Phenomenology of Nucleon Form Factors

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International Workshop on e^+e^- collisions from ϕ to ψ
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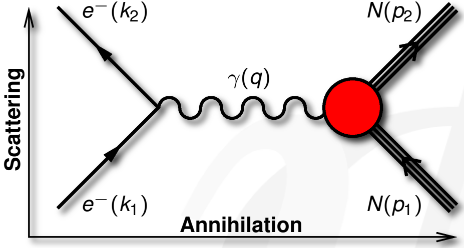
AGENDA

- Nucleon Electromagnetic Form Factors
  - Definition and properties
- The space-like region
  - Proton radius
  - Rosenbluth versus polarization
- The time-like region
  - Unphysical region
  - Threshold
  - An amazing effect
- The asymptotic region
- Conclusions

DIRAC AND PAULI FORM FACTORS



Scattering: $e^- N \to e^- N$
Space-like kinematic region

$$\boxed{q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \le 0}$$

Annihilation: $e^+e^- \leftrightarrow N\overline{N}$
Time-like kinematic region

$$\boxed{q^2 = 4\omega^2 > 0}$$

Scattering amplitude in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2}\,[e\,\overline{u}(k_2)\gamma_\mu u(k_1)]\,\underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = \mathcal{Q}_N$
- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

Sachs Form Factors

Breit frame

No energy exchanged

$$p_1 = (E, -\vec{q}/2)$$
$$p_2 = (E, \vec{q}/2)$$
$$q = (0, \vec{q})$$

Nucleon elecrtomagnetic four-current

$$J_N^\mu = (J_N^0, \vec{J}_N) \quad \begin{cases} \rho_q = J_N^0 = e\left[F_1^N + \dfrac{q^2}{4M_N^2}F_2^N\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\,\left[F_1^N + F_2^N\right] \end{cases}$$

Sachs Nucleon Form Factors

$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2) \qquad G_E^N(q^2) = F_1^N(q^2) + \frac{q^2}{4M_N^2}F_2^N(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

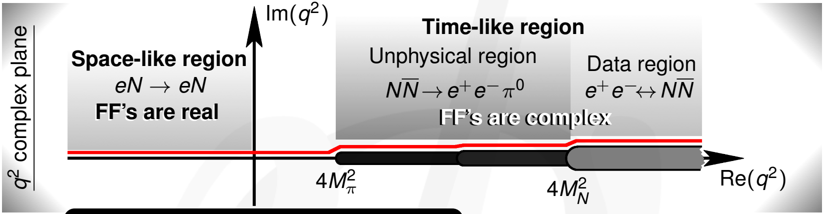
Normalization at $q^2 = 0$

$$G_E^N(0) = \mathcal{Q}_N$$
$$G_M^N(0) = \mu_N$$

$$\mu_N = \mathcal{Q}_N + \kappa_N$$

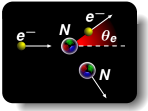
is the nucleon magnetic moment

Cross sections and analyticity



$$\text{Crossing: tot. helicity} = \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$$

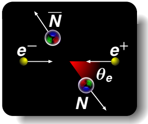
$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$

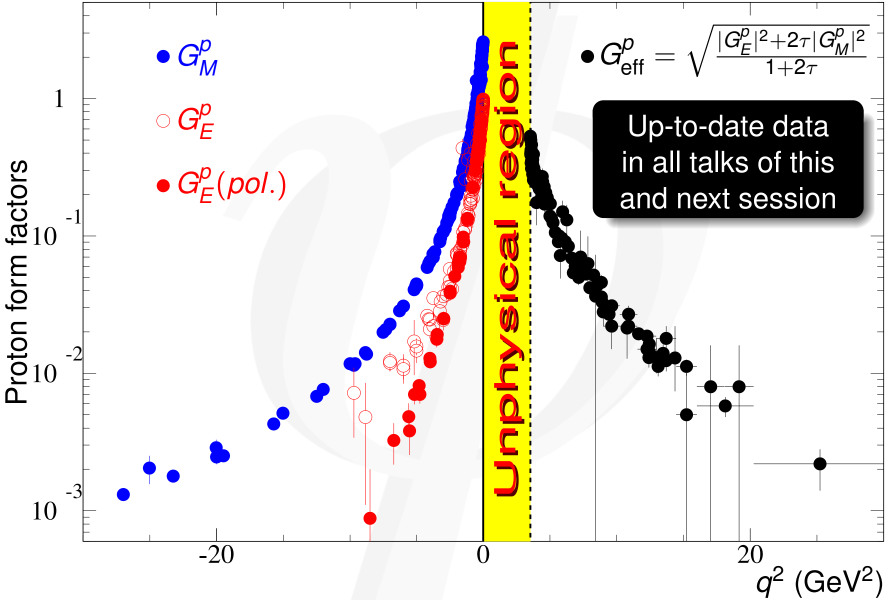


Annihilation

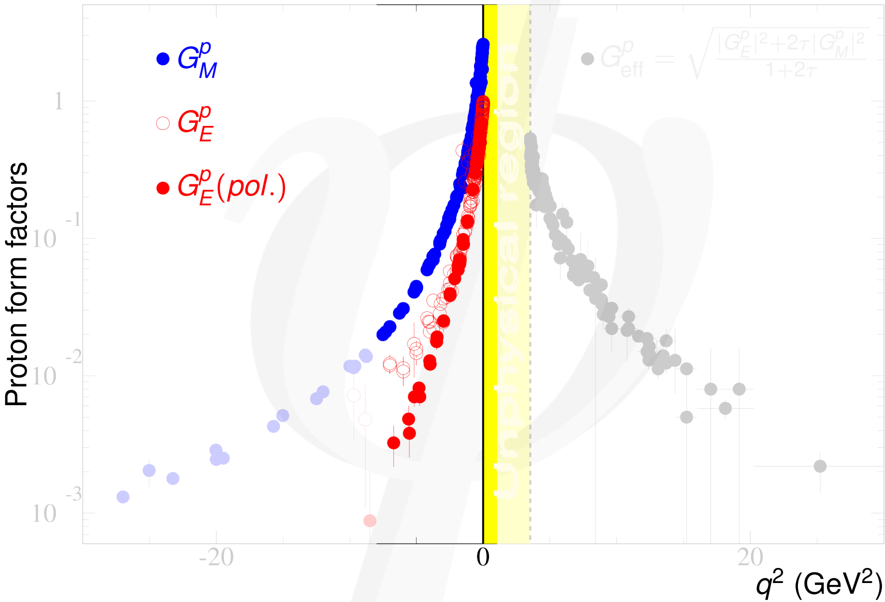
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2} \left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau} \sin^2\theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

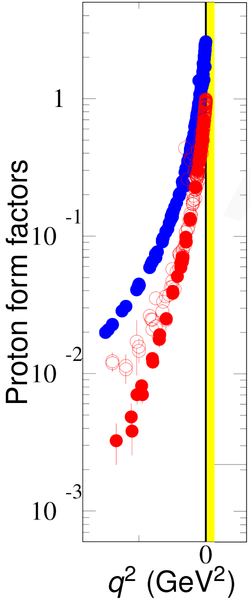
THE PROTON RADIUS



The proton radius



THE PROTON RADIUS



$$G_E^p(q^2) = \int d^3\vec{r}\,\rho(r)\,e^{i\,\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6}q^2\left\langle r_c^2\right\rangle + \mathcal{O}(q^4)$$

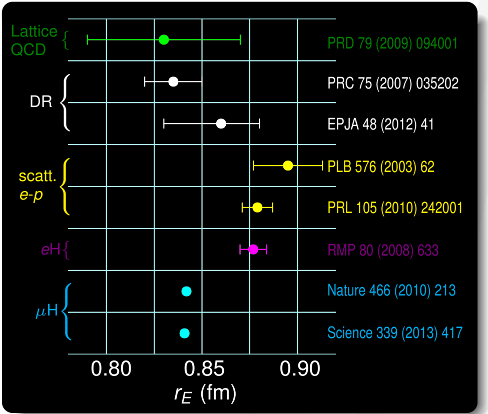
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2\rangle} = \sqrt{4\pi\int_0^\infty r^4\rho(r)\,dr} = \sqrt{\frac{6}{G_E^p(0)}\left.\frac{dG_E^p}{dq^2}\right|_{q^2=0}}$$

Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4/(q^2+\lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2/(q^2+\lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-r^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian

THE PROTON RADIUS



Ongoing discussions. . .

- $q^2 \to 0^-$ extrapolation
- Radiative corrections
- Two-photon exchange
- Coulomb corrections

Analyticity via dispersion relations and QCD counting rules can give directly the proton radius. . .

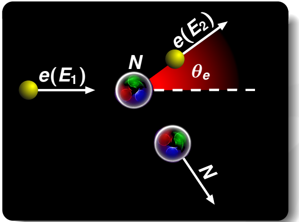
Logarithmic derivative of form factor at $q^2 = 0$ by means of dispersion relations for the logarithm

$$r_E^2 = \frac{12M_\pi^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\ln|G_E^p(t)/G_E^p(0)|}{t^2\sqrt{t-4M_\pi^2}}dt$$

ROSENBLUTH SEPARATION
POLARIZATION OBSERVABLES

N.M.Rosenbluth,Phys.Rev.79,615

A.I.Akhiezer,M.P.Rekalo,Sov.Phys.Dokl.13,572



Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{1}{1-\tau}\left[G_E^2 - \frac{\tau}{\epsilon} G_M^2\right] \qquad \tau = \frac{q^2}{4M_N^2}$$

- Mott pointlike cross section

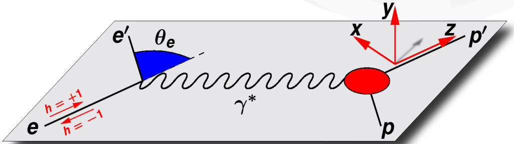
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{4\alpha^2}{(-q^2)^2}\frac{E_2^3}{E_1}\cos^2(\theta_e/2)$$

- Photon polarization

$$\epsilon = \left[1 + 2\,(1-\tau)\tan^2(\theta_e/2)\right]^{-1}$$

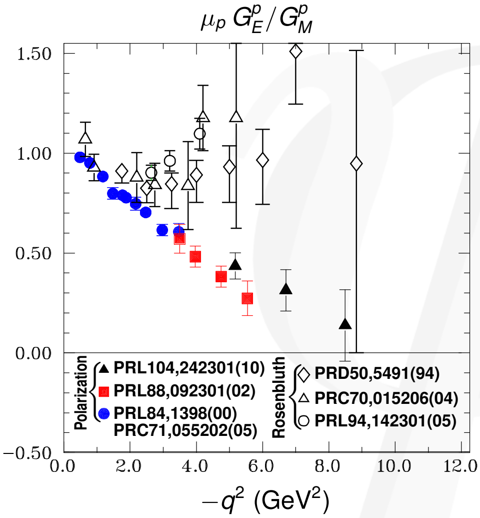
In case of **polarized electrons** ($h = \pm 1$) on unpolarized nucleon target:

$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon}G_M^2}\, G_E G_M \tan\left(\frac{\theta_e}{2}\right) \qquad P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M\,(G_E^2 - \frac{\tau}{\epsilon}G_M^2)}\, G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$



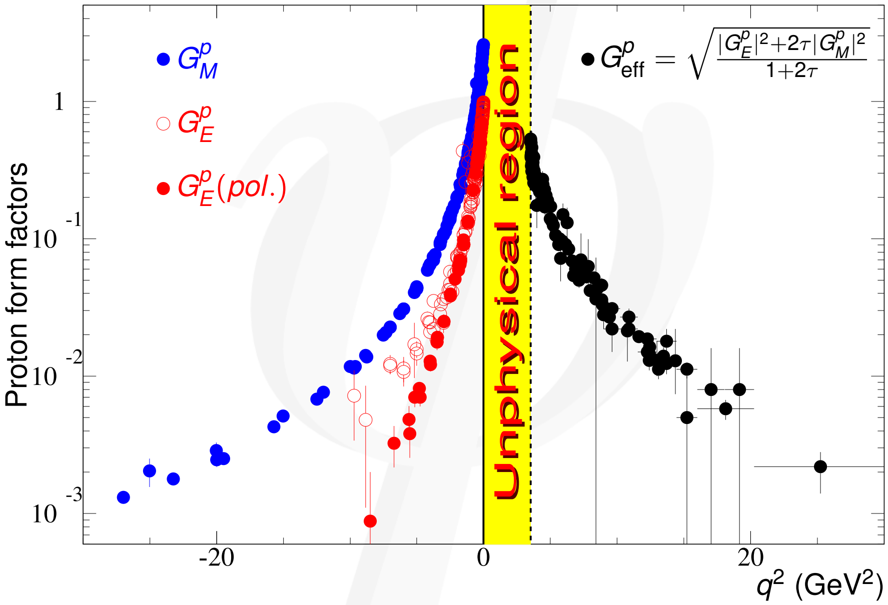
$$\frac{P'_x}{P'_z} = -\frac{2M\cot(\theta_e/2)}{E_e + E'_e}\,\frac{G_E}{G_M}$$

$\mu_p G_E^p / G_M^p$: Rosenbluth and Polarization Techniques

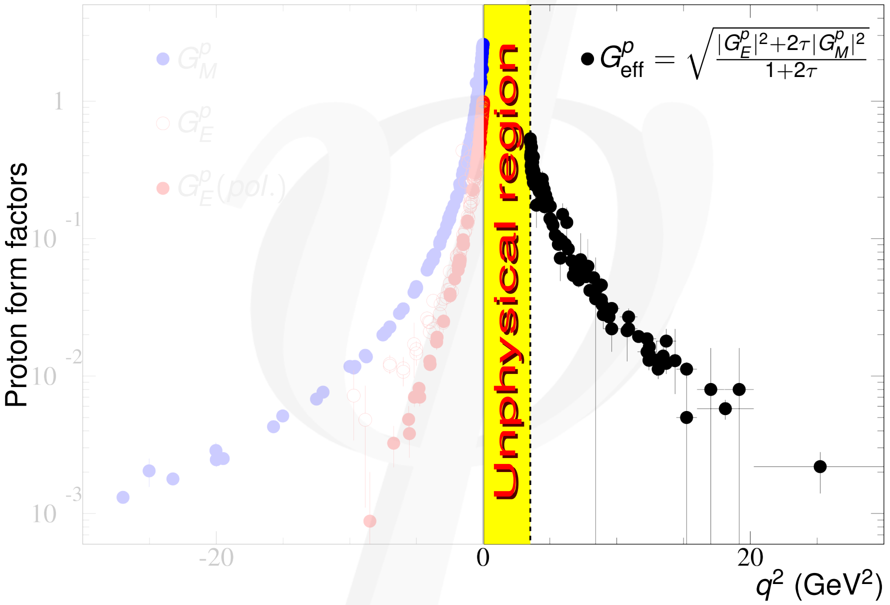


- "Standard" dipole for the proton magnetic form factors G_M^p
- Linear deviation from the dipole for the electric proton form factor G_E^p
- **QCD scaling still not reached**
- **Zero crossing for G_E^p**
- Polarization data do not agree with old Rosenbluth data (◇)
- New Rosenbluth separation data from JLab **still do not agree** with polarization data

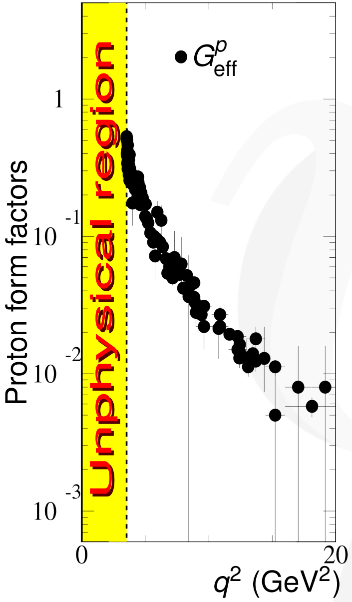
The time-like region



THE TIME-LIKE REGION



The time-like region



Differential cross section $e^+e^- \to p\bar{p}$

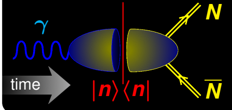
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M^p|^2 + \frac{1}{\tau}\sin^2\theta |G_E^p|^2\right]$$

Optical theorem

$\text{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

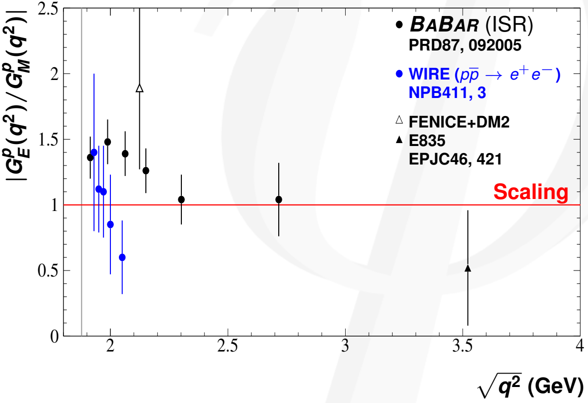


Form factors are complex for $q^2 > 4M_\pi^2$

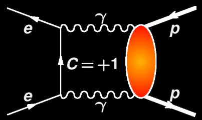
- The cross section is an even function of $\cos\theta$
- It does not depend on the form factor phases
- At high q^2 the $|G_E^p|^2$ contribution is suppressed
- The unphysical region is not accessible through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

TIME-LIKE $|G_E^p/G_M^p|$ MEASUREMENTS

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta\mathcal{C}}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2}\sin^2\theta\left|\frac{G_E^p}{G_M^p}\right|^2\right]$$



$\gamma\gamma$ exchange

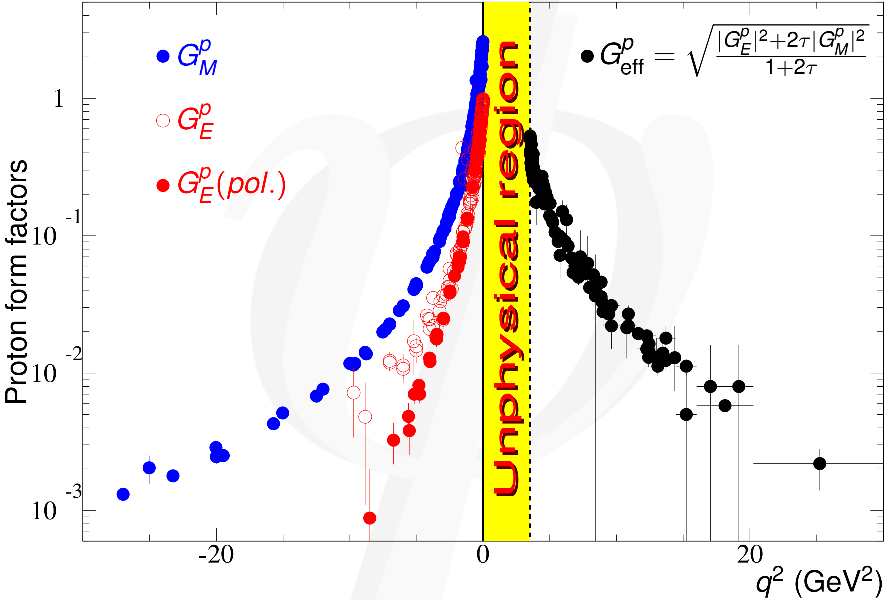


$\gamma\gamma$ exchange interferes with the Born term

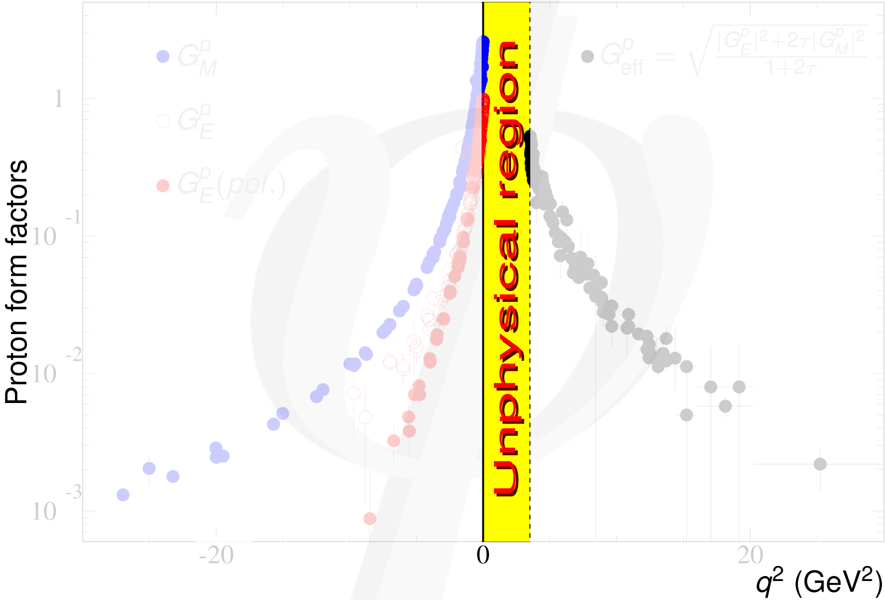
Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]

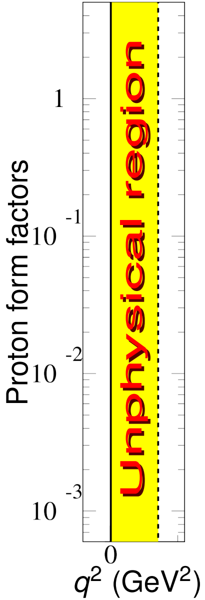
The unphysical region



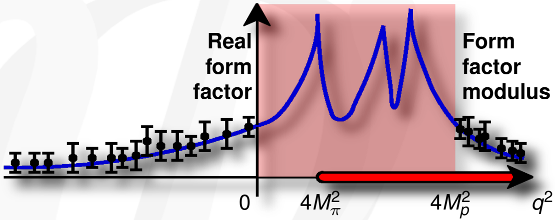
The unphysical region



The unphysical region



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$



In that region, form factors

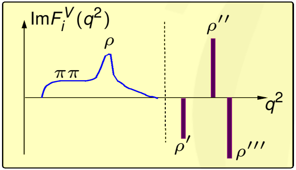
- are still well defined but not (directly) experimentally accessible
- are complex and, following VMD-based models, receive their main contribution from intermediate resonances

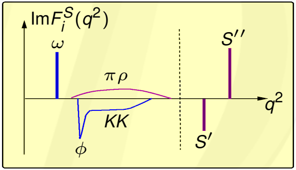
Handling the unphysical region[1]

Model dependent **disclosing** [Höler, Mergell, Meissner, Hammer]

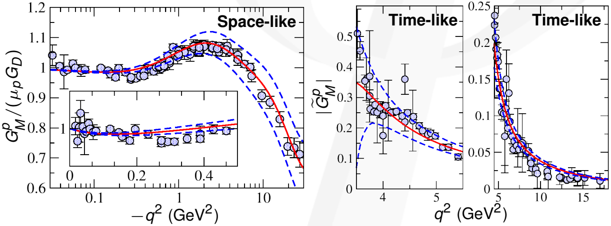
Optical theorem .... $\mathrm{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

Dispersion relations for the imaginary part .... $F(q^2_{\mathrm{SL}}) = \dfrac{1}{\pi}\displaystyle\int_{4M_\pi^2}^{\infty} \dfrac{\mathrm{Im}F(q^2_{\mathrm{TL}})}{q^2_{\mathrm{TL}} - q^2_{\mathrm{SL}}}\, dq^2_{\mathrm{TL}}$





- 2π and $2K$ continua are known
- The ρ resonance with finite width
- Dirac delta poles for higher mass states



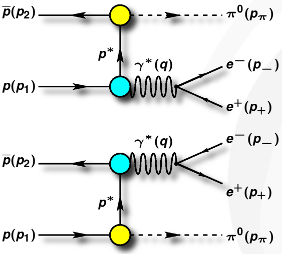
- Super convergence relations
$$\int_{4M_\pi^2}^{\infty} \mathrm{Im}\, F_{1,2}(q^2)dq^2 = 0$$
$$\int_{4M_\pi^2}^{\infty} q^2 \mathrm{Im}\, F_2(q^2)dq^2 = 0$$
- Asymptotic behaviors from perturbative QCD

HANDLING THE UNPHYSICAL REGION$_2$

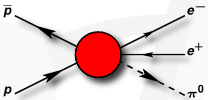
Accessing the unphysical region
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π-production

$$p\bar{p} \to \pi^0 e^+ e^-$$



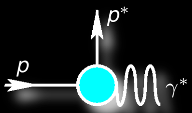
The process $p\bar{p} \to \pi^0 e^+ e^-$



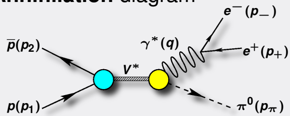
Described in general by **six** amplitudes which depend on **three** kinematical variables

Hadronic current [PRC75 045205]

- $J_\mu = \phi_\pi(p_\pi)\, \overline{v}(p_2)\, O_\mu\, u(p_1)$
- $O_\mu = O_\mu[\Gamma_\mu(q)]$
- $\langle N(p')|\Gamma_\mu(q)|N(p)\rangle = \bar{u}(p')\left[F_1(q^2)\gamma_\mu + \dfrac{i\,\sigma_{\mu\nu}q^\nu}{4M_p^2}F_2(q^2)\right]u(p)$



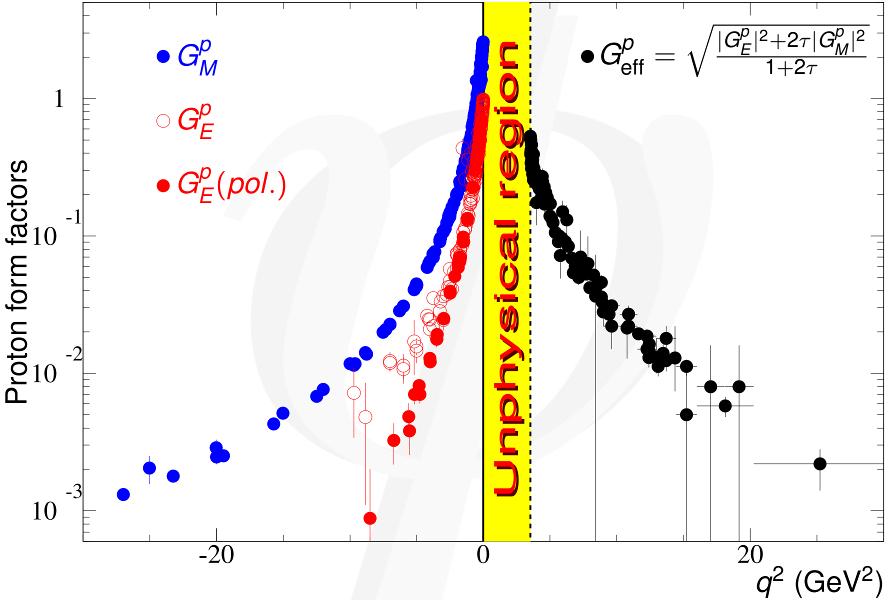
Background:
Annihilation diagram



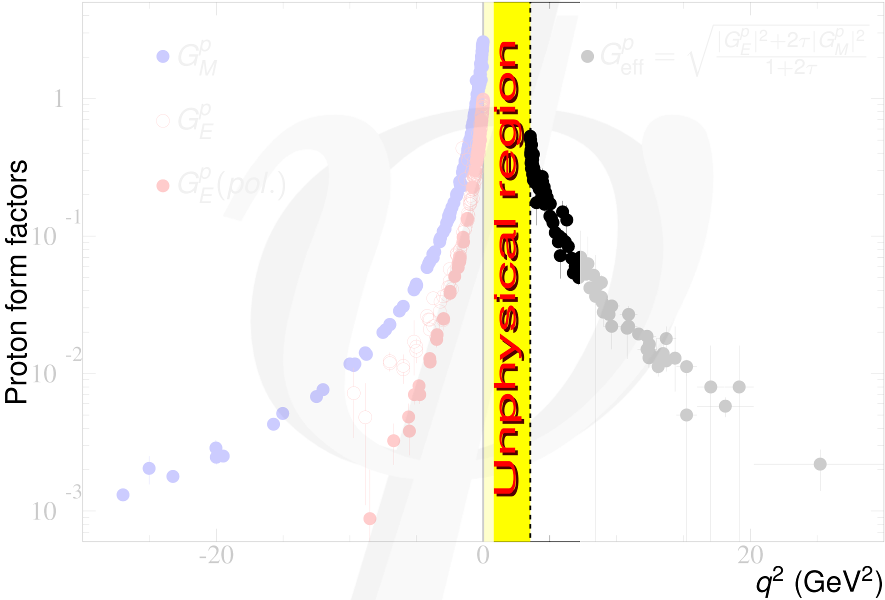
Polarization observables help in disentangle reaction mechanisms

[E. A. Kuraev *et al.*, J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

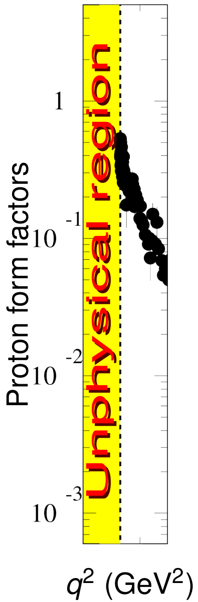
THE THRESHOLD REGION



The threshold region



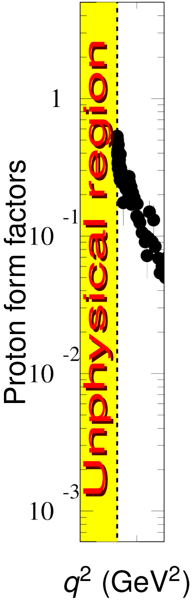
THE THRESHOLD REGION



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

THE THRESHOLD REGION



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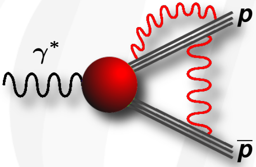


Enhancement factor $\mathcal{E} = \frac{\pi\alpha}{\beta}$

It is responsible for the one-photon exchange $p\bar{p}$ final state interaction, dominates at threshold and cancels the phase-space factor.

Resummation factor $\mathcal{R} = \frac{1}{1-e^{-\frac{\pi\alpha}{\beta}}}$

It is responsible for the multi-photon $p\bar{p}$ final state interaction, becomes ineffective few MeV above threshold and accounts also for gluon exchange.

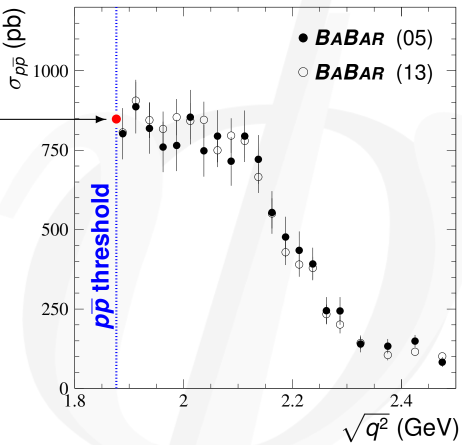
$$\mathcal{C} = \mathcal{E} \times \mathcal{R} = \frac{\pi\alpha/\beta}{1-\exp(-\pi\alpha/\beta)} \xrightarrow[\beta\to 0]{} \frac{\pi\alpha}{\beta} = \mathcal{E}$$

STEP AND PLATEAU IN *BABAR* DATA

Eur. Phys. J. A39 (2009) 315

Expected cross section with

$|G_S^p(4M_p^2)| = 1$



At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\cancel{\beta_p}}{\cancel{\beta_p}}\,|G_S^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G_S^p(4M_p^2)|^2 \text{ nb}$$

$\Longrightarrow$

$|G_S^p(4M_p^2)| \equiv 1$

as pointlike fermion pairs!

STEP AND PLATEAU IN *BABAR* DATA

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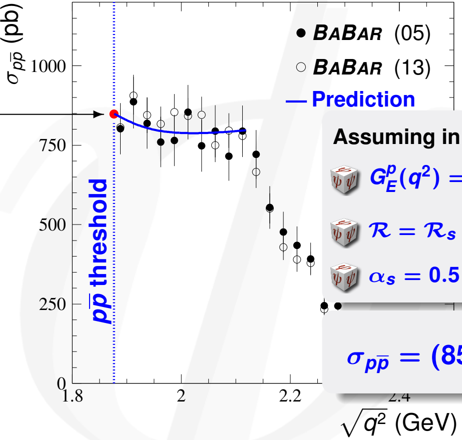
Expected cross section with $|G_S^p(4M_p^2)| = 1$



Assuming in $q^2 \leq (2.1\,\text{GeV})^2$:

- $G_E^p(q^2) = G_M^p(q^2) = 1$
- $\mathcal{R} = \mathcal{R}_s = \left(1 - e^{-\frac{\pi\alpha_s}{\beta}}\right)^{-1}$
- $\alpha_s = 0.5$

$$\sigma_{p\bar{p}} = (850\,\text{pb})\,\frac{1}{\tau}\,\mathcal{R}_s$$

At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\cancel{\beta_p}}{\cancel{\beta_p}}\,|G_S^p(4M_p^2)|^2$$

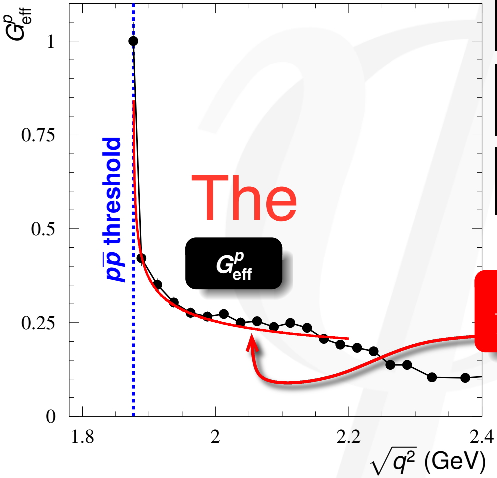
$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G_S^p(4M_p^2)|^2\ \text{nb}$$

$\Longrightarrow$

$|G_S^p(4M_p^2)| \equiv 1$

as pointlike fermion pairs!

G^p_{eff}, WE GET WHAT WE PUT IN...



$$G^p_{\text{eff}}(q^2) = \sqrt{\frac{1}{\mathcal{R}} \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}}$$

$$\mathcal{E}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right) \underset{\beta\to 0}{\simeq} \sigma_{p\bar{p}}(q^2)$$

$$G^p_{\text{eff}}(q^2) = \frac{1}{\sqrt{\mathcal{R}}}$$

$$\frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

Isotropy at the $p\bar{p}$ production threshold

$$G_E(4M^2) = G_M(4M^2)$$

- Electromagnetic current:

$$J^{\mu}(p_1,p_2) = \overline{U}(p_2)\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}F_2(q^2)\right]U(p_1)$$

- $F_1 = \dfrac{q^2 G_E - 4M^2 G_M}{q^2 - 4M^2}$ - $F_2 = 4M^2\dfrac{G_M - G_E}{q^2 - 4M^2}$

F_1 and F_2 "can" be analytic (pointlike limit: $F_1(q^2) = 1$ and $F_2(q^2) = 0$)

- Annihilation cross section $\left[\widetilde{G}_{E,M} \equiv G_{E,M}(4M^2)\right]$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta\mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right] \xrightarrow[q^2\to 4M^2]{|\widetilde{G}_E|=|\widetilde{G}_M|} \frac{\alpha^2\beta\mathcal{C}}{16M^2}\left[2|\widetilde{G}_M|^2\right]$$

- Partial wave form factors

- $G_S = \dfrac{2\sqrt{\frac{q^2}{4M^2}}G_M + G_E}{3} \xrightarrow[q^2\to 4M^2]{} \widetilde{G}_M$ - $G_D = \dfrac{\sqrt{\frac{q^2}{4M^2}}G_M - G_E}{3} \xrightarrow[q^2\to 4M^2]{} 0$

Anisotropy at the production threshold

$$G_E(4M^2) \neq G_M(4M^2)$$

- Dirac and Pauli form factors F_1 **and** F_2 **are not analytic**
- To preserve G_E and G_M analyticity, F_1 and F_2 must have **a simple pole** at the threshold with **opposite residues**

  - $F_1 = \dfrac{-4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_1^{an}$
  - $F_2 = \dfrac{4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_2^{an}$
  - $\Delta\widetilde{G} \equiv \widetilde{G}_E - \widetilde{G}_M$

  $F_{1,2}^{an}$ is the analytic part of $F_{1,2}$

- Annihilation cross section

$$\frac{d\sigma}{d\Omega} \xrightarrow[q^2 \to 4M^2]{|\widetilde{G}_E| \neq |\widetilde{G}_M|} \frac{\alpha^2 \beta C}{8M^2} \left[|\widetilde{G}_M|^2 + \mathrm{Re}\left(\Delta\widetilde{G}\, \widetilde{G}_M^* \right) \sin^2\theta \right]$$

Assuming $|\Delta\widetilde{G}| \ll |\widetilde{G}_M|$

- Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}}\, G_M + G_E}{3} \xrightarrow[q^2 \to 4M^2]{} \widetilde{G}_M + \frac{\Delta\widetilde{G}}{3}$$

$$G_D = \frac{\sqrt{\frac{q^2}{4M^2}}\, G_M - G_E}{3} \xrightarrow[q^2 \to 4M^2]{} -\frac{\Delta\widetilde{G}}{3}$$

Sources of Anisotropy

Coulomb/QCD interaction correction $\propto \beta^{-1}$ $\Longrightarrow$ **Singular** form factors at threshold $\Longrightarrow$ **Anisotropy**

Only Coulomb
Dmitriev, Milstein, PLB722 (13) 83

$$\tilde{G}_D \sim -\frac{\alpha^2}{8}$$

Very small
but not vanishing!

QCD Coulomb like
Brodsky, Hoang, Kuhn, Teubner, PB359 (95) 355

Large effect for heavy quarks

Anisotropy $\propto \beta^n$
No effect at threshold!

MEASURING ANISOTROPY AT THRESHOLD

$e^+e^- \to p\bar{p}$

SND | CMD3

Very difficult

Efficiency drops with proton antiproton velocity

$p\bar{p} \to e^+e^-$

PANDA

Very difficult

Normalization (Coulomb corrections...)

$e^+e^- \to p\bar{p}\gamma$

BABAR | BESIII

Difficult

ISR technique: not enough statistics

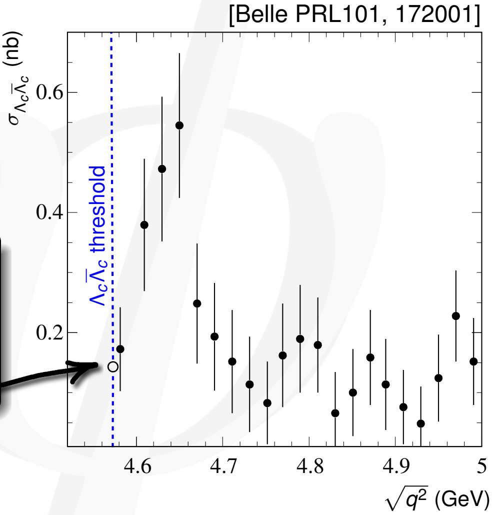
$e^+e^- \to H_B\overline{H_B}$

BESIII

Feasible with heavy baryons

The weak decay allows detection at threshold and polarization measurements (BESIII has...)

THE $e^+e^- \to \Lambda_c\overline{\Lambda}_c$ CROSS SECTION



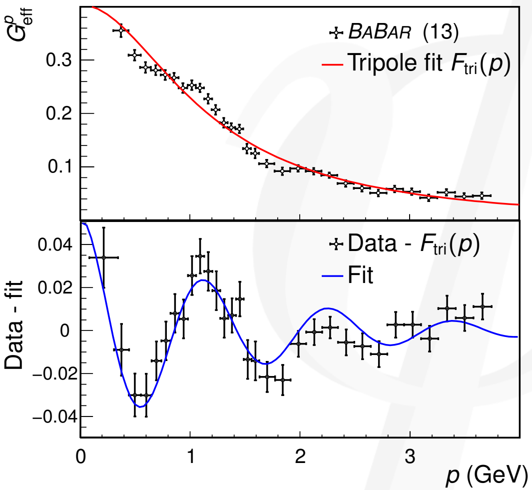
Cross section at threshold assuming

$$\left|G^{\Lambda_c}(4M_{\Lambda_c}^2)\right| = 1$$

BESIII is working on that
New results are coming soon

PERIODIC INTERFERENCE NEAR THRESHOLD

A. Bianconi, E. Tomasi-Gustafsson, Phys. Rev. Lett. 114, 232301



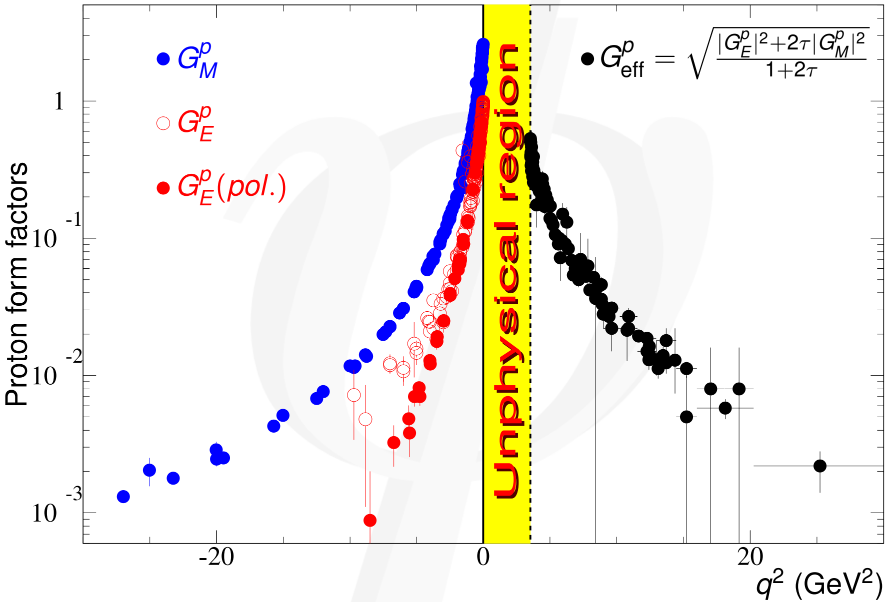
p is the momentum of the proton in the anti-proton rest frame.

The periodical behavior suggests an interference due to a rescattering of proton and antiproton at low kinetic energy and separation $\sim$ 1 fm.

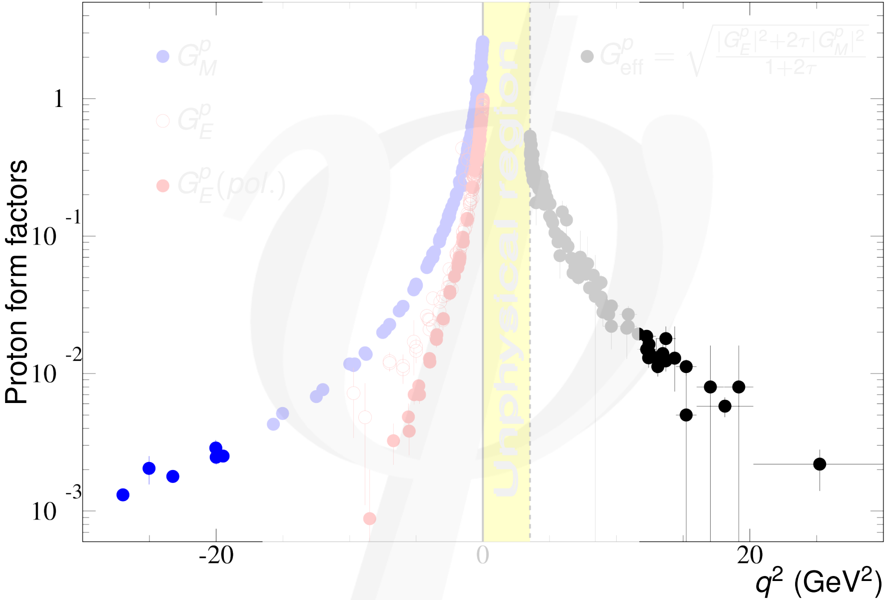
Proton and antiproton interact when they are almost phenomenological.

Unitarity implies a large imaginary part of form factors.

THE ASYMPTOTIC REGIONS$_1$

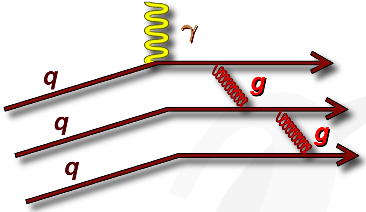


THE ASYMPTOTIC REGIONS$_1$

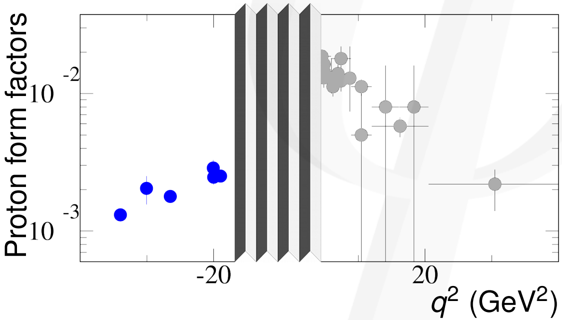


THE ASYMPTOTIC REGIONS$_1$

Space-like dimensional scaling



- pQCD: as $q^2 \to -\infty$, asymptotic behaviors of F_1 and F_2, and G_E and G_M must follow counting rules
- Valence quarks exchange gluons to distribute the momentum transfer q



- Dirac and Pauli form factors

$$F_i(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-1-i}$$

- Sachs form factors

$$G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$$

$$\frac{G_E(q^2)}{G_M(q^2)} \underset{q^2 \to -\infty}{\sim} \text{cost.}$$

THE ASYMPTOTIC REGIONS$_1$

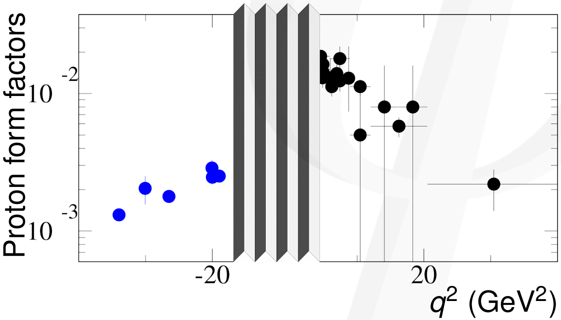
Time-like asymptotic behavior

Phragmèn Lindelöf theorem:

If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

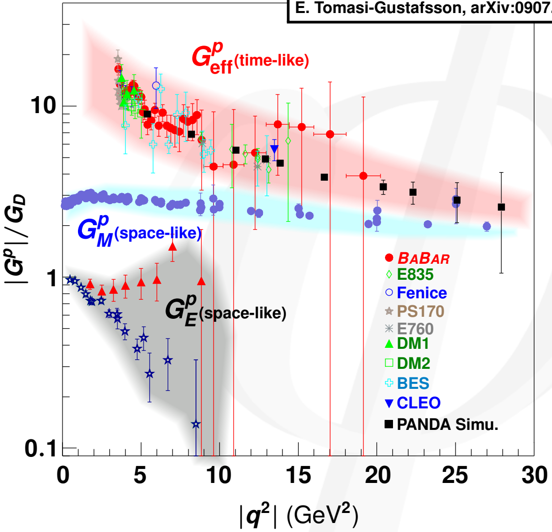
- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ $\boxed{\text{real}}$



THE ASYMPTOTIC REGIONS$_2$

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



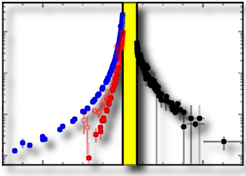
pQCD

$$G^p_{\rm eff}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{\rm eff}(q^2)}{G^p_M(-q^2)} = 1$$

Conclusions



Global models for proton and neutron, electric and magnetic form factors must be encouraged.
They can help in understanding:

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- ...

To measure:

- zero of G_E^p in space-like region
- moduli of G_E and G_M in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \to \pi^0 e^+ e^-$)
- ...

Experiments: now and future

Space-like region

Mainz

- G_E^n at $-q^2 = 1.5\ \text{GeV}^2$ (Pol. ^{3}He)
- G_E^p and G_M^p for $-q^2 \leq 2.0\ \text{GeV}^2$

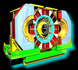
Jefferson Lab

- [Hall A] G_E^p / G_M^p up to 14 GeV^2
- [Hall A] G_M^n (ratio) up to 18 GeV^2
- [Hall A] G_E^n / G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2

Time-like region

at VEPP-2000
e^+e^- collider
$|G_{\text{eff}}^p|$, $|G_{\text{eff}}^n|$ (scan)
$q^2 \leq (4\ \text{GeV})^2$

BESIII



at BEPCII
e^+e^- collider
$|G_E^p|$, $|G_M^p|$, $|G_{\text{eff}}^n|$ (scan and ISR)
$q^2 \leq (3.5\ \text{GeV})^2$

at FAIR
$p\bar{p}$ collider
$|G_E^p|$, $|G_M^p|$, G_E^p / G_M^p phase ($\bar{p}$ polarization)
$(2.4\ \text{GeV})^2 \leq q^2 \leq (3.7\ \text{GeV})^2$

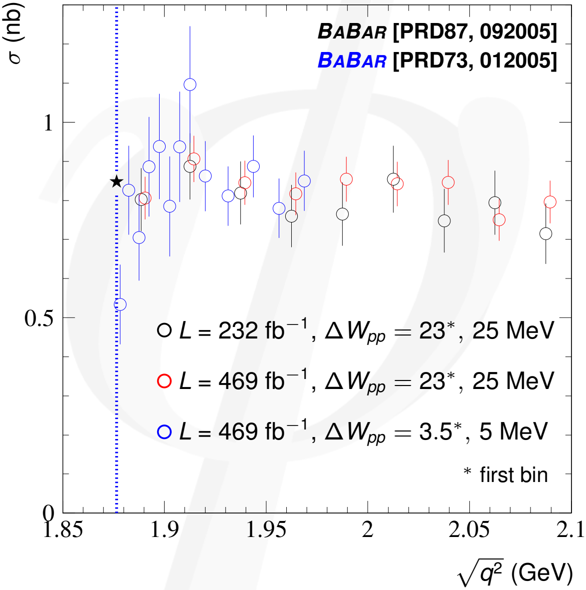
Belle II

?

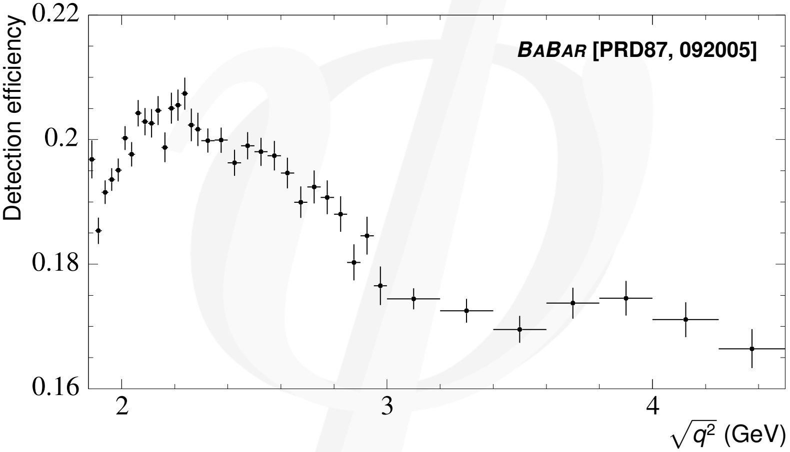
at SuperKEKB
e^+e^- collider
$|G_E^p|$, $|G_M^p|$, (ISR)
$q^2 \leq (4.5\ \text{GeV})^2$

Additional slides

THE THRESHOLD REGION3

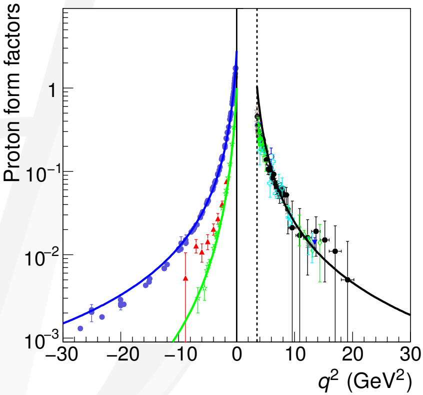


THE THRESHOLD REGION$_3$



AN INTERESTING MODEL

E. A. Kuraev, E. Tomasi-Gustafsson, A. Dbeyssi PLB712 240



Assumption
Pauli principle pulls away from the internal region of strong chromo-electromagnetic field quarks of same flavor because the color quantum number does not play any role (stochastic averaging).

Outer spatial region $|p\rangle = \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 1

Central region $|p\rangle \neq \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 0

space-like

Counting rule on the vector part of interaction

$$G_M^p(q^2) = \mu_p\, G_D(q^2)$$

A screening effect from the central region provides an additional suppression for the electric form factor

$$G_E^p(q^2) = \frac{G_D(q^2)}{1 - q^2/q_1^2}$$

time-like

$$G_M^p(q^2) = \frac{\theta(q^2 - 4M_p^2)}{\left[1 + (q^2 - 4M_p^2)^2/q_2^2\right]^2}$$

$$G_E^p(q^2) = \frac{G_M^p(q^2)\,\theta(q^2 - 4M_p^2)}{1 + (q^2 - 4M_p^2)^2/q_2^2}$$

PARTIAL WAVE FORM FACTORS

$$\begin{cases} P_\gamma = -1 \\ J_\gamma = 1 \end{cases} \qquad \begin{cases} P_{p\bar{p}} = (-1)^{L+1} \\ S_{p\bar{p}} = 0, 1 \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} L_{p\bar{p}} = 0, 2 \\ S_{p\bar{p}} = 1 \end{cases} \qquad \cancel{\begin{cases} L_{p\bar{p}} = 1 \\ S_{p\bar{p}} = 0 \end{cases}}$$

Partial wave form factors

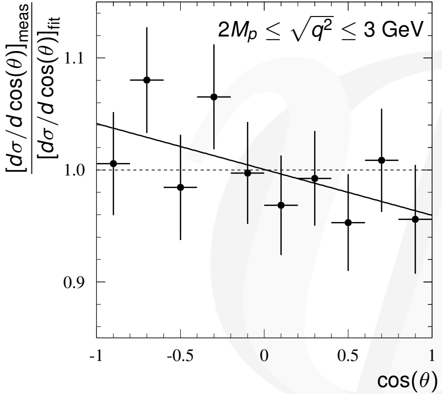
$$G_S^p = \frac{1}{3}\left(2G_M^p\sqrt{\frac{q^2}{4M_p^2}} + G_E^p\right) \qquad G_D^p = \frac{1}{3}\left(G_M^p\sqrt{\frac{q^2}{4M_p^2}} - G_E^p\right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta\frac{4M_p^2}{(q^2)^2}\left[\mathcal{C}\left|G_S^p(q^2)\right|^2 + 2\left|G_D^p(q^2)\right|^2\right]$$

$\gamma\gamma$ EXCHANGE FROM $e^+e^- \to p\bar{p}\gamma$ *BABAR* 2013 DATA

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
Phys. Rev. D87 (2013) 092005



Integrated over the $p\bar{p}$-CM energy from threshold up to 3 GeV

The MC-fit assumes **one-photon exchange**

Slope $= -0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

$$\langle\mathcal{A}\rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = \mathbf{-0.025 \pm 0.014 \pm 0.003}$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos\theta_p \gtrless 0$