

Analysis of the confinement string in (2+1)d Quantum Electrodynamics with quantum computing

Arianna Crippa,
Karl Jansen & Enrico Rinaldi

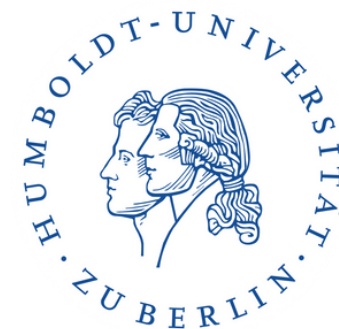


Perugia 8. May 2025



DESY.
QUANTUM

Center for
Quantum Technology
and Applications

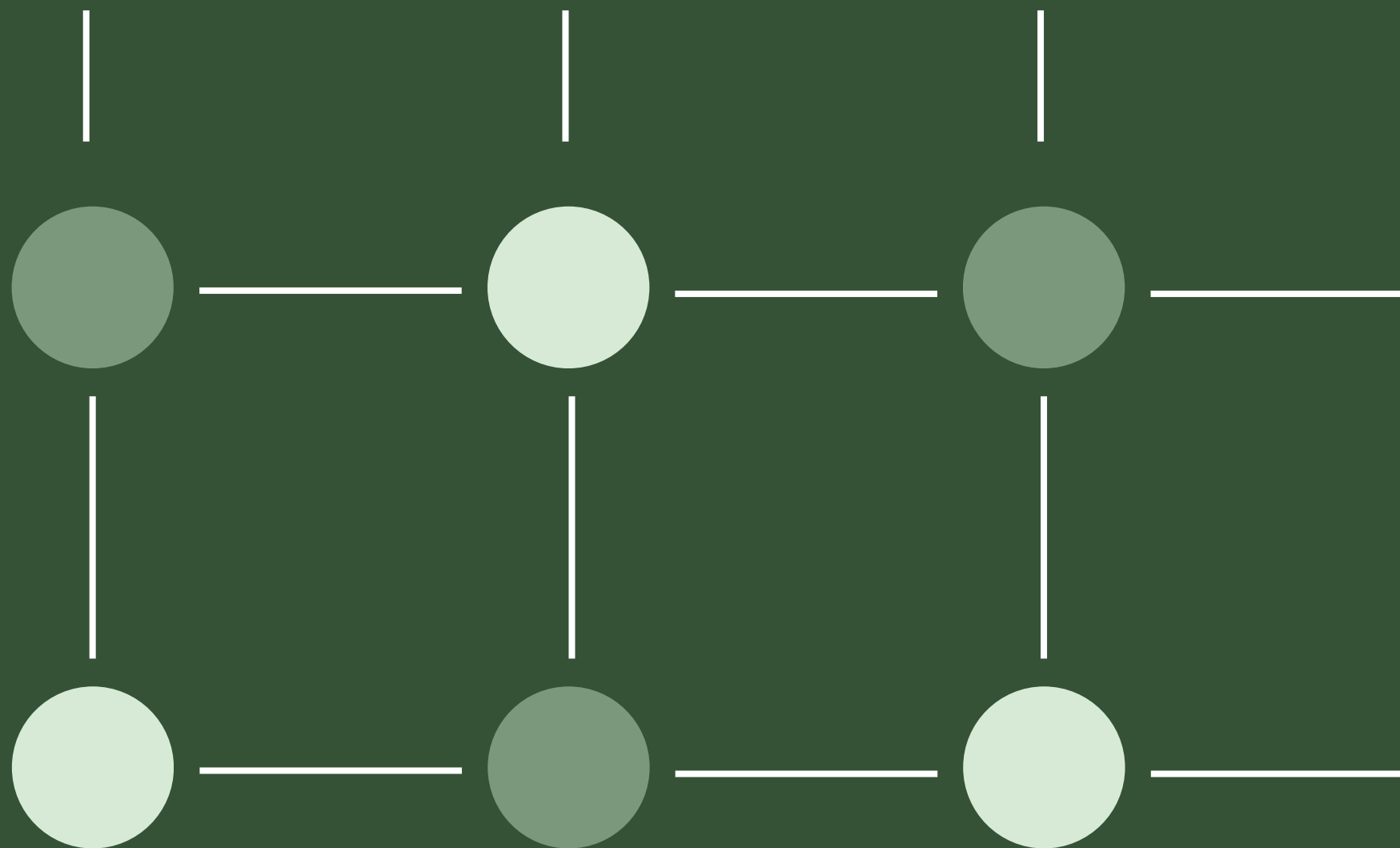


QUANTINUUM

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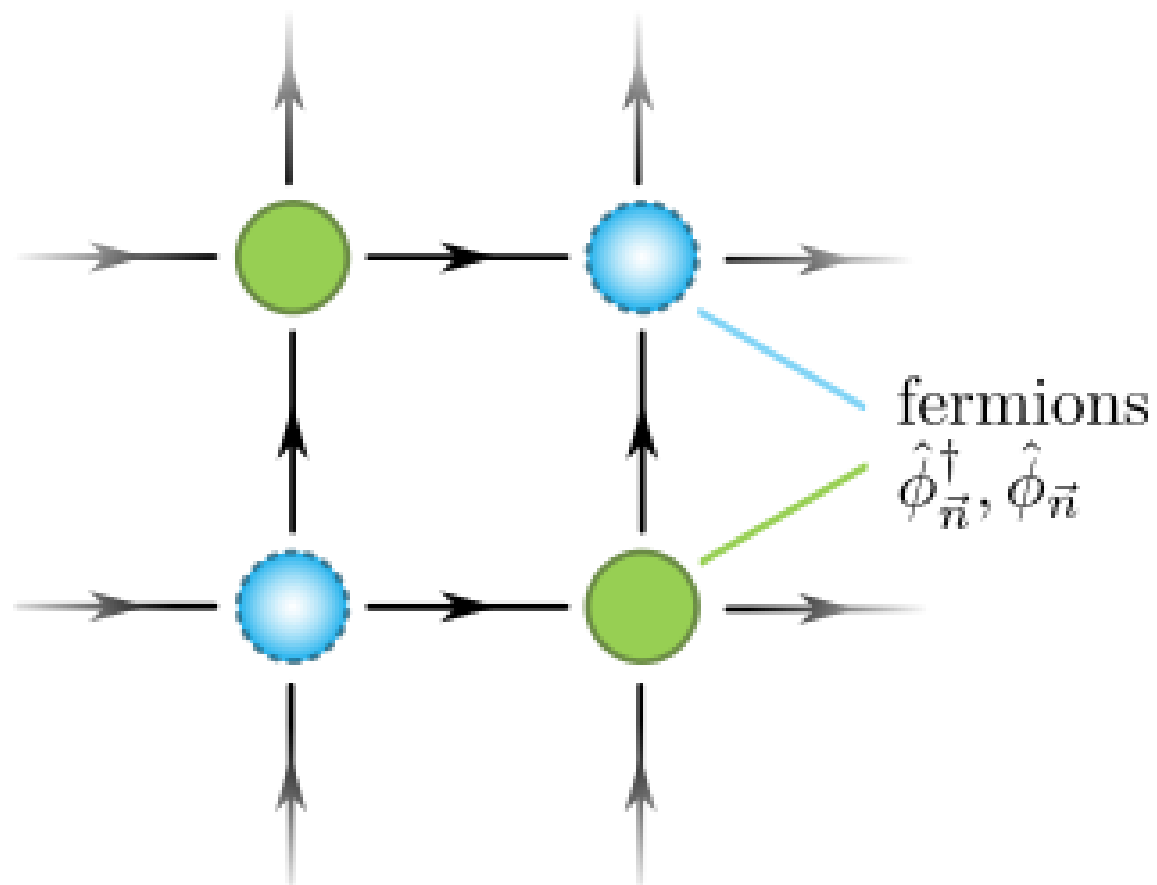
- 2+1D QED on the lattice
- Quantum computing methods
- Electric flux configurations of the static potential
 - Static potential
 - Quantum hardwares and circuits
 - Results
- Conclusions

2+1D QED on the lattice



Lattice Hamiltonian

Lattice Hamiltonian

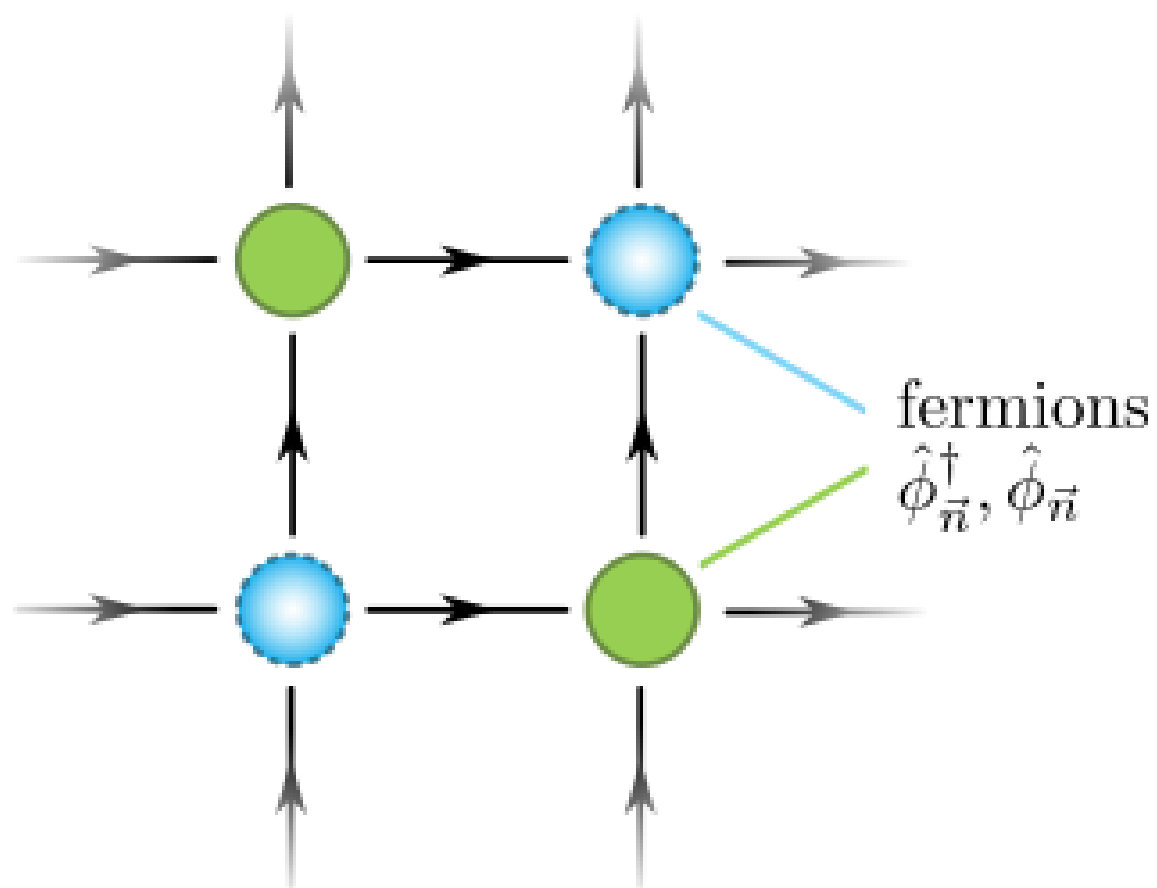


FERMIONIC HAMILTONIAN

$$\hat{H}_m = m \sum_{\vec{n}} (-1)^{n_x + n_y} \hat{\phi}_{\vec{n}}^\dagger \hat{\phi}_{\vec{n}}$$

$$\begin{aligned} \hat{H}_{kin} = & \frac{i}{2} \sum_{\vec{n}} (\phi_{\vec{n}}^\dagger \hat{U}_{\vec{n},x} \phi_{\vec{n}+x} - h.c.) \\ & - \frac{(-1)^{n_x + n_y}}{2} \sum_{\vec{n}} (\phi_{\vec{n}}^\dagger \hat{U}_{\vec{n},y} \phi_{\vec{n}+y} + h.c.) \end{aligned}$$

Lattice Hamiltonian



FERMIONIC HAMILTONIAN

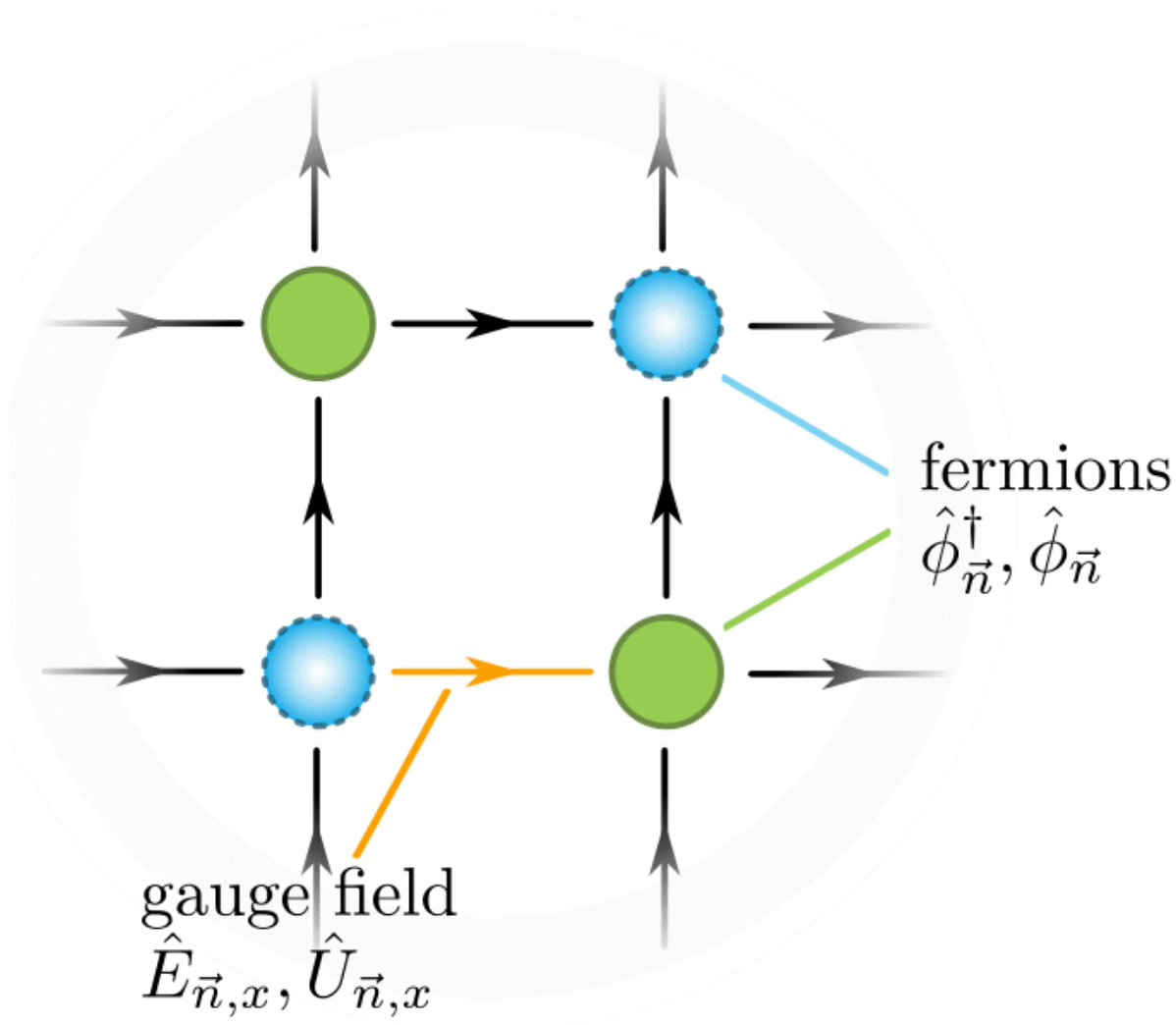
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Lattice Hamiltonian



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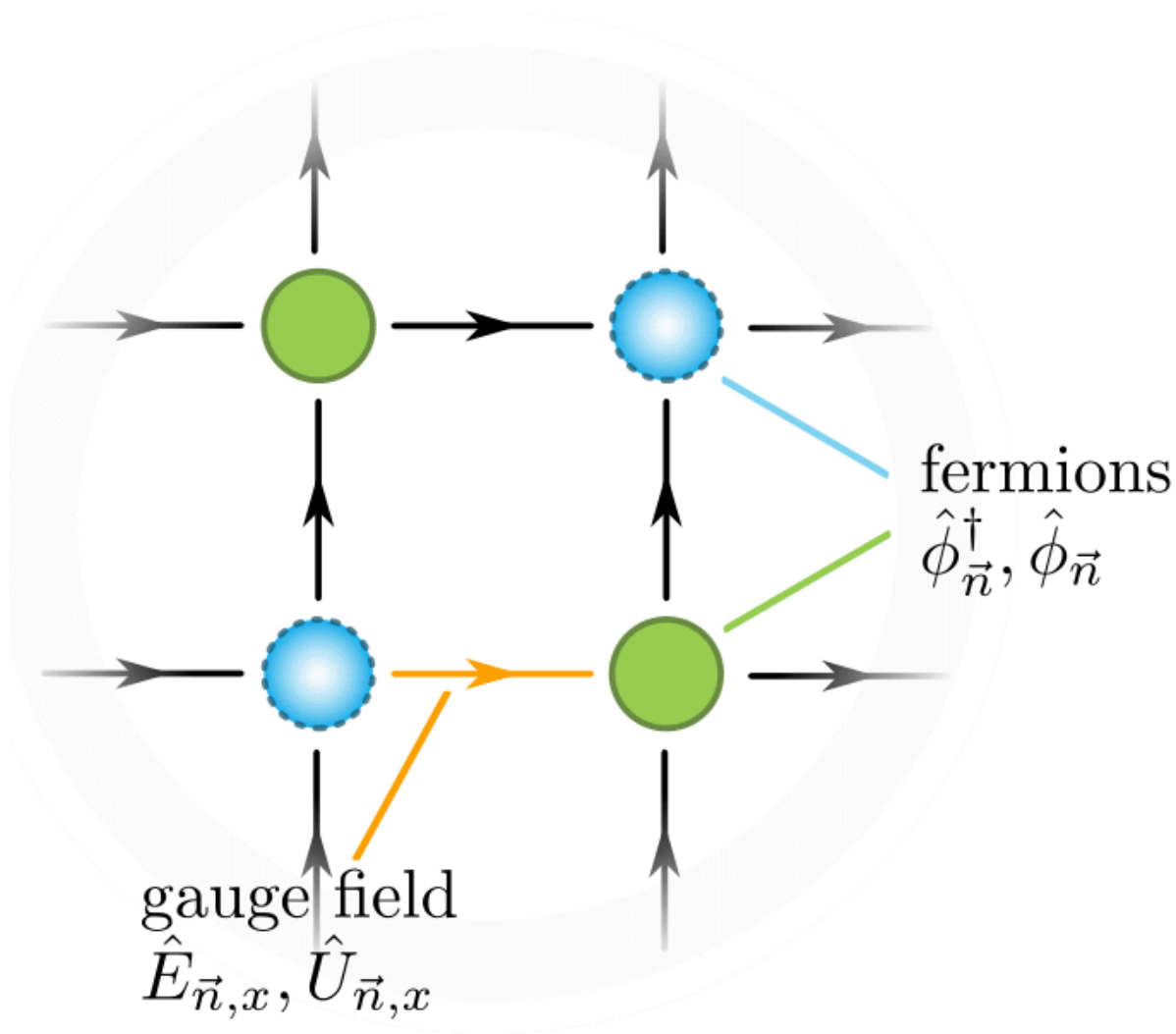


GAUGE HAMILTONIAN

$$\hat{H}_E = \frac{g^2}{2} \sum_{\vec{n}} (\hat{E}_{\vec{n},x}^2 + \hat{E}_{\vec{n},y}^2)$$

$$\hat{H}_B = -\frac{1}{2g^2} \sum_{\vec{n}} (\hat{P}_{\vec{n}} + \hat{P}_{\vec{n}}^\dagger)$$

Lattice Hamiltonian



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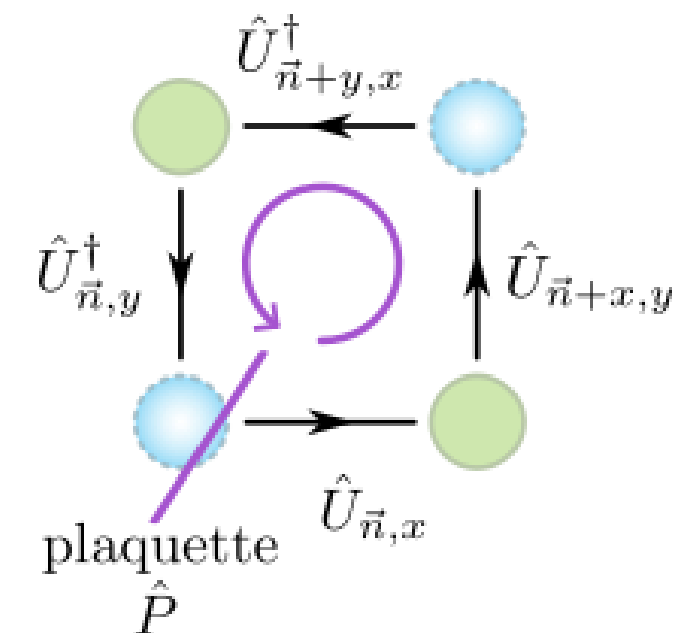
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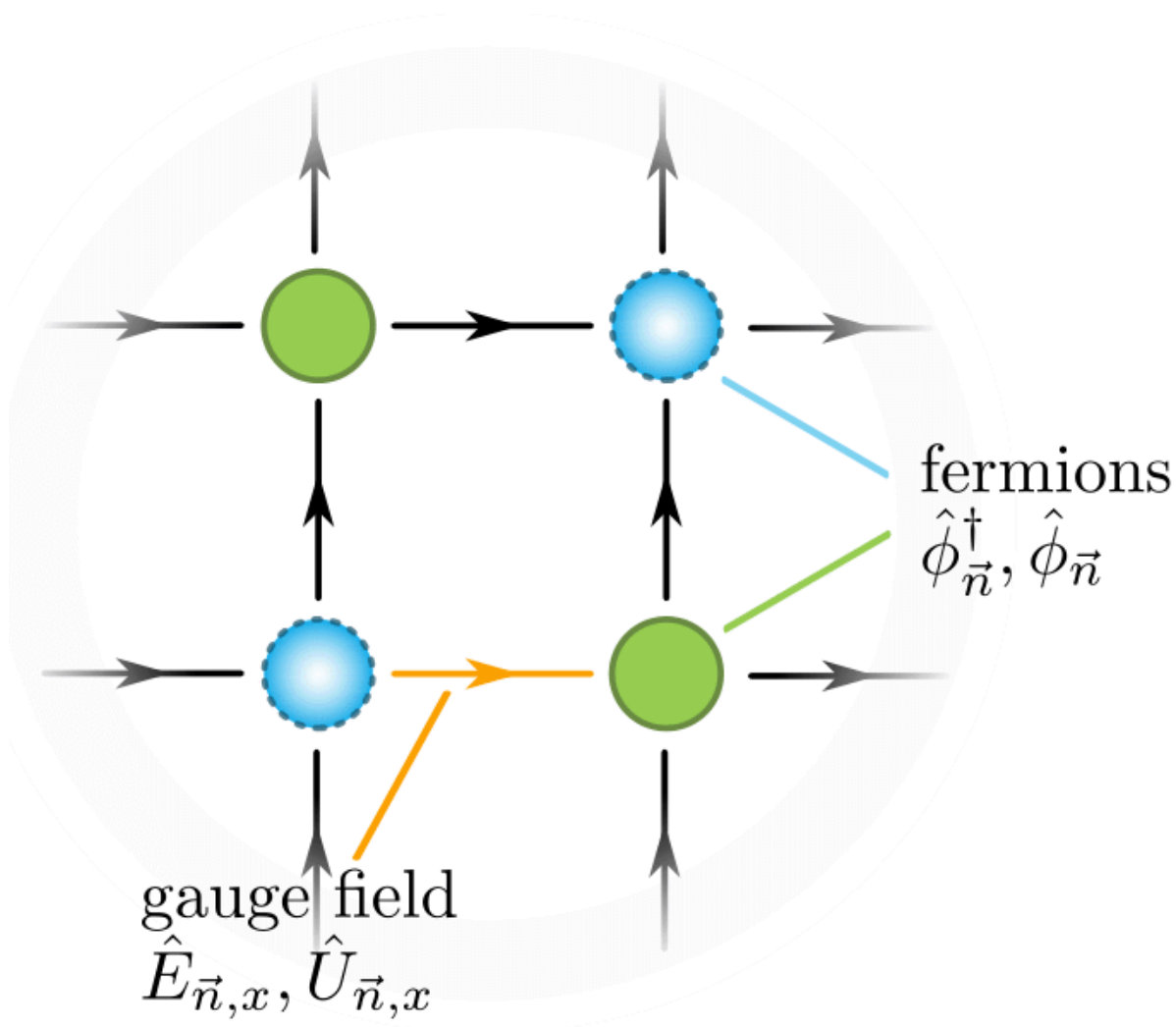
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Lattice Hamiltonian



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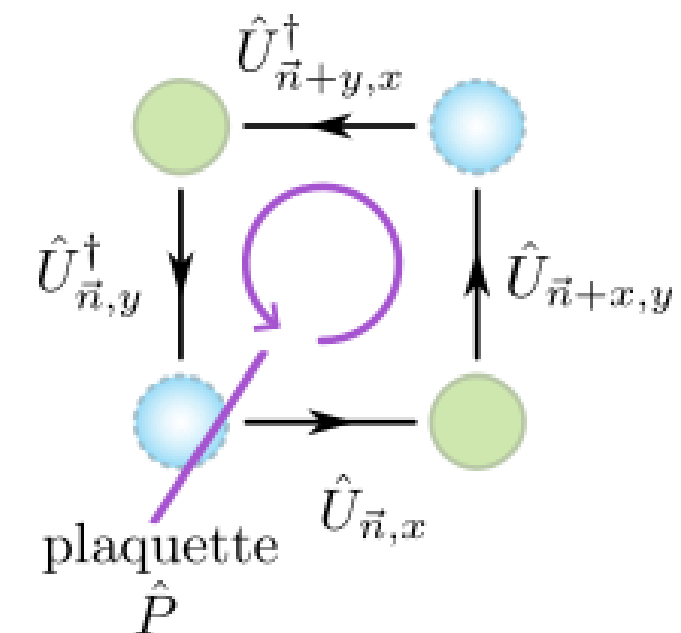
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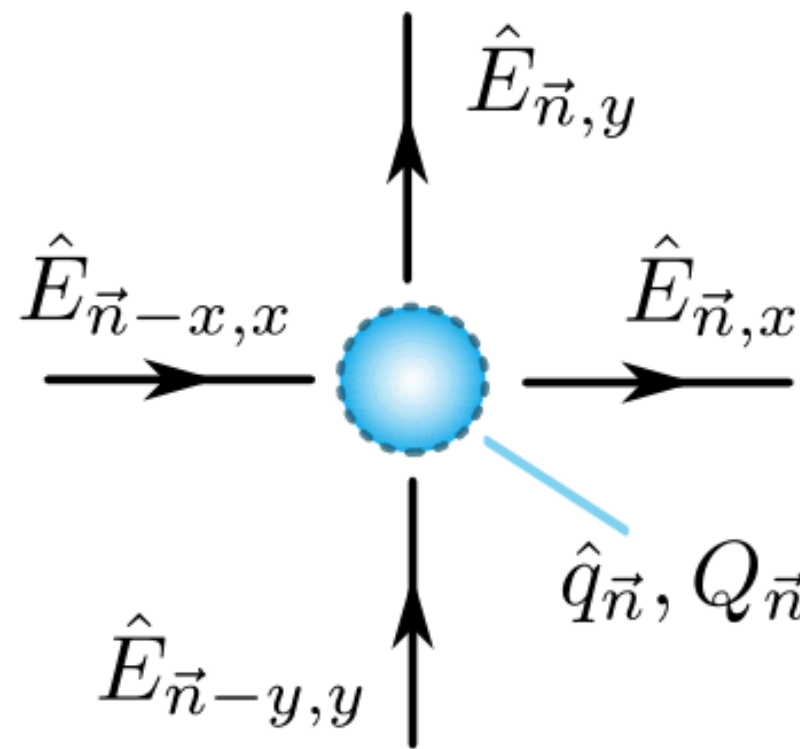
$$\hat{H}_B = -\frac{1}{2g^2} \sum_{\vec{n}} (\hat{P}_{\vec{n}} + \hat{P}_{\vec{n}}^\dagger)$$



For simplicity $a=1$.

Lattice Hamiltonian

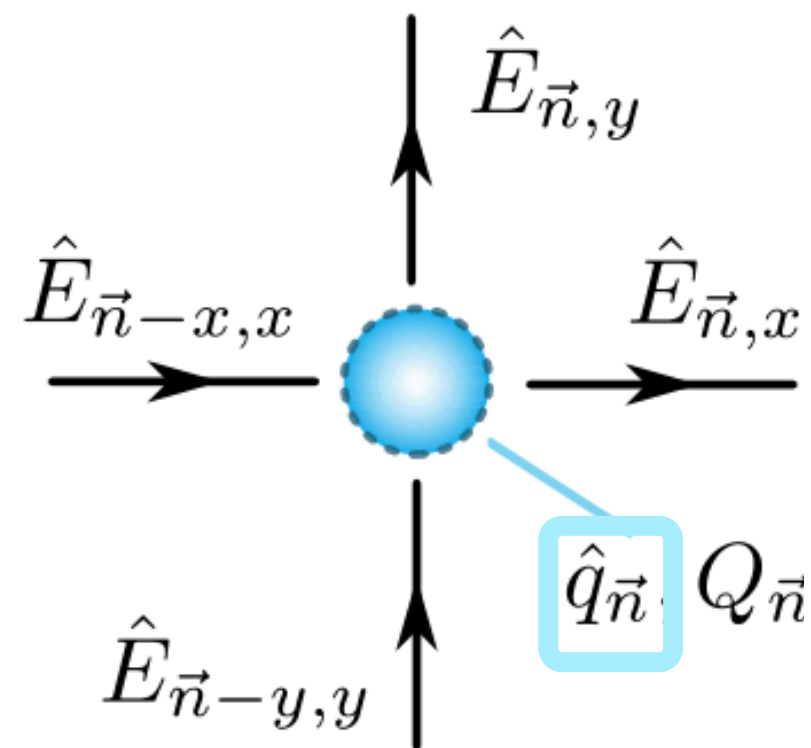
GAUSS' S LAW



$$\left[\sum_{\mu=x,y} \left(\hat{E}_{\vec{n},\mu} - \hat{E}_{\vec{n}-\mu,\mu} \right) - \hat{q}_{\vec{n}} - Q_{\vec{n}} \right] |\Phi\rangle = 0 \iff |\Phi\rangle \in \mathcal{H}_{\text{phys.}}$$

Lattice Hamiltonian

GAUSS' S LAW

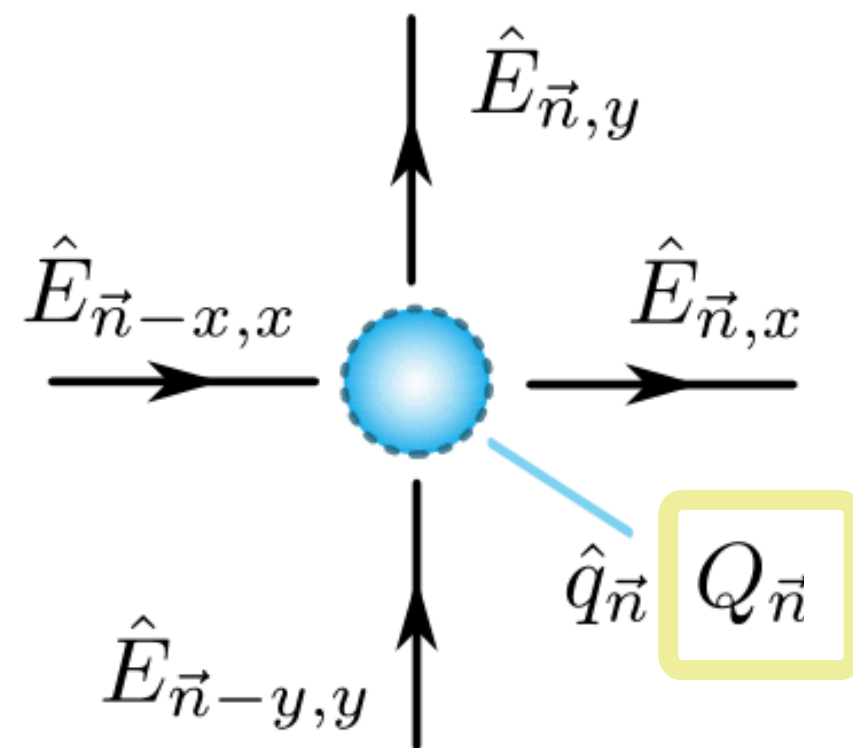


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DYNAMICAL CHARGES

Lattice Hamiltonian

GAUSS'S LAW



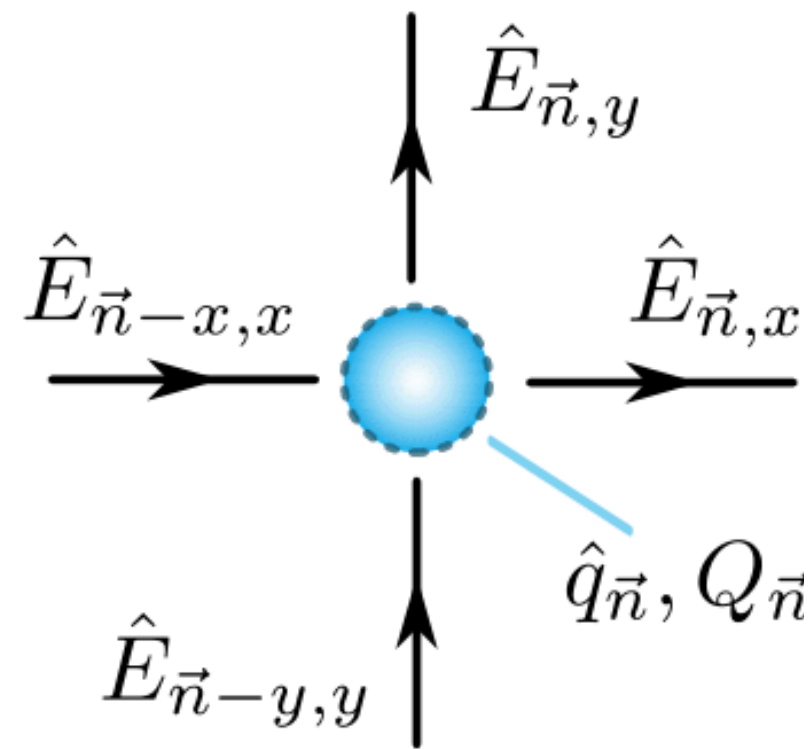
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STATIC CHARGES

DYNAMICAL CHARGES

Lattice Hamiltonian

GAUSS' S LAW

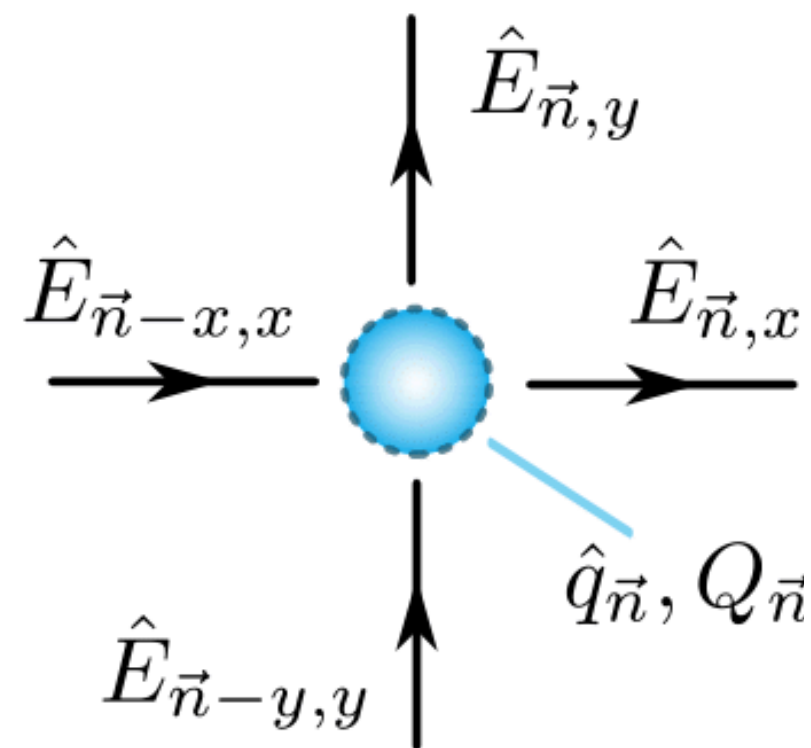


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Solve system of equations

Lattice Hamiltonian

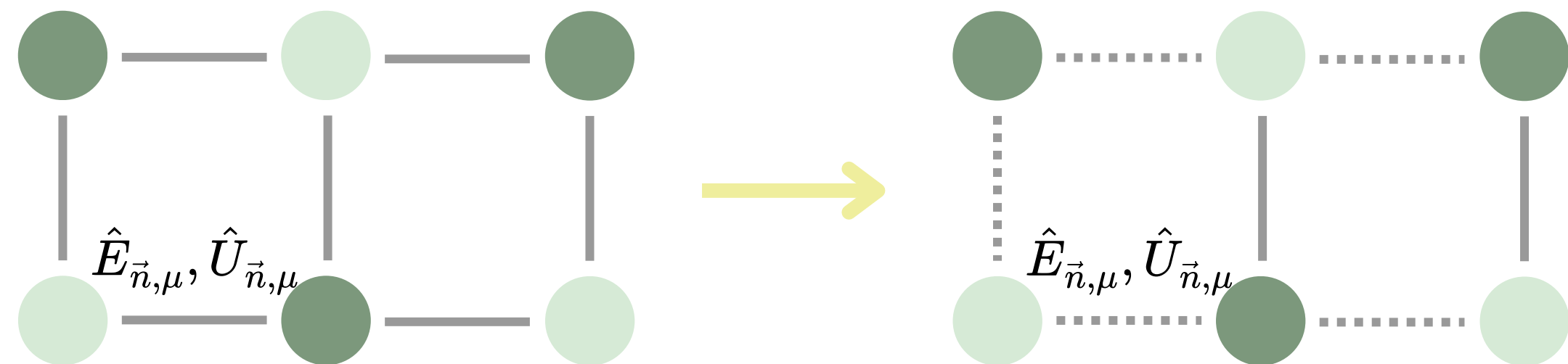
GAUSS' S LAW



$$\left[\sum_{\mu=x,y} \left(\hat{E}_{\vec{n},\mu} - \hat{E}_{\vec{n}-\mu,\mu} \right) - \hat{q}_{\vec{n}} - Q_{\vec{n}} \right] |\Phi\rangle = 0 \iff |\Phi\rangle \in \mathcal{H}_{\text{phys.}}$$

Solve system of equations

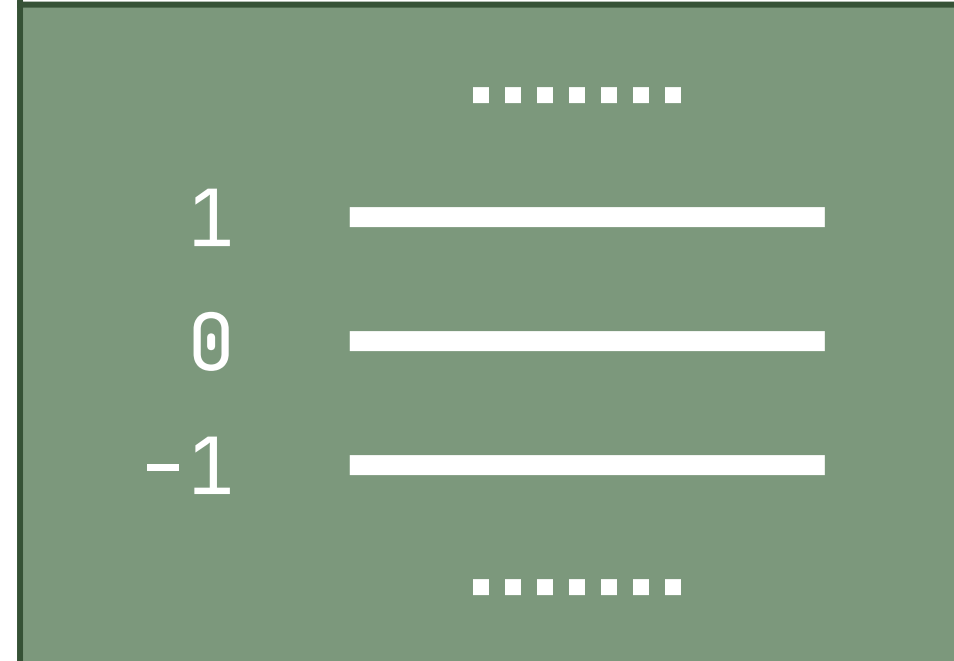
Subset of dynamical links



Truncation of gauge fields

Truncation of gauge fields

Compact U(1) group.

$$\hat{E}_{\vec{n},\mu} |e_{\vec{n},\mu}\rangle = e_{\vec{n},\mu} |e_{\vec{n},\mu}\rangle$$
$$e_{\vec{n},\mu} \in [-l, l]$$


The diagram illustrates the truncation of gauge field states. It features a green rectangular box containing three horizontal white bars. To the left of these bars are the integers 1, 0, and -1, representing the eigenvalues of the electric field operator. Above and below the bars are sets of six dots, indicating a finite range of states.

Truncation of gauge fields

Compact U(1) group.

$\hat{E}_{\vec{n},\mu} e_{\vec{n},\mu}\rangle = e_{\vec{n},\mu} e_{\vec{n},\mu}\rangle$ $e_{\vec{n},\mu} \in [-l, l]$	$\hat{U}_{\vec{n},\mu} e_{\vec{n},\mu}\rangle = e_{\vec{n},\mu} + 1\rangle$
<div>..... 1 _____ 0 _____ -1 _____</div>	<div>..... 1 _____ 0 _____ -1 _____</div> <div>↻ ↻ ↻</div>

$$[\hat{E}_{\vec{n},\mu}, \hat{U}_{\vec{m},\nu}] = \delta_{\vec{n},\vec{m}} \delta_{\mu,\nu} \hat{U}_{\vec{m},\nu}$$

Truncation of gauge fields

Compact U(1) group.

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<div> <div>.....</div> <div> <div>1</div> <div>0</div> <div>-1</div> </div> <div>.....</div> </div>	<div> <div>.....</div> <div> <div>1</div> <div>0</div> <div>-1</div> </div> <div>.....</div> </div>	<div> <div>.....</div> <div> <div>1</div> <div>0</div> <div>-1</div> </div> <div>.....</div> </div>

$$[\hat{E}_{\vec{n},\mu}, \hat{U}_{\vec{m},\nu}] = \delta_{\vec{n},\vec{m}}\delta_{\mu,\nu}\hat{U}_{\vec{m},\nu} \qquad [\hat{E}_{\vec{n},\mu}, \hat{U}_{\vec{m},\nu}^\dagger] = -\delta_{\vec{n},\vec{m}}\delta_{\mu,\nu}\hat{U}_{\vec{m},\nu}^\dagger$$

Quantum Computing methods




Quantum circuits for gauge fields

Example of truncation $l=1$:



Quantum circuits for gauge fields

Example of truncation $\ell=1$:

1 
0 
-1 

$$|-1\rangle_{\text{ph.}} \mapsto |00\rangle$$

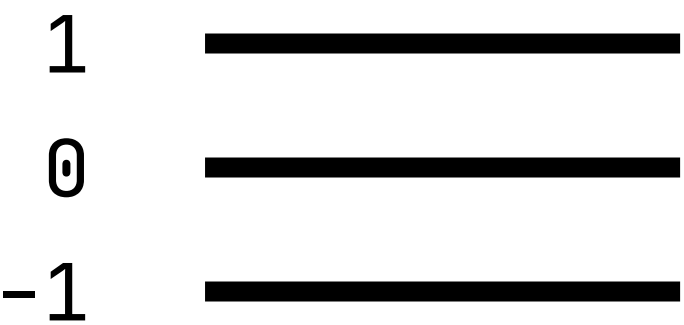
$$|0\rangle_{\text{ph.}} \mapsto |01\rangle$$

$$|1\rangle_{\text{ph.}} \mapsto |11\rangle$$

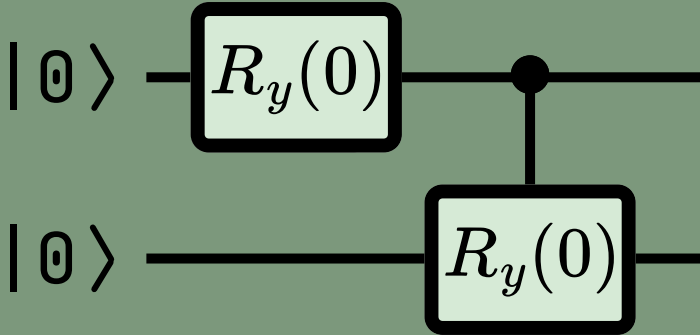
$$|10\rangle$$

Quantum circuits for gauge fields

Example of truncation $\mathfrak{l}=1$:



$$|-1\rangle_{\text{ph.}} \mapsto |00\rangle$$



$$|0\rangle_{\text{ph.}} \mapsto |01\rangle$$

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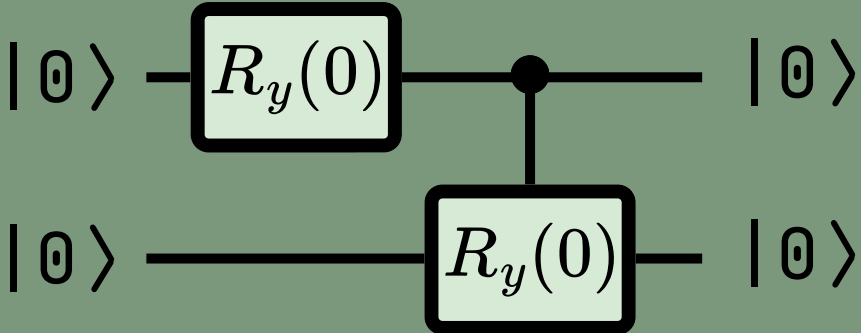
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Quantum circuits for gauge fields

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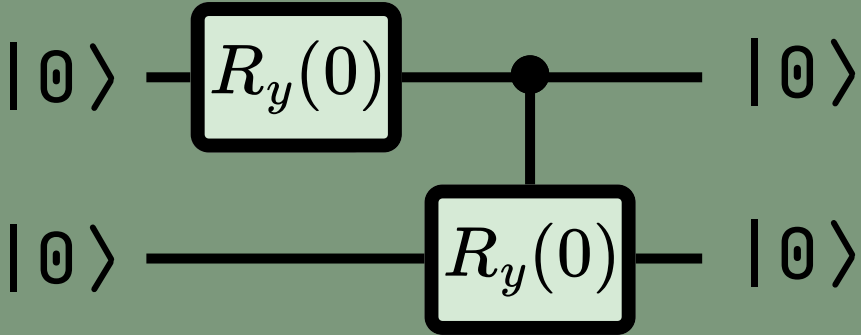
$$|10\rangle$$

Quantum circuits for gauge fields

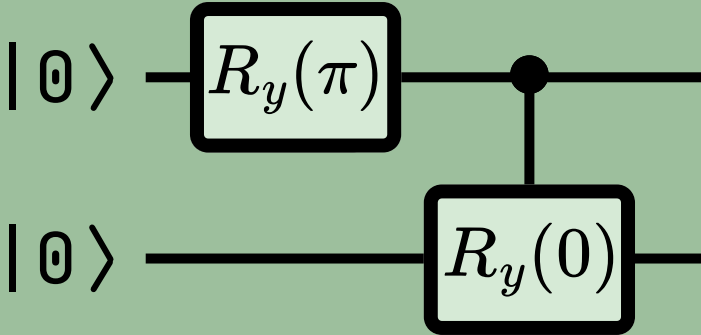
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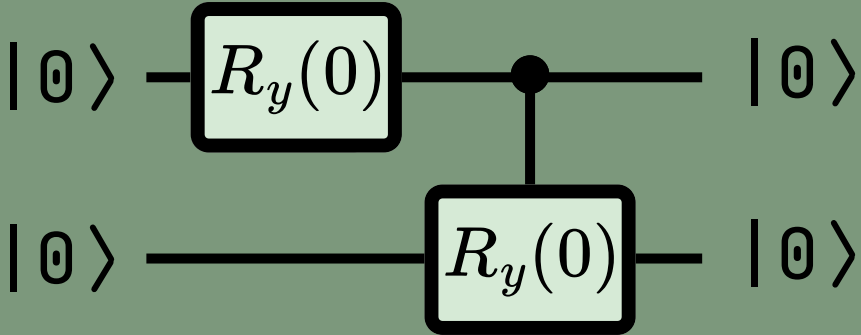
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Quantum circuits for gauge fields

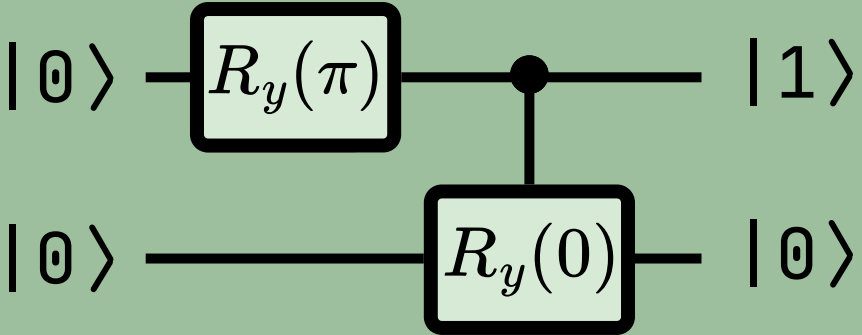
Example of truncation $\mathfrak{l}=1$:



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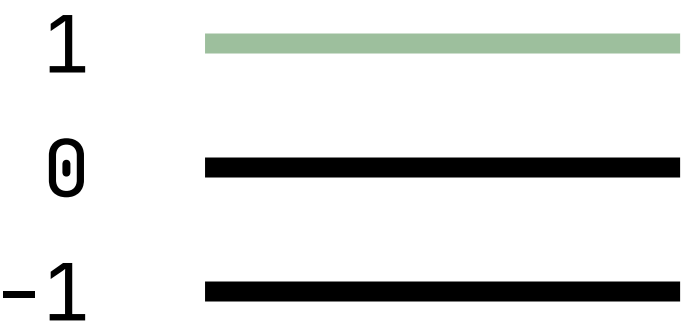


$$|1\rangle_{\text{ph.}} \mapsto |11\rangle$$

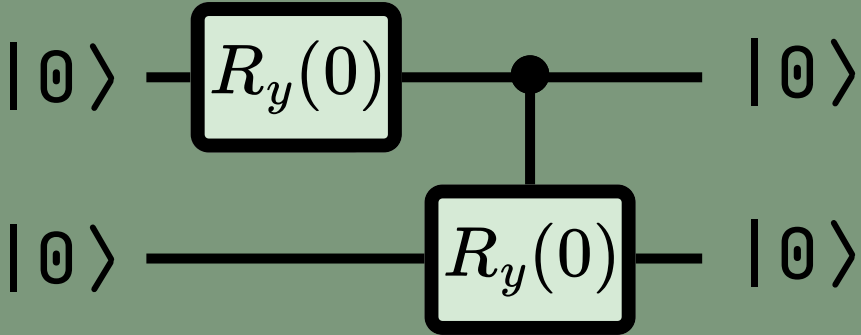
$$|10\rangle$$

Quantum circuits for gauge fields

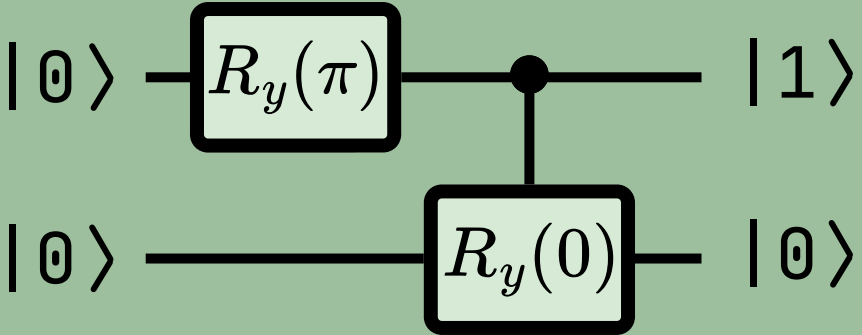
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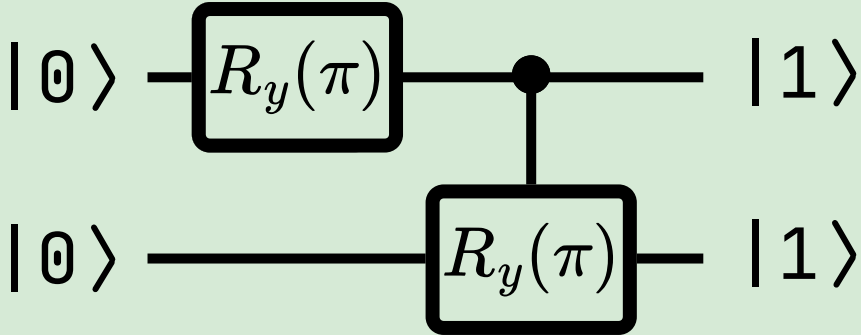
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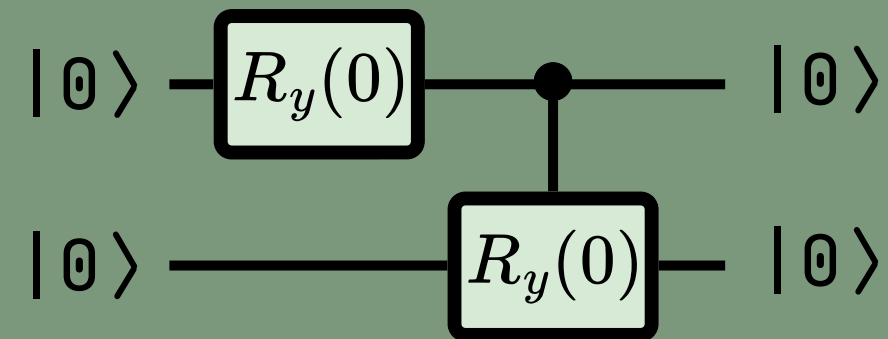
$$|10\rangle$$

Quantum circuits for gauge fields

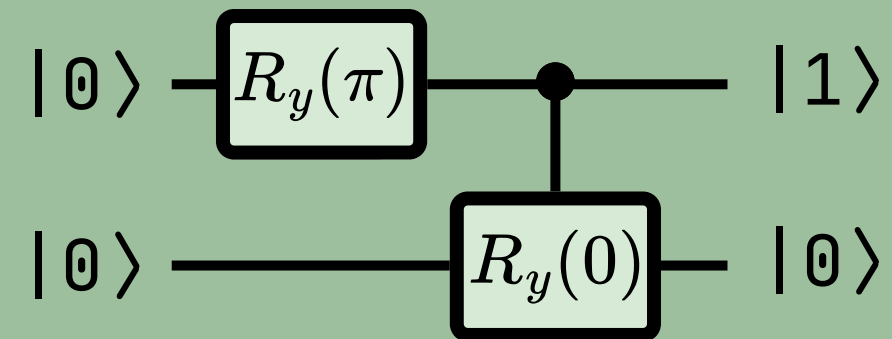
Example of truncation $\mathfrak{l}=1$:

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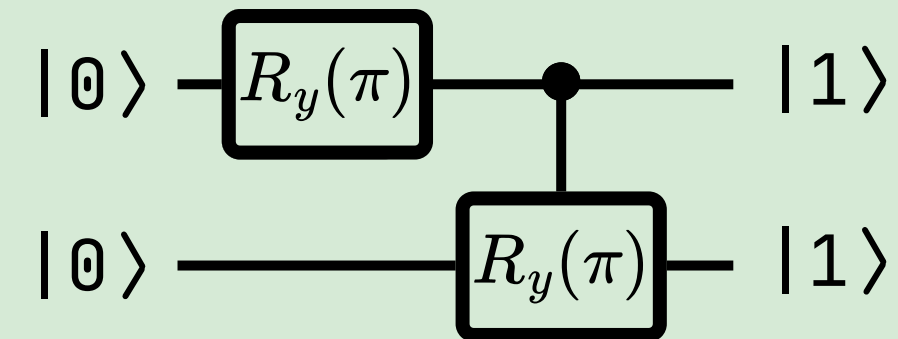
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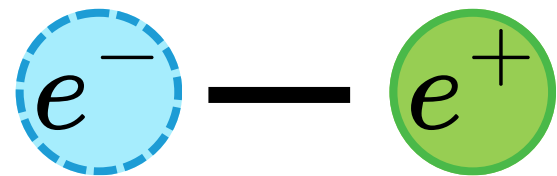
Quantum circuits for fermions

Quantum circuits for fermions

Zero total charge

EVEN

ODD



$|1\rangle$

$|0\rangle$



$|0\rangle$

$|1\rangle$

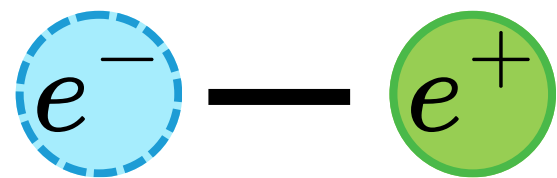
Quantum circuits for fermions

Zero total charge

Preserve parity

EVEN

ODD



$|1\rangle$

$|0\rangle$



$|0^n \dots 1^m\rangle$

$n = m$



$|0\rangle$

$|1\rangle$

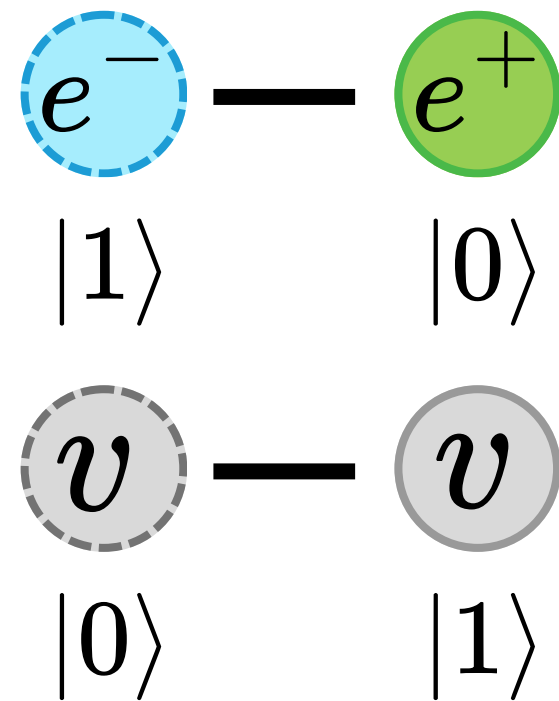
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EVEN

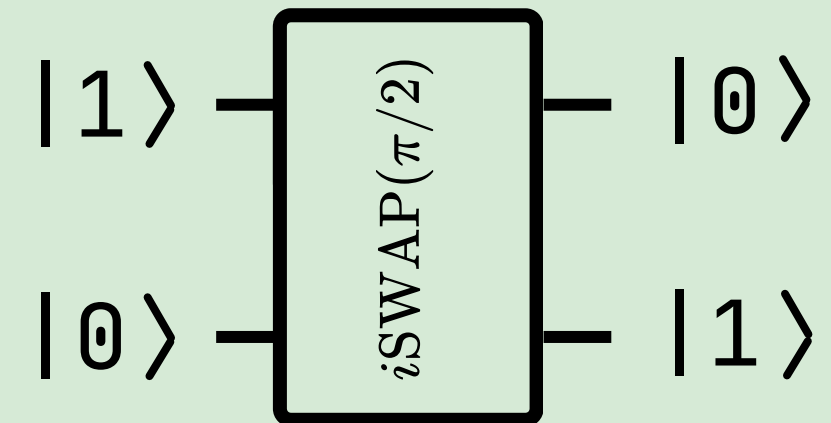
ODD



$$|0^n \dots 1^m\rangle$$

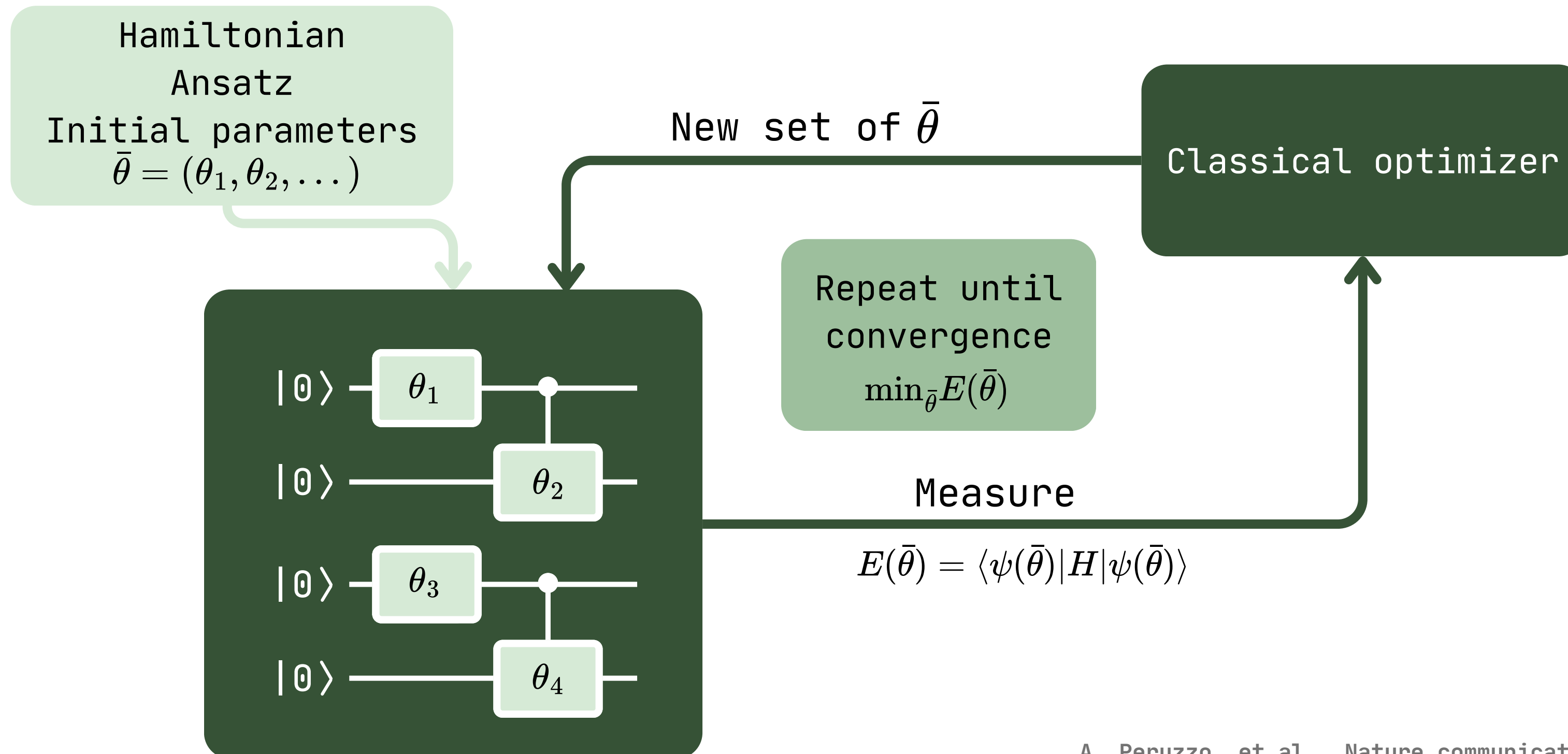
$$n = m$$

$$i\text{SWAP}_{j,k}(\theta) = e^{-i\frac{\theta}{4}(\sigma_j^x \sigma_k^x + \sigma_j^y \sigma_k^y)}$$



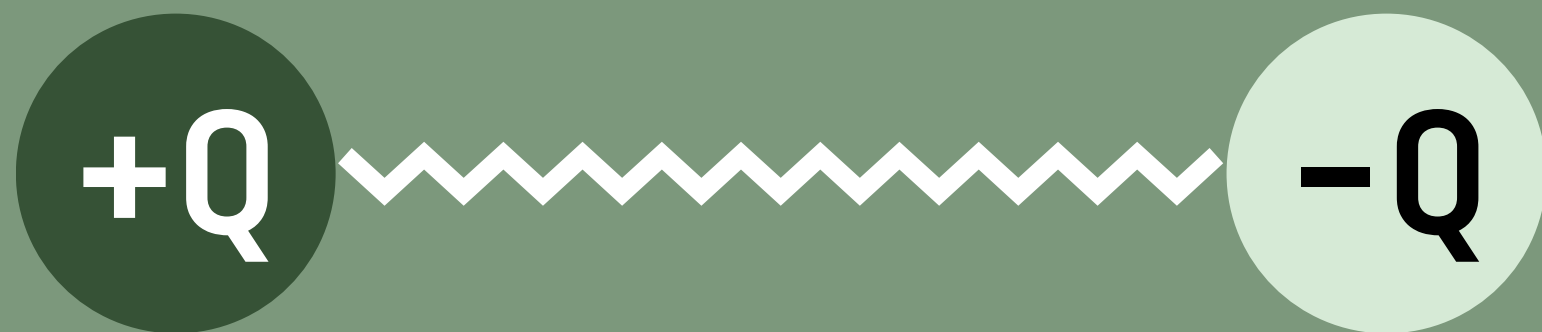
D. Paulson, et al., PRX Quantum 2,030334 (2021)

Variational quantum algorithm



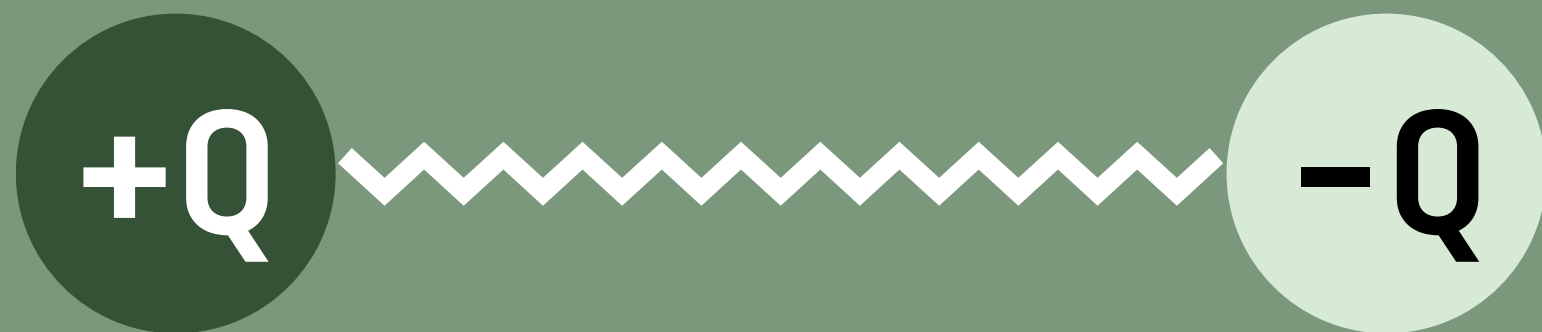
A. Peruzzo, et al., Nature communications 5, 4213 (2014)

Electric flux configurations of the static potential



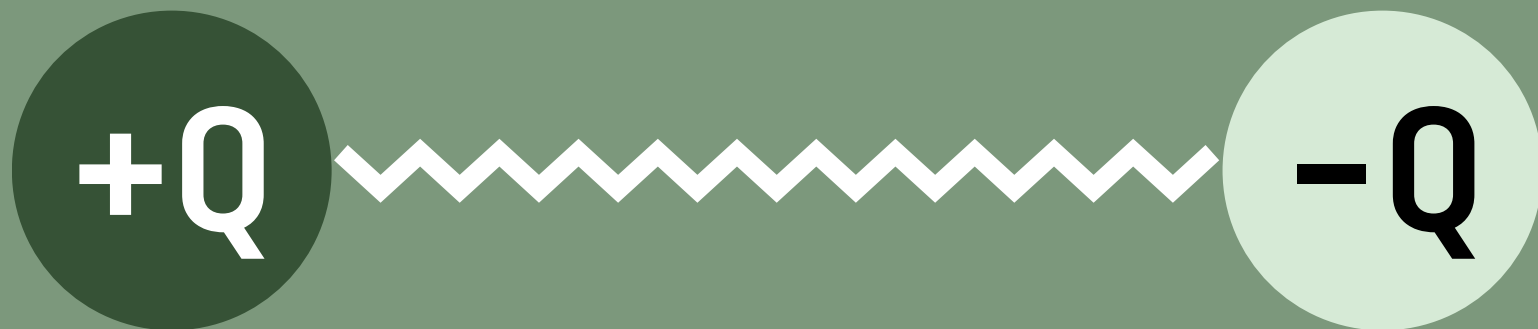
Electric flux configurations of the static potential

- Study confinement and string breaking phenomena.



Electric flux configurations of the static potential

- Study confinement and string breaking phenomena.
- Direct visualization of electric fluxes & probabilities of relevant states.



Static potential

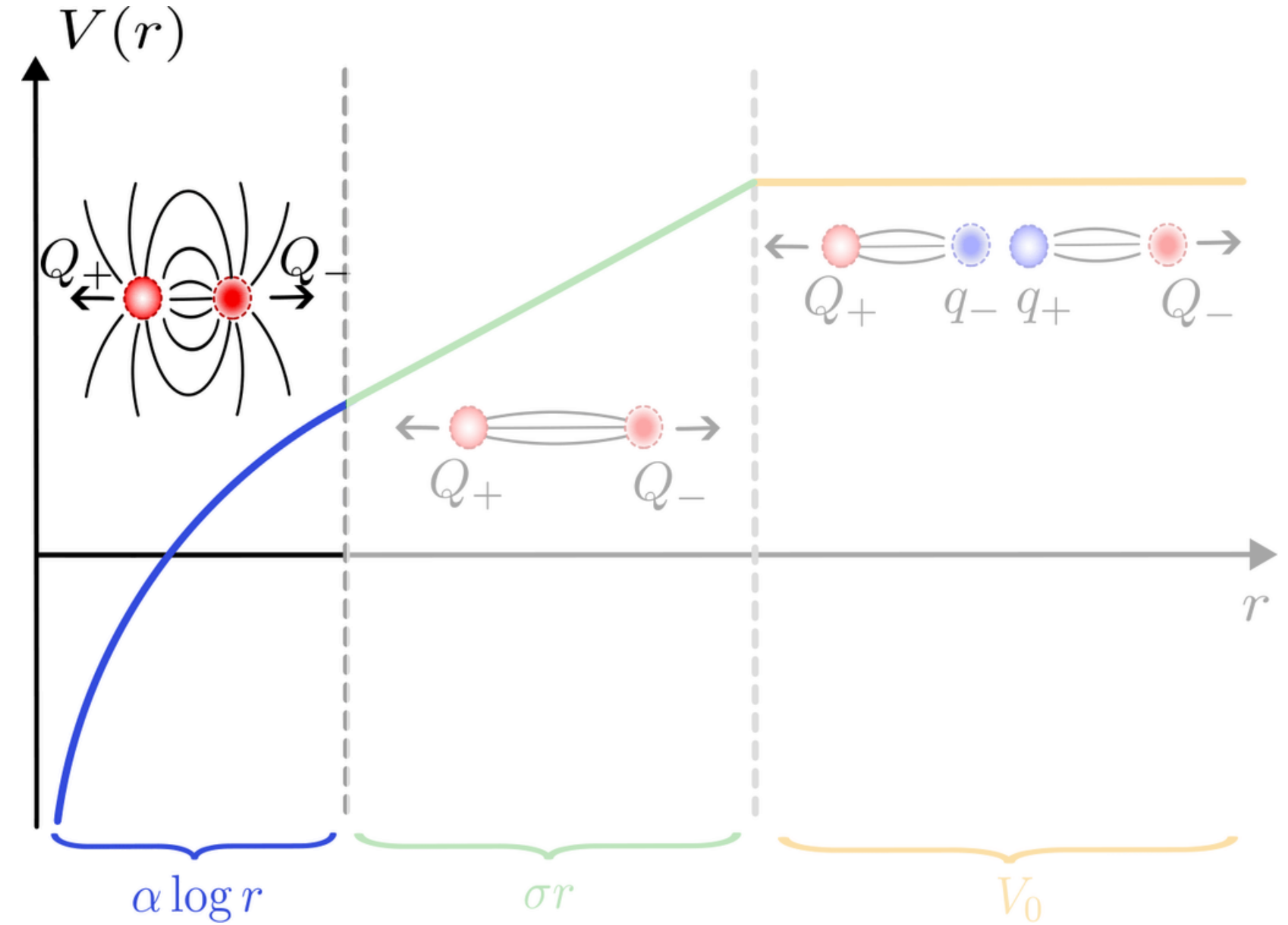
QED 2+1D

$$V(r) = \alpha \log r + \sigma r + V_0$$

Static potential

QED 2+1D

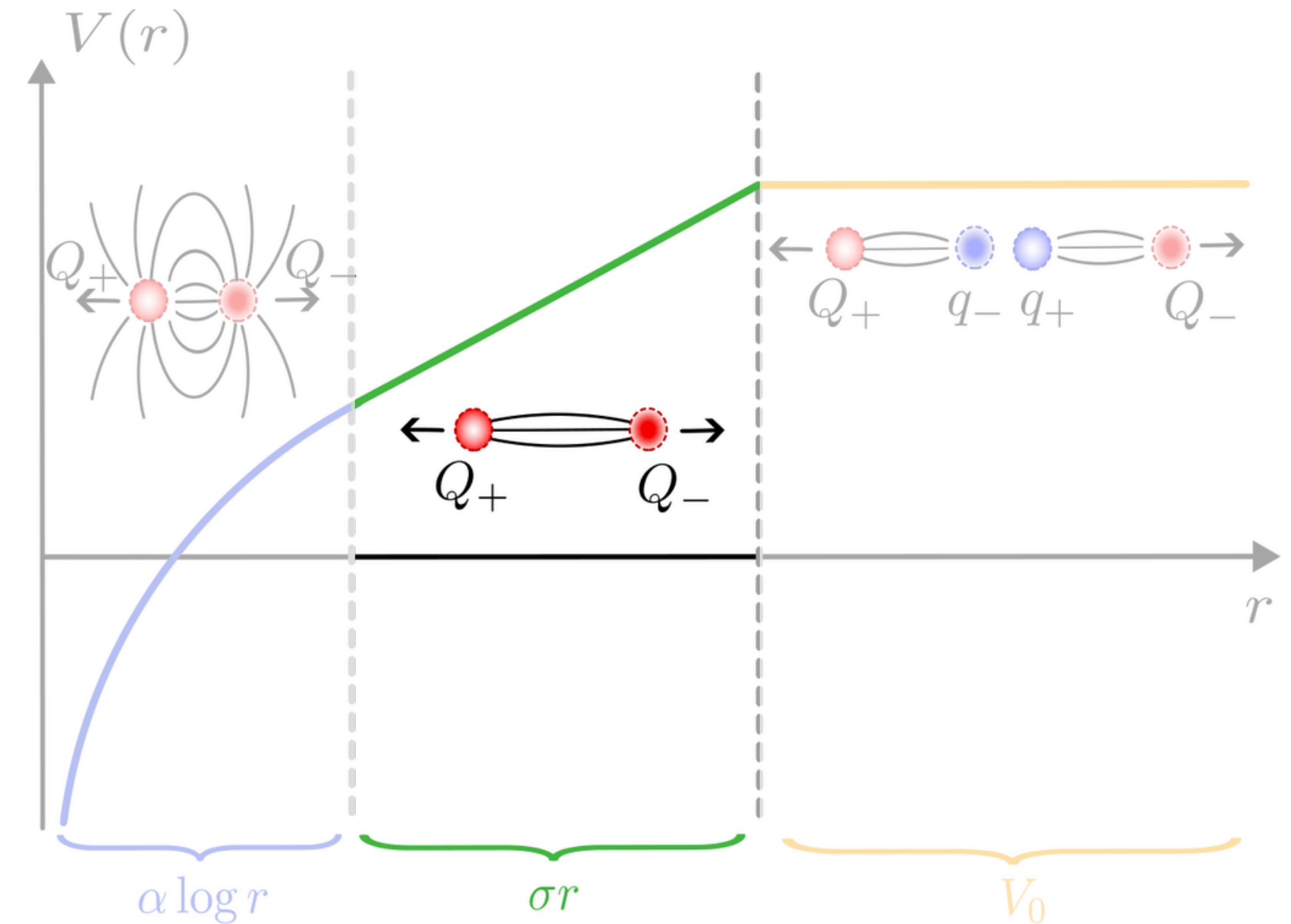
$$V(r) = \boxed{\alpha \log r} + \sigma r + V_0$$



Static potential

QED 2+1D

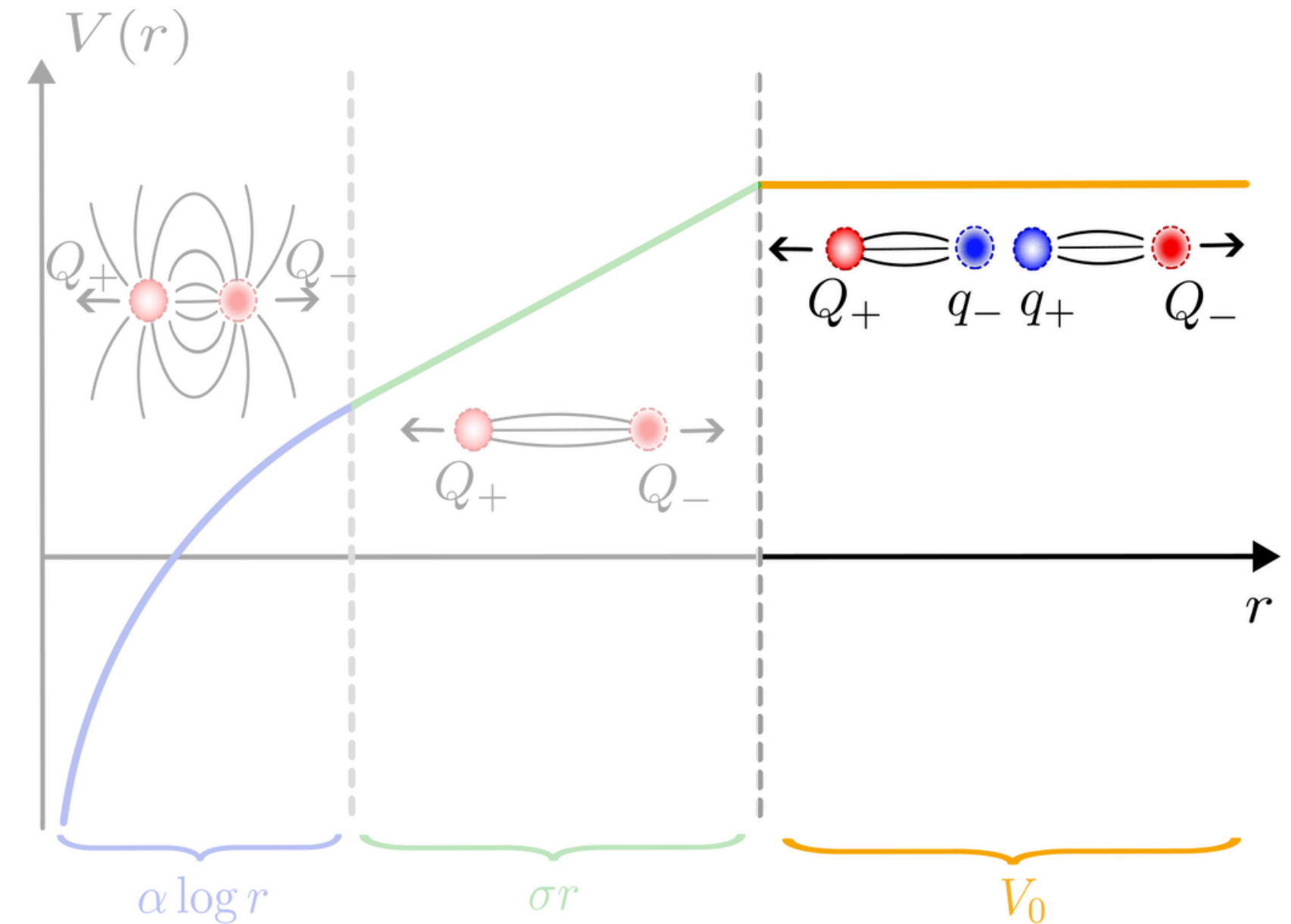
$$V(r) = \alpha \log r + \boxed{\sigma r} + V_0$$



Static potential

QED 2+1D

$$V(r) = \alpha \log r + \sigma r + \boxed{V_0}$$

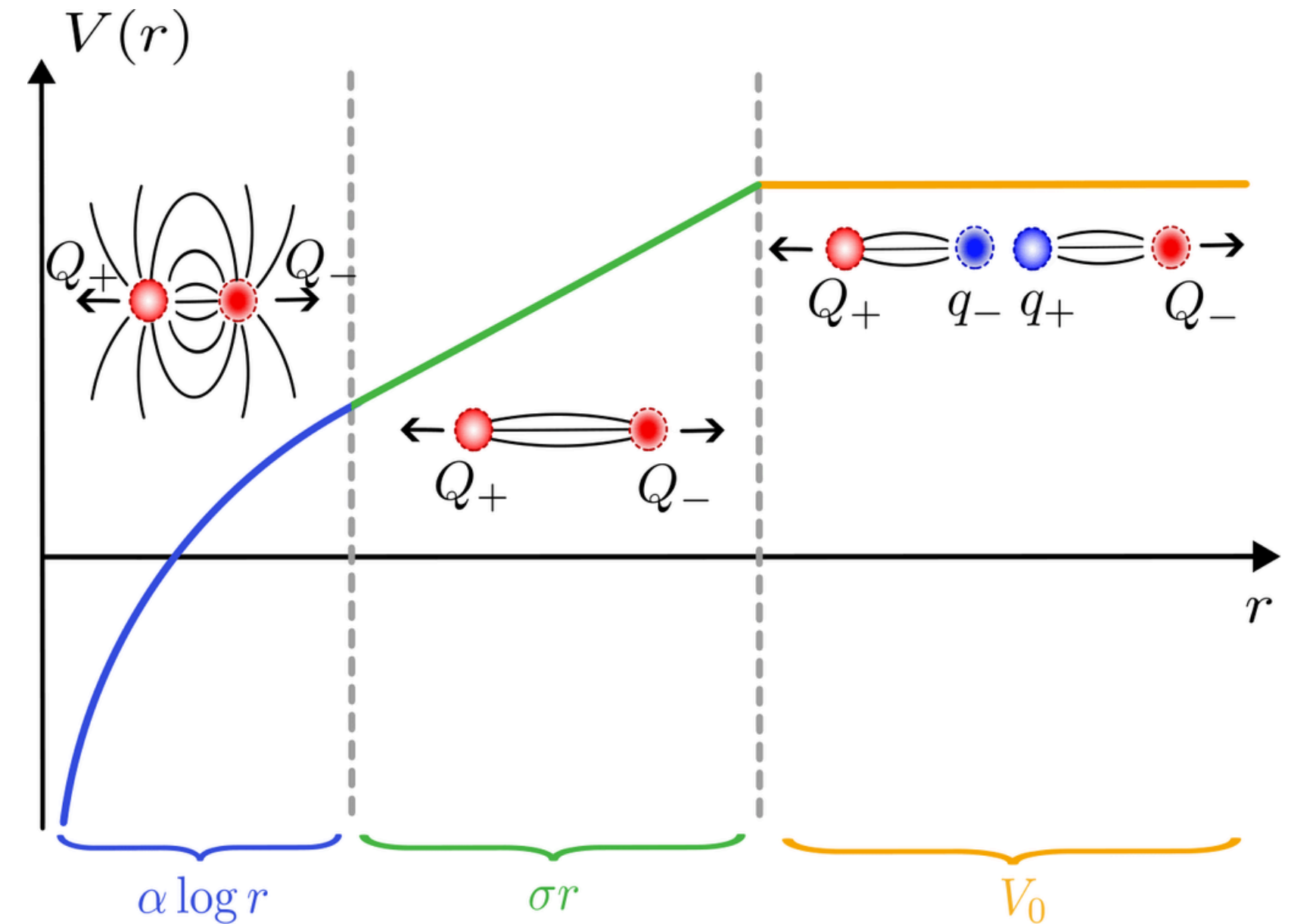


Static potential

QED 2+1D

$$V(r) = \alpha \log r + \sigma r + V_0$$

$$r = ar_{latt}$$



Static potential

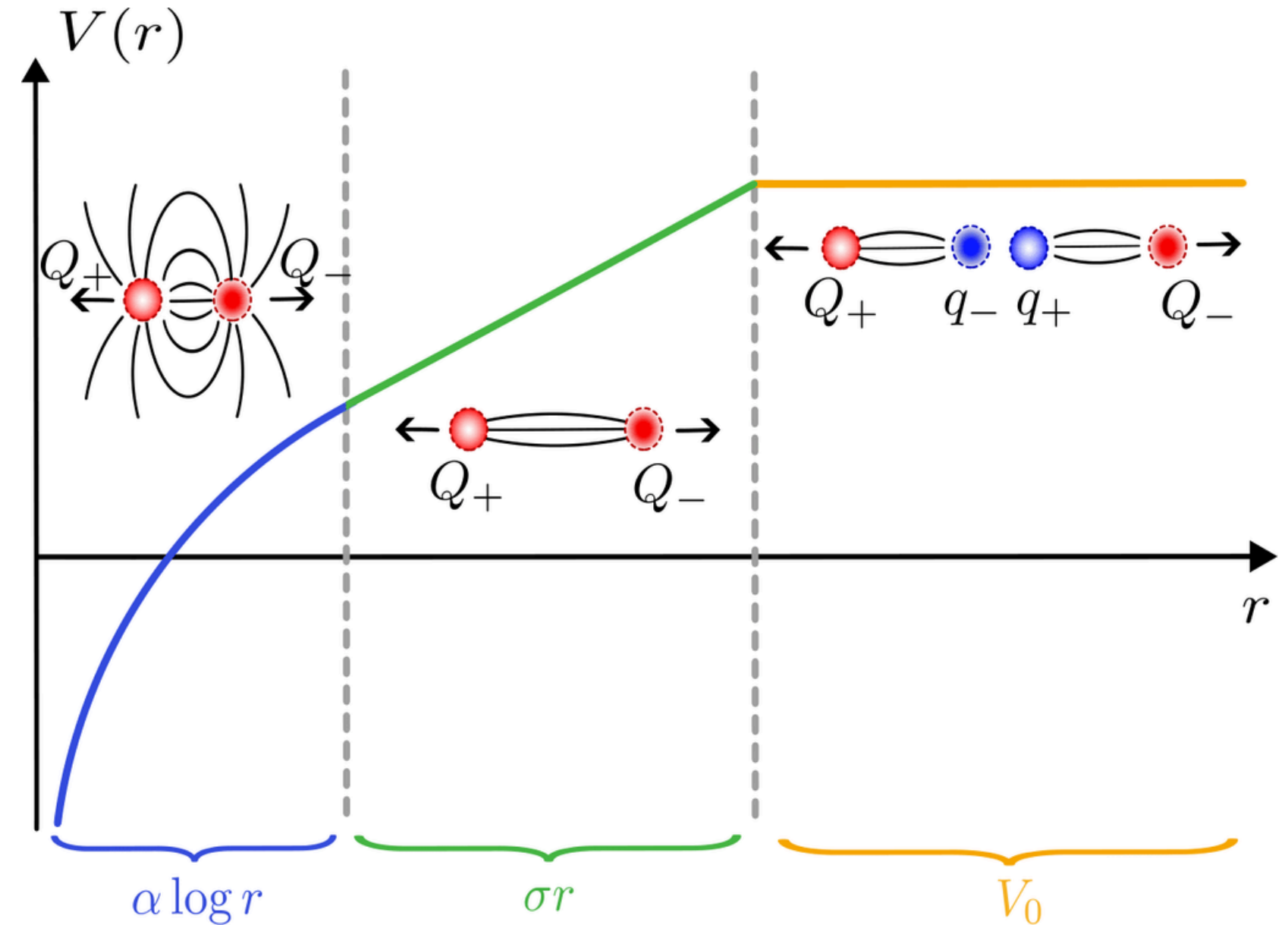
QED 2+1D

$$V(r) = \alpha \log r + \sigma r + V_0$$

$$r = a r_{latt}$$

\downarrow

$$g \mapsto g(a)$$



Static potential

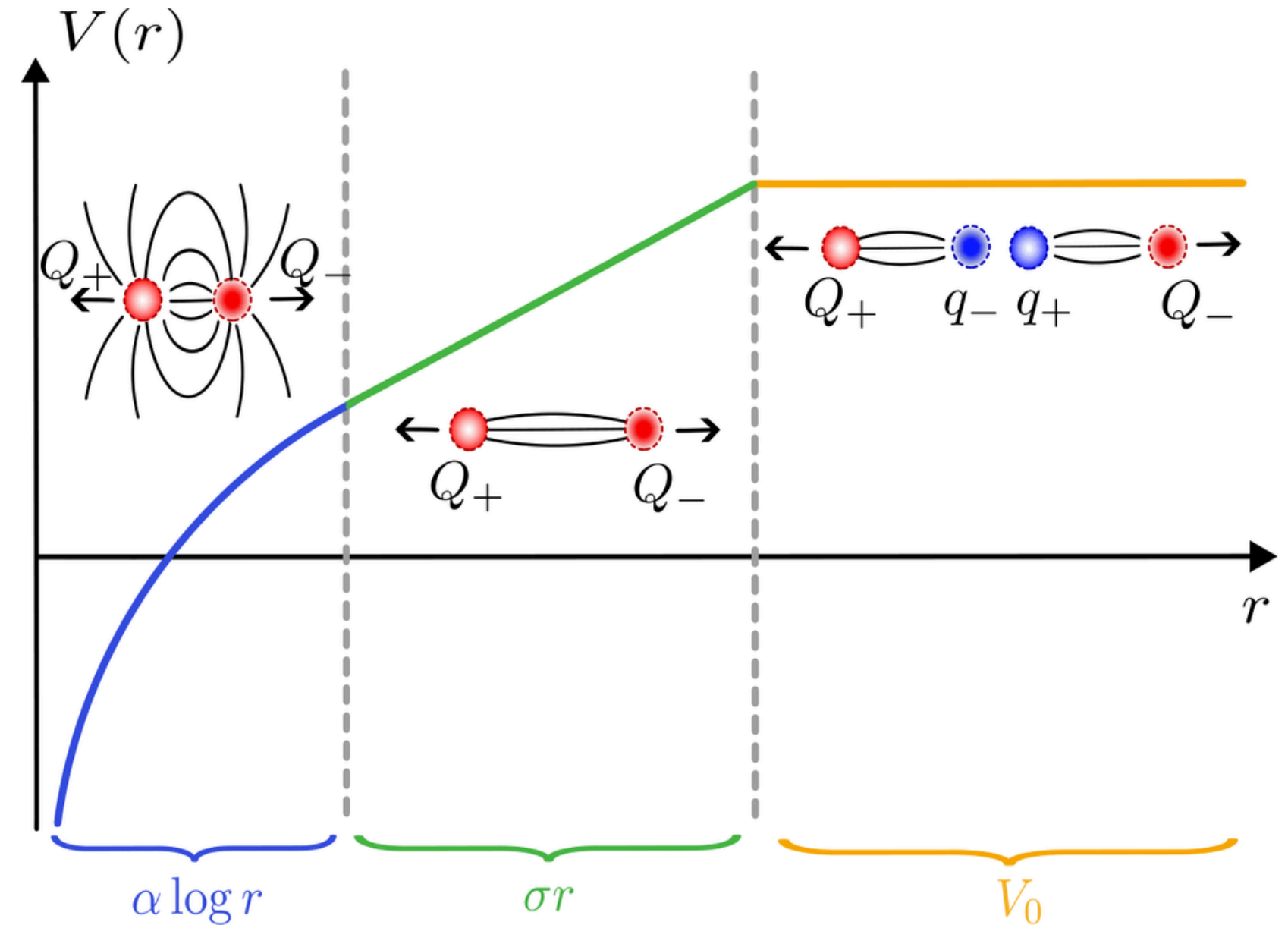
QED 2+1D

$$V(r) = \alpha \log r + \sigma r + V_0$$

$$r = a r_{latt}$$

$$g \mapsto g(a)$$

$$V(r) \rightarrow V(g)$$

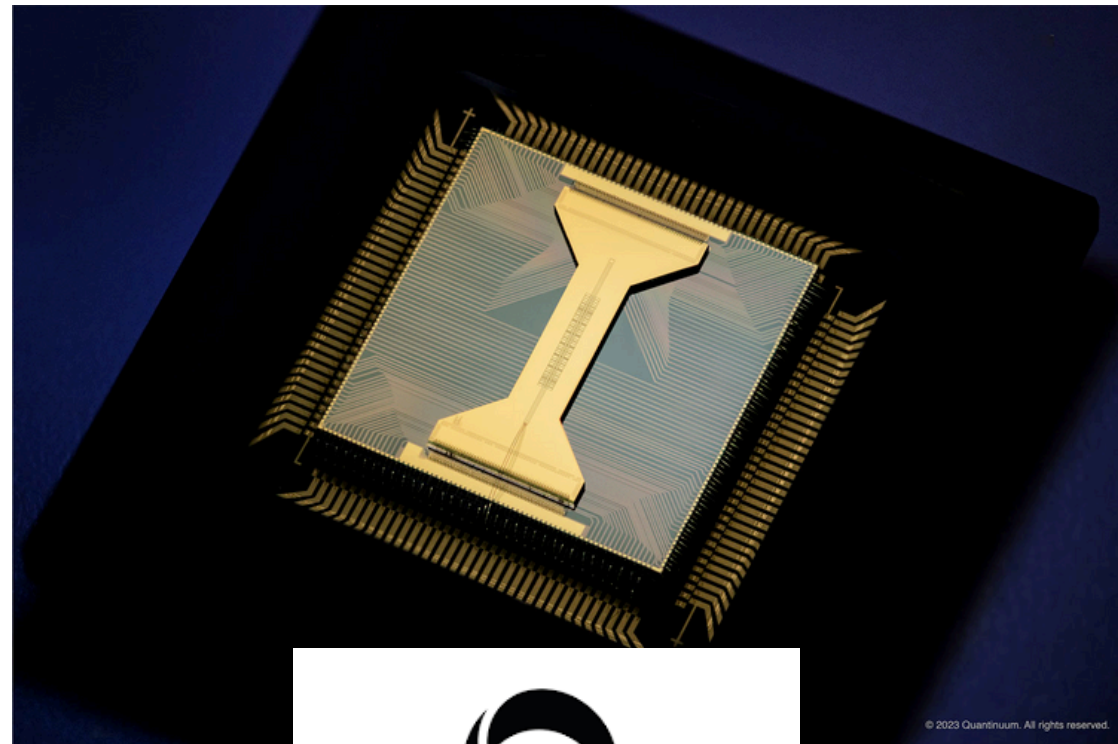


Quantum Hardware

Quantum Hardware

Ion trap

H1-1 20 qubits

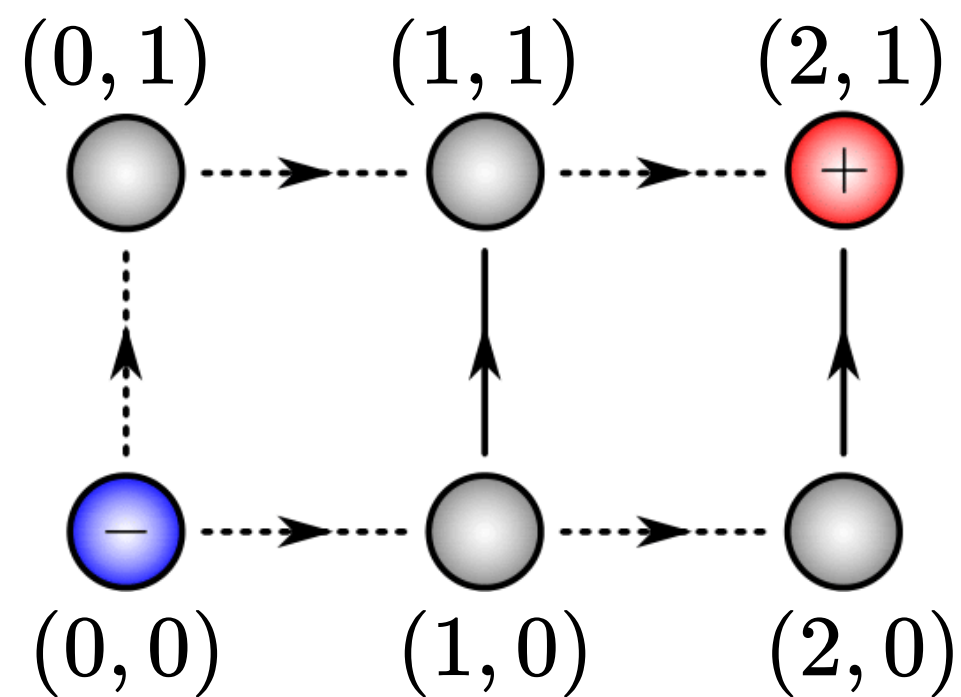
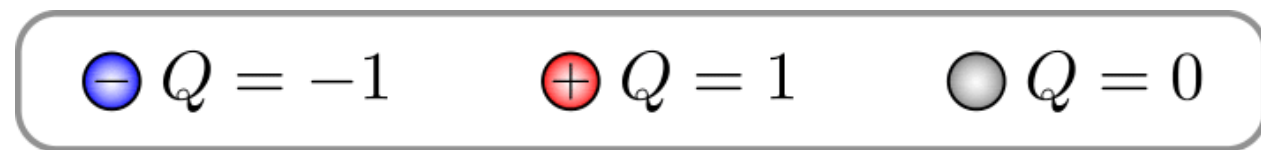


Superconducting

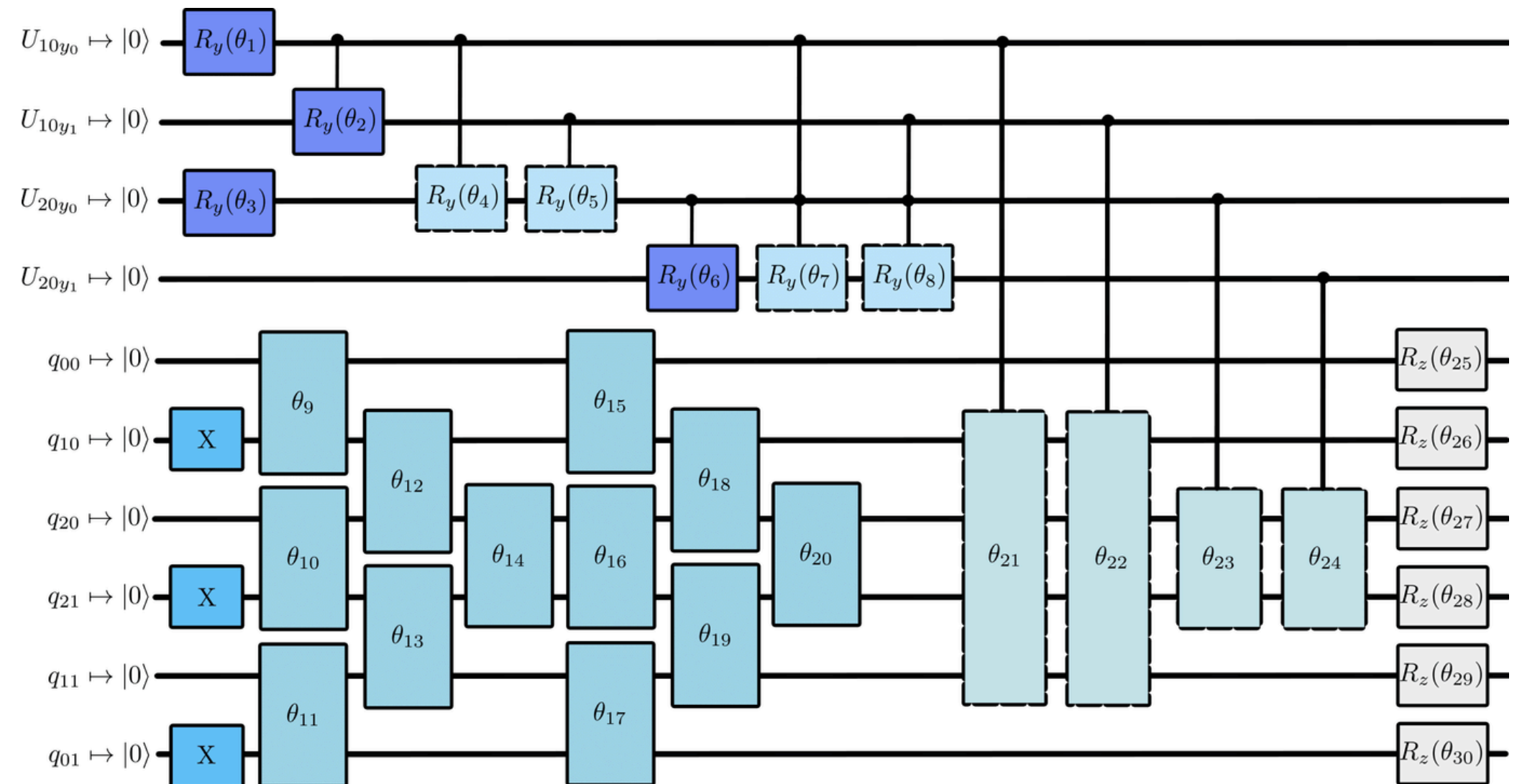
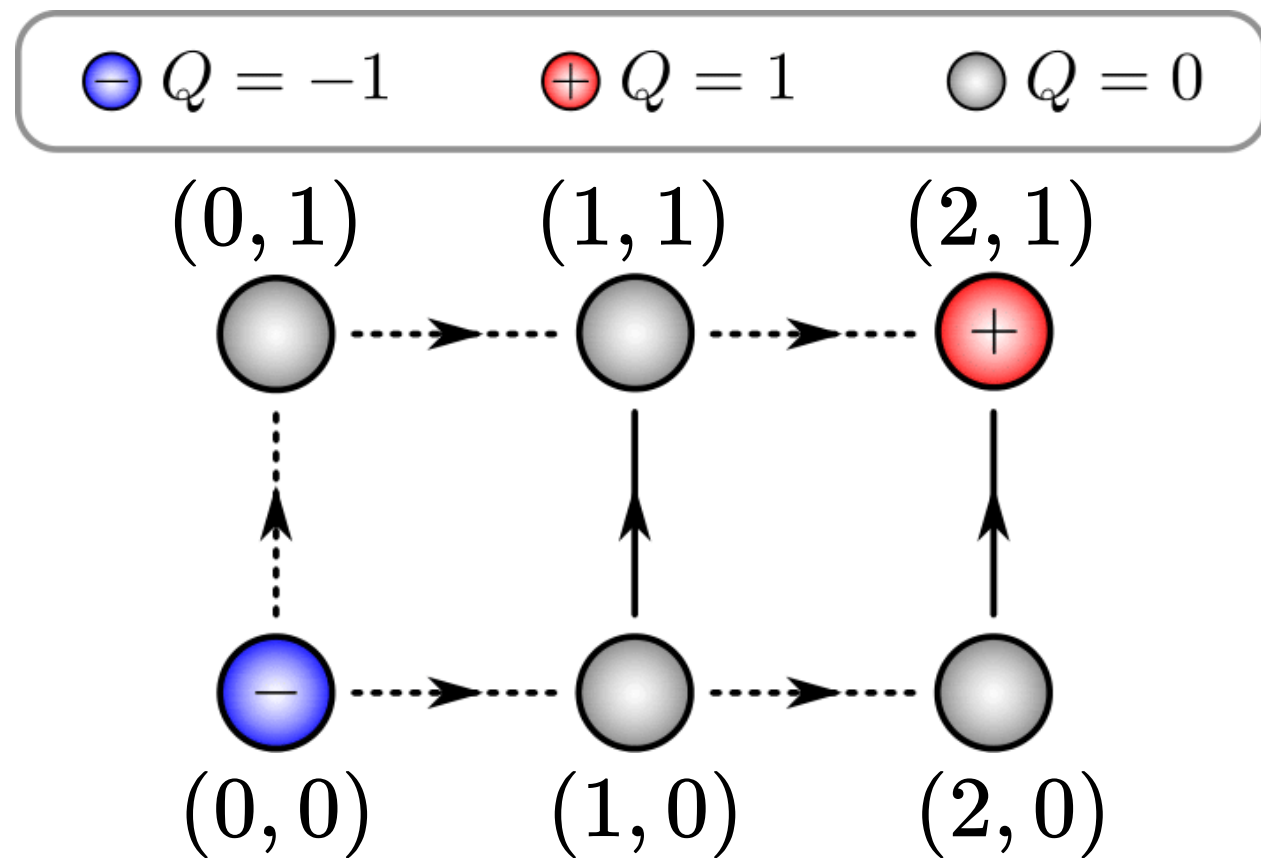
ibm_fez 156 qubits



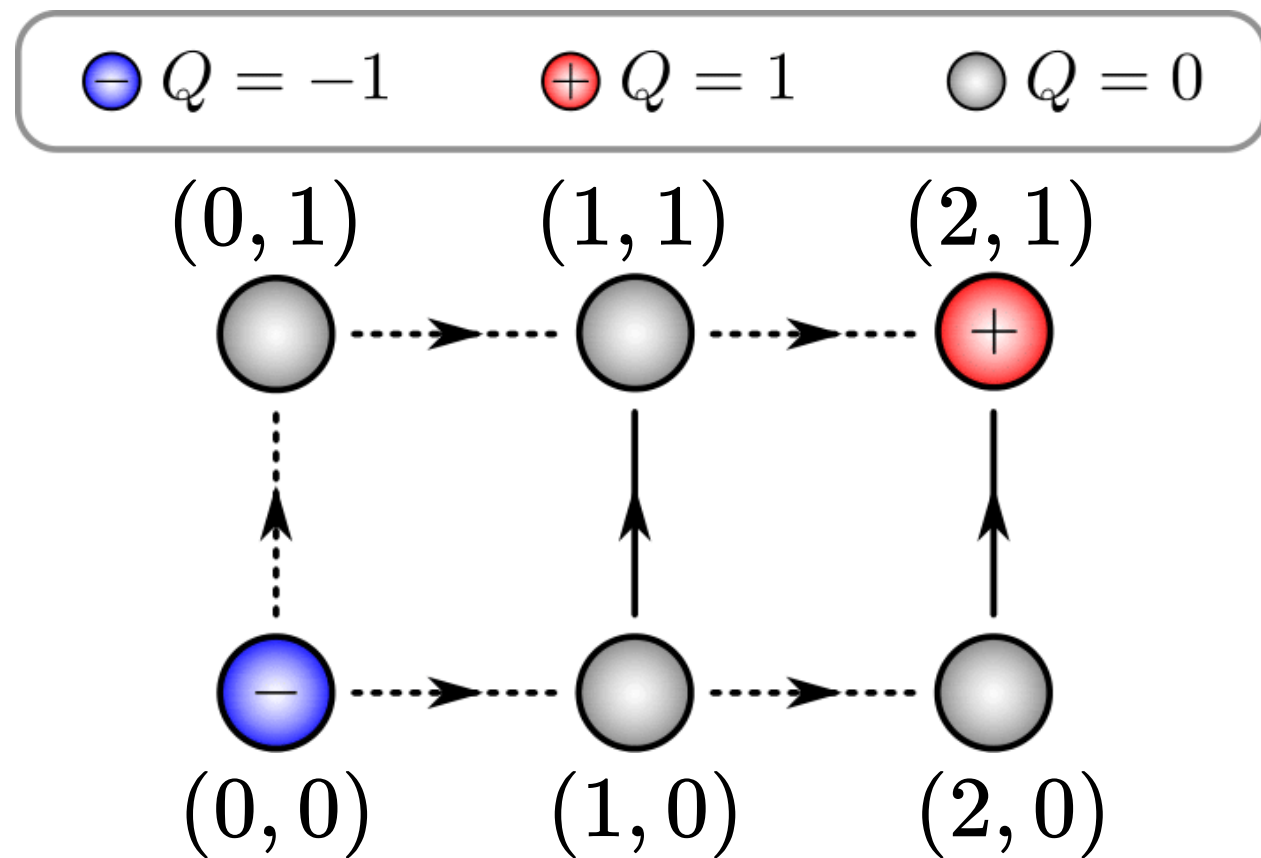
Quantum circuit



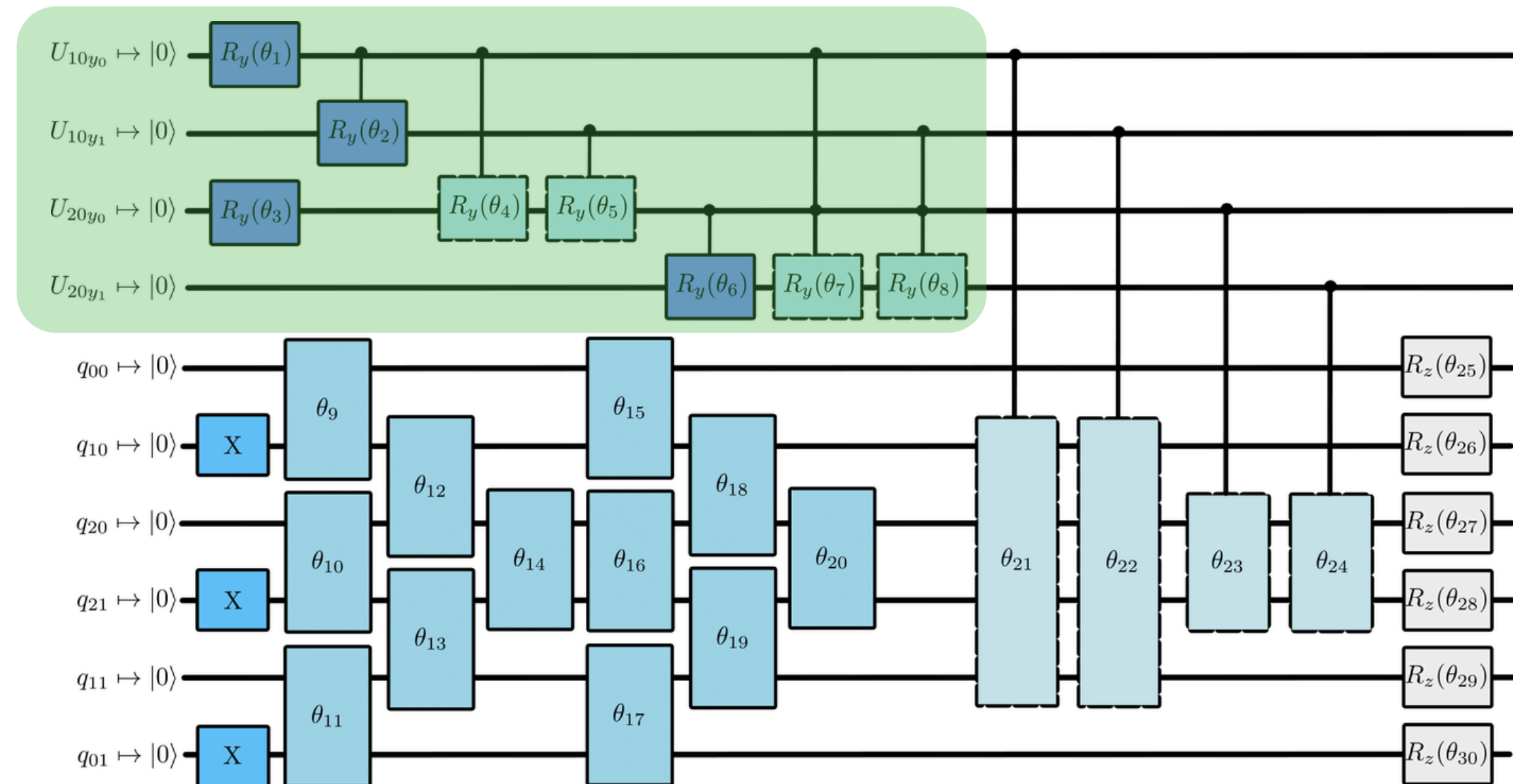
Quantum circuit



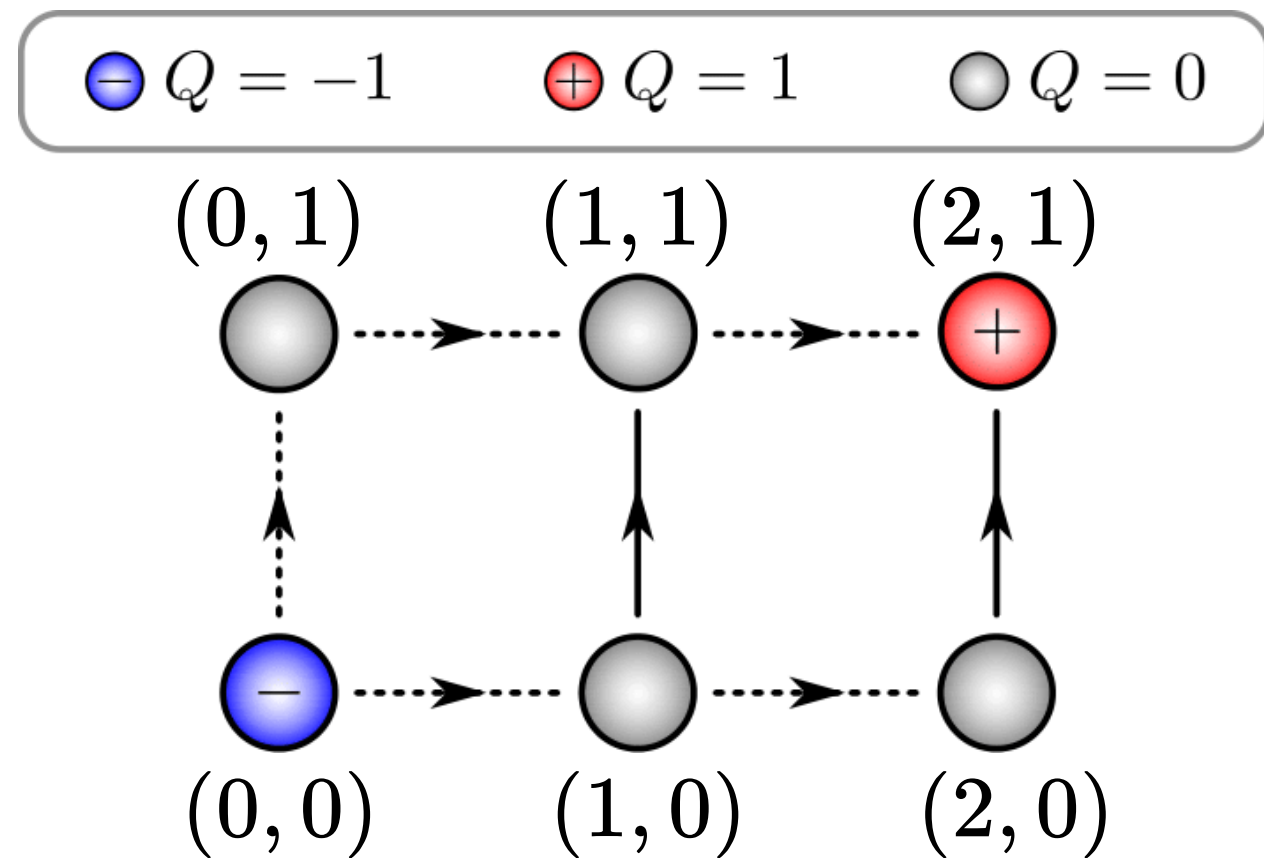
Quantum circuit



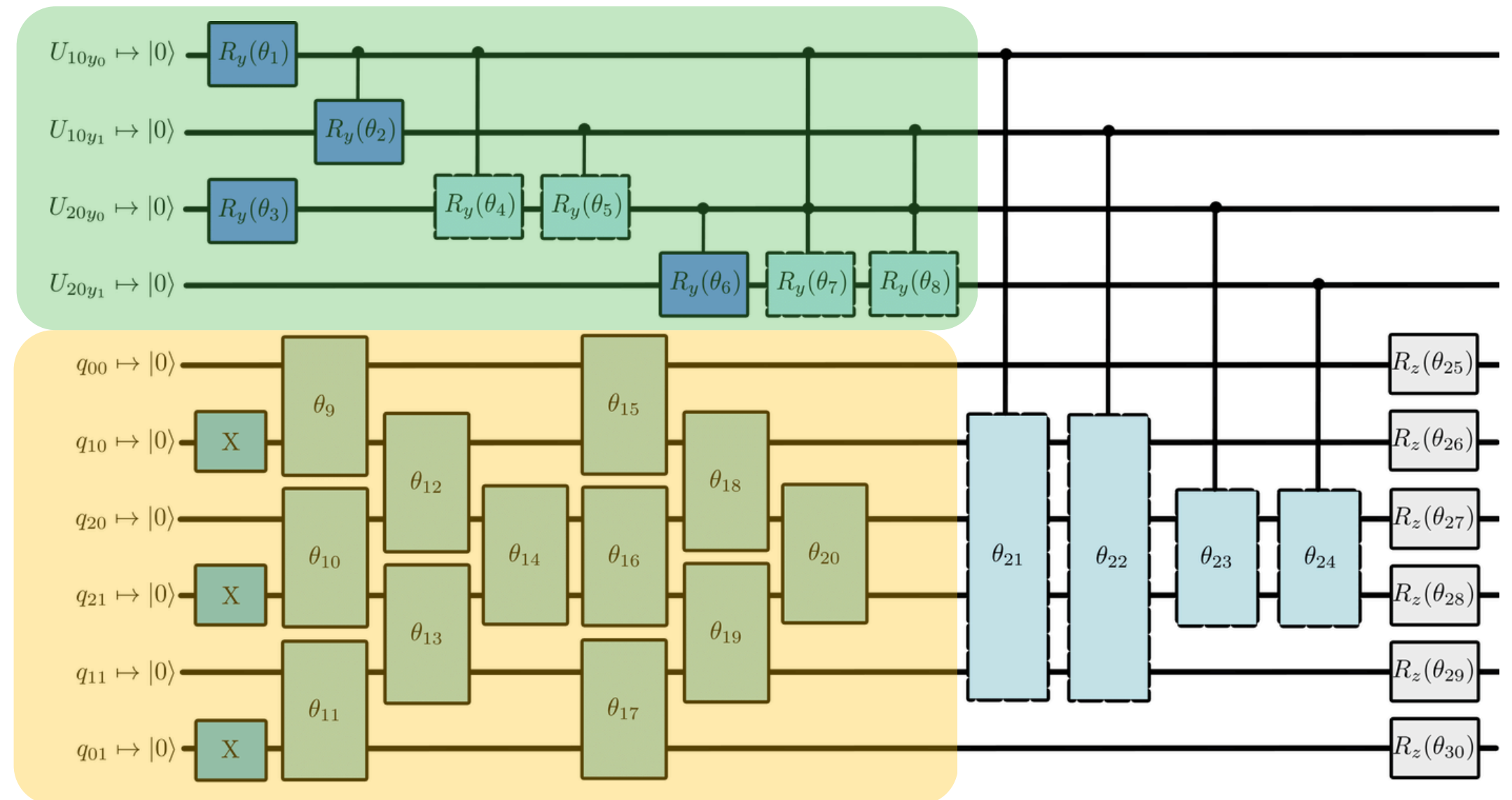
Gauge fields



Quantum circuit

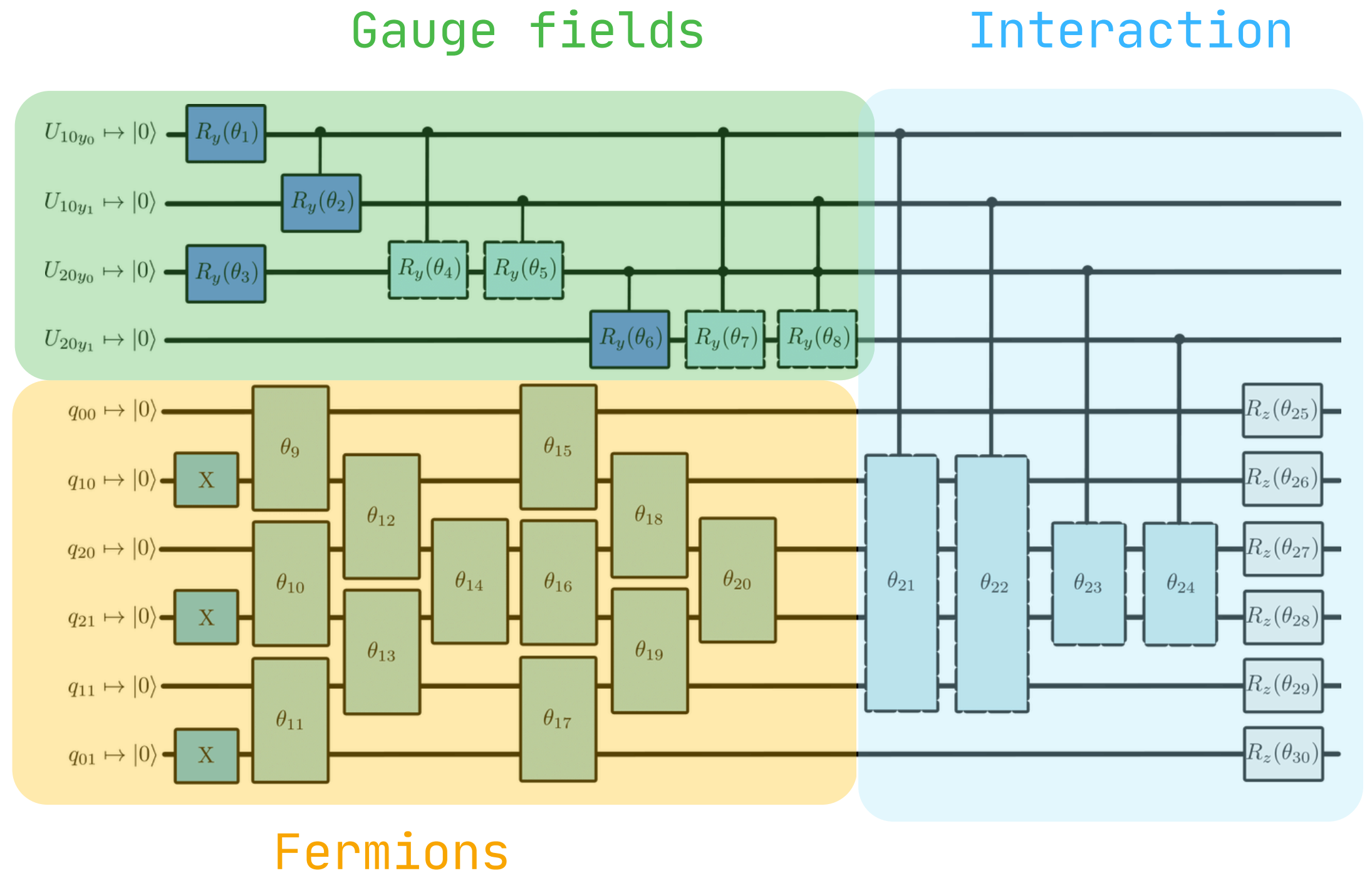
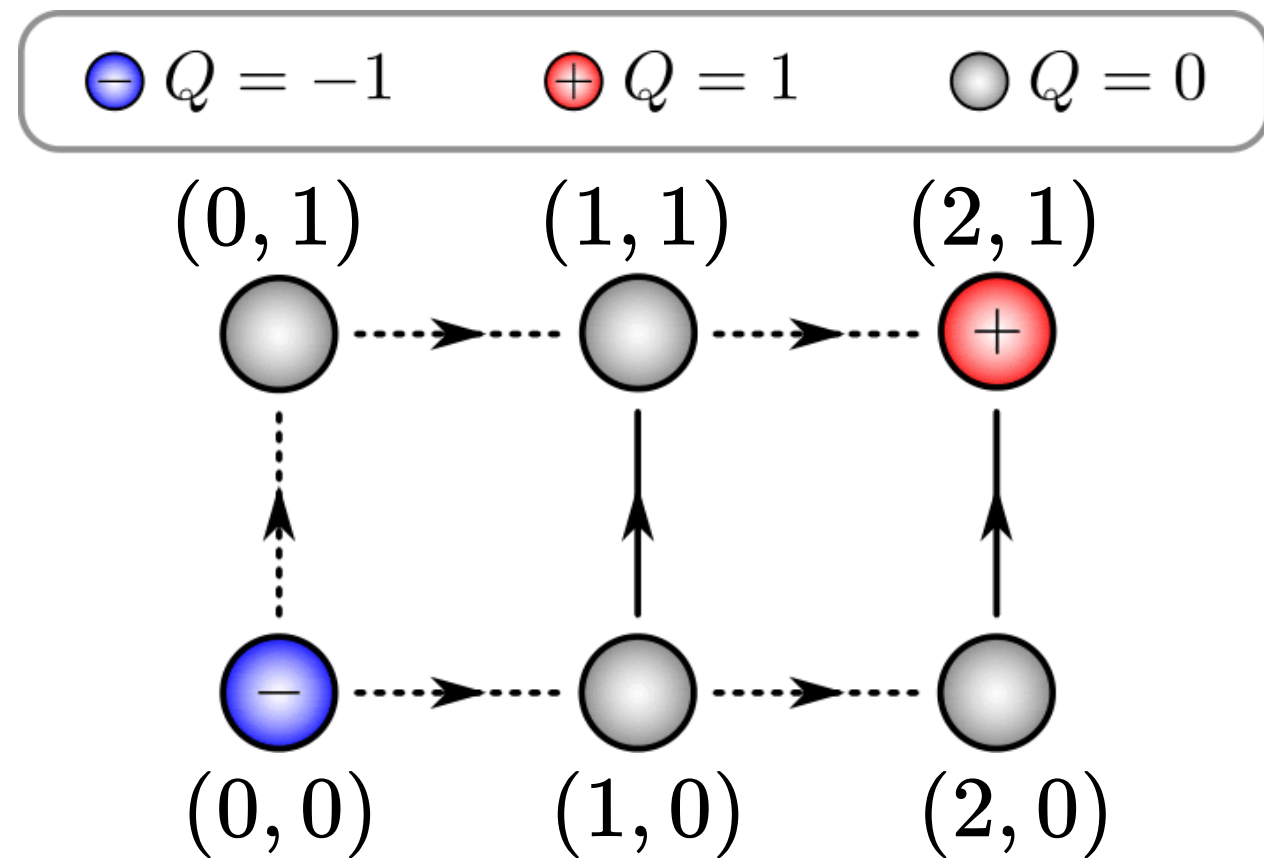


Gauge fields



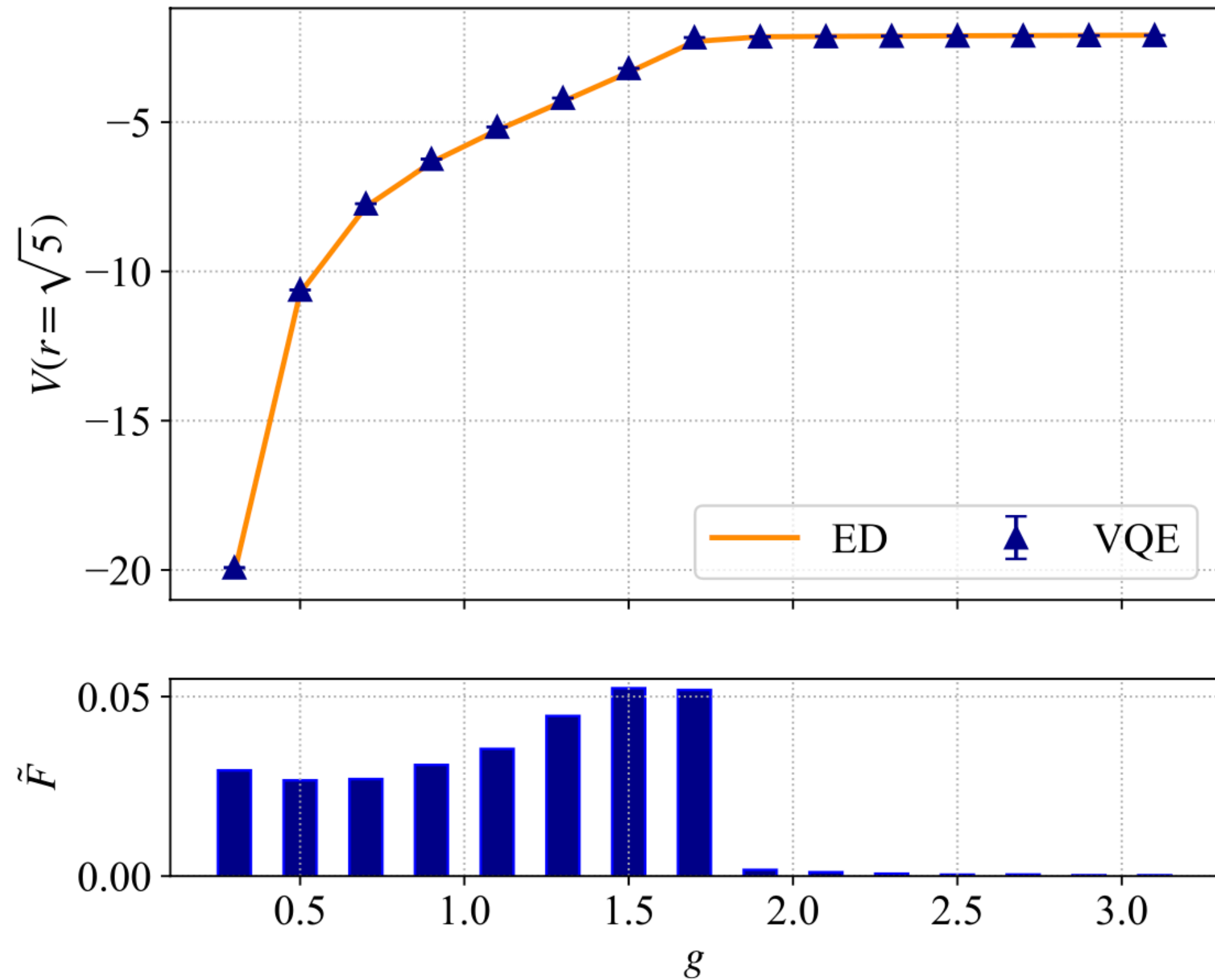
Fermions

Quantum circuit



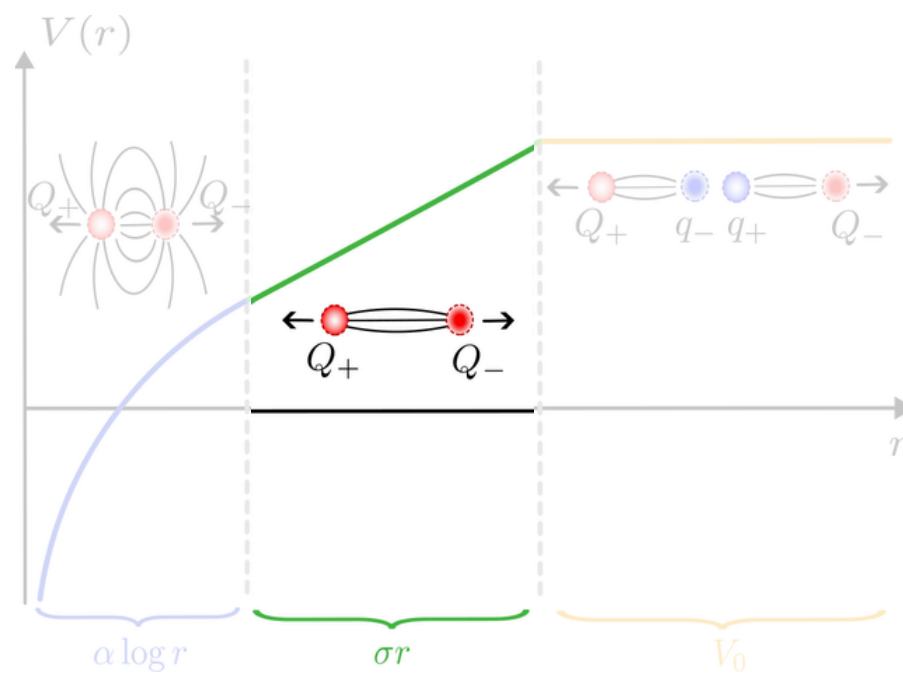
VQE results

Noiseless variational
quantum results
(NFT and 10^4 shots)



Quantum Hardware Results

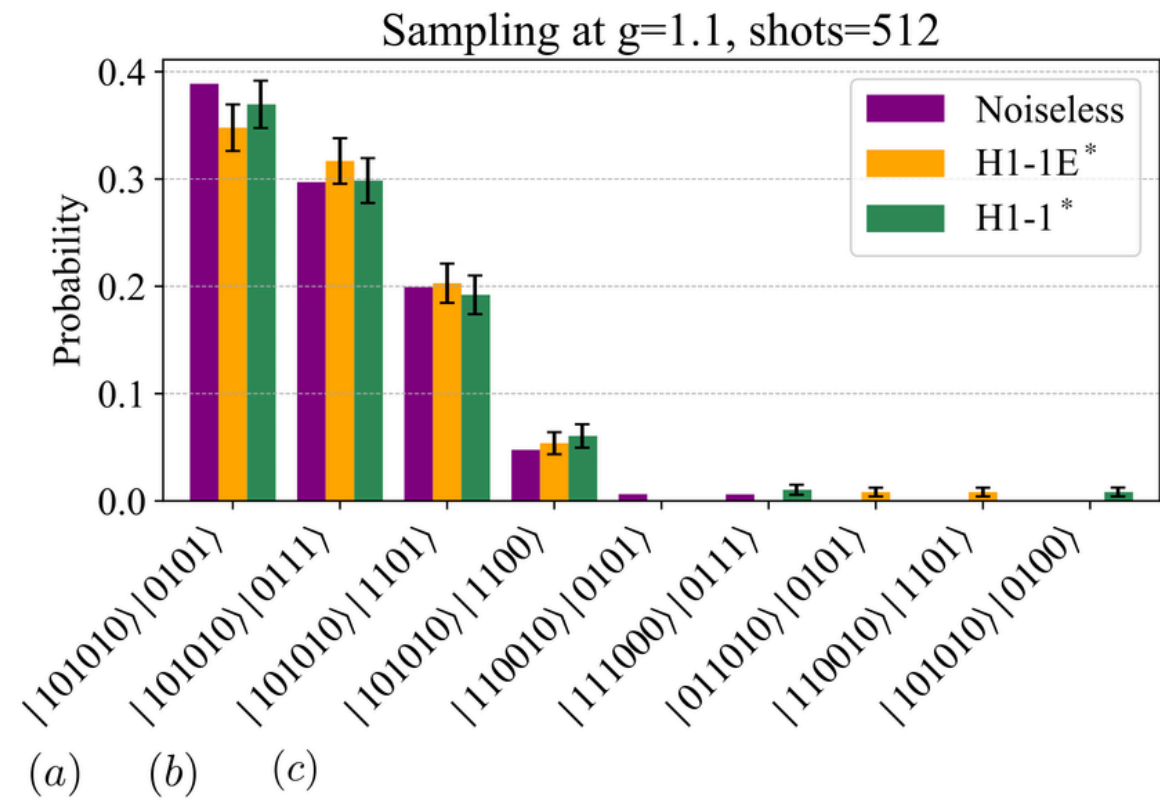
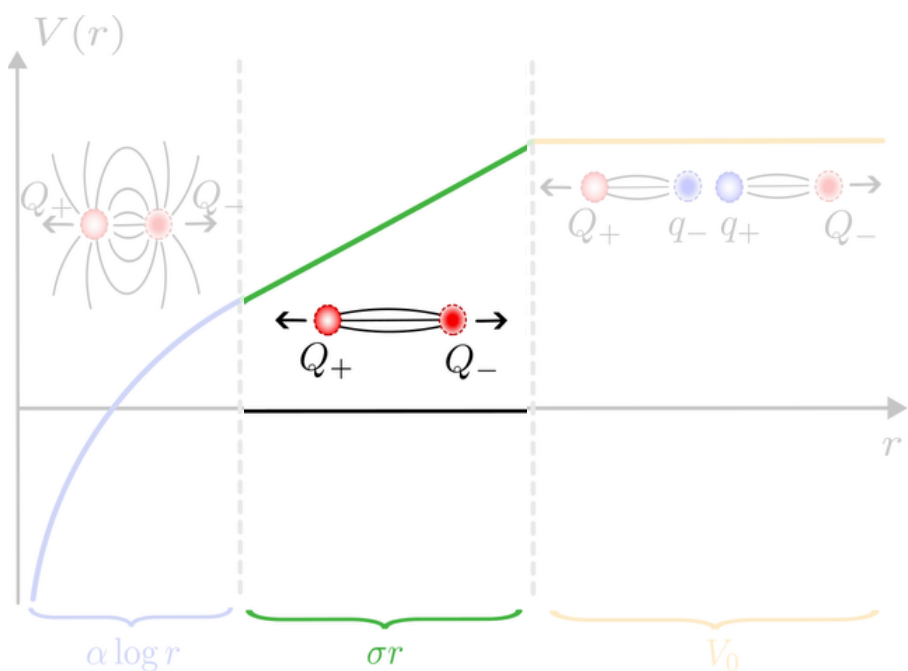
Quantum Hardware Results



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Quantum Hardware Results

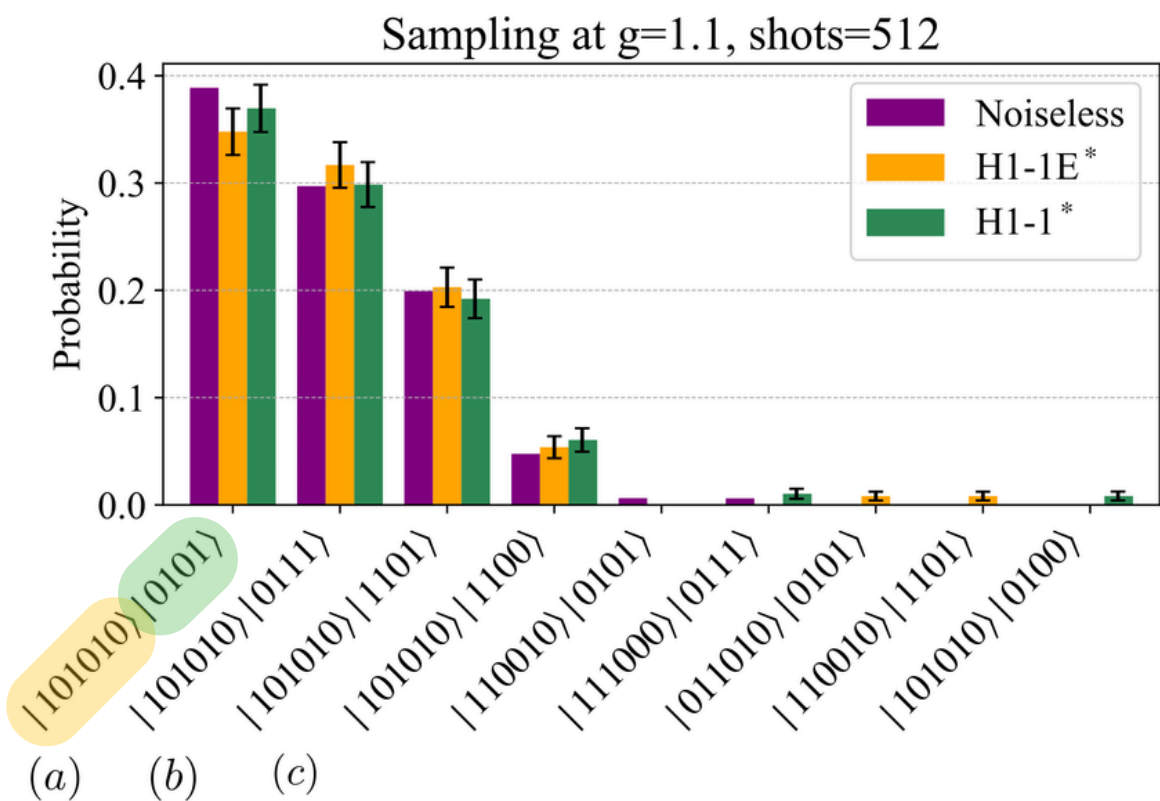
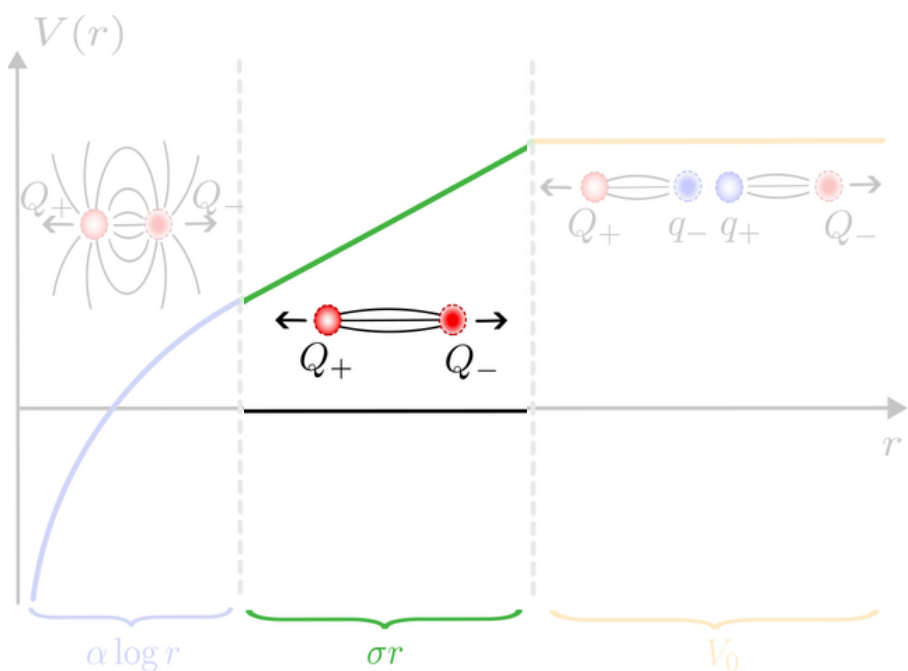
Ion trap



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Quantum Hardware Results

Ion trap

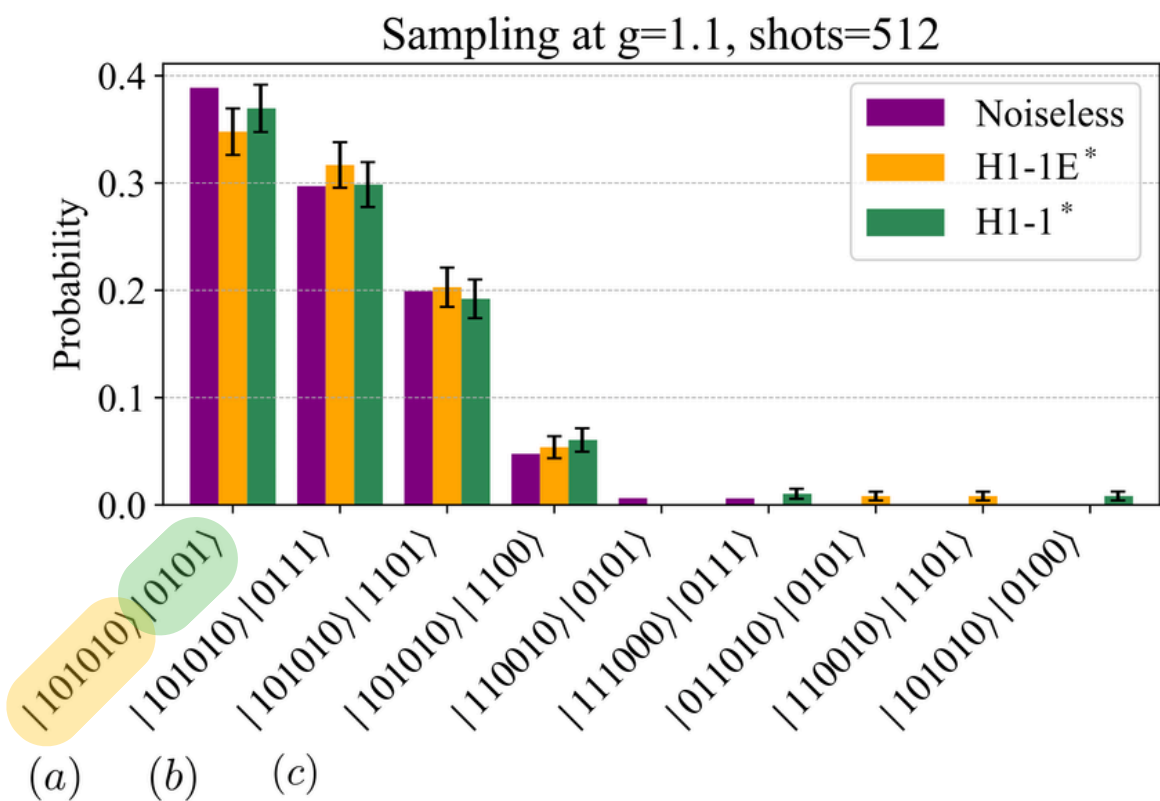
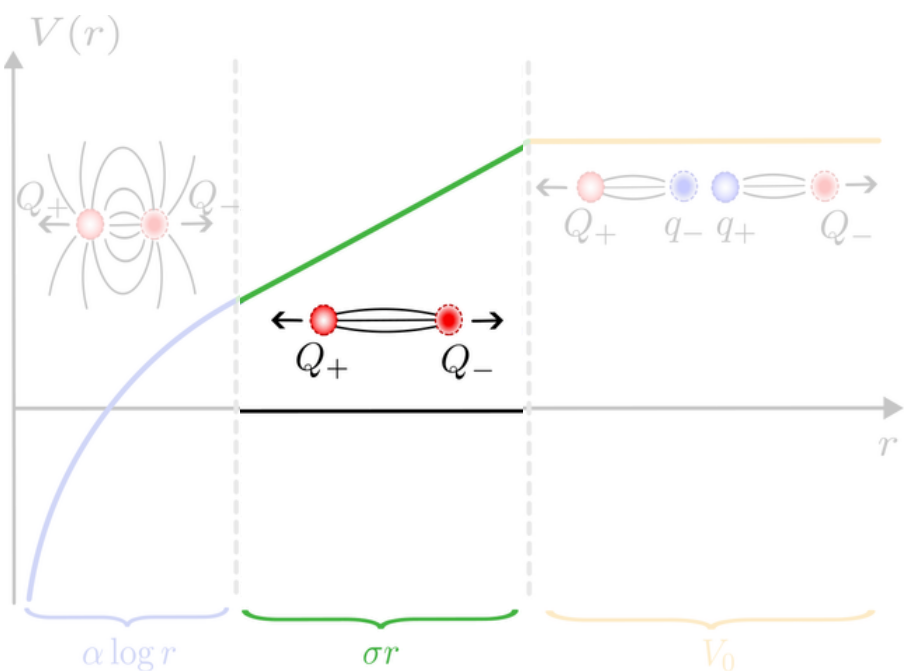


$$|101010\rangle \otimes |0101\rangle$$
$$|q_{01}q_{11}q_{21}q_{20}q_{10}q_{00}\rangle \otimes |U_{20y}U_{10y}\rangle$$

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Quantum Hardware Results

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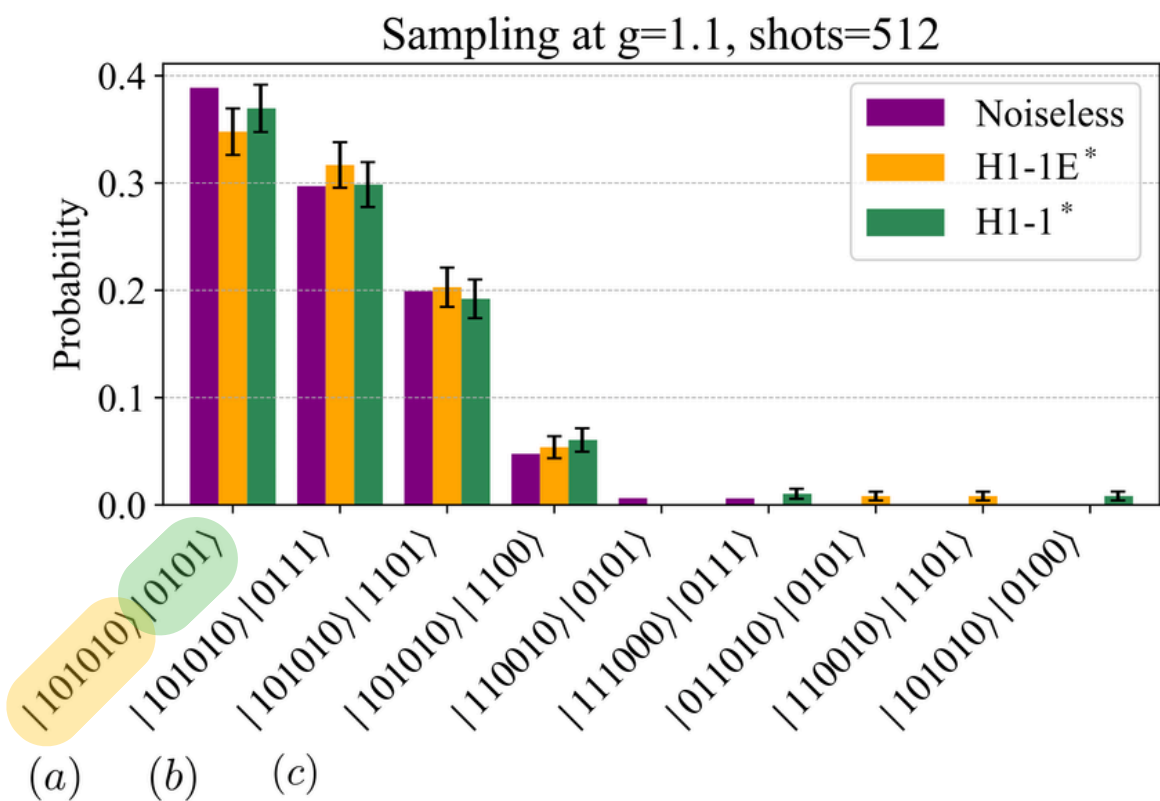
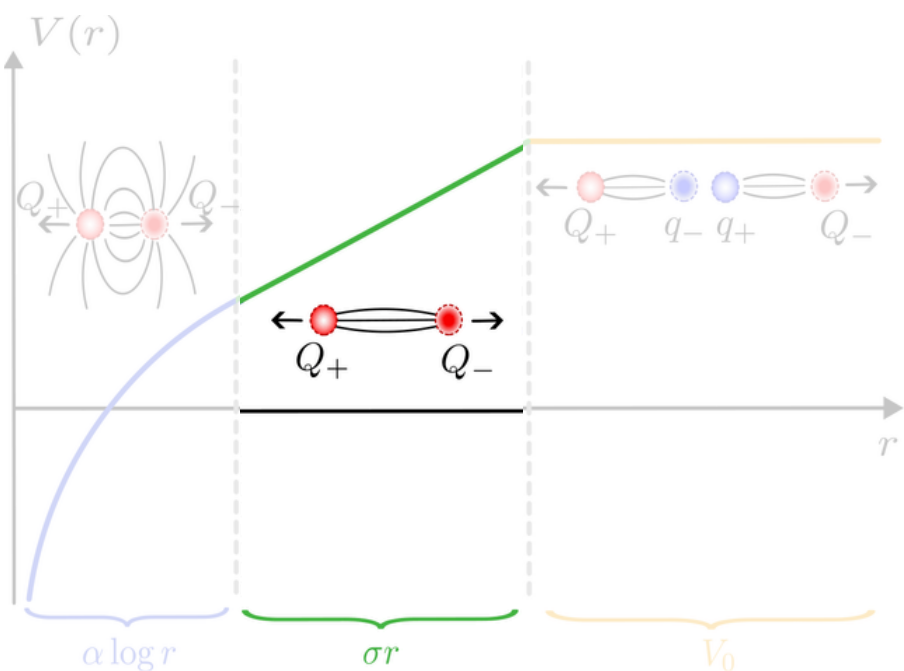
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EVEN $ 0\rangle$ v	ODD $ 1\rangle$ v
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Quantum Hardware Results

Ion trap



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EVEN	ODD
$ 0\rangle$	$ 1\rangle$
ν	ν

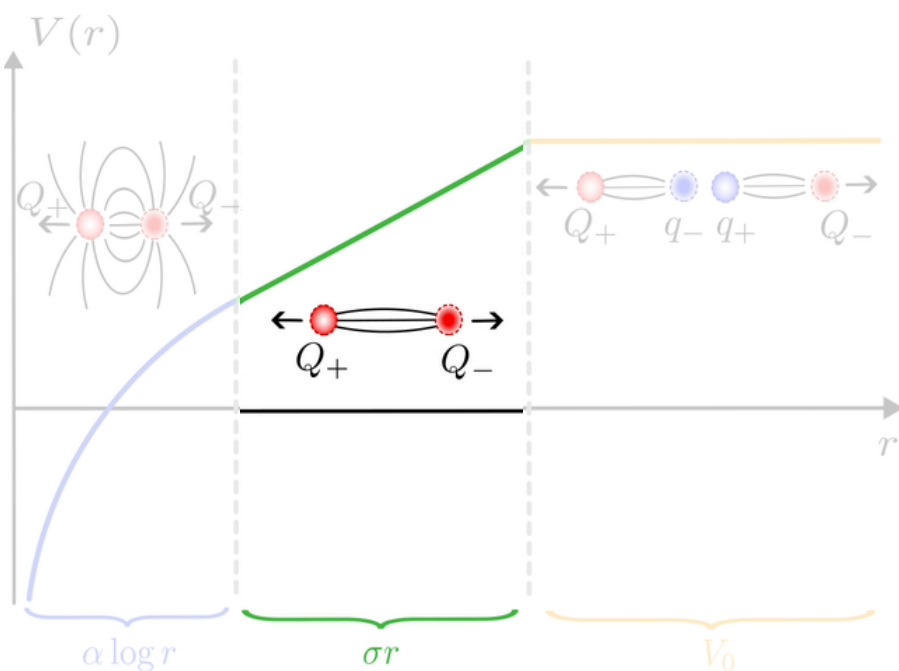
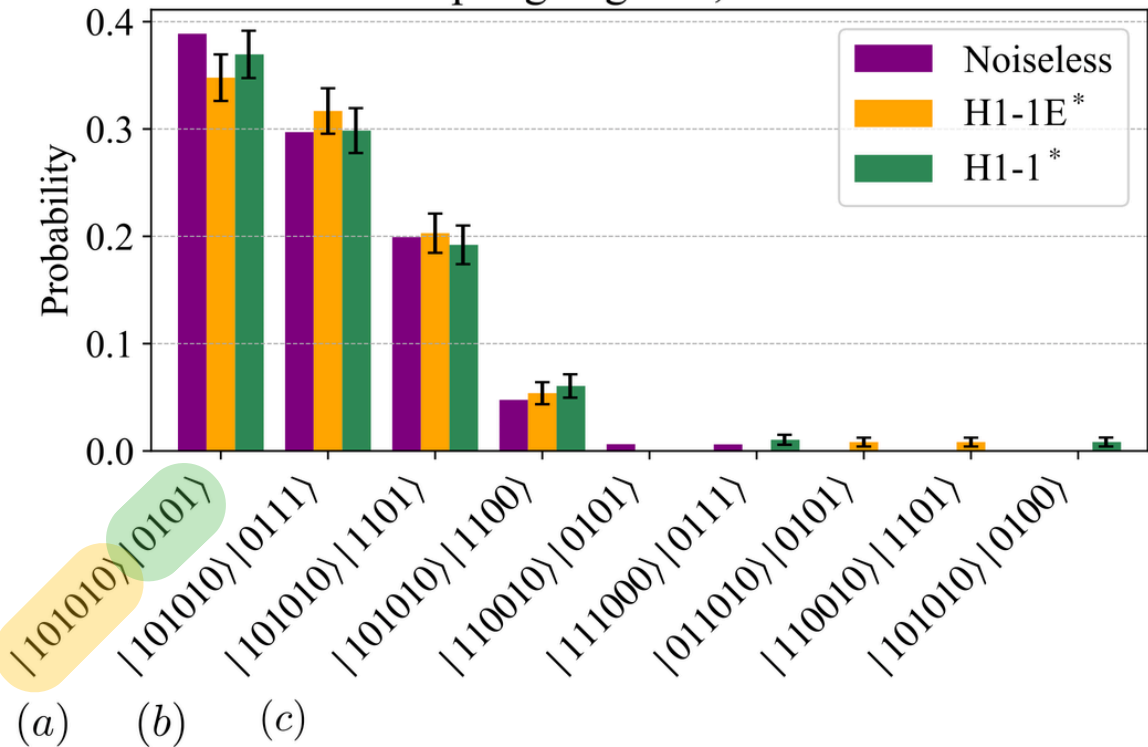
$$|01\rangle \mapsto |0\rangle_{\text{ph.}}$$

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Quantum Hardware Results

Ion trap

Sampling at $g=1.1$, shots=512



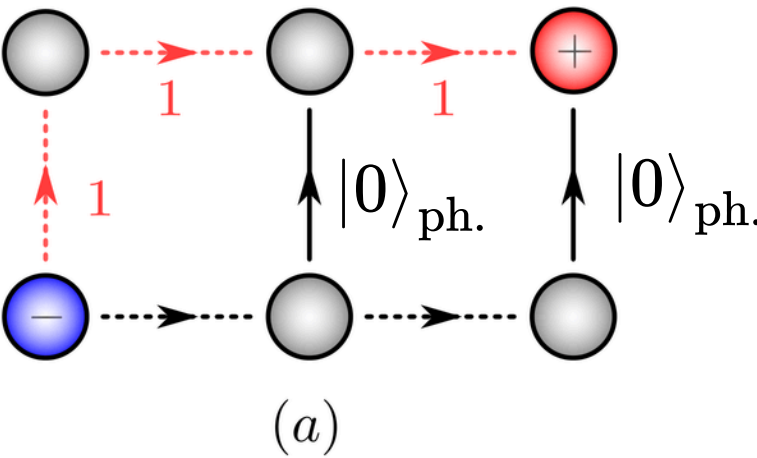
$$|101010\rangle \otimes |0101\rangle$$

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EVEN
 $|0\rangle$
 \mathcal{V}

ODD
 $|1\rangle$
 \mathcal{V}

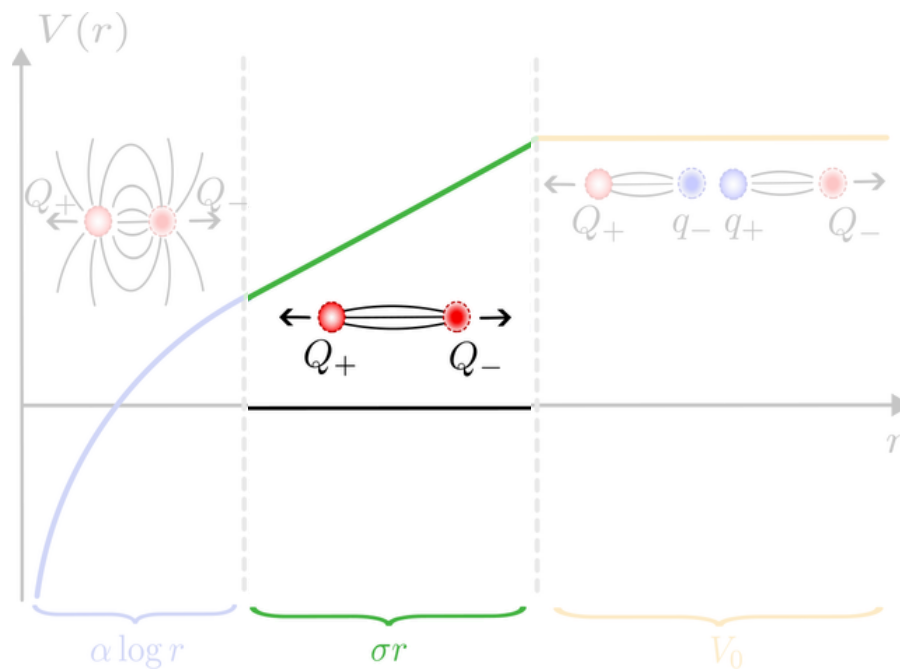
$|01\rangle \mapsto |0\rangle_{\text{ph.}}$



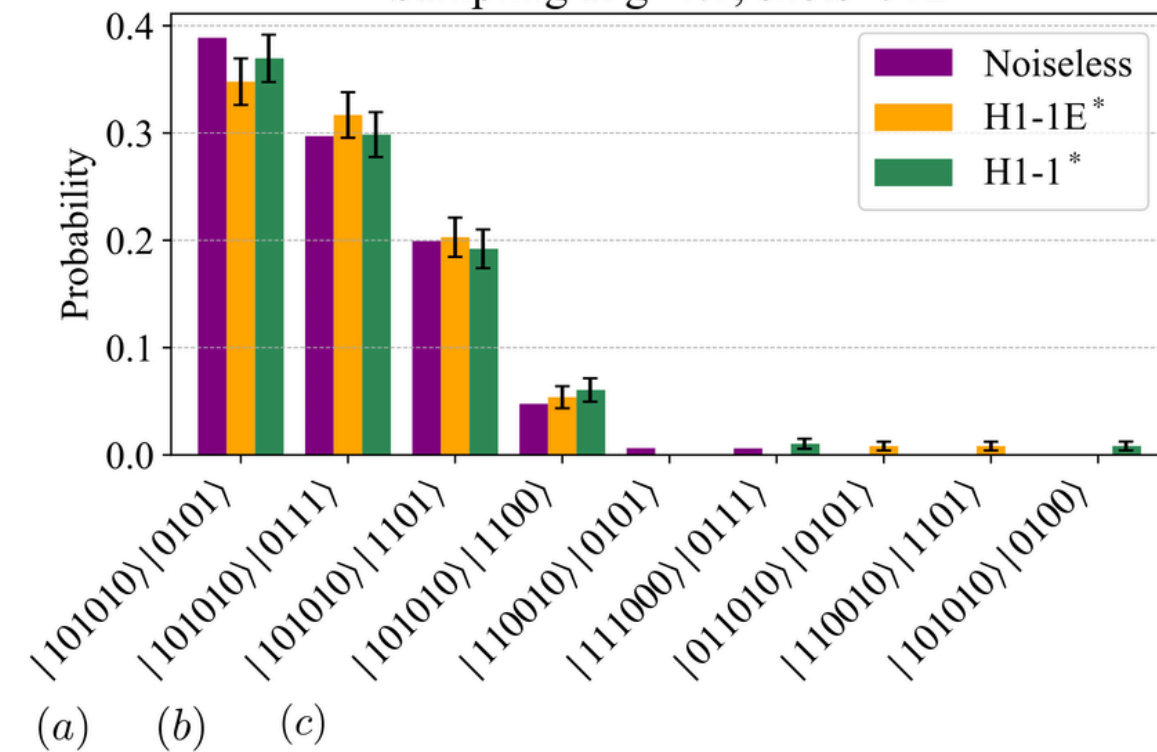
A. Crippa, K. Jansen, E. Rinaldi, arXiv:2411.05628

Quantum Hardware Results

Ion trap



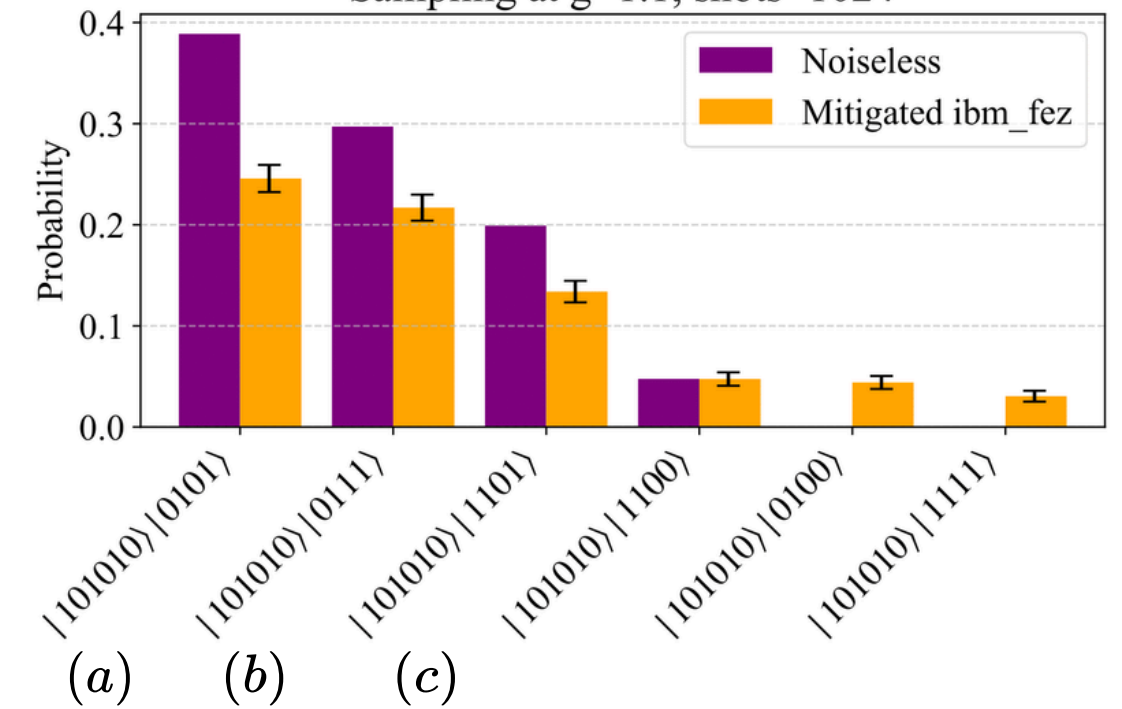
Sampling at $g=1.1$, shots=512



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Superconducting

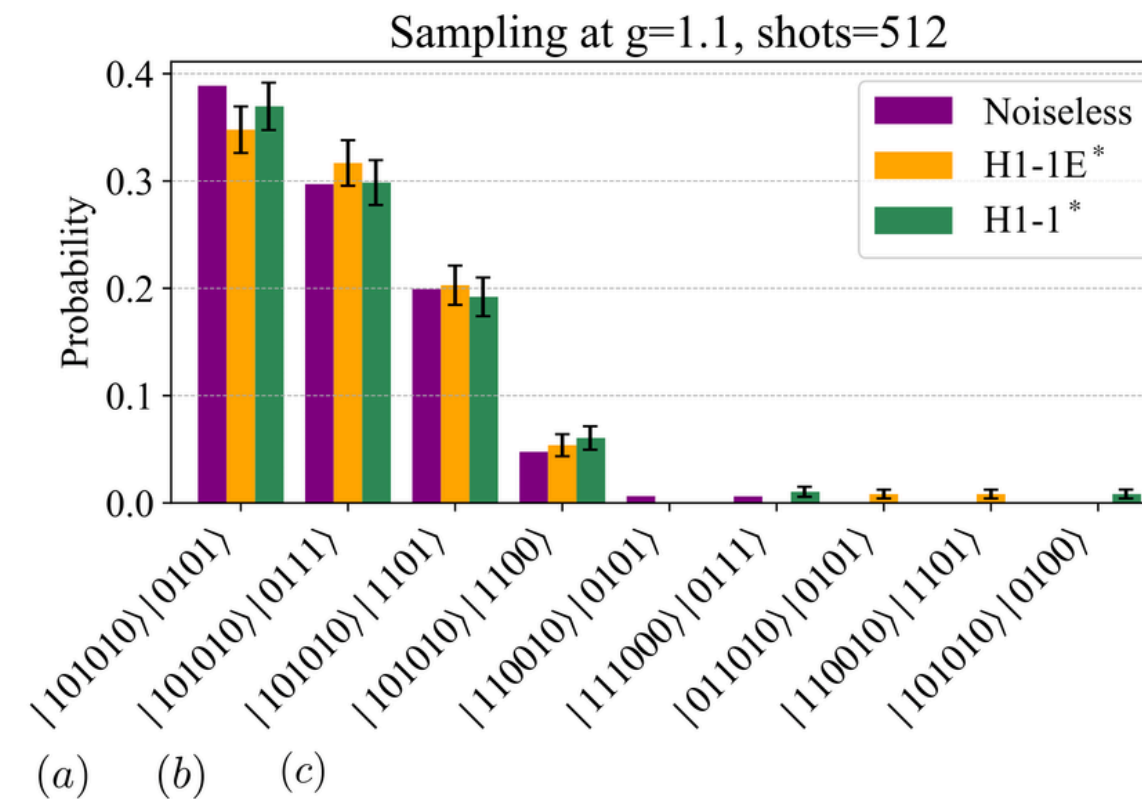
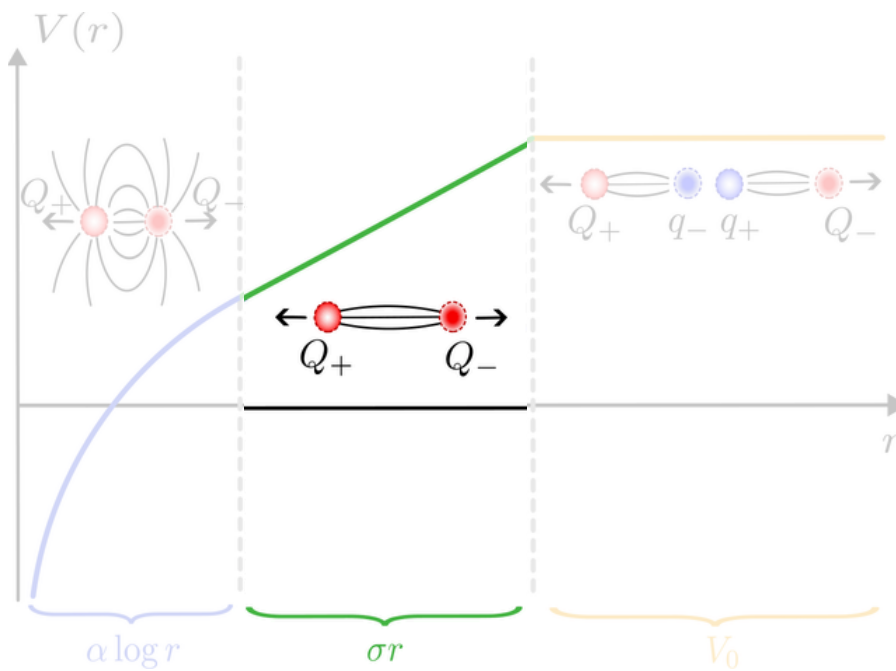
Sampling at $g=1.1$, shots=1024



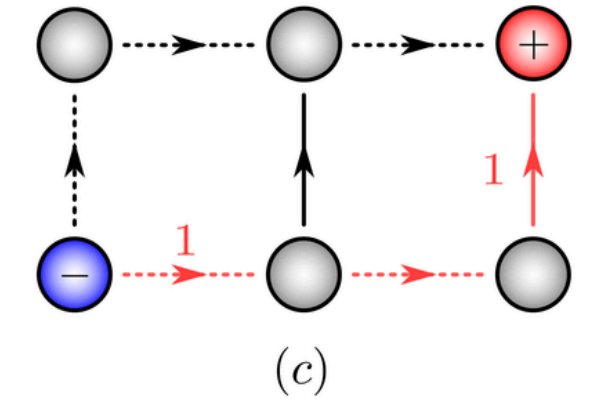
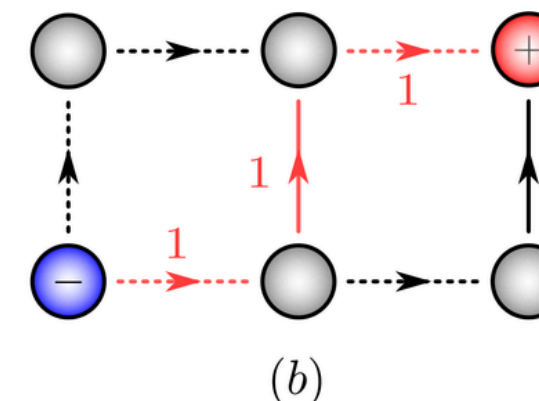
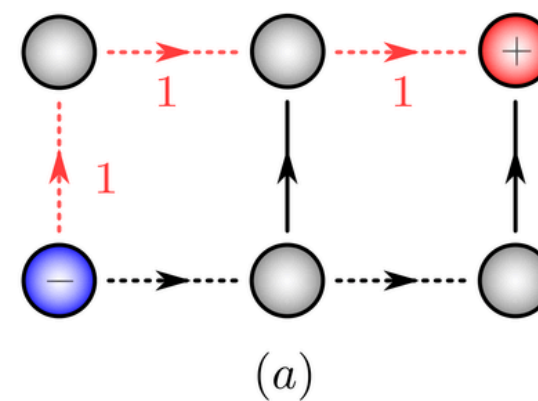
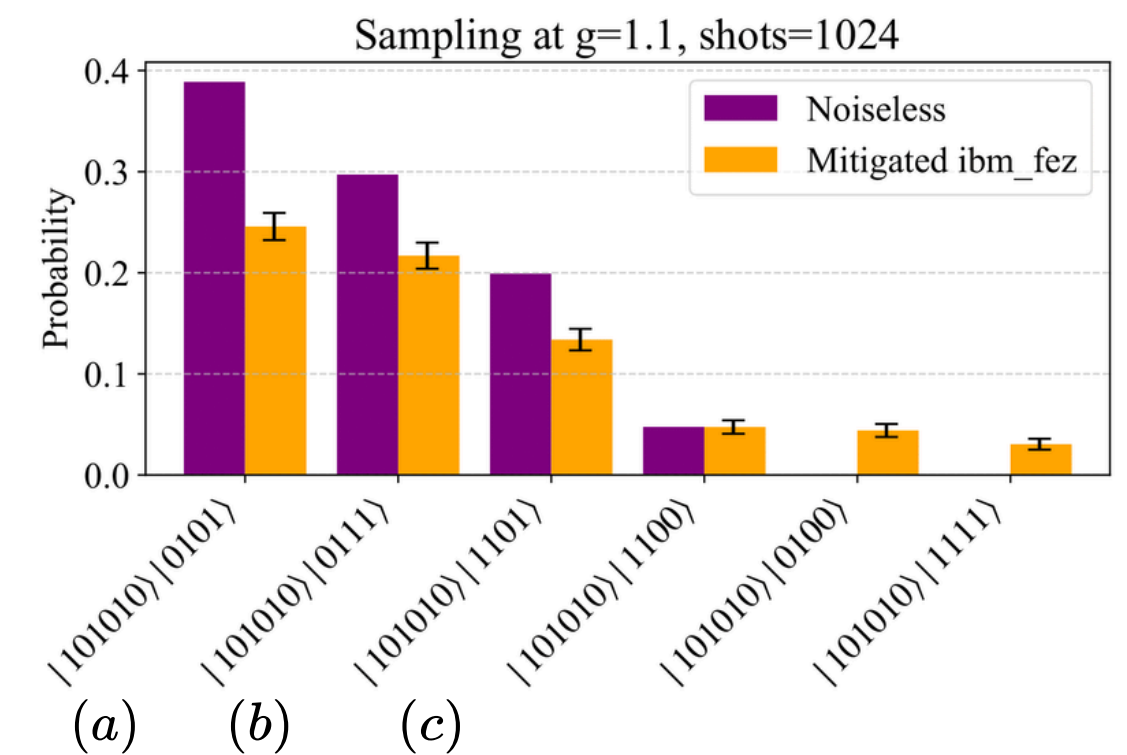
A. Crippa, PhD thesis

Quantum Hardware Results

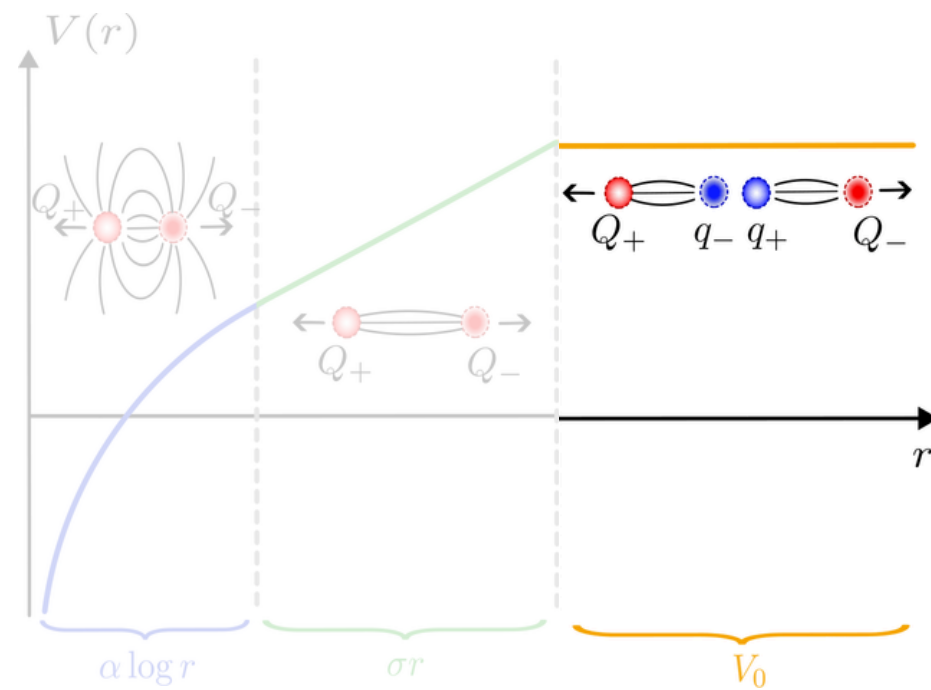
Ion trap



Superconducting

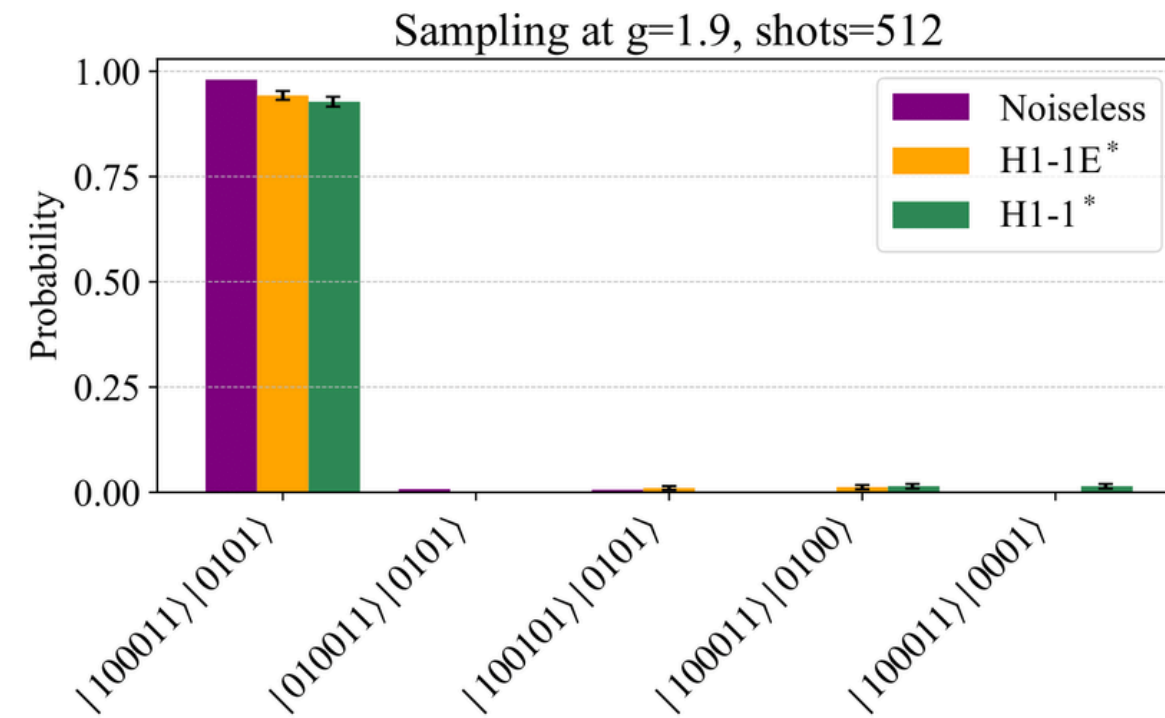
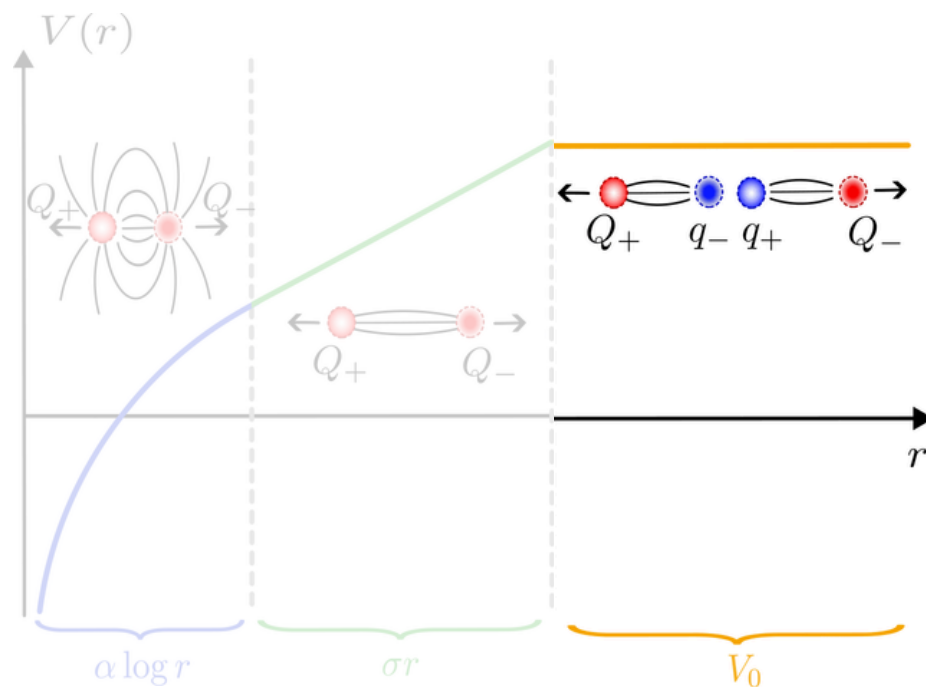


Quantum Hardware Results



Quantum Hardware Results

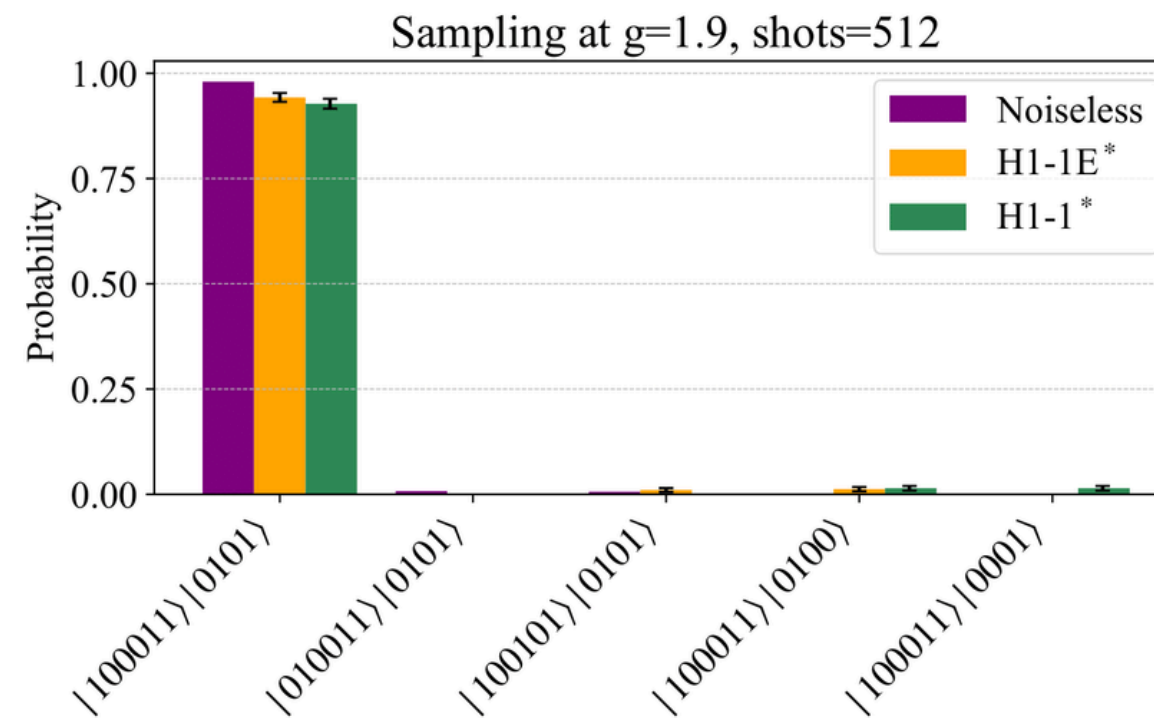
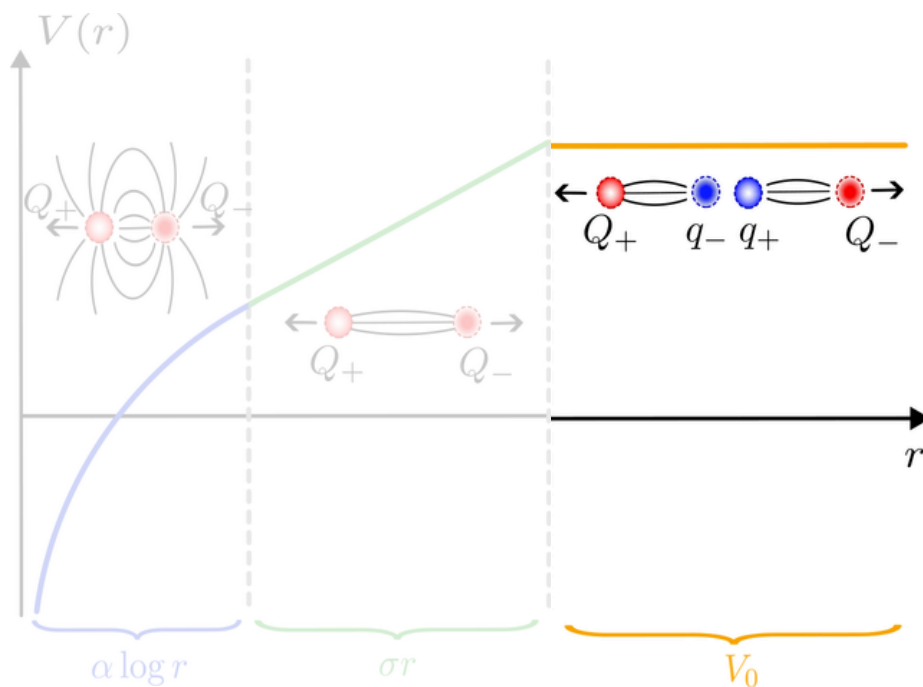
Ion trap



A. Crippa, K. Jansen, E. Rinaldi, arXiv:2411.05628

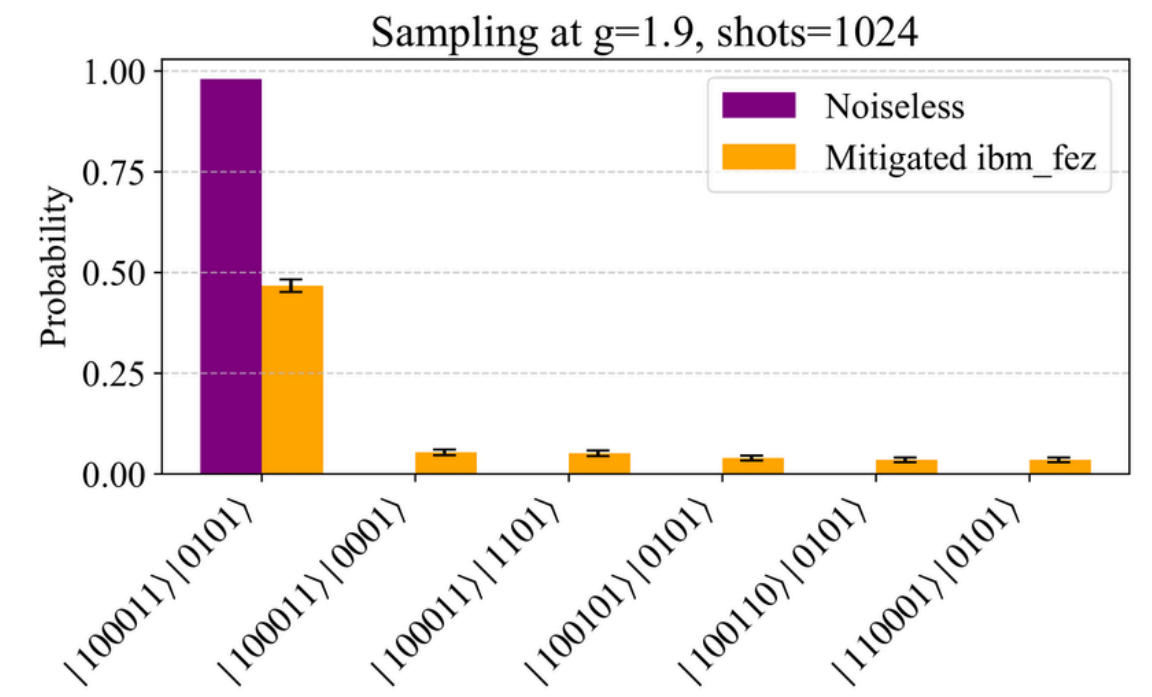
Quantum Hardware Results

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A. Crippa, K. Jansen, E. Rinaldi, arXiv:2411.05628

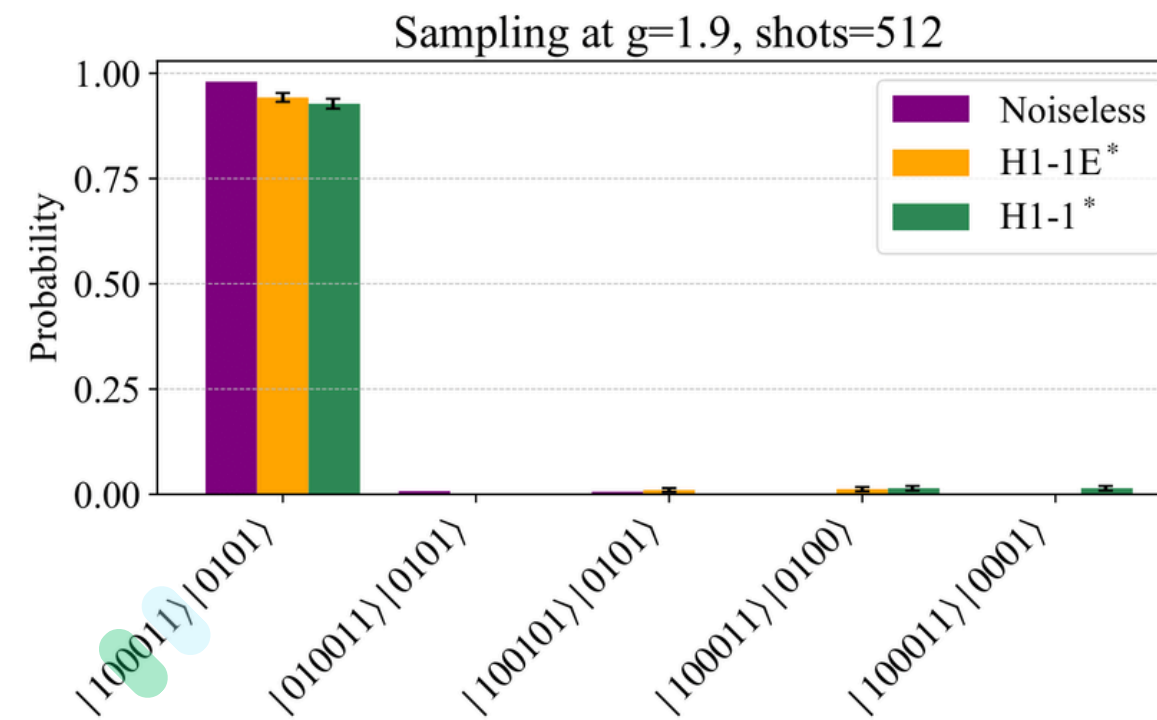
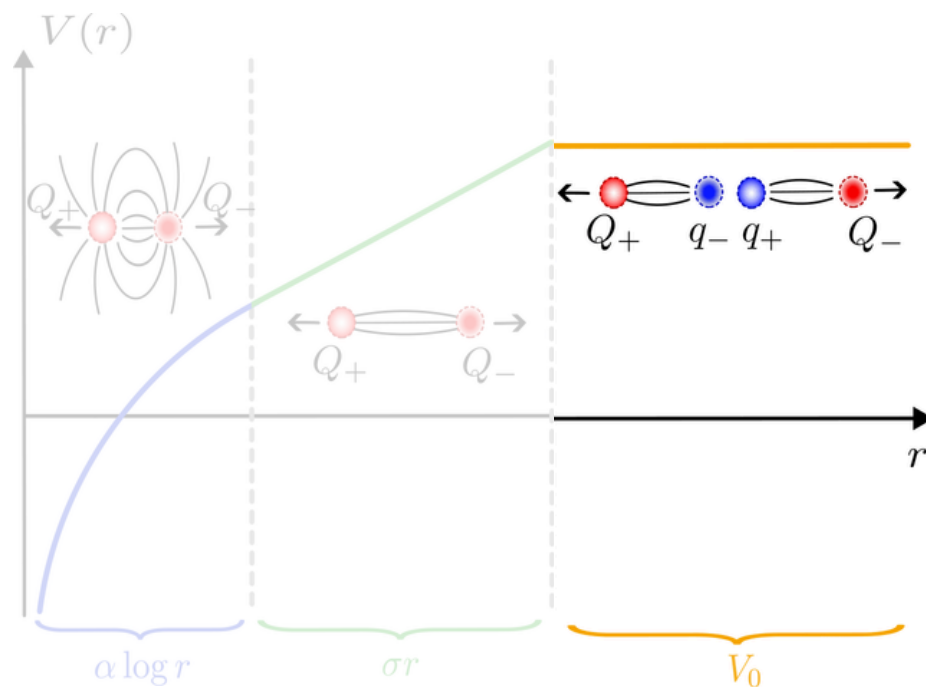
Superconducting



A. Crippa, PhD thesis

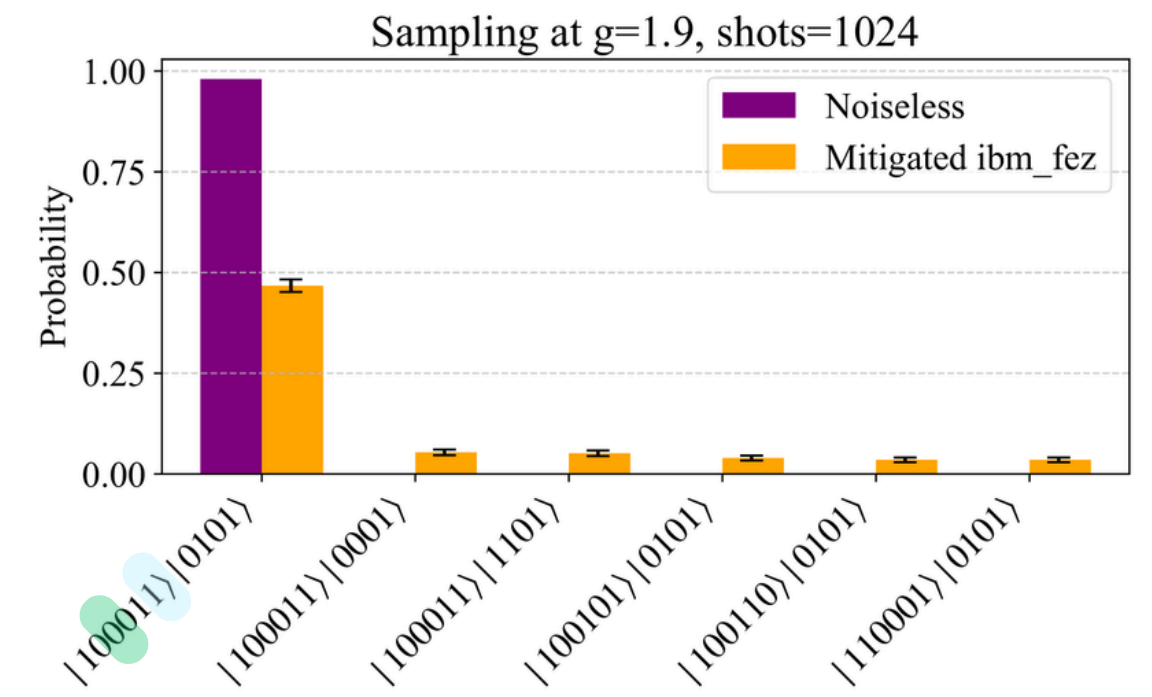
Quantum Hardware Results

Ion trap

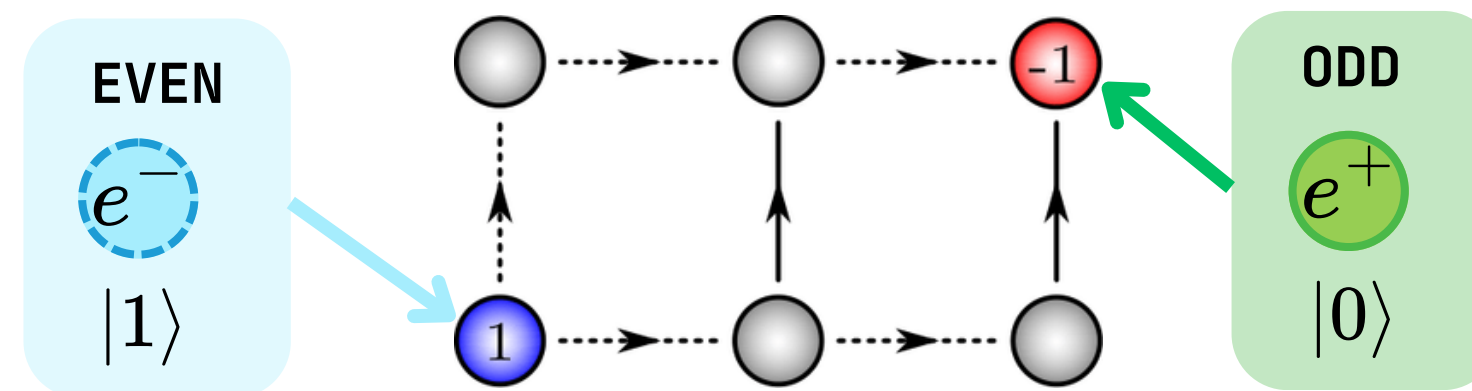


A. Crippa, K. Jansen, E. Rinaldi, arXiv:2411.05628

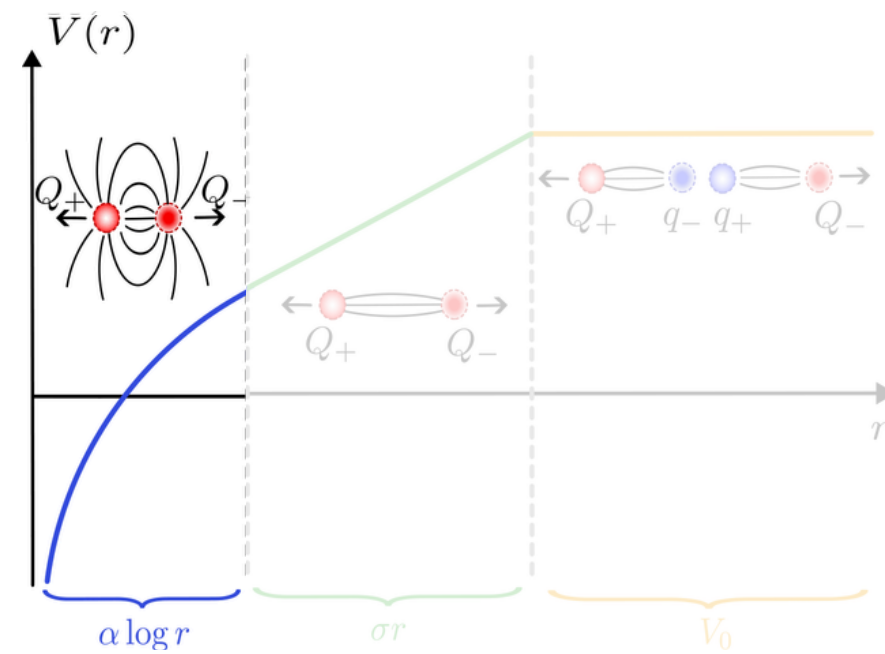
Superconducting



A. Crippa, PhD thesis

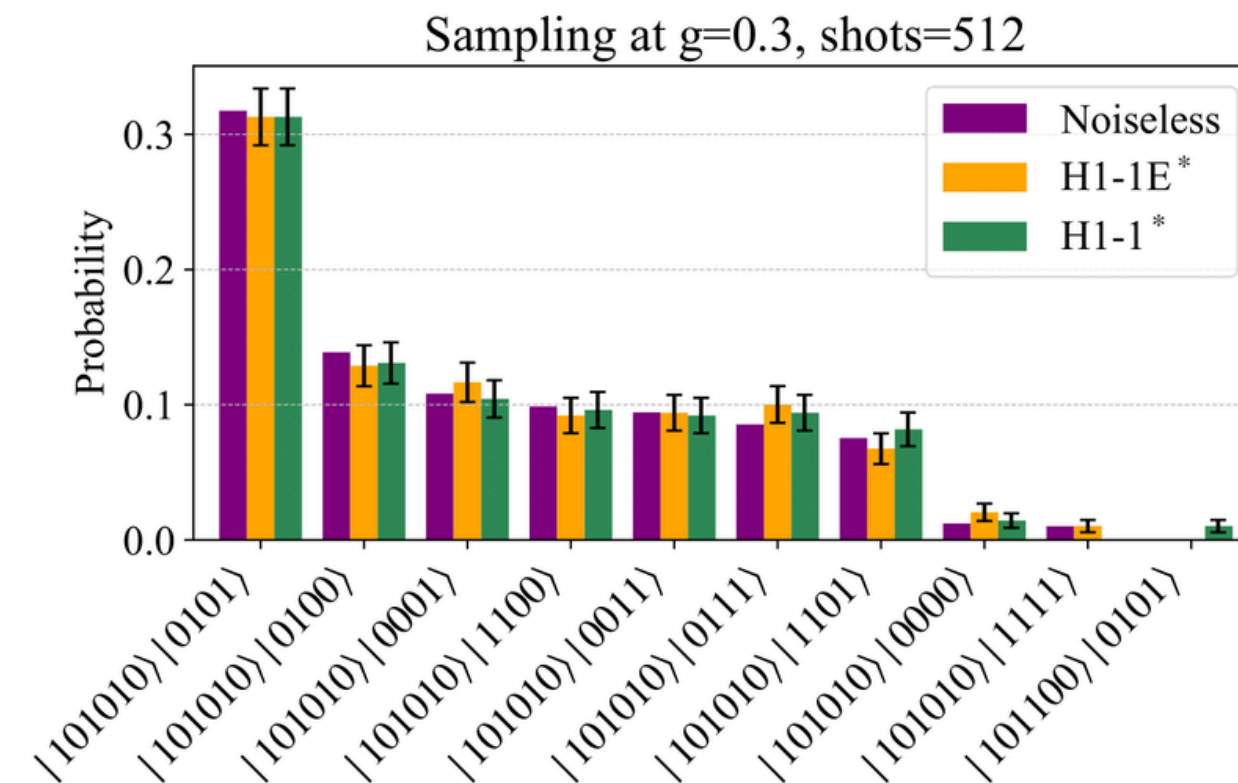
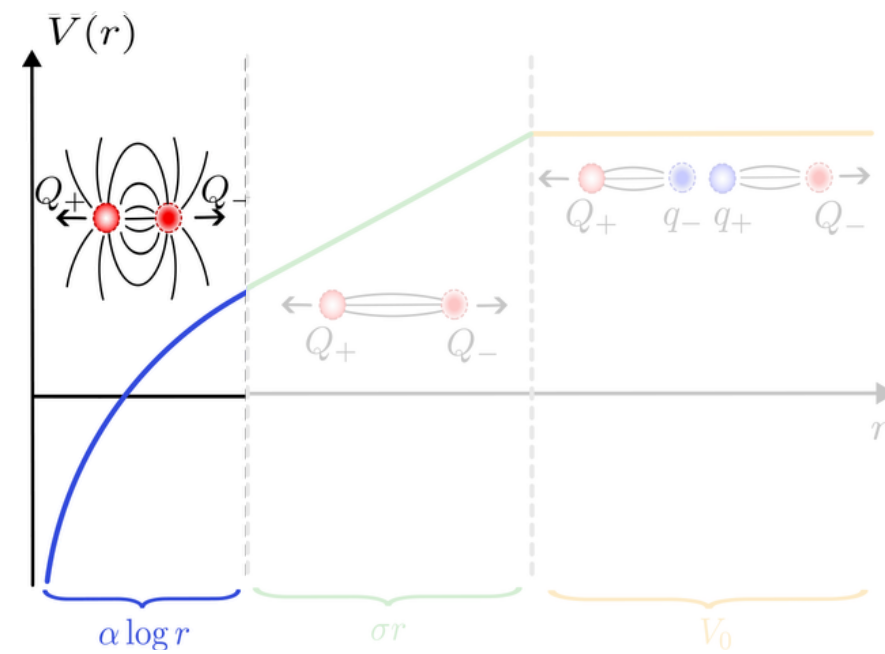


Quantum Hardware Results



Quantum Hardware Results

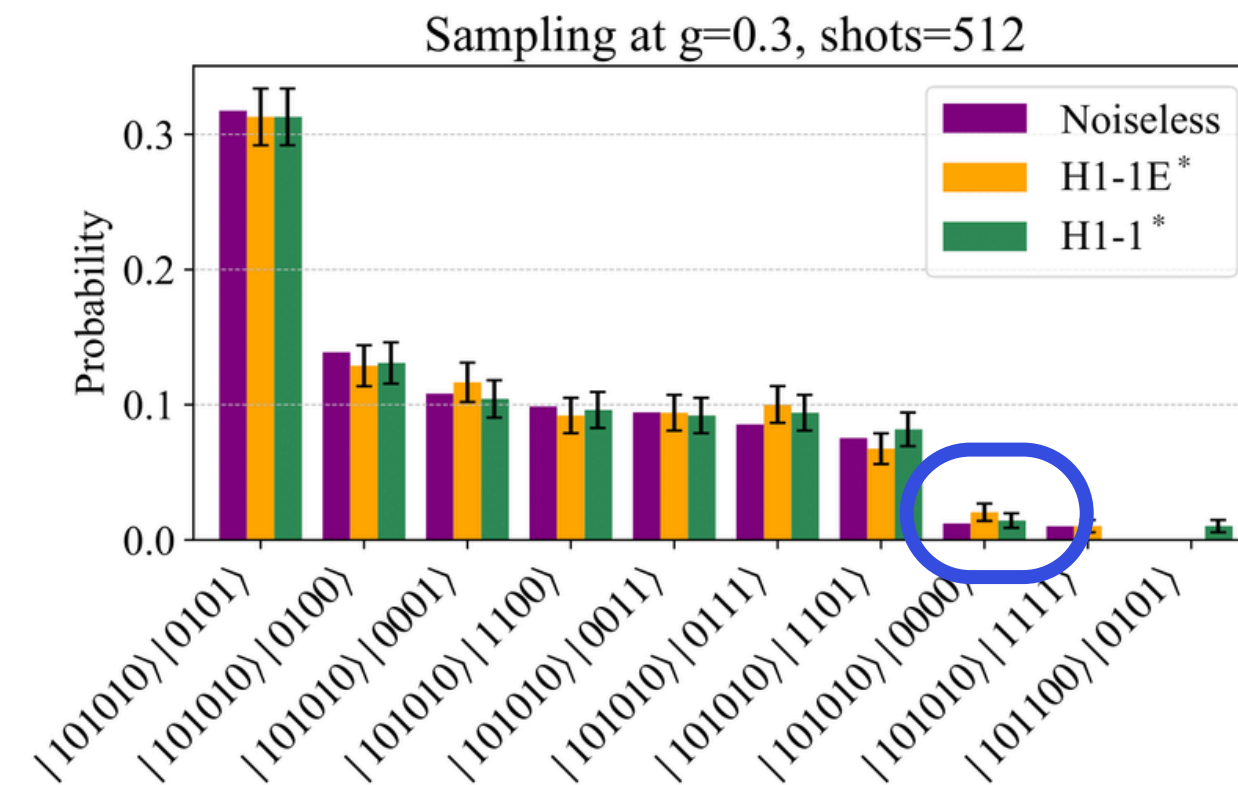
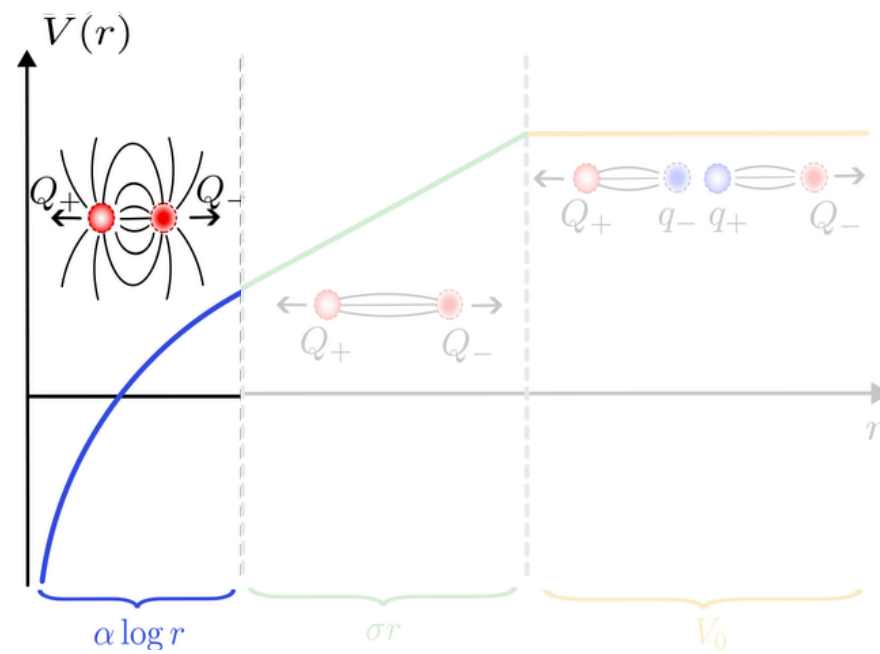
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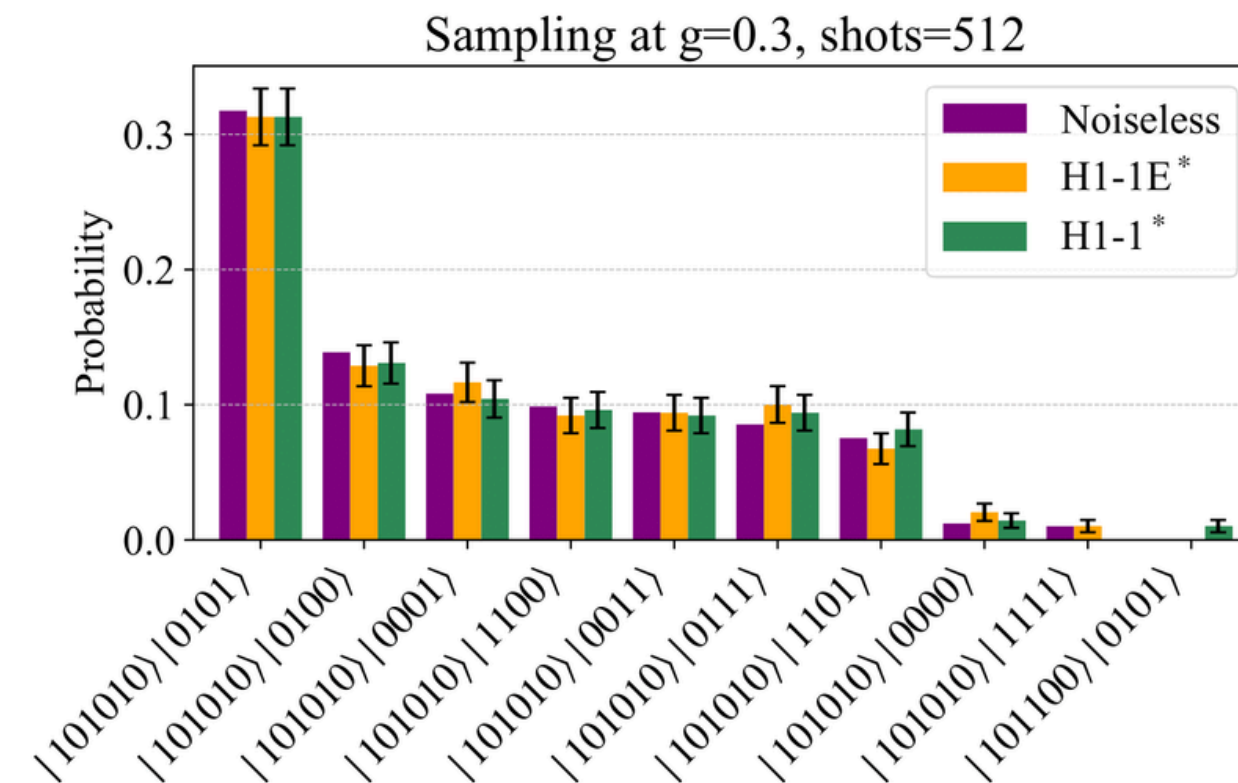
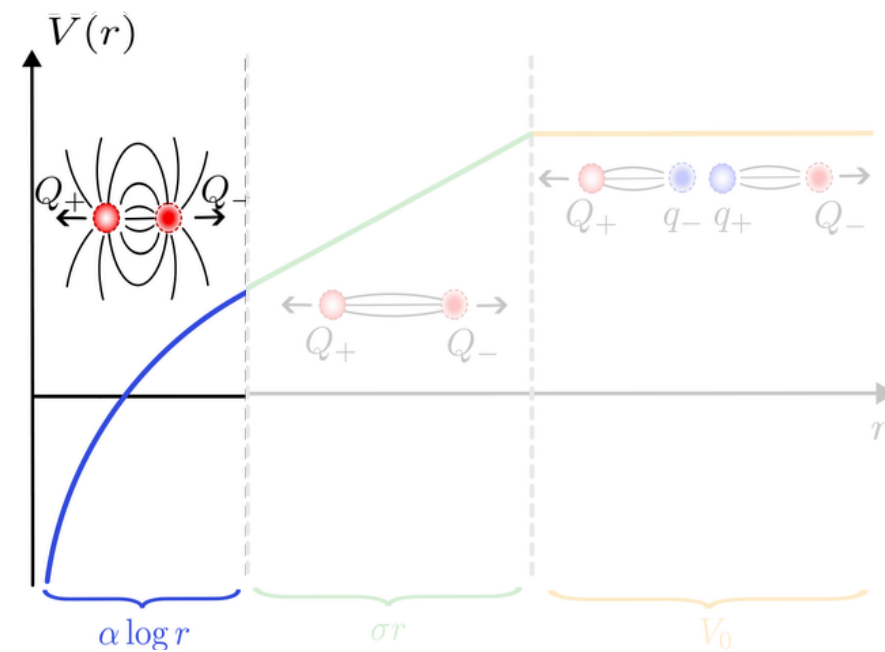
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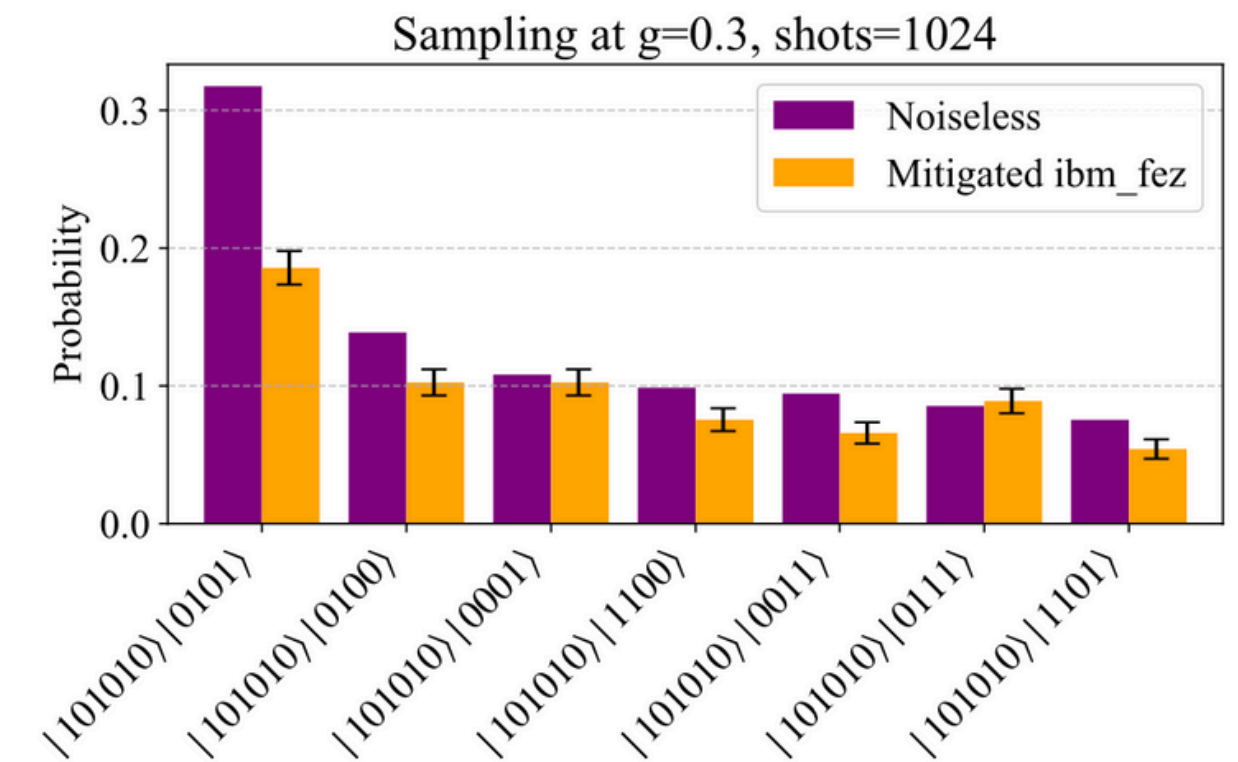
Quantum Hardware Results

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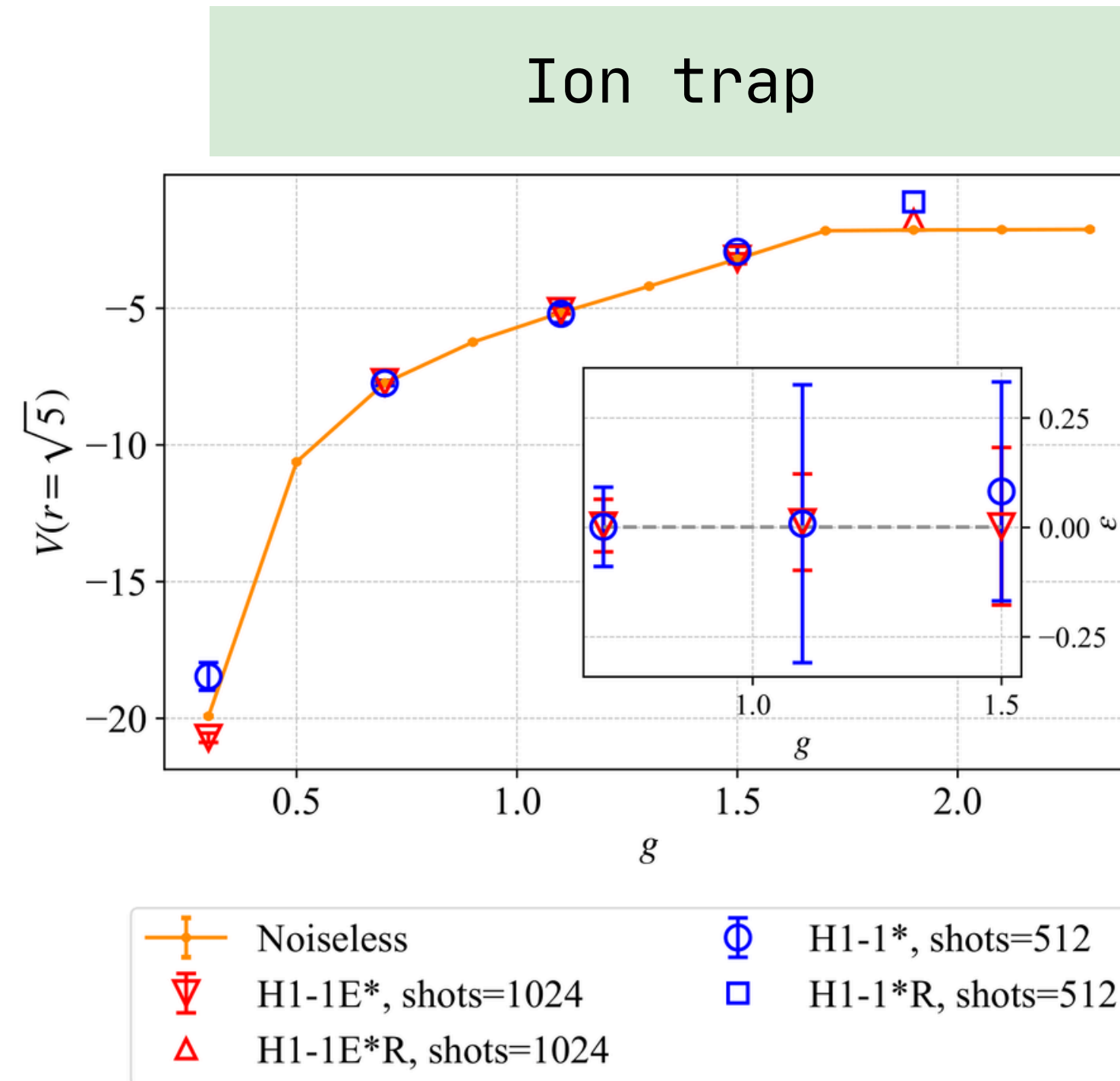
A. Crippa, PhD thesis

Quantum Hardware Results

Static potential

Quantum Hardware Results

Static potential

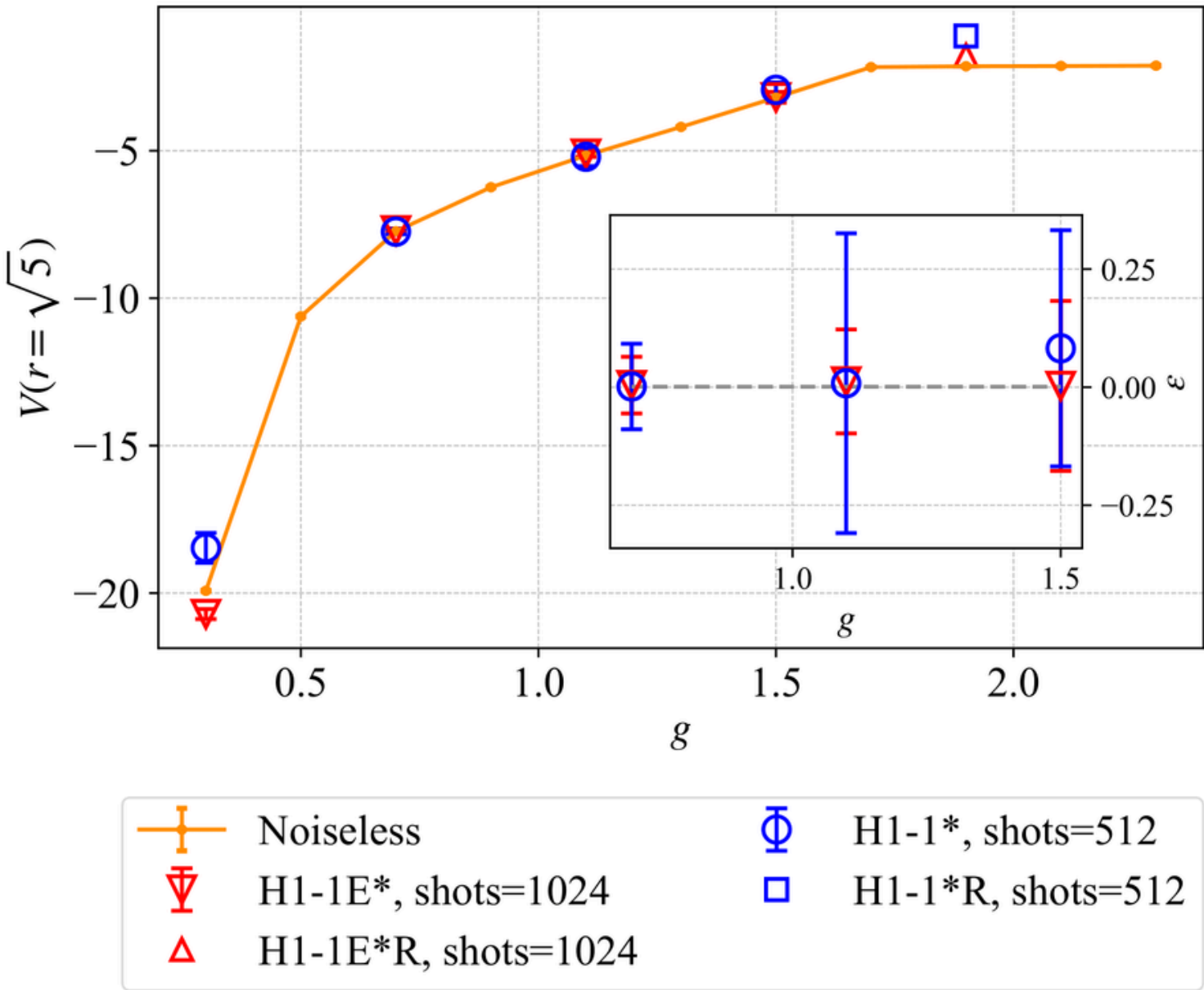


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Quantum Hardware Results

Static potential

Ion trap



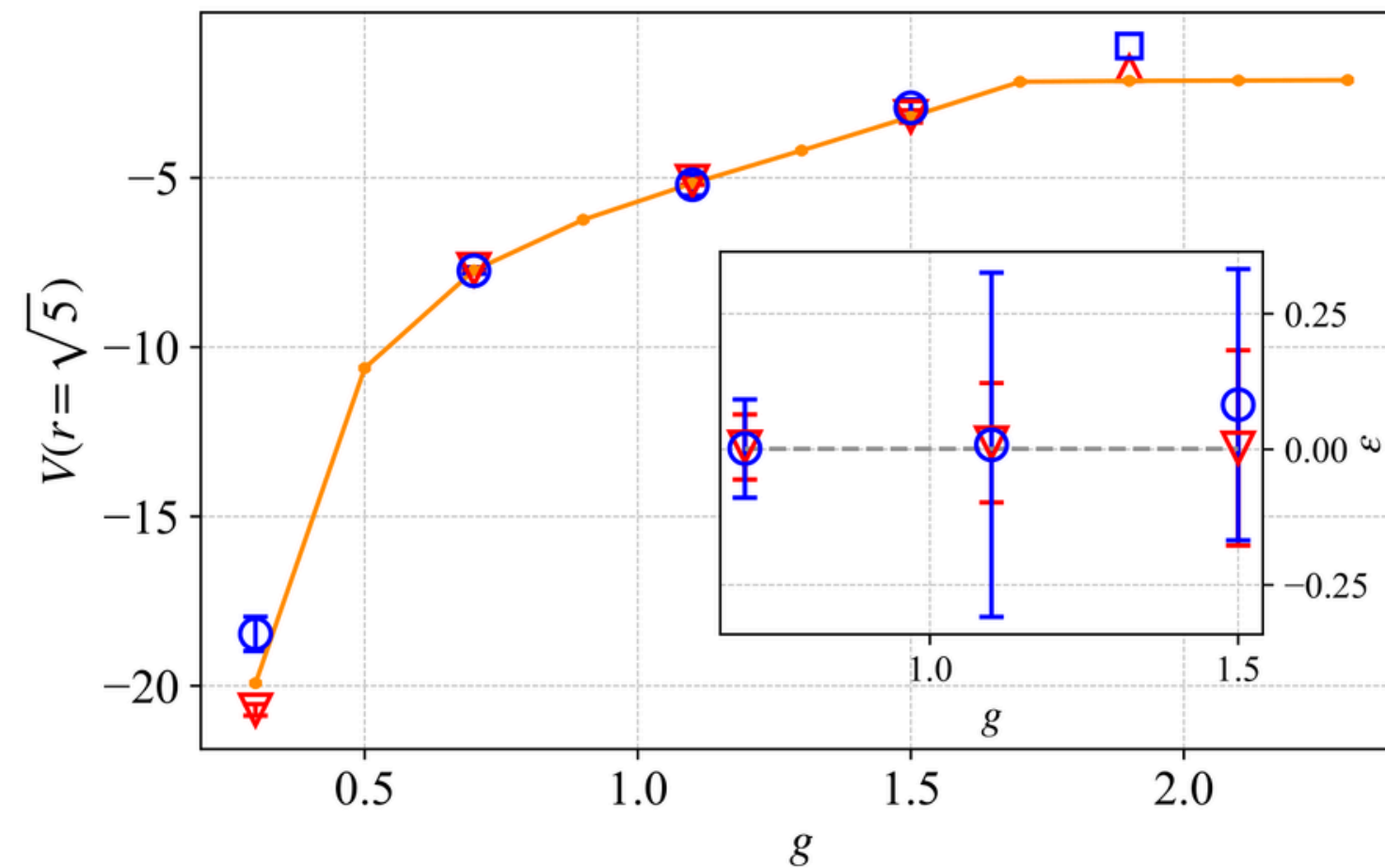
Emulator

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Quantum Hardware Results

Static potential

Ion trap



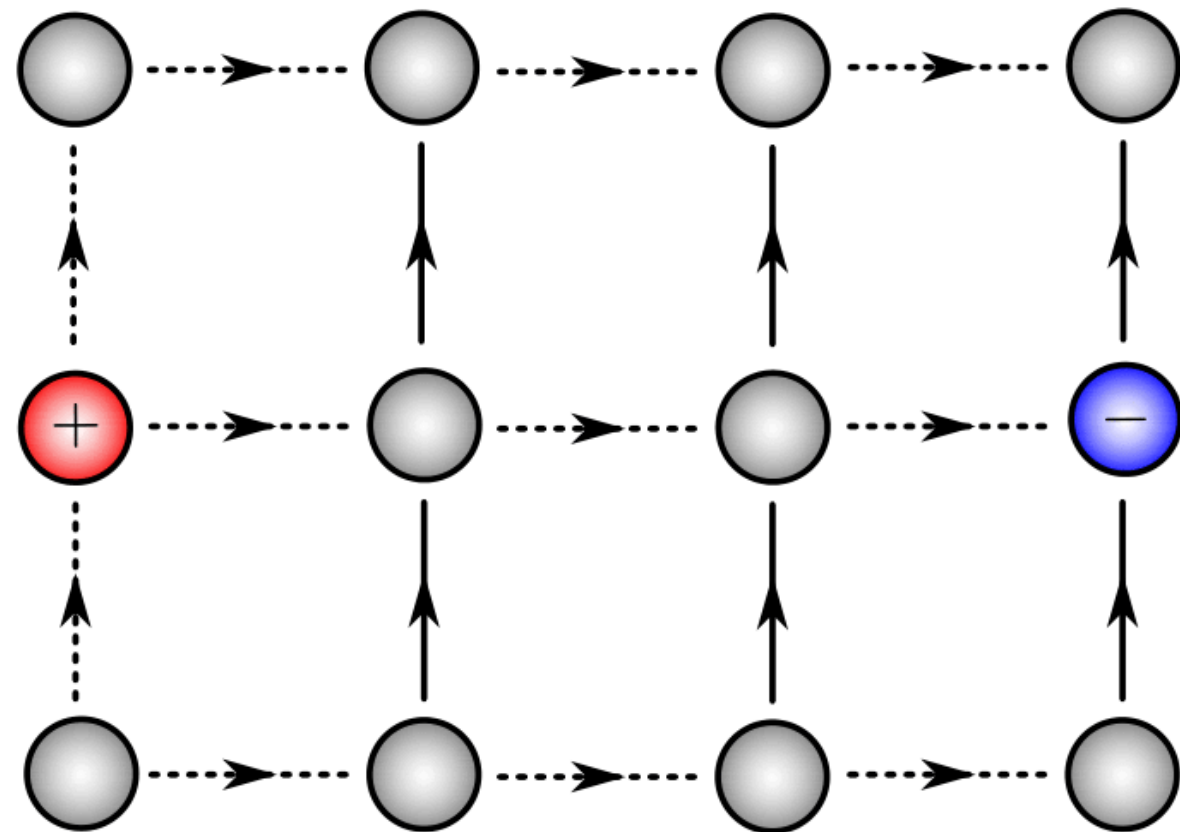
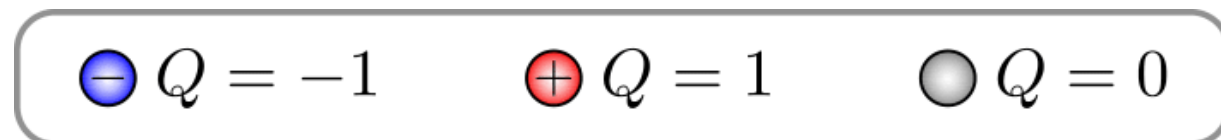
Emulator

Real hardware

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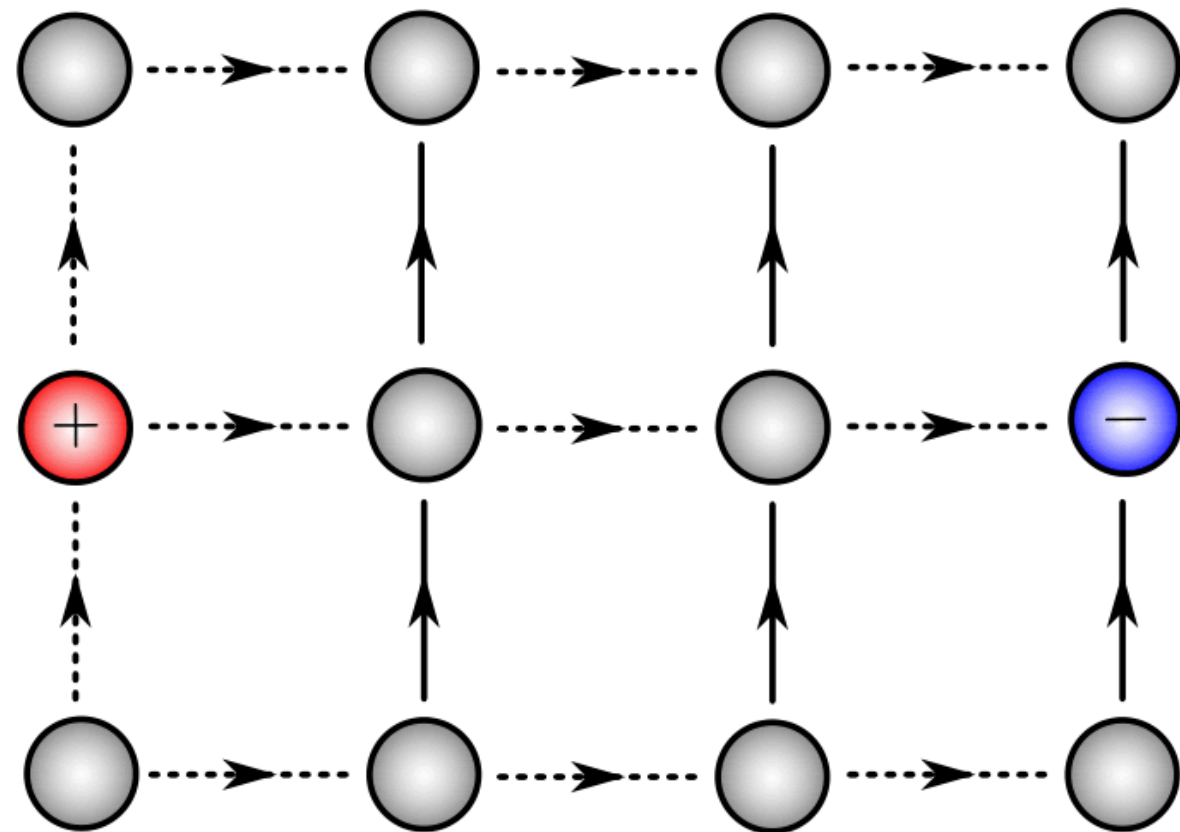
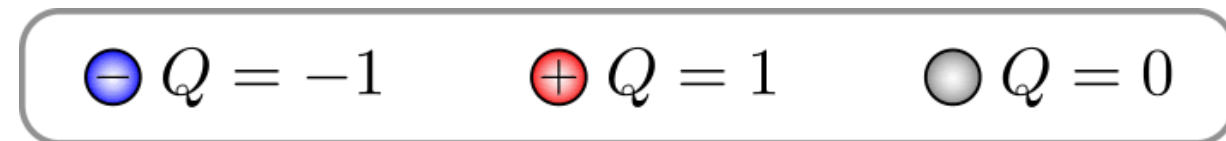
Quantum circuit

4x3 system



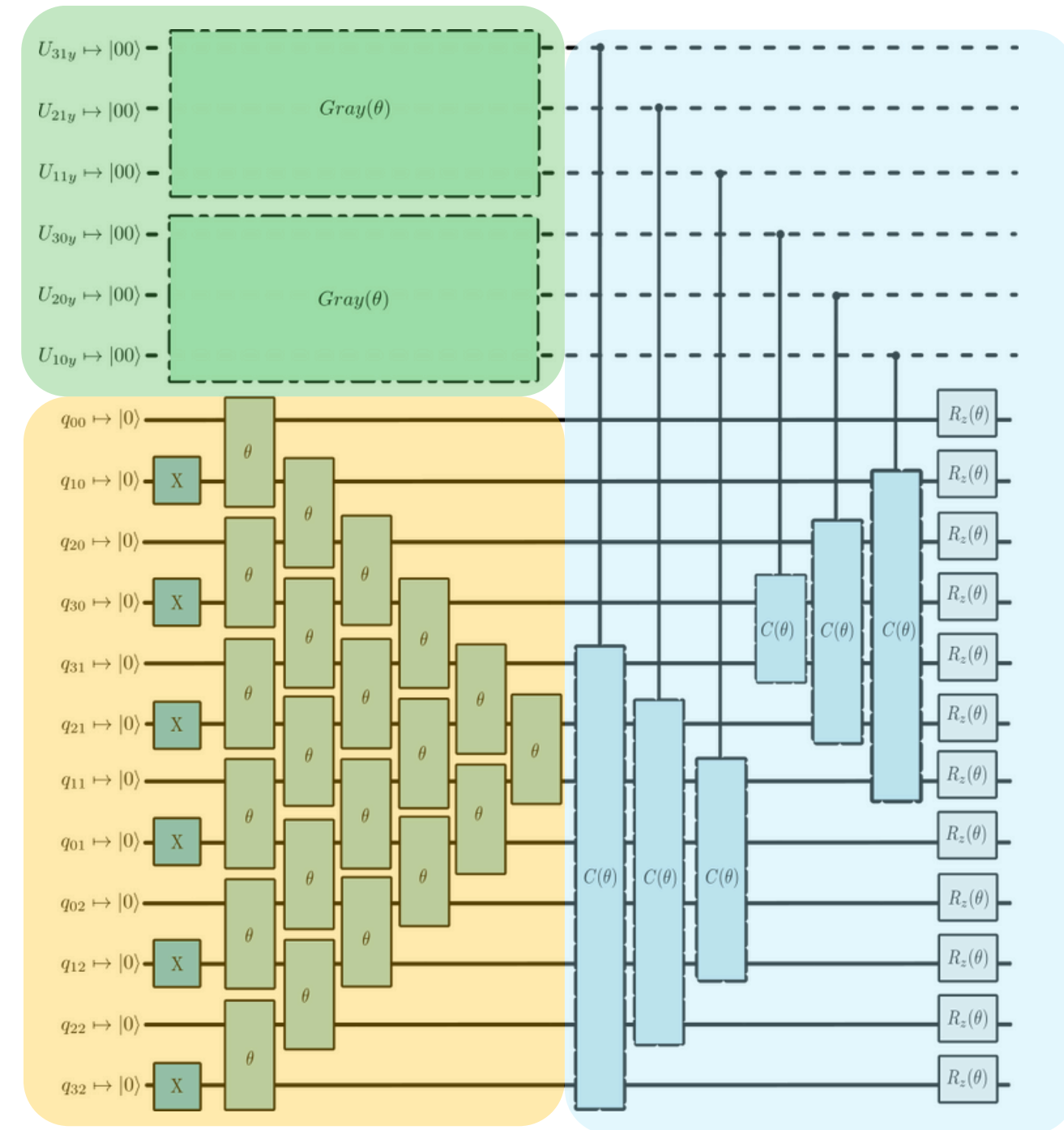
Quantum circuit

4x3 system



Gauge
fields

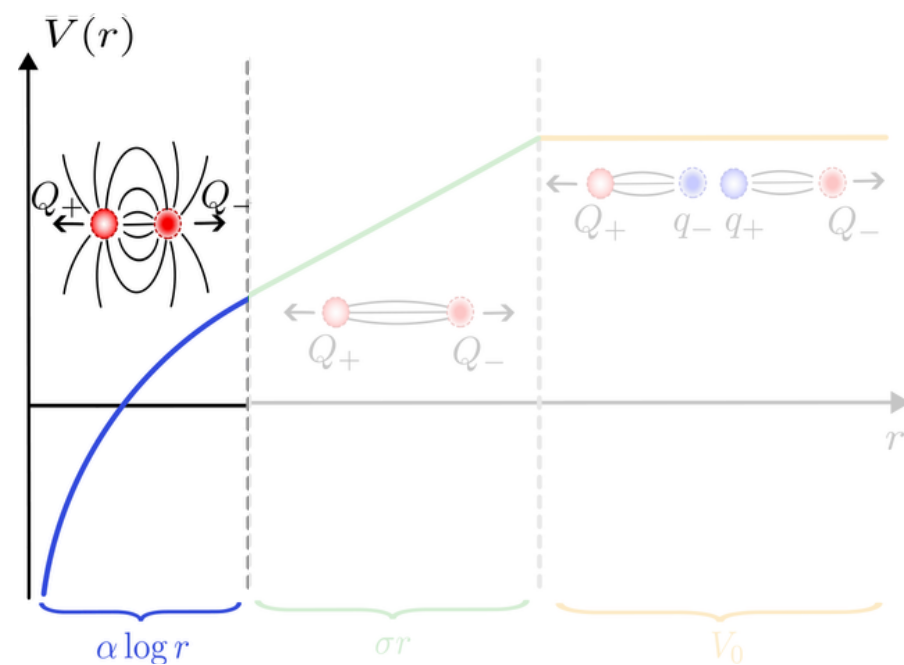
Fermions



Interaction

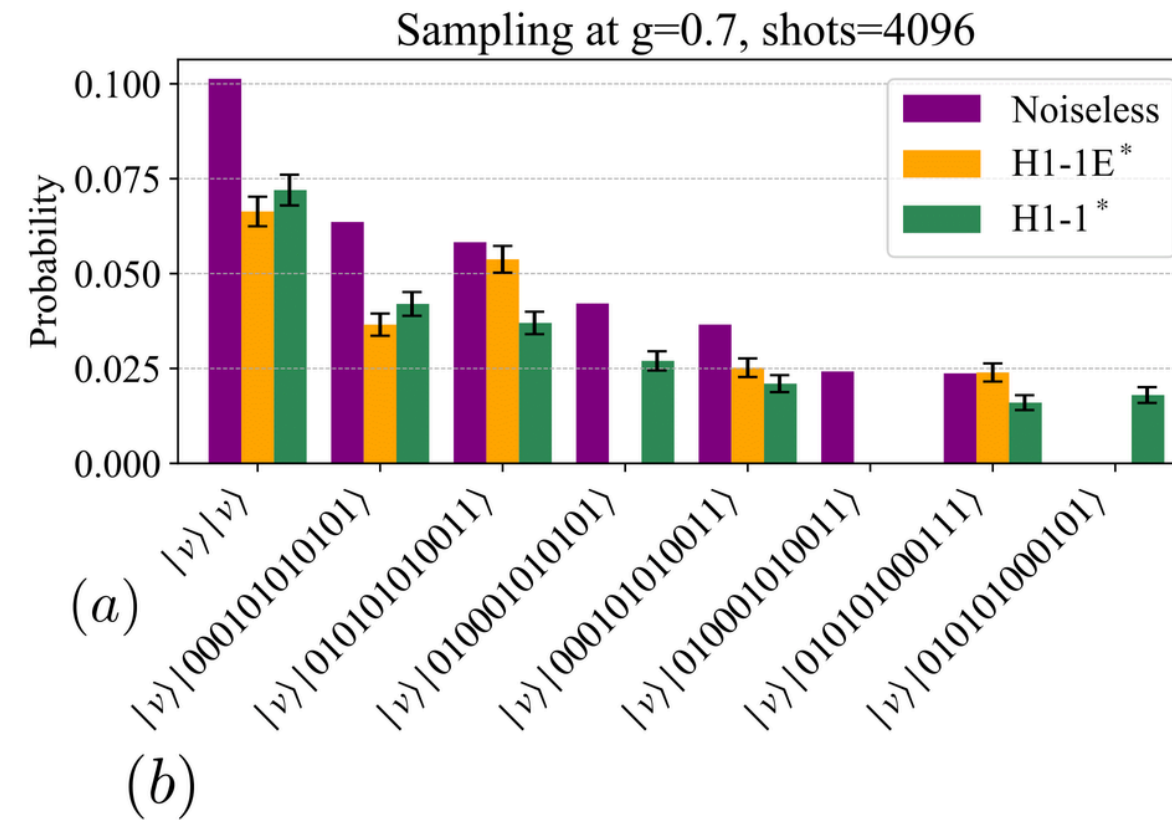
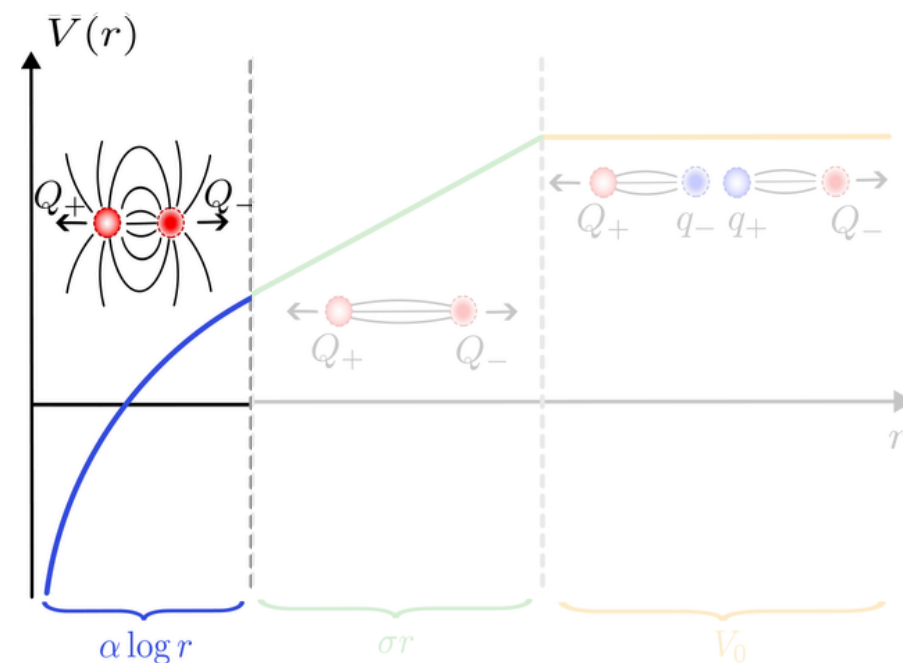
Quantum Hardware Results

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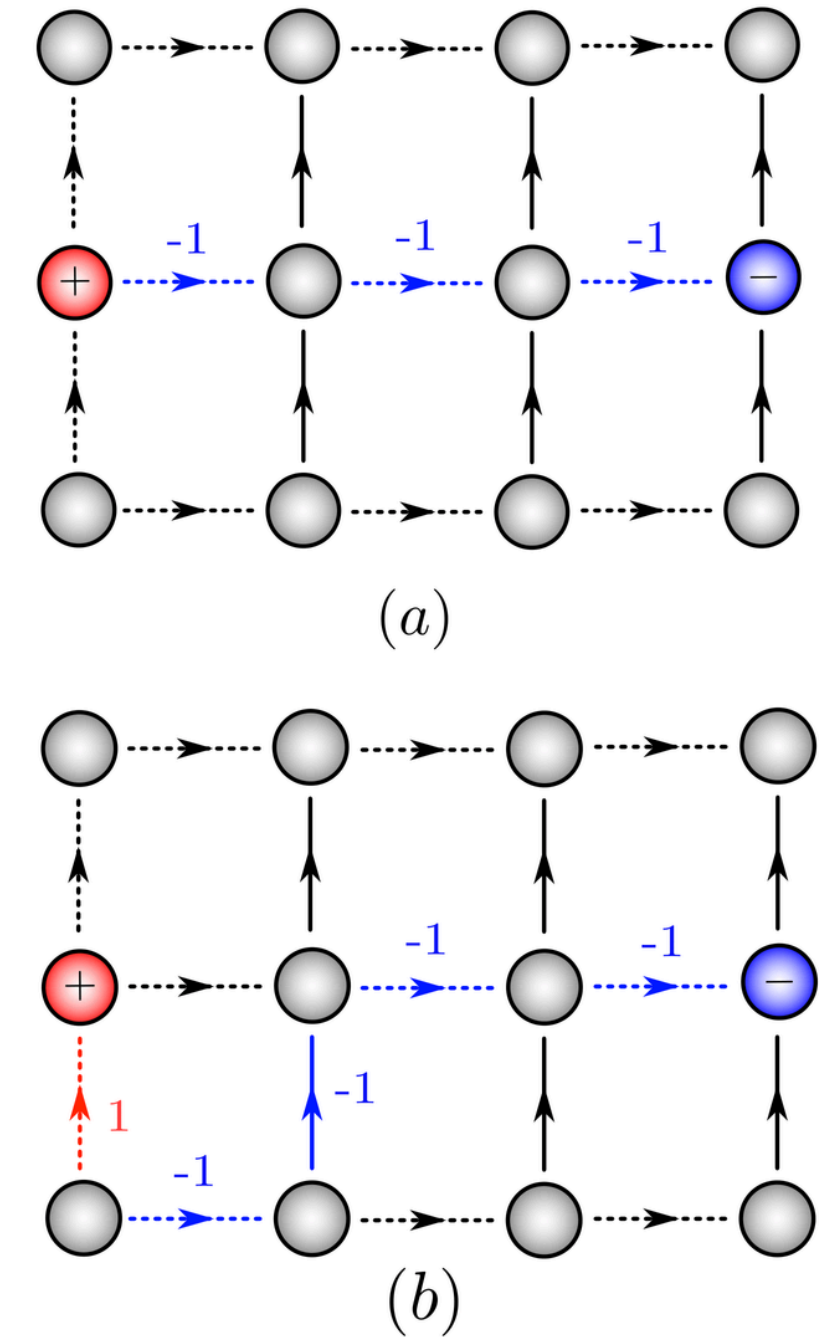
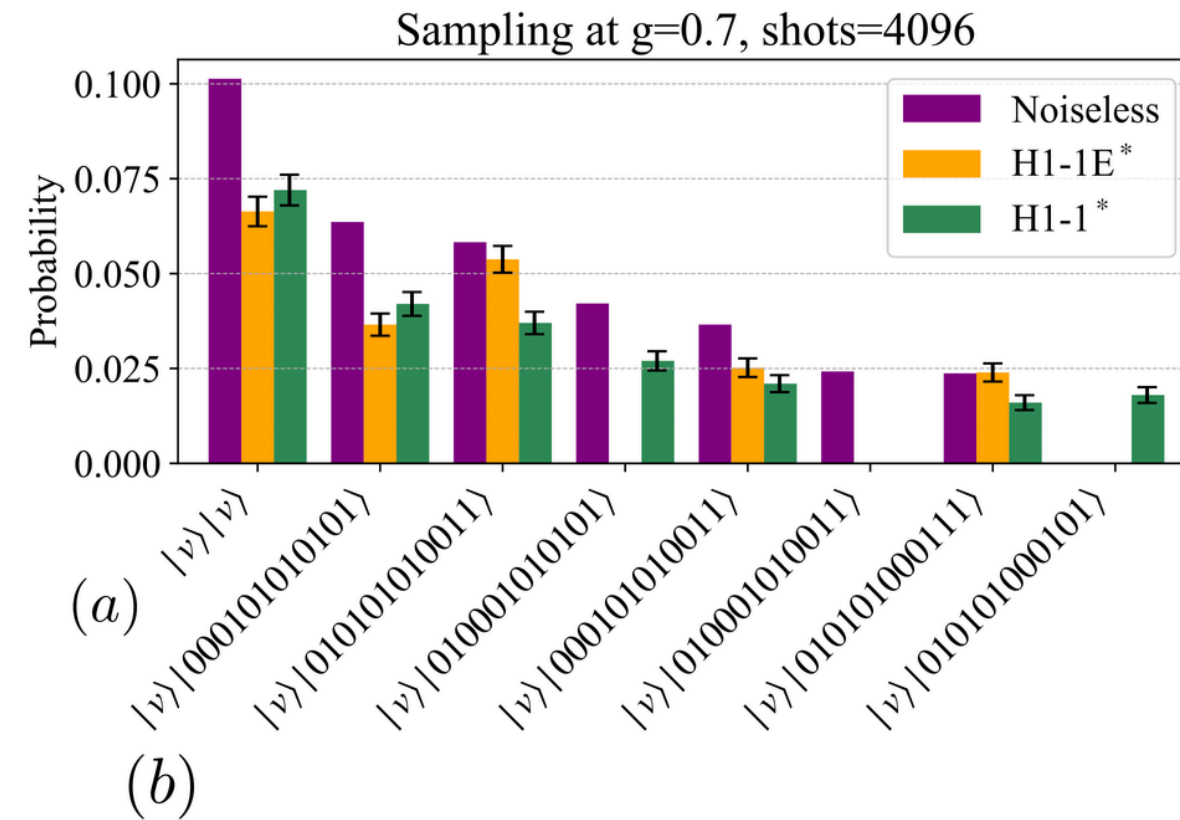
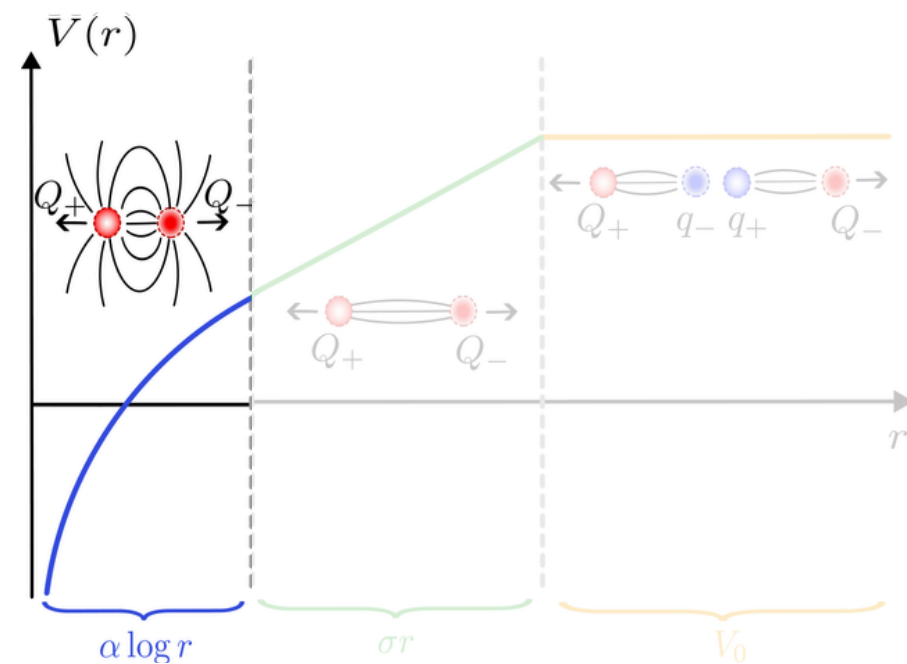
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Quantum Hardware Results



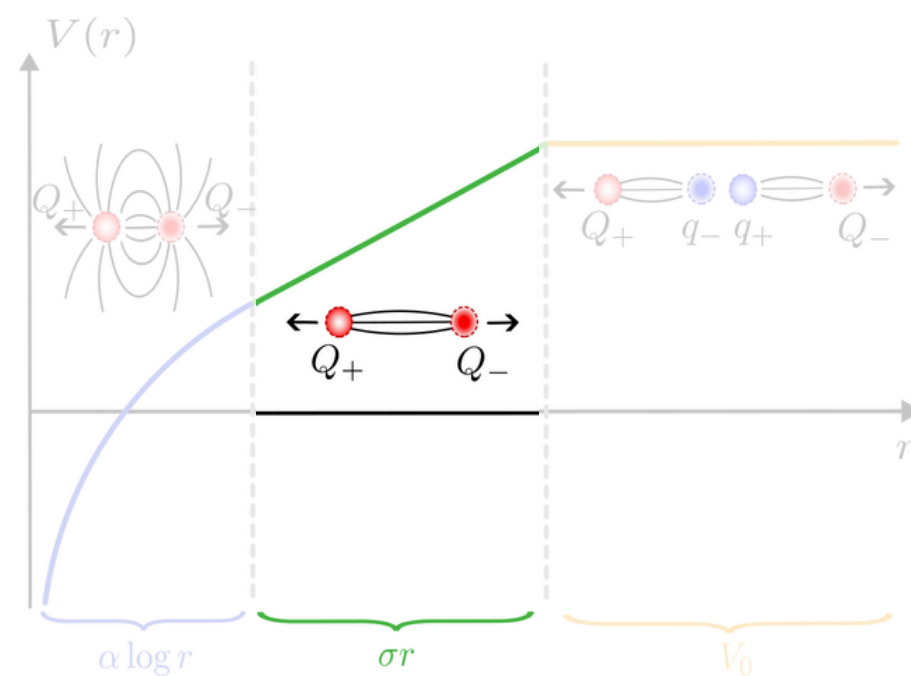
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Quantum Hardware Results



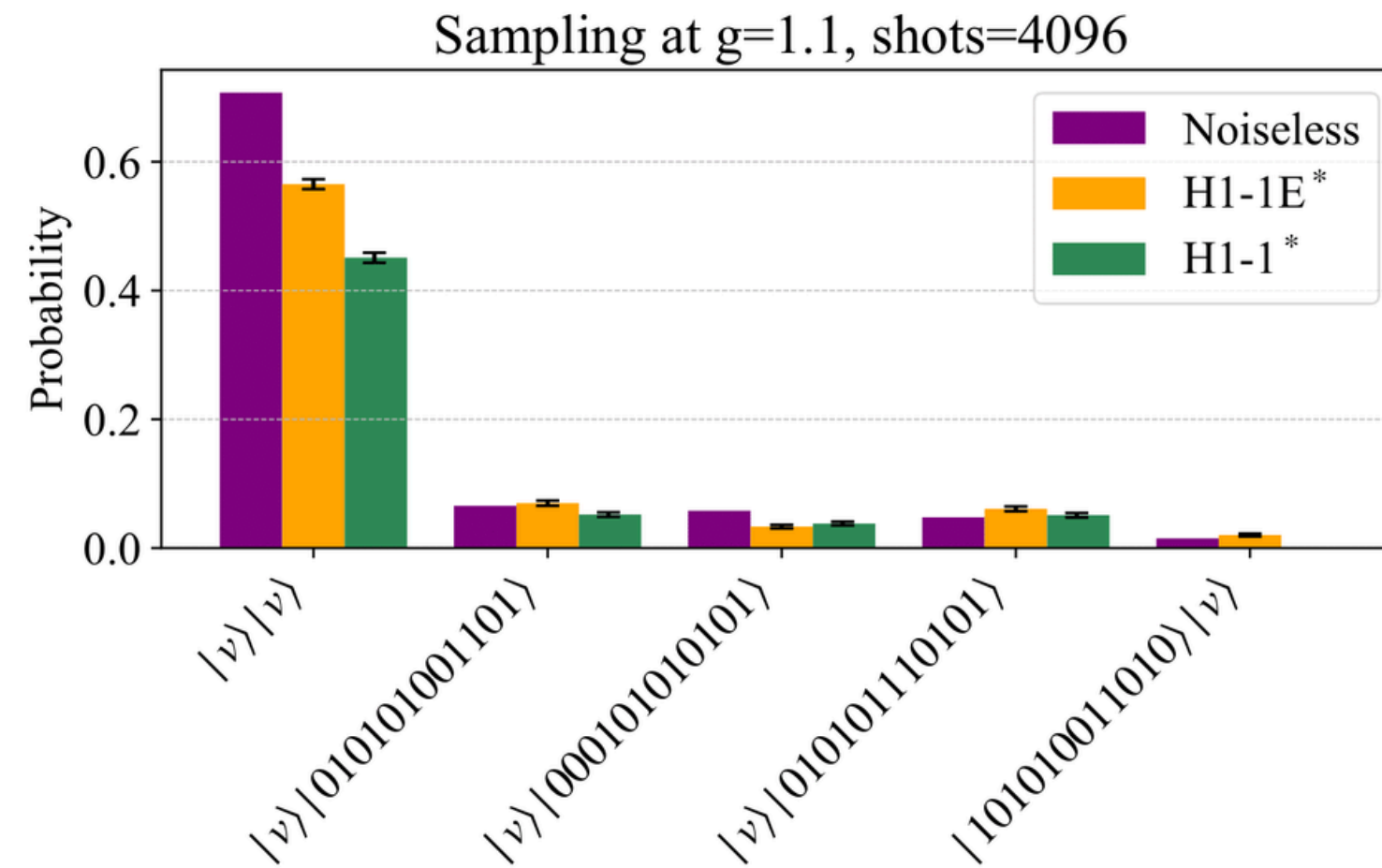
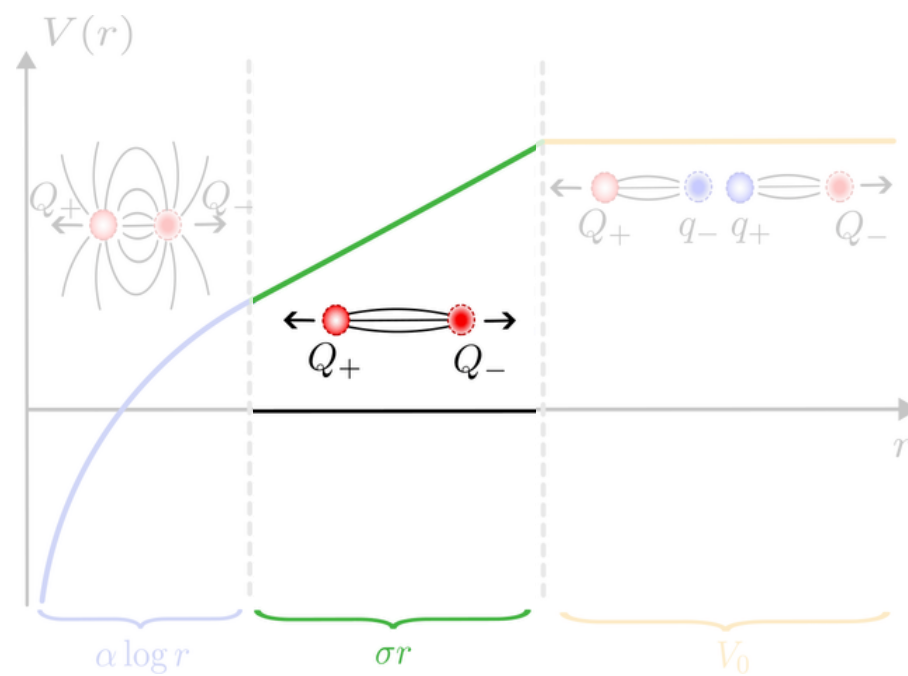
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Quantum Hardware Results



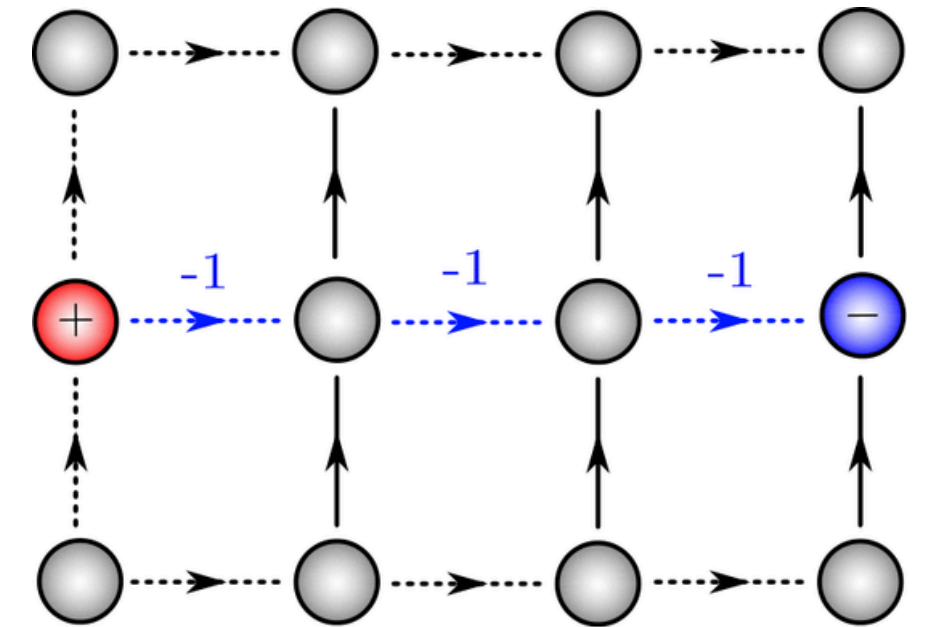
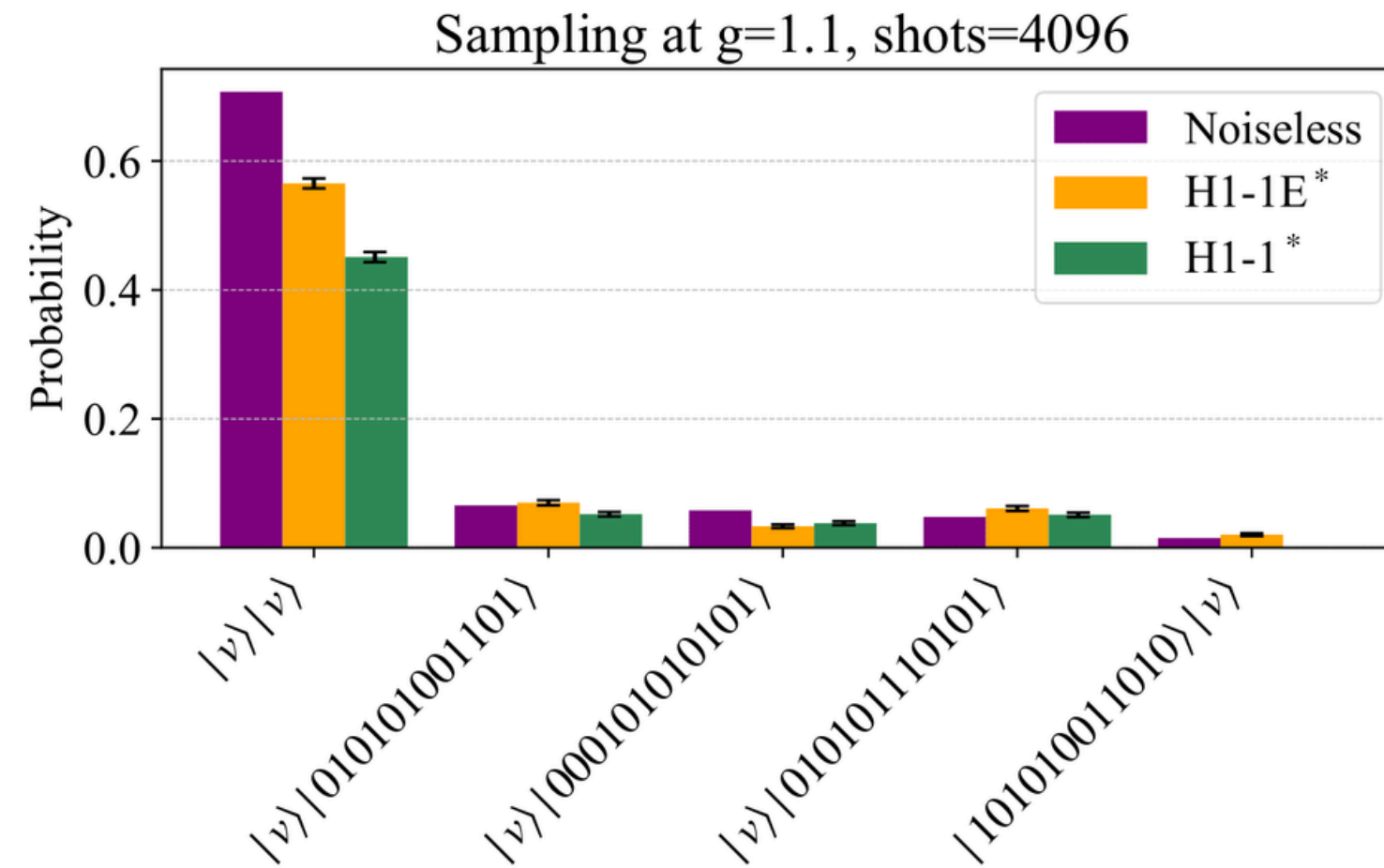
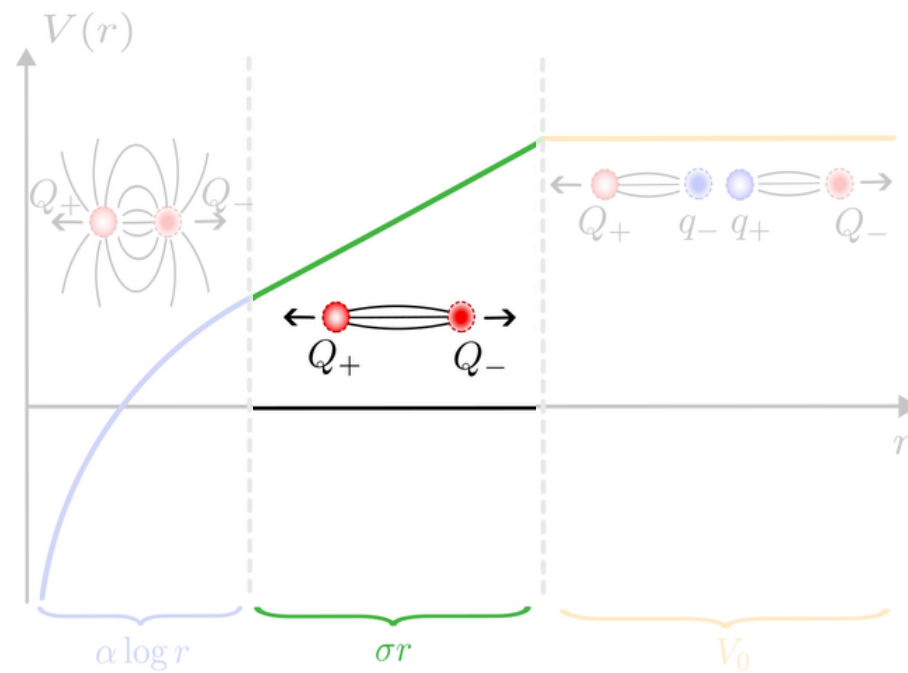
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Quantum Hardware Results



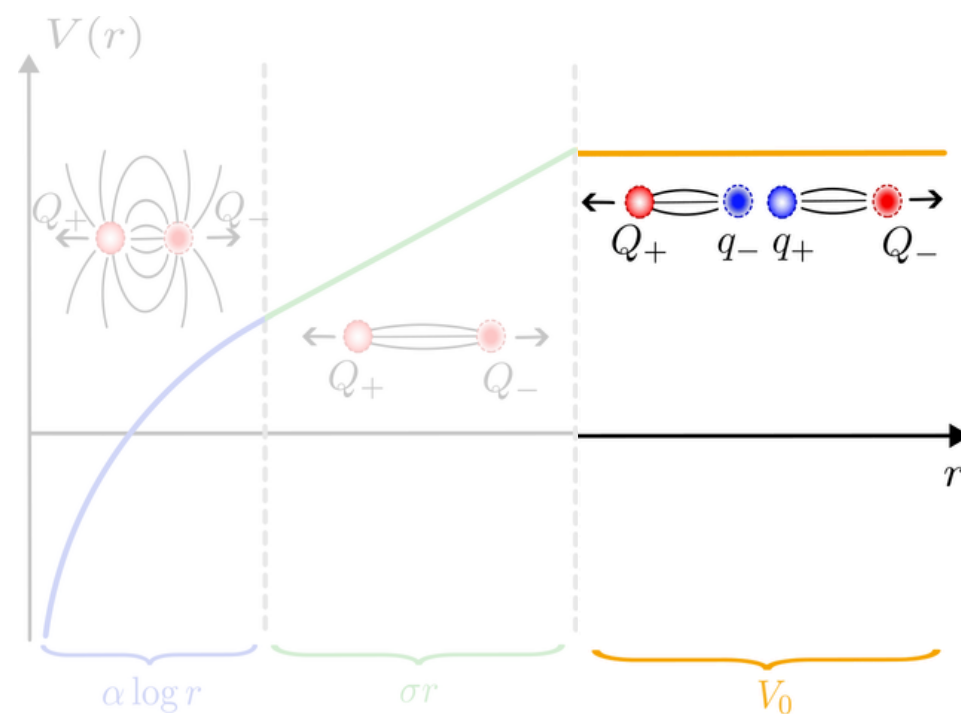
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Quantum Hardware Results



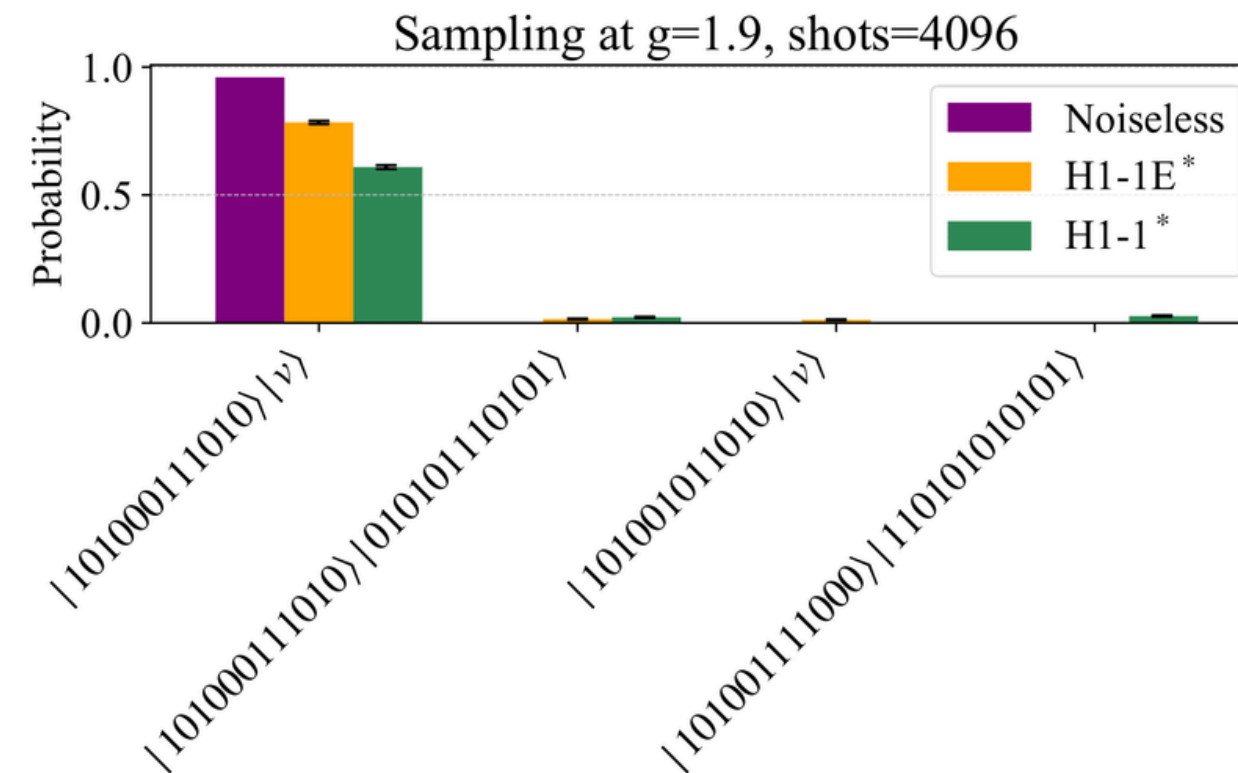
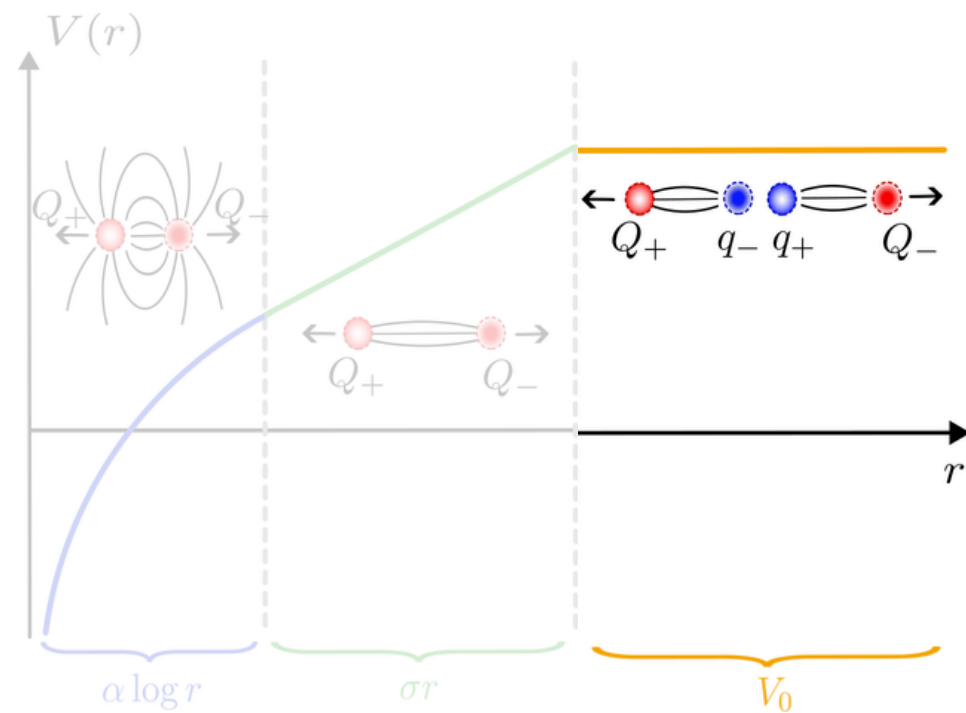
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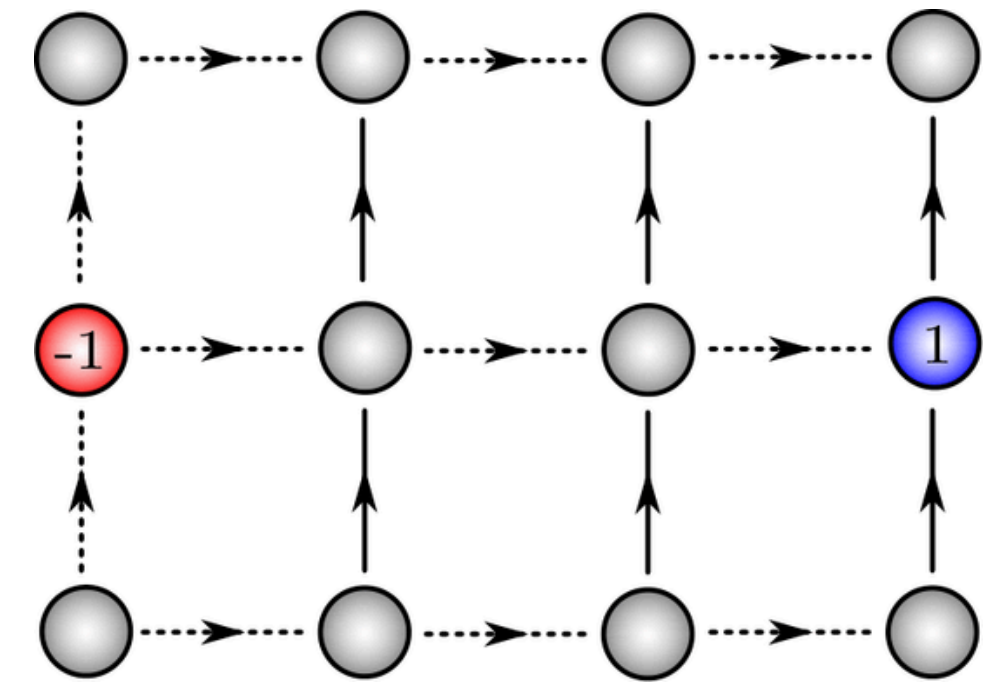
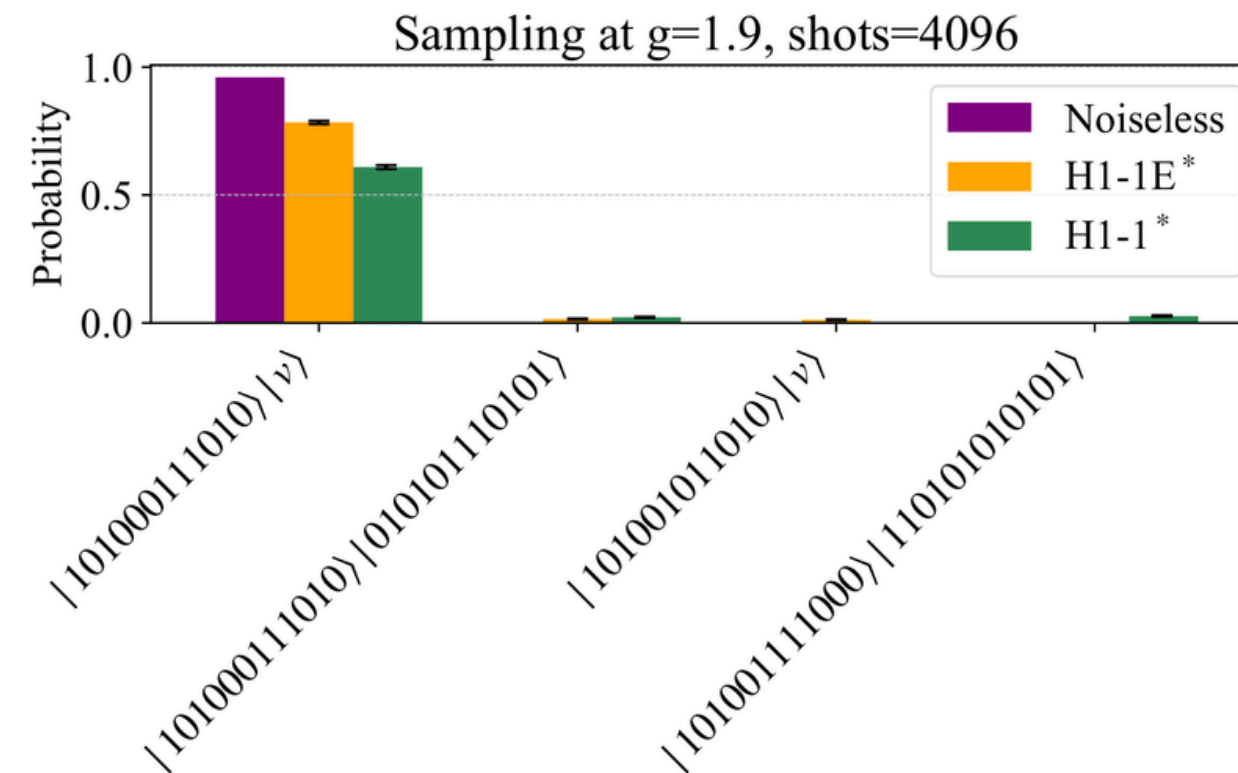
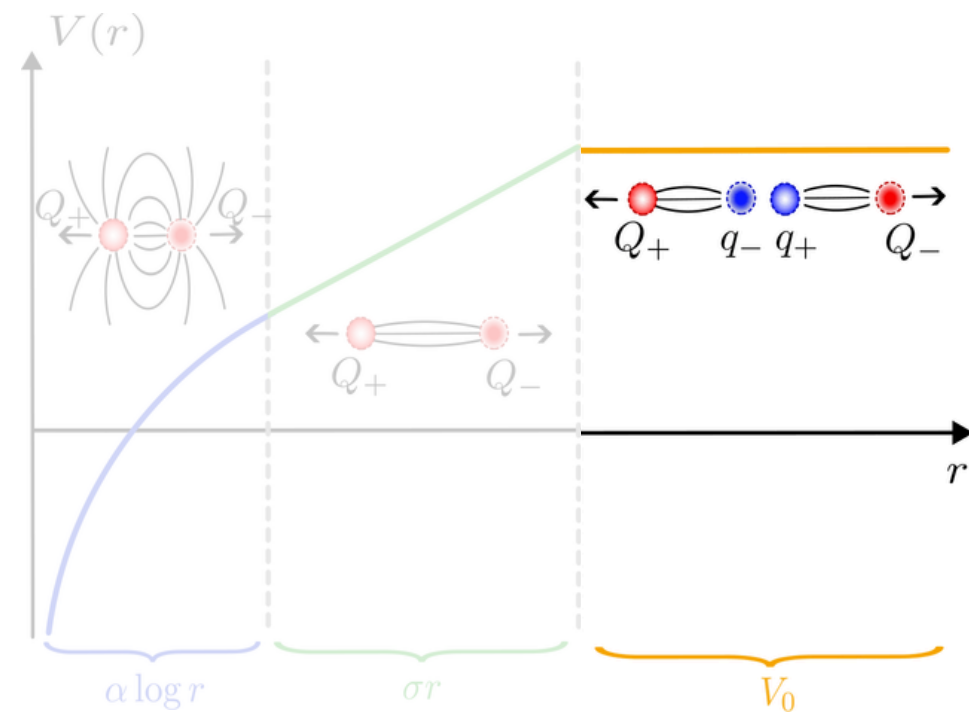
A. Crippa, K. Jansen, E. Rinaldi, arXiv:2411.05628

Quantum Hardware Results



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Conclusions

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- Visualize confining electric fluxes and string breaking.
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- Results with real quantum hardware.

Thank you for your attention!

BACKUP SLIDES

Encoding

Gauge fields

Map with Gray encoding

$$\hat{E} \mapsto -|00\rangle\langle 00| + |11\rangle\langle 11| = -\frac{1}{2}[\sigma_0^z + \sigma_1^z]$$

$$\hat{U}^\dagger \mapsto |00\rangle\langle 01| + |01\rangle\langle 11| = \frac{1}{2}[\sigma_0^-(I_1 + \sigma_1^z) + \sigma_1^-(I_0 - \sigma_0^z)]$$

Fermions

Map to spins with **Jordan-Wigner** transformation

$$\hat{\phi}_j^\dagger = \left[\prod_{k < j} (i\sigma_k^z) \right] \sigma_j^- \quad \hat{\phi}_j = \left[\prod_{k < j} (-i\sigma_k^z) \right] \sigma_j^+$$

U operator

Example $\ell=2$ U(lowering)

$$U_{\text{ladder}} = \begin{pmatrix} 0 & & & & \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & 1 & 0 & \\ & & & 1 & 0 \end{pmatrix} \quad U_{\text{spin}} = \frac{S^-}{\ell} = \frac{1}{2} \begin{pmatrix} 0 & & & & \\ 2 & 0 & & & \\ & \sqrt{6} & 0 & & \\ & & \sqrt{6} & 0 & \\ & & & 2 & 0 \end{pmatrix}$$

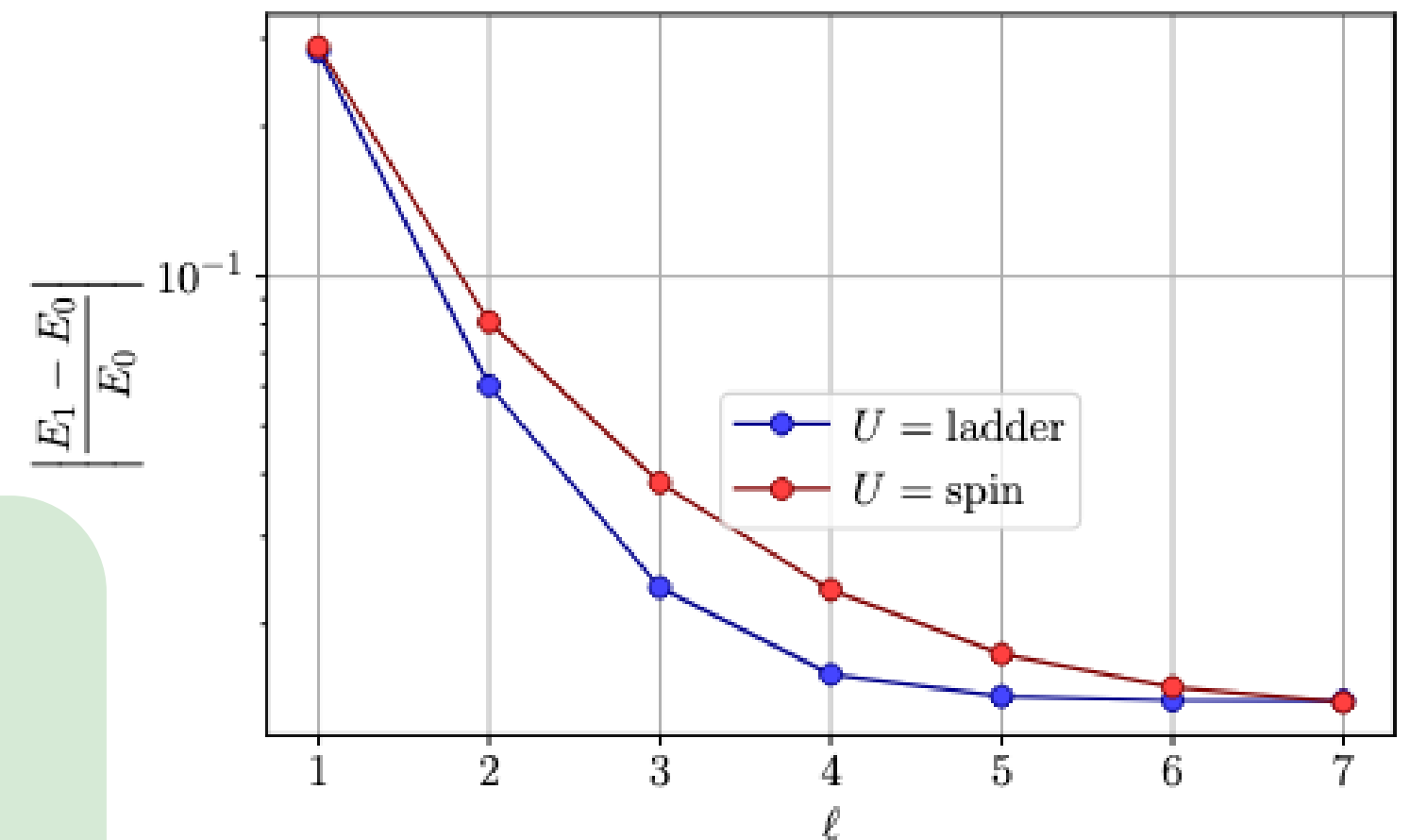


$$U_{\text{spin}} = \frac{1}{\ell} \sqrt{\ell(\ell+1) - m(m-1)} \delta_{m,m-1}$$

$$U_{\text{spin}}^\dagger = \frac{1}{\ell} \sqrt{\ell(\ell+1) - m(m+1)} \delta_{m,m+1}$$

$$E_{\text{spin}} = S^z = m \delta_{m,m} \quad m \in [-\ell, \ell]$$

Energy gap convergence

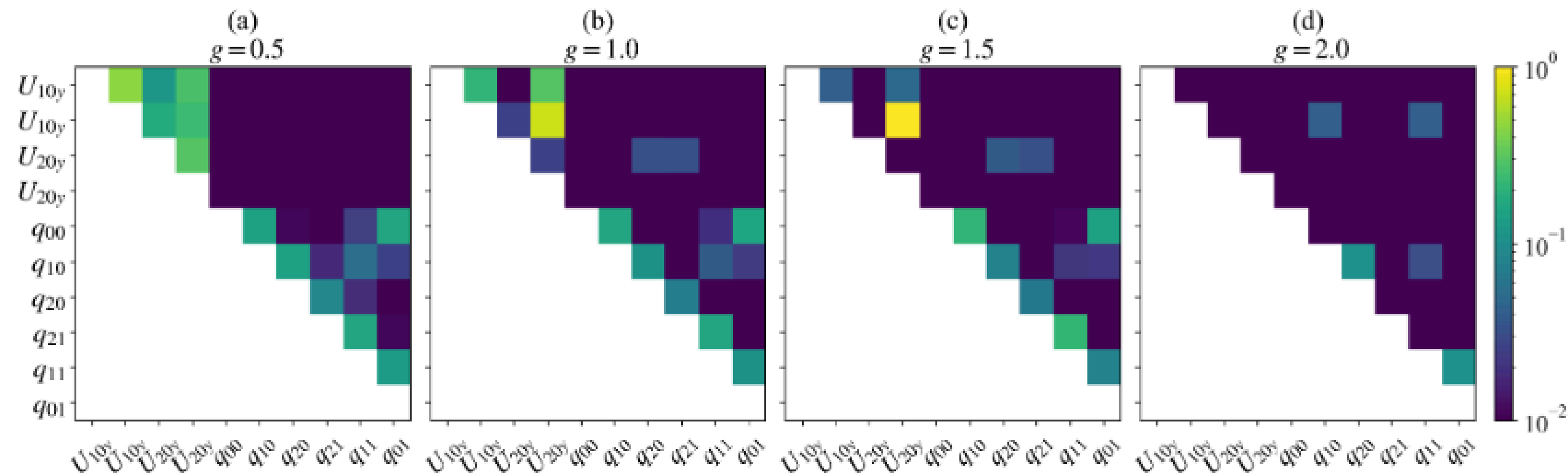


Mutual Information Ansatz

$$I(X; Y) = S(X) + S(Y) - S(X, Y)$$

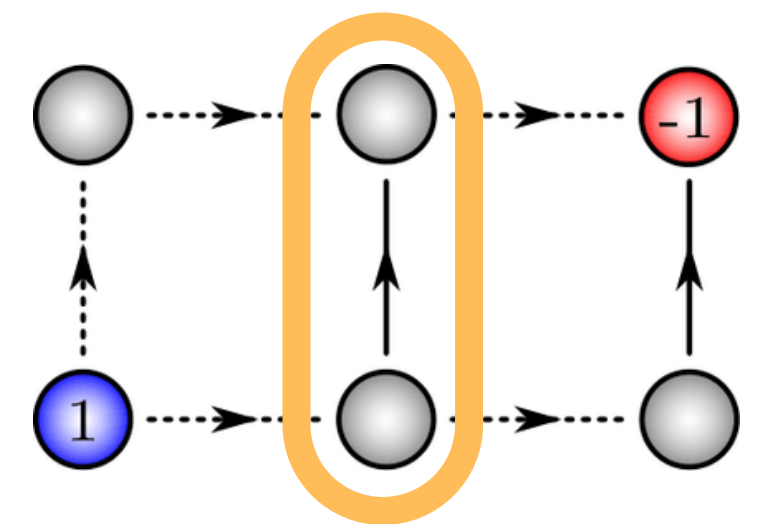
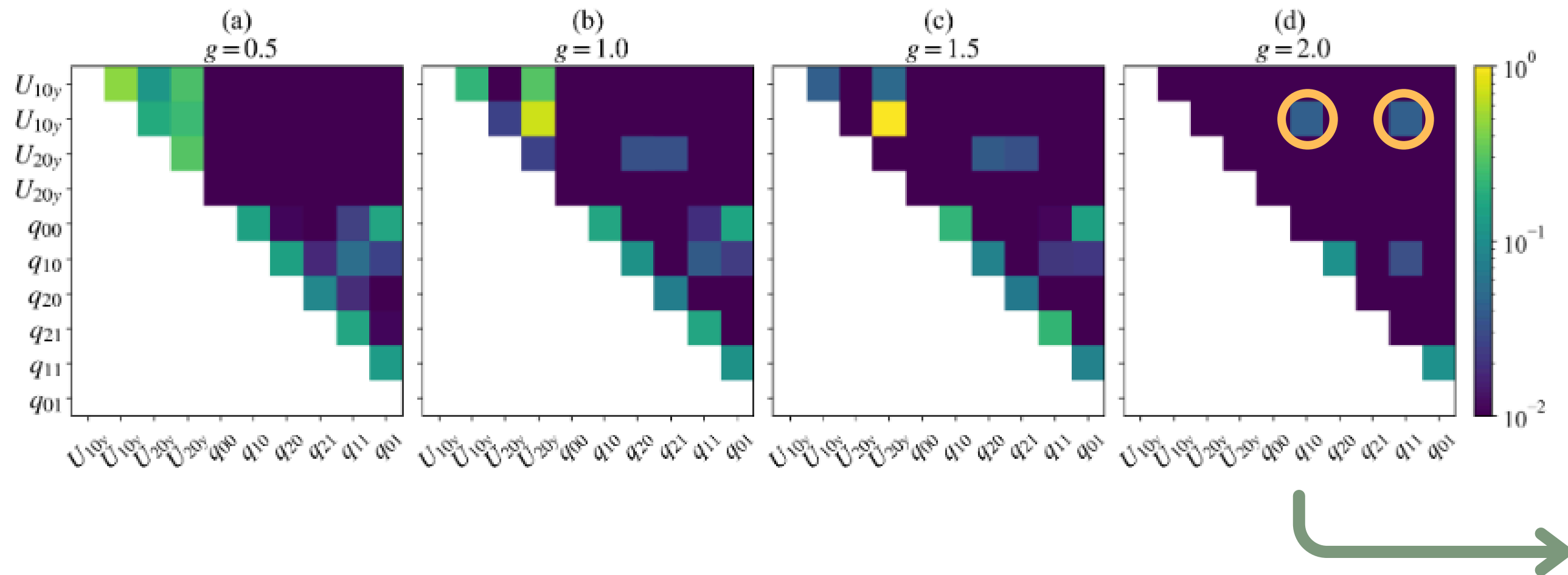
Mutual Information Ansatz

$$I(X; Y) = S(X) + S(Y) - S(X, Y)$$



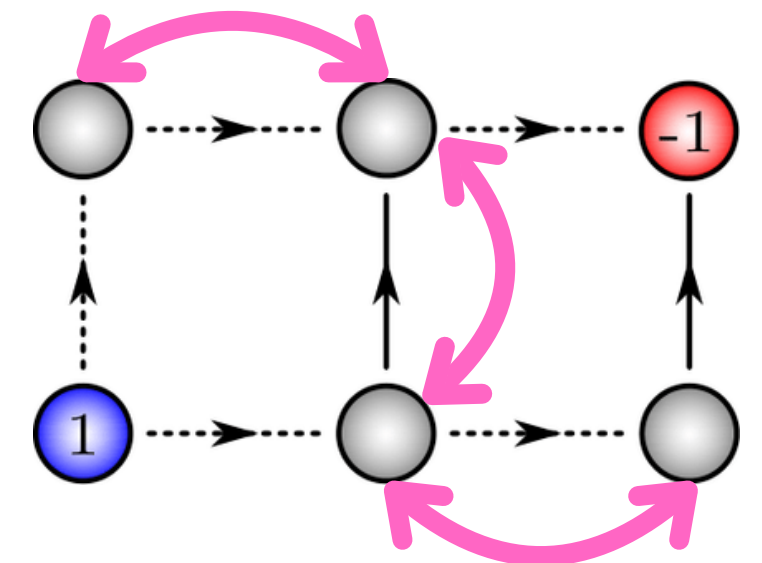
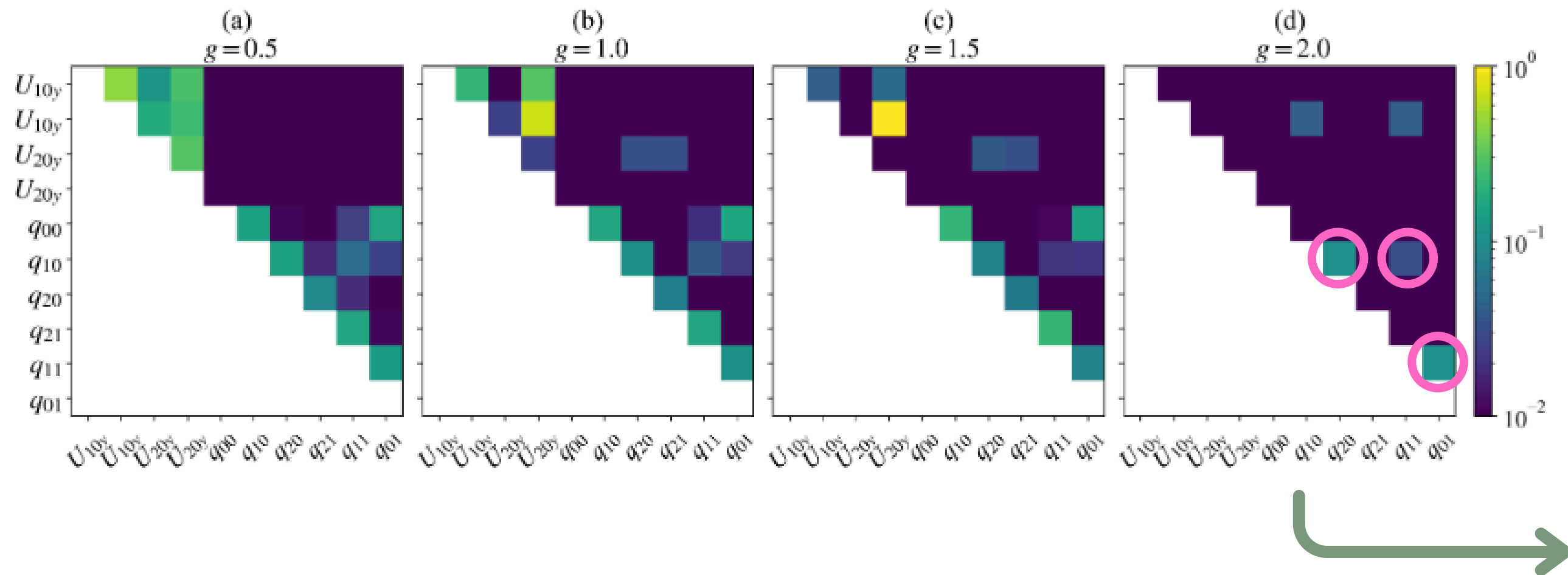
Mutual Information Ansatz

$$I(X; Y) = S(X) + S(Y) - S(X, Y)$$



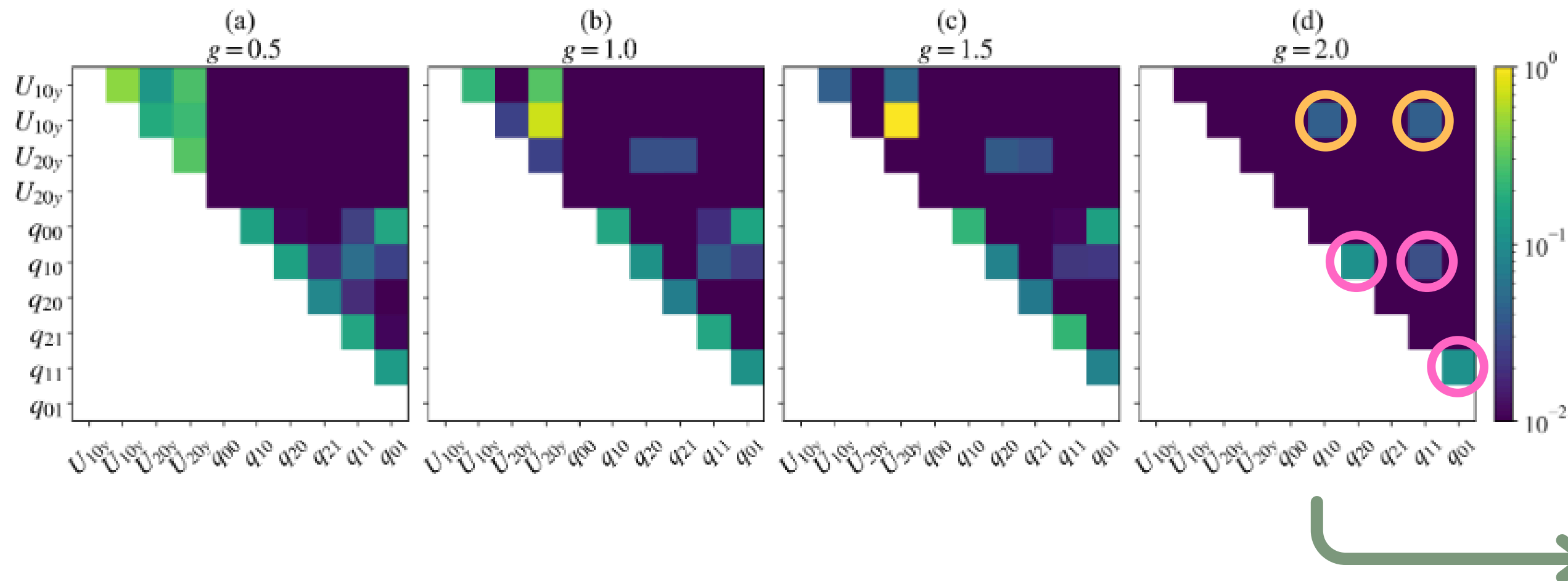
Mutual Information Ansatz

$$I(X; Y) = S(X) + S(Y) - S(X, Y)$$

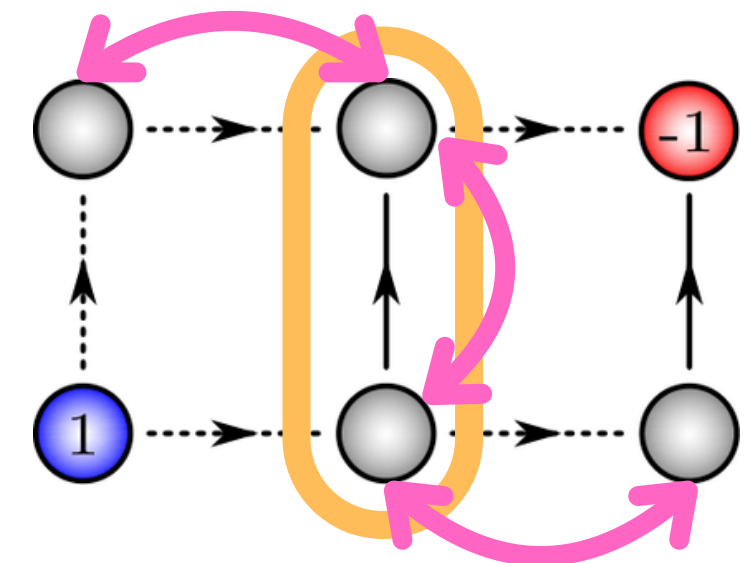


Mutual Information Ansatz

$$I(X; Y) = S(X) + S(Y) - S(X, Y)$$

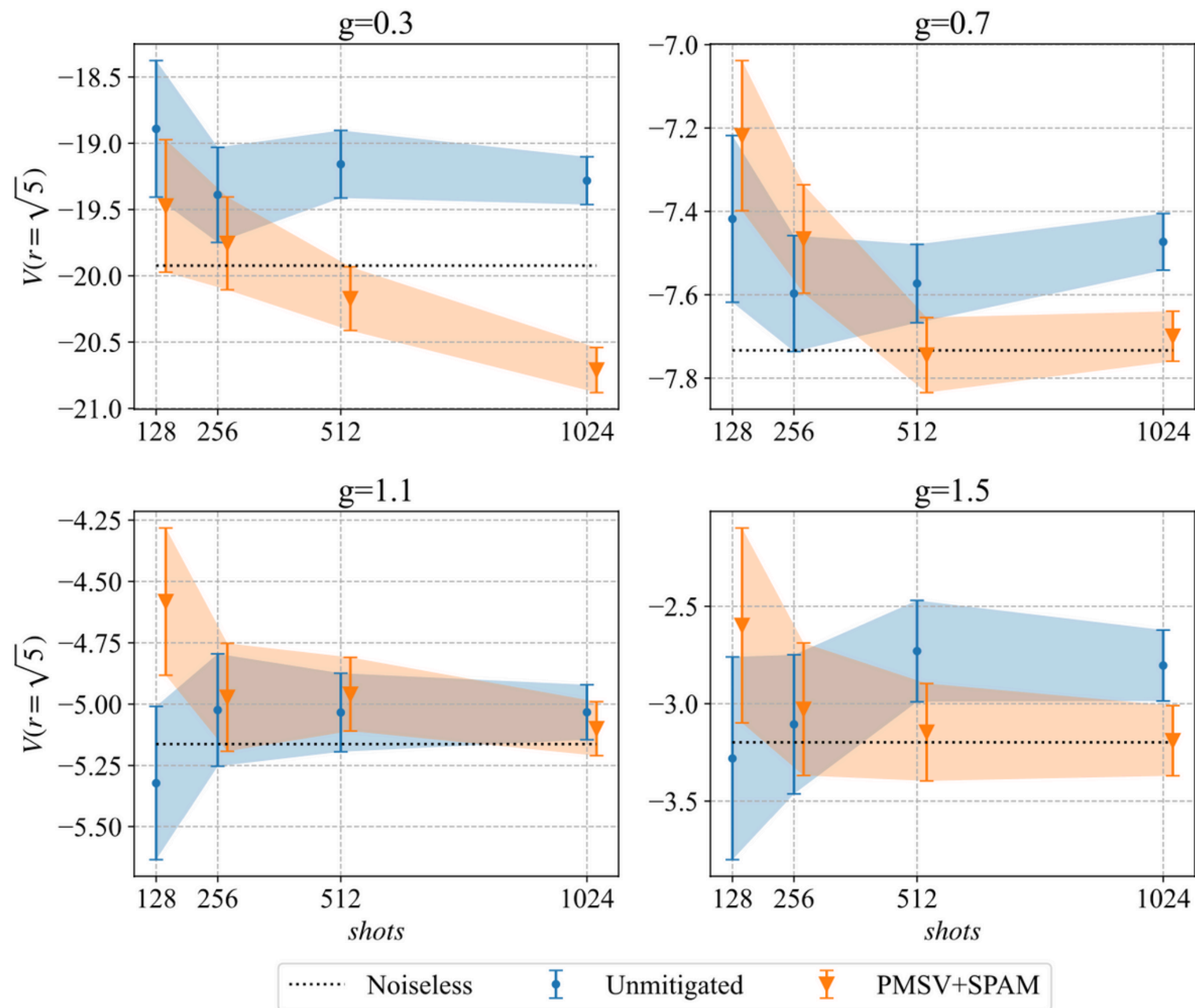


Dynamical charges+static charges are isolated.



H1-1E

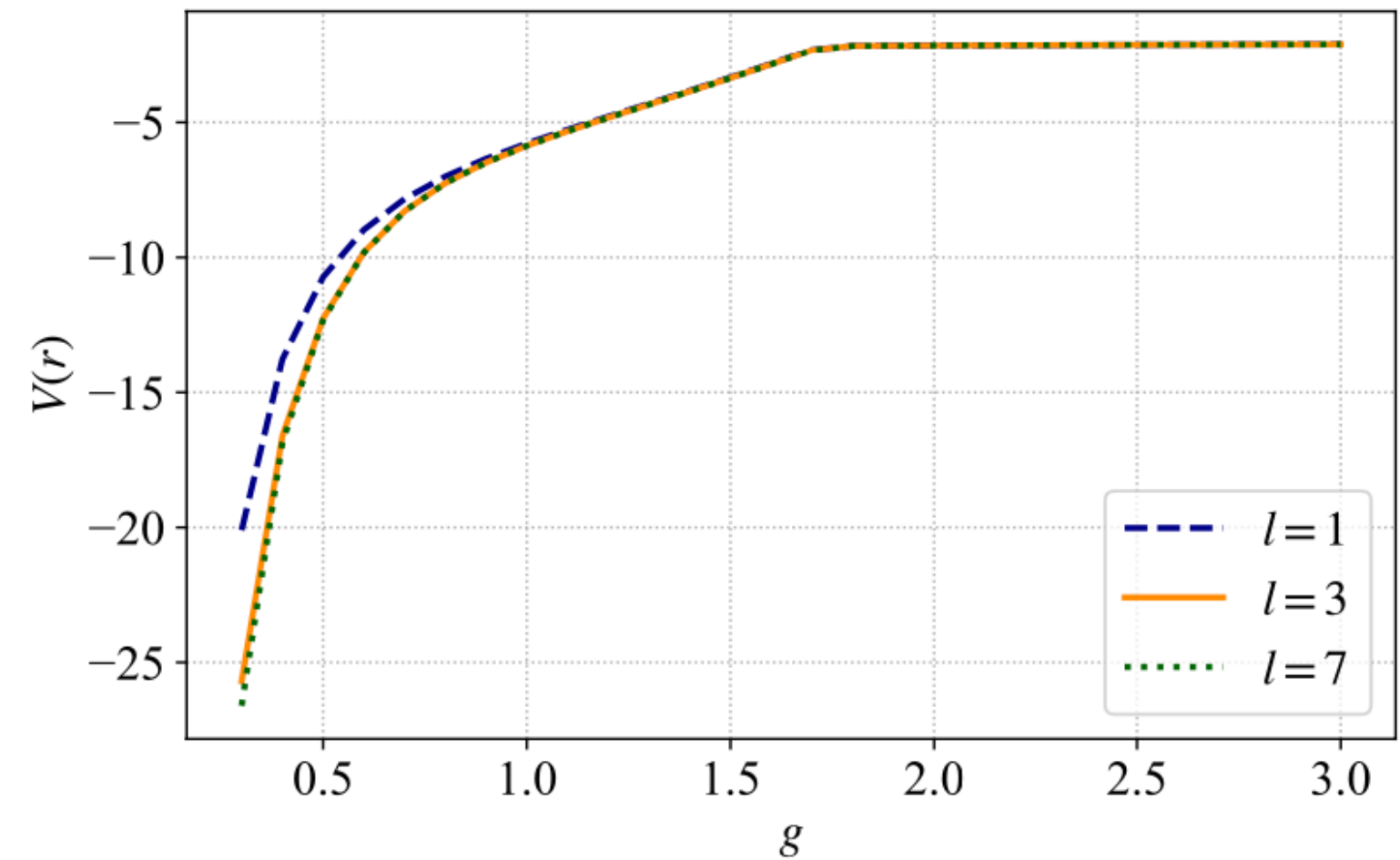
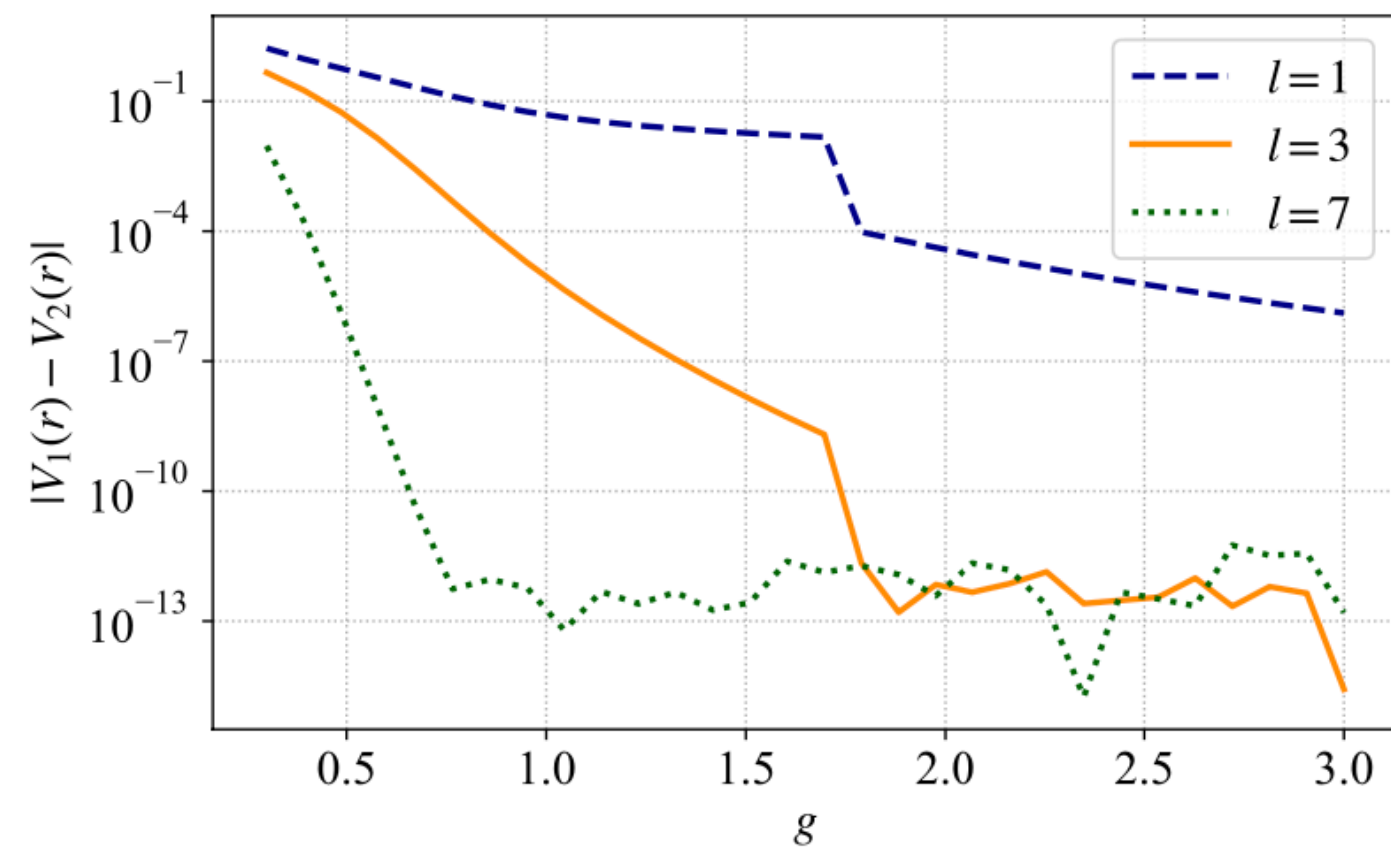
Study of the number of shots
at four values of the
coupling.



Truncation and Gauss's law

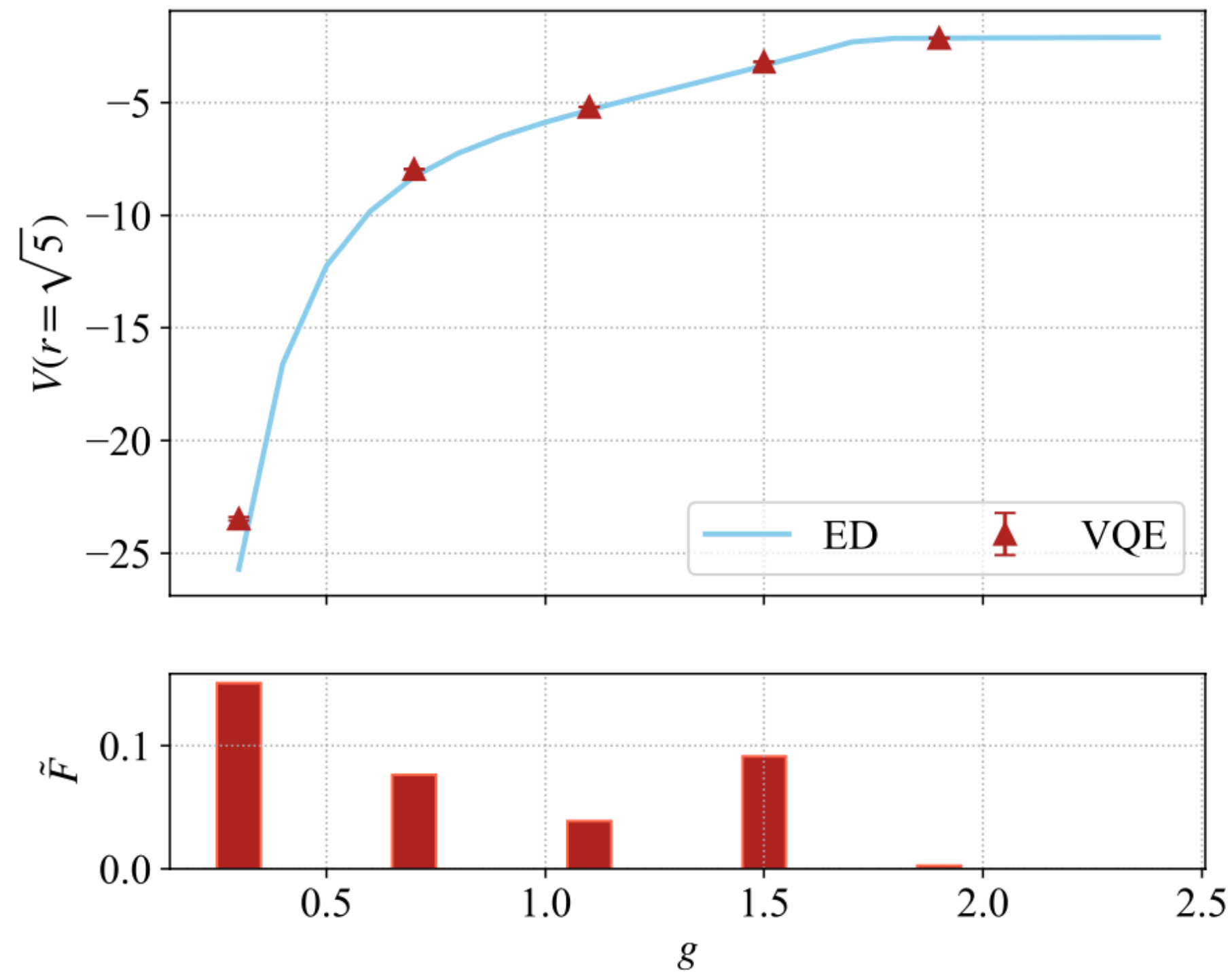
$$V_1(r) \rightarrow \{E_{10y}, E_{20y}\}$$

$$V_2(r) \rightarrow \{E_{00y}, E_{20y}\}$$



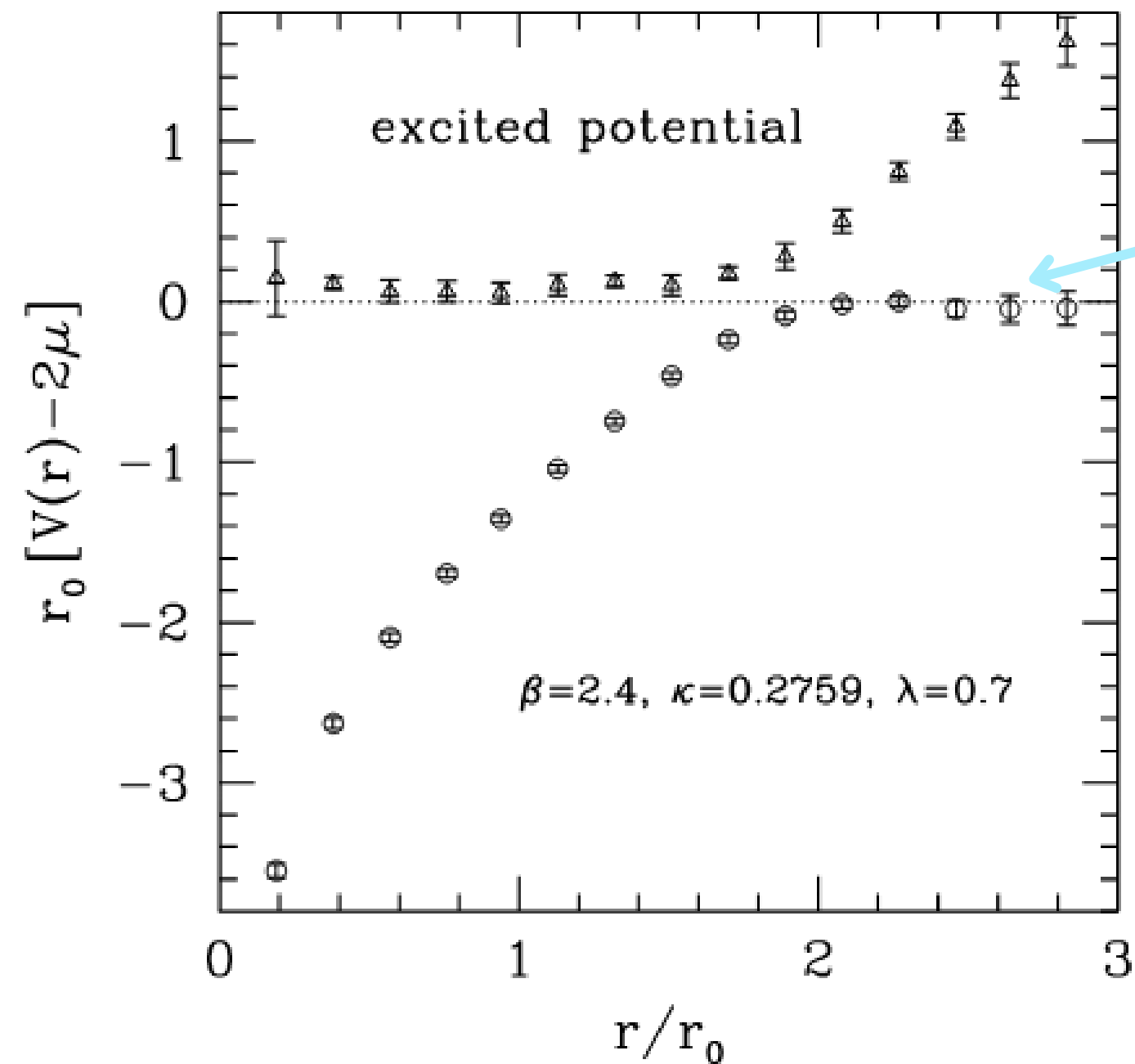
Static potential with different truncation values.

VQE Results with $l=3$



Noiseless variational
quantum results 3x2
system.
(NFT and 10^4 shots)

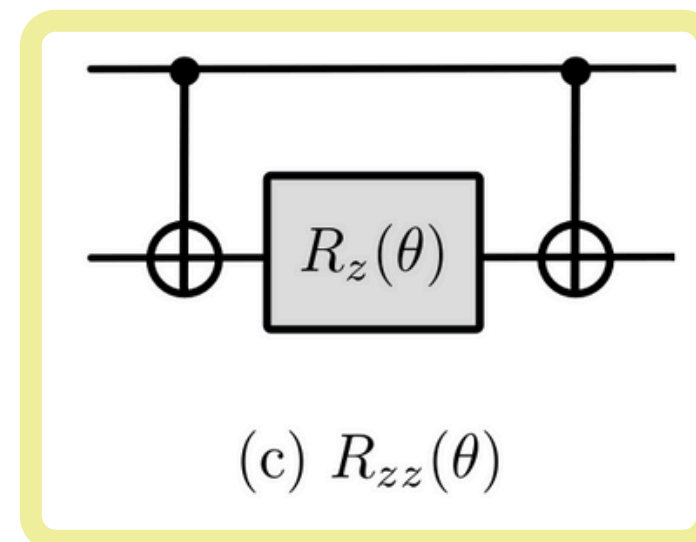
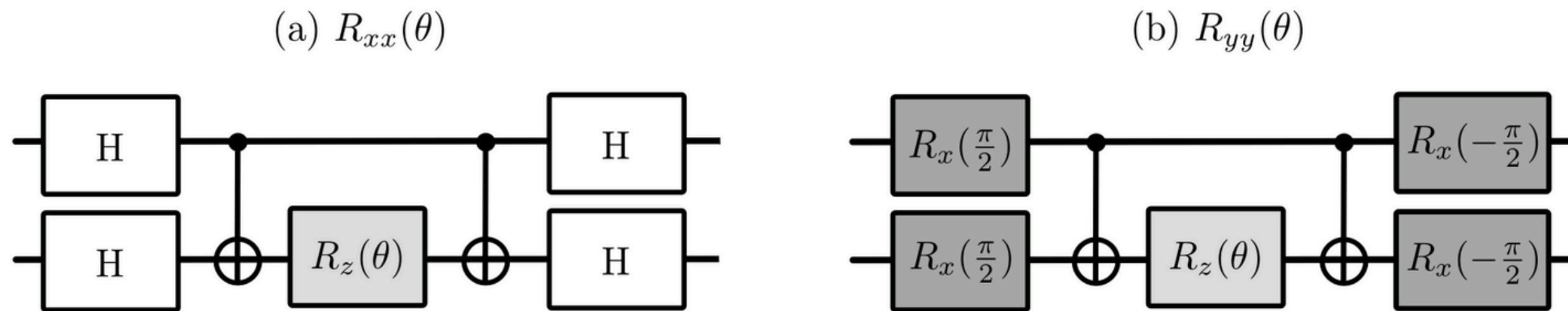
$V(r)$ with MC



Large distances the
potential
approaches the
asymptotic value
 2μ



Ground state and first excited state static potentials as functions of static quark separation. Simulations of SU(2).

iSWAP gates



Added for NFT optimizer

Quantum Hardware

Qubit Type	Pros	Cons	Examples
Superconducting	<ul style="list-style-type: none">• High gate speeds and fidelities.• Established technology.	<ul style="list-style-type: none">• Requires cryogenic cooling.• Short coherence times.	
Trapped Ions	<ul style="list-style-type: none">• Extremely high gate fidelities.• Long coherence times• No extreme cryogenic cooling needed.	<ul style="list-style-type: none">• Slow gate times and operations.• Difficulty in aligning and scaling lasers.• Ion charges may limit scalability.	

Error mitigations

Partition Measurement Symmetry Verification (PMSV)

Measurements of specific Pauli strings that encode the system's known **symmetries** (e.g. fermionic *zero-charge* sector).

Measurement outcomes not satisfying the symmetry are **discarded**.

State Preparation And Measurement error mitigation (SPAM)

Uses a **calibration matrix** that characterizes the **noise** profile of the quantum device.

The **inverse** of this matrix is applied to correct the measured expectation values.

Error mitigations

R-state selection

$$\langle \psi | \hat{H} | \psi \rangle = \sum_{m,n=0}^{2^N-1} \langle \psi | m \rangle \langle m | \hat{H} | n \rangle \langle n | \psi \rangle$$

$$= \sum_{m,n}' |\langle m | \psi \rangle|^2 |\langle n | \psi \rangle|^2 \frac{\langle m | \hat{H} | n \rangle}{\langle m | \psi \rangle \langle \psi | n \rangle}$$

with

$$|\psi\rangle = \sum_{n=0}^{2^N-1} \langle n | \psi \rangle |n\rangle$$

1. Sample state in computational basis.

2. Select R computational basis states (highest probability $|\langle n | \psi \rangle|^2$). Avoid noise from other states.

3. Calculate transition matrix elements classically,

$$\langle m | \hat{H} | n \rangle = \sum_{i=1}^M c_i \langle m | P_i | n \rangle,$$

(diag.=energy of R-th state).

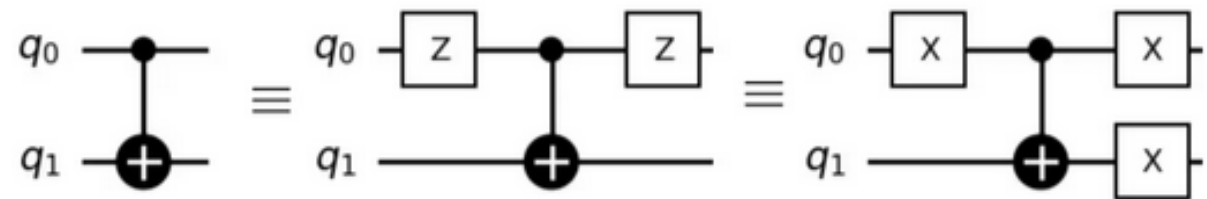
4. Calculate other terms.

5. Estimate final expectation value.

Error mitigations

PAULI TWIRLING

Transforms complex quantum noise into Pauli noise.



Random applications of **Pauli** gates before and after a gate.

Now stochastic errors, improved by **averaging** more.

READOUT ERROR MITIGATION

$$C := \begin{pmatrix} 1 - \Pr(0|1) & \Pr(0|1) \\ \Pr(1|0) & 1 - \Pr(1|0) \end{pmatrix}$$

$\Pr(\text{detected } j | \text{prepared } i)$

$$CP = P_{\text{noisy}}$$