Proton form factors
and
threshold behavior

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GdR PH-QCD

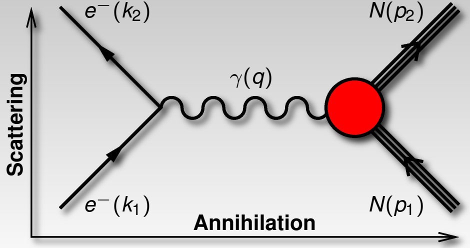
Recent highlights in hadron structure

IPN Orsay - October 6th-7th, 2014

Agenda

- Nucleon electromagnetic form factors
  - Definition and properties
- The space-like region
  - Proton radius
  - Rosenbluth versus polarization
- The time-like region
  - Unphysical region
- The threshold
- The asymptotic region
- Conclusions

Dirac and Pauli Form Factors



Scattering: $e^- N \to e^- N$

Space-like kinematic region

$$\boxed{q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \leq 0}$$

Annihilation: $e^+e^- \leftrightarrow N\overline{N}$

Time-like kinematic region

$$\boxed{q^2 = 4\omega^2 > 0}$$

Scattering amplitude in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2}\left[e\,\overline{u}(k_2)\gamma_\mu u(k_1)\right] \underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = \mathcal{Q}_N$
- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

Sachs Form Factors

Breit frame

No energy exchanged

$p_1 = (E, -\vec{q}/2)$

$p_2 = (E, \vec{q}/2)$

$q = (0, \vec{q})$

Nucleon electrtomagnetic four-current

$$J^\mu_N = (J^0_N, \vec{J}_N) \quad \begin{cases} \rho_q = J^0_N = e\left[F^N_1 + \dfrac{q^2}{4M^2}F^N_2\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\left[F^N_1 + F^N_2\right] \end{cases}$$

Sachs Nucleon Form Factors

$$G^N_M(q^2) = F^N_1(q^2) + F^N_2(q^2) \qquad G^N_E(q^2) = F^N_1(q^2) + \frac{q^2}{4M^2}F^N_2(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

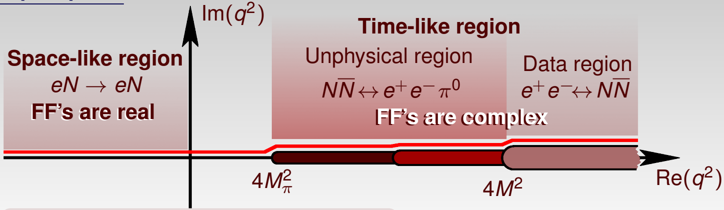
Normalization at $q^2 = 0$

- $G^N_E(0) = \mathcal{Q}_N$
- $G^N_M(0) = \mu_N$

$\mu_N = \mathcal{Q}_N + \kappa_N$

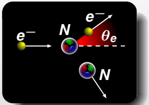
is the nucleon magnetic moment

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

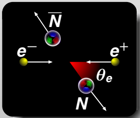
$G_E(4M^2) = G_M(4M^2)$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau}$$

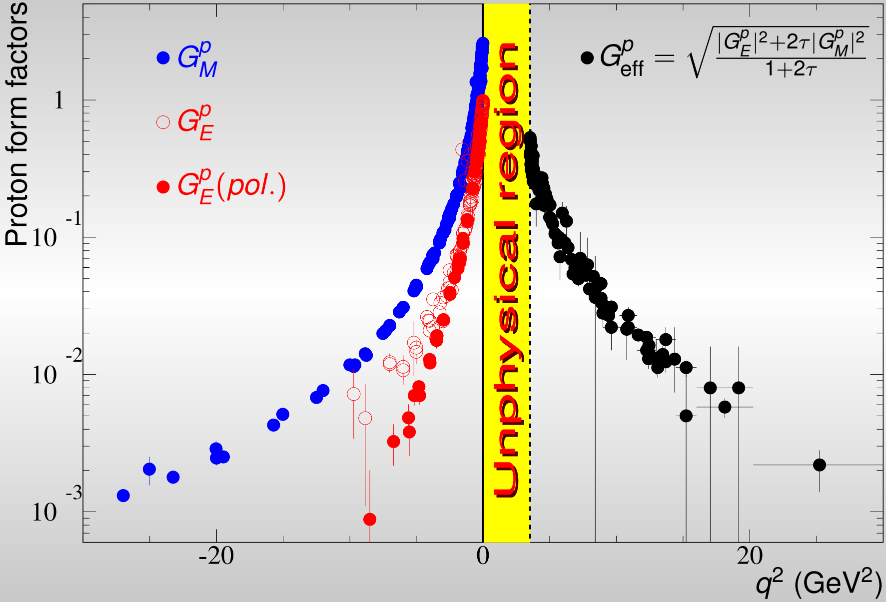
$$\tau = \frac{q^2}{4M^2}$$



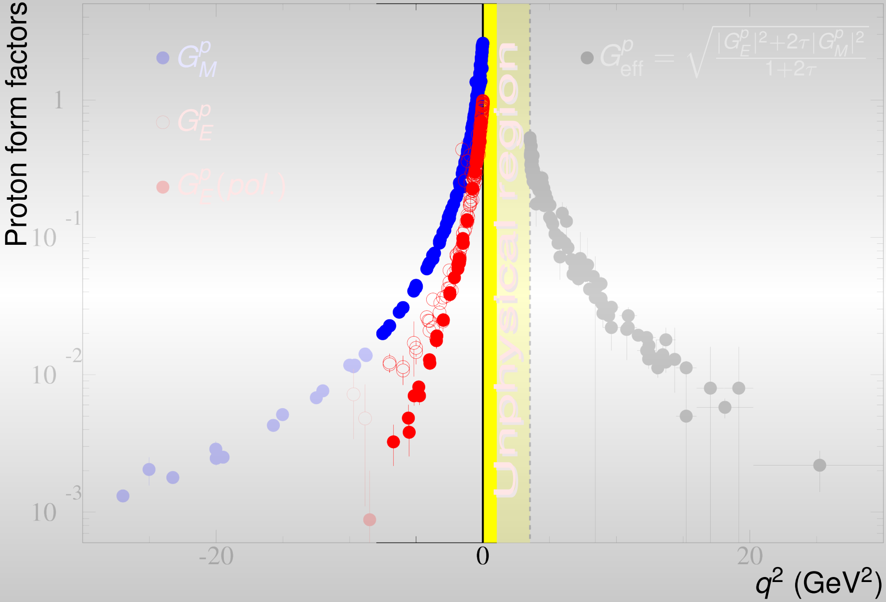
Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2} \left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau} \sin^2\theta |G_E|^2 \right]$$

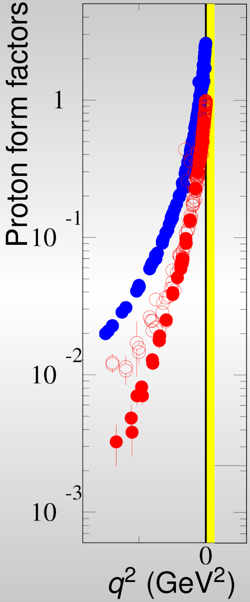
$$\beta = \sqrt{1 - \frac{1}{\tau}}$$


Proton form factors
G_M^p
G_E^p
$G_E^p(pol.)$
$G_{\rm eff}^p = \sqrt{\frac{|G_E^p|^2+2\tau|G_M^p|^2}{1+2\tau}}$
Unphysical region
1
10^{-1}
10^{-2}
10^{-3}
-20
0
20
q^2 (GeV^2)

The proton radius



The proton radius



$$G_E^p(q^2) = \int d^3\vec{r}\,\rho(r)\,e^{i\,\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6}q^2\left\langle r_c^2\right\rangle + \mathcal{O}(q^4)$$

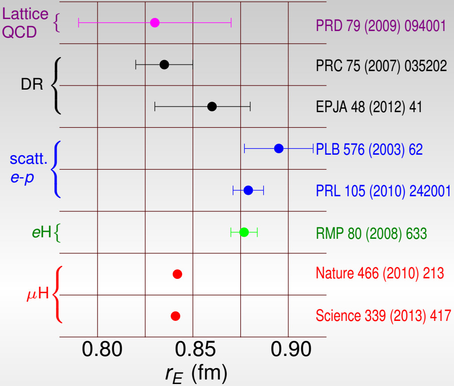
$\rho(r)$: normalized spherical charge density

The charge radius

$$r_E = \sqrt{\langle r_c^2\rangle} = \sqrt{4\pi\int_0^\infty r^4\rho(r)\,dr} = \sqrt{\frac{6}{G_E^p(0)}\,\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

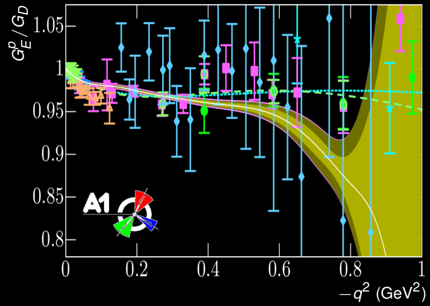
Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4/(q^2+\lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2/(q^2+\lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-r^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian

The proton radius

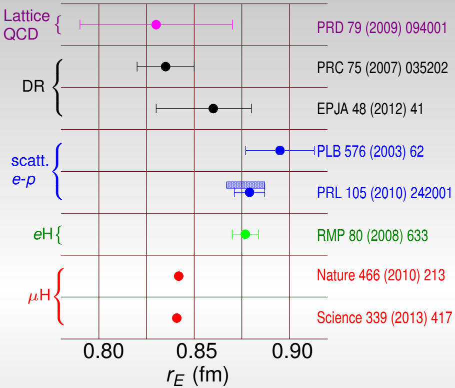


A1 Collaboration [arXiv:1307.6227]

- ∼1400 points
- $-q^2 \geq 0.003\ \mathrm{GeV}^2$



The proton radius



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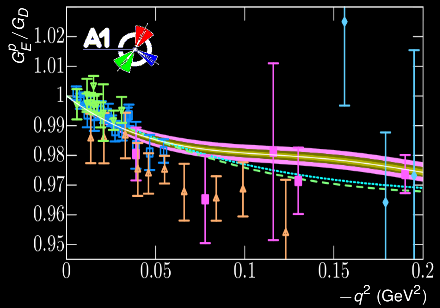
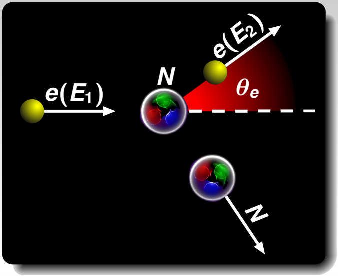


Ongoing discussions…

- Radiative corrections
- Two-photon exchange
- Coulomb corrections

Extrapolating $q^2 \to 0^-$

DR: $r_E^2 = \dfrac{12M_\pi^2}{\pi} \displaystyle\int_{4M_\pi^2}^{\infty} \dfrac{\ln |G_E^p(t)/G_E^p(0)|}{t^2\sqrt{t-4M_\pi^2}}\, dt$



Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1-\tau}\left[G_E^2 - \frac{\tau}{\epsilon}G_M^2\right] \qquad \tau = \frac{q^2}{4M^2}$$

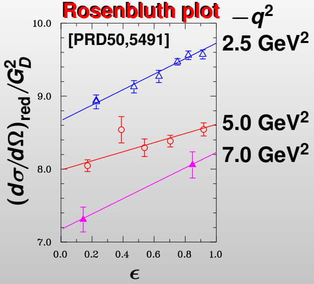
- Mott pointlike cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{4\alpha^2}{(-q^2)^2}\frac{E_2^3}{E_1}\cos^2(\theta_e/2)$$

- Photon polarization

$$\epsilon = \left[1 + 2\,(1-\tau)\tan^2(\theta_e/2)\right]^{-1}$$

Rosenbluth plot



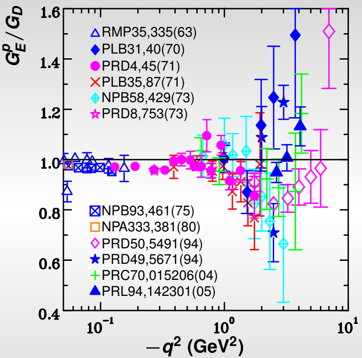
Reduced cross section

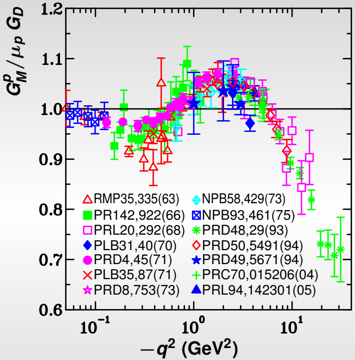
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{red}} = \frac{\epsilon(1-\tau)}{\tau}\frac{(d\sigma/d\Omega)_{\text{exp}}}{(d\sigma/d\Omega)_{\text{Mott}}} = G_M^2 - \frac{\epsilon}{\tau}G_E^2$$

- $(d\sigma/d\Omega)_{\text{red}}\ (\epsilon)$ slope $\longrightarrow G_E$
- $(d\sigma/d\Omega)_{\text{red}}\ (\epsilon)$ intercept $\longrightarrow G_M$

G_E^p and G_M^p with Rosenbluth separation

Dipole approximation $G_D(q^2) = \left(1 - q^2/M_D^2\right)^{-2}$ $M_D^2 = 0.71\ \text{GeV}^2$





Classical approach

Form factors, in nonrelativistic approximation, are Fourier transforms of charge and magnetic distributions

The dipole form factor corresponds to an exponential distribution $\rho(r) = \rho_0 e^{-r/r_0}$

$$M_D^2 = 0.71\ \text{GeV}^2 \Longrightarrow \begin{cases} r_0^2 = (0.24\ \text{fm})^2 \\ \langle r^2 \rangle = (0.81\ \text{fm})^2 \end{cases}$$

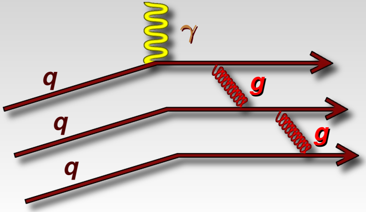
Lett. Nuovo Cim. 7 (1973) 719
Phys. Rev. Lett. 31 (1973) 1153
JETP Lett. 96 (2012) 6-12

Hadron form factor

$$F(q^2) = \frac{C_n}{\left(1 - q^2/M_n^2\right)^{n-1}}$$

- $M_n^2 = n\beta^2$
- β^2 = quark momentum squared
- n = number of constituent quarks

Dimensional scaling



- $F_\pi(q^2) = \dfrac{C_2}{1 - \dfrac{q^2}{0.471\ \mathrm{GeV}^2}}$ ....pion, $n = 2$
- $F_N(q^2) = \dfrac{C_3}{\left(1 - \dfrac{q^2}{0.71\ \mathrm{GeV}^2}\right)^2}$ ...nucleon, $n = 3$
- $F_d(q^2) = \dfrac{C_6}{\left(1 - \dfrac{q^2}{1.42\ \mathrm{GeV}^2}\right)^5}$ ..deuteron, $n = 6$

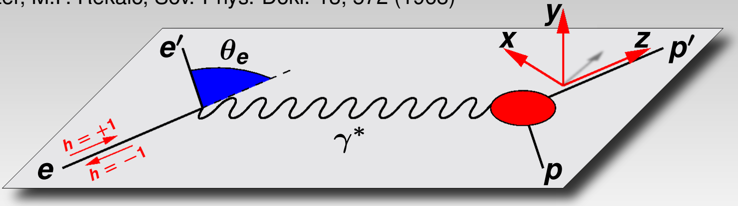
Pion form factor

$\Downarrow$

$\beta^2 = (0.471 \pm 0.010)\ \mathrm{GeV}^2$

Polarization observables

A.I. Akhiezer, M.P. Rekalo, Sov. Phys. Dokl. 13, 572 (1968)



- Elastic scattering of longitudinally polarized ($h = \pm 1$) electrons on nucleon target
- Hadronic tensor: $W_{\mu\nu} = \underbrace{W_{\mu\nu}(0)}_{\text{no pol.}} + \underbrace{W_{\mu\nu}(\vec{P}) + W_{\mu\nu}(\vec{P}')}_{\text{ini. or fin. pol. of } N} + \underbrace{W_{\mu\nu}(\vec{P}, \vec{P}')}_{\text{ini. and fin. pol. of } N}$
- In case of polarized ($h = \pm 1$) electrons on unpolarized nucleon target:

$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon} G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

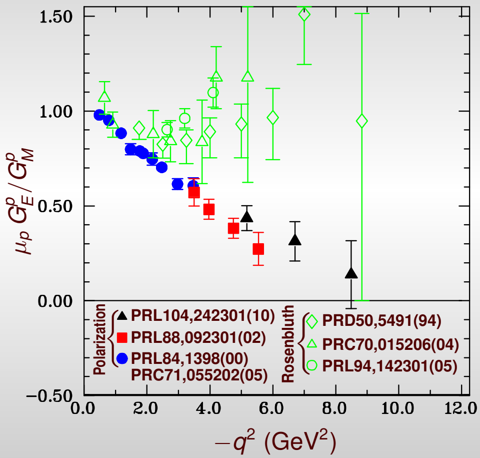
$$P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M\left(G_E^2 - \frac{\tau}{\epsilon} G_M^2\right)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$

$$\frac{P'_x}{P'_z} = -\frac{2M\cot(\theta_e/2)}{E_e + E'_e} \frac{G_E}{G_M}$$

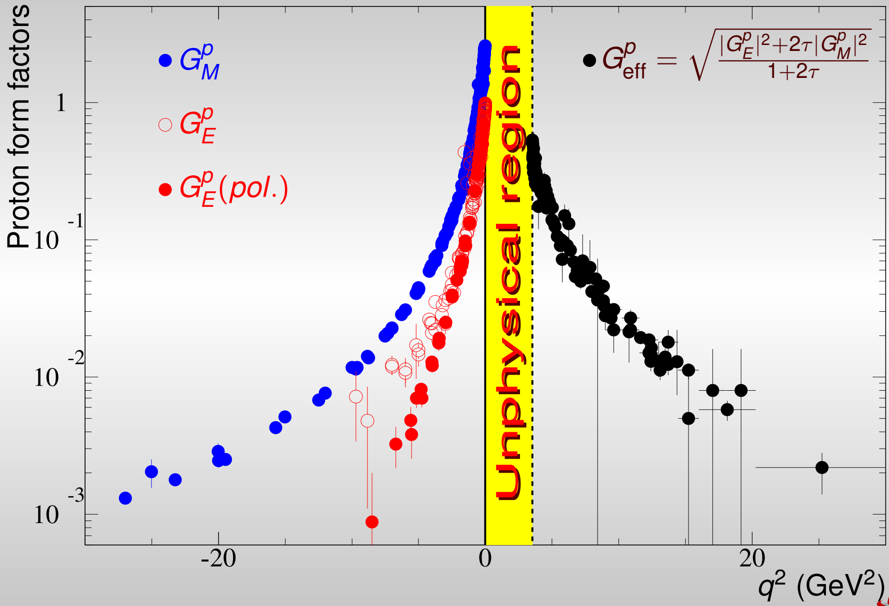
G_E^p / G_M^p in polarization transfer experiments

- "Standard" dipole for the proton magnetic form factors G_M^p
- Linear deviation from the dipole for the electric proton form factor G_E^p
- **QCD scaling still not reached**
- **Zero crossing for G_E^p**
- Polarization data do not agree with old Rosenbluth data (◇)
- New Rosenbluth separation data from JLab **still do not agree** with polarization data

C. Perdrisat *et al*.
JLab-GEp Collaboration



The time-like region



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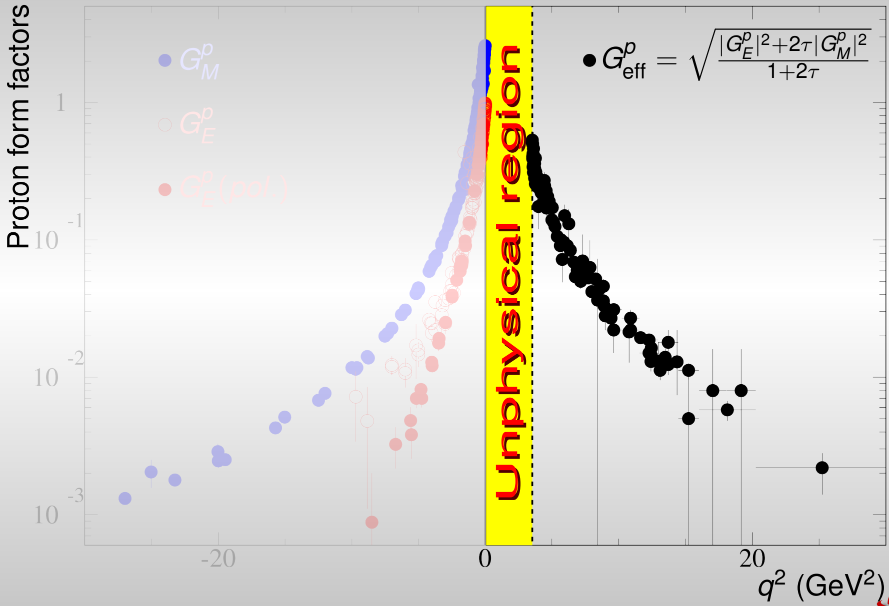
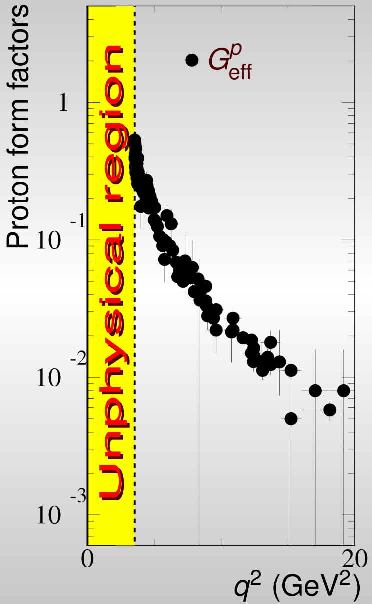




Differential cross section $e^+e^- \to p\bar{p}$

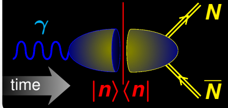
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M^p|^2 + \frac{1}{\tau}\sin^2\theta |G_E^p|^2\right]$$

Optical theorem

$\mathrm{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

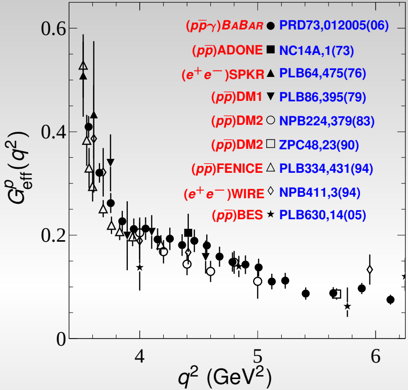
$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

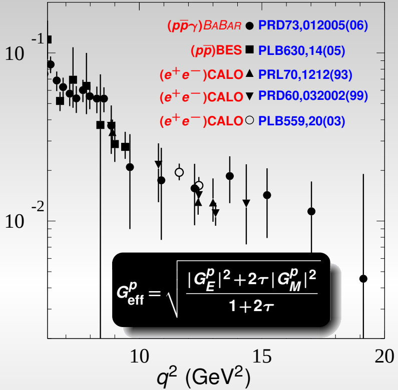


Form factors are complex for $q^2 > 4M_\pi^2$

- The cross section is an even function of $\cos\theta$
- The cross section does not depend on the form factor phases
- At high q^2 the $|G_E^p|^2$ contribution is suppressed
- The unphysical region is not accessible through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

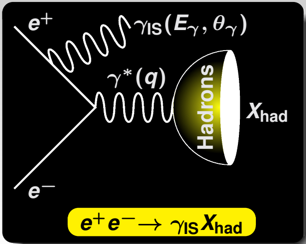
Proton effective form factor





- No individual determination of $|G_E^p|$ and $|G_M^p|$.
- Time-like proton form factors are larger (factor of two) than their space-like values at the same $|q^2|$.
- The threshold behavior is very steep.
- It is not smooth. Structures? Resonances?...

Initial State Radiation



- $$\frac{d^2\sigma}{dE_\gamma \, d\cos\theta_\gamma} = W(E_\gamma, \theta_\gamma)\,\sigma_{e^+e^- \to X_{had}}(s)$$
- $$W(E_\gamma, \theta_\gamma) = \frac{\alpha}{\pi x}\left(\frac{2 - 2x + x^2}{\sin^2\theta_\gamma} - \frac{x^2}{2}\right)$$

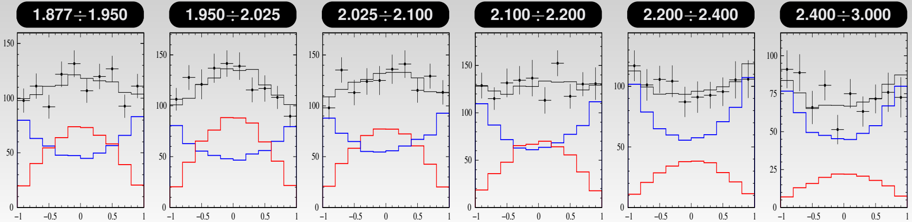
- $s = q^2$, q .... X_{had} momentum
- E_γ, θ_γ ..CM γ_{IS} energy, scatt. ang.
- E_{CM} .... CM e^+e^- energy
- $x = 2E_\gamma / E_{CM}$

- **All energies (q^2) at the same time** $\Rightarrow$ **Better control on systematics (greatly reduced point to point)**
- **Detected ISR at large angles** $\Rightarrow$ **full X_{had} angular coverage**
- **CM boost** $\Rightarrow$ $\left\{\begin{array}{l}\textbf{efficiency at threshold } \neq 0 \\ \textbf{energy resolution } \sim 1 \textbf{ MeV}\end{array}\right.$

$e^+e^- \to p\bar{p}$ angular distribution (BABAR)

PRD87, 092005

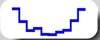
$\cos\theta_p$ distributions form threshold up to 3 GeV [intervals in $E_{CM} \equiv \sqrt{q^2}$ (GeV)]



Events/0.2 vs. $\cos\theta_p$

$$\frac{d\sigma}{d\cos\theta_p} = A\left[H_E(\cos\theta_p, q^2)\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right|^2 + H_M(\cos\theta_p, q^2)\right]$$

H_E and H_M from MC

- G_E^p 
- G_M^p 

At low q^2

$\sin^2\theta_p > 1 + \cos^2\theta_p$

$\Rightarrow$

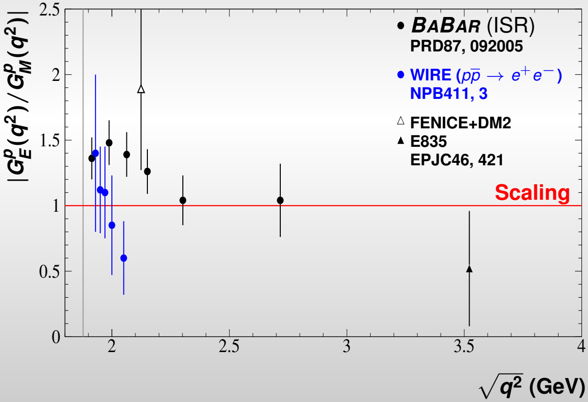
First observation!

$|G_E^p| > |G_M^p|$

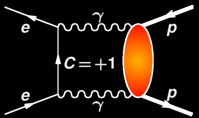
At higher q^2, $|G_E^p| \to |G_M^p|$

Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta\mathcal{C}}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta) + \frac{4M_p^2}{q^2}\sin^2\theta\left|\frac{G_E^p}{G_M^p}\right|^2\right]$$



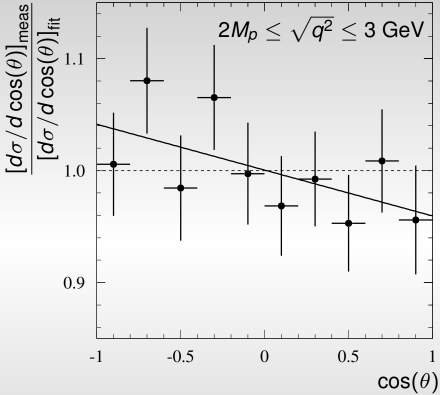
$\gamma\gamma$ exchange



$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]



Integrated over the $p\bar{p}$-CM energy from threshold up to 3 GeV

The MC-fit assumes **one-photon exchange**

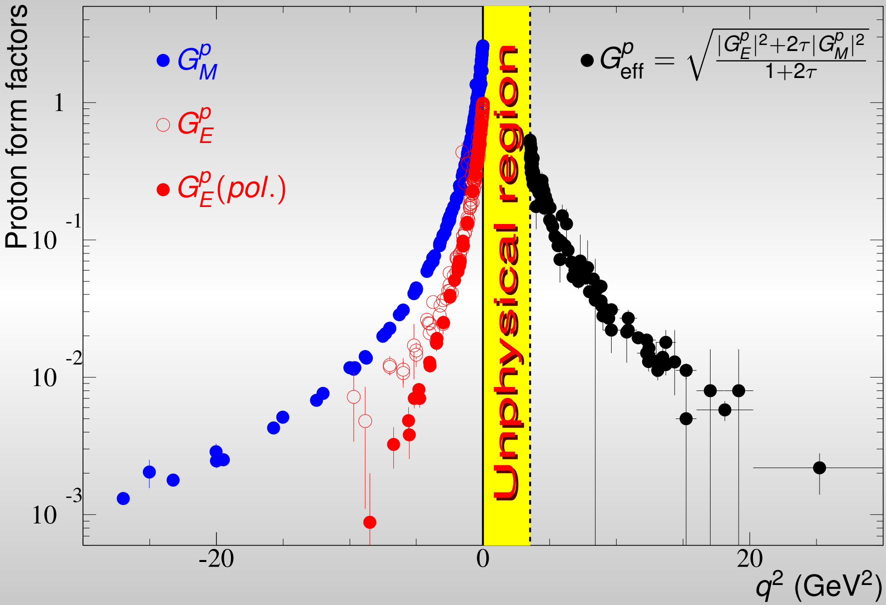
Slope $= -0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

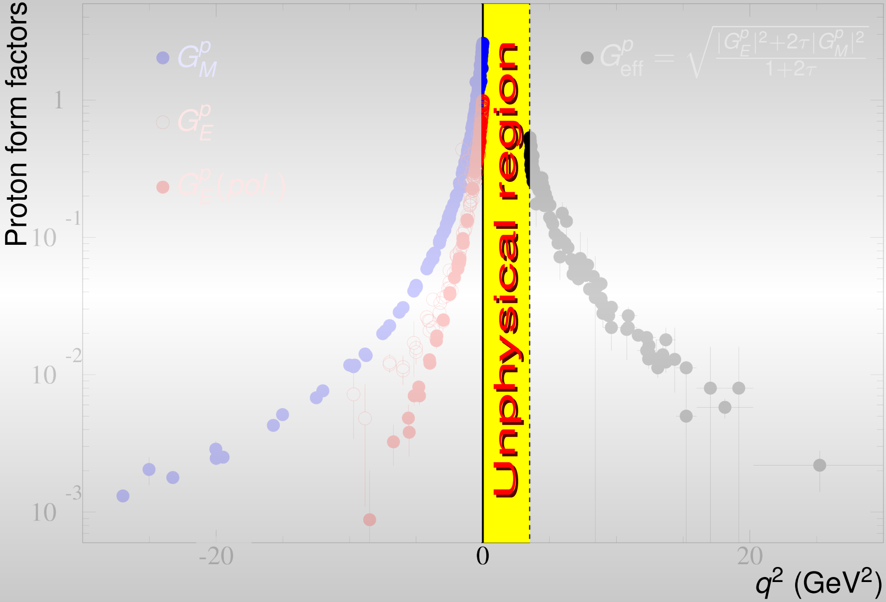
$$\langle\mathcal{A}\rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \le 3$ GeV and $\cos\theta_p \gtrless 0$

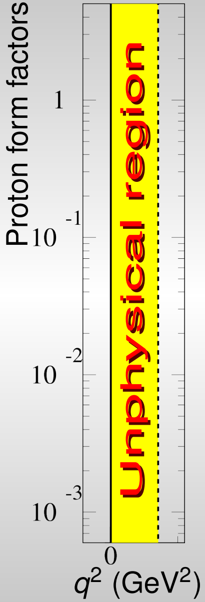
The unphysical region



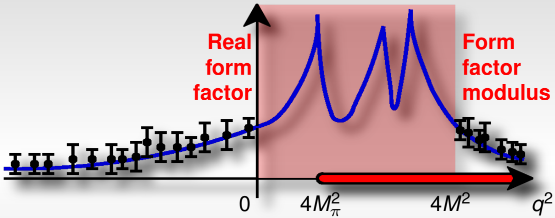
The unphysical region



The unphysical region



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M^2$



In that region, form factors

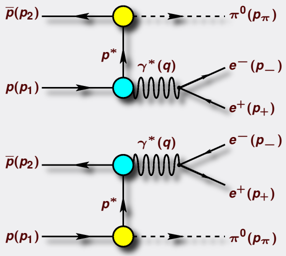
- are still well defined but not (directly) experimentally accessible
- are complex and, following VMD-based models, receive their main contribution from intermediate resonances

Accessing the unphysical region

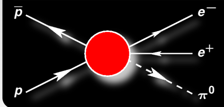
[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π-production

$$p\bar{p} \to \pi^0 e^+ e^-$$

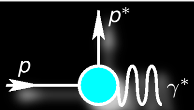


The process $p\bar{p} \to \pi^0 e^+ e^-$



Described in general by **six** amplitudes which depend on **three** kinematical variables

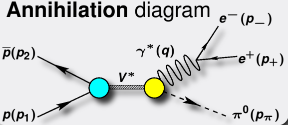
Hadronic current **[PRC75 045205]**



- $J_\mu = \phi_\pi(p_\pi)\,\bar{v}(p_2)\,O_\mu\,u(p_1)$
- $O_\mu = O_\mu[\Gamma_\mu(q)]$
- $\langle N(p')|\Gamma_\mu(q)|N(p)\rangle = \bar{u}(p')\left[F_1(q^2)\gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{4M_p^2}F_2(q^2)\right]u(p)$

Background

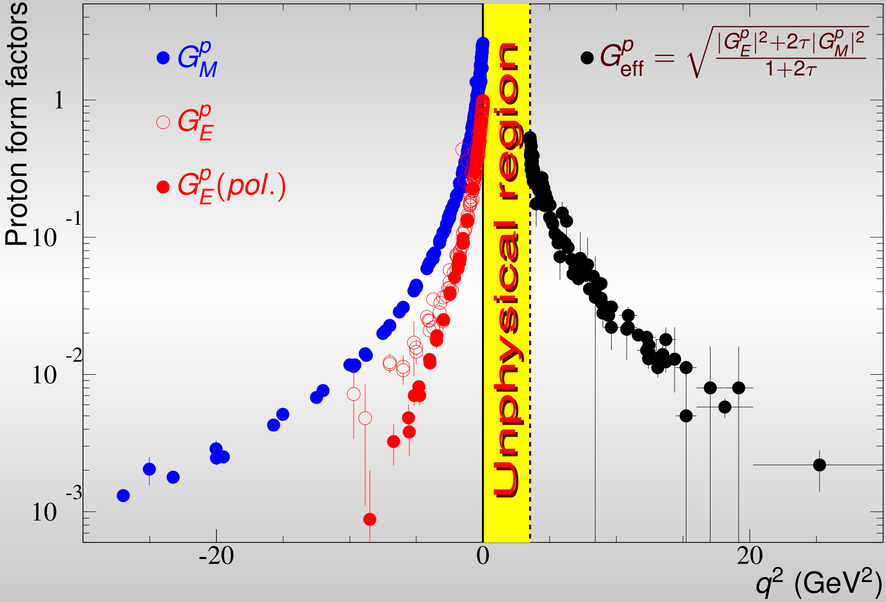
Annihilation diagram

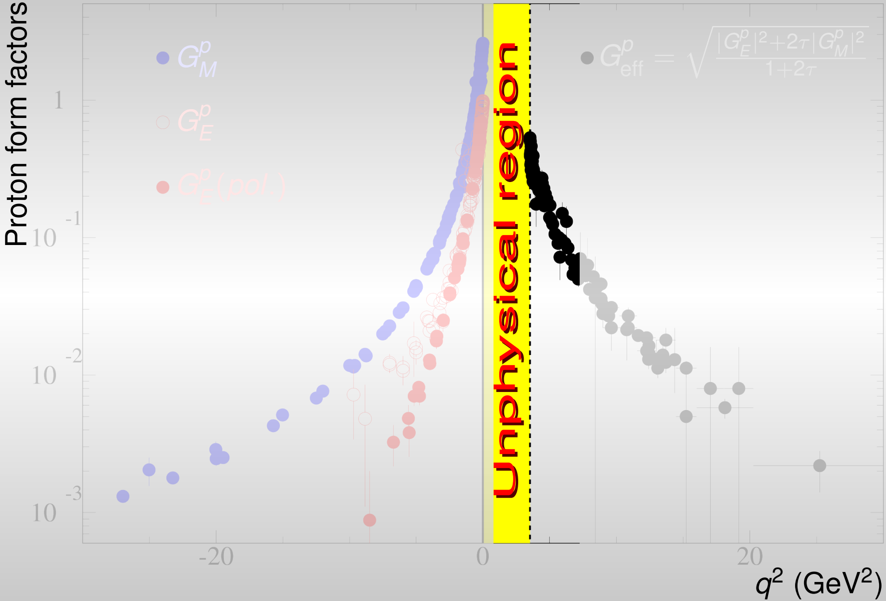


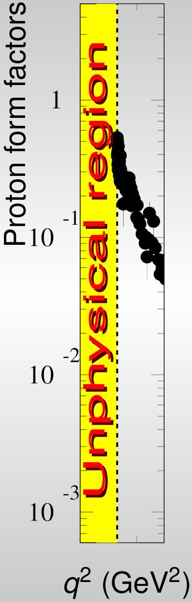
Polarization observables help in disentangle reaction mechanisms

[E. A. Kuraev *et al*., J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

The threshold region$_1$

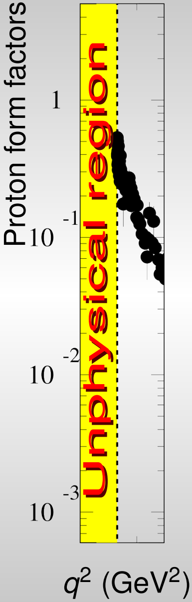



Proton form factors
G_M^p
G_E^p
$G_E^p(pol.)$
$G_{\rm eff}^p = \sqrt{\frac{|G_E^p|^2+2\tau|G_M^p|^2}{1+2\tau}}$
Unphysical region
1
10^{-1}
10^{-2}
10^{-3}
-20
0
20
q^2 (GeV^2)



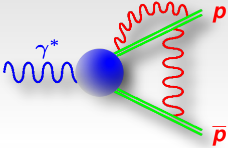
Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$



Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$



$p\bar{p}$ Coulomb interaction as FSI

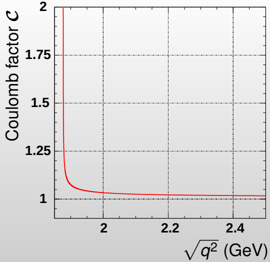
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Schrödinger wave function

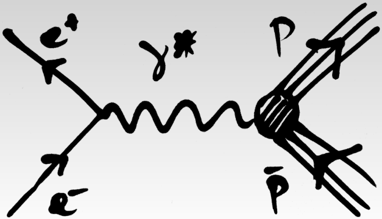
$$\mathcal{C} = |\Psi_{\mathrm{Coul}}(0)|^2$$

- **S-wave:** $\mathcal{C} = \dfrac{\frac{\pi\alpha}{\beta}}{1-\exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $\mathcal{C} = 1$

No Coulomb factor for boson pairs (P-wave)



Partial wave form factors



$$\begin{cases} P_\gamma = -1 \\ J_\gamma = 1 \end{cases} \qquad \begin{cases} P_{p\bar{p}} = (-1)^{L+1} \\ S_{p\bar{p}} = 0, 1 \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} L_{p\bar{p}} = 0, 2 \\ S_{p\bar{p}} = 1 \end{cases} \qquad \cancel{\begin{cases} L_{p\bar{p}} = 1 \\ S_{p\bar{p}} = 0 \end{cases}}$$

Partial wave form factors

$$G_S^p = \frac{1}{3}\left(2G_M^p\sqrt{\frac{q^2}{4M_p^2}} + G_E^p\right) \qquad G_D^p = \frac{1}{3}\left(G_M^p\sqrt{\frac{q^2}{4M_p^2}} - G_E^p\right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta\,\frac{4M_p^2}{(q^2)^2}\left[\mathcal{C}\,\left|G_S^p(q^2)\right|^2 + 2\left|G_D^p(q^2)\right|^2\right]$$

Enhancement and Resummation Factors

Coulomb factor

Enhancement factor

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

Resummation factor

Enhancement factor

$$\mathcal{E} = \frac{\pi\alpha}{\beta}$$

- It is responsible for the **one-photon exchange $p\bar{p}$ FSI**
- It dominates close to threshold: $\mathcal{C} \underset{\beta\sim 0}{\simeq} \mathcal{E}$
- It cancels the phase-space factor $\Rightarrow$ $\boxed{\text{stepwise cross section at threshold}}$

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\cancel{\beta}}{\cancel{\beta}}|G_S^p(4M_p^2)|^2 = 0.85\,|G_S^p(4M_p^2)|^2 \text{ nb}$$

Resummation factor

$$\mathcal{R} = \frac{1}{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

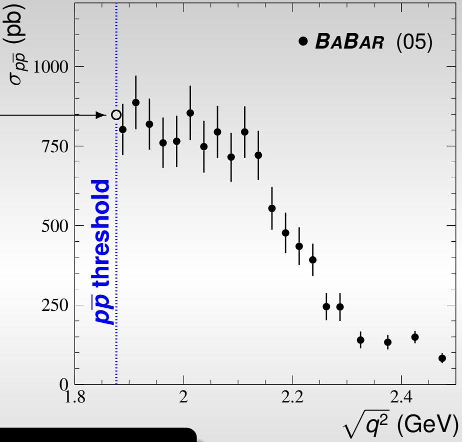
- It is responsible for the **multi-photon exchange $p\bar{p}$ FSI**
- No effective few MeV above threshold: $\mathcal{R} \underset{\beta > 0}{\simeq} 1$
- It must account also for gluon exchange

$$\boxed{\mathcal{R} \to \mathcal{R}_s = \left[1 - \exp\left(-\pi\alpha_s/\tilde{\beta}\right)\right]^{-1}} \quad \boxed{\alpha_s \simeq 0.5} \quad \boxed{\tilde{\beta} = \beta/(1-\beta)}$$

Step and plateau in *BaBar* data

Expected cross section with

$\left|G_S^p(4M_p^2)\right| = 1$



At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\cancel{\beta_p}}{\cancel{\beta_p}}\left|G_S^p(4M_p^2)\right|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\left|G_S^p(4M_p^2)\right|^2 \text{ nb}$$

$\Longrightarrow$

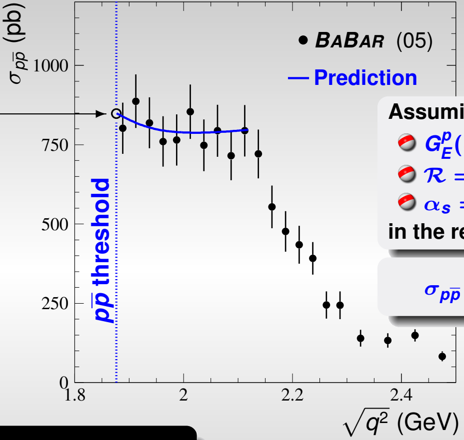
$\left|G_S^p(4M_p^2)\right| \equiv 1$

as pointlike fermion pairs!

Step and plateau in BABAR data

Expected cross section with

$$|G_S^p(4M_p^2)| = 1$$



Assuming:

- $G_E^p(q^2) = G_M^p(q^2) = 1$
- $\mathcal{R} = \mathcal{R}_s$
- $\alpha_s = 0.5$

in the region $q^2 \leq (2.1\text{GeV})^2$

$$\sigma_{p\bar{p}} = (850\,\text{pb})\,\frac{1}{\tau}\,\mathcal{R}_s$$

At threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\cancel{\beta_p}}{\cancel{\beta_p}}\,|G_S^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G_S^p(4M_p^2)|^2\ \text{nb}$$

$\Longrightarrow$

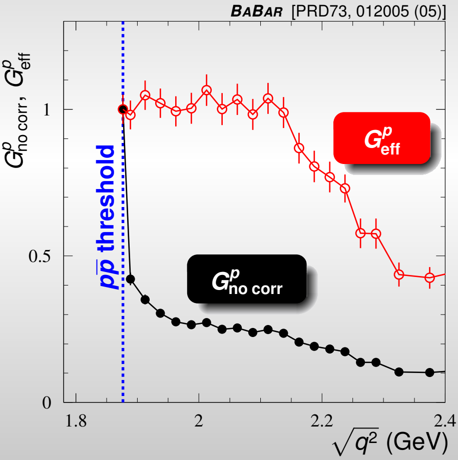
$$|G_S^p(4M_p^2)| \equiv 1$$

as pointlike fermion pairs!

BaBar: G^p_{eff} including threshold effects

$$\left[G^p_{\text{no corr}}(q^2)\right]^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E}\mathcal{R}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

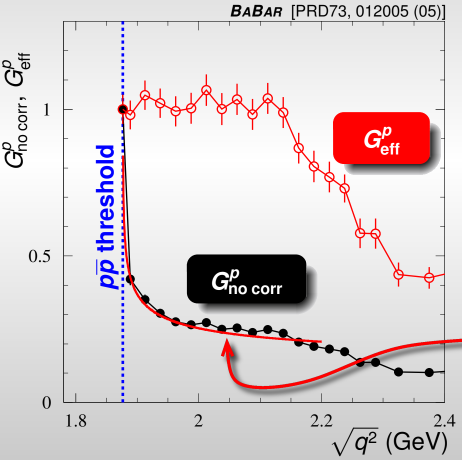
$$\left[G^p_{\text{eff}}(q^2)\right]^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{E}\mathcal{R}_s\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$



BABAR: $G^p_{\rm eff}$ including threshold effects

$$\left[G^p_{\rm no\,corr}(q^2)\right]^2 = \frac{\sigma_{p\bar p}(q^2)}{\mathcal{E}\mathcal{R}\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

$$\left[G^p_{\rm eff}(q^2)\right]^2 = \frac{\sigma_{p\bar p}(q^2)}{\mathcal{E}\mathcal{R}_s\,\frac{16\pi\alpha^2}{3}\,\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$



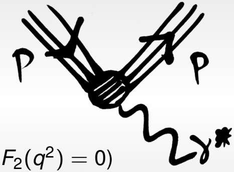
$$\frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - e^{-\frac{\pi\alpha}{\beta}}}$$

Isotropy at the $p\bar{p}$ production threshold

$$G_E(4M^2) = G_M(4M^2)$$

- Electric form factor $G_E \longrightarrow$ p and $\bar{p}$ have **antiparallel** spins
- Magnetic form factor $G_M \longrightarrow$ p and $\bar{p}$ have **parallel** spins
- Electromagnetic current:

$$J^\mu(p_1,p_2) = \overline{U}(p_2)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]U(p_1)$$



  - $F_1 = \dfrac{q^2 G_E - 4M^2 G_M}{q^2 - 4M^2}$
  - $F_2 = 4M^2\dfrac{G_M - G_E}{q^2 - 4M^2}$

F_1 and F_2 "can" be analytic (pointlike limit: $F_1(q^2) = 1$ and $F_2(q^2) = 0$)

- Annihilation cross section $\left[\widetilde{G}_{E,M} \equiv G_{E,M}(4M^2)\right]$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta\mathcal{C}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right] \xrightarrow[q^2\to 4M^2]{|\widetilde{G}_E|=|\widetilde{G}_M|} \frac{\alpha^2\beta\mathcal{C}}{16M^2}\left[2|\widetilde{G}_M|^2\right]$$

- Partial wave form factors
  - $G_S = \dfrac{2\sqrt{\frac{q^2}{4M^2}}G_M + G_E}{3} \xrightarrow[q^2\to 4M^2]{} \widetilde{G}_M$
  - $G_D = \dfrac{\sqrt{\frac{q^2}{4M^2}}G_M - G_E}{3} \xrightarrow[q^2\to 4M^2]{} 0$

$$G_E(4M^2) \neq G_M(4M^2)$$

- Dirac and Pauli form factors F_1 **and** F_2 **are not analytic**
- To preserve G_E and G_M analyticity, F_1 and F_2 must have **a simple pole** at the threshold with **opposite residues**

$$F_1 = \frac{-4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_1^{an} \qquad F_2 = \frac{4M^2 \Delta\widetilde{G}}{q^2 - 4M^2} + F_2^{an} \qquad \Delta\widetilde{G} \equiv \widetilde{G}_E - \widetilde{G}_M$$

$F_{1,2}^{an}$ is the analytic part of $F_{1,2}$

- Annihilation cross section

$$\frac{d\sigma}{d\Omega} \xrightarrow[q^2 \to 4M^2]{|\widetilde{G}_E| \neq |\widetilde{G}_M|} \frac{\alpha^2 \beta C}{8M^2}\left[|\widetilde{G}_M|^2 + \mathrm{Re}\left(\Delta\widetilde{G}\,\widetilde{G}_M^*\right)\sin^2\theta\right]$$

Assuming $|\Delta\widetilde{G}| \ll |\widetilde{G}_M|$

- Partial wave form factors

$$G_S = \frac{2\sqrt{\frac{q^2}{4M^2}}G_M + G_E}{3} \xrightarrow[q^2 \to 4M^2]{} \widetilde{G}_M + \frac{\Delta\widetilde{G}}{3} \qquad G_D = \frac{\sqrt{\frac{q^2}{4M^2}}G_M - G_E}{3} \xrightarrow[q^2 \to 4M^2]{} -\frac{\Delta\widetilde{G}}{3}$$

Sources of anisotropy

Coulomb/QCD interaction correction $\propto \beta^{-1}$ $\Longrightarrow$ **Singular** form factors at threshold $\Longrightarrow$ **Anisotropy**

Only Coulomb
Dmitriev, Milstein, PLB722 (13) 83

$$\widetilde{G}_D \sim -\frac{\alpha^2}{8}$$

Very small
but not vanishing!

QCD Coulomb like
Brodsky, Hoang, Kuhn, Teubner, PB359 (95) 355

Large effect for heavy quarks

Anisotropy $\propto \beta^n$
No effect at threshold!

Measuring anisotropy at threshold

$e^+e^- \to p\bar{p}$

SND | CMD3

Very difficult

Efficiency drops with proton antiproton velocity

$p\bar{p} \to e^+e^-$

PANDA

Very difficult

Normalization (Coulomb corrections...)

$e^+e^- \to p\bar{p}\gamma$

BABAR | BESIII

Difficult

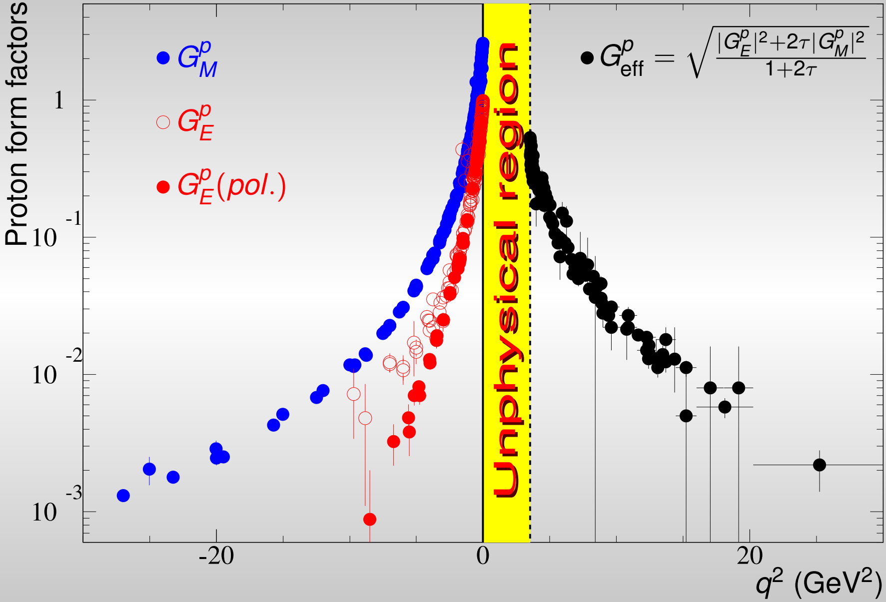
ISR technique: not enough statistics

$e^+e^- \to H_B\overline{H}_B$

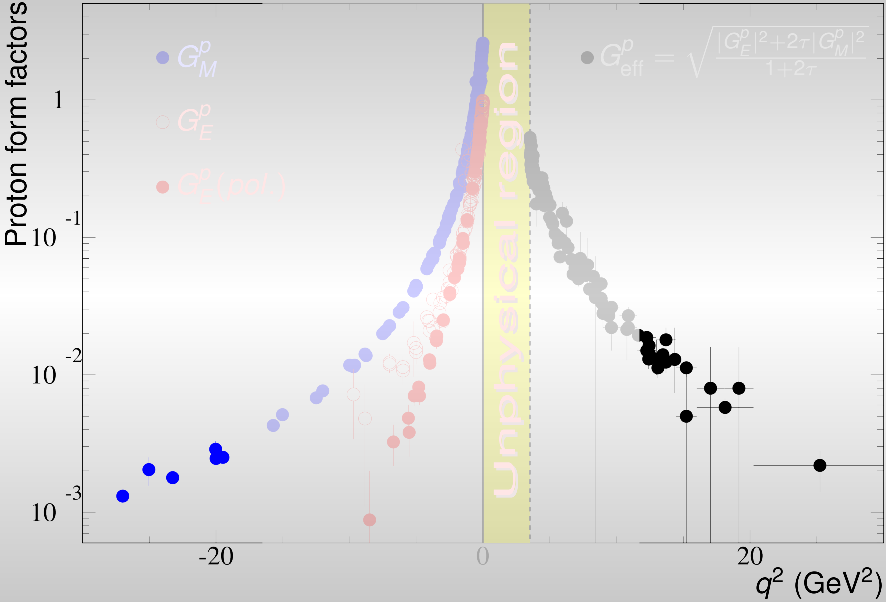
BESIII

Feasible with heavy baryons

The weak decay allows detection at threshold and polarization measurements (BESIII has...)



Proton form factors
G_M^p
G_E^p
$G_E^p(pol.)$
Unphysical region
$G_{\rm eff}^p = \sqrt{\frac{|G_E^p|^2+2\tau|G_M^p|^2}{1+2\tau}}$
1
10^{-1}
10^{-2}
10^{-3}
-20
0
20
q^2 (GeV^2)


Proton form factors
G_M^p
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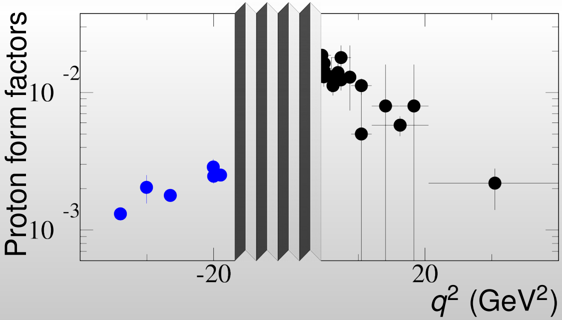
$G_{\rm eff}^p = \sqrt{\frac{|G_E^p|^2+2\tau|G_M^p|^2}{1+2\tau}}$
Unphysical region
1
10^{-1}
10^{-2}
10^{-3}
-20
0
20
q^2 (GeV^2)

Time-like asymptotic behavior

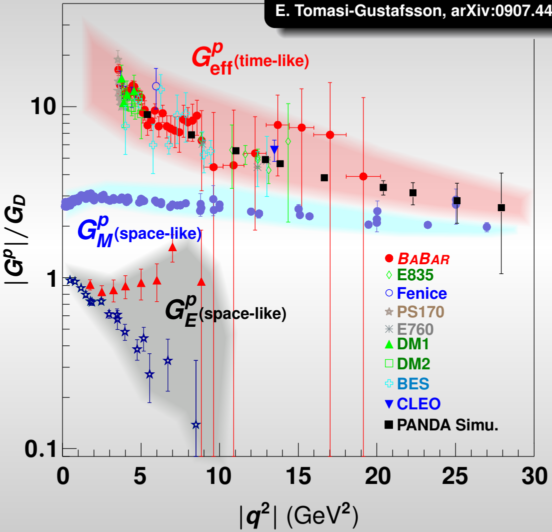
Phragmèn Lindelöf theorem:

If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

- $$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space—like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time—like}}$$
- $$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$ **real**



E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



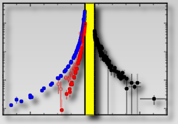
pQCD

$$G^p_{\text{eff}}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{\text{eff}}(q^2)}{G^p_M(-q^2)} = 1$$

Conclusions



Global models for proton and neutron, electric and magnetic form factors must be encouraged. They can help in understanding. . .

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- . . .

To measure. . .

- zero of G_E^p in space-like region
- moduli of G_E and G_M in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \to \pi^0 e^+ e^-$)
- . . .

Experiments: now and future

Space-like region

A1 Mainz

- G_E^n at $-q^2 = 1.5$ GeV^2 (Pol. ^{3}He)
- G_E^p and G_M^p for $-q^2 \leq 2.0$ GeV^2

Jefferson Lab

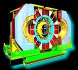
- [Hall A] G_E^p/G_M^p up to 14 GeV^2
- [Hall A] G_M^n (ratio) up to 18 GeV^2
- [Hall A] G_E^n/G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2

Time-like region

SND, CMD

at VEPP-2000
e^+e^- **collider**
$|G_{\text{eff}}^p|$, $|G_{\text{eff}}^n|$ (scan)
$q^2 \leq (4\text{ GeV})^2$

BESIII



at BEPCII
e^+e^- **collider**
$|G_E^p|$, $|G_M^p|$, $|G_{\text{eff}}^n|$ (scan and ISR)
$q^2 \leq (3.5\text{ GeV})^2$

panda

at FAIR
$p\bar{p}$ **collider**
$|G_E^p|$, $|G_M^p|$, G_E^p/G_M^p phase ($\bar{p}$ polarization)
$(2.4\text{ GeV})^2 \leq q^2 \leq (3.7\text{ GeV})^2$

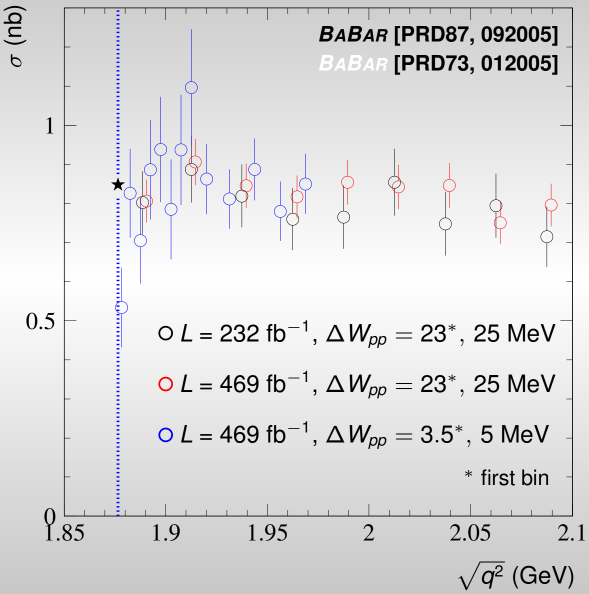
Belle II

?

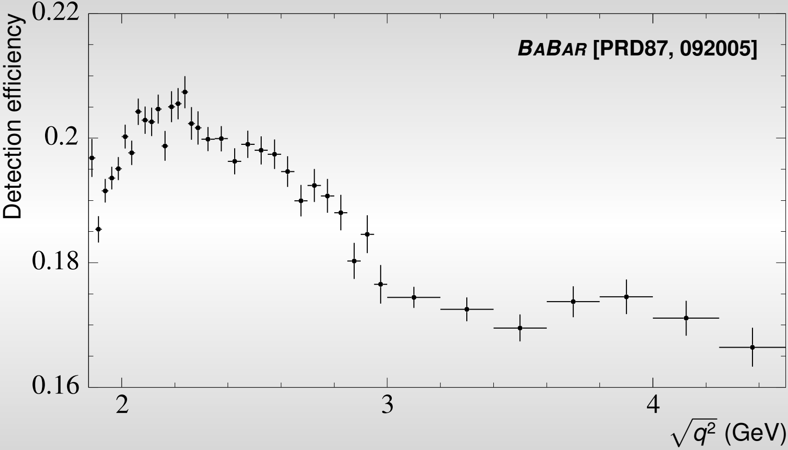
at SuperKEKB
e^+e^- **collider**
$|G_E^p|$, $|G_M^p|$, (ISR)
$q^2 \leq (4.5\text{ GeV})^2$

Additional slides

The threshold region$_3$



The threshold region$_3$

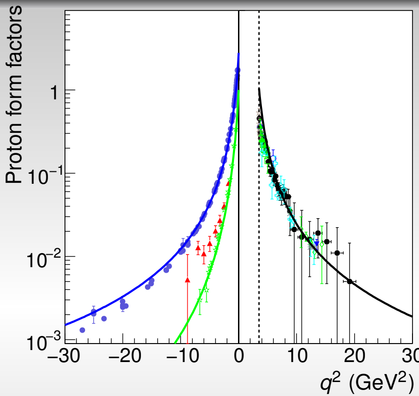


An interesting model

Assumption
Pauli principle pulls away from the internal region of strong chromo-electromagnetic field quarks of same flavor because the color quantum number does not play any role (stochastic averaging).

Outer spatial region $|p\rangle = \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 1

Central region $|p\rangle \neq \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 0



Counting rule on the vector part of interaction

space-like

A screening effect from the central region provides an additional suppression for the electric form factor

$$G_M^p(q^2) = \mu_p \, G_D(q^2)$$

$$G_E^p(q^2) = \frac{G_D(q^2)}{1 - q^2/q_1^2}$$

time-like

$$G_M^p(q^2) = \frac{\theta(q^2 - 4M_p^2)}{\left[1 + (q^2 - 4M_p^2)^2/q_2^2\right]^2}$$

$$G_E^p(q^2) = \frac{G_M^p(q^2)\,\theta(q^2 - 4M_p^2)}{1 + (q^2 - 4M_p^2)^2/q_2^2}$$