On the Physical Meaning of Sachs Form Factors, amplitudes of a proton current with spin-flip and non spin-flip and on the Violation of the Dipole Dependence of G_E and G_M on Q^2

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The Rosenbluth formula in the laboratory reference frame

Our notations for the process of ep elastic scattering in the Born approximation:

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2)\,, \tag{1}$$

$$M_{ep\to ep} = \overline{u}(p_2)\gamma^{\mu}u(p_1)\cdot\overline{u}(q_2)\Gamma_{\mu}(q^2)u(q_1)\;\frac{1}{q^2}\;, \tag{2}$$

$$(J_p)_{\mu} = \overline{u}(q_2)\Gamma_{\mu}u(q_1)\,, \Gamma_{\mu}(q^2) = F_1\;\gamma_{\mu} + \frac{F_2}{4M}(\;\hat{q}\gamma_{\mu} - \gamma_{\mu}\hat{q}\;)\;, \tag{3}$$

where $\overline{u}(p_i)u(p_i) = 2\,m_e,\; \overline{u}(q_i)u(q_i) = 2\,M\;(i=1,2)$, $p_i^2 = m_e^2,\; q_i^2 = M^2$, m_e and M – electron and proton mass, $q = q_2 - q_1$, s_1 and s_2 – spin 4-vectors for initial and final protons with: $s_1q_1 = s_2q_2 = 0$, $s_1^2 = s_2^2 = -1$. The Rosenbluth formula for $q_1 = (M, \vec{0})$ and $m_e = 0$ read as:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2\cos^2(\theta_e/2)}{4E_1^3\sin^4(\theta_e/2)}\frac{1}{1+\tau}\left(G_E^2 + \frac{\tau}{\varepsilon}\,G_M^2\right)\,. \tag{4}$$

$$G_E = F_1 + \frac{q^2}{4M^2}\,F_2\,,\; G_M = F_1 + F_2\,. \tag{5}$$

Here $\tau = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1E_2\sin^2(\theta_e/2)$, $\alpha = 1/137$ - fine structure constant, $\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$, **$\varepsilon$ is the degree of the virtual photon linear polarization !!! see authors paper [arXiv:1210.0634 [nucl-th]]**

The Rosenbluth formula in the arbitrary reference frame

[A.I. Akhiezer and V.B. Berestetsky, Quantum Electrodynamics, Nauka, Moscow, 1969, in Russian, eq.(34.3.3), page 475.]

The Rosenbluth formula in the arbitrary reference frame read as:

$$d\sigma = \frac{\alpha^2 do}{4w^2}\frac{1}{1+\tau}\,(\,G_E^2\,Y_I + \tau\,G_M^2\,Y_{II}\,)\,\frac{1}{q^4}\,, \tag{6}$$

$$Y_I = (p_+q_+)^2 + q_+^2 q^2, \qquad Y_{II} = (p_+q_+)^2 - q_+^2(q^2+4m_e^2)\,,$$

$$p_+ = p_1 + p_2\,,\; q_+ = q_1 + q_2\,.$$

The Rosenbluth formulas in an arbitrary reference frame (6) as well as in the laboratory reference frame (4) are expressed only through the squares of the form factors (FFs) Sachs G_E^2 and G_M^2.

It is the question arises: whether there is any physical meaning in the decomposition of G_E^2 and G_M^2 in Rosenbluth's cross section?

Physical meaning in decomposition of squares G_E^2 and G_M^2.

Summed over the polarizations of the initial and final protons the Rosenbluth cross section (4) is actually the sum of the cross sections corresponding to the transition without spin-flip ($\sigma^{\delta,\delta}$) and with spin-flip ($\sigma^{-\delta,\delta}$) of the initial proton:

$$\frac{d\sigma}{d\Omega} = \kappa \left(G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right) = \kappa \, (\sigma^{\delta,\delta} + \sigma^{-\delta,\delta}), \tag{7}$$

$$\sigma^{\delta,\delta} = G_E^2, \;\; \sigma^{-\delta,\delta} = \frac{\tau}{\varepsilon} G_M^2 \, . \tag{8}$$

where κ is the factor in front of the parentheses in Eq. (7). The spin projections axis for initial and final proton are identical and coincide with the direction of the final proton momentum $\vec{q_2}$ ($q_2 = (q_{20}, \vec{q_2})$):

$$s_1 = (0, \vec{n_2}), \; s_2 = (|\vec{v_2}|, v_{20} \, \vec{n_2}), \; \vec{c}_1 = \vec{c}_2 = \vec{n}_2 = \vec{q_2}/|\vec{q_2}| \, . \tag{9}$$

Spin 4-vectors s_1 and s_2 for protons with 4-momentum q_1, q_2 are standard:

$$s_i = (s_{i0}, \vec{s}_i), s_{0i} = \vec{v}_i \, \vec{c}_i, \vec{s}_i = \vec{c}_i + \frac{(\vec{c}_i \vec{v}_i) \, \vec{v}_i}{1 + v_{i0}}, v_i = (v_{i0}, \vec{v}_i) = q_i/M, i = 1, 2. \tag{10}$$

Physical meaning in decomposition of squares G_E^2 and G_M^2.

$$\sigma^{\delta,\delta} = G_E^2, \;\; \sigma^{-\delta,\delta} = \frac{\tau}{\varepsilon} G_M^2 \; . \tag{11}$$

The terms $\sigma^{\delta,\delta}$ and $\sigma^{-\delta,\delta}$ in Eq. (7), (11) are the cross sections without and with the spin-flip for the case where the initial and final protons are fully polarized in the direction of the motion of the final proton.
In the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = \vec{n}_2$ we have $\sigma^{\delta,\delta} \rightarrow$ for non-spin-flip transition.
In the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = -\vec{n}_2$ we have $\sigma^{-\delta,\delta} \;\; \rightarrow$ for spin-flip transition.

Consequently for the matrix elements of the proton current $J_p^{\pm\delta,\delta}$ we have

$$J_p^{\delta,\delta} = \sqrt{\sigma^{\delta,\delta}} \sim G_E \, , \, J_p^{-\delta,\delta} = \sqrt{\sigma^{-\delta,\delta}} \sim \sqrt{\tau}\, G_M \, . \tag{12}$$

To prove the relations (11) and (12) we can offer three ways:

- Using the method for calculating of the QED matrix elements in the so-called "Diagonal Spin Basis" (DSB).
- Using the standard method for calculation of the QED cross sections.
- Using the textbook of F. Halzen and A. Martin "Quarks and leptons. An Introductory Course in Modern Particle Physics", 1984 (Page 178).

Diagonal spin basis (DSB)

[S. Sikach, Izvestia AN BSSR, s.f.-m.n, 2, 84 (1984)]
In the diagonal spin basis (DSB) spin 4-vectors s_1 and s_2 of protons with 4-momenta q_1 and q_2 ($s_1 q_1 = s_2 q_2 = 0, s_1^2 = s_2^2 = -1$) have the form:

$$s_1 = -\frac{(v_1 v_2) v_1 - v_2}{\sqrt{(v_1 v_2)^2 - 1}}\,, \quad s_2 = \frac{(v_1 v_2) v_2 - v_1}{\sqrt{(v_1 v_2)^2 - 1}}\,, v_1 = \frac{q_1}{M}, \, v_2 = \frac{q_2}{M}, \tag{13}$$

The spin vectors (13) obviously do not change under transformations of the Lorentz little group (little Wigner group) common to particles with 4-momenta q_1 and q_2: $L_{q_1,q_2} q_1 = q_1$, $L_{q_1,q_2} q_2 = q_2$. It is a one-parameter subgroup of the rotation group SO_3 with axis whose direction is determined by the 3-vector [F.I. Fedorov, TMF **2**, 3, 343 (1970)]:

$$\vec{a} = \vec{q}_1 / q_{10} - \vec{q}_2 / q_{20} \,. \tag{14}$$

The direction of $\vec{a}$ (14) have property that the projections of the spins of both particles on it simultaneously have definite values. Therefore, the DSB naturally makes it possible to describe the spin states of systems of any two particles by means of the spin projections on the common direction given by the 3-vector (14).

Diagonal spin basis (DSB)

Since vector $\vec{a}$ (14) is the difference of two vectors and the geometrical image of the difference of two vectors is the diagonal of a parallelogram, hence the name "diagonal spin basis" given by academician F.I. Fedorov.

Let us consider the realization DSB in the rest frame of the initial proton, where $q_1 = (M, \vec{0})$. Here $\vec{a}$ (14) equal $\vec{a} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|$, i.e. common direction for spin projection is the direction of the motion of the final proton, thus this final proton polarization state is a helicity and spin 4-vectors $s_1 \;\; s_2$ (13) have the form:

$$s_1 = (0, \vec{n}_2),\; s_2 = (|\vec{v}_2|, v_{20}\,\vec{n_2}), \vec{c}_1 = \vec{c}_2 = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,, \tag{15}$$

axis of spin projections $\vec{c}_1$ and $\vec{c}_2$ is coincide with the direction of the final proton. Breit system, where $\vec{q_2} = -\vec{q_1}$, is a special case of DSB. In the Breit system where $q_1 = (q_0, -\vec{q}), q_2 = (q_0, \vec{q})$, the spin states of the initial and final protons are helicity, so they spin 4-vectors $s_1 \;\; s_2$ in DSB have the form:

$$s_1 = (-|\vec{v}|, v_0\vec{n}_2),\; s_2 = (|\vec{v}|, v_0\vec{n}_2)\,, \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,. \tag{16}$$

Spin operators in the DSB

[M. Galynskii and S. Sikach, Phys.Part.Nucl. **29** (1998) 469-495, hep-ph/9910284]
In the DSB all spin operators for initial and final proton have the same form:

$$\sigma = \sigma_1 = \sigma_2 = \gamma^5 \hat{s_1} \hat{v_1} = \gamma^5 \hat{s_2} \hat{v_2} = \gamma^5 \hat{b}_0 \hat{b}_3 = i \hat{b}_1 \hat{b}_2 \ , \tag{17a}$$

$$\sigma^{\pm\delta} = \sigma_1^{\pm\delta} = \sigma_2^{\pm\delta} = -i/2 \gamma^5 \hat{b}_{\pm\delta}, \ b_{\pm\delta} = b_1 \pm i\delta b_2 \ , \ \delta = \pm 1 \ , \tag{17b}$$

$$\sigma u^\delta(q_i) = \delta u^\delta(q_i) \ , \ \sigma^{\pm\delta} u^{\mp\delta}(q_i) = u^{\pm\delta}(q_i). \tag{17c}$$

The set of unit 4-vectors b_0, b_1, b_2, b_3 is an orthonormal basis of 4-vectors b_A, $b_A b_B = g_{AB}$ $(A, B = 0, 1, 2, 3)$:

$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma, \ (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa p_1^\sigma / \rho \, , b_3 = \frac{q_-}{\sqrt{-q_-^2}}, \ b_0 = \frac{q_+}{\sqrt{q_+^2}} \ , \tag{18}$$

where $q_- = q_2 - q_1$, $q_+ = q_2 + q_1$, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), ρ is determined from the normalization conditions $b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1$.

Calculation of QED matrix elements in the DSB

[M. Galynskii and S. Sikach, Phys.Part.Nucl. **29** (1998) 469-495, hep-ph/9910284]
A matrix elements of QED processes have a standard form

$$M^{\pm\delta,\delta} = \overline{u}^{\pm\delta}(q_2) Q u^{\delta}(q_1) \,, \tag{19}$$

where Q is the interaction operator, and $u^{\delta}(q_1)$ and $u^{\pm\delta}(q_2)$ are the bispinors of the initial and final states, with $\overline{u}^{\delta}(q_i)\, u^{\delta}(p_i) = 2M \ , \ q_i^2 = M^2 \ , \ (i = 1, 2)$.
In our covariant approach the calculation of matrix elements of the form (19) reduces to computation the trace from the product of Dirac operators:

$$M^{\pm\delta,\delta} = Tr(P_{21}^{\pm\delta,\delta} Q) \ , \ P_{21}^{\pm\delta,\delta} = u^{\delta}(q_1)\, \overline{u}^{\ \pm\delta}(q_2) \ . \tag{20}$$

The operators $P_{21}^{\pm\delta,\delta}$ determine the structure of the spin dependence of the matrix elements (19) in the case of transitions without spin-flip ($P_{21}^{\delta,\delta}$) and with spin-flip ($P_{21}^{-\delta,\delta}$). They have the form:

$$P_{21}^{\delta,\delta} = (\hat{q}_1 + M)\, \hat{b}_\delta\, \hat{b}_0\, \hat{b}_\delta^{*}/4 \,, \tag{21}$$

$$P_{21}^{-\delta,\delta} = \delta(\hat{q}_1 + M)\, \hat{b}_\delta\, \hat{b}_3/2 \,, \tag{22}$$

where $b_\delta^{*} = b_1 - i\delta b_2$.

The matrix elements of the proton current in the DSB

[S. Sikach, Izvestia AN BSSR, s.f.-m.n, 2, 84 (1984)]
Let us consider the process of elastic ep scattering

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2)\,, \tag{23}$$

where s_1 and s_2 are spin 4-vectors for initial and final protons in the DSB (13).
Matrix elements (amplitudes) for proton current defined as:

$$(J_p^{\pm\delta,\delta})_\mu = \overline{u}^{\pm\delta}(q_2)\Gamma_\mu(q^2)u^\delta(q_1)\,,\ \ \Gamma_\mu(q^2) = F_1\ \gamma_\mu + \frac{F_2}{4M}(\ \hat{q}\gamma_\mu - \gamma_\mu\hat{q}\ )\,. \tag{24}$$

They were calculated by S.Sikach (1984):

$$(J_p^{\delta,\delta})_\mu = 2G_E M(b_0)_\mu\,, \tag{25}$$

$$(J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}G_M(b_\delta)_\mu\,. \tag{26}$$

For the point particles with mass m_q the amplitude of the currents have the form

$$(J_q^{\delta,\delta})_\mu = 2\,m_q\,(b_0)_\mu\,, \tag{27}$$

$$(J_q^{-\delta,\delta})_\mu = -2\,m_q\,\delta\sqrt{\tau_q}\,(b_\delta)_\mu\ \,. \tag{28}$$

where $\tau_q = Q_q^2/4m_q^2$.

Standard method for calculation QED cross sections

[Galynskii, Kuraev, Bystritskiy, JETP Lett. 88 (2008) 481-486, arXiv: 0805.0233]. In the standard method calculation of the cross section for the process $ep \to ep$ with taken into account the polarization of initial in final protons reduces to determination of product of lepton ($L^{\mu\nu}$) and proton ($W_{\mu\nu}$) tensors

$$\sigma \sim |M_{ep\to ep}|^2 = |\overline{u}(p_2)\gamma^{\mu}u(p_1)\cdot\overline{u}(q_2)\Gamma_{\mu}(q^2)u(q_1)|^2\,, \tag{29}$$

$$\sigma(s_1,s_2) \sim \mathrm{Tr}(\tau_2^e\gamma^{\mu}\tau_1^e\gamma^{\nu})\cdot\mathrm{Tr}(\tau_2^p\,\Gamma_{\mu}\tau_1^p\,\overline{\Gamma}_{\nu})\;= L^{\mu\nu}\,W_{\mu\nu}\,, \tag{30}$$

$$L^{\mu\nu} = 2\cdot\mathrm{Tr}(\tau_2^e\gamma^{\mu}\tau_1^e\gamma^{\nu}),\, W_{\mu\nu} = \mathrm{Tr}(\tau_2^p\,\Gamma_{\mu}\tau_1^p\,\overline{\Gamma}_{\nu})\,, \tag{31}$$

$$\tau_1^e = \frac{1}{2}(\hat{p_1}+m_e),\, \tau_2^e = \frac{1}{2}(\hat{p_2}+m_e)\,, \tag{32}$$

$$\tau_1^p = \frac{1}{2}(\hat{q_1}+M)(1-\delta_1\gamma_5\hat{s}_1)\,, \tau_2^p = \frac{1}{2}(\hat{q_2}+M)(1-\delta_2\gamma_5\hat{s}_2)\,, \tag{33}$$

$$s_1 = -\,\frac{(v_1v_2)v_1 - v_2}{\sqrt{(v_1v_2)^2-1}}\,,\;\; s_2 = \frac{(v_1v_2)v_2 - v_1}{\sqrt{(v_1v_2)^2-1}}\;\;,$$

Lepton tensor $L^{\mu\nu}$ (31) have the standard form

$$L^{\mu\nu} = 2\,(p_1^{\mu}p_2^{\nu} + p_2^{\mu}p_1^{\nu}) + q^2g^{\mu\nu}\,. \tag{34}$$

We note Eq. (34) is valid for both massive and massless cases.

Lepton and proton tensors

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]
Leptonic tensor $L^{\mu\nu}$ (34) in terms $p_+ = p_2 + p_1$ and $p_- = p_2 - p_1$ have a form

$$L^{\mu\nu} = p_+^\mu p_+^\nu + q^2 g^{\mu\nu} \,. \tag{35}$$

Note that for the case of unpolarized leptons (initial and the scattered) the asymmetry part of the proton tensor $W_{\mu\nu}$ (or the imaginary part of it) do not contribute to the cross section of process $ep \to ep$. So for tensors $W_{\mu\nu}$ for the unpolarized leptons we have

$$W_{\mu\nu} \equiv W_{\mu\nu}^{\delta_1\delta_2} = \frac{1+\delta_1\delta_2}{2}\, W_{\mu\nu}^{\delta,\delta} + \frac{1-\delta_1\delta_2}{2}\, W_{\mu\nu}^{-\delta,\delta}, \tag{36}$$

$$W_{\mu\nu}^{\delta,\delta} = \frac{4M^2G_E^2}{q_+^2}\,(q_+)_\mu(q_+)_\nu \,, \tag{37}$$

$$W_{\mu\nu}^{-\delta,\delta} = \frac{4M^2\tau G_M^2}{q_+^2}\,\{(q_+)_\mu(q_+)_\nu - q_+^2 g_{\mu\nu}\} \,. \tag{38}$$

where tensors $W_{\mu\nu}^{\delta,\delta}$ and $W_{\mu\nu}^{-\delta,\delta}$ corresponds to the cases without spin-flip and with spin-flip transition.

DSB cross section in arbitrary reference frame for $ep \longrightarrow ep$

Forming the product of leptonic tensor (35) and the proton one (36) with help of (37) and (38)) we obtain cross section for the elastic process $ep \to ep$ in arbitrary reference frame:

$$\sigma_{s_1,s_2} = \frac{(1+\delta_1\delta_2)}{2} W^{\delta,\delta}_{ep\to ep} + \frac{(1-\delta_1\delta_2)}{2} W^{-\delta,\delta}_{ep\to ep}, \tag{39}$$

$$W^{\delta,\delta}_{ep\to ep} = \frac{4M^2G_E^2}{q_+^2}\left[(p_+q_+)^2 + q_+^2q_-^2\right], \tag{40}$$

$$W^{-\delta,\delta}_{ep\to ep} = \frac{4M^2\tau G_M^2}{q_+^2}\left[(p_+q_+)^2 - q_+^2(q_-^2 + 4m_e^2)\right]. \tag{41}$$

Thus the differential cross section for the $ep \to ep$ process in the DSB (ONLY!) naturally splits into the sum of two terms containing only the squares of the Sachs form factors and corresponding to the contribution of transition without ($\sim G_E^2$) and with ($\sim G_M^2$) proton spin-flip.
The expressions (39), (40) and (41) in a unpolarized case leads to the cross section, which coincides with result for arbitrary reference frame in monography [A.I. Akhiezer and V.B. Berestetsky, Quantum Electrodynamics, Nauka, Moscow, 1969, in Russian]

On the violation of dipole dependence G_E and G_M

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]

Since $|b_0| = 1$ and $|b_\delta b_\delta^*| = 2$ and they are does not depend on Q^2, then from the (25), (26), (27), (28) we can easily obtain the dependence on Q^2 for (absolute) values of the matrix elements of proton currents $J_p^{\pm\delta,\delta}$ and point particles $J_q^{\pm\delta,\delta}$:

$$J_p^{\delta,\delta} \sim 2\,M\,G_E,\; J_p^{-\delta,\delta} \sim 2\,M\,\sqrt{\tau}\,G_M\,, \tag{42}$$

$$J_q^{\delta,\delta} \sim 2\,m_q\,,\, J_q^{-\delta,\delta} \sim 2\,m_q\,\sqrt{\tau_q}\,. \tag{43}$$

Note that the factorization of $2M$ and $2m_q$ in the expressions (42), (43) is caused by the normalization bispinors $\bar{u}_i u_i = 2m_i$. Below during the computation is more convenient to use the normalization of $\bar{u}_i u_i = 1$, and instead of (42), (43) we will use the expressions:

$$J_p^{\delta,\delta} \sim\; G_E, J_p^{-\delta,\delta} \sim\; \sqrt{\tau}\,G_M\,, \tag{44}$$

$$J_q^{\delta,\delta} \sim 1\,,\;\; J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,. \tag{45}$$

Expressions (44), (45) we will be used below to explain the dependence FFs G_E and G_M on Q^2.

On the violation of dipole dependence G_E and G_M

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]

Let us consider the hard scattering mechanism of elastic ep scattering. It is commonly accepted in frames of QCD that in the region $Q^2 \gg 1$ GeV^2 the hard part (kernel) of the proton current (24) can be presented as a summ of contributions where proton is replaced by a set of three almost on mass shell quarks. Each of the relevant Feynman amplitudes contains two gluon Green functions, of order of magnitude $1/Q^2$ and, besides two quark Green functions of order $1/Q$. Below, we will employ that the respective absolute values of the proton current matrix elements $J_p^{\pm\delta,\delta}$ (44) are the product of three point-quark current amplitudes $J_q^{\pm\delta,\delta}$ (45) divided by Q^6

$$J_p^{\pm\delta,\delta} \sim J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} / Q^6 \,. \tag{46}$$

It is necessary to note that representation (46) is valid in the region $Q^2 \gg 1$ GeV^2. Below we will suppose the masses of quarks m_q to be equal to $1/3$ of the proton mass M and the fraction of the transfer momenta of them to be equal. So we have

$$\tau_q = \tau \,. \tag{47}$$

On the violation of dipole dependence G_E and G_M

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]

There are two possibilities for a proton non-spin-flip transition: (i) none of the three quarks undergoes a spin-flip transition, and (ii) two quarks undergo a spin-flip transition, while the third does not. We denote the number of such ways as $n_q^{\delta,\delta} = [0,2]$ in accordance with the number of quarks involved in a spin-flip process (none or two). Proton spin-flip can also proceed in two ways: (i) one quark undergoes a spin-flip transition, while the other two do not, and (ii) all three quarks undergo a spin-flip transition. We denote the number of such ways by $n_q^{-\delta,\delta} = [1,3]$ in accordance with the number of quarks involved in a spin-flip process (one or three). Thus, there are in all four combinations to be considered:

$$n_q^{\delta,\delta} \times n_q^{-\delta,\delta} = (0,1) + (0,3) + (2,1) + (2,3)\,. \tag{48}$$

Of these, the fourth, (2,3), corresponds to the dipole dependence of the form factors G_E and G_M on Q^2, in which case two of the quarks reverses a spin upon the proton non-spin-flip transition (the first number in parentheses is two); at the same time, the proton spin-flip is due to the spin-flip for all three quarks (the second number in parentheses is equal to three). We obtain $G_E/G_M \sim 1$ for the (0,1) and (2,3) sets in (48), $Q^2 G_E/G_M \sim 4\,M^2$ for the (0,3) set, and $Q^2 G_M/G_E \sim 4\,M^2$ for the (2,1) set.

The set (0,1), $G_E/G_M \sim 1$, but $G_E, G_M \sim 1/Q^6$

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]

Let us consider the first (0,1) set. We use for the amplitudes of protons and point-like quarks currents expressions:

$$J_p^{\delta,\delta} \sim G_E, J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \,, \tag{49}$$

$$J_q^{\delta,\delta} \sim 1 \,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau} \,. \tag{50}$$

It is convenient to represent this case in the form of the following diagrams:

$$J_p^{\delta,\delta} \sim G_E \quad \sim \quad \begin{array}{ccc} + & \to\to * \to\to\to\to & + \\ - & \to\to\to * \to\to\to & - \\ + & \to\to\to\to * \to\to & + \end{array} \quad \text{non spin-flip,} \tag{51}$$

$$J_p^{\delta,\delta} \sim G_E \quad \sim \quad 1 \times 1 \times 1 \times \frac{1}{Q^6} \,, \quad \Rightarrow \quad G_E \sim \frac{1}{Q^6} \,, \tag{52}$$

where the factors "$1''$ correspond to transitions without spin-flip [see Eq. (50)] of three point quarks and Q^6 in the denominator arises because of two gluon and two quark propagators.

The set (0,1), $G_E/G_M \sim 1$, but $G_E, G_M \sim 1/Q^6$

$$J_p^{\delta,\delta} \sim G_E, \quad J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M\,,$$
$$J_q^{\delta,\delta} \sim 1\,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau}\,.$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \quad \sim \quad \begin{array}{ccc} + & \rightarrow\rightarrow * \rightarrow\rightarrow\rightarrow\rightarrow & - \\ - & \rightarrow\rightarrow\rightarrow * \rightarrow\rightarrow\rightarrow & - \\ + & \rightarrow\rightarrow\rightarrow\rightarrow * \rightarrow\rightarrow & + \end{array} \quad \text{spin-flip}\,. \tag{53}$$

The diagram (53) corresponds to a transition in which the spin of the top quark is flipped, while the bottom two is not, that generally corresponds to the transition of a proton with spin flip. As a result, we have

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times 1 \times 1 \times \frac{1}{Q^6} \Rightarrow G_M \sim \frac{1}{Q^6}\,. \tag{54}$$

$$G_E \sim \frac{1}{Q^6}, G_M \sim \frac{1}{Q^6}\,, \frac{G_E}{G_M} \sim 1\,. \tag{55}$$

Therefore, the form factor ratio G_E/G_M behaves in just the same way as in the dipole model. However, the dependence $G_E \sim 1/(Q^6)$ and the dependence $G_M \sim 1/(Q^6)$ are not of the dipole character (the dipole dependence correspond to $G_E \sim 1/Q^4$ and $G_M \sim 1/Q^4$).

The set (0,3), dependence $G_E/G_M \sim 4M^2/Q^2$

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]
Let us consider the (0,3) set, in which case spin-flip transitions for all three quarks contribute to $J_p^{-\delta,\delta}$. For this purpose we write equalities similar to (52) and (54); that is,

$$J_p^{\delta,\delta} \sim G_E \sim 1 \times 1 \times 1 \, \times \frac{1}{Q^6} \,, \tag{56}$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \, \times \frac{1}{Q^6} \,. \tag{57}$$

From here, we obtain

$$G_E \sim \frac{1}{Q^6} \,, \;\; G_M \sim \frac{\tau}{Q^6} \,, \; \frac{G_E}{G_M} \sim \frac{1}{\tau} \sim \frac{4M^2}{Q^2} \,, \tag{58}$$

$$Q^2 \, \frac{G_E}{G_M} \sim 4M^2 = \text{const} \,. \tag{59}$$

Note that the relation (59) is sometimes called in the literature as the Brodsky saturation law. Obviously really it correspond to the maximal possible number of the quarks spin-flip transition in which case of the proton transition with spin-flip.

The set (2,1), dependence $G_E/G_M \sim Q^2/4M^2$

[M.Galynskii, E.Kuraev, arXiv:1210.0634 [nucl-th]]
Let us now consider the (2,1) spin combination in the set (48). It is generated by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for only one quark in the case of the contribution to $J_p^{-\delta,\delta}$. Following the same line of reasoning as above, one can readily show that, for the (2,1) set, G_E and G_M have the form

$$G_E \sim \frac{\tau}{Q^6} \sim \frac{1}{Q^4}\;,\; G_M \sim \frac{1}{Q^6}\;, \tag{60}$$

that is, the ratio G_E/G_M behaves as

$$\frac{G_E}{G_M} \sim \tau \sim \frac{Q^2}{4M^2}\;,\; Q^2\,\frac{G_M}{G_E} \sim 4M^2 = \text{const}\,. \tag{61}$$

The set (2,3), $G_E/G_M \sim 1$, dipole dependence

Let us consider the (2,3) spin combination in the set (48). It is generated by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for all three quarks in the case of the contribution to $J_p^{-\delta,\delta}$. In the case being considered, we have

$$J_p^{\delta,\delta} \sim G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1 \times \frac{1}{Q^6}\,, \tag{62}$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \times \frac{1}{Q^6}\,. \tag{63}$$

Whence we obtain

$$G_E \sim \frac{\tau}{Q^6},\;\; G_M \sim \frac{\tau}{Q^6}\,, G_E \sim \frac{1}{Q^4},\; G_M \sim \frac{1}{Q^4},\; \frac{G_E}{G_M} \sim 1\,. \tag{64}$$

Thus, the dipole dependence in the behavior of the form factors arises owing to the contribution by spin-flip transitions for two quarks in the case of the contribution to $J_p^{\delta,\delta}$ and by spin-flip transitions for all three quarks in the case of the contribution to $J_p^{-\delta,\delta}$. The dipole dependence can be realized at high Q^2 in the case when the quark spin-flip transitions become dominant. In other words it take place for the case when the number of quark transitions with spin-flip is maximal, i.e. the saturation take place.

The spin parametrization for the ratio G_E/G_M.

The non-spin-flip and spin-flip proton-current amplitudes ($J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$) can be represented as the linear combinations

$$J_p^{\delta,\delta} = \alpha_0\, J_q^{\delta,\delta} J_q^{-\delta,-\delta} J_q^{\delta,\delta} + \alpha_2\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{\delta,\delta}, \tag{65}$$

$$J_p^{-\delta,\delta} = \beta_1 J_q^{-\delta,\delta} J_q^{\delta,\delta} J_q^{-\delta,-\delta} + \beta_3\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{-\delta,\delta}, \tag{66}$$

where the coefficients $\alpha_0, \alpha_2\;\; \beta_1, \beta_3$ determine the number of quarks with spin-flip, contributing to the proton transitions without and with spin-flip. With help of (65), (66) we get a general expression for the ratio G_E/G_M:

$$\frac{G_E}{G_M} = \frac{\alpha_0\; + \alpha_2\, \tau}{\beta_1\; + \beta_3\, \tau}\;. \tag{67}$$

It can be the basis for the spin parametrization and fitting of experimental data on measurement of the ratio G_E/G_M. Because the set (0,1) where $G_E/G_M \sim 1$ is realized at small τ, the coefficients α_0 and β_1 in Eq. (67) must obviously be close to unity: $\alpha_0 \sim 1$ and $\beta_1 \sim 1$. With allowance for this comment, we expand the right-hand side of (67) in a power series for $\tau < 1$. As a result, we arrive at the law of a linear decrease in the ratio G_E/G_M as Q^2 increases;

$$\frac{G_E}{G_M} \sim 1 - \frac{(\beta_3 - \alpha_2)}{4M^2}\; Q^2\;. \tag{68}$$

Polarization transfer experiments JLab data for G_E^p/G_M^p

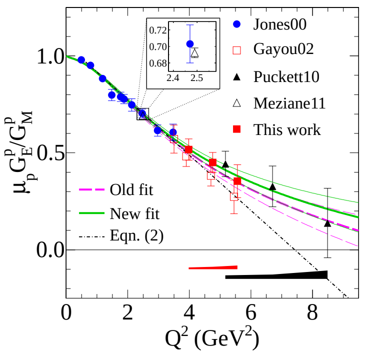

Figure : Polarization transfer data for G_E^p/G_M^p from Jones00, Gayou02, Puckett10, Meziane11 and work Puckett12 (red squares). Curves are global proton form factor fits using the originally published GEp-II data Gayou02 (Old fit) and work Puckett12 (New fit). Both fits include the GEp-III data. The linear fit is shown for comparison.

We take this figure from paper A. Puckett et al., Phys. Rev. **C85**, (2012) 045203.

Conclusion

In this work we discuss questions related to the interpretation of unexpected results of measurements of the proton form factors ratio G_E/G_M in the high-precision double polarization experiments done in JLab in the region of $0.5 \le Q^2 \le 8.5$ GeV^2. For this purpose, in the case of the hard scattering mechanism we calculated the hard kernel of the proton current matrix elements $J_p^{\pm\delta,\delta}$ for the full set of spin combinations corresponding to the number of the spin-flipped quarks, which contribute to the proton transition without spin-flip ($J_p^{\delta,\delta}$) and with the spin-flip ($J_p^{-\delta,\delta}$). This set is: (0,1), (0,3), (2,1), (2,3), where the first number in parentheses is the number of the spin-flipped quarks, which contribute to the $J_p^{\delta,\delta}$, and the second one is the number of the spin-flipped quarks which contribute to the $J_p^{-\delta,\delta}$. For the sets of (0,1) and (2,3), we found that the ratio $G_E/G_M \sim 1$, and the form factors G_E and G_M behave for the set of (0,1) as $G_E, G_M \sim 1/Q^6$, and for the set of (2,3) as $G_E, G_M \sim 1/Q^4$. At the same time the set of (0,1) is realized for $\tau \ll 1$, and the set (2,3) for $\tau \gg 1$ ($\tau = Q^2/4M^2$).

Conclusion

This allows us to suppose that:
1) at the lower boundary of the experimental measurements of the ratio G_E/G_M not dipole dependence appears but the law of $G_E, G_M \sim 1/Q^6$;
2) the conditions for the observation of the dipole dependence in the experiments has not yet been achieved;
3) since for quarks $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau}$, then the dipole dependence is realized when $\tau \gg 1$ in the case when the quark transitions with spin-flip are dominate;
4) the law of the linear decrease of G_E/G_M at $\tau < 1$ is due to additional contributions to the $J_p^{\delta,\delta}$ by spin-flip transitions of two quarks and an additional contribution to $J_p^{-\delta,\delta}$ by spin-flip transitions of three quarks.
5) The fundamental physical meaning of electric and magnetic FFs of G_E and G_M consists in that they define the matrix elements of the proton current in the case of transitions without ($J_p^{\delta,\delta}$) and with spin-flip ($J_p^{-\delta,\delta}$) of the proton, respectively. They have already factored in the amplitudes $J_p^{\delta,\delta}$, $J_p^{-\delta,\delta}$ of the proton current.
6) We believe that the presented above interpretation can be considered as a possible solution of the G_E/G_M problem. One of our predictions is the realization (restoration) of a dipole dependence of form factors and the value $R = 1$ for higher values of Q^2 (at $Q^2 \gg 4m^2$).

Thanks

THANK FOR YOUR ATTENTION