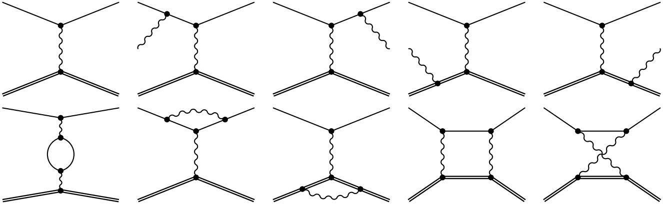
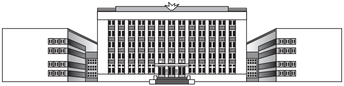
Radiative corrections to elastic *ep* scattering: application to experiments

Alexander Gramolin
Budker Institute of Nuclear Physics, Novosibirsk, Russia

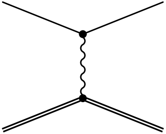


Workshop "Scattering and annihilation electromagnetic processes"



Elastic electron-proton scattering

Differential cross section for elastic *ep* scattering is given by the Rosenbluth formula:



$$\frac{d\sigma_{\text{Ros}}}{d\Omega_\ell} = \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\,\mathrm{tg}^2\,\frac{\theta_\ell}{2}\right]\frac{d\sigma_{\text{Mott}}}{d\Omega_\ell},$$

$$\frac{d\sigma_{\text{Mott}}}{d\Omega_\ell} = \frac{\alpha^2}{4E_\ell^2}\,\frac{\cos^2(\theta_\ell/2)}{\sin^4\theta_\ell/2}\,\frac{E'_\ell}{E_\ell},$$

where $\tau = Q^2/(4M^2)$, $Q^2 = 2M(E_\ell - E'_\ell)$, $d\sigma_{\text{Mott}}/d\Omega_\ell$ — Mott cross section, $G_E(Q^2)$ and $G_M(Q^2)$ — electric and magnetic form factors of the proton.

G_E and G_M are functions of the 4-momentum transfer squared (Q^2) only and describe the distributions of charge and magnetic moment inside the proton.

Introducing the variable ε (virtual photon polarization)

$$\varepsilon = \left[1 + 2(1+\tau)\,\mathrm{tg}^2\,\frac{\theta_\ell}{2}\right]^{-1},$$

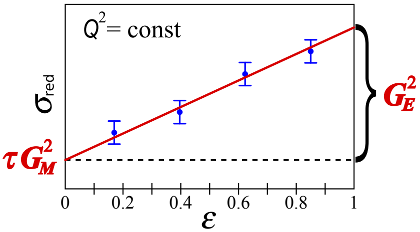
the Rosenbluth formula can be written as follows:

$$\frac{d\sigma_{\text{Ros}}}{d\Omega_\ell} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right] = \frac{\sigma_{\text{red}}}{\varepsilon(1+\tau)},$$

where σ_{red} (reduced cross section) is a linear function of ε if $Q^2 = \text{const}$.

The proton's form factors, two methods of measuring

$$\sigma_{\text{red}} = \varepsilon(1+\tau)\frac{d\sigma}{d\Omega_\ell} = \varepsilon G_E^2 + \tau G_M^2$$



Rosenbluth method

It consists in measuring of $d\sigma/d\Omega_\ell$ for fixed Q^2, but with different E_ℓ, θ_ℓ.
$\Rightarrow$ Dipole formula for G_E and G_M:

$$G_E(Q^2) \approx \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2},$$

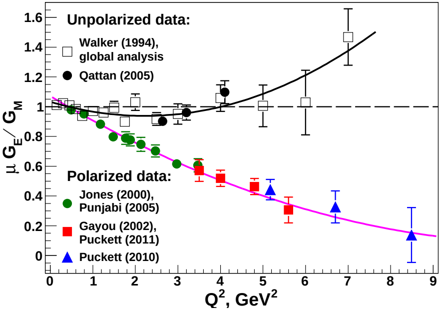
$$G_M(Q^2) \approx \mu G_E(Q^2).$$

Polarization transfer method

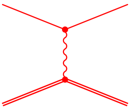
(Akhiezer and Rekalo, 1968)

The ratio G_E/G_M is proportional to the ratio of transverse P_T and longitudinal P_L polarization components of the recoil proton in reaction $\vec{e}p \to e'\vec{p}'$:

$$\frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{E_\ell + E'_\ell}{2M}\,\text{tg}\,\frac{\theta}{2}.$$

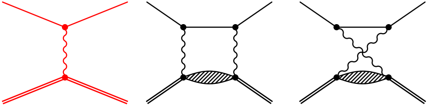


Born cross section and radiative corrections of order α



$\mathcal{M}_{\text{Born}}$

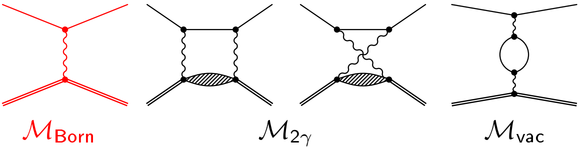
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2$$



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right)$$

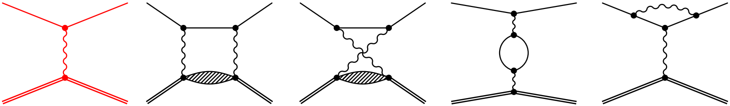
Born cross section and radiative corrections of order α



$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\, 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right)$$

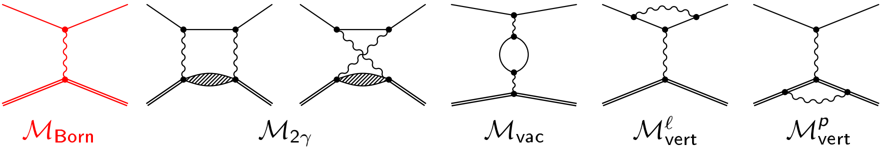
Born cross section and radiative corrections of order α



$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\mathrm{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$

$$+ 2\,\mathrm{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\mathrm{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right)$$

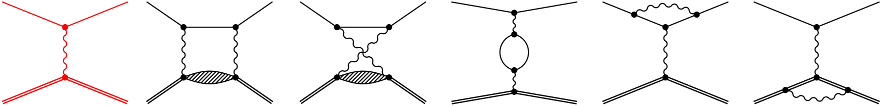
Born cross section and radiative corrections of order α



$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$
$$+\,2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right)$$

Born cross section and radiative corrections of order α

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + \\ + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right)$$

Born cross section and radiative corrections of order α

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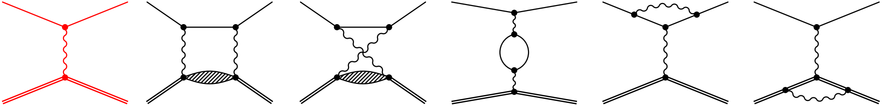
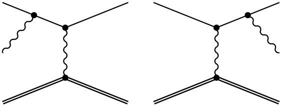


$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

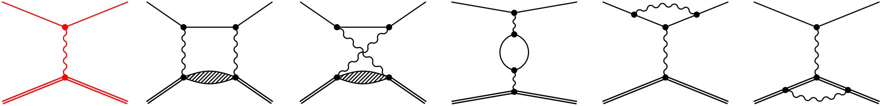
Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brems}}$

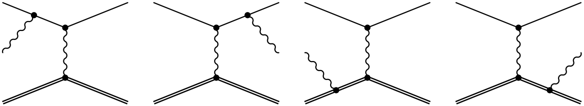
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + \\ + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) + \\ + |\mathcal{M}^{\ell}_{\text{brems}}|^2$$

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brems}}$ $\mathcal{M}^{p}_{\text{brems}}$

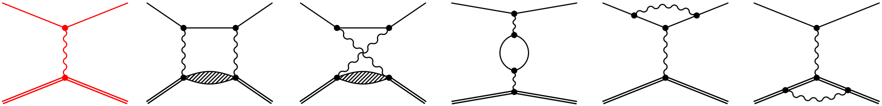
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\,2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+\,|\mathcal{M}^{\ell}_{\text{brems}}|^2 + |\mathcal{M}^{p}_{\text{brems}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brems}}\mathcal{M}^{p}_{\text{brems}}\right)$$

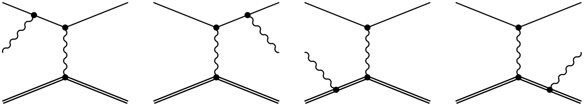
Born cross section and radiative corrections of order α

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brems}}$ $\mathcal{M}^{p}_{\text{brems}}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$

$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+ |\mathcal{M}^{\ell}_{\text{brems}}|^2 + |\mathcal{M}^{p}_{\text{brems}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brems}}\mathcal{M}^{p}_{\text{brems}}\right)$$

✓ Cancellation of infrared divergences (corresponding terms are marked in color)

✓ Some of the terms are of different signs ("$\pm$") for $e^{+}p$ and $e^{-}p$ scattering

Asymmetry A and ratio R for the cross sections

$$\boxed{A = \frac{\sigma(e^+p) - \sigma(e^-p)}{\sigma(e^+p) + \sigma(e^-p)}} \qquad \boxed{R = \frac{\sigma(e^+p)}{\sigma(e^-p)}}$$

How are they related?

$$A = \frac{R-1}{R+1} \approx \frac{R-1}{2} \qquad R = \frac{1+A}{1-A} \approx 1 + 2A$$

After taking into account the radiative corrections:

$$A \approx 2\frac{\mathrm{Re}\left(\mathcal{M}^{\dagger}_{\mathrm{Born}}\mathcal{M}_{2\gamma}\right)}{|\mathcal{M}_{\mathrm{Born}}|^2} \qquad R \approx 1 + 4\frac{\mathrm{Re}\left(\mathcal{M}^{\dagger}_{\mathrm{Born}}\mathcal{M}_{2\gamma}\right)}{|\mathcal{M}_{\mathrm{Born}}|^2}$$

How to take into account the radiative corrections?

$$A = A_{\mathrm{exp}} - A_{\mathrm{MC}}$$

(exp = experimental,

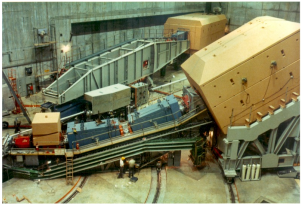
MC = Monte Carlo)

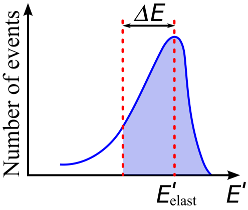
$$R = \frac{R_{\mathrm{exp}}R_{\mathrm{MC}} + 3R_{\mathrm{exp}} - R_{\mathrm{MC}} + 1}{R_{\mathrm{exp}}R_{\mathrm{MC}} - R_{\mathrm{exp}} + 3R_{\mathrm{MC}} + 1}$$

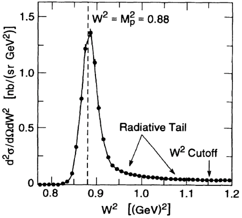
$$R \approx R_{\mathrm{exp}} - R_{\mathrm{MC}} + 1$$

The asymmetry is more natural, but the ratio is used more often.

Radiative corrections in single arm experiments







To select elastic events the following condition is commonly used:

$$E' > E'_{\rm elast}(E,\theta_\ell) - \Delta E, \text{ where } E'_{\rm elast}(E,\theta_\ell) = \frac{ME}{M + E(1-\cos\theta_\ell)}.$$

Or another condition:

$$W^2 < W^2_{\rm cut}, \text{ where } W^2 = M^2 + 2M(E-E') - 4EE'\sin^2\frac{\theta_\ell}{2},$$

and W^2 is the missing mass squared. Typically used value is $W^2_{\rm cut} = 1.1 \div 1.15$ GeV^2 (and $W^2 = M^2 = 0.88$ GeV^2 in the case of purely elastic scattering). It is easy to express ΔE through $W^2_{\rm cut}$:

$$\Delta E = \frac{W^2_{\rm cut} - M^2}{2M + 4E\sin^2(\theta_\ell/2)}.$$

Radiative corrections in single arm experiments

When only electron is detected (so we are measuring its energy E' and scattering angle θ_ℓ) the procedure of elastic scattering event selection can be described with a single parameter ΔE. Then the following simple formula is used for taking into account the radiative corrections:

$$\boxed{\frac{d\sigma_{\text{exp}}}{d\Omega_\ell} = \left[1 + \delta_{\text{virt}} + \delta_{\text{brems}}(\Delta E)\right] \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell}}.$$

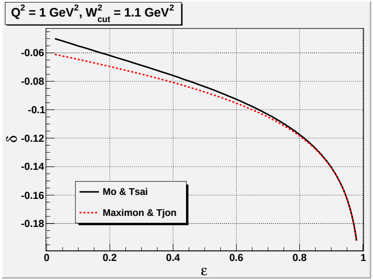
In this case, integration over kinematic parameters of recoil proton and bremsstrahlung photon can be done analytically (using some approximations). Then theorists write in their papers simple formulas for experimentalists to calculate δ_{virt} and δ_{brems} from E, E', θ_ℓ and ΔE.

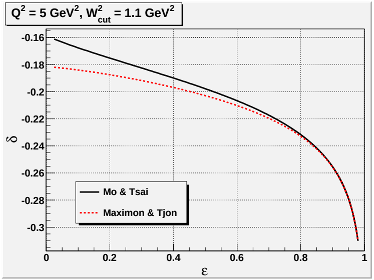
The main theoretical works about radiative corrections for such experiments:

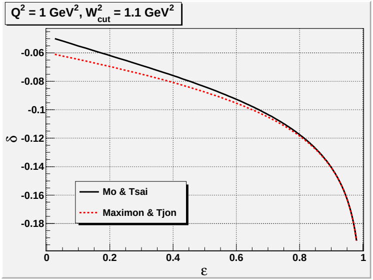
- *Yung-Su Tsai,* Phys. Rev. **122** (1961) 1898.
- *L. W. Mo, Y. S. Tsai,* Rev. Mod. Phys. **41** (1969) 205.
- *L. C. Maximon, J. A. Tjon,* Phys. Rev. C **62** (2000) 054320.

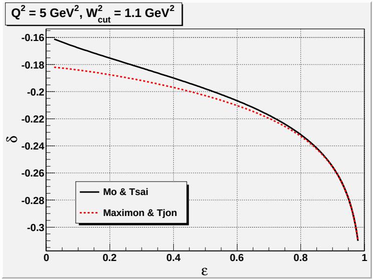
Formulas of Mo & Tsai was a standart recipe for taking into account RC during a few decades (and are still in use)!

Comparison between Mo–Tsai and Maximon–Tjon

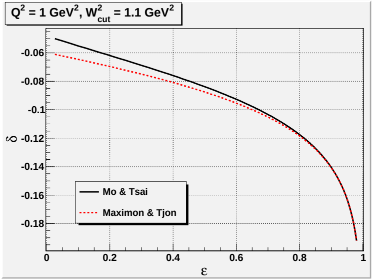


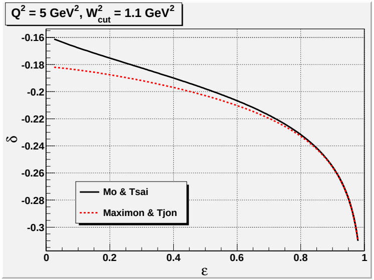






$$\sigma_{\mathsf{red}} \frac{1+\delta_{\mathsf{MTs}}}{1+\delta_{\mathsf{MTj}}} :$$





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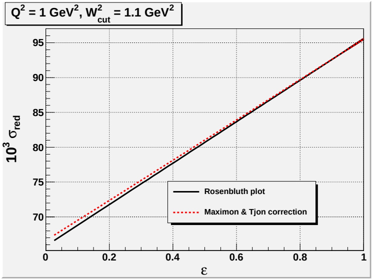

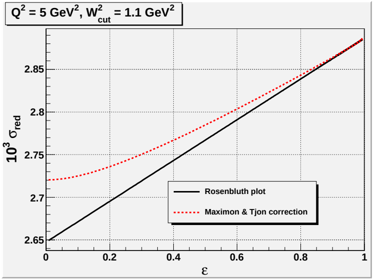

Comparison between Mo–Tsai and Maximon–Tjon

$$\delta' = \delta_{\mathrm{IR}}(\mathrm{MTj}) - \delta_{\mathrm{IR}}(\mathrm{MoT}) =$$

$$= -\frac{\alpha}{\pi}\left[\ln\left(\frac{E}{E'}\right)\ln\left(\frac{Q^4}{4M^2EE'}\right) + 2\Phi\left(1-\frac{M}{2E}\right) - 2\Phi\left(1-\frac{M}{2E'}\right)\right],$$

$$\Phi(x) = -\int\limits_0^x \frac{\ln|1-x|}{x}dx.$$

Reference: *J. Arrington, et al.* Prog. Part. Nucl. Phys. **66** (2011) 782.

Comparison between Mo–Tsai and Maximon–Tjon

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$$\sigma_{\mathsf{red}}\frac{1+\delta_{\mathsf{MoT}}}{1+\delta_{\mathsf{MTj}}-\delta'}:$$

Comparison between Mo–Tsai and Maximon–Tjon

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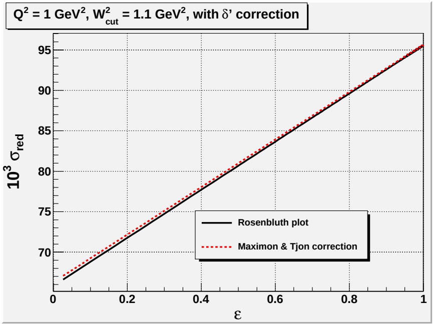
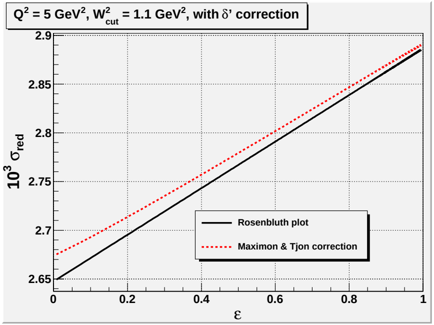




✓ In general, magnitude of radiative corrections depends on the type of detector (magnetic or not), detector geometry, its spatial and energy resolutions and the kinematic cuts used in event selection. It is impossible to consider all these factors using only single parameter ΔE. So we need to have an event generator and to conduct Monte Carlo simulation of the detector.

✓ It is important also, that this approach allows us for bremsstrahlung replace analytical integration over kinematic variables of electron, proton and photon on numerical. And numerical integration allows us avoid the use of soft photon approximation and other simplifications. We need an analytical integration only in the kinematic region where photons are very soft (this is necessary to separate infrared divergent terms).

✓ Another advantage is an opportunity to take into account some complex processes, for example bremsstrahlung with the delta-isobar $\Delta(1232)$ excitation. We need to know only the square of the amplitude of the process. Analytical integration is not required. Modern computer algebra packages can be used for the calculation of amplitudes (for example, `Mathematica` + `FeynCalc`).

A new event generator

A new general-purpose event generator ($\ell p \to \ell' p'$ and $\ell p \to \ell' p' \gamma$), called ESEPP (Elastic Scattering of Electrons and Positrons by Protons), has been developed for Monte Carlo simulation of unpolarized elastic scattering of electrons and positrons (as well as muons and antimuons) on hydrogen target.

The source code of the generator is freely available (under the GNU GPL license) on the page http://www.inp.nsk.su/~gramolin/esepp/.

The main advantages of ESEPP:

✓ Four types of incident particles are possible: e^-, e^+, μ^-, μ^+;

✓ All the kinematic parameters of the final particles are known $\Rightarrow$ ESEPP is a general-purpose generator;

✓ We use an accurate calculation for the first-order bremsstrahlung instead of the usual soft-photon approximation;

✓ Not only the lepton bremsstrahlung is considered, but also the proton bremsstrahlung and the interference term;

✓ We do not use the ultrarelativistic approximation $m \ll Q^2$ (except for the lepton vertex correction); it should be important for experiments with muons (PSI) and for measurements with extremely small values of Q^2 (JLab).

The main disadvantage:

✓ Only the first-order bremsstrahlung is taken into account.

ESEPP event generator: events $\ell p \to \ell' p'$

Events of two different types are generated: without a photon in the final state (when $E_\gamma < E_\gamma^{\text{cut}} \approx 1$ MeV) and with all three particles in the final states.

$$\left.\frac{d\sigma_{\text{elast}}}{d\Omega_\ell} + \frac{d\sigma_{\text{brems}}}{d\Omega_\ell}\right|_{E_\gamma < E_\gamma^{\text{cut}}} = \left(1 + \delta_{\text{virt}} + \delta_{\text{brems}}\right) \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell},$$

$$\delta_{\text{virt}} = \delta_{\text{vac}} + \delta_{\text{vert}} - z\,\delta', \quad \delta_{\text{brems}} = \delta^{\ell\ell}_{\text{brems}} + \delta^{pp}_{\text{brems}} - z\,\delta^{\ell p}_{\text{brems}},$$

$$\delta_{\text{vac}} = 2\,\text{Re}\,\mathcal{P}(q^2),$$

$$\delta_{\text{vert}} = \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{-q^2}{m^2} - 2\right),$$

$$\delta^{\ell\ell}_{\text{brems}} = -2\alpha\left[\widetilde{B}(\ell, \ell, E_\gamma^{\text{cut}}) - 2\widetilde{B}(\ell, \ell', E_\gamma^{\text{cut}}) + \widetilde{B}(\ell', \ell', E_\gamma^{\text{cut}})\right],$$

$$\delta^{pp}_{\text{brems}} = -2\alpha\left[\widetilde{B}(p, p, E_\gamma^{\text{cut}}) - 2\widetilde{B}(p, p', E_\gamma^{\text{cut}}) + \widetilde{B}(p', p', E_\gamma^{\text{cut}})\right],$$

$$\delta^{\ell p}_{\text{brems}} = 4\alpha\left[\widetilde{B}(\ell, p, E_\gamma^{\text{cut}}) - \widetilde{B}(\ell, p', E_\gamma^{\text{cut}}) - \widetilde{B}(\ell', p, E_\gamma^{\text{cut}}) + \widetilde{B}(\ell', p', E_\gamma^{\text{cut}})\right],$$

where $z = -1$ for e^-, μ^-; and $z = 1$ for e^+, μ^+.

Reference: `http://cmd.inp.nsk.su/~ignatov/vpl/` (vacuum polarization)

$$\widetilde{B}\left(p_i, p_j, E_\gamma^{\text{cut}}\right) = \frac{p_i \cdot p_j}{4\pi} \int_0^1 \frac{dx}{p_x^2} \left(\ln \frac{4\left(E_\gamma^{\text{cut}}\right)^2}{p_x^2} + \frac{p_x^0}{|\mathbf{p}_x|} \ln \frac{p_x^0 - |\mathbf{p}_x|}{p_x^0 + |\mathbf{p}_x|} \right),$$

where we have introduced the four-momentum p_x, which is equal to

$$p_x = \left(p_x^0,\, \mathbf{p}_x\right) = x p_i + (1-x) p_j.$$

The soft photon approximation was used for the integration. In this approximation the differential cross section of the process $\ell p \to \ell' p' \gamma$ is expressed through the Rosenbluth cross section $d\sigma_{\text{Ros}}/d\Omega_\ell$ by the following simple formula:

$$\frac{d\sigma_{\text{brems}}}{dE_\gamma\, d\Omega_\gamma\, d\Omega_\ell} = -\frac{\alpha E_\gamma}{4\pi^2} \left[z\, \frac{\ell}{k \cdot \ell} - z\, \frac{\ell'}{k \cdot \ell'} + \frac{p}{k \cdot p} - \frac{p'}{k \cdot p'} \right]^2 \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell}.$$

Reference: *R. Ent, et al.*, Phys. Rev. C **64** (2001) 054610.

ESEPP event generator: events $\ell p \to \ell' p' \gamma$

The differential cross section of the process $\ell p \to \ell' p' \gamma$ is expressed in terms of the square of the amplitude $\left|\mathcal{M}_{\text{brems}}\right|^2$ as follows:

$$\frac{d\sigma_{\text{brems}}}{dE_\gamma\, d\Omega_\gamma\, d\Omega_\ell} = \frac{1}{(2\pi)^5}\,\frac{1}{32I}$$

$$\times\ \frac{E_\gamma\left(E_\ell'^{\,2} - m^2\right)}{\left|E_\ell'\left(E_\gamma\cos\psi - \sqrt{E_\ell^2 - m^2}\cos\theta_\ell\right) + (E_\ell + M - E_\gamma)\sqrt{E_\ell'^{\,2} - m^2}\right|}\left|\mathcal{M}_{\text{brems}}\right|^2,$$

where

$$I = \sqrt{(\ell\cdot p)^2 - m^2M^2},$$

$$\left|\mathcal{M}_{\text{brems}}\right|^2 = \mathcal{M}^{\ell}_{\text{brems}} + \mathcal{M}^{p}_{\text{brems}} - z\mathcal{M}^{\ell p}_{\text{brems}}.$$

Expressions for the terms $\mathcal{M}^{\ell}_{\text{brems}}$, $\mathcal{M}^{p}_{\text{brems}}$ and $\mathcal{M}^{\ell p}_{\text{brems}}$ can be easily obtained in the framework of QED without using the soft-photon approximation.

Lepton bremsstrahlung term

$$\mathcal{M}^{\ell}_{\text{brems}} = \frac{e^6}{q_1^4}\left(\mathcal{L}_{1\mu\nu} + \mathcal{L}_{2\mu\nu}\right)\mathcal{P}^{\mu\nu},$$

where

$$\begin{aligned}
\mathcal{L}_{1\mu\nu} &= \frac{1}{2}\,\mathrm{tr}\left[(\not{\ell}' + m)\gamma^{\alpha}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\mu}(\not{\ell} + m)\gamma_{\alpha}\frac{\not{\ell} - \not{k} + m}{2(k\cdot\ell)}\gamma_{\nu}\right] \\
&- \frac{1}{2}\,\mathrm{tr}\left[(\not{\ell}' + m)\gamma^{\alpha}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\mu}(\not{\ell} + m)\gamma_{\nu}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\alpha}\right], \\
\mathcal{P}^{\mu\nu} &= \frac{1}{2}\,\mathrm{tr}\left[(\not{p} + M)\left\{\left(F_1(q_1^2) + F_2(q_1^2)\right)\gamma^{\nu} - \frac{F_2(q_1^2)}{2M}P^{\nu}\right\}\right. \\
&\left.(\not{p}' + M)\left\{\left(F_1(q_1^2) + F_2(q_1^2)\right)\gamma^{\mu} - \frac{F_2(q_1^2)}{2M}P^{\mu}\right\}\right],
\end{aligned}$$

and an expression for the tensor $\mathcal{L}_{2\mu\nu}$ is obtained from the expression for $\mathcal{L}_{1\mu\nu}$ after changing $\ell \leftrightarrow -\ell'$.

Squares of the four-momenta transferred to the proton:

$$q_1^2 = \left(p' - p\right)^2 = 2M\left(E'_{\ell} - E_{\ell} + E_{\gamma}\right),$$

$$q_2^2 = \left(\ell - \ell'\right)^2 = 2\sqrt{\left(E_{\ell}^2 - m^2\right)\left(E_{\ell}'^2 - m^2\right)}\,\cos\theta_{\ell} - 2E_{\ell}E'_{\ell} + 2m^2.$$

Proton bremsstrahlung term 1

$$\mathcal{M}^p_{\text{brems}} = \frac{e^6}{q_2^4}\,\mathcal{L}_{\mu\nu}\left(\mathcal{P}_1^{\mu\nu} + \mathcal{P}_2^{\mu\nu}\right),$$

where

$$\mathcal{L}_{\mu\nu} = \frac{1}{2}\,\mathrm{tr}\left[(\not{\ell} + m)\gamma_\nu(\not{\ell}' + m)\gamma_\mu\right],$$

$$\begin{aligned}
\mathcal{P}_1^{\mu\nu} = \frac{1}{2}\,\mathrm{tr}\Big[(\not{p}' + M)\Big\{&(F_1(0) + F_2(0))\gamma^\alpha \\
&- \frac{F_2(0)}{2M}\left[(2p' + k)^\alpha - \gamma^\alpha(\not{p}' + \not{k} - M)\right]\Big\}\frac{\not{p}' + \not{k} + M}{2(k\cdot p')}
\end{aligned}$$

$$\Big\{(F_1(q_2^2) + F_2(q_2^2))\gamma^\mu - \frac{F_2(q_2^2)}{2M}\left[P_+^\mu - (\not{p}' + \not{k} - M)\gamma^\mu\right]\Big\}(\not{p} + M)$$

$$\Big\{(F_1(0) + F_2(0))\gamma^\alpha - \frac{F_2(0)}{2M}\left[(2p - k)^\alpha - \gamma^\alpha(\not{p} - \not{k} - M)\right]\Big\}\frac{\not{p} - \not{k} + M}{2(k\cdot p)}$$

$$\Big\{(F_1(q_2^2) + F_2(q_2^2))\gamma^\nu - \frac{F_2(q_2^2)}{2M}\left[P_-^\nu - (\not{p} - \not{k} - M)\gamma^\nu\right]\Big\}\Big]$$

Proton bremsstrahlung term 2

$$
\begin{aligned}
&-\frac{1}{2}\,\mathrm{tr}\Big[(\not p' + M)\Big\{(F_1(0)+F_2(0))\gamma^\alpha \\
&-\frac{F_2(0)}{2M}\left[(2p'+k)^\alpha - \gamma^\alpha(\not p' + \not k - M)\right]\Big\}\frac{\not p' + \not k + M}{2(k\cdot p')} \\
&\Big\{(F_1(q_2^2)+F_2(q_2^2))\gamma^\mu - \frac{F_2(q_2^2)}{2M}\left[P_+^\mu - (\not p' + \not k - M)\gamma^\mu\right]\Big\}(\not p + M) \\
&\Big\{(F_1(q_2^2)+F_2(q_2^2))\gamma^\nu - \frac{F_2(q_2^2)}{2M}\left[P_+^\nu - \gamma^\nu(\not p' + \not k - M)\right]\Big\}\frac{\not p' + \not k + M}{2(k\cdot p')} \\
&\Big\{(F_1(0)+F_2(0))\gamma^\alpha - \frac{F_2(0)}{2M}\left[(2p'+k)^\alpha - (\not p' + \not k - M)\gamma^\alpha\right]\Big\}\Big],
\end{aligned}
$$

and an expression for the tensor $\mathcal{P}_2^{\mu\nu}$ is obtained from the expression for $\mathcal{P}_1^{\mu\nu}$ after changing $p \leftrightarrow -p'$.
Some notations:

$$
P = p + p', \quad P_+ = p + p' + k, \quad P_- = p + p' - k.
$$

Interference term

$$\mathcal{M}^{\ell p}_{\text{brems}} = \frac{e^6}{q_1^2 q_2^2}\left(\frac{1}{k\cdot\ell'}\,\mathcal{L}^{\alpha}_{1\mu\nu} + \frac{1}{k\cdot\ell}\,\mathcal{L}^{\alpha}_{2\mu\nu}\right)\left(\mathcal{P}^{\mu\nu}_{1\alpha} - \mathcal{P}^{\mu\nu}_{2\alpha}\right),$$

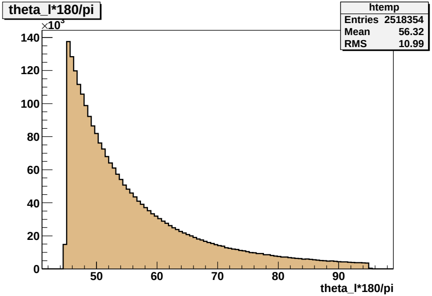
where

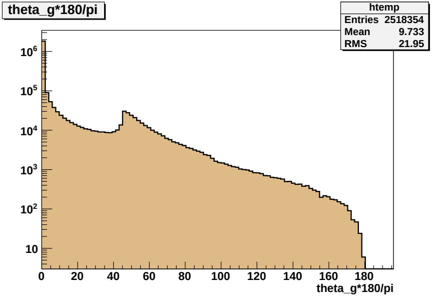
$$\mathcal{L}^{\alpha}_{1\mu\nu} = \frac{1}{2}\,\text{tr}\left[(\not{\ell}' + m)\gamma^{\alpha}(\not{\ell}' + \not{k} + m)\gamma_{\mu}(\not{\ell} + m)\gamma_{\nu}\right],$$

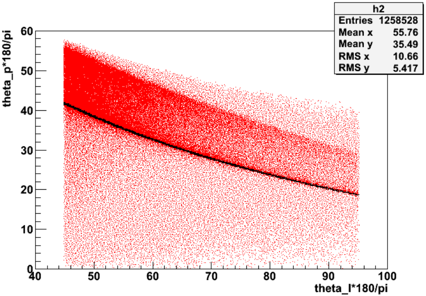
$$\mathcal{P}^{\mu\nu}_{1\alpha} = \frac{1}{2}\,\text{tr}\Bigg[(\not{p}' + M)\left\{\left(F_1(q_1^2) + F_2(q_1^2)\right)\gamma^{\mu} - \frac{F_2(q_1^2)}{2M}P^{\mu}\right\}(\not{p} + M)$$
$$\left\{\left(F_1(q_2^2) + F_2(q_2^2)\right)\gamma^{\nu} - \frac{F_2(q_2^2)}{2M}\left[P^{\nu}_{+} - \gamma^{\nu}(\not{p}' + \not{k} - M)\right]\right\}\frac{\not{p}' + \not{k} + M}{2(k\cdot p')}$$
$$\left\{\left(F_1(0) + F_2(0)\right)\gamma_{\alpha} - \frac{F_2(0)}{2M}\left[\left(2p' + k\right)_{\alpha} - (\not{p}' + \not{k} - M)\gamma_{\alpha}\right]\right\}\Bigg],$$

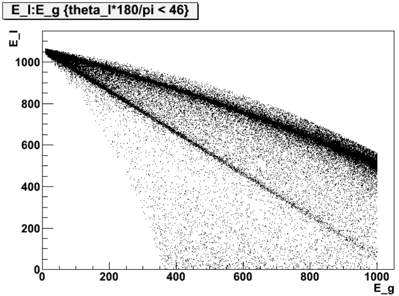
and expressions for the lepton $\mathcal{L}^{\alpha}_{2\mu\nu}$ and proton $\mathcal{P}^{\mu\nu}_{2\alpha}$ tensors are obtained from the expressions for $\mathcal{L}^{\alpha}_{1\mu\nu}$ and for $\mathcal{P}^{\mu\nu}_{1\alpha}$ after changing $\ell \leftrightarrow -\ell'$ and $p \leftrightarrow -p'$ respectively.

Some examples of events generated



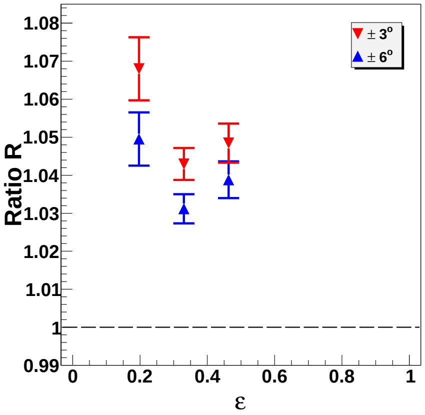




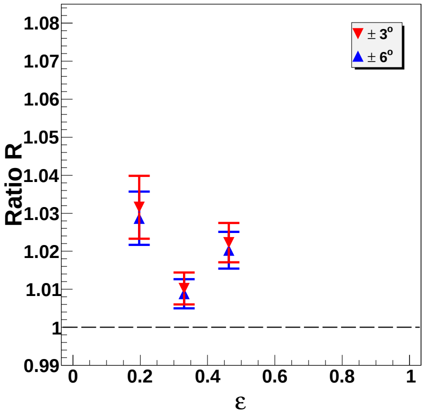


Ratio R and RC depend both on the kinematic cuts used

Raw data for the ratio R:



Radiatively corrected ratio R:



Experimentally measured ratio R is shown before (left figure) and after (right figure) taking into account the radiative corrections (accurate QED calculation). Red markers correspond to the cut $\Delta\theta = \Delta\phi = 3^\circ$ on the angular correlations, blue markers correspond to the cut $\Delta\theta = \Delta\phi = 6^\circ$ (data for LA range of the run II).

Conclusion and acknowledgements

✓Not only the two-photon exchange, but also bremsstrahlung can affect the results of Rosenbluth measurements. This point still requires study.

✓Comprehensive simulation of the detector (for example, using the Geant4 toolkit) with a realistic event generator is the best way to do radiative correction in modern experiments.

✓We developed the new event generator for elastic scattering of charged leptons by protons, which allows to take into account bremsstrahlung more accurately. A detailed description of the generator will be published later.

- I'm very grateful to V. S. Fadin, A. L. Feldman, R. E. Gerasimov, D. M. Nikolenko, I. A. Rachek, and D. K. Toporkov for their participation in this work.
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Thank you for your attention!

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