larmor radius of a proton since it is on this length scale that the particles are deflected by a magnetic field

$$\lambda \approx r_B = \frac{mvc}{eB}$$

Assuming a typical proton speed in a supernova explosion of ~ 10^4 km/s in the typical galactic magnetic field of 10^{-6} G, we have

$$\lambda \approx (10^{10}\ cm) \times (\frac{v}{10^4\ kms^{-1}})(\frac{10^{-6}\ G}{B})$$

If compared with typical length involved in SN explosions (~ light year), the shock thickness is very small and the infinitely thin layer approximation can be safely used

The shocks where the randomization mechanism of velocity vector is the interaction with electro-magnetic fields are called "non-collisional" shocks

Limiting cases: weak and strong shocks

Weak shock: pressure & density change by small amount

This happens when $M_s \geq 1$

$$r = \frac{\rho_2}{\rho_1} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2} \equiv \frac{\Gamma p_2 + p_1}{\Gamma p_1 + p_2}$$

With $\rho_2 = \rho_1 + \Delta\rho$ and $p_2 = p_1 + \Delta p$,
$\Delta\rho \ll \rho_1$, $\Delta p \ll p_1$

Weak shocks

$$\frac{\rho_1 + \Delta\rho}{\rho_1} = \frac{\Gamma(p_1 + \Delta p) + p_1}{\Gamma p_1 + p_1 + \Delta p} \Rightarrow 1 + \frac{\Delta\rho}{\rho_1} = \frac{\Gamma + 1 + \Gamma \Delta p / p_1}{\Gamma + 1 + \Delta p / p}$$

$$= \frac{1 + \left(\frac{\Gamma}{\Gamma+1}\right)\Delta p/p_1}{1 + \left(\frac{1}{\Gamma+1}\right)\Delta p/p_1} \approx \left[1 + \frac{\Gamma}{\Gamma+1}\left(\frac{\Delta p}{p_1}\right)\right]\left[1 - \left(\frac{1}{\Gamma+1}\right)\frac{\Delta p}{p_1}\right]$$

Power serie for denominator

$$\approx 1 - \frac{1}{\Gamma+1}\left(\frac{\Delta p}{p_1}\right) + \frac{\Gamma}{\Gamma+1}\left(\frac{\Delta p}{p_1}\right) = 1 + \frac{\Gamma-1}{\Gamma+1}\left(\frac{\Delta p}{p_1}\right)$$

Neglect 2nd order terms in $\Delta p/p$

$$\Gamma + 1 = \frac{\gamma+1}{\gamma-1} + 1 = \frac{2\gamma}{\gamma-1} \qquad \Gamma - 1 = \frac{2}{\gamma-1}$$

$$\Rightarrow \frac{\Gamma-1}{\Gamma+1} = \frac{1}{\gamma}$$

$$\Rightarrow 1 + \left(\frac{\Delta\rho}{\rho_1}\right) = 1 + \frac{1}{\gamma}\left(\frac{\Delta p}{p_1}\right) \Rightarrow \Delta p = \left(\frac{\gamma p_1}{\rho_1}\right)\Delta\rho = c_{s1}^2 \Delta\rho$$

Weak shocks

<u>Weak shock:</u> pressure & density change by <u>small</u> amount

$$\left.\begin{aligned} r \equiv \rho_2 / \rho_1 &= \frac{\left(\frac{\gamma+1}{\gamma-1}\right)P_2 + P_1}{\left(\frac{\gamma+1}{\gamma-1}\right)P_1 + P_2} \\ \rho_2 &= \rho_1 + \Delta\rho \text{ with } \Delta\rho \ll \rho_1 \\ P_2 &= P_1 + \Delta P \text{ with } \Delta P \ll P_1 \end{aligned}\right\} \Rightarrow \Delta P = \left(\frac{\gamma P_1}{\rho_1}\right)\Delta\rho = c_s^2 \Delta\rho$$

The relation between pressure and density changes is exactly the same as for small perturbations

Weak shock can be considered as a strong sound wave!

Strong shocks

In many astrophysical applications, the normal Mach number is large, $M_s >> 1$

In this limit the Rankine-Hugoniot jump conditions simplify considerably

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)}{2/M_s^2 + (\gamma-1)} \quad \Longrightarrow \quad \frac{\rho_2}{\rho_1} \approx \frac{(\gamma+1)}{(\gamma-1)}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma-1)}{(\gamma+1)} \quad \Longrightarrow \quad \frac{p_2}{p_1} \approx 2\gamma M_s^2$$

$$\frac{V_2}{V_1} = \frac{2 + (\gamma-1)M_s^2}{(\gamma+1)M_s^2} \quad \Longrightarrow \quad \frac{V_2}{V_1} \approx \frac{\gamma-1}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma-1)M_s^2][2\gamma M_s^2 - (\gamma-1)]}{(\gamma+1)^2 M_s^2} \quad \Longrightarrow \quad \frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_s^2$$

$$M_{s2} = \frac{2 + (\gamma-1)M_s^2}{2\gamma M_s^2 - \gamma + 1} \quad \Longrightarrow \quad M_{s2} \approx \frac{\gamma-1}{2\gamma}$$

Strong shocks

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma+1)}{(\gamma-1)}$$

Fluid density changes moderately

$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma+1)}$$

Fluid is strongly compressed

$$\frac{V_2}{V_1} \approx \frac{\gamma-1}{\gamma+1}$$

Speed flow is slowed

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_s^2$$

Fluid temperature is highly increased

$$M_{s2} \approx \frac{\gamma-1}{2\gamma}$$

Mach number is <1→ fluid is subsonic

Summary shock physics

Across an <u>infinitely thin steady</u> shock you have in the shock frame where the shock is at rest:

Mass-flux conservation

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

Momentum-flux conservation

$$\rho_1 \left(V_{n1}\right)^2 + P_1 = \rho_2 \left(V_{n2}\right)^2 + P_2$$

$$V_{t1} = V_{t2}$$

Energy-flux conservation

$$\tfrac{1}{2}\left(V_{n1}\right)^2 + \frac{\gamma P_1}{(\gamma - 1)\rho_1} = \tfrac{1}{2}\left(V_{n2}\right)^2 + \frac{\gamma P_2}{(\gamma - 1)\rho_2}$$

Summary: Rankine-Hugoniot relations (for normal shock)

Fundamental parameter:
Mach Number

$$M_s \equiv \frac{\text{shock speed}}{\text{sound speed}} = \frac{V_1}{c_{s1}}$$

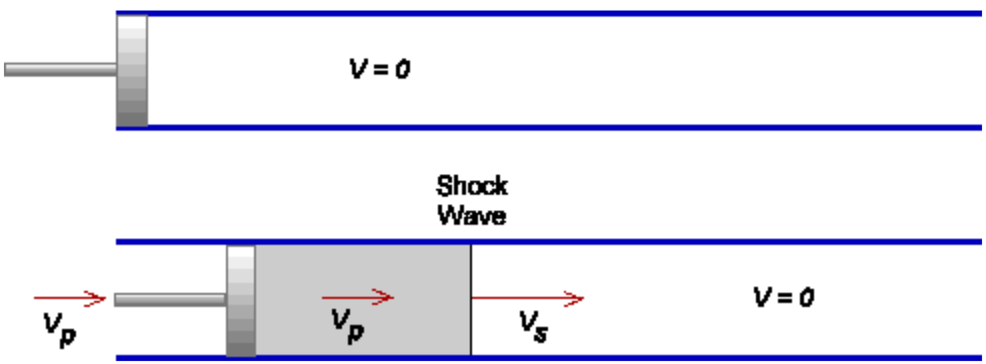
R-H Jump Conditions relate the up- and downstream quantities at the shock:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_s^2}{(\gamma-1)M_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma M_s^2-(\gamma-1)}{\gamma+1}$$

The supersonic piston

A common situation in high energy astrophysics is one in which an object is driven supersonically into a gas, or equivalently, a supersonic flow past a stationary object (ie what is important is the relative speed between fluid and object), as for istance in the case of supernovae explosions



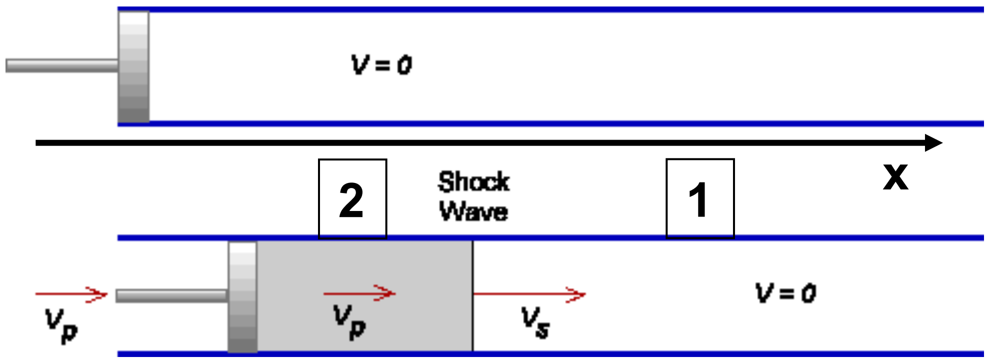
A good illustrative example is a piston driven supersonically into a tube filled with stationary gas

Basically, a hollow tube is filled with a uniform gas at rest and fitted with a piston at one end. At time $t = 0$, the piston is suddenly put into motion with a constant speed, V_p ($>c_s$).

The motion of the piston creates a shock wave, ahead of the piston, that moves in the same direction as the piston, but at faster, constant speed, V_s (faster than the speed of sound).

The problem is to calculate the shock wave speed

The supersonic piston



Let work out the problem in the shock wave reference frame, which is moving in the observer frame at speed $+V_s$

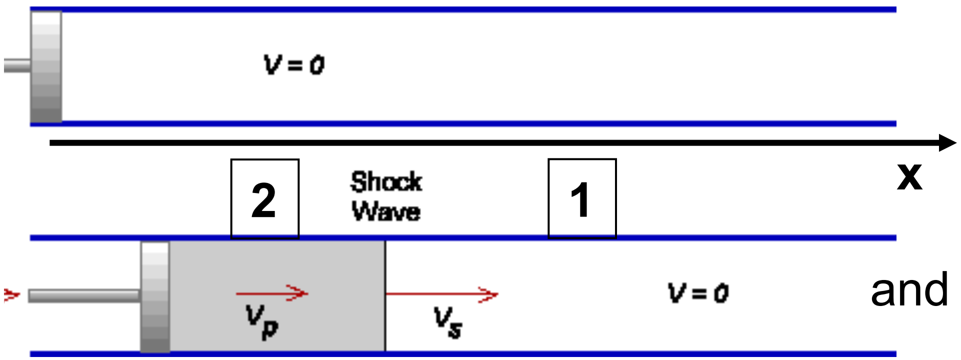
In the obs frame, the pre-shock fluid is at rest $V_1=0$, while for the post-shock fluid is reasonable to assume that is moving at the same speed of the piston $V_2=V_p>c_s$ because the particles of the fluid are swept up when the piston reach them, very much like snow is swept up by a snowplow

The galileo's tranformation is $\mathbf{V_{sh}=V_{obs} - V_s}$

⇩

In the shock frame, the pre-shock fluid moves with speed $V_1'=-V_s$ and the post shock fluid moves at (unknown) speed V_2'

The supersonic piston



From mass flux conservation

$$V_1'^2 = J^2 v_1^2$$

and $J^2 = \dfrac{p_2 - p_1}{v_1 - v_2} = \dfrac{p_2 - p_1}{v_1(1 - v_2/v_1)}$ (cfr. pag 233)

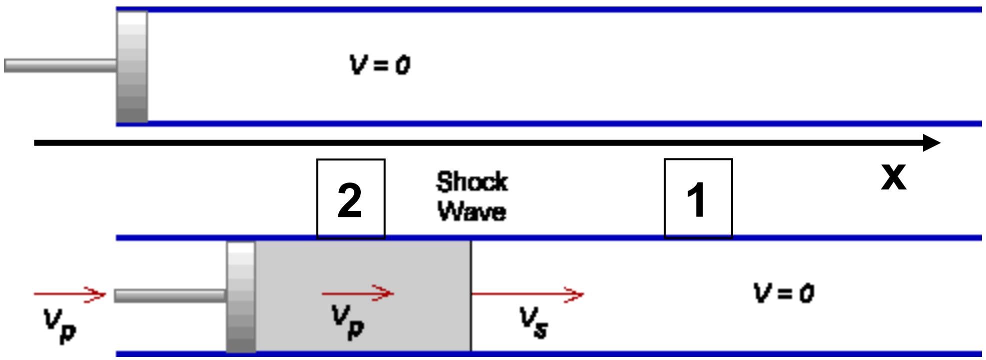
but $\dfrac{v_1}{v_2} = \dfrac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2}$ (cfr. pag 234) $\Rightarrow$ $J^2 = \dfrac{(\gamma-1)p_1 + (\gamma+1)p_2}{2v_1}$

$$\Rightarrow \quad V_1^2 = \frac{v_1}{2}(\gamma-1)p_1 + (\gamma+1)p_2 = \frac{p_1 v_1}{2}[(\gamma-1) + (\gamma+1)(p_2/p_1)]$$

But $c_1^2 = \gamma p_1 v_1$ $\Rightarrow$ $V_1^2 = \dfrac{c_1^2}{2\gamma}[(\gamma-1) + (\gamma+1)(p_2/p_1)]$

Then we have to find the pressure ratio

The supersonic piston



We dont know V_2', but we know the speed difference

$$V_1' - V_2' = V_p$$

From Galileo's transformation

From mass flux conservation we have $\rho_1 V_1' = \rho_2 V_2' \equiv J \Rightarrow (\rho_1/\rho_2) V_1' = V_2'$

$$V_1' - V_2' = V_p = V_1' - (\rho_1/\rho_2) V_1'$$

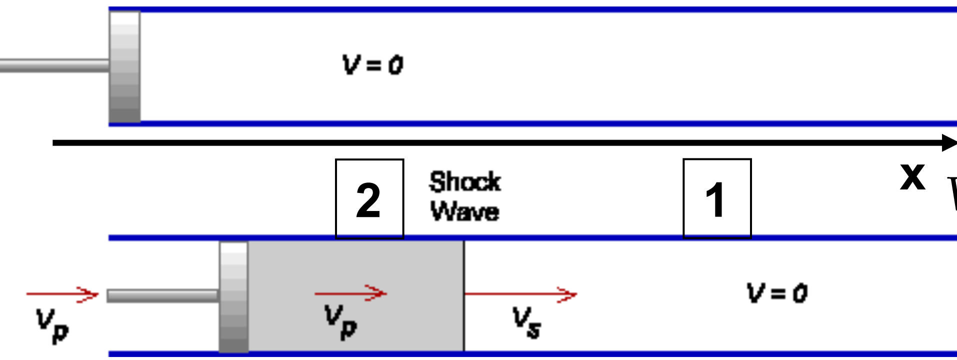
$$V_1' - V_2' = V_p = V_1'[1 - (\rho_1/\rho_2)] = \rho_1 V_1'[1/\rho_1 - 1/\rho_2] = \rho_1 V_1'[v_1 - v_2] = J[v_1 - v_2]$$

$$\Rightarrow \frac{V_1' - V_2'}{v_1 - v_2} = J \qquad \text{but} \qquad J^2 = \frac{p_2 - p_1}{v_1 - v_2}$$

(See pag 234)

$$\frac{(V_1' - V_2')^2}{(v_1 - v_2)^2} = \frac{p_2 - p_1}{v_1 - v_2} \Rightarrow V_1' - V_2' = (p_2 - p_1)(v_1 - v_2)^{1/2}$$

The supersonic piston



$$V_1^{'} - V_2^{'} = V_p$$

$$V_1^{'} - V_2^{'} = (p_2 - p_1)(v_1 - v_2)^{1/2} = V_p \quad \text{(a)}$$

Now eliminate specific volume v_2 in (a) by using

$$\frac{v_1}{v_2} = \frac{(\frac{\gamma+1}{\gamma-1})p_2 + p_1}{(\frac{\gamma+1}{\gamma-1})p_1 + p_2} \quad \text{(cfr. pag 234)}$$

Then square (a) and solve for for p_2/p_1

$$(p_2/p_1)^2 - (p_2/p_1)[2 + (\gamma+1)\frac{V_p^2}{2p_1 v_1^{'}}] + [1 - \frac{(\gamma-1)V_p^2}{2p_1 v_1^{'}}] = 0$$

The sound speed in the pre-shock region is $\gamma \mathbf{v_1' p_1 = c_1^2}$ ➔ we can solve for p_2/p_1

$$(p_2/p_1) = 1 + \frac{\gamma(\gamma+1)V_p^2}{4c_1^2} + \frac{\gamma V_p}{c_1}[1 + \frac{(\gamma+1)^2 V_p^2}{16c_1^2}]^{1/2}$$

The supersonic piston

$$(p_2/p_1) = 1 + \frac{\gamma(\gamma+1)V_p^2}{4c_1^2} + \frac{\gamma V_p}{c_1}[1 + \frac{(\gamma+1)^2 V_p^2}{16c_1^2}]^{1/2} \qquad \text{(a)}$$

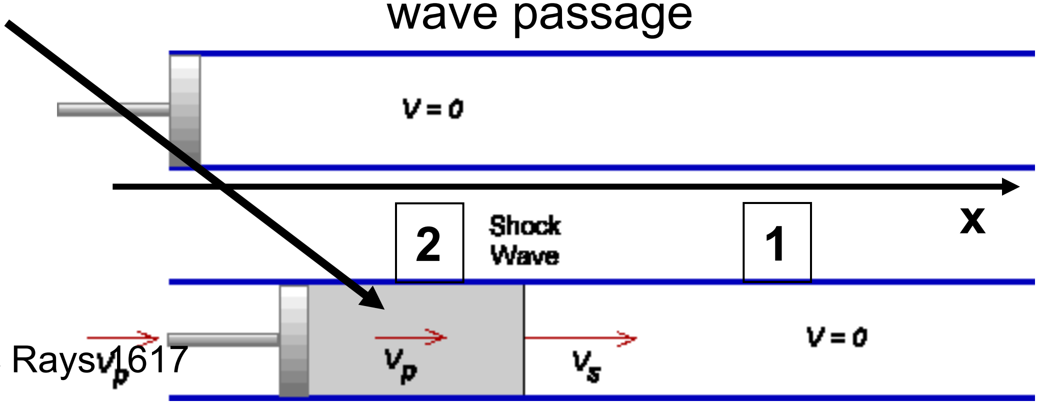
$$V_1^2 = \frac{c_1^2}{2\gamma}[(\gamma-1) + (\gamma+1)(p_2/p_1)] \qquad \text{(b)}$$

Inserting (a) into (b) after some simple but tedious algebra we have

$$V_1' \equiv |V_s| = \frac{(\gamma+1)}{4}V_p + [c_1^2 + \frac{(\gamma+1)^2 V_p^2}{16}]^{1/2}$$

The shock wave travels at higher speed with respect to the piston

➔ there is a layer of shocked material in between the shock wave and the piston, traveling at the piston speed, compressed and heated by the shock wave passage



The supersonic piston

$$V_1^{'} \equiv |V_s| = \frac{(\gamma+1)}{4}V_p + [c_1^2 + \frac{(\gamma+1)^2 V_p^2}{16}]^{1/2}$$

In the limit of strong shocks, the expression reduces to $|V_s| \approx \frac{(\gamma+1)}{2}V_p$

→ the ratio between the shock position and the piston position is $|V_s|/V_p \approx \frac{(\gamma+1)}{2}$

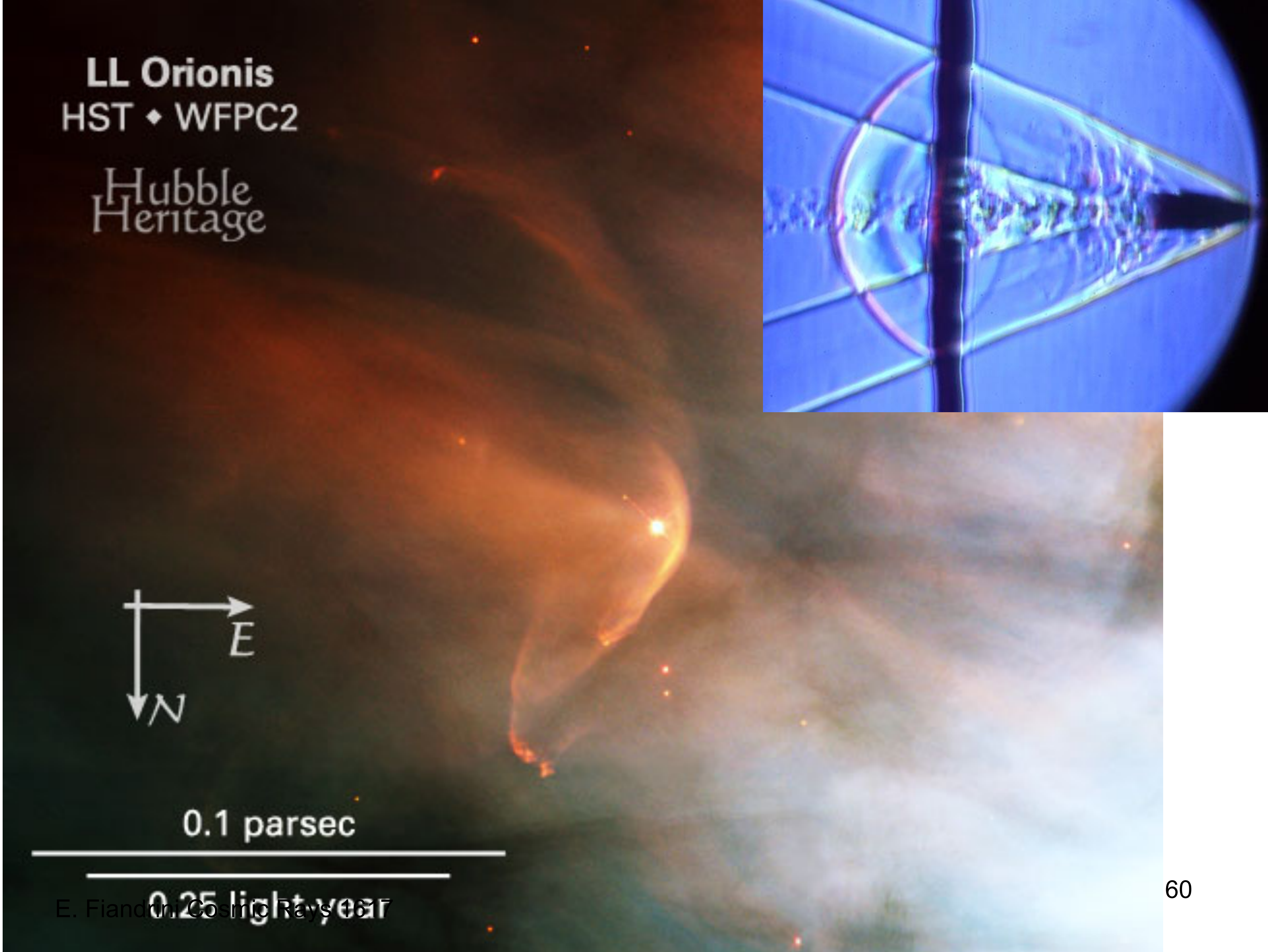
E.g. for a monoatomic gas γ=5/3 → **V_s/V_p=4/3**

All the gas originally in the tube between x=0 and the shock position is squeezed into a smaller distance $(V_s-V_p)t$

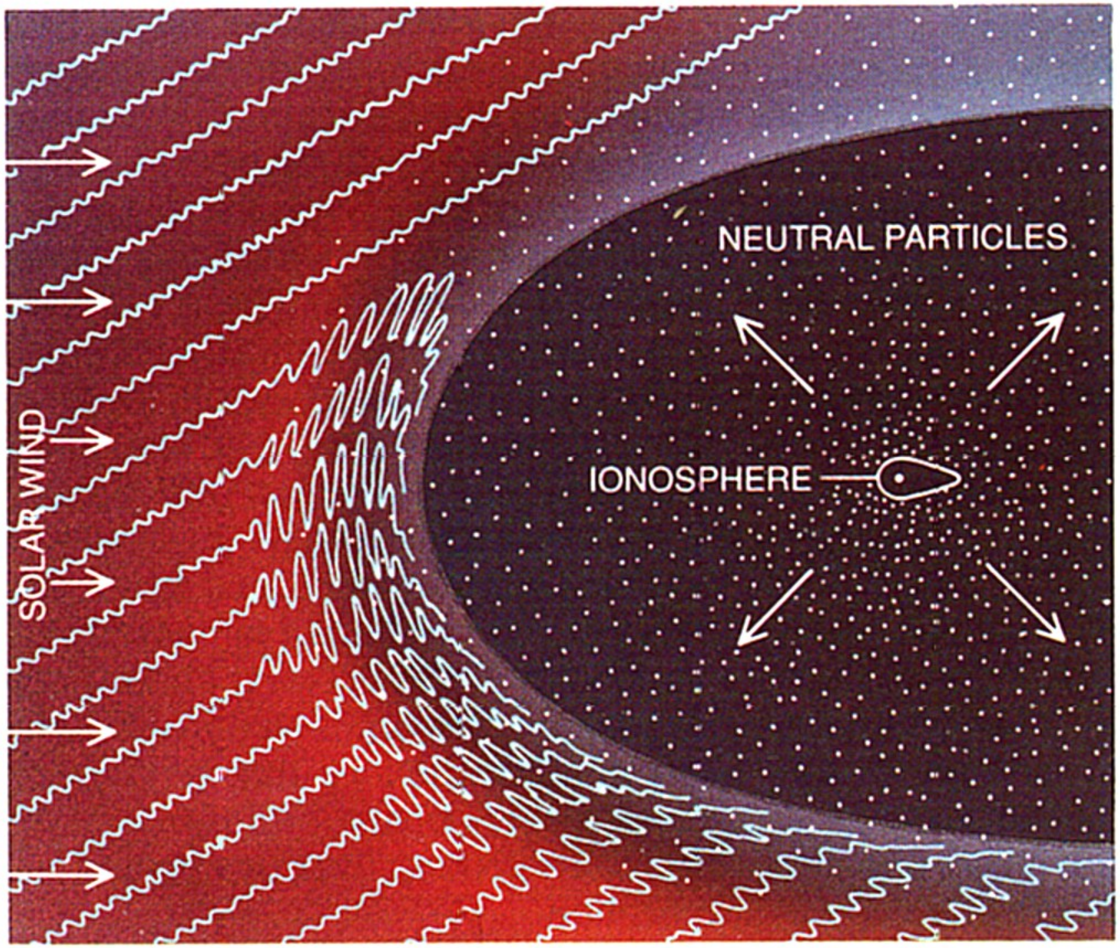
So behind the shock wave and ahead of the obstacle (the piston in this example) there is a layer of material compressed and heated

It is also seen that there is a stand-off distance of a shock front from a blunt object placed in the flow, as for istance in the case of solar wind past the Earth's magnetic dipole

This is what is expected to occur when a supernova ejects a sphere of hot gas into the ISM


LL Orionis
HST ◆ WFPC2
Hubble Heritage
E
N
0.1 parsec
0.25 light year

Examples of Astrophysical shocks



Cometary bow-shocks

Blast shock waves

A blast shock wave is a shock wave formed by a hot gas bubble expanding supersonically in the ambient medium

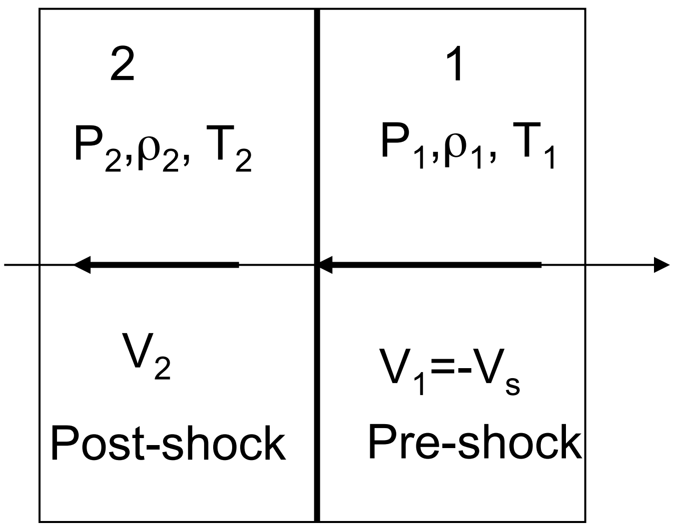
Let us assume that the expansion occurs in a uniform stationary polytropic medium with density ρ_o and pressure p_o

how does the shock wave evolve in time?

First, since the surrounding medium is uniform, the expansion will have spherical symmetry

Blast shock waves

We worked out the physics of the (strong) shock in the shock reference frame, where it is stationary



Shock rest frame

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma+1)}{(\gamma-1)}$$

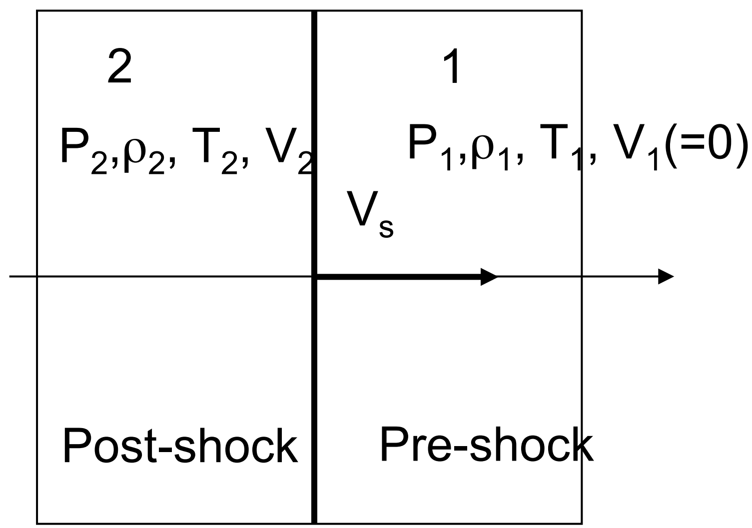
$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma+1)}$$

$$\frac{V_2}{V_1} \approx \frac{\gamma-1}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_s^2$$

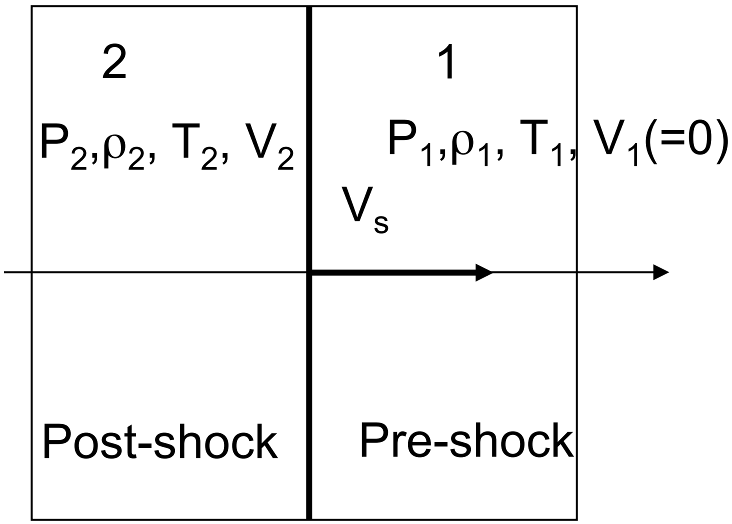
Blast shock waves

Now we have an (supersonic) expansion in a uniform, stationary (V_1=0) polytropic medium with density $\rho_1 = \rho_o$ and pressure $p_1 = p_o$ and a supersonic shock wave propagating to the right (+x dir) with speed V_s ahead the expanding gas, as in the case of the supersonic piston

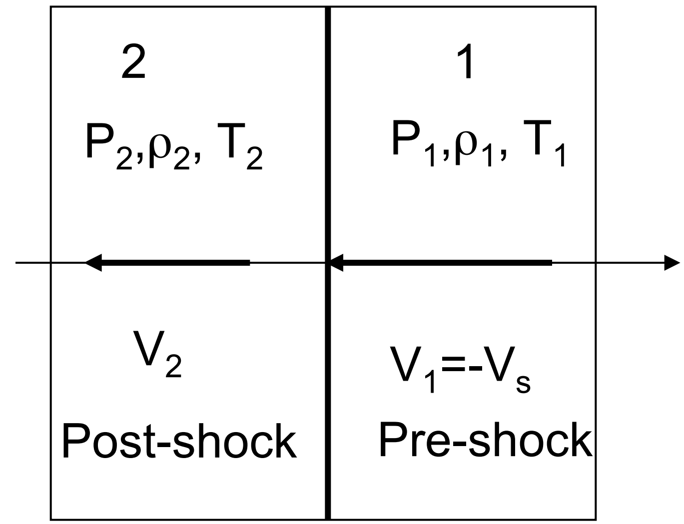


As in the supersonic piston example, the situation in a reference frame where the shock is traveling with speed V_s can be obtained, for non-relativistic shocks, from a simple galileian transformation

Blast shock waves



Observer frame



Shock rest frame

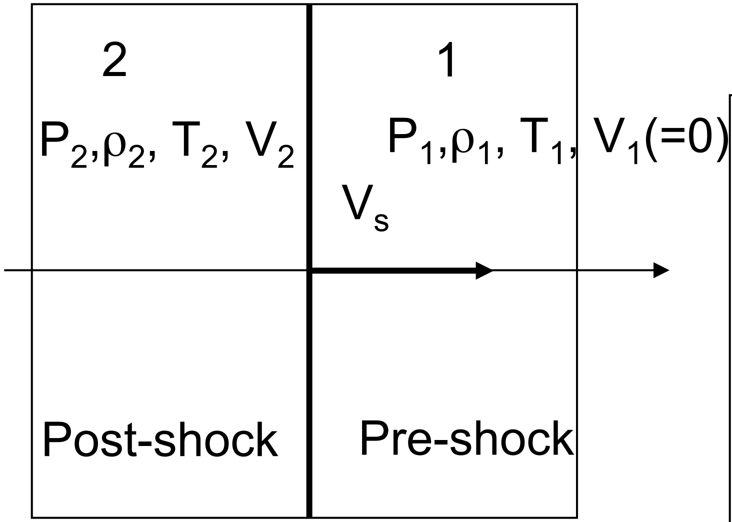
The RH conditions can be still applied, provided one interprets the speeds V_1 and V_2 as relative speeds with respect to the shock, ie apply a galileian velocity tranformation

$$\vec{V} \rightarrow \vec{V}_{rel} = \vec{V} - \vec{V}_s$$

For a shock propagating with velocity $\mathbf{V}_s$ into a medium at rest, $\mathbf{V}$=0, one has $\underline{\mathbf{V_1=-V_s}}$ and θ_s=0 (as for the supersonic piston) ➔ in this case any shock is a normal shock, with $\mathbf{V}_t$=0, even when the shock surface itself is not a plane!

➔ spherical shocks are normal shocks

Blast shock waves



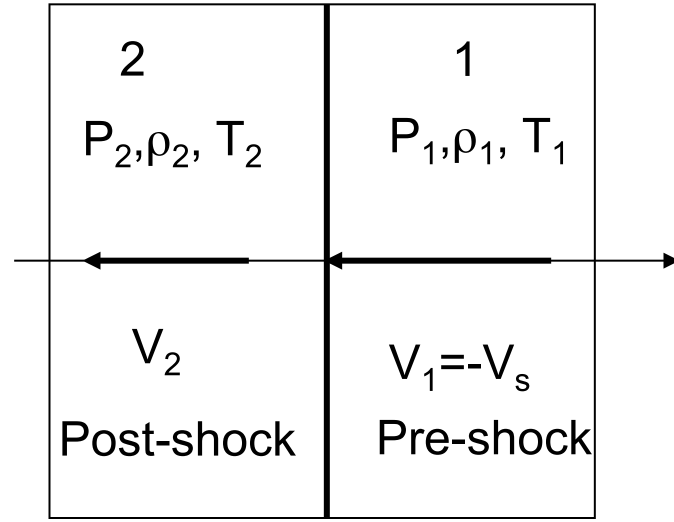
$$\vec{V}_{sh} = \vec{V}_{obs} - \vec{V}_s$$

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma+1)}{(\gamma-1)}$$

$$\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma+1)}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_s^2$$

$$\frac{V_2}{V_1} \approx \frac{\gamma-1}{\gamma+1}$$

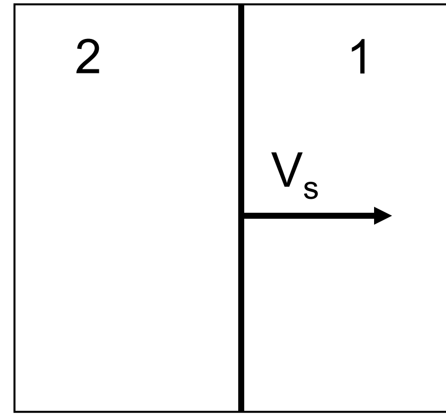


All the conditions may be directly applied, except the speed for which we need to make the substitution $V_2^{sh}=V_2^{obs} - V_s$ and $V_1=-V_s$

$$\frac{V_2}{V_1} \rightarrow \frac{V_2 - V_s}{-V_s} \approx \frac{\gamma-1}{\gamma+1} \quad \Rightarrow \quad V_2 \approx \frac{2V_s}{\gamma+1}$$

Is the post shock speed in the obs frame

Blast shock waves



Let assume also that the explosion occurs in a uniform stationary polytropic medium with density $\rho_1 = \rho_o$ and pressure $p_1 = p_o$

The strong shock satisfies the relation $M_s^2 = (\frac{V_s}{c_s})^2 = \frac{\rho_o V_s^2}{\gamma p_o} \gg 1$

The RH relations then give

$$\frac{\rho_2}{\rho_1} \approx \frac{(\gamma+1)}{(\gamma-1)}$$

$$\boxed{\frac{p_2}{p_1} \approx \frac{2\gamma M_s^2}{(\gamma+1)}}$$

$$V_2 \approx \frac{2V_s}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_s^2$$

We can have the pressure p_2 immediately behind the shock

$$p_2 \approx \frac{2\gamma M_s^2}{(\gamma+1)} p_o = \frac{2\rho_o V_s^2}{(\gamma+1)}$$

Inverting this relation, one can calculate the shock speed as a function of the post-shock pressure and the pre-shock density

$$V_s = (\frac{\gamma+1}{2})^{1/2} (\frac{P_2}{\rho_o})^{1/2}$$

This result can be applied for the formation of high pressure bubbles in a stationary medium

As for istance SuperNova remnants (SNRs) and stellar wind bubbles in the interstellar medium

(...and to nuclear explosions too, unfortunately)