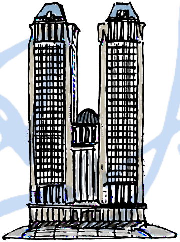
Baryon form factors

Rinaldo Baldini Ferroli, Alessio Mangoni and Simone Pacetti

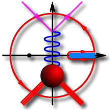


Workshop on form factor, polarization and CP violation in quantum-correlated hyperon-antihyperon production

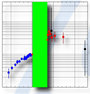
Fudan University, Shanghai, People’s Republic of China

July 7th and 8th, 2019

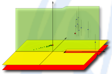
Agenda



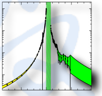
Baryon form factors and dispersion relations



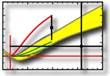
Space-like and time-like data on G_E/G_M



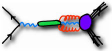
Space-like and time-like G_E/G_M via DR's



Asymptotic G_M from a DR sum rule



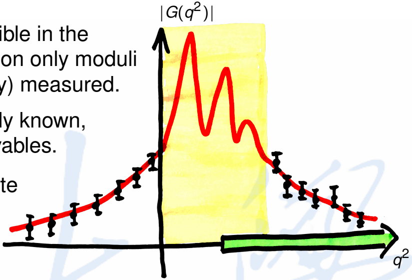
Hints for the ratio of Λ form factors



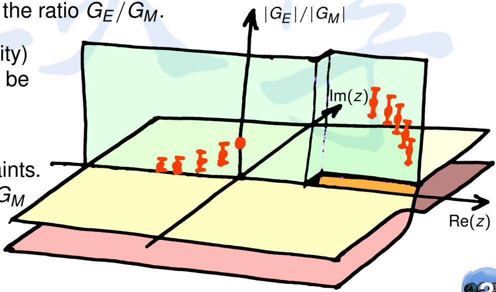
Clues from the J/ψ decays

About proton form factors

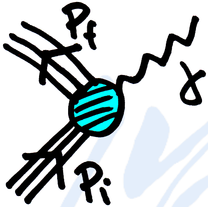
- Proton form factors are completely accessible in the space-like region while in the time-like region only moduli above the physical threshold can be (easily) measured.
- In the space-like region they are individually known, especially by means of polarization observables.
- Only recently, attempts to measure separate value of moduli in the time-like region have become stronger.



- So far, the better known quantity is the ratio G_E/G_M.
- Using dispersion relations (analyticity) space-like and time-like values can be exploited to extract information on phase and asymptotic behavior.
- Analyticity imposes serious constraints. A space-like zero for the ratio G_E/G_M does require an asymptotic phase of 180 degrees.



Proton-photon vertex



Nucleon electromagnetic four-current ($q = p_f - p_i$)

$$\langle P_f | J^\mu_{\rm EM}(0) | P_i \rangle = e\, \overline{u}(p_f) \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} F_2(q^2) \right] u(p_i)$$

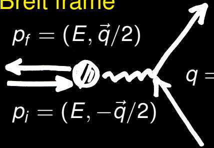
$F_1(q^2)$ and $F_2(q^2)$ are the Dirac and Pauli form factors

$F_1(0) = \mathcal{Q}_p$ $\qquad$ $F_2(0) = \kappa_p$

$\mathcal{Q}_p$ = electric charge $\qquad$ κ_p = anomalous magnetic moment

Breit frame

$p_f = (E, \vec{q}/2)$



$q = (0, \vec{q})$

$p_i = (E, -\vec{q}/2)$

$$\langle P_f | J^\mu_{\rm EM}(0) | P_i \rangle \equiv J^\mu_{\rm EM} = \left(J^0_{\rm EM}, \vec{J}_{\rm EM} \right)$$

- $J^0_{\rm EM} = e \left(F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2) \right)$
- $\vec{J}_{\rm EM} = e\, \overline{u}(p_f) \vec{\gamma} u(p_i) \left(F_1(q^2) + F_2(q^2) \right)$

Sachs form factors

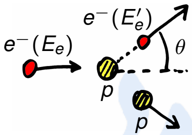
- $G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2)$
- $G_M(q^2) = F_1(q^2) + F_2(q^2)$

Normalizations

- $G_E(0) = \mathcal{Q}_p$
- $G_M(0) = \mu_p = \kappa_p + \mathcal{Q}_p$

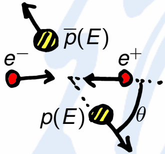
μ_p = total magnetic moment

Cross sections and Coulomb correction



Elastic scattering cross section (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\left(\frac{\theta}{2}\right)}{4E_e^3 \sin^4\left(\frac{\theta}{2}\right)} \left[G_E^2 - \tau \left(1 + 2(1-\tau)\tan^2\left(\frac{\theta}{2}\right)\right) G_M^2 \right] \frac{1}{1-\tau}$$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \, \mathcal{C}}{16E^2} \left[\left(1 + \cos^2(\theta)\right) |G_M|^2 + \frac{1}{\tau} \sin^2(\theta) \, |G_E|^2 \right]$$

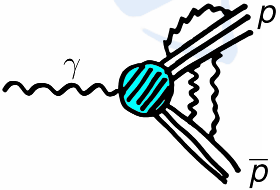
$$\tau = E^2 / M_p^2$$

$$\beta = \sqrt{1 - 1/\tau}$$

Coulomb correction

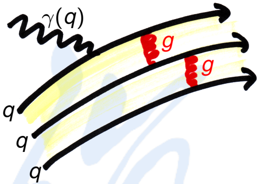
$$\mathcal{C} = \frac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}}$$

- $p\bar{p}$ Coulomb interaction as FSI
- Only S-wave



pQCD asymptotic behavior
Space-like region

V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze, LNC7 (1973) 719
S. J. Brodsky, G. R. Farrar, PRL31 (1973) 1153
M. V. Galynsky, E. A. Kuraev JETPL96 (2012) 6



- **pQCD**: as $q^2 \to -\infty$, F_1, F_2, G_E, G_M follow power laws driven by counting rules
- Valence quarks exchange gluons to distribute the photon momentum transfer q

Non-helicity-flip current $J^{\lambda,\lambda}(q^2)$

- $J^{\lambda,\lambda}(q^2) \propto G_M(q^2)$
- Two gluon propagators
- $G_M(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

Helicity-flip current $J^{\lambda,-\lambda}(q^2)$

- $J^{\lambda,-\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$
- [Two gluon propagators]/$\sqrt{-q^2}$
- $G_E(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$

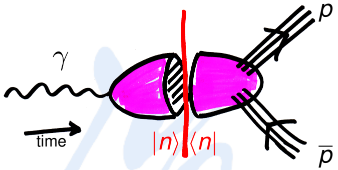
Dirac and Pauli form factors

- $F_1(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-2}$
- $F_2(q^2) \underset{q^2\to-\infty}{\sim} \left(q^2\right)^{-3}$

Ratio of Sachs form factors

- $\dfrac{G_E(q^2)}{G_M(q^2)} \underset{q^2\to-\infty}{\sim}$ constant

Nucleon form factors
Time-like region ($q^2 > 0$)



- Crossing symmetry:

$$\langle P(p')|j^\mu|P(p)\rangle \rightarrow \langle \overline{P}(p')P(p)|j^\mu|0\rangle$$

- Form factors are complex functions of q^2

Optical theorem

$$\mathrm{Im}\langle \overline{P}(p')P(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{P}(p')P(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \Longrightarrow \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, $\ldots$

Time-like asymptotic behavior

Phragmèn Lindelöf theorem

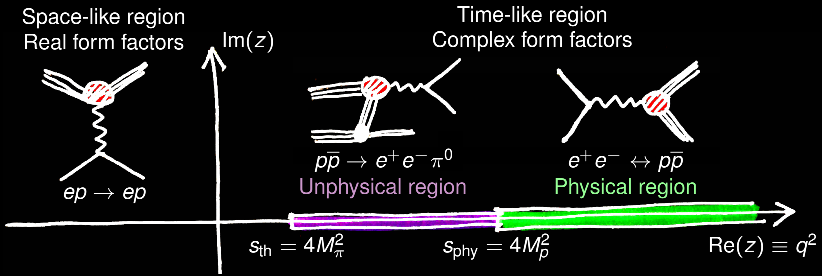
If $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ as $z \rightarrow \infty$ along a straight line, and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{b}$ as $z \rightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{f(z)} \longrightarrow \boldsymbol{a}$ **uniformly in this angle**.

- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ **Must be real**

Analyticity of form factors

q^2-complex plane

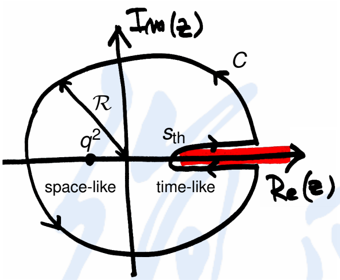


Only the real axis of the q^2-complex plane is experimentally accessible

Space-like region $q^2 < 0$	Time-like region* $s_{th} < q^2 \leq s_{phy}$	Time-like region $q^2 \geq s_{phy}$	
$ep \to ep$	$p\bar{p} \to e^+e^-\pi^0$	$e^+e^- \leftrightarrow p\bar{p}$	$e^+e^- \leftrightarrow p\bar{p}$ (pol.)
G_E, G_M	$\|G_E\|, \|G_M\|$	$\|G_E\|, \|G_M\|$	$\|G_E\|, \|G_M\|, \arg(G_E/G_M)$

*C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC75, 045205
E. A. Kuraev et al., JETP115, 93
G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh PRC86, 025204

Dispersion relations



- The form factors are analytic on the q^2-plane with a multiple cut ($s_{\rm th} = 4M_\pi^2, \infty$)
- Dispersion relation for the imaginary part ($q^2 < 0$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}\, G(s) ds}{s - q^2}$$

- Dispersion relation for the logarithm ($q^2 < 0$)
  B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{\rm th} - q^2}}{\pi} \int_{s_{\rm th}}^{\infty} \frac{\ln|G(s)| ds}{(s - q^2)\sqrt{s - s_{\rm th}}}$$

Experimental inputs

- Space-like data on the real values of form factors from: $ep \to ep$ and $e^{\uparrow} p \to e^- p^{\uparrow}$, with polarization
- Time-like data on form factor moduli from: $e^+e^- \leftrightarrow p\bar{p}$
- Time-like data on G_E/G_M phase from: $e^+e^- \leftrightarrow p^{\uparrow}\bar{p}$ (pol.)

Theoretical ingredients

- Analyticity $\Rightarrow$ convergence relations
- Normalization and threshold values
- Asymptotic behavior $\Rightarrow$ super-convergence relations

Advantages and drawbacks of dispersive approaches

Advantages

- DR's are based on unitarity and analyticity ⇒ model-independent approach
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\textbf{space-like}\\ \textbf{form factor}\\ ep \to ep\end{array}\right] = \int_{s_{\rm th}}^{\infty} \left[\begin{array}{c}\textbf{Im(form factor) or ln|form factor|}\\ \textbf{over the time-like cut } (s_{\rm th}, \infty)\\ e^+e^- \to p\bar{p} + \textbf{theory}\end{array}\right]$$

- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane

Drawbacks

- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted DR's

- No data in the unphysical region, crucial in dispersive analyses

Advantages and drawbacks of dispersive approaches

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- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane
- Poles cancel out in the ratio!

Drawbacks

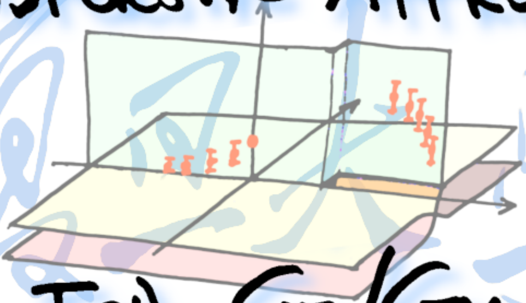
- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted DR's

- No data in the unphysical region, crucial in dispersive analyses

A DISPERSIVE APPROACH



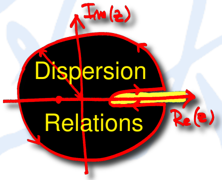
FOR GE/GM

Dispersive approach for the ratio $R = \mu_p G_E / G_M$

We start from the imaginary part of the ratio $R(q^2)$, written in the most general and model-independent way as

$$I(q^2) \equiv \text{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Theoretical constraints can be applied directly on this function $I(q^2)$



The function $R(q^2)$ is reconstructed in time and space-like regions

Additional theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of $R(q^2)$

Parametrization for $R = \mu_p G_E / G_M$

The imaginary part of $R(q^2)$ is parametrized by two series of orthogonal polynomials

$$\mathrm{Im}\left[R(q^2)\right] \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\mathrm{phy}} - s_{\mathrm{th}}}{s_{\mathrm{phy}} - s_{\mathrm{th}}} & s_{\mathrm{th}} \le q^2 \le s_{\mathrm{phy}} \\ \sum_j D_j T_j(x') & x' = \frac{2 s_{\mathrm{phy}}}{q^2} - 1 & q^2 > s_{\mathrm{phy}} \end{cases}$$

Theoretical conditions on $\mathrm{Im}\left[R(q^2)\right]$

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_p^2)$ is real $\Longrightarrow I(4M_p^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

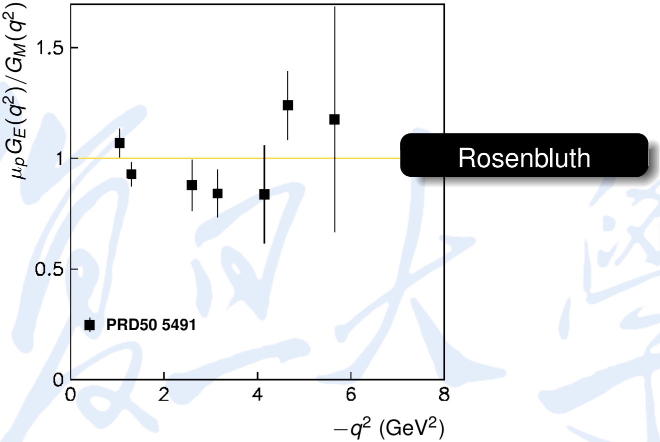
Theoretical conditions on $R(q^2)$

- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_p^2)$ is real and $\mathrm{Re}\left[R(4M_p^2)\right] = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

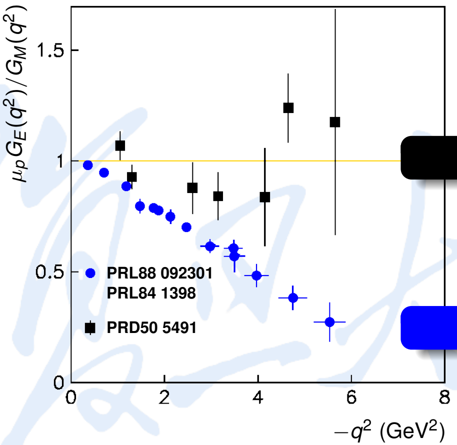
- Space-like region ($q^2 < 0$) data for R from JLab and MIT-Bates
- Time-like region ($q^2 \ge 4M_p^2$) data for $|R|$ from FENICE+DM2, *BABAR*, and E835

Space-like data on $R = \mu_p G_E / G_M$



Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

Space-like data on $R = \mu_p G_E / G_M$



Rosenbluth

Polarization

Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

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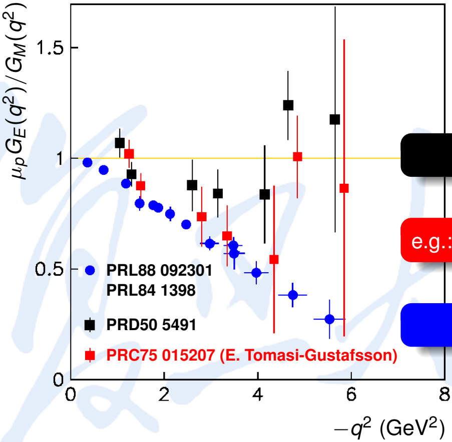


Rosenbluth

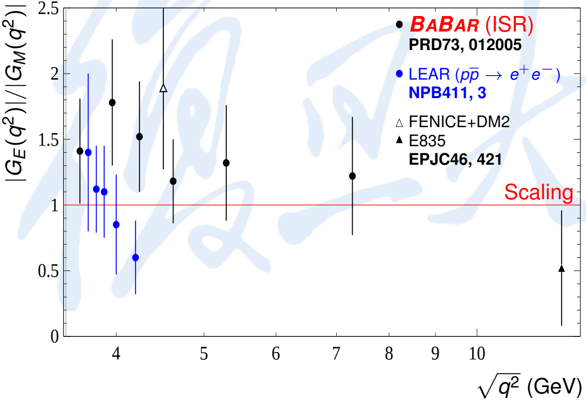
e.g.: Ros. + RC, 2γ,...

Polarization

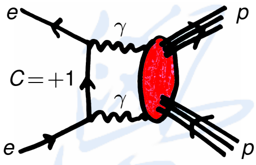
Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

Time-like data on $|G_E/G_M|$

$$\frac{d\sigma}{d\cos(\theta)} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M|^2\left[(1+\cos^2(\theta)) + \frac{4M_p^2}{q^2}\sin^2(\theta)\left|\frac{G_E}{G_M}\right|^2\right]$$



Two-photon exchange



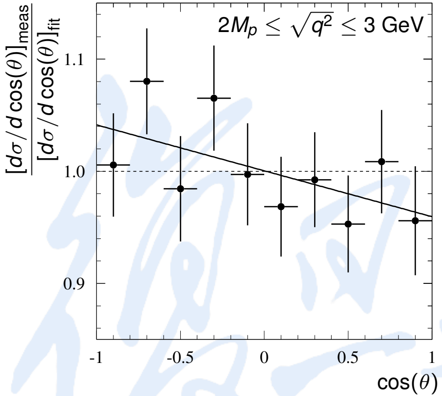
$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson and Q. H. Zhou]

Time-like two-photon contribution?

Phys. Lett. B659 (2008) 197
arXiv:1302.0055



Integrated over the $p\bar{p}$ - CM energy from threshold up to 3 GeV

The MC-fit assumes one-photon exchange

Slope $= -0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

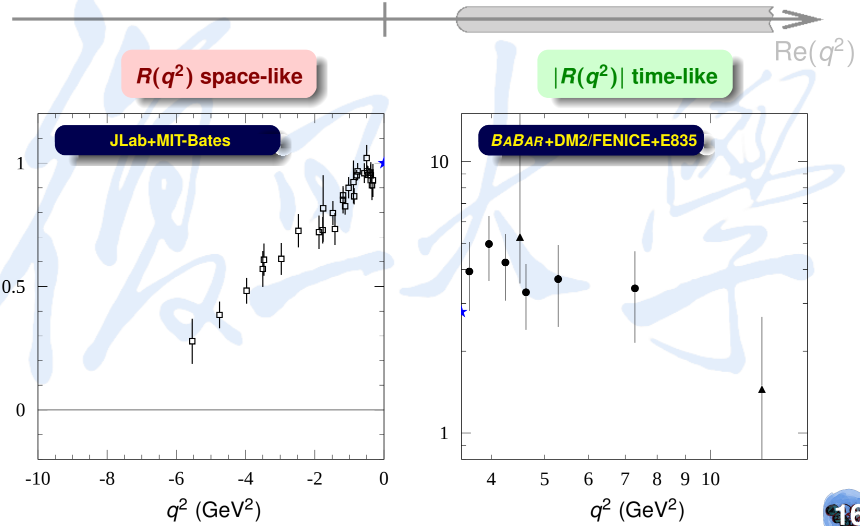
$$\langle\mathcal{A}\rangle_{\cos(\theta)} = \frac{\sigma(\cos(\theta) > 0) - \sigma(\cos(\theta) < 0)}{\sigma(\cos(\theta) > 0) + \sigma(\cos(\theta) < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos(\theta) \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos(\theta) \gtrless 0$

The result for $R = \mu_p G_E / G_M$

EPJA32 421 (2007)

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\,[R(s)]}{s(s-q^2)} ds$$



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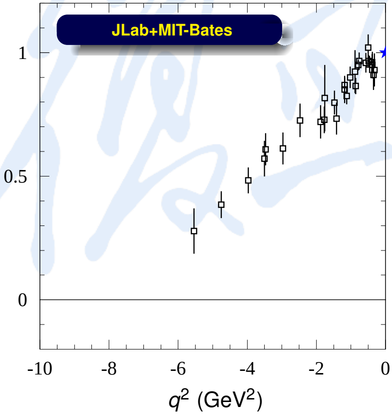
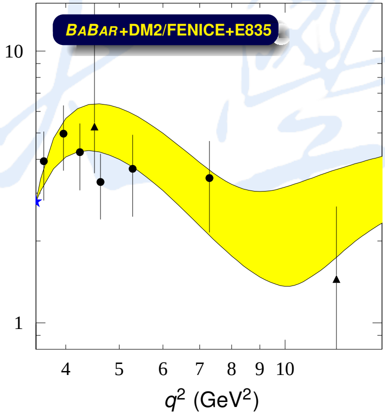
$R(q^2)$ space-like



$|R(q^2)|$ time-like



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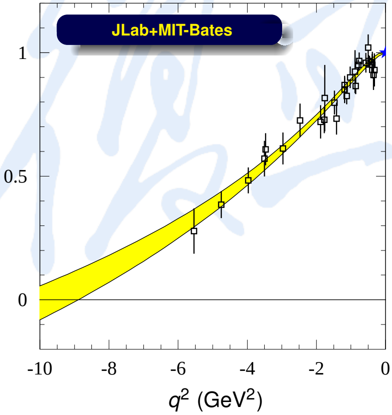
Req^2

$R(q^2)$ space-like



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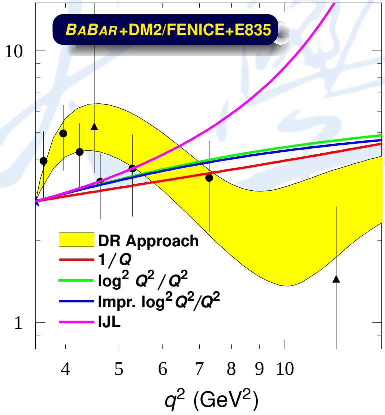

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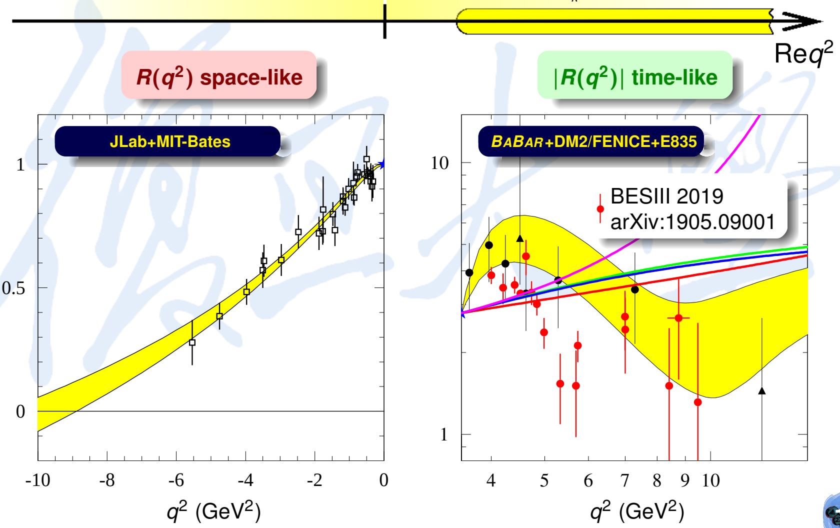
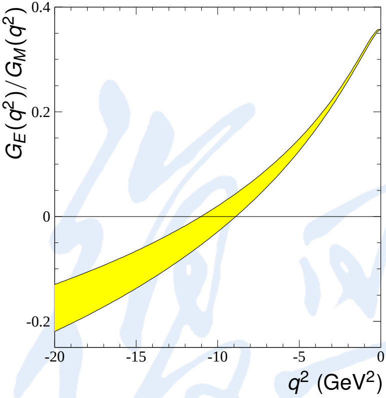
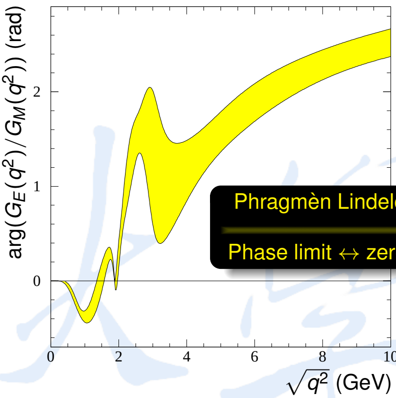


Space-like zero and phase





Phragmèn Lindelöf

Phase limit $\leftrightarrow$ zeros

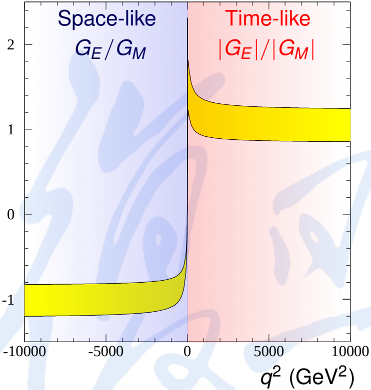
Space-like zero

$$t_0^{BABAR} = (-10 \pm 1)\ \text{GeV}^2$$

Phase from dispersion relations

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{\text{th}}}}{\pi}\ \text{Pr}\int_{s_{\text{th}}}^{\infty} \frac{\ln|R(s)|\,ds}{\sqrt{s - s_{\text{th}}}(s - q^2)}$$

Asymptotic G_E^p/G_M^p



Real asymptotic values for G_E/G_M

- $\frac{G_E}{G_M} \xrightarrow[|q^2|\to\infty]{} -1.0 \pm 0.2$

Asymptotic behaviour of F_2/F_1

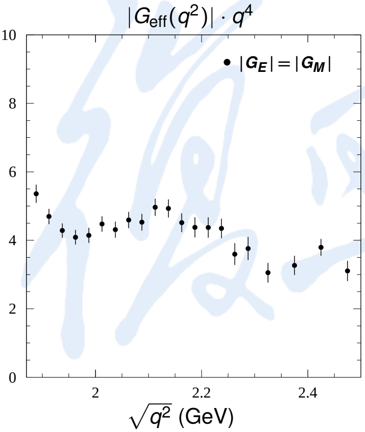
- $\left| \frac{q^2}{4M_N^2} \frac{F_2}{F_1} \right| \xrightarrow[|q^2|\to\infty]{} \left| \frac{G_E}{G_M} - 1 \right| = 2.0 \pm 0.2$

pQCD prediction

$$\left| \frac{G_E(q^2)}{G_M(q^2)} \right| \xrightarrow[|q^2|\to\infty]{} 1$$

$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421 (2007)

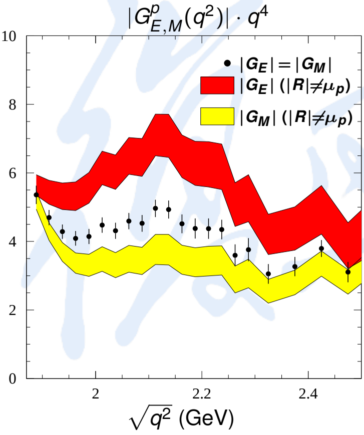


$$|G_{\rm eff}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta\mathcal{C}}{3s}}\left(1+\frac{1}{2\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \to p\bar{p})$ is the effective time-like form factor $|G^p_{\rm eff}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$

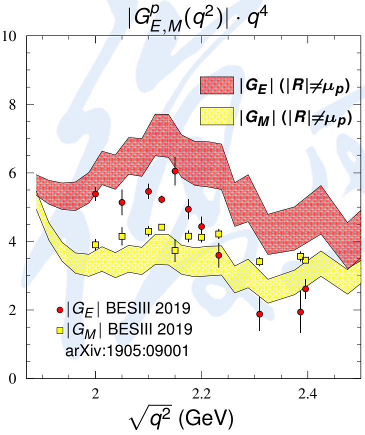
$|G_E^p|$ and $|G_M^p|$ from $p\bar{p}$ cross section and dispersion relations

EPJA32, 421 (2007)



$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}}\left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \to p\bar{p})$ is the effective time-like form factor $|G^p_{\text{eff}}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- Using our parametrization for R and the **BABAR** data on $\sigma(e^+e^- \to p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled



$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

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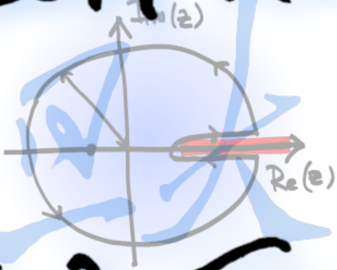
- Using our parametrization for R and the ***BaBar*** data on $\sigma(e^+e^- \to p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

A SUM RULE



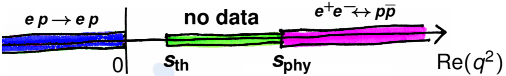
FOR G_M

Dispersion relations and sum rules

Geshkenbeĭn, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- DR's connect space and time values of a form factor $G(q^2)$

$$G(q^2) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}G(s)ds}{s-q^2}$$



Drawbacks

- The imaginary part is not experimentally accessible
- There are no data in the unphysical region $[s_{\rm th}, s_{\rm phy}]$
- We need to know the asymptotic behavior

- They applied the DR for the imaginary part to the function

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{\rm th}-z}} \quad \text{with} \quad \int_0^{s_{\rm phy}} f^2(z)dz << 1$$

Advantages

- The DR integral contains the modulus $|G(s)|$
- The unphysical region contribution is suppressed

Drawback

- Zeros of $G(z)$ are poles for $\phi(z)$

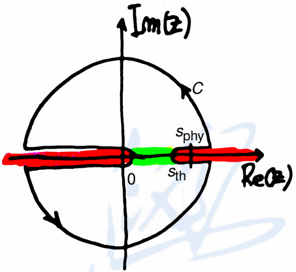
Attenuated DR and sum rule

Assuming $G(q^2) \neq 0$ and using the Cauchy theorem, we have the new DR

$$\oint_C \phi(z)dz = 0$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}_{\text{Time-like}}$$



Convergence relation to find the asymptotic power-law behavior of G_M

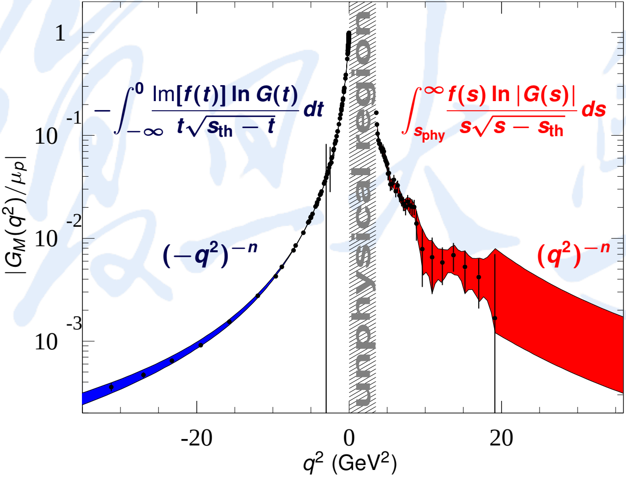
$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like data } + (-t)^{-n}} = \int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds \approx \underbrace{\int_{s_{\rm phy}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}_{\text{Time-like data } + s^{-n}}$$

n is the only free parameter

Sum rule: result for G_M

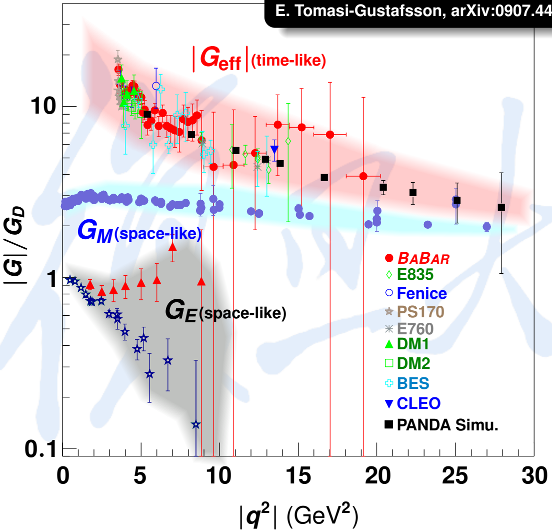
Chin. Phys. C34 874 (2010)

$$G_M(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm0.36)}$$



Asymptotic behaviors

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504, 291
E. Tomasi-Gustafsson, arXiv:0907.4442



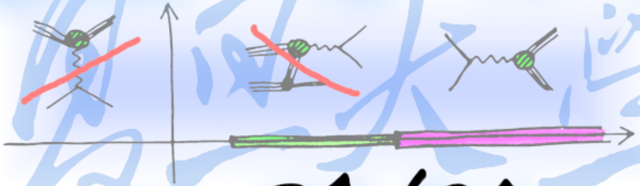
pQCD

$$G^{p}_{\rm eff}(q^2) \underset{q^2\to\infty}{\sim} G_M(q^2)$$

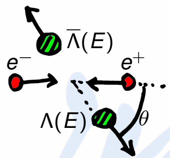
Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G_{\rm eff}(q^2)}{G_M(-q^2)} = 1$$

Phase and Modulus of $\hat{G}_E/\hat{G}_M$



Λ form factors



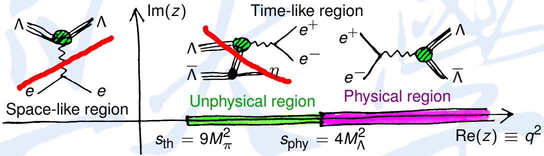
Same definitions, but for labels and Coulomb factor. . .

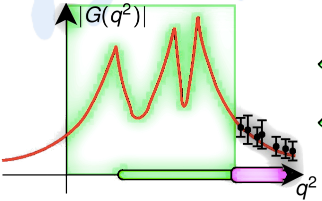
Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \,\mathcal{C}}{16E^2}\left[\left(1+\cos^2(\theta)\right)|G_M^\Lambda|^2 + \frac{1}{\tau}\sin^2(\theta)\,|G_E^\Lambda|^2\right]$$

$$\tau = E^2/M_\Lambda^2 \qquad \beta = \sqrt{1-1/\tau}$$

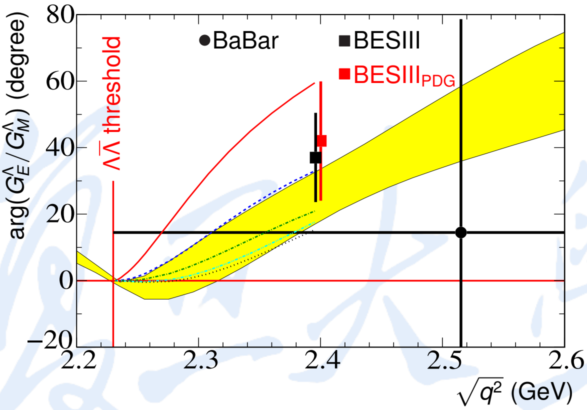
- Same analyticity as for nucleons.
- Difficult to measure in space-like and unphysical regions.
- Relative phase from weak decay.





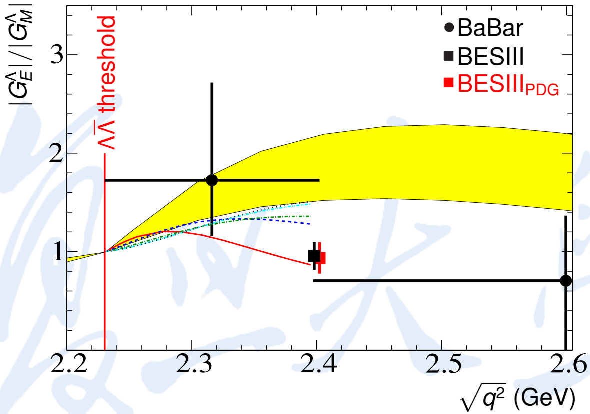
- Same unitarity and intermediate states contributions, but for the isospin.
- Form factors have not vanishing imaginary part above the theoretical threshold.

Phase of $G_E^\Lambda / G_M^\Lambda$



- Theoretical prediction based considering only $\Lambda\bar{\Lambda}$ FSI
  [J. Haidenbauer, U.-G. Meissner, PLB 761 (2016) 456]
- Data from BaBar and BESIII (preliminary)
  [PRD 76 (2007) 092006, arXiv:1903.09421 [hep-ex]]
- "Lambdization" of proton, i.e., proton results with $\sqrt{q^2} \to \sqrt{q^2} + 2(M_\Lambda - M_p)$
  [EPJA32 421 (2007)]

Modulus of $G_E^\Lambda / G_M^\Lambda$



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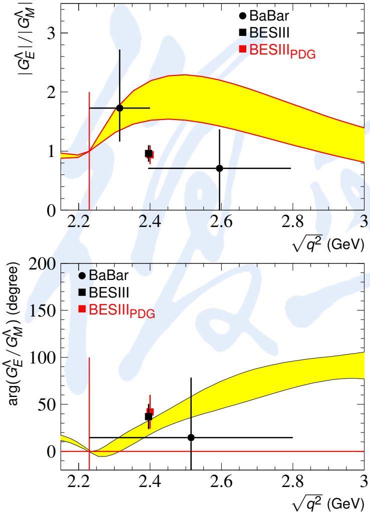
  [EPJA32 421 (2007)]

Modulus and phase of $G_E^\Lambda / G_M^\Lambda$

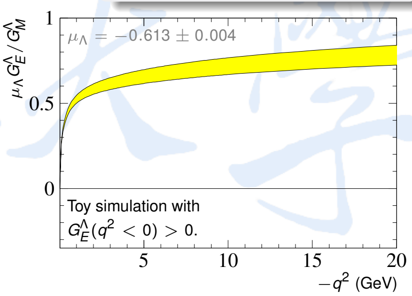
Time-like region



Space-like region

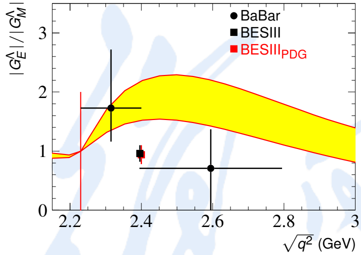
- The space-like FF ratio can be obtained by means of the two dispersion relations for the logarithm and phase.
- The cancellation of poles in the ratio reduces the dependence on the unknown unphysical region.
- **Since $G_E^\Lambda(0) = 0$ the ratio has a zero at $q^2 = 0$.**

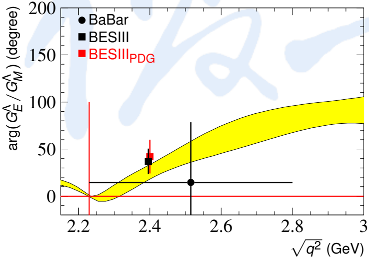
$$\Longrightarrow \quad \arg\left[G_E^\Lambda(q^2)/G_M^\Lambda(q^2)\right] \underset{q^2\to\infty}{\longrightarrow} \pi$$



Modulus and phase of $G_E^\Lambda / G_M^\Lambda$

Time-like region





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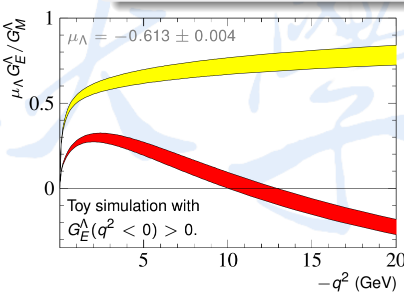


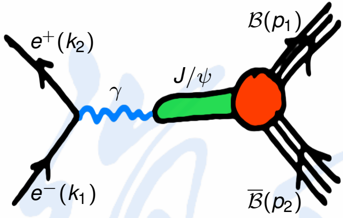
In case of an additional zero at $q^2 < 0$

$$\arg\left[G_E^\Lambda(q^2)/G_M^\Lambda(q^2)\right] \underset{q^2\to\infty}{\longrightarrow} 2\pi$$

CLUES

FROM THE J/Ψ DECAYS

J/ψ decay amplitude



Baryon four-current ($q = p_1 + p_2$)

$$J^\mu_{\mathcal{B}} = \overline{u}(p_1)\left[\gamma^\mu f_1^{\mathcal{B}} + \frac{i\sigma^{\mu\nu} q_\nu}{2M_{\mathcal{B}}} f_2^{\mathcal{B}}\right] v(p_2)$$

$f_1^{\mathcal{B}}$ and $f_2^{\mathcal{B}}$ are the Dirac and Pauli couplings

Amplitude of $e^+e^- \to J/\psi \to \mathcal{B}\overline{\mathcal{B}}$

$$\mathcal{M} = -ie^2 J^\mu_{\mathcal{B}} D_\psi(q^2)\, \overline{v}(k_2)\gamma_\mu u(k_1)$$

$D_\psi(q^2)$ is the J/ψ propagator

Differential cross section of $e^+e^- \to J/\psi \to \mathcal{B}\overline{\mathcal{B}}$

$$\frac{d\sigma_{e^+e^-\to J/\psi\to \mathcal{B}\overline{\mathcal{B}}}}{d\cos(\theta)} = \frac{\pi\alpha^2\beta}{2M_\psi^2}\left(\left|g_M^{\mathcal{B}}\right|^2 + \frac{4M_{\mathcal{B}}^2}{M_\psi^2}\left|g_E^{\mathcal{B}}\right|^2\right)\left(1 + \alpha_\psi^{\mathcal{B}}\cos^2(\theta)\right)$$

Polarization parameter

$$\alpha_\psi^{\mathcal{B}} = \frac{M_\psi^2\left|g_M^{\mathcal{B}}\right|^2 - 4M_{\mathcal{B}}^2\left|g_E^{\mathcal{B}}\right|^2}{M_\psi^2\left|g_M^{\mathcal{B}}\right|^2 + 4M_{\mathcal{B}}^2\left|g_E^{\mathcal{B}}\right|^2}$$

Sachs couplings

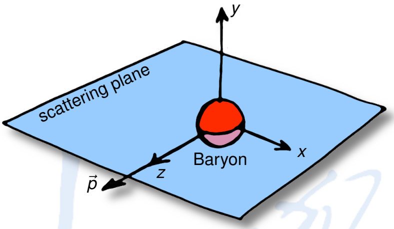
$$g_E^{\mathcal{B}} = f_1^{\mathcal{B}} + \frac{M_\psi^2}{4M_{\mathcal{B}}^2} f_2^{\mathcal{B}} \qquad g_M^{\mathcal{B}} = f_1^{\mathcal{B}} + f_2^{\mathcal{B}}$$

Polarization formulae in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq 4M_\pi^2$

$$\frac{G_E(q^2)}{G_M(q^2)} = \frac{|G_E(q^2)|}{|G_M(q^2)|} e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

- $\mathcal{P}_y = -\frac{\sin(2\theta)\sin(\rho)}{D\sqrt{\tau}} \frac{|G_E|}{|G_M|} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \mathcal{A}_y$ — Does not depend on P_e
- $\mathcal{P}_x = -P_e \frac{2\sin(2\theta)\cos(\rho)}{D\sqrt{\tau}} \frac{|G_E|}{|G_M|}$
- $\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D}$ — Does not depend on ρ

$$D = 1+\cos^2(\theta) + \frac{|G_E|^2}{|G_M|^2} \frac{\sin^2(\theta)}{\tau}$$

$$\tau = \frac{q^2}{4M_{\mathcal{B}}^2}$$

- P_e is the electron polarization
- θ is the scattering angle

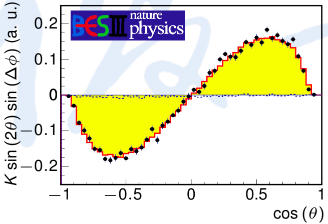
$J/\psi\Lambda\bar{\Lambda}$ Sachs coupling constants

- If the relative phase between g_E^Λ and g_M^Λ is not an integer multiple of π it follows

$\Delta\phi \equiv \arg\left(g_E^{\mathcal{B}}/g_M^{\mathcal{B}}\right) \neq k\pi \quad \forall k \in \mathbb{Z}$ ⇒ $\sin(\Delta\phi) \neq 0$ ⇒ $\mathcal{P}_y \neq 0$

- The polarization of Λ baryons can be measured through their weak decay

Angular distribution $\frac{dN(\Lambda \to p\pi^-)}{d\cos(\theta)}$ ⇒ Λ polarization $\vec{\mathcal{P}} = (\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z)$ ⇒ Relative phase $\sin(\Delta\phi)$



- $\Delta\phi = 42.4 \pm 0.6 \pm 0.5^\circ$
- $\alpha_\psi^\Lambda = 0.461 \pm 0.006 \pm 0.007$
- $\frac{|g_E^\Lambda|}{|g_M^\Lambda|} = \frac{M_\psi}{2M_\Lambda}\sqrt{\frac{1-\alpha_\psi^\Lambda}{1+\alpha_\psi^\Lambda}} = 0.84 \pm 0.01$

Sachs coupling constants g_E^Λ ane g_M^Λ of the J/ψ are complex

Baryon form factors at the J/ψ mass[1]

- Total cross section of $e^+e^- \to J/\psi \to \mathcal{B}\overline{\mathcal{B}}$

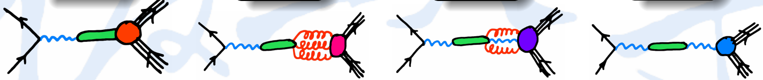
$$\sigma_{e^+e^- \to J/\psi \to \mathcal{B}\overline{\mathcal{B}}} = \frac{4\pi\alpha^2\beta}{3M_\psi^2}\left(\left|g_M^{\mathcal{B}}\right|^2 + \frac{2M_{\mathcal{B}}^2}{M_\psi^2}\left|g_E^{\mathcal{B}}\right|^2\right) = \frac{4\pi\alpha^2\beta}{3M_\psi^2}\left(1 + \frac{2M_{\mathcal{B}}^2}{M_\psi^2}\right)\left|g_{\text{eff}}^{\mathcal{B}}\right|^2$$

- Decay rate of $J/\psi \to \mathcal{B}\overline{\mathcal{B}}$

$$\Gamma_{J/\psi \to \mathcal{B}\overline{\mathcal{B}}} = \frac{M_\psi\beta}{12\pi}\left(\left|g_M^{\mathcal{B}}\right|^2 + \frac{2M_{\mathcal{B}}^2}{M_\psi^2}\left|g_E^{\mathcal{B}}\right|^2\right) = \frac{M_\psi\beta}{12\pi}\left(1 + \frac{2M_{\mathcal{B}}^2}{M_\psi^2}\right)\left|g_{\text{eff}}^{\mathcal{B}}\right|^2$$

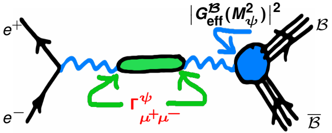
- The effective coupling constant $g_{\text{eff}}^{\mathcal{B}}$ has three contributions

$$g_{\text{eff}}^{\mathcal{B}} = g_{ggg}^{\mathcal{B}} + g_{gg\gamma}^{\mathcal{B}} + g_{\gamma}^{\mathcal{B}}$$



- $$\Gamma_{J/\psi \to \mathcal{B}\overline{\mathcal{B}}}(g_\gamma^{\mathcal{B}}) = \Gamma_{\mu^+\mu^-}^{\psi}\frac{\sigma_{\mathcal{B}\overline{\mathcal{B}}}(M_\psi^2)}{\sigma_{\mu^+\mu^-}(M_\psi^2)}$$

$$\left|G_{\text{eff}}^{\mathcal{B}}(M_\psi^2)\right|^2 = \frac{\Gamma_{J/\psi \to \mathcal{B}\overline{\mathcal{B}}}(g_\gamma^{\mathcal{B}})}{\beta\,\Gamma_{\mu^+\mu^-}^{\psi}}$$



Chin. Phys. C43, 023103 (2019)
arXiv:1905.01069

Effective Lagrangian density

$$\mathcal{L} = \mathcal{L}\,(B; G_0, D_e, D_m, F_e, F_m, R)$$

- Spin 1/2 SU(3) baryon octet

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{pmatrix}$$

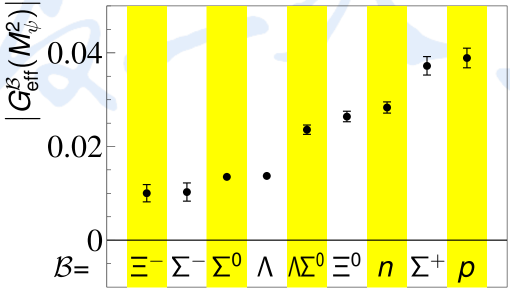
- Coupling constants: G_0, D_e, D_m, F_e, F_m

$J/\psi \to p\bar{p}$

- $g^p_{ggg} = (G_0 - D_m + F_m)e^{i\varphi}$
- $g^p_{gg\gamma} = R(G_0 - D_m + F_m)e^{i\varphi}$
- $g^p_{\gamma} = D_e + F_e$

$J/\psi \to n\bar{n}$

- $g^n_{ggg} = (G_0 - D_m + F_m)e^{i\varphi}$
- $g^n_{gg\gamma} = 0$
- $g^n_{\gamma} = -2D_e$



Λ effective form factor at the J/ψ mass

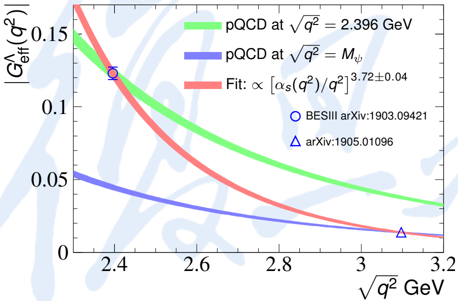
Chin. Phys. C43, 023103 (2019)
arXiv:1905.01069

The model

- Same $g^{\mathcal{B}}_{gg\gamma}/g^{\mathcal{B}}_{ggg}$ for all SU(3) octet baryons.
- $g^{\mathcal{B}}_{gg\gamma} = 0$ for all neutral baryons.
- $g^{\mathcal{B}}_{gg\gamma}$ and $g^{\mathcal{B}}_{ggg}$ have the same phase.

$J/\psi \to \Lambda\bar{\Lambda}$

- $g^{\Lambda}_{ggg} = (G_0 - 2D_m)e^{i\varphi}$
- $g^{\Lambda}_{gg\gamma} = 0$
- $g^{\Lambda}_{\gamma} = -D_e$



Perturbative QCD

$$|G^{\Lambda}_{\rm eff}(q^2)| \propto \left(\frac{\alpha_s(q^2)}{q^2}\right)^2$$

Observed behavior

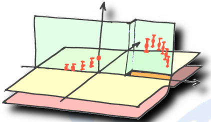
$$|G^{\Lambda}_{\rm eff}(q^2)| \propto \left(\frac{\alpha_s(q^2)}{q^2}\right)^{3.72\pm0.04}$$

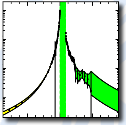
The asymptotic regime is not attained.

There is still dynamical activity.

This is proven by the rising trend of the relative phase.

Final considerations





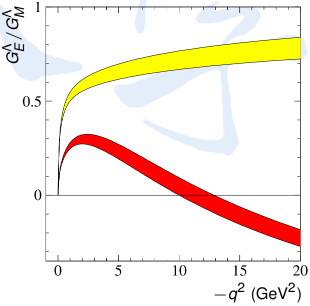
- Space-like zero for G_E^p
- Time-like phase of G_E^p/G_M^p goes to 180^o
- Time-like form factors separation

Space-like and time-like "fixed" data on $|G_M^p|$ and analyticity

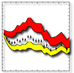
$\Downarrow$

Confirmation of the pQCD asymptotic behavior

- Relative phase and modulus for the ratio G_E^Λ/G_M^Λ agree with (only) FSI interaction
- However, due to the zero of the ratio G_E^Λ/G_M^Λ at the $q^2=0$, **the phase has to tend to 180° as $q^2 \to \infty$**
- An asymptotic relative phase of 360° would imply the presence of an **additional space-like zero** for G_E^Λ

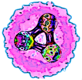


“To do and to do better” list



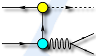
Time-like $|G_E|$-$|G_M|$ separation

DR and data

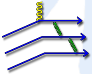


Understanding threshold effect(s):

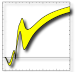
Dispersive analyses: integral equation, sum rule,. . .



Experimental observation in $p\bar{p} \to \pi^0 l^+ l^-$
[PRC75,045205(07)]



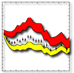
Asymptotic behavior: DR and data for the phase



Zeros $\leftrightarrow$ phases

DR and data

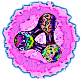
“To do and to do better” list



Time-like $|G_E|$-$|G_M|$ separation

DR and data

BESIII · panda · CMD · SND



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BESIII · CMD · SND

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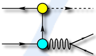

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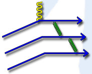
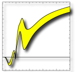
panda



Asymptotic behavior: DR and data for the phase

panda



Zeros $\leftrightarrow$ phases

DR and data

Jefferson Lab · BESIII · panda

Thank you