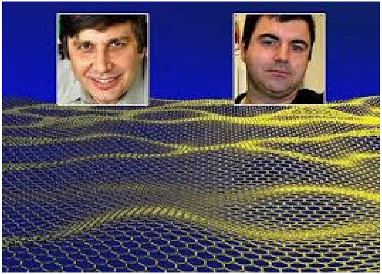
Holographic Description of Graphene

PhD Project

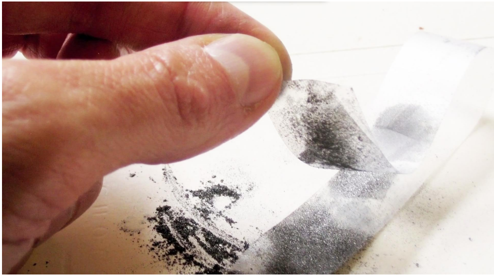
Adriano-Costantino Pigna

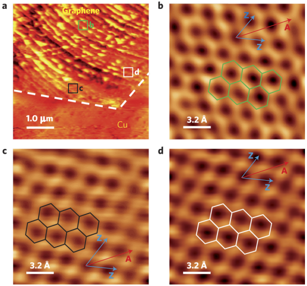
February 13, 2015

Supervisor: Prof. Gianluca Grignani

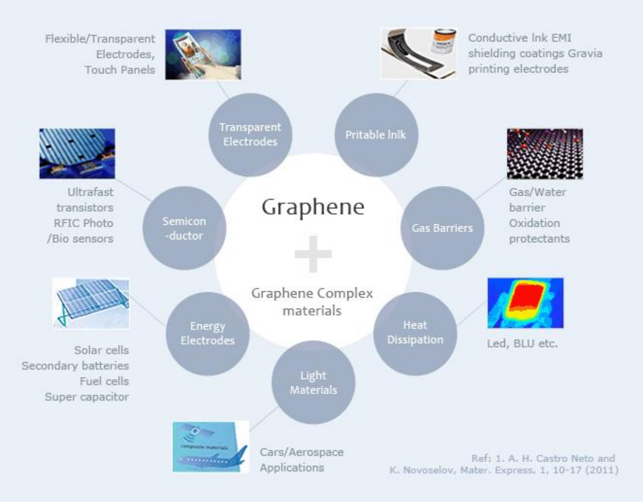


2004 → Isolation of graphene (through the Scotch Tape Method), a strictly two-dimensional carbon-based material by A. Geim, K. Novoselov. Nobel prize in 2010.

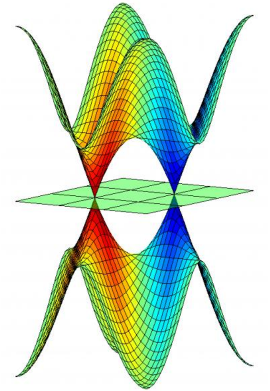


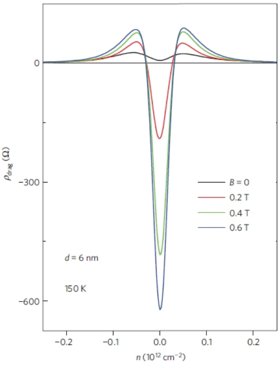


Interesting technology applications in electronic devices (ultra-fast switches and dispersionless field-effect transistors).



Its charge carriers are massless Dirac fermions and experiments clearly show it presents electromagnetic **strong interactions** due to the presence of electron-hole condensates.





$$\epsilon(\kappa) = \pm\hbar v_F \kappa, \quad \alpha_{graphene} = \frac{e^2 c}{4\pi v_F \hbar c} = \alpha \; c/v_F \approx \frac{300}{137} \approx 2.2$$

Motivations

- Perturbative methods fail in describing the physics of strongly interacting systems.
- AdS/CFT correspondence, duality between strongly coupled gauge theories and weakly coupled gravity (string) theories.
- We can introduce gravity by means of dynamical extended objects called *Dp-branes.* A Dp-brane is a $(p+1)$-dimensional hyperplane extended in p spatial directions.
  D-branes are described in terms of the geometry they generate gravitationally.
- Description of graphene sheets through a D-brane model.
- The model I studied reveals the presence of phase transitions as functions of physical parameters (magnetic field, temperature, chemical potential) together with the formation of fermionic condensates (electron-hole bound states).
- Idea $\rightarrow$ Interpretation of these phase transitions and possible analogies with graphene.

D-branes

A Dp-brane is a $(p+1)$-dimensional hyperplane extended in p spatial directions.
Like a point particle, whose action is the length of its world-line

$$S_{pointlike} = -m \int \sqrt{-g_{\mu\nu} dx^{\mu} dx^{\nu}},$$

the action for the Dp-brane, called Dirac-Born-Infeld (DBI) action is the volume spanned by its coordinates:

$$S_{DBI} = -T_{Dp} \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\det(g_{ab} + 2\pi l_s^2 F_{ab})}$$

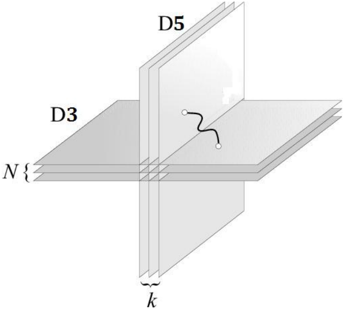
where g_{ab} is the space-time metric, F_{ab} the field strength of the gauge fields living on the Dp-brane, T_{Dp} is the Dp-brane tension and $\mathcal{W}_{p+1}$ is the $(p+1)$-dimensional worldvolume of the Dp-brane.

AdS/CFT correspondence

- Duality relating the field theory of strongly correlated systems to a gravity (string) theory in one higher dimension.
- Field theory side $\rightarrow$ coupling constant g^2.
- Gravity theory side $\rightarrow$ number of D3-branes N.
- Introduce 't Hooft coupling $\lambda = g^2 N$.
- The perturbative regime of the gauge theory valid only when $\lambda \ll 1$.
- Gravity theory valid only when $\lambda \gg 1$.

From the string theory side the model I used is simple and exactly resolvable.

A brief look at the dual field theory



An appropriate choice of the branes coordinates gives a gauge theory living in the $(2+1)$-dimensional intersection of the branes.

We report the form of this gauge theory action

$$S_3 \sim \int d^3x \Big((D^k q^m)^\dagger D_k q_m - i\bar{\Psi}^i \rho^k D_k \Psi_i \Big) + \int d^3x \, (\bar{q}q \text{ terms}) + \int d^3x \, (i(\bar{q}\Psi) - i(\bar{\Psi}q))$$

with q scalar fields and Ψ fermion fields. There is a mass operator that breaks a bosonic symmetry for which only the fermions remain massless.

My Master Thesis Work

- Two D5-branes in the space-time generated by a stack of N D3-branes in the limit $\lambda \gg 1$.
- Space-time created by D3-branes $\rightarrow$ Black Hole $AdS_5 \times S^5$:

$$ds^2 = \sqrt{\lambda}\alpha' \Big(\frac{dr^2}{r^2 h(r)} + r^2(-h(r)dt^2 + dx^2 + dy^2 + dz^2) + d\psi^2 + \sin^2\psi d^2\Omega_2 + \cos^2\psi d^2\tilde{\Omega}_2 \Big),$$

with

$$h(r) = 1 - \frac{r_h^4}{r^4}, \quad r_h = \pi T.$$

- Parameters $\rightarrow$ External magnetic field, charge density, finite temperature.
- Chiral symmetry breaking $\rightarrow$ Condensates formation like in graphene.

Worldvolume gauge field strength:

$$2\pi\alpha'\mathcal{F} = \sqrt{\lambda}\alpha' \left(\frac{d}{dr} a_0(r) dr \wedge dt + b dx \wedge dy \right).$$

Hence, we have a constant external magnetic field

$$B = \frac{\sqrt{\lambda}}{2\pi} b$$

and a charge density q

$$q = \frac{1}{V_{2+1}} \frac{2\pi}{\lambda} \frac{\delta S}{\delta \frac{d}{dr} a_0(r)}$$

where

$$a_0(r) = \mu - \frac{q}{r} + \dots$$

is the temporal world-volume gauge field with μ and q related to the chemical potential and the charge density, respectively.

D5-brane worldvolume coordinates ansatz:

	t	**x**	**y**	**z**	**r**	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
D3	$\times$	$\times$	$\times$	$\times$						
$\mathbf{D5 - \bar{D}5}$	$\times$	$\times$	$\times$	$z(r)$	$\times$	$\psi(r)$	$\times$	$\times$		

The worldvolume metric can be written as

$$ds^2 = \sqrt{\lambda}\alpha' \Big[r^2(-h(r)dt^2 + dx^2 + dy^2) + \sin^2\psi \left(d\theta^2 + \sin^2\theta d\phi^2 \right) + \frac{dr^2}{h(r)r^2}\left(1 + h(r)\left(\left(r\frac{d\psi}{dr} \right)^2 + \left(r^2\frac{dz}{dr} \right)^2 \right) \right) \Big]$$

The DBI action for either the D5- or anti-D5-brane becomes

$$S = \mathcal{N}_5 \int dr \sin^2\psi \sqrt{r^4 + b^2} \sqrt{1 + h(r)((r\psi')^2 + (r^2 z')^2) - a_0'^2}.$$

Determine the functions $z(r)$, $\psi(r)$ and $a_0(r)$ by varying the action using the boundary condition for $r \to \infty$

$$\psi(r) = \frac{\pi}{2} + \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

where

- $c_1 \propto$ mass term for fermions $\to$ $c_1 = 0$ massless fermions,
- $c_2 \propto$ expectation value of the *intra-layer* chiral condensate $\to$ $\langle \bar{f} f \rangle$ condensation between fermionic species on the same brane (electron-hole condensates on a single graphene layer).

The boundary conditions for the function $z(r)$ for $r \to \infty$ are

$$z(r) \to \pm\frac{L}{2} \mp \frac{f}{r^5} + \dots$$

for the D5-brane and anti-D5-brane, where f is proportional to the expectation value for the *inter-layer* chiral condensate $\to$ $\langle \bar{f} g \rangle$ condensation between a fermion species on a brane and a fermion species on the other brane (electron-hole condensates between two graphene layers).

This gives us the lagrangian

$$\mathcal{L}[L,\mu] = r^2(1+r^4)\sin^4\psi\sqrt{\frac{h(r)(1+r^2h(r)\psi'^2)}{-f^2+r^4h(r)[q^2+(1+r^4)\sin^4\psi]}}.$$

Solve Euler-Lagrange equations to find equations of motion for $\psi(r)$:

$$\begin{aligned}
&2r^2\Big(f^2r_h^4+q^2(-r^8+r_h^8)+(-2r^{12}+r_h^8+r^8(-1+2r_h^4))\sin^4\psi\Big)\psi' \\
&+2r^3(1+r^4)(r^4-r_h^4)^2\cos\psi\ \sin^3\psi\ \psi'^2 \\
&-\Big((r^4-r_h^4)^2(f^2+q^2(r^4+r_h^4)+(r^4+3r^8+r_h^4-r^4r_h^4)\sin^4\psi)\Big)\psi'^3 \\
&+r^3(r^4-r_h^4)\Big(2r^2(1+r^4)\cos\psi\ \sin^3\psi \\
&+(f^2+q^2(-r^4+r_h^4)-(1+r^4)(r^4-r_h^4)\sin^4\psi)\Big)\psi''=0
\end{aligned}$$

Minkowski vs Black Hole solutions

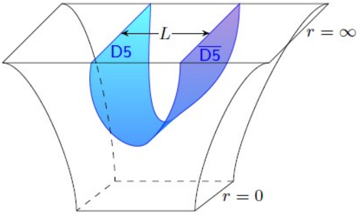
If $f \neq 0$ and $c_2 = 0$ ($\psi = \frac{\pi}{2}$) the solution for $z(r)$ is:

$$z(r) = f \int_{r_0}^{\infty} dr \frac{f\sqrt{h(r)(1 + r^2 h(r)\psi'^2)}}{r^2\sqrt{h(r)}\sqrt{-f^2 + r^4 h(r)[q^2 + (1 + r^4)\sin^4\psi]}}$$

with r_0 satisfying the condition

$$-f^2 + r_0^4 h(r_0)[q^2 + (1 + r_0^4)\sin^4\psi(r_0)] = 0.$$

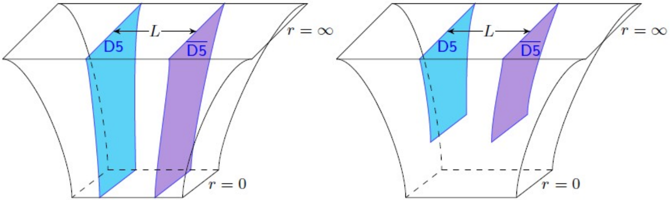
- To find a sensible solution the D5/antiD5 glue smoothly at $r = r_0$ $\rightarrow$ connected solution.
- $f_{D5} = -f_{\bar{D}5}$ and $q_{D5} = -q_{\bar{D}5}$ $\Leftrightarrow$ overall system is neutral.
- Inter-layer condensate.



Minkowski embedding

If $f = 0$ and $c_2 \neq 0$ the solution is trivial $\rightarrow z = \pm L/2$.

Unconnected solution



"Black hole" embedding Minkowski embedding

- Black Hole solution for $q \neq 0$, $\rightarrow$ the solution reaches the horizon.
- Minkowski solution only for $q = 0$, $\rightarrow$ the solution caps off before reaching the horizon.
- Intra-layer condensate.

If $f \neq 0$ and $c_2 \neq 0$ there are no solutions. $\rightarrow$ no mixed intra- /inter-layer condensates.

Free energies. Lengths. Chemical potentials

- To see which configuration is favored compare the free energies of the different solutions at the same L and μ.
- The right quantity to define the free energy is the action evaluated on solutions

$$\mathcal{F}[L,\mu] = \int_{r_0}^{\infty} r^2(1+r^4)\sin^4\psi \sqrt{\frac{h(r)(1+r^2h(r)\psi'^2)}{-f^2+r^4h(r)[q^2+(1+r^4)\sin^4\psi]}}.$$

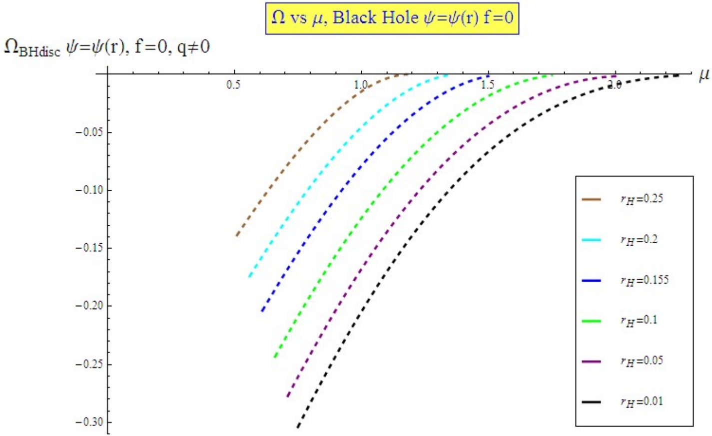
- This free energy is UV divergent for all solutions.
- Introduce a *regularization* procedure.
- Subtract to the free energy of each solution the energy of the BH disconnected constant solution, that is

$$\mathcal{F}_{reg} = \int_{r_0}^{\infty} \frac{1+r^4}{\sqrt{1+q^2+r^4}}.$$

The regularized energy becomes $\Omega = \mathcal{F} - \mathcal{F}_{reg}$.

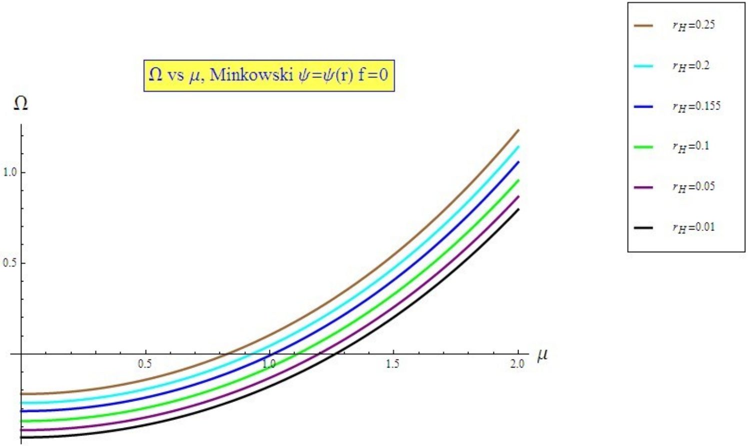
BH disconnected r-dependent solutions free energies

I picked the following r_h values $r_h = (0.25, 0.2, 0.155, 0.1, 0.05, 0.01)$.
Ω^{disc}_{BH} as a function of μ:

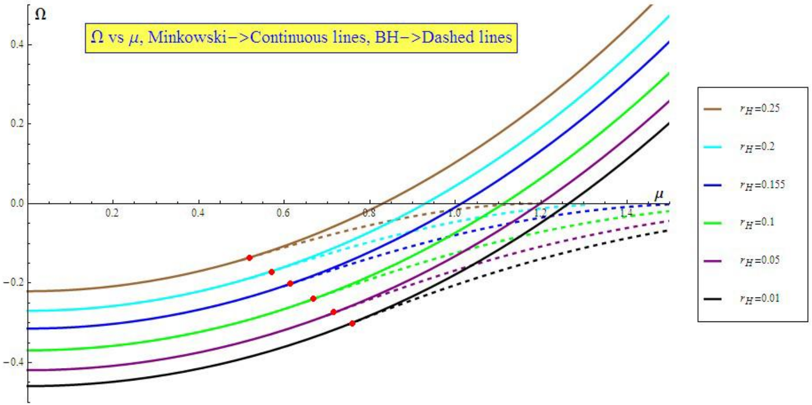


Minkowski disconnected r-dependent solutions free energies

Same values of r_h. Ω as a function of μ:



If we look at the Ω vs μ plots for disconnected r-dependent Minkowski and Black Hole solutions and we fuse them together we obtain



Second order phase transition between disconnected r-dependent Minkowski and BH solutions in correspondence of a particular value $\mu_{transition}$ of the chemical potential.

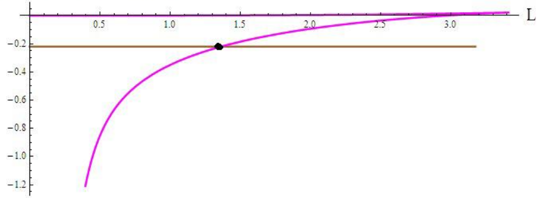
Finally I want to draw the (Ω, L) plots with a fixed chemical potential for the three solutions:

1. the disconnected r-dependent Minkowski $(q = 0)$ solution,
2. the disconnected r-dependent BH $(q \neq 0)$ solution,
3. the connected constant Minkowski solution.

For $\mu < \mu_{transition}$ I consider type 1) and 3) solutions. For $\mu > \mu_{transition}$ I consider type 2) and 3) solutions.

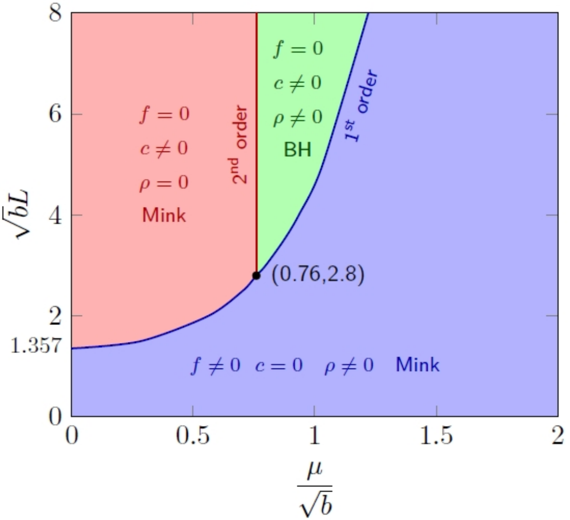
Both for $\mu < \mu_{transition}$ and $\mu > \mu_{transition}$ I find

$\Omega_{\mathrm{MINKdisc}}(\psi=\psi(\mathrm{r}), r_H=0.25, \mu=0, \mathrm{q}\neq 0)$



There is a transition length $L_{transition}$. For $L < L_{transition}$ the preferred configuration is the connected constant Minkowski one while for $L > L_{transition}$ the favourite is the disconnected Minkowski (BH) one for $\mu < \mu_{transition}$ ($\mu > \mu_{transition}$). The transition is of the first order.
We are ready to draw the phase diagrams.

Phase diagrams



Phase diagram description

- $L < L_{tr}$
  - Minkowski connected constant solution.
  - $f \neq 0 \to$ nonzero expectation value for the inter-layer condensate
  - $\langle \bar{f} g \rangle$ condensation in field theory.
  - Graphene layers strongly interact because of the close distance.
- $L > L_{tr}$
  - Minkowski disconnected r-dependent solution for $\mu < \mu_{tr}$
  - BH disconnected r-dependent solution for $\mu > \mu_{tr}$
  - For both $c_2 \neq 0 \to$ nonzero expectation value for the intra-layer condensate
  - Two monolayer phases ($z = \pm L/2$) with a $\langle \bar{f} f \rangle$ condensation on each brane.
  - When two graphene layers are separated at a distance $L > L_{tr}$ they exert no more reciprocal forces.
  - Strong interactions are localized on a single graphene layer.
- No phase with both intra- and inter-layer condensation.

PhD Project

"Old" D5-branes coordinates

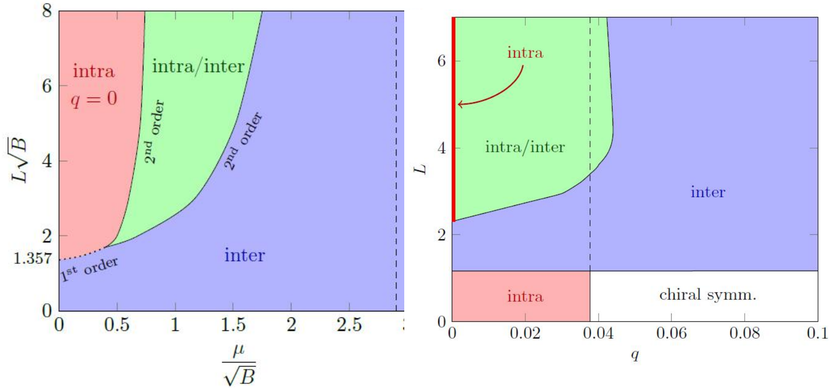
	t	**x**	**y**	**z**	**r**	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
D3	$\times$	$\times$	$\times$	$\times$						
$\mathbf{D5}-\mathbf{\bar{D}5}$	$\times$	$\times$	$\times$	$z(r)$	$\times$	$\psi(r)$	$\times$	$\times$		

Change of variables

$$\rightarrowtail \quad \rho = r\sin\psi, \qquad l = r\cos\psi \quad \leftarrowtail$$

"New" D5-branes coordinates

	t	**x**	**y**	**z**	ρ	θ	ϕ	**l**	$\tilde{\theta}$	$\tilde{\phi}$
D3	$\times$	$\times$	$\times$	$\times$						
$\mathbf{D5}-\mathbf{\bar{D}5}$	$\times$	$\times$	$\times$	$z(\rho)$	$\times$	$\times$	$\times$	$l(\rho)$		



This is the zero temperature case.

- *Extension to finite temperatures* $\rightarrow$ BH factor $h(r) = 1 - \frac{r_h^4}{r^4}$.

Recent papers have shown the possibility for the D5-branes to "blow up" to D7-branes for high enough values of the charge density, together with the introduction of a magnetic flux term in the gauge field strength.

- *Drawing of a complete phase diagram including both the D5-branes and D7-branes contributions*.

Thanks for your attention!!

Dimensional analysis

- Magnetic units

$$r_h \to \frac{r_h}{\sqrt{b}}, \quad L \to L\sqrt{b}, \quad \mu \to \frac{\mu}{\sqrt{b}}$$

- $b = \frac{2\pi}{\sqrt{\lambda}} B$, B in natural units
- Inverse relations

$$r_h = \frac{\bar{r}_h \sqrt{2\pi} B^{1/2}}{\lambda^{1/4}}, \quad L = \frac{\bar{L}\lambda^{1/4}}{\sqrt{2\pi} B^{1/2}}, \quad \mu = \frac{\bar{\mu}\sqrt{2\pi} B^{1/2}}{\lambda^{1/4}}$$

with $\bar{L}$, $\bar{r}_h$, $\bar{\mu}$ are the numerical values.

- Introduce appropriate $\hbar c$ factors to convert to Gaussian cgs units.

We find

$$L[\mathrm{cm}] = \frac{\bar{L}\lambda^{1/4}}{\sqrt{2\pi}B^{1/2}[\mathrm{Gauss}]}(\hbar c)^{1/4}.$$

We choose $\bar{L} \simeq 1$, $B \simeq 1T = 10^4\mathrm{Gauss}$.
Inserting $(\hbar c)^{1/4} \simeq 7.5 \cdot 10^{-5}\ \mathrm{erg} \times \mathrm{cm}$ we find

$$L \simeq 3\lambda^{1/4} \cdot 10^{-7}\mathrm{cm}.$$

Then we have

$$k_B T[\mathrm{erg}] = \frac{\bar{r}_h\sqrt{2\pi}B^{1/2}(\hbar c)^{3/4}}{\pi\lambda^{1/4}}.$$

We choose $\bar{r}_h \sim 0.1$ and we find

$$T \simeq \frac{10^4}{\lambda^{1/4}}\mathrm{K}$$

$\lambda \rightarrow$ one free parameter in the theory.

Once the function $\psi(r)$ is known, the separation of the D5 branes is given by

$$L = \int_{r_0}^{\infty} dr\, z'(r) = 2f \int_{r_0}^{\infty} dr \frac{f\sqrt{h(r)(1+r^2 h(r)\psi'^2)}}{r^2\sqrt{h(r)}\sqrt{-f^2 + r^4 h(r)[q^2 + (1+r^4)\sin^4\psi]}}$$

where r_0 is the turning point.
Analogously, the chemical potential is related to the integral of the gauge field strength on the brane in the $(r, 0)$ directions

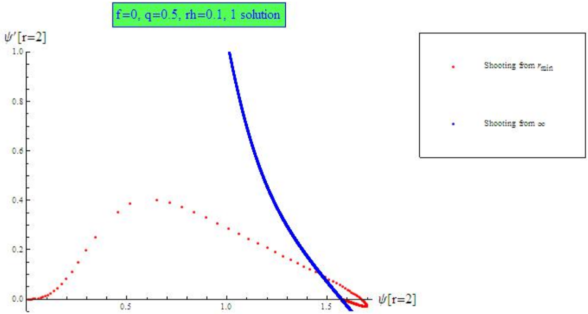
$$\mu = \int_{r_0}^{\infty} dr\ a_0'(r) = \int_{r_0}^{\infty} dr \frac{qr^2\sqrt{h(r)(1+r^2 h(r)\psi'^2)}}{\sqrt{-f^2 + r^4 h(r)[q^2 + (1+r^4)\sin^4\psi]}}.$$

Shooting technique

The differential EOMs can be solved from two directions: from a r_{min} or from the boundary at $r = \infty$.

- Fix some $\bar{r}$ value of r in the middle (say $\bar{r} = 2$).
- Start with shooting from the horizon. For the appropriate values of ψ at the horizon r_h, integrate the solution outwards to $\bar{r}$ and compute ψ and its derivative at $\bar{r}$.
- For each solution, put a point on a plot of $\psi'(\bar{r})$ vs $\psi(\bar{r})$, (red dots).
- Do the same thing starting from the boundary at $r = \infty$, and varying the coefficient c_2 of the expansion around infinity (blue dots).

When the two solutions, coming from r_{min} and from $r = \infty$ meet at the intermediate point, then there is a solution, provided that $\psi(\bar{r}) < \frac{\pi}{2}$.



Technically I set up a *for* loop of about 200 iterations. Accordingly with the embeddings I consider, I have different initial conditions for the $\psi(r_{min})$ and its derivative $\psi'(r_{min})$. At every iteration I change the initial conditions by a small quantity (as a function of the current iteration number) keeping fixed the f, q, r_h values obtaining the red dots in figure.

Shootings (Black Hole embeddings)

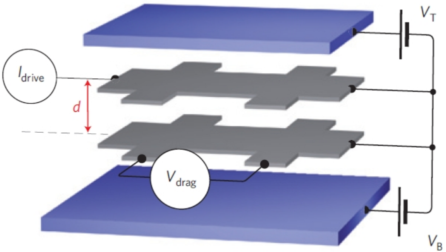
Analitically, for a BH embedding, I have to impose a boundary condition at the horizon given by

$$\frac{d}{dr}\psi = \frac{1}{2r_h}\frac{(1+r_h^4)\sin^3\psi\cos\psi}{q^2+(1+r_h^4\sin^4\psi)},$$

where $\psi = \psi|_{r=r_h}$. By varying the value of the function at the horizon $\psi(r_h)$ with an appropriate *for* loop in the range $[0, \frac{\pi}{2}]$ we obtain the red dots. I numerically interpolate all the red and blue points to verify if the two curves meet in $[0, \frac{\pi}{2}]$. If they intersect, I record the couple of values $(\psi(\bar{r}), \psi'(\bar{r}))$ and we insert them back in the equation of motion as initial conditions (providing numerical values for f, q and r_h) to find the solution we are looking for.

Strong Coulomb drag in double-layer graphene

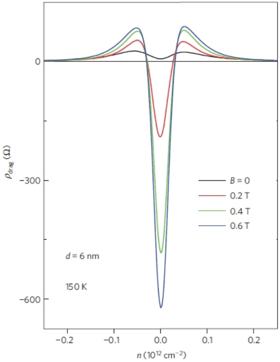
Coulomb drag → frictional coupling between electric currents flowing in spatially separated conducting layers. Caused by **inter-layer electron-electron interactions**.



Apply current I_{drive} through one graphene layer, and measure the induced voltage V_{drag} in the second layer. Observe linear response $V_{drag} \propto I_{drive}$.

Magneto-drag

Even relatively small B causes Coulomb drag to increase. This is an overwhelmingly strong effect such that $|\rho_{drag}|$ increases by a factor of > 100 in $B < 1T$.



For strong magneto-drag, the inter-layer state is described in terms of condensation of excitons that consist of an electron in one layer bound to a vacancy-like state in the other layer. In a magnetic field inter-layer exciton-like correlations offer a tentative explanation for strong drag and invite one to search for excitonic condensates.

AdS/CFT at finite temperature

For a nonzero temperature T, the geometry is:

$$ds^2 = \frac{r^2}{R^2}(-fdt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{fr^2}dr^2 + R^2 d\Omega_5^2,$$

product of a black hole in AdS_5 with S^5, with

$$f = 1 - \frac{r_o^4}{r^4}.$$

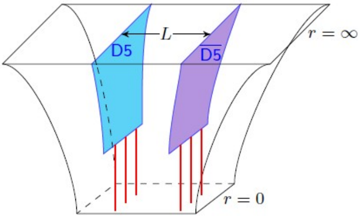
The horizon is located at $r = r_0$. The temperature "hides" in $r_0 \propto T$.

If the charge density $q \neq 0$ only BH embeddings are allowed.

$$a_0'(r) = \frac{qr^2\sqrt{h(r)(1+r^2h(r)\psi'^2)}}{\sqrt{-f^2+r^4h(r)[q^2+(1+r^4)\sin^4\psi]}}.$$

$a_0'(r)$ is singular at $r = r_0$. This means that there must be charge sources.

- Density of F-strings suspended between the D5 and the horizon.
- F-strings always have a larger tension than the D5-brane, and they pull the D5 brane to the horizon.
- Minkowski unconnected ($f = 0$) embeddings are allowed only if $q = 0$.



Connected $f \neq 0$ r-dependent solution with $q = 0$

In this case we look for a D5-brane z(r)-profile that joins at the turning point r_0 the corresponding antiD5-brane. Recall the general expression for $z'(r)$ for $q = 0$

$$z'(r) = \frac{f\sqrt{h(r)(1 + r^2 h(r)\psi'^2)}}{r^2\sqrt{h(r)}\sqrt{-f^2 + r^4 h(r)(1 + r^4)\sin^4\psi}}.$$

At r_0 we must have $z'(r_0) \to \infty$ and r_0 can be determined by imposing that the denominator is zero, that reads

$$-f^2 + (r_0^4 - r_h^4)(1 + r_0^4)\sin^4\psi = 0.$$

This yields

$$\psi(r_0) = \sin^{-1}\left(\sqrt[4]{\frac{f^2}{(r_0^4 - r_h^4)(1 + r_0^4)}}\right).$$

I insert $\psi(r_0)$ back in the equation of motion. I find a condition on $\psi'(r_0)$ that reads

$$\psi'(r_0) = \frac{(1+r_0^4)^{3/4}\sqrt{\frac{(r_0^4-r_h^4)(1+r_0^4)-f\sqrt{(r_0^4-r_h^4)(1+r_0^4)}}{f}}}{r_0(r_0^4-r_h^4)^{1/4}(1+2r_0^4-r_h^4)}$$

I repeated the shooting procedure for many values of f and r_h.

Solutions

I find three kinds of solutions for $c_1 = 0$ (that is, with massless fermions):

1. Unconnected $f = 0$ r-dependent Minkowski solutions (intra-layer condensate),
2. Unconnected $f = 0$ r-dependent Black Hole solutions (intra-layer condensate),
3. Connected $f \neq 0$ constant $\psi = \frac{\pi}{2}$ Minkowski solutions (inter-layer condensate).

It is possible to define an effective Hamiltonian

$$H_{eff} = \hbar\, v_F \begin{pmatrix} \alpha \cdot \kappa & 0 \\ 0 & \alpha^* \cdot \kappa \end{pmatrix}$$

acting on a four-component spinor

$$\psi(\kappa) = (\psi_{A,+}(\kappa), \psi_{B,+}(\kappa), \psi_{A,-}(\kappa), \psi_{B,-}(\kappa)).$$

We can independently resolve the two blocks of H_{eff} giving

$$\tilde{H}_{\pm}\psi_{\pm}(\kappa) = \epsilon(\kappa)\psi_{\pm}(\kappa)$$

with $\psi_{\pm}(\kappa) = (\psi_{A,\pm}(\kappa), \psi_{B,\pm}(\kappa))$. The dispersion relation becomes

$$\epsilon(\kappa) = \pm\frac{3a_0 t}{2}|\kappa| = \pm\hbar v_F \kappa.$$

Estimate of the strength of the Coulomb interaction in graphene. Define a graphene fine structure constant as the ratio of potential energy and kinetic energy of a massless electron with wavelength λ.
Potential energy $\rightarrow U = \frac{e^2}{4\pi\lambda}$
Kinetic energy $\rightarrow T = \frac{\hbar v_F}{\lambda}$.
Hence the graphene structure constant is

$$\alpha_{graphene} = \frac{U}{T} = \frac{e^2/4\pi\lambda}{\hbar v_F/\lambda} = \frac{e^2}{4\pi v_F \hbar} = \frac{e^2 c}{4\pi v_F \hbar c} = \alpha\ c/v_F \approx \frac{300}{137} \approx 2.2$$