Nucleon Form Factors in time-like and space-like regions

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Probing transverse nucleon structure at high momentum transfer

ECT* - European Center for Theoretical Studies in Nuclear Physics and Related Areas

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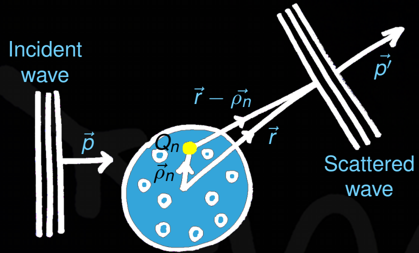
AGENDA

- Nucleon Electromagnetic Form Factors
  - Definition and properties
- The space-like region
  - Proton radius
  - Rosenbluth versus Akhiezer-Rekalo
- The time-like region
  - Unphysical region
  - Threshold
  - An amazing effect
- The asymptotic region
- Analytic $R = \mu G_E / G_M$
- Conclusions

SEMI-CLASSICAL DEFINITION



Amplitude of the scattered wave

$$\mathcal{A}_n(\vec{r}) = Q_0\, e^{i\,\vec{p}'\cdot\vec{r}}\, Q_n\, e^{i\,(\vec{p}-\vec{p}')\cdot\vec{\rho}_n}$$

$$\mathcal{A}(\vec{r}) = \sum_n \mathcal{A}_n(\vec{r}) = Q_0\, e^{i\,\vec{p}'\cdot\vec{r}} \sum_n Q_n\, e^{i\,\vec{q}\cdot\vec{\rho}_n}$$

Form factor

$$F(\vec{q}) = \frac{1}{Q_0} \langle\phi| \sum_n Q_n\, e^{i\,\vec{q}\cdot\vec{\rho}_n} |\phi\rangle$$

The **unpolarized** differential cross section on an extended object

$$\frac{d\sigma}{d\Omega} = |F(\vec{q})|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}}$$

Form factor **modulus** is obtained by comparing theory and experiment

In the Breit frame the Fourier transform of the form factor

$$\rho(\vec{r}) = \frac{Q_0}{(2\pi)^3} \int d^3\vec{q}\, F(\vec{q})\, e^{-i\,\vec{q}\cdot\vec{r}}$$

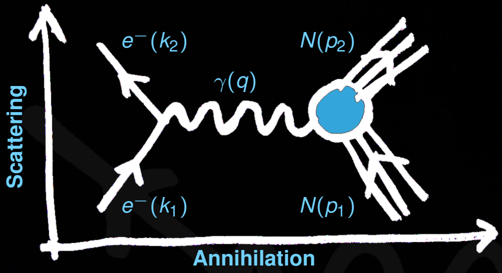
is the **charge spatial distribution**.
The form factor is Fourier transform of $\rho(\vec{r})$

$$F(\vec{q}) = \frac{1}{Q_0} \int d^3\vec{r}\, \rho(\vec{r})\, e^{i\,\vec{q}\cdot\vec{r}}$$

Nucleon electromagnetic form factors

- Form factors characterize the internal structure of a hadron $\Rightarrow F_{\text{point-like}} = \text{constant}$.
- Elastic form factors contain information on the hadron ground state.
- In a parity and T-invariant theory, the electromagnetic structure of a particle of spin $S\hbar$ is defined by **$2S+1$ form factors**.
- Neutron and proton form factors are different.
- Playground for **theory** and **experiment**:
  - at low q^2 probe the size of the hadron;
  - at high q^2 test the QCD counting rule.

Dirac and Pauli Form Factors



Scattering: $e^- N \to e^- N$
Space-like kinematic region

$$q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \leq 0$$

Annihilation: $e^+e^- \leftrightarrow N\overline{N}$
Time-like kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2}\left[e\,\overline{u}(k_2)\gamma_\mu u(k_1)\right]\underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = \mathcal{Q}_N$
- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

Sachs Form Factors

Breit frame

No energy exchanged

$$p_1 = (E, -\vec{q}/2)$$
$$p_2 = (E, \vec{q}/2)$$
$$q = (0, \vec{q})$$

Nucleon elecrtomagnetic four-current

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \begin{cases} \rho_q = J_N^0 = e\left[F_1^N + \frac{q^2}{4M_N^2}F_2^N\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\left[F_1^N + F_2^N\right] \end{cases}$$

Sachs Nucleon Form Factors

$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2) \qquad G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2}F_2^N(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

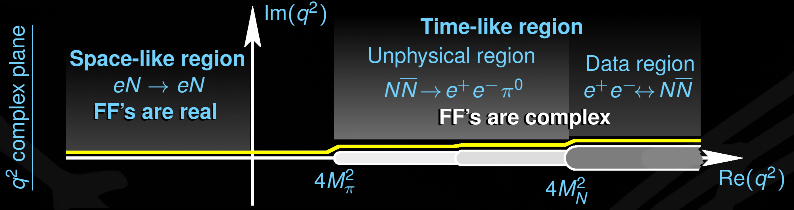
Normalization at $q^2 = 0$

- $G_E^N(0) = \mathcal{Q}_N$
- $G_M^N(0) = \mu_N$

$$\mu_N = \mathcal{Q}_N + \kappa_N$$

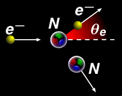
is the nucleon magnetic moment

CROSS SECTIONS AND ANALYTICITY



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

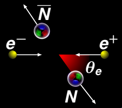
$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$

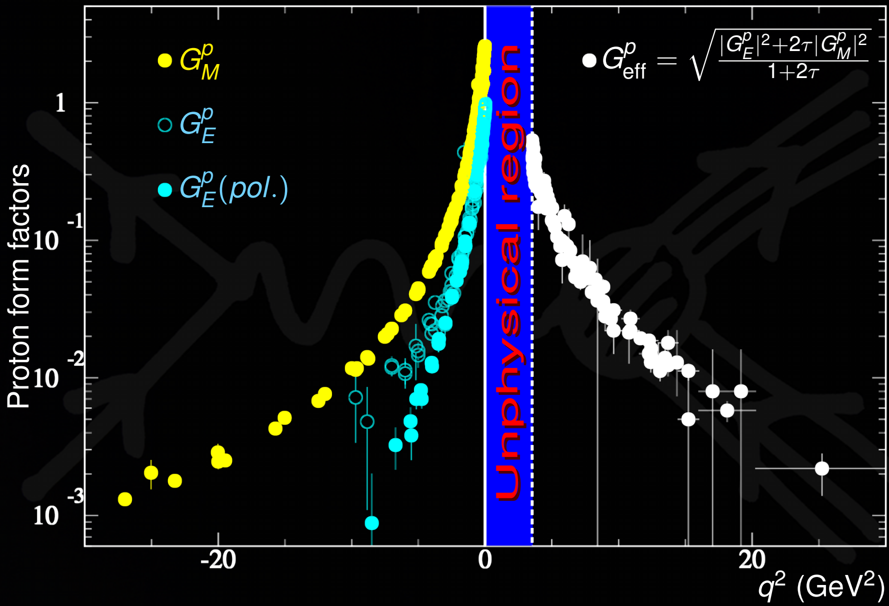


Annihilation

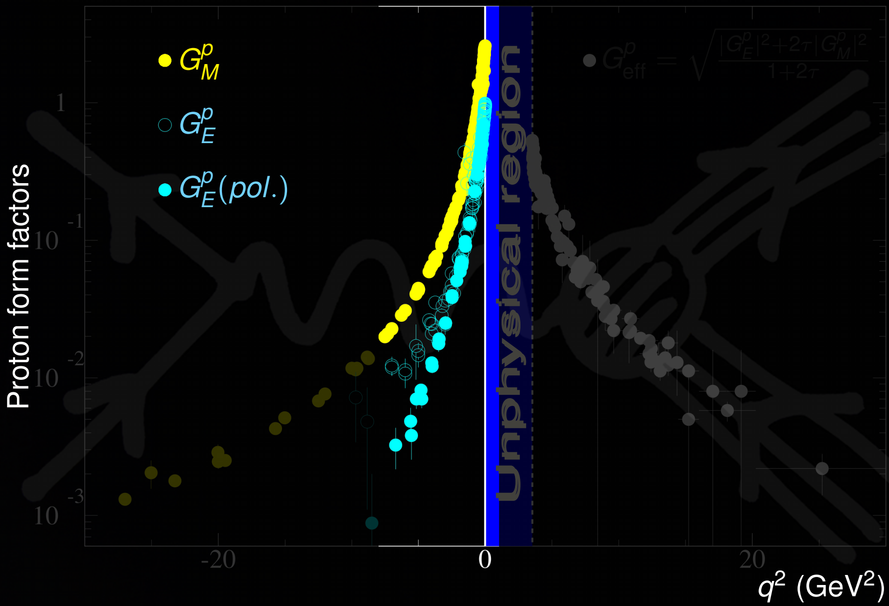
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2} \left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau} \sin^2\theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

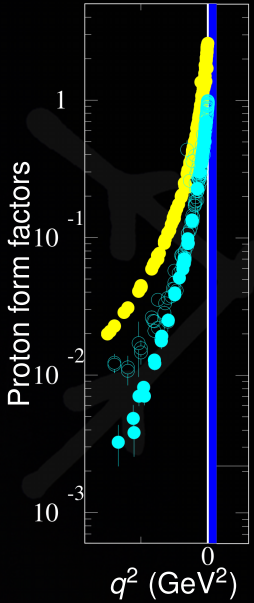
The proton radius



The proton radius



THE PROTON RADIUS



$$G_E^p(q^2) = \int d^3\vec{r}\,\rho(r)\,e^{i\,\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6}q^2\left\langle r_c^2\right\rangle + \mathcal{O}(q^4)$$

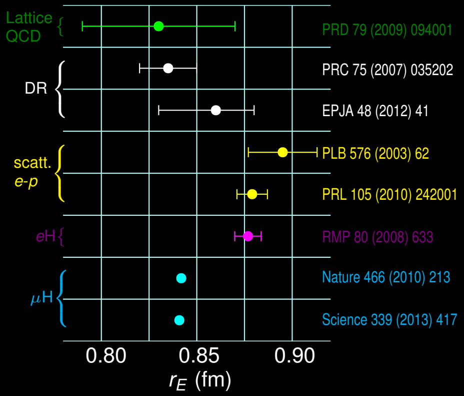
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2\rangle} = \sqrt{4\pi\int_0^\infty r^4\rho(r)\,dr} = \sqrt{\frac{6}{G_E^p(0)}\left.\frac{dG_E^p}{dq^2}\right|_{q^2=0}}$$

Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4/(q^2+\lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2/(q^2+\lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-q^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian

THE PROTON RADIUS



Ongoing discussions. . .

- $q^2 \to 0^-$ extrapolation
- Radiative corrections
- Two-photon exchange
- Coulomb corrections

Analyticity via dispersion relations and QCD counting rules can give directly the proton radius. . .

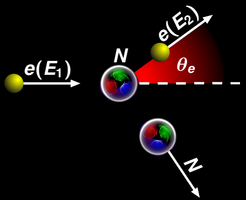
Logarithmic derivative of form factor at $q^2 = 0$ by means of dispersion relations for the logarithm

$$r_E^2 = \frac{12M_\pi^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\ln|G_E^p(t)/G_E^p(0)|}{t^2\sqrt{t-4M_\pi^2}}\,dt$$

UNPOLARIZED CROSS SECTION
POLARIZATION OBSERVABLES

N.M.Rosenbluth,Phys.Rev.79,615

A.I.Akhiezer,M.P.Rekalo,Sov.Phys.Dokl.13,572



Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1-\tau}\left[G_E^2 - \frac{\tau}{\epsilon} G_M^2\right] \qquad \tau = \frac{q^2}{4M_N^2}$$

Mott pointlike cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{4\alpha^2}{(-q^2)^2}\frac{E_2^3}{E_1}\cos^2(\theta_e/2)$$

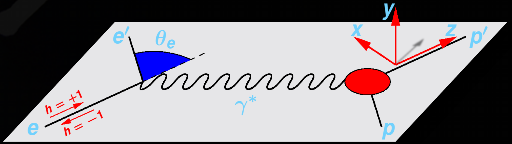
Degree of linear polarization of the virtual photon

$$\epsilon = \left[1 + 2(1-\tau)\tan^2(\theta_e/2)\right]^{-1}$$

In case of **polarized electrons** ($h = \pm 1$) on unpolarized nucleon target and measuring the polarization of the outgoing proton:

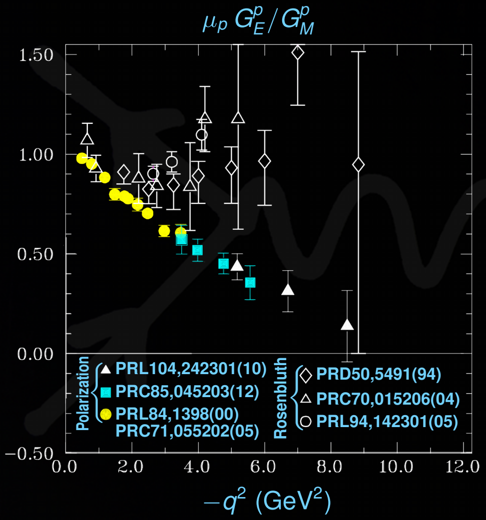
$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon}G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M\left(G_E^2 - \frac{\tau}{\epsilon}G_M^2\right)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$



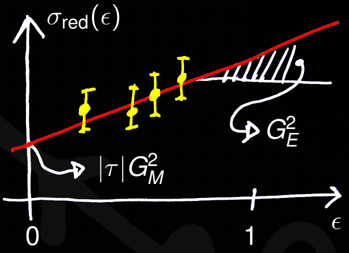
$$\frac{P'_x}{P'_z} = -\frac{2M\cot(\theta_e/2)}{E_e + E'_e}\frac{G_E}{G_M}$$

$\mu_p G_E^p / G_M^p$: Rosenbluth and Polarization Techniques



- "Standard" dipole for the proton magnetic form factors G_M^p
- Linear deviation from the dipole for the electric proton form factor G_E^p
- **QCD scaling still not reached**
- **Zero crossing for G_E^p**
- Polarization data do not agree with old Rosenbluth data (◇)
- New Rosenbluth separation data from JLab **still do not agree** with polarization data

THE ROSENBLUTH METHOD

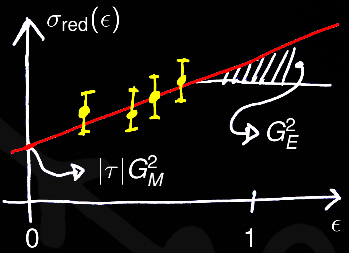


Reduced cross section $\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}}\epsilon(1-\tau)$

$$\sigma_{\text{red}}(\epsilon;\theta,Q^2) = \epsilon G_E^2 + |\tau|\, G_M^2$$

- Function of ϵ at fixed θ and Q^2
- Slope $\rightarrow G_E^2$
- Intercept $\rightarrow G_M^2$

THE ROSENBLUTH METHOD



Reduced cross section $\dfrac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}}\epsilon(1-\tau)$

$$\sigma_{\text{red}}(\epsilon;\theta,Q^2) = \epsilon G_E^2 + |\tau|\, G_M^2$$

- Function of ϵ at fixed θ and Q^2
- Slope $\to G_E^2$
- Intercept $\to G_M^2$

$$\sigma_{\text{red}}(\varepsilon) = \underbrace{\epsilon G_E^2}_{<3\%} + \underbrace{|\tau| G_M^2}_{>97\%}$$

For $Q^2 \geq 4$ GeV^2 the G_E^2 contribution to the reduced cross section can be of the same order of the error

$\Longrightarrow$ Large correlation

$\Longrightarrow$ Large errors

$\Longrightarrow$ Large rad. corr.

$$\delta\sigma_{\text{red}}(\varepsilon) \sim 1.5\,\%$$

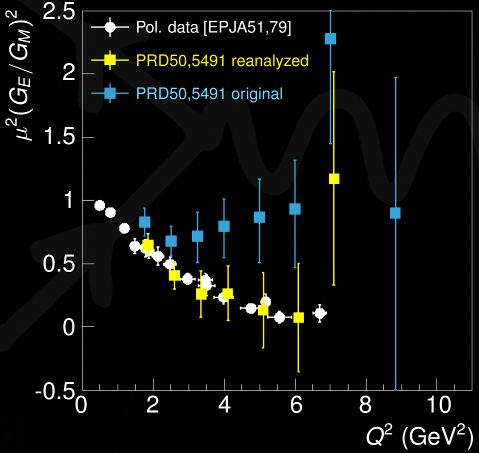
THE ROSENBLUTH RE-ANALYSIS[1]

ARXIV:1604.02421

"New" fit function

$$\sigma_{red}(\epsilon) = G_M^2 \left(R^2 \epsilon + |\tau|\right)$$

- It reduces the effect of the corrections on the individual form factors
- The general normalization and systematic errors absorbed by G_M^2



$$R^2 = \frac{G_E^2}{G_M^2}$$

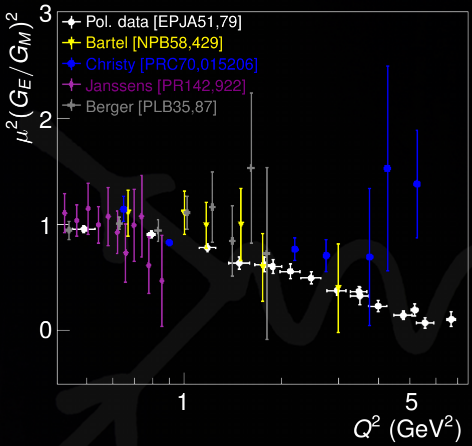
Reference data from PRD50,5491

Good agreement with pol. data

No other reaction mechanisms (two-photon exchange) are needed

The Rosenbluth re-analysis$_2$

arXiv:1604.02421



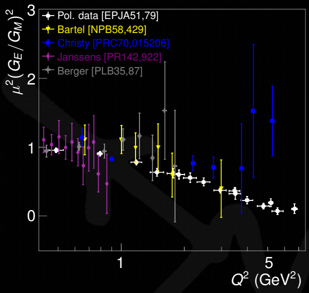
Other sets of data agree with polarization measurements

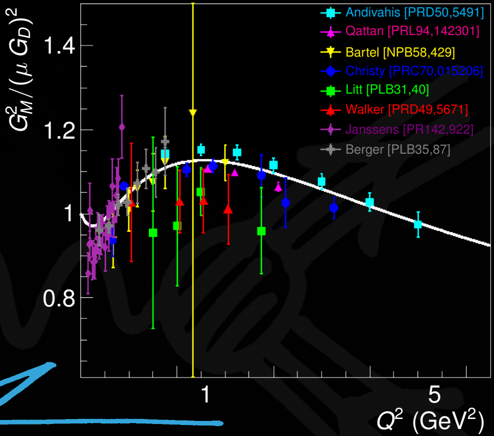
Those not in agreement, Qattan [PRL94, 142301] and Walker [RD49, 5671], show values of R^2 growing with Q^2

For these experiments radiative corrections and/or correlations are especially large

THE ROSENBLUTH RE-ANALYSIS$_2$

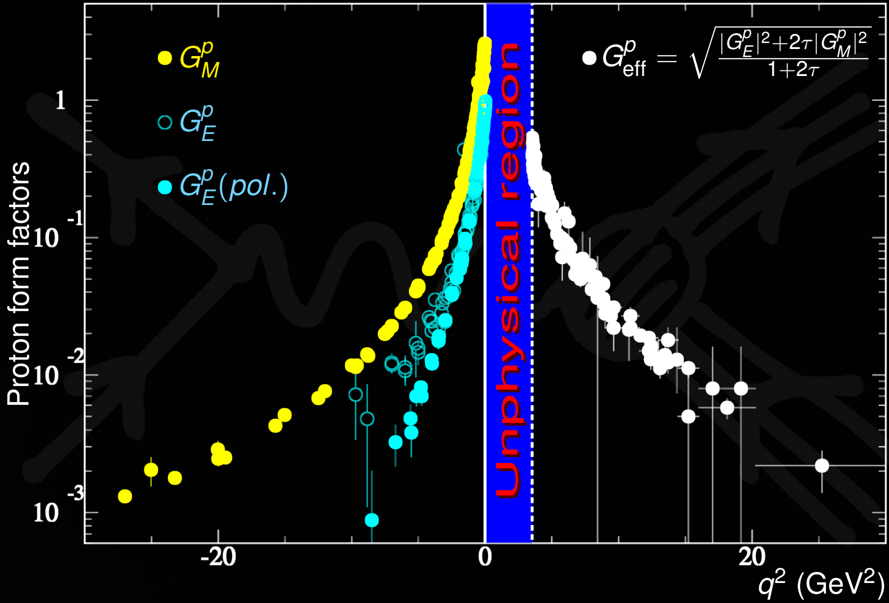
ARXIV:1604.02421



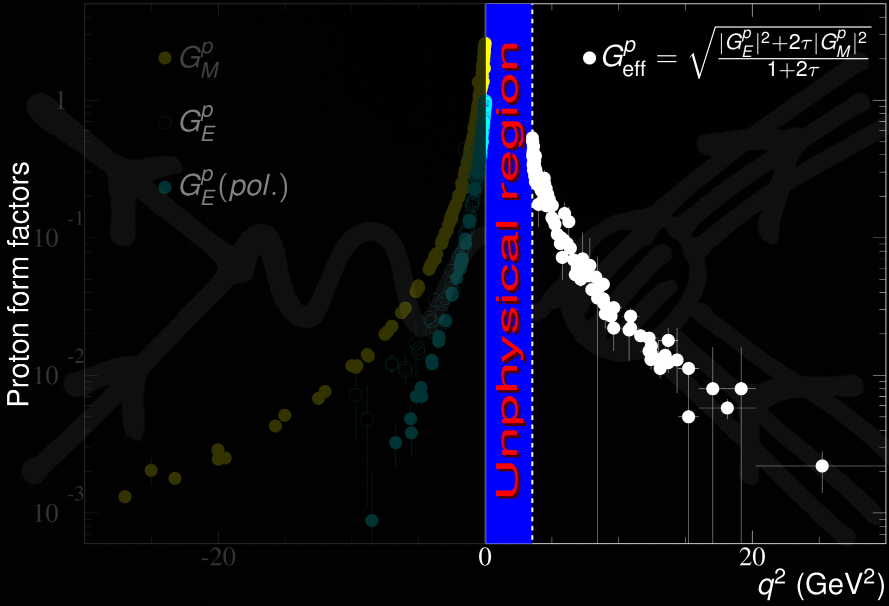


- The parameter G_M^2 is better determined than R^2 by the Rosenbluth fit
- Its values are well described by two component nucleon model based on VMD [PRC69, 068201]
- Values consistent with the model that represents, in fact, a global fit to the data

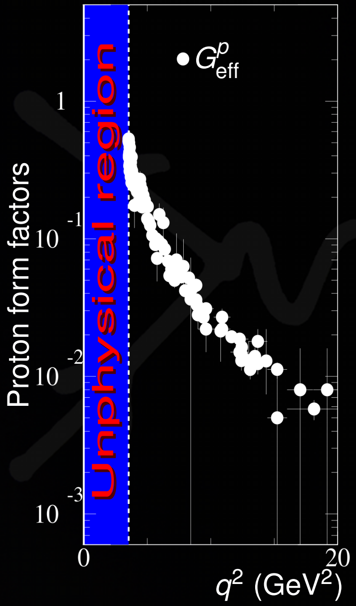
THE TIME-LIKE REGION



The time-like region



The time-like region



Differential cross section $e^+e^- \to p\bar{p}$

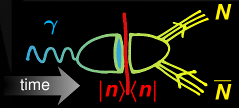
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta\mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M^p|^2 + \frac{1}{\tau}\sin^2\theta|G_E^p|^2\right]$$

Optical theorem

$${\rm Im}\langle\overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle\overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...



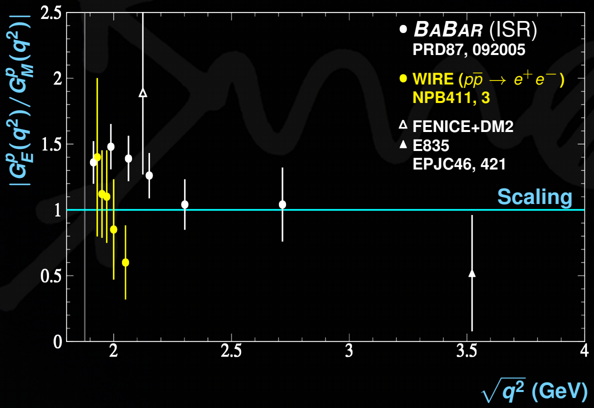
Form factors are complex for $q^2 > 4M_\pi^2$

- The cross section is an even function of $\cos\theta$
- It does not depend on the form factor phases
- At high q^2 the $|G_E^p|^2$ contribution is suppressed
- The unphysical region is not accessible through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

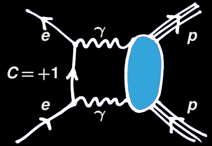
TIME-LIKE $|G_E^p/G_M^p|$ MEASUREMENTS

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta\mathcal{C}}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2}\sin^2\theta\left|\frac{G_E^p}{G_M^p}\right|^2\right]$$



$\gamma\gamma$ exchange



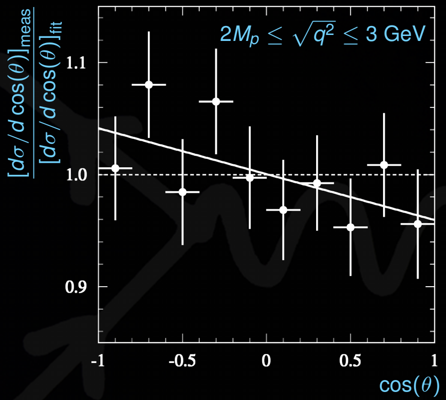
$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]

$\gamma\gamma$ EXCHANGE FROM $e^+e^- \to p\bar{p}\gamma$ BABAR 2013 DATA

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
BABAR Phys. Rev. D87 (2013) 092005



See next talk by Vladimir Druzhinin

Integrated over the $p\bar{p}$-CM energy from threshold up to 3 GeV

The MC-fit assumes **one-photon exchange**

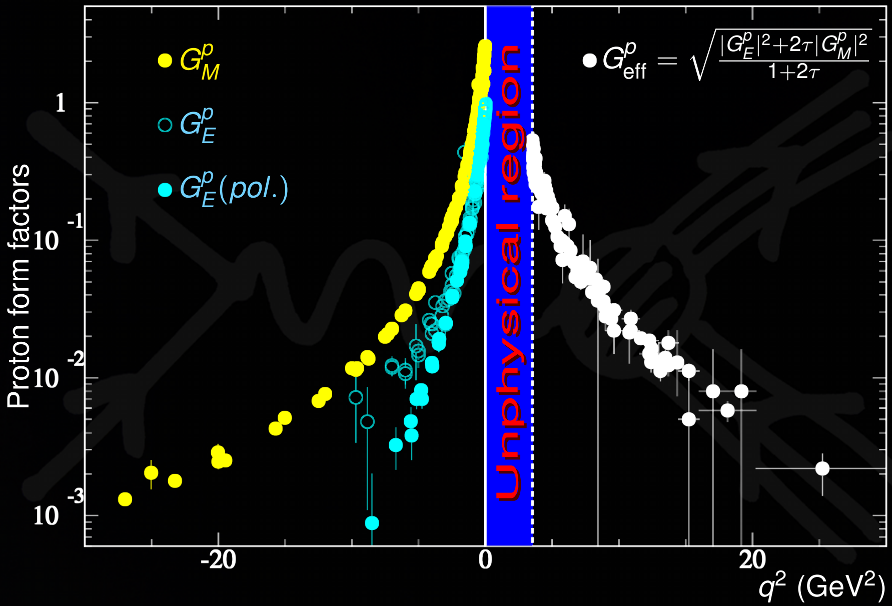
Slope $= -0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

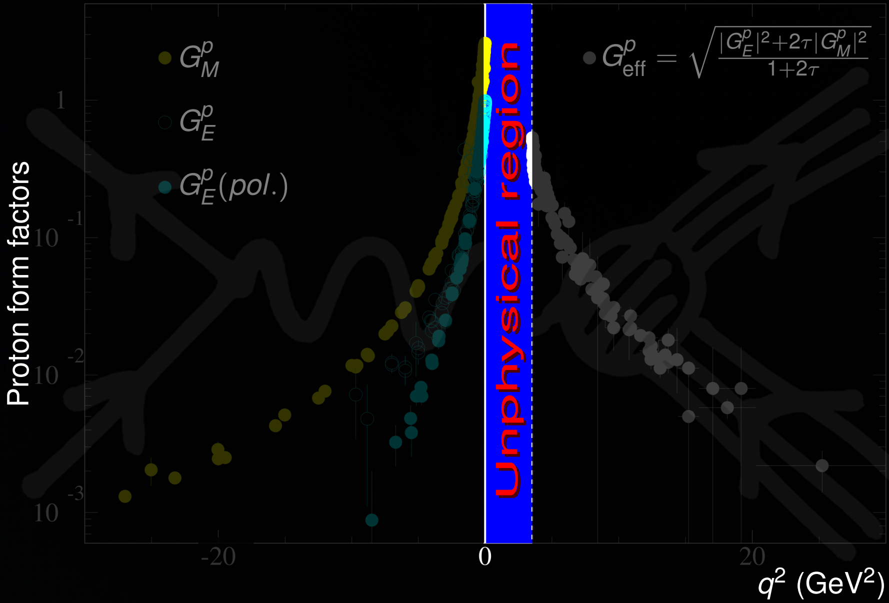
$$\langle \mathcal{A} \rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos\theta_p \gtrless 0$

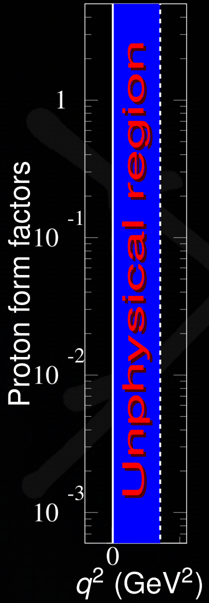
THE UNPHYSICAL REGION



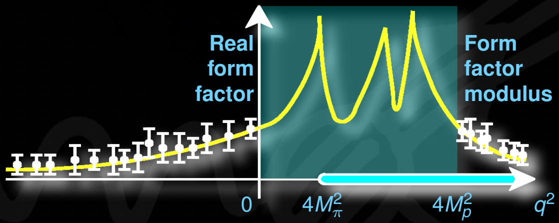
The unphysical region



THE UNPHYSICAL REGION



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$



In that region, form factors

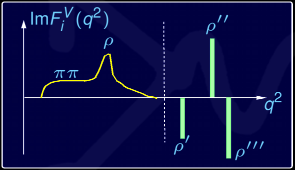
- are still well defined but not (directly) experimentally accessible
- are complex and, following VMD-based models, receive their main contribution from intermediate resonances

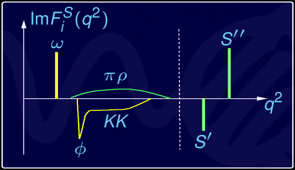
HANDLING THE UNPHYSICAL REGION$_1$

Model dependent disclosing [Höler, Mergell, Meissner, Hammer]

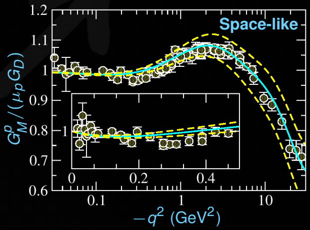
Optical theorem .... $\mathrm{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

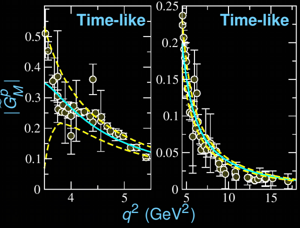
Dispersion relations for the imaginary part .... $F(q^2_{\mathrm{SL}}) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}F(q^2_{\mathrm{TL}})}{q^2_{\mathrm{TL}} - q^2_{\mathrm{SL}}}\, dq^2_{\mathrm{TL}}$





- 2π and $2K$ continua are known
- The ρ resonance with finite width
- Dirac delta poles for higher mass states





- Super convergence relations
$$\int_{4M_\pi^2}^{\infty} \mathrm{Im}\, F_{1,2}(q^2)dq^2 = 0$$
$$\int_{4M_\pi^2}^{\infty} q^2 \mathrm{Im}\, F_2(q^2)dq^2 = 0$$
- Asymptotic behaviors from perturbative QCD

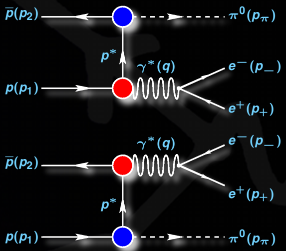
HANDLING THE UNPHYSICAL REGION$_2$

Accessing the unphysical region

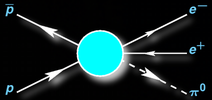
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π-production

$p\bar{p} \to \pi^0 e^+ e^-$



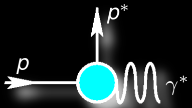
The process $p\bar{p} \to \pi^0 e^+ e^-$



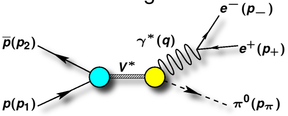
Described in general by **six** amplitudes which depend on **three** kinematical variables

Hadronic current **[PRC75 045205]**

- $J_\mu = \phi_\pi(p_\pi)\, \overline{v}(p_2)\, O_\mu\, u(p_1)$
- $O_\mu = O_\mu[\Gamma_\mu(q)]$
- $\langle N(p')|\Gamma_\mu(q)|N(p)\rangle = \overline{u}(p')\left[F_1(q^2)\gamma_\mu + \frac{i\,\sigma_{\mu\nu}q^\nu}{4M_p^2}F_2(q^2)\right]u(p)$



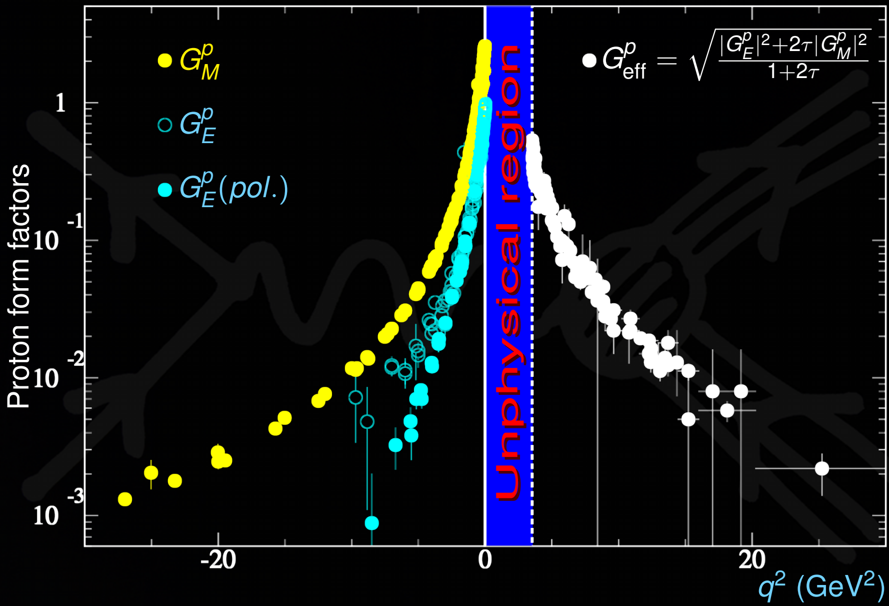
Background:
Annihilation diagram



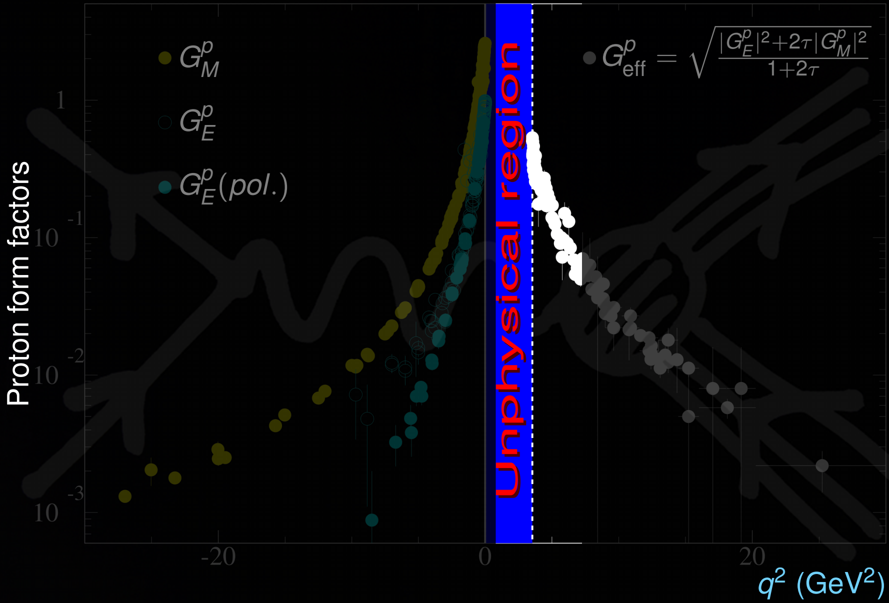
Polarization observables help in disentangle reaction mechanisms

[E. A. Kuraev *et al*., J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

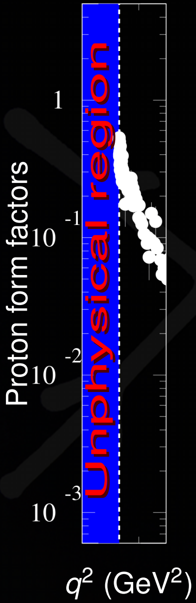
The threshold region



THE THRESHOLD REGION



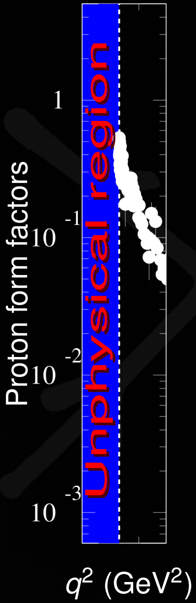
THE THRESHOLD REGION



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

THE THRESHOLD REGION



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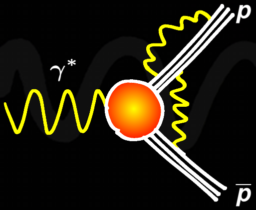


Enhancement factor $\mathcal{E} = \dfrac{\pi\alpha}{\beta}$

It is responsible for the one-photon exchange $p\bar{p}$ final state interaction, dominates at threshold and cancels the phase-space factor.

Resummation factor $\mathcal{R} = \dfrac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$

It is responsible for the multi-photon $p\bar{p}$ final state interaction, becomes ineffective few MeV above threshold and accounts also for gluon exchange.

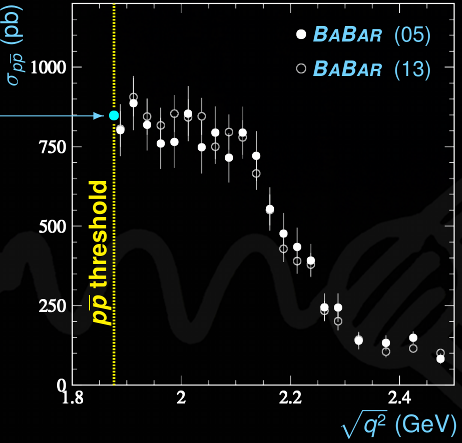
$$\mathcal{C} = \mathcal{E} \times \mathcal{R} = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} \xrightarrow[\beta\to 0]{} \frac{\pi\alpha}{\beta} = \mathcal{E}$$

STEP AND PLATEAU IN BABAR DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with

$\left|G^p(4M_p^2)\right| = 1$



At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\beta_p}{\beta_p}\left|G^p(4M_p^2)\right|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\left|G^p(4M_p^2)\right|^2 \text{ nb}$$

$\Rightarrow$

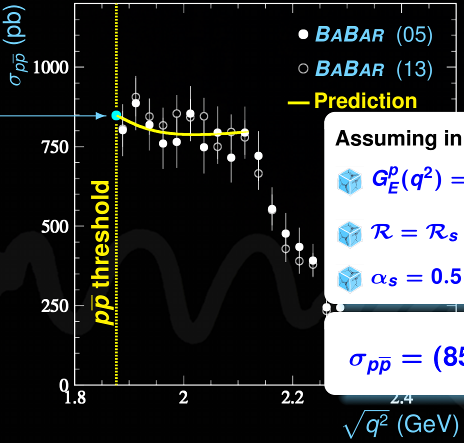
$\left|G_S^p(4M_p^2)\right| \equiv 1$

as pointlike fermion pairs!

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with $|G^p(4M_p^2)| = 1$



Assuming in $q^2 \leq (2.1\text{GeV})^2$:

- $G_E^p(q^2) = G_M^p(q^2) = 1$
- $\mathcal{R} = \mathcal{R}_s = \left(1 - e^{-\frac{\pi\alpha_S}{\beta}}\right)^{-1}$
- $\alpha_s = 0.5$

$$\sigma_{p\bar{p}} = (850\,\text{pb})\,\frac{1}{\tau}\,\mathcal{R}_s$$

At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\beta_p}{\beta_p}\,|G^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G^p(4M_p^2)|^2\ \text{nb}$$

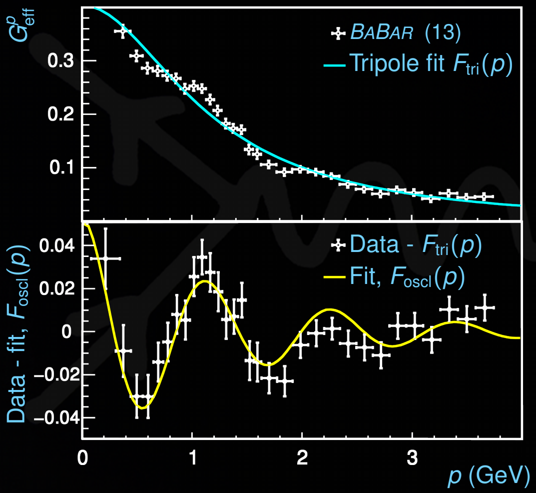
$\Longrightarrow$

$|G_S^p(4M_p^2)| \equiv 1$

as pointlike fermion pairs!

PERIODIC INTERFERENCE NEAR THRESHOLD

A. BIANCONI, E. TOMASI-GUSTAFSSON, PHYS. REV. LETT. 114, 232301



$$F_{\rm oscl}(p) = Ae^{-Bp}\cos(Cp + D)$$

$A \ll 1$	B damp. par.
$C = r < 1$ fm	D tresh. shift

p is the momentum of the proton in the anti-proton rest frame.

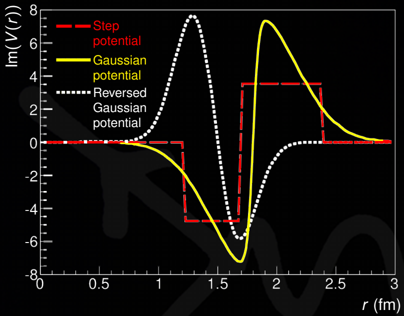
The periodical behavior suggests an interference due to a rescattering of proton and antiproton at low kinetic energy and separation $\sim$ 1 fm.

Proton and antiproton interact when they are almost phenomenological.

Unitarity implies a large imaginary part of form factors.

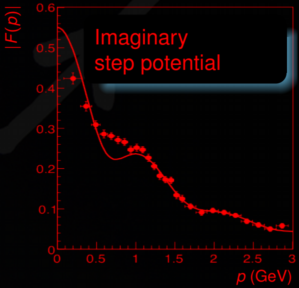
Double hollow potentials

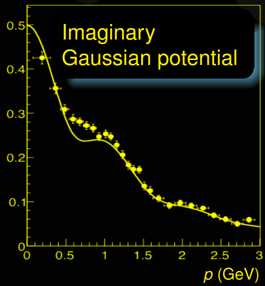
A. Bianconi, E. Tomasi-Gustafsson, PRC93, 035201

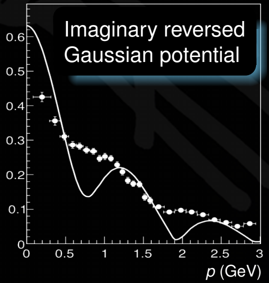


Double-layer optical potentials are phenomenologically required

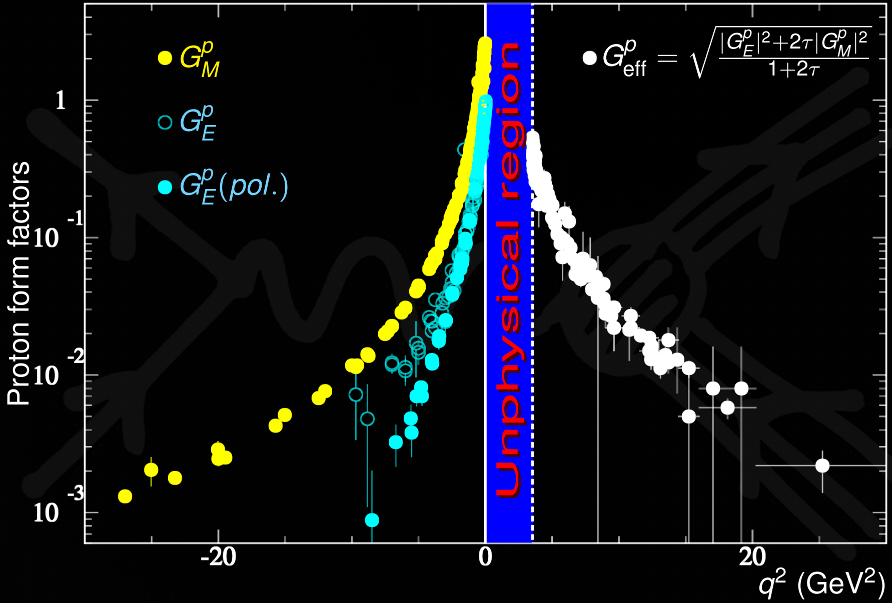
- Purely absorptive at $r > 1.7$ fm
- Flux-generating at $r < 1.7$ fm
- Quick (< 2 fm) transition absorption to flux-generating to get the period
- Maximum at $p = 0$ $\Rightarrow$ threshold enhancement
- Reversed potentials reproduce the oscillation amplitude not the period



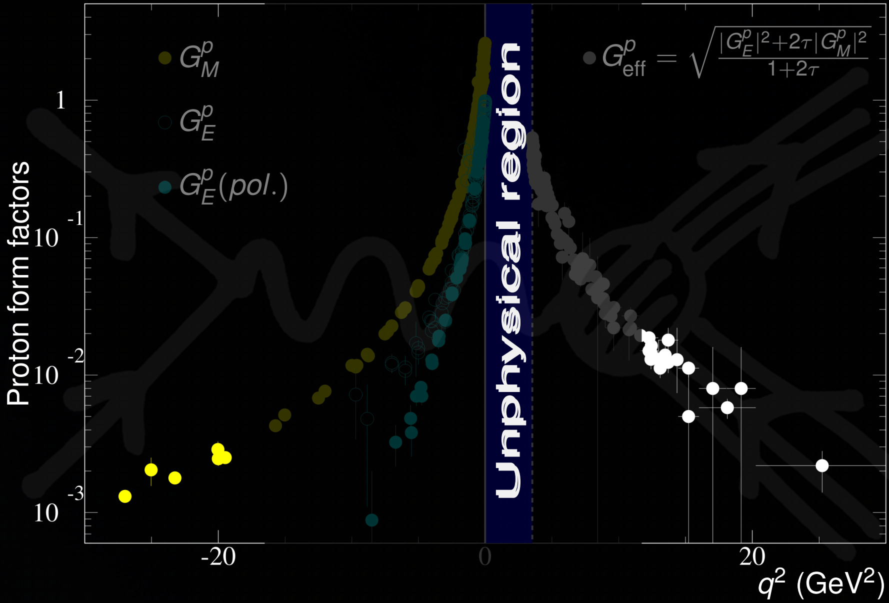




THE ASYMPTOTIC REGIONS$_1$

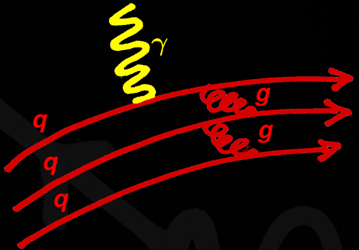


THE ASYMPTOTIC REGIONS$_1$

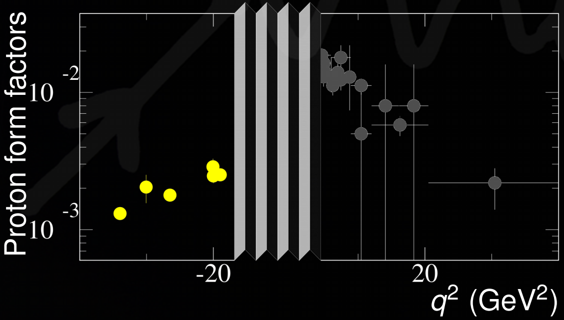


THE ASYMPTOTIC REGIONS$_1$

Space-like dimensional scaling



- pQCD: as $\boldsymbol{q}^2 \to -\infty$, asymptotic behaviors of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$, and $\boldsymbol{G_E}$ and $\boldsymbol{G_M}$ must follow counting rules
- Valence quarks exchange gluons to distribute the momentum transfer $\boldsymbol{q}$



- Dirac and Pauli form factors

$$F_i(q^2) \underset{q^2\to-\infty}{\sim} (-q^2)^{-1-i}$$

- Sachs form factors

$$G_{E,M}(q^2) \underset{q^2\to-\infty}{\sim} (-q^2)^{-2}$$

$$\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2\to-\infty}{\sim} \text{cost.}$$

THE ASYMPTOTIC REGIONS$_1$

Time-like asymptotic behavior

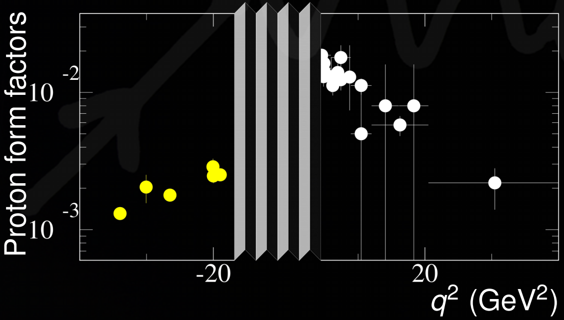
Phragmèn Lindelöf theorem:

If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

$$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

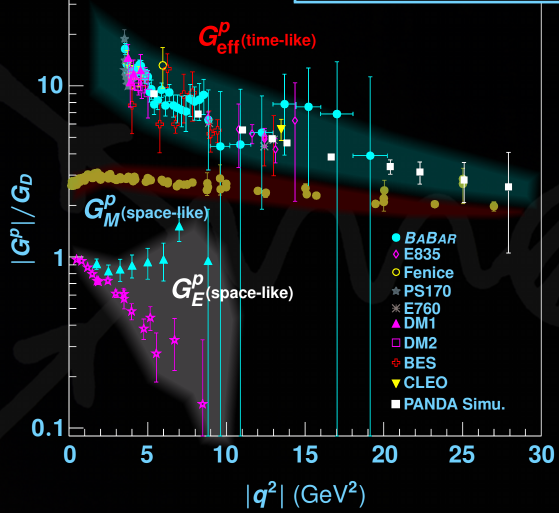
$$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$

real



THE ASYMPTOTIC REGIONS$_2$

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



pQCD

$$G^p_{eff}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

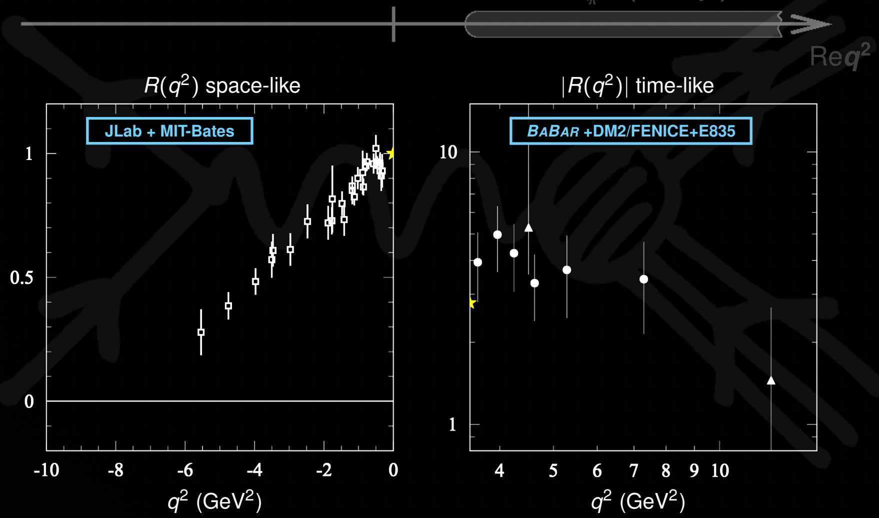
Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{eff}(q^2)}{G^p_M(-q^2)} = 1$$

ANALYTIC $R = \mu G_E / G_M$

Eur. Phys. J. A32 (2007) 421

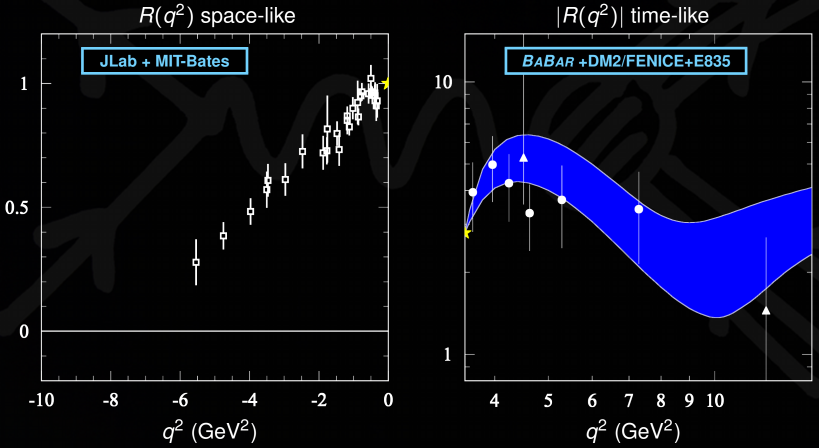
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$



Analytic $R = \mu G_E / G_M$

Eur. Phys. J. A32 (2007) 421

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\,R(s)}{s(s-q^2)} ds$$



ANALYTIC $R = \mu G_E / G_M$

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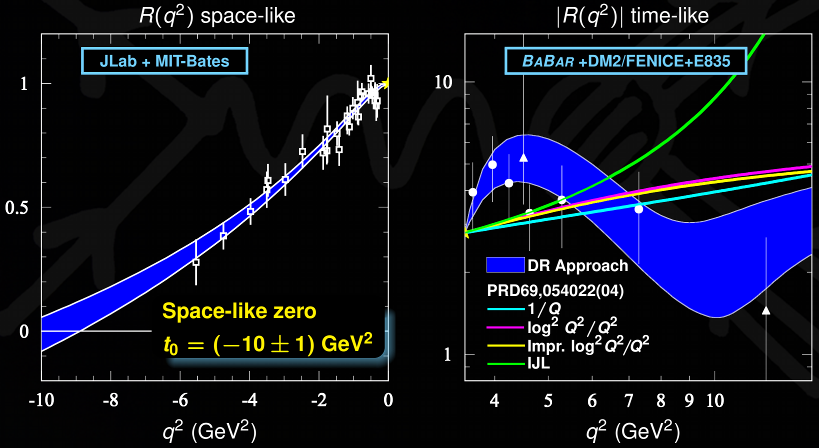
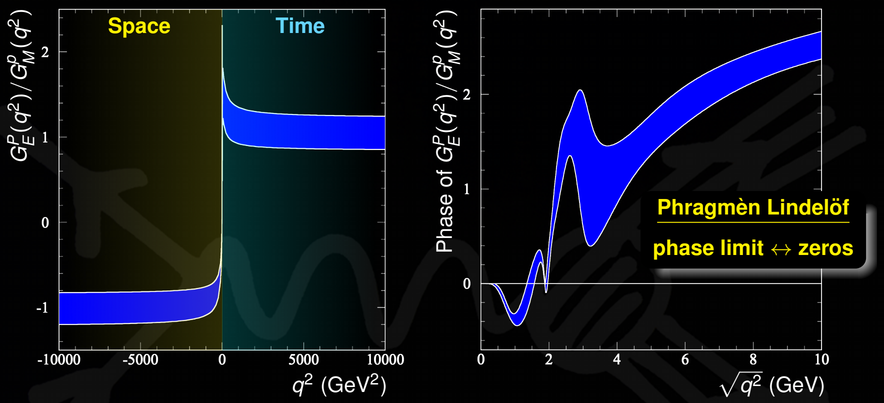


ASYMPTOTIC G_E/G_M AND PHASE



pQCD prediction

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \underset{|q^2|\to\infty}{\longrightarrow} -1$$

Phase from DR

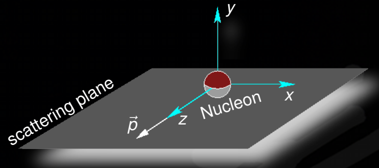
$$\phi(q^2) = -\frac{\sqrt{q^2 - s_{\rm th}}}{\pi}\,\mathrm{Pr}\int_{s_{\rm th}}^{\infty} \frac{\ln|R(s)|\,ds}{\sqrt{s-s_{\rm th}}\,(s-q^2)}$$

POLARIZATION FORMULAE IN THE TIME-LIKE REGION

The ratio $R(q^2)$ is complex for $q^2 \geq 4M_\pi^2$

$$R(q^2) = \mu_p \frac{G_E(q^2)}{G_M(q^2)} = |R(q^2)| e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



[A.Z. Dubnickova, S. Dubnicka, M.P. Rekalo, NCA109,241(96)]

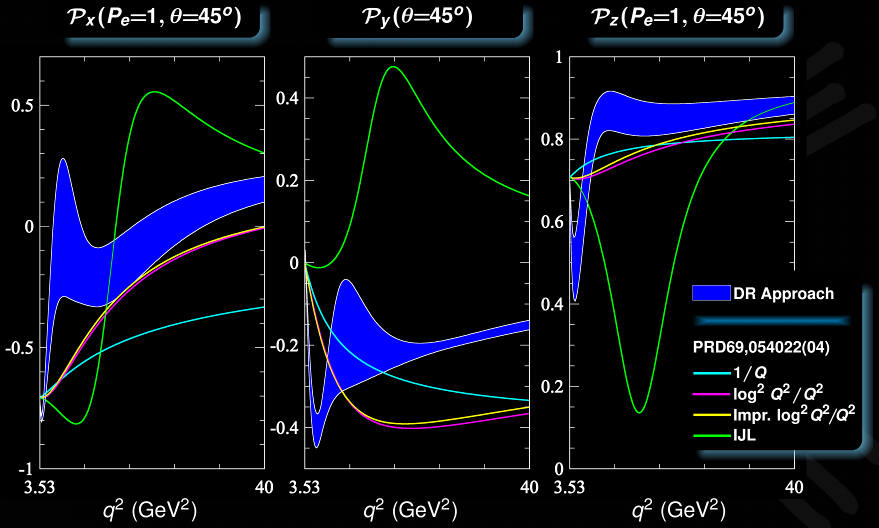
$$\mathcal{P}_y = - \frac{\sin(2\theta)|R|\sin(\rho)}{D\sqrt{\tau}} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \mathcal{A}_y \Big\} \text{ Does not depend on } P_e$$

$$\mathcal{P}_x = - P_e \frac{2\sin(2\theta)|R|\cos(\rho)}{D\sqrt{\tau}}$$

$$\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D} \Big\} \text{ Does not depend on } \rho$$

$$D = \frac{1 + \cos^2\theta + \frac{1}{\tau}|R|^2 \sin^2\theta}{\mu}, \quad \tau = \frac{q^2}{4M^2}, \quad P_e = \text{electron polarization}$$

Single Polarization



Conclusions Phys. Rev. **550-551** (2015) 1

Global models in space and time-like regions for proton and neutron, electric and magnetic form factors must be encouraged.

They can help in understanding:

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- . . .

To measure:

- zero of G_E^p in space-like region
- moduli of G_E and G_M in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \to \pi^0 e^+ e^-$)
- . . .

Experiments: now and future

See next talks by
Vladimir Druzhinin
Cristina Morales
Iris Zimmermann

Space-like region

A1 Mainz

- G_E^n at $-q^2 = 1.5$ GeV^2 (Pol. ^{3}He)
- G_E^p and G_M^p for $-q^2 \leq 2.0$ GeV^2

Jefferson Lab

- [Hall A] G_E^p / G_M^p up to 14 GeV^2
- [Hall A] G_M^n (ratio) up to 18 GeV^2
- [Hall A] G_E^n / G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2

Time-like region

SND, CMD-3 at VEPP-2000
e^+e^- collider
$|G_{\text{eff}}^p|$, $|G_{\text{eff}}^n|$ (scan)
$q^2 \leq (4\text{ GeV})^2$

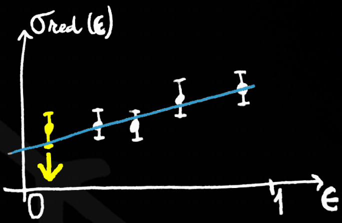
BESIII at BEPCII
e^+e^- collider
$|G_E^p|$, $|G_M^p|$, $|G_{\text{eff}}^n|$ (scan and ISR)
$q^2 \leq (3.5\text{ GeV})^2$

PANDA at FAIR
$p\bar{p}$ collider
$|G_E^p|$, $|G_M^p|$, G_E^p / G_M^p phase ($\bar{p}$ polarization?)
$(2.4\text{ GeV})^2 \leq q^2 \leq (3.7\text{ GeV})^2$

Belle II ? at SuperKEKB
e^+e^- collider
$|G_E^p|$, $|G_M^p|$, (ISR)
$q^2 \leq (4.5\text{ GeV})^2$

Additional slides

The Andivahis normalization



Andivahis data on $\sigma_{\text{red}}(\epsilon)$ at lowest ϵ's, $\{Q_j^2, \epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$, have been obtained with a different experimental setup.

The corresponding reduced cross section values are normalized in order to be aligned with the other points at the same Q^2.

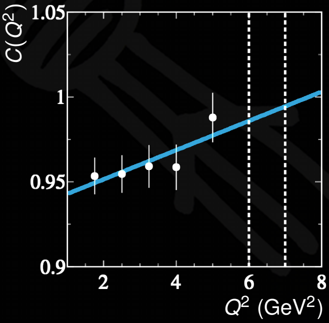
The normalization factor $\overline{C}$ is obtained as the mean value of

$$C(Q_j^2) = \frac{G_{M0}^2 (R_0^2 \epsilon_{0,j} + |\tau_j|)}{\sigma_{0,j}}$$

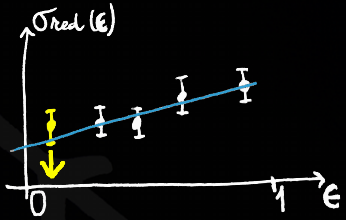
G_{M0}^2 and R_0^2 are the parameters obtained by not including $\{\epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$.

We find that:

- the correction factor depends on Q^2
- values $C(Q_i^2)$ are obtained by a linear fit
- the correction, if needed, vanishes at high Q^2
- the first two points are slightly affected by the procedure



THE ANDIVAHIS NORMALIZATION



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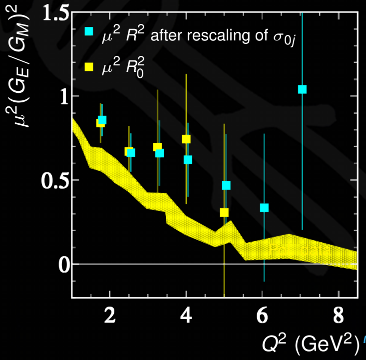
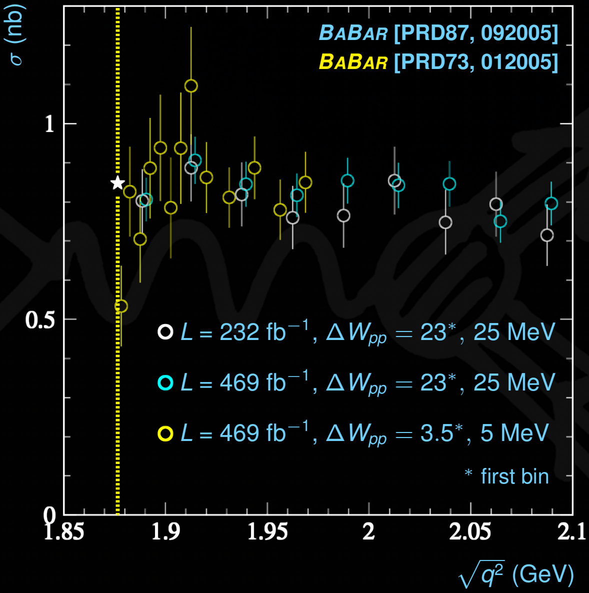
G_{M0}^2 and R_0^2 are the parameters obtained by not including $\{\epsilon_{0j}, \sigma_{0j}, \delta\sigma_{0j}\}$.

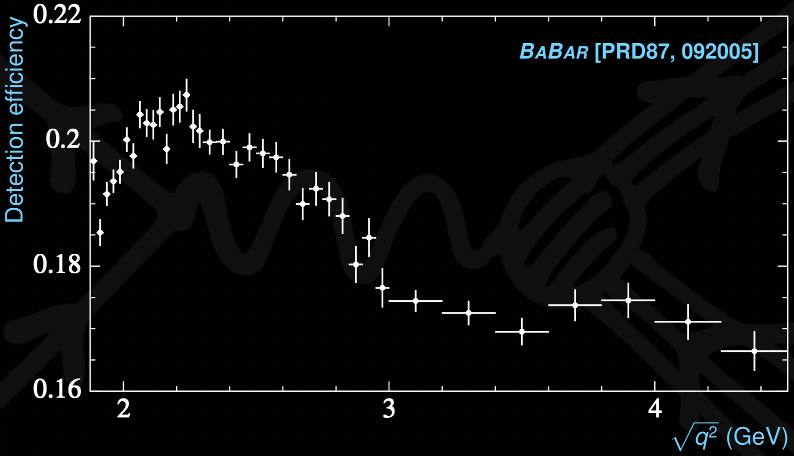
- The correction does not change the tendency
- Values of R^2 at high Q^2 do not follow the scaling hypothesis



THE THRESHOLD REGION₃



THE THRESHOLD REGION$_3$



Partial wave form factors

$$\begin{cases} P_\gamma = -1 \\ J_\gamma = 1 \end{cases} \qquad \begin{cases} P_{p\bar{p}} = (-1)^{L+1} \\ S_{p\bar{p}} = 0, 1 \end{cases}$$

$$\Downarrow \qquad \Downarrow$$

$$\begin{cases} L_{p\bar{p}} = 0, 2 \\ S_{p\bar{p}} = 1 \end{cases} \qquad \begin{cases} L_{p\bar{p}} = 1 \\ S_{p\bar{p}} = 0 \end{cases}$$

Partial wave form factors

$$G_S^p = \frac{1}{3}\left(2G_M^p\sqrt{\frac{q^2}{4M_p^2}} + G_E^p\right) \qquad G_D^p = \frac{1}{3}\left(G_M^p\sqrt{\frac{q^2}{4M_p^2}} - G_E^p\right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta\frac{4M_p^2}{(q^2)^2}\left[\mathcal{C}\left|G_S^p(q^2)\right|^2 + 2\left|G_D^p(q^2)\right|^2\right]$$

ENHANCEMENT AND RESUMMATION FACTORS

Coulomb factor $\mathcal{C}$ for S-wave only

- **Partial wave FF**......... $G_S = (2G_M\sqrt{\tau} + G_E)/3\,, \qquad G_D = (G_M\sqrt{\tau} - G_E)/3$
- **Cross section**................. $\sigma(q^2) = 2\pi\alpha^2 \dfrac{\beta}{\tau\, q^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$
- **$\mathcal{E}$nhancement and $\mathcal{R}$esummation factors** .... $\mathcal{C} = \mathcal{E} \times \mathcal{R}$

Enhancement factor

$$\mathcal{E} = \frac{\pi\alpha}{\beta}$$

- It is responsible for the **one-photon exchange $p\bar{p}$ FSI**
- dominates close to threshold: $\mathcal{C} \underset{\beta\sim 0}{\simeq} \mathcal{E}$
- cancels the phase-space factor $\Rightarrow$ stepwise cross section at threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\not\beta}{\not\beta}|G_S^p(4M_p^2)|^2 = 0.85\,|G_S^p(4M_p^2)|^2 \text{ nb}$$

Resummation factor

$$\mathcal{R} = \frac{1}{1-e^{-\frac{\pi\alpha}{\beta}}}$$

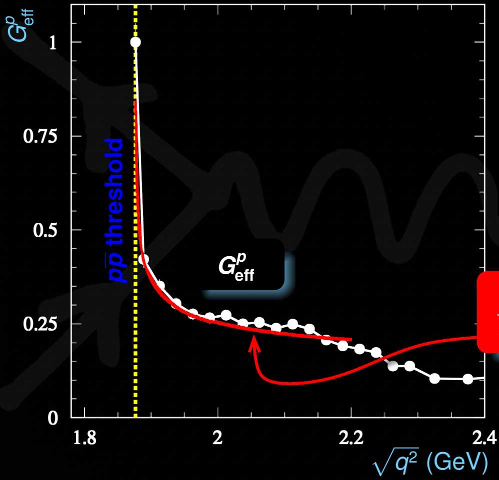
- It is responsible for the **multi-photon exchange $p\bar{p}$ FSI**
- becomes ineffective few MeV above threshold: $\mathcal{R} \underset{\beta>0}{\simeq} 1$
- must account also for gluon exchange

$$\mathcal{R} \to \mathcal{R}_s = \left[1 - \exp\left(-\pi\alpha_s/\tilde{\beta}\right)\right]^{-1}$$

$\alpha_s \simeq 0.5$

$\tilde{\beta} = \beta/(1-\beta)$

$G_{\rm eff}^p$, WE GET WHAT WE PUT IN...



$$G_{\rm eff}^p(q^2) = \sqrt{\frac{1}{\mathcal{R}}\,\frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}}$$

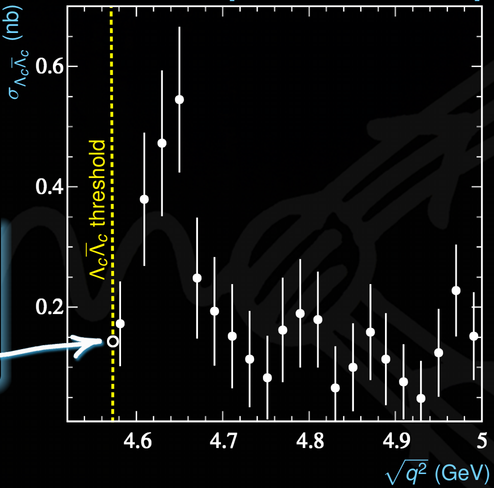
$$\mathcal{E}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right) \underset{\beta\to 0}{\simeq} \sigma_{p\bar{p}}(q^2)$$

$$G_{\rm eff}^p(q^2) = \frac{1}{\sqrt{\mathcal{R}}}$$

$$\frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

The $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ cross section

[Belle PRL101, 172001]



Cross section at threshold assuming

$$\left|G^{\Lambda_c}(4M^2_{\Lambda_c})\right| = 1$$

BESIII is working on that
New results are coming soon

Isotropy at the $p\bar{p}$ production threshold

$$G_E(4M^2) = G_M(4M^2)$$

- Electromagnetic current:

$$J^\mu(p_1, p_2) = \overline{U}(p_2)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2)\right] U(p_1)$$

- $\boldsymbol{F_1 = \dfrac{q^2 G_E - 4M^2 G_M}{q^2 - 4M^2}}$
- $\boldsymbol{F_2 = 4M^2 \dfrac{G_M - G_E}{q^2 - 4M^2}}$

F_1 and F_2 "can" be analytic (pointlike limit: $F_1(q^2) = 1$ and $F_2(q^2) = 0$)

- Annihilation cross section $\left[\widetilde{G}_{E,M} \equiv G_{E,M}(4M^2)\right]$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right] \xrightarrow[q^2 \to 4M^2]{|\widetilde{G}_E| = |\widetilde{G}_M|} \frac{\alpha^2 \beta \mathcal{C}}{16M^2}\left[2|\widetilde{G}_M|^2\right]$$

- Partial wave form factors

- $$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}}\, G_M + G_E}{3} \xrightarrow[q^2 \to 4M^2]{} \widetilde{G}_M$$
- $$G_D = \frac{\sqrt{\frac{q^2}{4M^2}}\, G_M - G_E}{3} \xrightarrow[q^2 \to 4M^2]{} 0$$

Anisotropy at the production threshold

$$G_E(4M^2) \neq G_M(4M^2)$$

- Dirac and Pauli form factors **F_1 and F_2 are not analytic**
- To preserve G_E and G_M analyticity, F_1 and F_2 must have **a simple pole** at the threshold with **opposite residues**

$$F_1 = \frac{-4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_1^{an} \qquad F_2 = \frac{4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_2^{an} \qquad \Delta\widetilde{G} \equiv \widetilde{G}_E - \widetilde{G}_M$$

$F_{1,2}^{an}$ is the analytic part of $F_{1,2}$

- Annihilation cross section

$$\frac{d\sigma}{d\Omega} \xrightarrow[q^2 \to 4M^2]{|\widetilde{G}_E| \neq |\widetilde{G}_M|} \frac{\alpha^2 \beta \mathcal{C}}{8M^2}\left[|\widetilde{G}_M|^2 + \mathrm{Re}\left(\Delta\widetilde{G}\,\widetilde{G}_M^*\right)\sin^2\theta\right]$$

Assuming $|\Delta\widetilde{G}| \ll |\widetilde{G}_M|$

- Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}}G_M + G_E}{3} \xrightarrow[q^2 \to 4M^2]{} \widetilde{G}_M + \frac{\Delta\widetilde{G}}{3} \qquad G_D = \frac{\sqrt{\frac{q^2}{4M^2}}G_M - G_E}{3} \xrightarrow[q^2 \to 4M^2]{} -\frac{\Delta\widetilde{G}}{3}$$

Sources of Anisotropy

Coulomb/QCD interaction correction $\propto \beta^{-1}$ ⟹ **Singular** form factors at threshold ⟹ **Anisotropy**

Only Coulomb
Dmitriev, Milstein, PLB722 (13) 83

$$\tilde{G}_D \sim -\frac{\alpha^2}{8}$$

Very small
but not vanishing!

QCD Coulomb like
Brodsky, Hoang, Kuhn, Teubner, PB359 (95) 355

Large effect for heavy quarks

Anisotropy $\propto \beta^n$
No effect at threshold!

MEASURING ANISOTROPY AT THRESHOLD

$e^+e^- \to p\bar{p}$

SND | CMD3

Very difficult

Efficiency drops with proton antiproton velocity

$p\bar{p} \to e^+e^-$

PANDA

Very difficult

Normalization (Coulomb corrections...)

$e^+e^- \to p\bar{p}\gamma$

BABAR | BESIII

Difficult

ISR technique: not enough statistics

$e^+e^- \to H_B\overline{H}_B$

BESIII

Feasible with heavy baryons

The weak decay allows detection at threshold and polarization measurements (BESIII has...)