A Realistic Calculation of the Effects of Nucleon-NucleonCorrelations in High-Energy Scattering Processes Off Nuclei

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Abstract. – *A new linked cluster expansion for the calculation of ground state observables of complex nuclei with realistic interactions has been developed [1, 2, 3]; using the $V8'$ potential the ground state energy, density and momentum distribution of complex nuclei have been calculated and found to be in good agreement with the results of [4], obtained within the Fermi Hyper Netted Chain, and Variational Monte Carlo [5] approaches. Using the same cluster expansion, with wave function and correlations parameters fixed from the ground state calculation, the semi-inclusive reaction of type A (e, e2p) X has been calculated taking final state interaction effects into account within a Glauber type calculation [6]; the comparison between the resulting distorted and undistorted momentum distributions provides an estimate of the transparency of the nuclear medium to the propagation of the hit proton. The effect of color transparency has also been considered within the approach of [8, 9]; it is shown that at high values of Q^2 finite formation time effects strongly reduce the final state interaction, consistently with the idea of a reduced interaction of the hadron produced inside the nucleus [7]. The total neutron-nucleus cross section at the energies has also been calculated [10] by considering the effects of nucleon-nucleon correlations, which are found to increase the total cross section by about 10% in disagreement with the experimental data. The inclusion of inelastic shadowing effects [11, 12] decreases the cross section, leading to a final good agreement between experimental data and theoretical calculations.*

The knowledge of the nuclear wave function, in particular its most interesting and unknown part, *viz* the short range one, as predicted by a realistic description in terms of realistic models of the nucleon-nucleon (NN) interaction is not only a prerequisite for understanding the details of bound hadronic systems, but is becoming at present a necessary ingredient for a correct description of medium and high energy scattering processes off nuclear targets; the latter in fact represent a current way of investigating short range effects in nuclei as well as those QCD effects (e.g. color transparency, hadronization, dense hadronic matter, etc) which manifest themselves in the nuclear medium. The necessity to treat nuclear effects in medium and high energy scattering within a realistic many body description, becomes therefore clear. The problem is not trivial for one has first to solve the many body problem and then has to find a way to apply it to scattering processes. The difficulty mainly arises because even if a reliable and manageable many-body description of the ground state is developed, the problem remains of the calculation of the final state. In the case of few-body systems, a consistent treatment of initial state correlations (ISC) and final state interaction (FSI) is nowadays possible at low energies by solving the Schrödinger equation for the bound and continuum states but, at high energies, the Schrödinger approach becomes impractical and other method shave to be employed. In the case of complex nuclei, much remains to be done, also in view that the results of very sophisticated calculations (*e.g.* the variational Monte Carlo ones [5]), show that the wave function which minimizes the expectation value of the Hamiltonian provides a very poor nuclear density; moreover, the structure of the best trial wave function is so complicated, that its use in the calculation of various processes at intermediate and high energies appears to be not easy task. It is for this reason that the evaluation of nuclear effects in medium and high energy scattering processes is usually carried out within simplified models of nuclear structure. As a matter of fact, initial state correlations (*ISC*) are often introduced by a procedure which has little to recommend itself, namely the expectation value of the transition operator is evaluated with shell model (SM) uncorrelated wave functions and the final state two-nucleon SM wave function is replaced by a phenomenological correlated wave function; to date, a consistent treatment of both ISC and FSI in intermediate and high energy scattering off complex nuclei is far from being completed, so that a quantitative and unambiguous evaluationof the role played ISC is still lacking. For such a reason we have undertaken the calculation of the ground-state properties (energies, densities and momentum distributions) of complex nuclei within a framework which can be easily generalized to the treatment of various scattering processes, keeping the basic features of ISC as predicted by the structure of realistic Nucleon-Nucleon (NN) interactions.

1. The cluster expansion

The evaluation of the expectation value of the nuclear Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = -\frac{\hbar^2}{2M_N}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j}\hat{v}_2(\boldsymbol{x}_i,\boldsymbol{x}_j)\,, \tag{1}$$

where the vector $\boldsymbol{x}$ denotes the set of nucleonic degrees of freedom, is object of intensive activity which in the last few years has produced considerable results; nevertheless, the level of complexity of these calculations often prevents the WF to be used with reasonable ease in other nuclear-related problems, such as nuclear reactions. Our goal is to present a more economical, but effective method for the calculation of the expectation value of any quantum mechanical operator $\hat{\mathcal{O}}$ in the many-body ground-state described by the WF ψ_o, *i.e.*:

$$\langle \hat{\mathcal{O}} \rangle = \frac{\langle \psi_o | \hat{\mathcal{O}} | \psi_o \rangle}{\langle \psi_o | \psi_o \rangle} ; \tag{2}$$

with ψ_o having the following structure

$$\psi_o(\boldsymbol{x}_1, ..., \boldsymbol{x}_A) = \hat{F}(\boldsymbol{x}_1, ..., \boldsymbol{x}_A)\, \phi_o(\boldsymbol{x}_1, ..., \boldsymbol{x}_A) , \tag{3}$$

where ϕ_o is a Shell-Model (SM), mean-field WF, and $\hat{F}$ is a symmetrized correlation operator, which generates correlations into the mean field WF. In the present paper we are going to introduce a cluster expansion technique in order to evaluate Eq. (2). We take the correlation operator $\hat{F}$ in (3), as

$$\hat{F}^2 = \prod_{i<j} \hat{f}^2(r_{ij}) = \prod_{i<j} \big(1 + \hat{\eta}(r_{ij})\big)\, 1 + \sum_{i<j} \hat{\eta}_{ij} + \sum_{(ij)<(kl)} \hat{\eta}_{ij}\, \hat{\eta}_{kl} + ... \tag{4}$$

where $\hat{\eta}_{ij} \equiv \hat{f}^2_{ij} - 1$; $\langle |\hat{\eta}|^2 \rangle$ plays the role of a *small* expansion parameter. We can now perform the expansion of the expectation value in Eq. (2); the quantity $\hat{F}^\dagger \hat{F}$ is expanded both in the numerator and the denominator of Eq. (2), obtaining, to 2-nd order in η:

$$\mathcal{O}_0 \equiv \langle \hat{\mathcal{O}} \rangle , \tag{6a}$$

$$\mathcal{O}_1 = \langle \sum_{ij} \hat{\eta}_{ij}\, \hat{\mathcal{O}} \rangle - \mathcal{O}_0 \langle \sum_{ij} \hat{\eta}_{ij} \rangle , \tag{6a}$$

$$\mathcal{O}_2 = \langle \sum_{ij<kl} \hat{\eta}_{ij}\, \hat{\eta}_{kl}\, \hat{\mathcal{O}} \rangle - \langle \sum_{ij} \hat{\eta}_{ij}\, \hat{\mathcal{O}} \rangle \langle \sum_{ij} \hat{\eta}_{ij} \rangle + \tag{6a}$$

$$- \mathcal{O}_0 \left(\langle \sum_{ij<kl} \hat{\eta}_{ij}\, \hat{\eta}_{kl} \rangle - \langle \sum_{ij} \hat{\eta}_{ij} \rangle^2 \right) ; \tag{6a}$$

more in general, the term of order n contains $\hat{\eta}$ ($\hat{f}$) up to the n-th (2n-th) power.

We show in Fig. 1 the nucleon momentum distribution calculated at first order of η-expansion as a sample result from our approach; the one- and two-body densities are the other basic quantities needed to calculate ground state observables, *e.g.* the binding energy, as shown in Refs. [1]. The momentum distribution is indeed a key quantity in describing scattering processes as $A(e, e'p)X$(see [2, 3]); it can be seen in Fig. 1 that taking into account short range correlations produce high-momentum components that by no means would have been obtained with a shell-model picture of the nucleus and, as anticipated, we found a very good agreement with the calculation performed within the FHNC ([4]) and VMC ([5]) approaches.

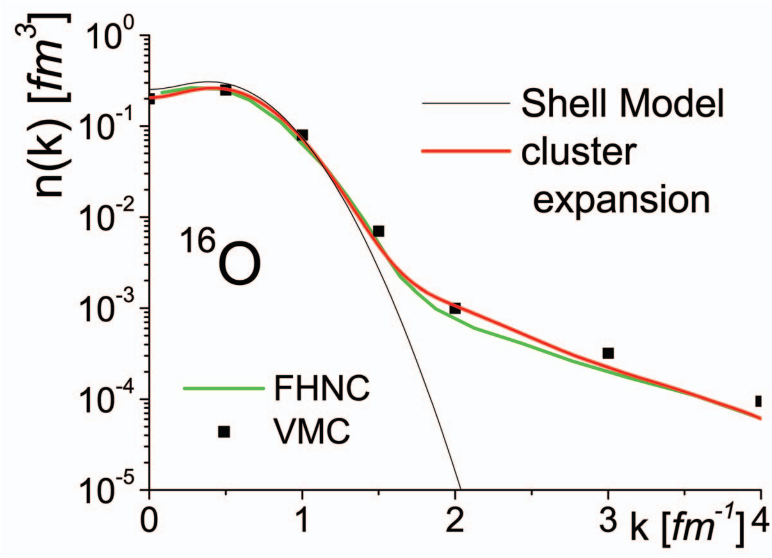

FIGURE 1. – ^{16}O *momentum distribution calculated within η-expansion.*

2. Applications. I - calculation of *color transparency in A(e, e′ p)X* semi-inclusive reactions within the *Finite Formation Time* approach

The usual description of semi-inclusive $A(e, e'p)X$ scattering process is carried over in the framework of Glauber the multiple scattering approach, in which the interaction between the hit proton and the residual systemis parameterized by a final state interaction operator

$$S_G(\boldsymbol{b}; \boldsymbol{b}_1, ..., \boldsymbol{b}_{A-1}) = \prod_j \left[1 - \theta(z_j - z_1)\,\Gamma(\boldsymbol{b} - \boldsymbol{b}_j)\right] =$$
$$= \prod_j \left[1 - \theta(z_j - z_1)\,\frac{\sigma_{pN}^{tot}}{4\pi B^2}\,(1 - i\,\alpha_{pN})\,e^{-(\boldsymbol{b}-\boldsymbol{b}_j)^2/2B}\right]$$

which depends upon the impact parameter $\boldsymbol{b} - \boldsymbol{b_j}$ of each proton-nucleon pair and upon the parameters $\sigma_{pN}^{tot}(p)$, $\alpha_{pN}(p)$ and $B(p)$ (p is the proton momentum).

It is then known taht at high values of Q^2 the interaction of the virtual photon with the nucleus produces an intermediate object which is believed to have very different interaction with the nuclear medium, as compared with a free proton, and it takes a definite amount of time for it to evolve into a physical proton ([7]).

The FFT ([8]) approach consists in the implementation of a final state interaction operator which differs from the Glauberone in that it takes into account an in-medium effective interaction with the residual nucleus, *i.e.*:

$$J(\boldsymbol{b}, \boldsymbol{b}_j) = 1 - \theta(z_j - z)\left(1 - e^{-x\frac{mM^\star}{Q^2}z}\right)\Gamma(\boldsymbol{b} - \boldsymbol{b_j}) \tag{7}$$

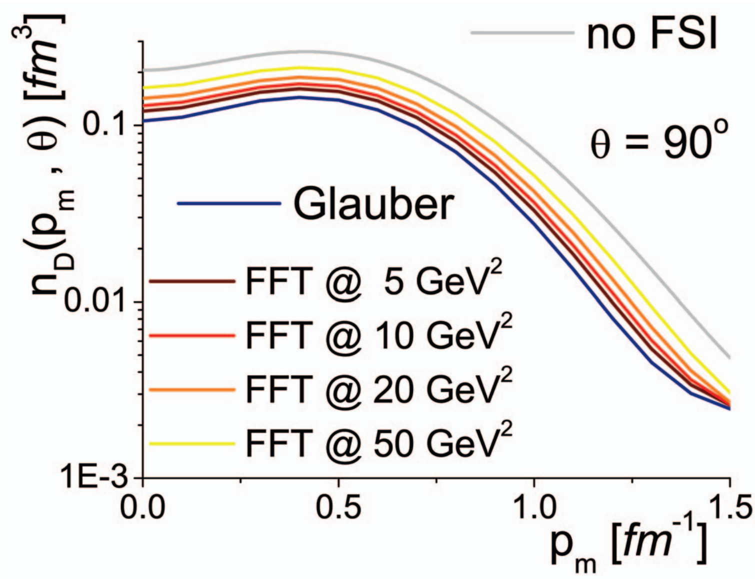

FIGURE 2. – ^{16}*OFSI-distorted momentum ditribution calculated within* η*-expansion.*

with $x \equiv x_{Bj} = Q^2/2m\nu = 1$, m the nucleon mass, $M^\star$ an intermediate excited state effetcive mass and z the longitudinal distance travelled by the proton. One has, from Eq. (7), for $Q^2 \;>>\; m\,M^\star\,x$ a strong reduction of the strength of final state interaction as predicted by QCD inspired calculations. We show results of preliminary calculations

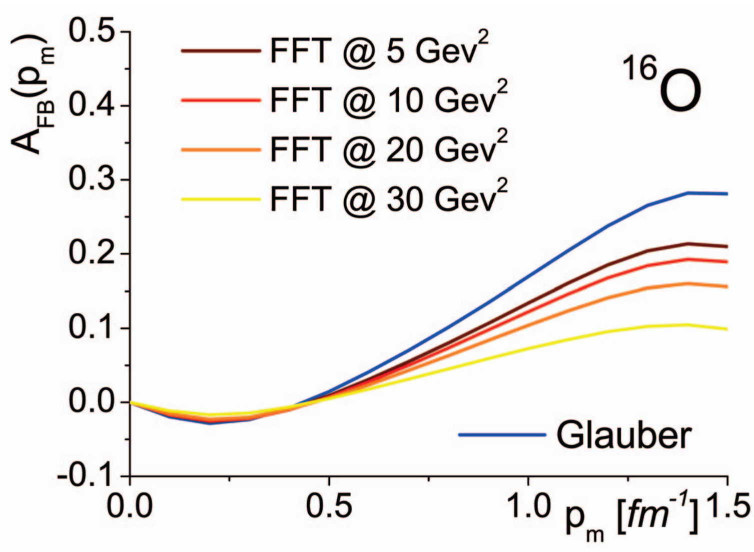

FIGURE 3. – *Forward-backward asymmetry for the process* $A(e, e'p)X$.

in Figs. 2 and 3; Fig. 2 shows the momentum distribution for ^{16}O in the cases of no final state interaction, with the inclusion of the Glauber operator and the calculation with FFT at various values of the transferred momentum Q^2. In Fig. 3 the results for the *forward-backward* asymmetry for the cross-section of the process, namely

$$A_{FB} = \frac{\sigma(p_m, \theta = 0^o) - \sigma(p_m, \theta = 180^o)}{\sigma(p_m, \theta = 0^o) + \sigma(p_m, \theta = 180^o)} \tag{8}$$

is shown.

3. Applications. I - calculation the total σ_{nA}^{tot} neutron-nucleus cross section

We start from the *optical theorem*:

$$\sigma_{tot} = \frac{4\pi}{k} Im[F_{00}(0)]\,; \tag{9}$$

with the elastic scattering amplitude in the eikonal approximation:

$$F_{00}(0) = \frac{ik}{2\pi} \int e^{i\mathbf{q}\cdot\mathbf{b_n}} \langle\psi_o|\{1 - \hat{S}_G\}|\psi_o\rangle = \frac{ik}{2\pi} \int d^2 b_n e^{i\mathbf{q}\cdot\mathbf{b_n}} \left[1 - e^{i\,\chi_{opt}(\mathbf{b}_n)}\right] \tag{10}$$

in which the optical *phase shift* is:

$$\mathbf{e}^{\mathbf{i}\,\chi_{\rm opt}(\boldsymbol{b}_{\rm n})} = \int \prod_{j=1}^{A} d\boldsymbol{r}_j\, G(\boldsymbol{b}_n, \boldsymbol{s}_j)\, |\psi_o(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)|^2 \delta\Big(\frac{1}{A}\sum \mathbf{r}_j\Big) \tag{11}$$

and using the wave function of Section 1, we obtained the cross-section at zeroth-order (*i.e.* mean field) η-expansion, and first-order η-expansion (*i.e.* with correlations). The wave function in terms of density distributions can be written as:

$$|\psi(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)|^2\, \delta\Big(\frac{1}{A}\sum \mathbf{r}_j\Big) = \prod_{j=1}^{A} \rho(\boldsymbol{r}_j) + \sum_{i<j=1}^{A} \Delta(\boldsymbol{r}_i, \boldsymbol{r}_j) \prod_{k\neq(il)}^{A} \rho(\boldsymbol{r}_k) + ... \tag{12}$$

where $\rho(\boldsymbol{r})$ is the one-body density distribution and the *two-body contraction* Δ is defined in terms of one- and two- body density distributions, $\rho^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$:

$$\Delta(\boldsymbol{r}_1, \boldsymbol{r}_2) = \left[\rho^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) - \rho^{(1)}(\boldsymbol{r}_1)\, \rho^{(1)}(\boldsymbol{r}_2)\right]; \tag{13}$$

in the limit $A >> 1$, Eq. (11) can be approximated by

$$\begin{aligned} \mathbf{e}^{\mathbf{i}\,\chi_{\rm opt}(\boldsymbol{b}_{\rm n})} \simeq exp\Bigg[&-A \int d\boldsymbol{r}_1\, \rho(\boldsymbol{r}_1)\, \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_1) + \\ &+ A^2\, \frac{\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\, \Delta(\boldsymbol{r}_1, \boldsymbol{r}_2)\, \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_1)\, \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_2)}{1 - \int d\boldsymbol{r}_1\, \rho(\boldsymbol{r}_1)\, \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_1)}\Bigg]; \end{aligned} \tag{14}$$

where we can distinguish a one body term, in the first line of Eq. (14), and a two body term, in the second line. In the expression above the wave function obtained in Section

1 can be inserted, *i.e.*, the one- and -two body density distributions calculated within the η-expansion approach can be used.

In order to have an overall agreement with experimental data for the total cross section, inelastic shadowing have to be taken into account ([11]); it can be done by complementing the elastic phase-shift with:

$$e^{i\tilde{\chi}^A_{el}} \longrightarrow e^{i\chi^A_{el}} + e^{i\chi^{A-2}_{el}}\, e^{i\chi_{in}}\,, \tag{15}$$

$$e^{i\chi^{A-2}_{el}}\, e^{i\chi_{in}} = \prod_{j=3}^{A}\left[1 - \Gamma^{\mathbf{el}}(\boldsymbol{b}-\boldsymbol{s}_j)\right]\theta(z_2-z_1)\,\Gamma^{\mathbf{in}}(\boldsymbol{b}-\boldsymbol{s}_1)\,\Gamma^{\mathbf{in}}(\boldsymbol{b}-\boldsymbol{s}_2)\,. \tag{16}$$

which provides a new term in the transition amplitude:

$$\begin{aligned}\Delta f_{ii} \propto \int \prod_i^A di \Bigg\{ & \prod_j^A \rho(j) + \Delta(12)\prod_{j\neq 12}^A \rho(j) + \\ & + 2(A-2)\Delta(13)\prod_{j\neq 13}^A \rho(j) + \frac{(A-2)(A-3)}{2}\Delta(34)\prod_{j\neq 34}^A \rho(j)\Bigg\} \times \\ & \times\ \theta(z_2-z_1)\,\Gamma^{in}(\boldsymbol{b}-\boldsymbol{s}_1)\,\Gamma^{in}(\boldsymbol{b}-\boldsymbol{s}_2)\times \\ & \times \left[1 - \sum_{p=3}^A \Gamma^{el}_p + \sum_{3=p<q}^A \Gamma^{el}_p\Gamma^{el}_q - \sum_{3=p<q<r}^A \Gamma^{el}_p\Gamma^{el}_q\Gamma^{el}_r + \dots\right].\end{aligned} \tag{17}$$

The results of calculations are shown in Figs. 4 and 5 for ^{12}C and ^{208}Pb target nuclei, respectively. It should be noticed how the inclusion of correlations in the target wave

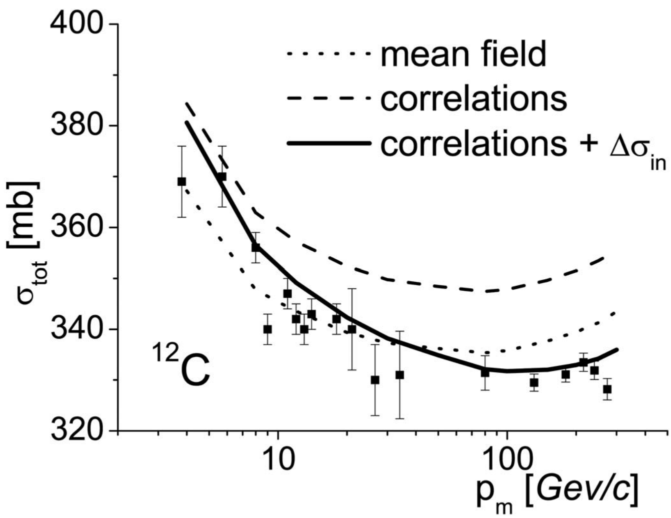

FIGURE 4. – ^{12}C *total neutron-nucleus cross section, calculated using the cluster expansion approximation for the target wave function.*

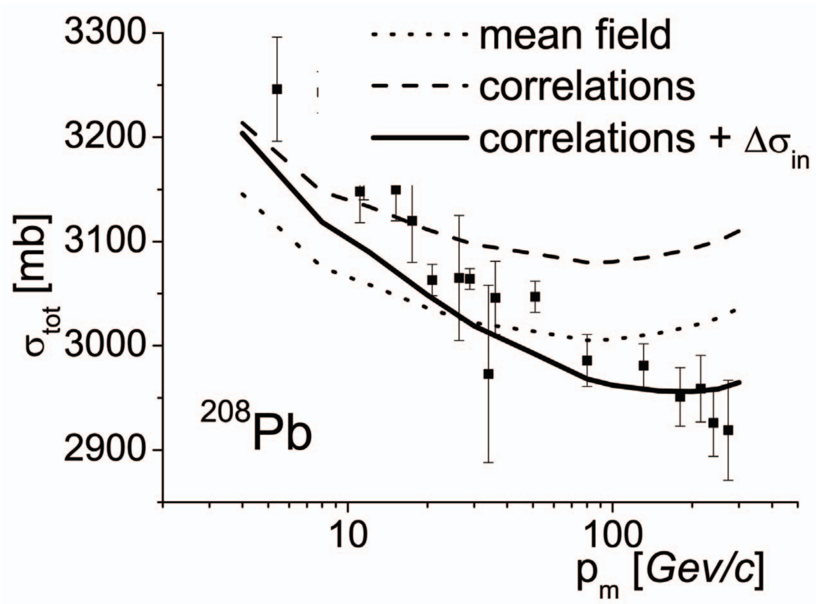

FIGURE 5. – ^{208}Pb *total neutron-nucleus cross section, calculated using the cluster expansion approximation for the target wave function.*

function produce an enhancement of the cross section of about 10% with respect to the mean field result, which turns out to be a necessary ingredient in order to achieve agreement with data.

4. Summary and perspectives

In this paper we have outlined a cluster expansion technique developed in such a way to calculate high-energy scattering processes off complex nuclei. To date, our approach has been used to calculate ground state properties of ^{16}O and ^{40}Ca at first order of the described η-expansion. It is our intention to carry out calculations for the ground state energy of ^{16}O to next order of η-expansion in order to establish on solid grounds the basic features of our method; nonetheless, it should be mentioned that calculations at first order are already very satisfying and make us confident about the reliability of our wave functions. Our method has been to be very effective in the calculation of $A(e,e'p)X$ reactions and neutron-nucleus cross section σ_{nA}^{tot} and it could be extended to the calculation of two-nucleon emission processes, which would be particularly valuable in order to investigate short range correlations in the nuclear wave function. To this respect, it should be mentioned that $A(e,e'2N)X$ reactions are one of the current experimental research interests of the Glasgow group which hosted M. Alvioli during the HPC Europa program at EPCC, and this is very likely to produce some future collaboration between the two groups.

References

[1] M. Alvioli, C. Ciofi degli Atti, H. Morita, ArXiv://nucl-th/0506054 (2005).

[2] M. Alvioli, *PhD Thesis*, *University of Perugia* (2003).

[3] M. Alvioli, C. Ciofi degli Atti, H. Morita, *Fizica* **B13** 585 (2004).

[4] A. Fabrocini, F. Arias de Saavedra and G. Co', *Phys. Rev.* **C61**, 044302 (2000) and *Private Communication.*

[5] S.C. Pieper, R.B. Wiringa and V.R. Pandharipande, *Phys. Rev.* **C46**, 1741 (2000).

[6] C. Ciofi degli Atti, and D. Treleani, *Phys. Rev* **C60** (1999) 024602.

[7] S. J. Brodsky and A. H. Mueller, *Phys. Lett.* **B206** (1988) 685.

[8] M. A. Braun, C. Ciofi degli Atti and D. Treleani, *Phys. Rev.* **C62** (2001) 034606.

[9] H. Morita, M. A. Braun, C. Ciofi degli Atti and D. Treleani, *Nucl. Phys.* **A699** (2002) 328c.

[10] M. Alvioli, C. Ciofi degli Atti and I. Marchino, *in preparation*

[11] V. N. Gribov, *Sov JETP* **29** (1969) 483; J. Pumplin, M. Ross, *Phys. Rev. Lett.* **21** (1968) 1778; V. A. Karmanov, L. A. Kondratyuk, *Jetp letters* **18** (1973) 451.

[12] B. K. Jennings and G. A. Miller, *Phys. Rev.* **C49** (1999) 2637.